EW Sudakov Logarithms in $e^+e^- \to t\bar{t}$

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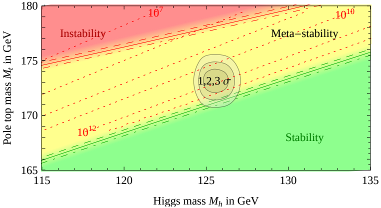
Particle Seminar 24/01/2017

Contents

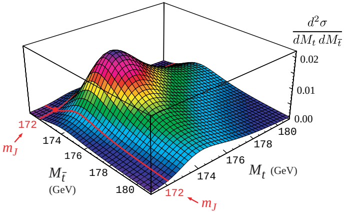
Introduction

Top Physics

- Process: $e^+e^- \to t\bar{t}$
- Top is unique particle in SM:
  - Large mass ($m_{\text{top}} \sim \Lambda_{\text{EW}} \gg \Lambda_{\text{QCD}}$)
  - $\tau_{\text{top}} \ll 1/\Lambda_{QCD}$ (does not hadronize)
  - Weak decay ($t \to W^+ b$)
  - Connects various sectors of SM
- Top parameters important
  - Precision test of SM
  - Vacuum stability, ...
  - BSM searches
- Top mass from mass-sensitive observables



[Degrassi et al '12]



[Hoang et al '07]

Electroweak effects in top physics

- QCD-corrections extensively treated, Factorization known
- Goal: understand and quantify EW effects in this framework
- $\alpha \ll \alpha_s \quad \Rightarrow$ EW effects small?!
- Problem: logarithmic enhancement for $s \gg \Lambda^2_{\text{EW}}$

$$
\begin{aligned}
O &= O^{(0)} + \alpha O^{(1)} + \alpha^2 O^{(2)} + \ldots \\
&= O^{(0)} + \alpha \ln^2\left(\frac{s}{\Lambda^2_{\text{EW}}}\right) c_1 + \alpha \ln\left(\frac{s}{\Lambda^2_{\text{EW}}}\right) c_2 + \alpha^2 \ln^4\left(\frac{s}{\Lambda^2_{\text{EW}}}\right) c_3 + \ldots
\end{aligned}
$$

- Need for resummation in precision physics at collider energies

Contents

Theory setup and building blocks

Theory setup

- Work in broken phase of GWS-theory (gauge fields $= A, Z, W^{\pm}$)
- Assumptions: massless initial-state leptons, stable tops
- R_ξ-gauge ($\Rightarrow$ appearance of massive Goldstones + ghosts) with $\{\xi_W, \xi_Z, \xi_A\}$
- Gauge and Goldstone/ghost propagators:

$$(D_V)_{\mu\nu} = \frac{-i}{p^2 - M_V^2 + i0^+}\left(g_{\mu\nu} - \frac{(1-\xi_V)p_\mu p_\nu}{p^2}\right),$$

$$D_G = \frac{i}{p^2 - \xi_V M_V^2 + i0^+}$$

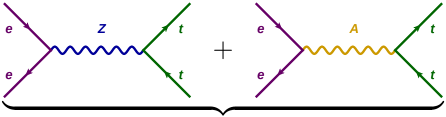
- Focus on weak radiative corrections, exclude pure QED
- Previous calculations
  - [**Beenakker et al '91**]: $l\bar{l} \to f\bar{f}$ for $\xi = 1$
  - [**Bardin et al '02**] : $l\bar{l} \to f\bar{f}$ in R_ξ (no analytical R_ξ-results)
  - [**Manohar et al '08**]: EW-EFT-approach to $q\bar{q}$-production (vector-like current)

Goals

- Investigate structure (gauge and UV) of 1-loop $e^+e^- \to t\bar{t}$ matrix element

$$\mathcal{M}(e^+e^- \to t\bar{t}) = \mathcal{M}_{\text{tree}} + \delta\mathcal{M}_{\text{rad}}$$

$i\mathcal{M}_{\text{tree}} =$



$\mathcal{O}(\alpha)$

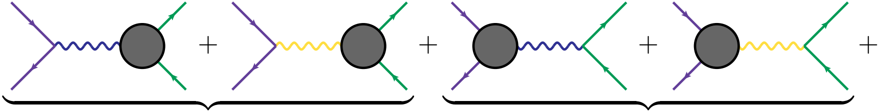
- Analytical results, relative numerical size of various radiative contributions
- Extract high-energy limit $\Rightarrow$ Sudakov logs

$$\ln^n\left(-\frac{s}{\Lambda_{\text{EW}}^2}\right)$$

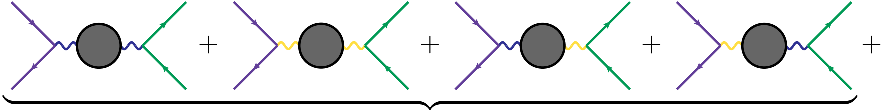
- Assumption: $\{|s|,|t|,|u|\} \gg \{m_t^2, M_H^2, M_Z^2, M_W^2\} \sim \Lambda_{\text{EW}}^2$

Schematic 1-loop amplitude

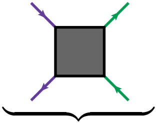
$i\delta\mathcal{M}_{\text{rad}} =$



virtual final-state corrections $\mathcal{O}(\alpha^2)$ — virtual initial-state corrections $\mathcal{O}(\alpha^2)$



oblique corrections $\mathcal{O}(\alpha^2)$

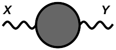


$+$ real radiation $+\ \mathcal{O}(\alpha^3)$

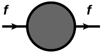
boxes $\mathcal{O}(\alpha^2)$

Building blocks

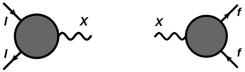
- Bosonic 2-point functions ($X \to Y$)



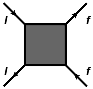
- Fermionic 2-point functions ($f \to f$)



- Amputated 3-point functions ($X \to f\bar{f}, l\bar{l} \to X$)



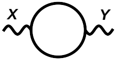
- 4-point functions ($l\bar{l} \to f\bar{f}$)



Bosonic 2-point functions

- Boson 2-point corrections

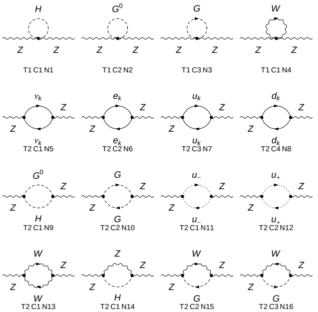
- Transverse and longitudinal parts



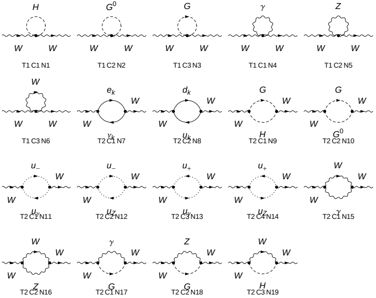
$$= i\Pi^{XY}_{\mu\nu} = i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi^{XY}_T + i\frac{p_\mu p_\nu}{p^2}\Pi^{XY}_L$$

- Counterterms consist of Π^{XY}_T and/or $(\Pi^{XY}_T)'$ evaluated on mass shells
- Bosonic counterterms are ingredients for $\delta e, \delta c_W, \delta v_f, \delta a_f$

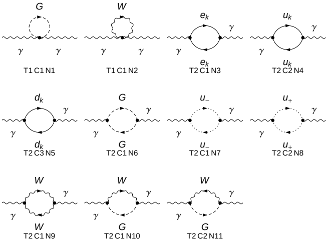
$Z \rightarrow Z$



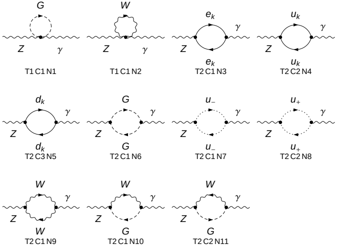
$W \rightarrow W$



$\gamma \rightarrow \gamma$



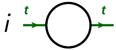
$Z \rightarrow \gamma$



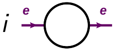
Fermionic 2-point functions

- Fermion 2-point corrections (top and electron)

- Dirac structure



$$= \Sigma_t(\not{p}) = \not{p}\,(A_V + \gamma_5 A_A) + m\,(B_S + \gamma_5 B_P)$$

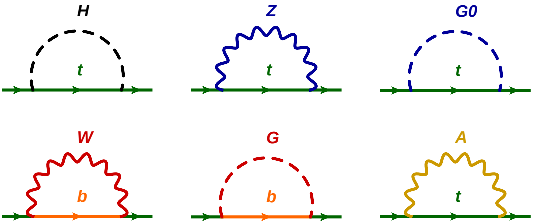


$$= \Sigma_e(\not{p}) = \not{p}\,(a_V + \gamma_5 a_A)$$

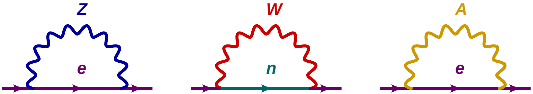
- Counterterms: $A_i/B_i/a_i$ and/or $(A_i)'/(B_i)'/(a_i)'$ evaluated on mass shell
- Fermionic field strengths $\delta Z_L, \delta Z_R$ important for local vertex counterterms

Fermionic 2-point diagrams

- Top

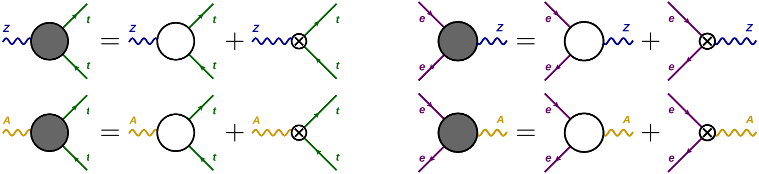


- Electron

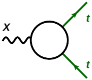


Amputated vertices

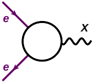
- Amputated vertices



- Form factor decomposition



$$= ie\bar{u}_t \left[\gamma^\mu \left(F_1 + \gamma_5 G_1\right) + \frac{p^\mu}{m}\left(F_2 + \gamma_5 G_2\right) + \frac{q^\mu}{m}\left(F_3 + \gamma_5 G_3\right)\right] v_t$$



$$= ie\bar{v}_e \left[\gamma^\mu \left(f_1 + \gamma_5 g_1\right) + \frac{p^\mu}{m}\left(f_2 + \gamma_5 g_2\right) + \frac{q^\mu}{m}\left(f_3 + \gamma_5 g_3\right)\right] u_e$$

- Number of relevant form factors is 3 (final) and 2 (initial)

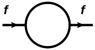
Amputated vertices

- Local counterterms for 3-point vertices directly from 2-point counterterms
- e.g. for Zff-vertex: $\hat{F}_1^Z = F_1^Z + \delta F_1^Z$:

$$(\delta F_1^{Zff})_{\text{fer}} = \frac{v_f}{2}\,(\delta Z_L + \delta Z_R) + \frac{a_f}{2}\,(\delta Z_L - \delta Z_R)$$

$$(\delta F_1^{Zff})_{\text{bos}} = v_f\left(\delta e + \frac{\delta Z_{ZZ}}{2} + \frac{c_W^2 - s_W^2}{s_W^2}\,\delta c_W\right) - Q_f\left(\frac{\delta Z_{AZ}}{2} - 2\frac{c_W}{s_W}\,\delta c_W\right)$$

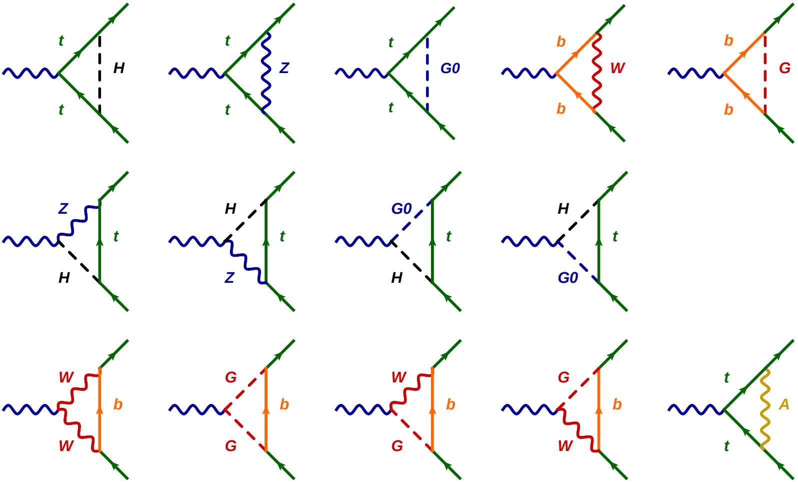
- Fermionic and bosonic parts

$(\delta F_1^{Zff})_{\text{fer}} \sim$ 

$(\delta F_1^{Zff})_{\text{bos}} \sim$ 

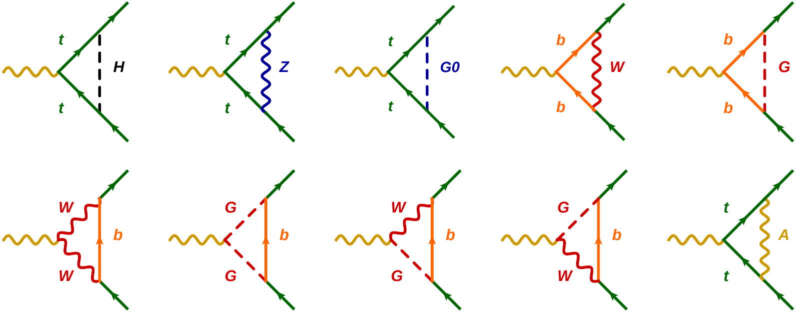
Boson-fermion 3-point diagrams

- $Zt\bar{t}$



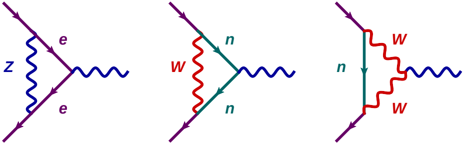
Boson-fermion 3-point diagrams

- $At\bar{t}$

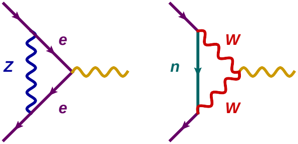


Boson-fermion 3-point diagrams

- $Ze\bar{e}$

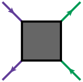


- $Ae\bar{e}$

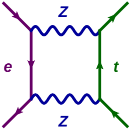


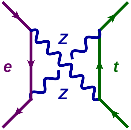
Boxes

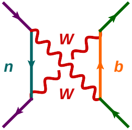
- Box diagrams are UV-finite
- Necessary for gauge independence
- Box decomposition with chirality amplitudes (ρ, κ ... top, electron chirality)



$$= \sum_{\rho,\kappa=\pm} \sum_{i=1}^{4} (F_{\text{box}})_i^{\rho\kappa} M_i^{\rho\kappa}$$







+ QED-boxes

Useful Packages

- FeynArts [**Hahn '90**]: automatic generation of Feynman diagrams and amplitudes
- FeynCalc [**Mertig '92**]: symbolic manipulations of Dirac algebra/Feynman integrals and calculation of 1-loop amplitudes (symbolically and numerically)
- LoopTools [**van Oldenborgh '90**]: numerical evaluation of ($n \leq 5$)-point tensor integral coefficients
- Package-X [**Patel '15**]: analytical and numerical tool for HEP computations (tree + 1-loop)
  - Dirac algebra simplifications (traces, projection operators,...)
  - Analytical computation of full diagrams with only 2 Mathematica commands
  - Analytical PV-reduction including explicit IR- and UV-regularization with DIMREG
  - Analytical and numerical implementation of scalar ($n \leq 4$)-integrals (automatic handling of $i\epsilon$-prescription, special cases,...)
  - ...

Renormalization scheme

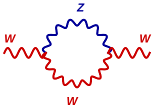
- On-shell scheme [Denner '93]
- OS-conditions for all masses and residues (for real parts)
- Non-relativistic OS-condition for electric charge
- Full inclusion of Higgs tadpoles
- No Dyson summation for bosonic self energies

Contents

Gauge independence

Clustering

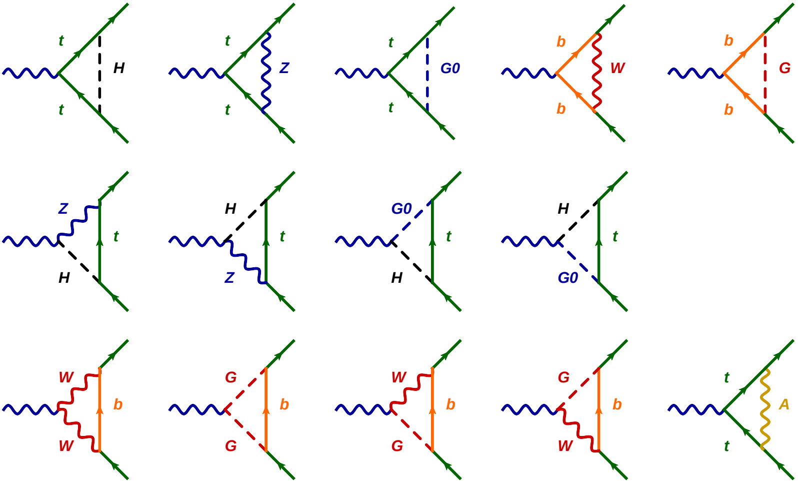
- Virtual particles determine ξ-dependence of diagrams
- Vertices and fermion self-energies: natural formation of subsets (clusters)
  - Z-cluster: internal Z- or associated Goldstone bosons (ξ_Z)
  - W-cluster: internal W- or associated Goldstone bosons (ξ_W)
  - A-cluster: "pure QED" (ξ_A)
  - H-cluster: internal Higgs bosons
- For gauge boson self energies the assignment is not unique, e.g.



- BUT: only ξ_W-dependence in $(\delta F_1^{Zff})_{\text{bos}}$

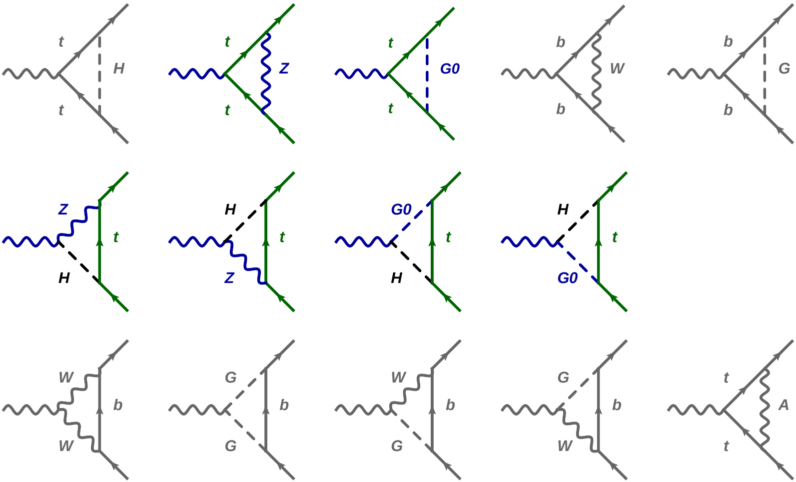
Clustering

- $Zt\bar{t}$



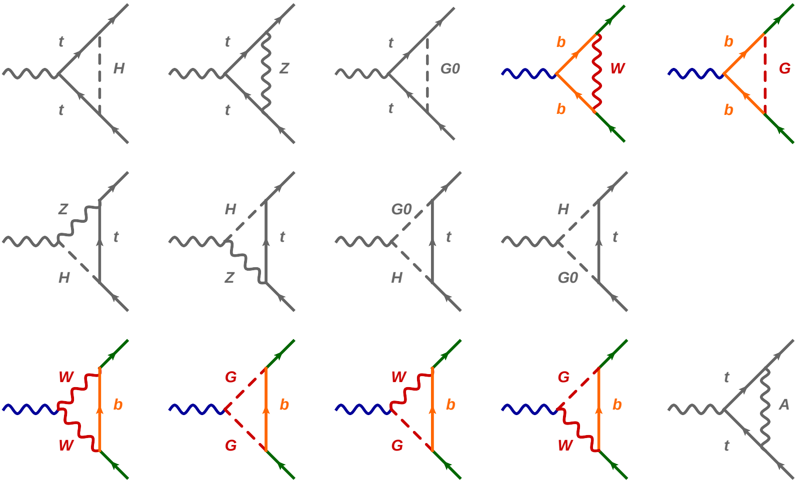
Clustering

- $Zt\bar{t}$ - $\xi_Z \Longrightarrow$ Z-Cluster



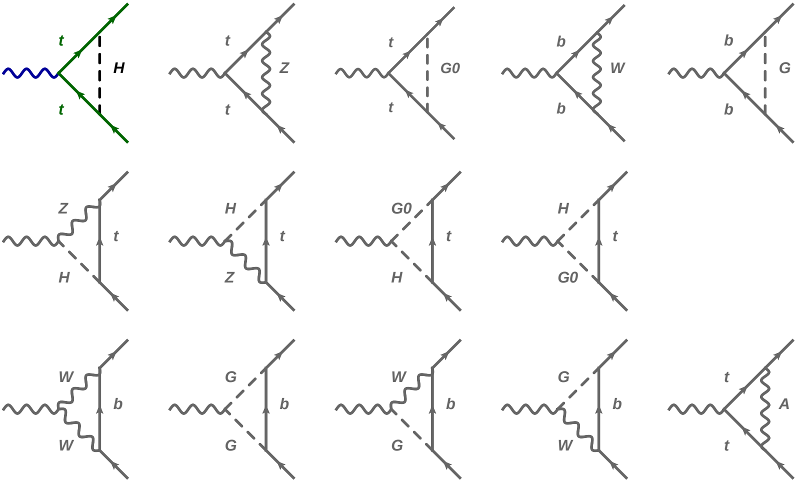
Clustering

- $Zt\bar{t}$ - $\xi_W \Longrightarrow$ W-Cluster



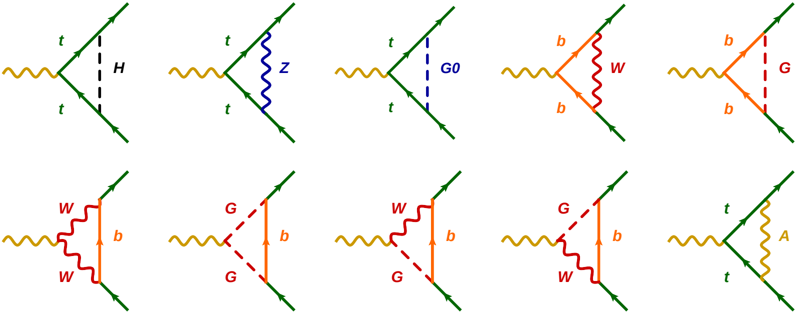
Clustering

- $Zt\bar{t}$ - H-Cluster



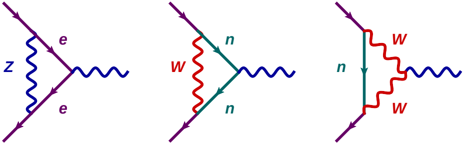
Clustering

- $At\bar{t}$

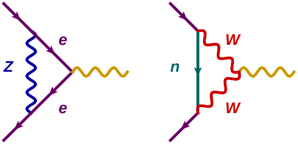


Clustering

- $Ze\bar{e}$

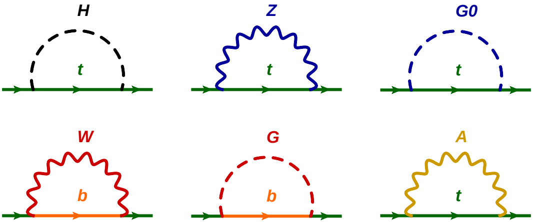


- $Ae\bar{e}$

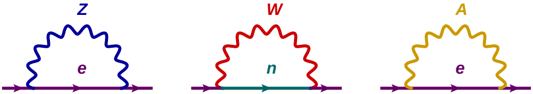


Clustering

- Top

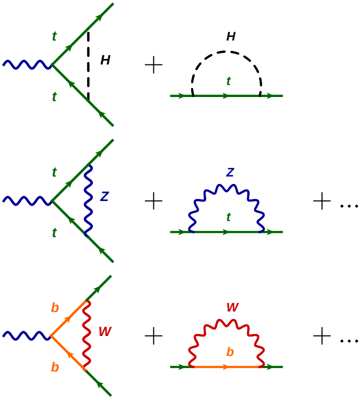


- Electron



Gauge independence overview for final-state corrections

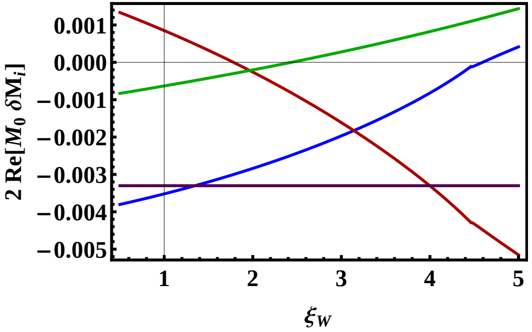
$\mathbf{Zt\bar{t}}$	UV	ξ_W	ξ_Z	ξ_A
$(F_1^{Ztt})_{H\text{-cl}} + (\delta Z_{F_1}^H)_{\text{fer}}$	✓	✓	✓	✓
$(G_1^{Ztt})_{H\text{-cl}} + (\delta Z_{G_1}^H)_{\text{fer}}$	✗	✓	✓	✓
$(F_1^{Ztt})_{Z\text{-cl}}^{\text{ab}} + (\delta Z_{F_1}^Z)_{\text{fer}}$	✓	✓	✓	✓
$(G_1^{Ztt})_{Z\text{-cl}}^{\text{ab}} + (\delta Z_{G_1}^Z)_{\text{fer}}$	✗	✓	✓	✓
$(F_1^{Ztt})_{Z\text{-cl}}^{\text{na}}$	✓	✓	✓	✓
$(G_1^{Ztt})_{Z\text{-cl}}^{\text{na}}$	✗	✓	✓	✓
$(F_1^{Ztt})_{W\text{-cl}}^{\text{ab}} + (\delta Z_{F_1}^W)_{\text{fer}}$	✗	✗	✓	✓
$(G_1^{Ztt})_{W\text{-cl}}^{\text{ab}} + (\delta Z_{G_1}^W)_{\text{fer}}$	✗	✗	✓	✓
$(F_1^{Ztt})_{W\text{-cl}}^{\text{na}}$	✗	✗	✓	✓
$(G_1^{Ztt})_{W\text{-cl}}^{\text{na}}$	✗	✗	✓	✓
$(F_1^{Ztt})_{W\text{-cl}}^{\text{total}} + (\delta Z_{F_1})_{\text{bos}}$	✓	✗	✓	✓
$(G_1^{Ztt})_{W\text{-cl}}^{\text{total}} + (\delta Z_{G_1})_{\text{bos}}$	✓	✗	✓	✓



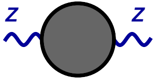
- UV-cancellations not only within clusters
- $\xi_A \to$ almost trivial, $\xi_Z \to$ straightforward, $\xi_W \to$ non-trivial

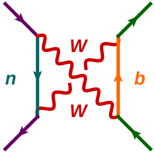
Gauge independence

- *Z*-clusters ✓
- *W*-clusters ✗ $\longrightarrow$ residual ξ_W
- Missing pieces: oblique and box corrections



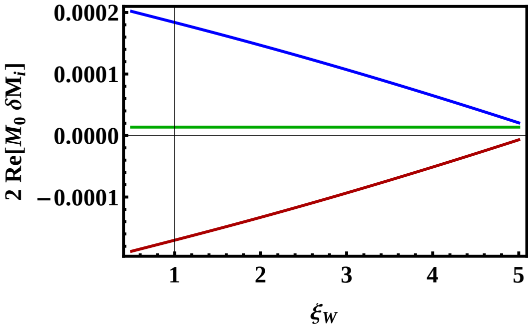
{boxes, vertices, oblique, total}



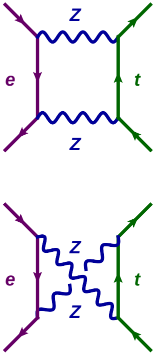


Gauge independence

- No ξ_Z-dependence in oblique corrections
- Z-boxes: ξ_Z cancels

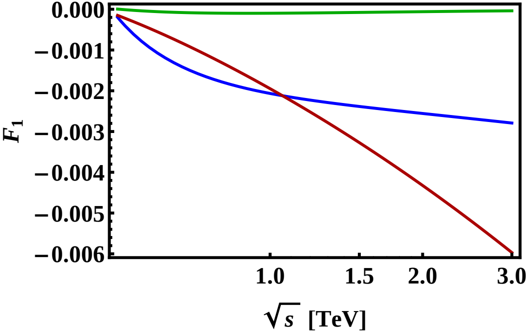


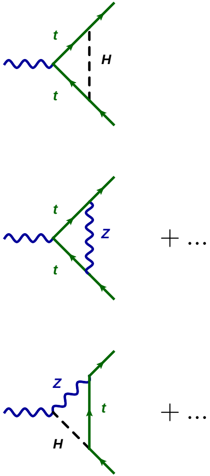
{direct, crossed, total}



Example: numerical size of H- and Z-clusters of $Zt\bar{t}$

{H-cluster, abelian Z-cluster, non-abelian Z-cluster}





- Different logarithmic structure of clusters shifts numerical importance!

Contents

High-energy expansions

High-energy expansions

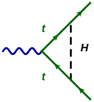
- Assumption: $\{|s|, |t|, |u|\} \gg \{m_t^2, M_H^2, M_Z^2, M_W^2\} \sim \Lambda_{\text{EW}}^2$
- Expectation:

$$a \ln^2\left(-\frac{\mu^2}{s}\right) + b \ln\left(-\frac{\mu^2}{s}\right) + c + \mathcal{O}\left(\frac{\Lambda_{\text{EW}}^2}{s}\right)$$

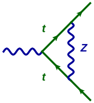
- c contains no logarithmic s-dependence (but e.g. $s/t \sim \mathcal{O}(1)$)
- Counterterms $\longrightarrow c \Rightarrow$ Sudakov-log extraction from bare diagrams
- Exemplary look in Feynman gauge $\xi = 1$ at:
  - $Zt\bar{t}$ vertex diagrams
  - Z-box diagrams

High-energy expansions (H- and abelian Z-clusters of $Zt\bar{t}$)

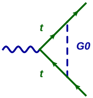
F_1 $\qquad$ G_1 $\qquad$ F_2



$$= \left\{ \frac{v_t y_t^2}{2} L_s + c, \frac{a_t y_t^2}{2} L_s + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$



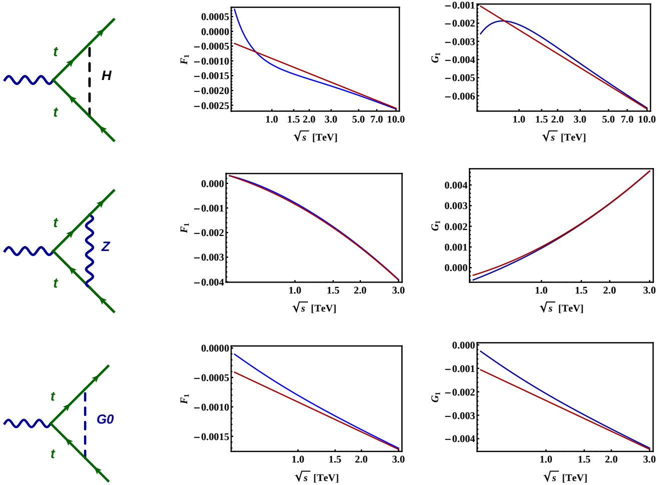
$$\begin{aligned} = \Big\{ & v_t \left(v_t^2 + 3a_t^2\right) \left(-L_s^2 + (-3 + 2L_{M_Z})L_s\right) + c, \\ & - a_t \left(a_t^2 + 3v_t^2\right) \left(-L_s^2 + (-3 + 2L_{M_Z})L_s\right) + c,\ 0 \Big\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right) \end{aligned}$$



$$= \left\{ \frac{v_t y_t^2}{2} L_s + c, \frac{a_t y_t^2}{2} L_s + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$

$$L_s \equiv \ln\left(-\frac{\mu^2}{s}\right), \quad L_M \equiv \ln\left(\frac{\mu^2}{M^2}\right),$$

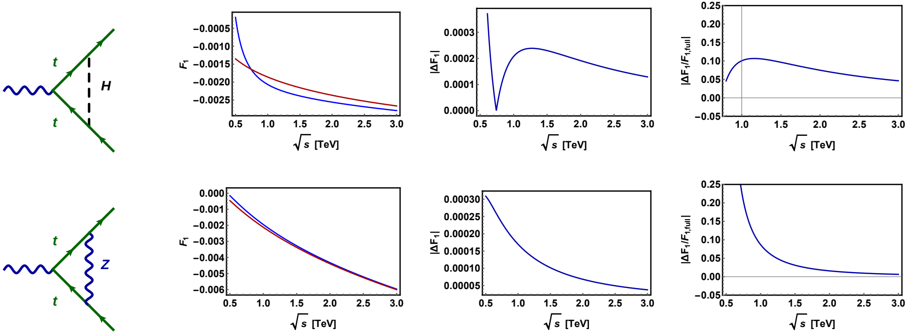
High-energy numerics (H- and abelian Z-cluster of $Zt\bar{t}$)



$\{$full, expanded$\}$

Accuracy of expansion

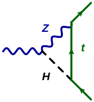
- Compare expanded with full result
- E.g. for (renormalized) F_1 of H- and abelian Z-cluster of $Zt\bar{t}$-vertex



$\{$full, expanded$\}$

$\Rightarrow \quad \sim 90\,\%$ accuracy for $\sqrt{s} \sim 1$ TeV

High-energy expansions (non-abelian Z-clusters of $Zt\bar{t}$)



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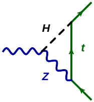
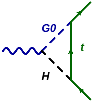


$$= \{0,\ 0,\ 0\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$



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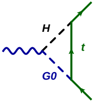
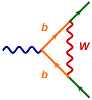


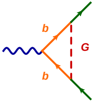
$$= \left\{0, -a_t y_t^2\, L_s + c,\ 0\right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$

High-energy expansions (abelian W-clusters of $Zt\bar{t}$)



$$= \left\{ \frac{v_b + a_b}{4s_W^2} \left(-L_s^2 + (-3 + 2L_{M_W}) L_s \right) + c, \right.$$

$$\left. - \frac{v_b + a_b}{4s_W^2} \left(-L_s^2 + (-3 + 2L_{M_W}) L_s \right) + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\text{EW}}^2}{s} \right)$$

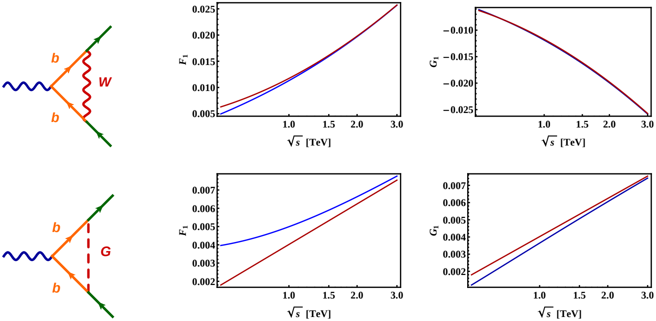


$$= \left\{ \frac{(v_b + a_b) y_t^2 + (v_b - a_b) y_b^2}{2} L_s + c, \right.$$

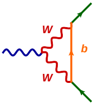
$$\left. \frac{(v_b + a_b) y_t^2 - (v_b - a_b) y_b^2}{2} L_s + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\text{EW}}^2}{s} \right)$$

$$L_s \equiv \ln\left(-\frac{\mu^2}{s} \right), \quad L_M \equiv \ln\left(\frac{\mu^2}{M^2} \right),$$

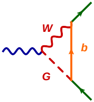
High-energy numerics (abelian W-cluster of $Zt\bar{t}$)



$\{$full, expanded$\}$



$$= \left\{ -\frac{c_W}{4s_W^3}\, L_s + c,\ \frac{c_W}{4s_W^3}\, L_s + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$



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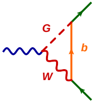
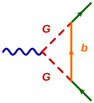


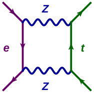
$$= \left\{ 0,\ 0,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$



$$= \left\{ \frac{(c_W^2 - s_W^2)(y_t^2 + y_b^2)}{4c_W s_W}\, L_s + c,\ \frac{(c_W^2 - s_W^2)(y_t^2 - y_b^2)}{4c_W s_W}\, L_s + c,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{EW}}^2}{s}\right)$$

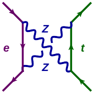
$$L_s \equiv \ln\left(-\frac{\mu^2}{s}\right), \quad L_M \equiv \ln\left(\frac{\mu^2}{M^2}\right),$$

High-energy expansions (Z-boxes)



$$= \frac{2(v_t - \rho a_t)^2 (v_e - \kappa a_e)^2}{s} \times$$

$$\left\{ \delta^{\rho}_{-\kappa}(-L_t^2 + 2L_{M_Z} L_t + c),\ \frac{\delta^{\rho}_{\kappa}}{t}(-L_t^2 + 2L_{M_Z} L_t + c),\ 0,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{EW}}}{s}\right)$$

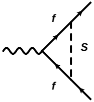


$$= \frac{-2(v_t - \rho a_t)^2 (v_e - \kappa a_e)^2}{s} \times$$

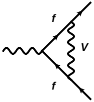
$$\left\{ \delta^{\rho}_{\kappa}(-L_u^2 + 2L_{M_Z} L_u + c),\ \frac{\delta^{\rho}_{-\kappa}}{u}(L_u^2 - 2L_{M_Z} L_u + c),\ 0,\ 0 \right\} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{EW}}}{s}\right)$$

$$L_{t,u} \equiv \ln\left(-\frac{\mu^2}{t, u}\right), \quad L_M \equiv \ln\left(\frac{\mu^2}{M^2}\right),$$

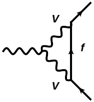
Particle content determines Sudakov logarithmic structure



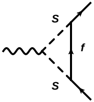
$\sim L_s$



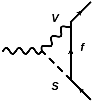
$\sim -L_s^2 + (-3 + 2L_{M_V})L_s,$



$\sim L_s$



$\sim L_s$



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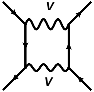
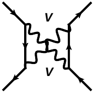


$\sim -L_t^2 + 2L_{M_V}L_t$



$\sim -L_u^2 + 2L_{M_V}L_u$

Contents

Summary and Outlook

Summary

- EW 1-loop corrections to $e^+e^- \to t\bar{t}$ in R_ξ-gauge
- $\exists$ various n-point contributions: 1-, 2-, 3-, 4-point
- Gauge independence and UV-finitness ties sectors together
- Structure of gauge- and UV-cancellations
- Logarithmic (+ constant) high-energy structure ($\xi = 1$) $\Rightarrow$ Sudakov logs
- Sizable Sudakov log effects for $\sqrt{s} \sim 1$ TeV
- $\Rightarrow$ no clear hierachy of single contributions in this energy regime

Outlook

Short-term

- Explore gauge-structure in the high-energy regime
- Construct (toy-)EFT (analogous to SCET) for
  - H-contributions
  - Neutral sector ($H-$ and Z-contributions)
- QED

Long-term

- Full EFT for EW effects in physics involving jets
- Finite-lifetime effects
  - Top decay
  - Nonzero boson widths
- 2-loop (mixed EW-QCD at $\mathcal{O}(\alpha\alpha_s)$)