Fun with Renormalization and Regularization

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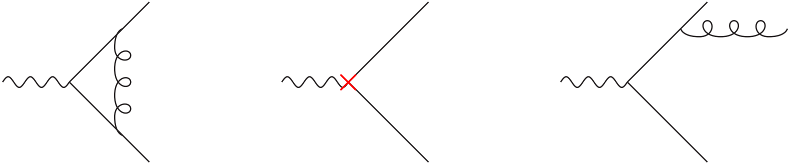
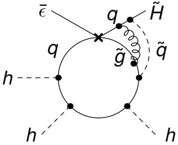
Outline



- DREG, FDH and DRED:
  Consistency, UV and IR properties
  [Gnendiger, Signer, DS; Broggio, Gnendiger, Signer, DS, Visconti;
  DS, Unger (preliminary)]



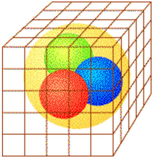
- Renormalization of VEVs [Sperling, DS, Voigt]

Outline

1. Regularization: DREG, FDH, DRED: UV, IR
   - Precise definitions of all schemes possible
   - puzzles can be resolved
   - Simple point of view for DRED — understand IR structure
   - Is DRED supersymmetric???
2. Renormalization of VEVs
3. Conclusions

Motivation

Regularization necessary to define QFT at the quantum level



cutoff-scale Λ

$\int_{|p|<\Lambda} d^4p$

DREG

$\mu^{4-D} \int d^Dp$

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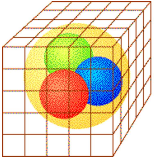

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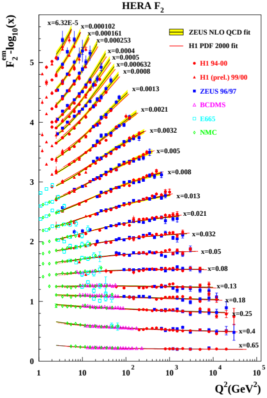
- Choice of regularization is unphysical



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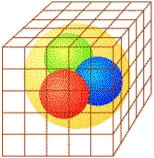

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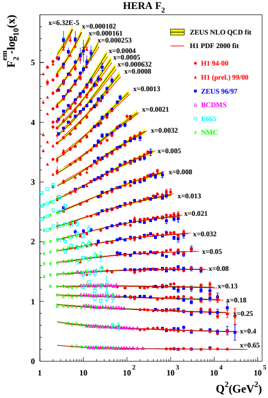
- Unitarity/causality determine physics
- e.g. anomaly/breaking of scale invariance



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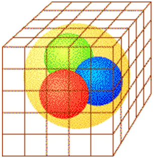

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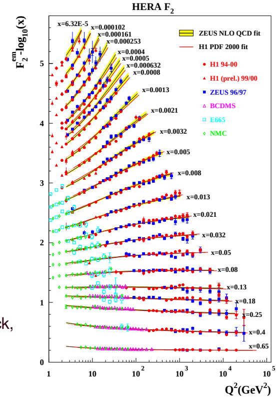
- e.g. anomaly/breaking of scale invariance
- regularization must be consistent with unitarity/causality (proven for DREG: [Speer '74][Breitenlohner, Maison '77], equivalence of DRED, no inconsistency: [Jack, Jones, Roberts '94][DS '05])



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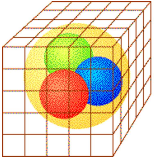

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History: puzzles, problems

- DREG breaks SUSY
- "DRED is mathematically inconsistent [Siegel '80]"
- "DRED has IR factorization problem [van Neerven, Smith, et al '88 and '05][Zerwas et al]"
- "No DRED IR factorization problem found [Kunszt, Signer, Trocsanyi '94; Catani et al '97]"
- "DRED violates unitarity ['t Hooft, van Damme '84]"
- "Some published results therefore wrong [Harlander, Kant, Mihaila, Steinhauser '06; Kilgore '11]"

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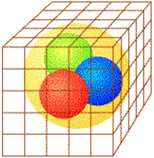
Regularization necessary to define QFT at the quantum level



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Aims:

- resolve inconsistencies, study symmetry properties of schemes
- today: consistent definitions of DREG, DRED, FDH; IR relations; SUSY breaking of DRED(?)

Definitions/explicit construction

4S: ordinary 4-dimensional Minkowski/momentum space, metric $\bar{g}^{\mu\nu}$

QDS: "D-dimensional space" [Wilson'73],[Collins] :=
truly ∞-dimensional space with some D-dim characteristics:

- D-dimensional Integral = linear mapping $\int d^D k e^{-k^2} = \pi^{D/2}$
- $g^{\mu\nu}_{(D)}$: bilinear form $\mu = 0, 1, 2, \ldots \infty, \quad g_{(D)}{}^{\mu}{}_{\mu} = D$
- γ-matrices similar

Q2ϵS: "2ϵ-dimensional space" analogous

Q4S: "quasi-4-dimensional space" Q4S := QDS $\oplus$ Q2ϵS

hierarchy of spaces 4S $\subset$ QDS $\subset$ Q4S [DS '05]

although $\mu = 0, 1, 2, \ldots \infty$, useful relations hold, e.g.

$$g^{\mu\nu} = g^{\mu\nu}_{(D)} + g^{\mu\nu}_{(2\epsilon)}, \qquad g^{\mu}{}_{\mu} = 4, \qquad g^{\mu\nu} k_{(D)\nu} = k^{\mu}_{(D)}$$

How does Q4S avoid Siegel's inconsistency?

Siegel: "With

$$\epsilon^{(4)}_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{(4)}_{\nu_1\nu_2\nu_3\nu_4} \propto \det((g^{(4)}_{\mu_i\nu_j}))$$

calculate

$$\epsilon^{(D)\mu\nu\rho\sigma}\,\epsilon^{(\epsilon)}{}_{\alpha\beta\gamma\delta}\,\epsilon^{(D)}{}_{\mu\nu\rho\sigma}\,\epsilon^{(\epsilon)\alpha\beta\gamma\delta}$$

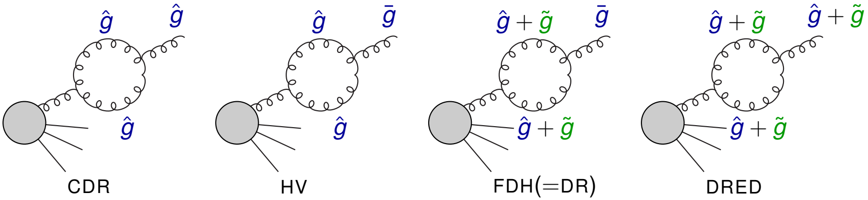
in two different ways

$$\Rightarrow 0 = D(D-1)^2(D-2)^2(D-3)^2(D-4)$$

different calculational steps lead to different results,

mathematical inconsistency!!!" [Siegel'80]

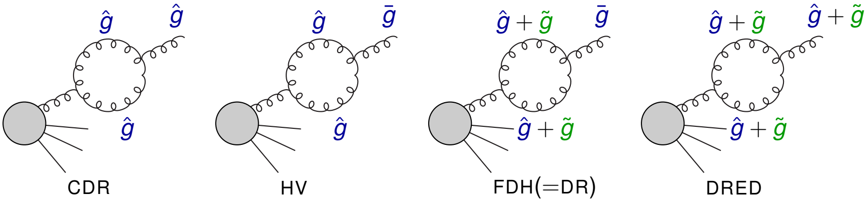
Don't allow explicit index counting (step one) any more, because $g^{(4)}{}_{\mu\nu} \in$quasi-4-dim space!



Define four commonly used schemes [Signer, DS '08]

$$Q4S = QDS \oplus Q2\epsilon S, \qquad g^{\mu\nu} = \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu}$$

- difference: internal and/or external gauge fields



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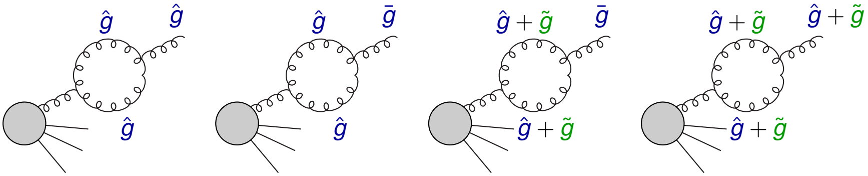
- Note: van Neerven, Smith, Zerwas et al used DRED, Kunszt, Signer, Trocsanyi, Catani et al used DR=FDH$\Rightarrow$ resolves half of factorization problem



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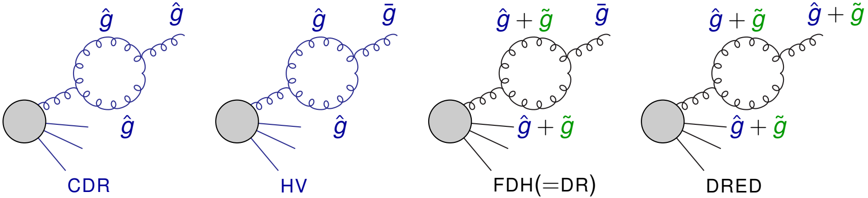
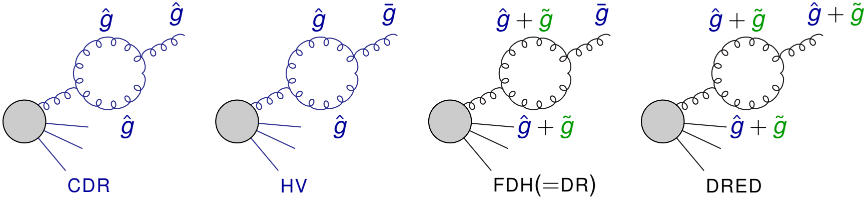
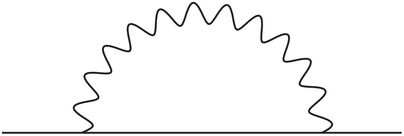
- focus first on UV: CDR=HV$\equiv$ DREG, FDH=DRED



Dim. Regularization (DREG)

D dimensions
D Gluon/photon-components
4 Gluino/photino-components





$$\hat{\gamma}^{\mu}\hat{\gamma}^{\rho}p_{\rho}\hat{\gamma}_{\mu} = (2-D)\hat{\gamma}^{\rho}p_{\rho}$$

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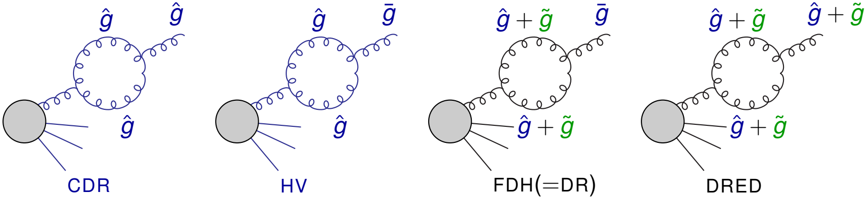

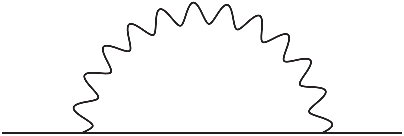

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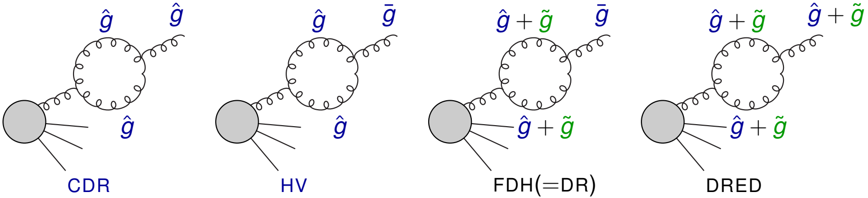
SUSY broken, need SUSY-restoring counterterms



4-dim. scheme (try 4S ⊂ QDS)

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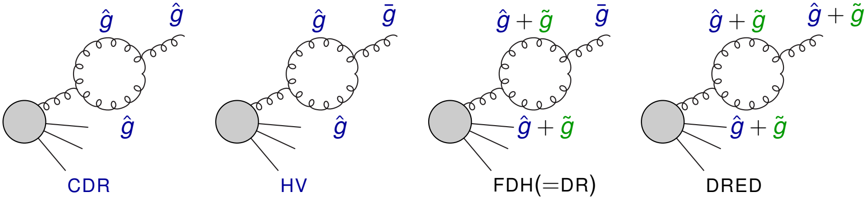

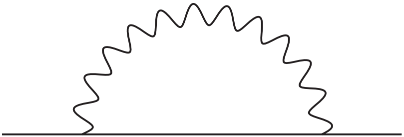

$$\bar{\gamma}^{\mu}\hat{\gamma}^{\rho}p_{\rho}\bar{\gamma}_{\mu} = 2\bar{\gamma}^{\rho}p_{\rho} - 4\hat{\gamma}^{\rho}p_{\rho}$$

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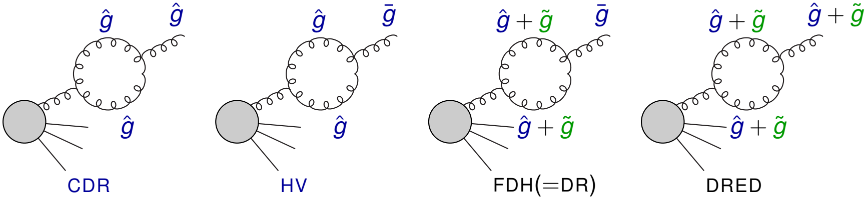

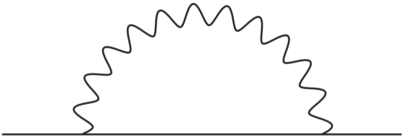

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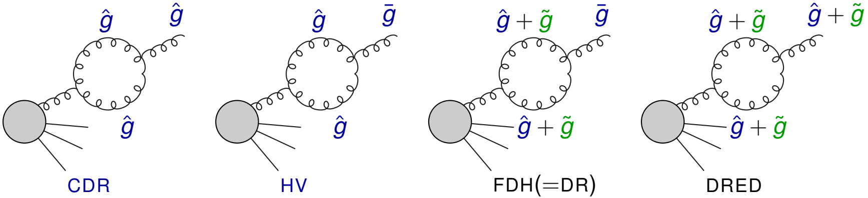
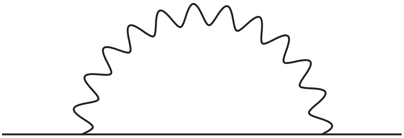
complicated; no *D*-dim. gauge covariant derivative possible





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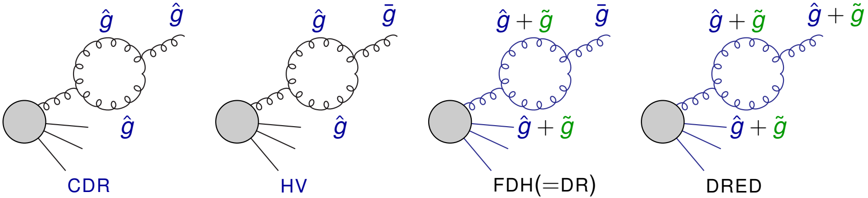
sometimes used in literature, e.g. for chiral anomaly [Anselmi '14]



FDH/DRED (use Q4S)

D dimensions

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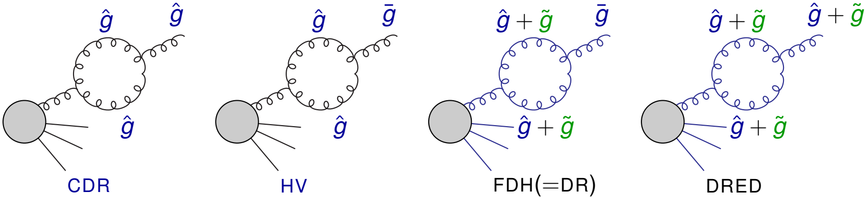

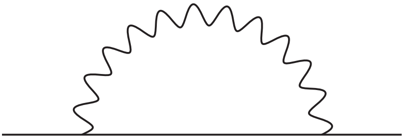

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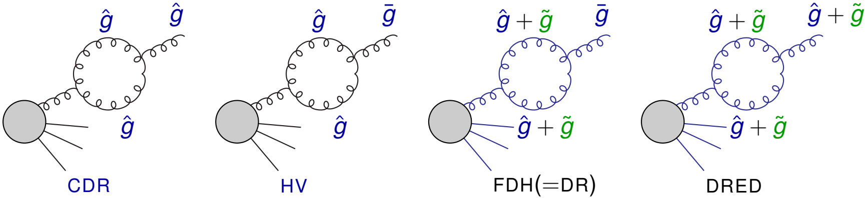

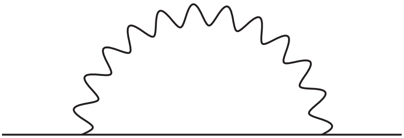

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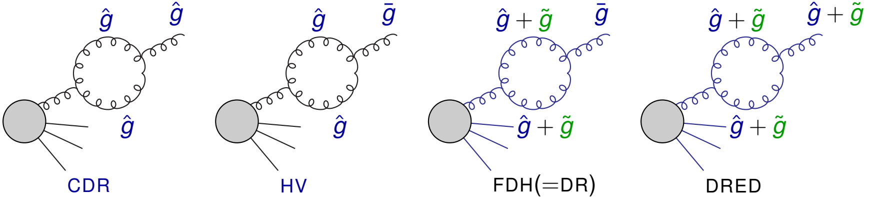
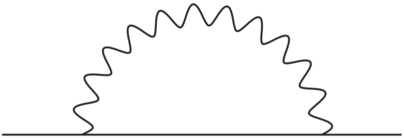
simple; *D*-dim gauge invariance holds





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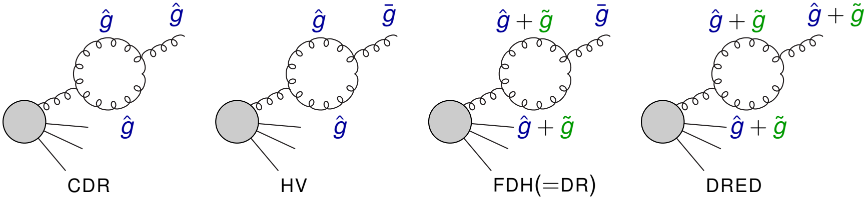
Lorentz invariance only in *D*-dim; not in full Q4S



Summary regarding UV regularization:

- CDR and HV only need QDS, simpler but break SUSY
- FDH and DRED require Q4S to preserve gauge invariance

Common formulation of FDH (Bern, Dixon, Freitas; Kilgore, . . .):

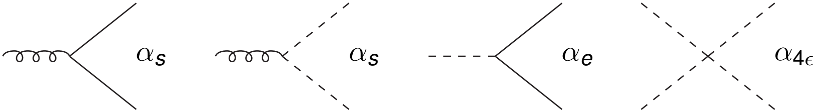
- "$4 < D < N_s$, internal gluons are N_s-dim.; at the end $N_s = 4$"

Important complication: Renormalization in FDH/DRED

- ϵ-scalars not "protected" by gauge invariance

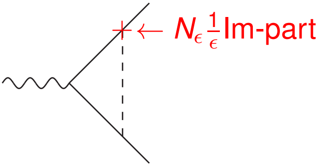
$$D^\mu = \hat{\partial}^\mu + igA^\mu = \quad \hat{\partial}^\mu + ig\hat{A}^\mu \quad + \quad ig\tilde{A}^\mu$$

4-component Gluon in DRED g = D-component gauge field $\hat{g}$ + ϵ-scalars $\tilde{g}$



- Different couplings,
  especially $\delta\alpha_s \neq \delta\alpha_e, \beta^s \neq \beta^e, \ldots$
- Distinction required, otherwise divergent/non-unitary results

  [Jack, Jones, Roberts '94][Harlander, Kant, Mihaila, Steinhauser '06][Kilgore '11]



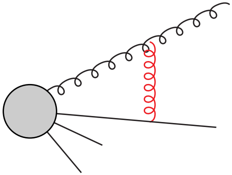
FDH or DRED

$\Updownarrow$

HV or CDR of theory with new, ϵ-scalar of multiplicity $N_\epsilon = 2\epsilon$ with independent couplings

This point of view also allows to understand IR structure in these schemes

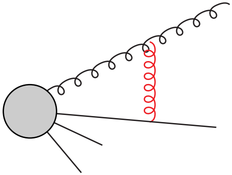
1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

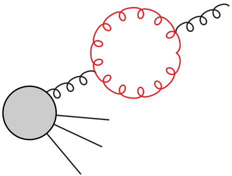


soft/soft-collinear:

$$\propto \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{\gamma^{\text{cusp}}}{4\epsilon^2} + \frac{\gamma^{\text{cusp}}}{2\epsilon} \frac{1}{2} \ln \frac{\mu^2}{-s_{ij}} \right)$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





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collinear:

$$\propto \frac{\gamma^i}{2\epsilon}$$

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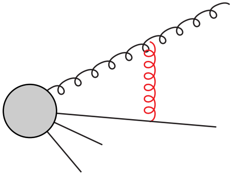

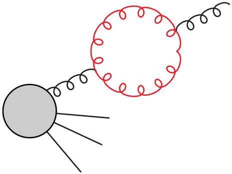

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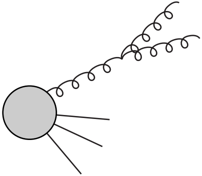
collinear:

$$\propto \frac{\gamma^i}{2\epsilon}$$

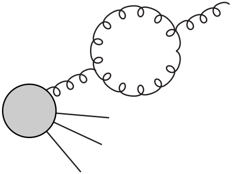
CDR: Three constants describe all IR divergences

$$\gamma^{\text{CDR}}_{\text{cusp}} = \frac{\alpha_s}{4\pi}(\,4\,) \qquad \gamma^{\text{CDR}}_{q} = \frac{\alpha_s}{4\pi}(-3\,C_F) \qquad \gamma^{\text{CDR}}_{g} = \frac{\alpha_s}{4\pi}\left[-\frac{11}{3}C_A + \frac{2}{3}N_F\right]$$

Side remark about real corrections

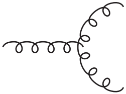


$\propto \frac{P_{i\to jk}}{p_j \cdot p_k}$

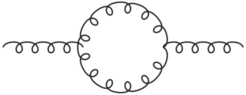


$\propto \frac{\gamma^i}{\epsilon}$

Unitarity:



$\leftrightarrow$

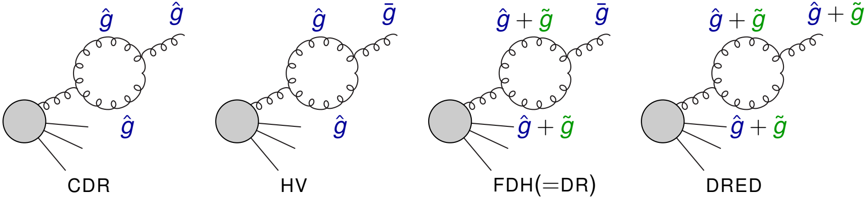


real $\leftrightarrow$ virtual coll. sing.

$$\int P_{i\to\text{anything}} \quad = \quad \gamma(i)$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

[Signer, DS '08]



$\gamma_g^{\rm CDR} = \frac{\alpha_s}{4\pi}\left[-\frac{11}{3}C_A + \frac{2}{3}N_F\right]$

...

$\gamma_g^{\rm FDH} = \dots$

$+\frac{\alpha_s}{4\pi}N_\epsilon\frac{C_A}{6}$

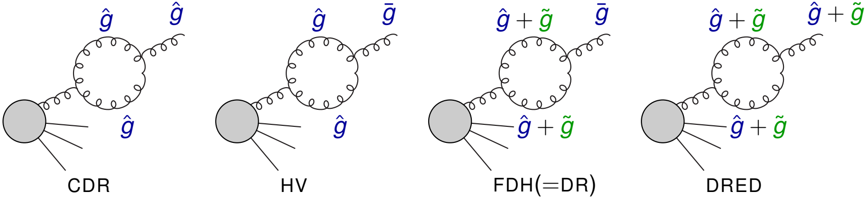
$\gamma_g^{\rm DRED} = \dots$

...

$\gamma_{\tilde{g}}^{\rm DRED} = \frac{\alpha_s}{4\pi}\,(-4\,C_A) + \frac{\alpha_e}{4\pi}\,(2N_F T_R)$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

[Signer, DS '08]



$\gamma_g^{\mathrm{CDR}} = \frac{\alpha_s}{4\pi}\left[-\frac{11}{3}C_A + \frac{2}{3}N_F\right]$

. . .

$\gamma_g^{\mathrm{FDH}} = \dots$

$+ \frac{\alpha_s}{4\pi} N_\epsilon \frac{C_A}{6}$

$\gamma_g^{\mathrm{DRED}} = \dots$

. . .

$\gamma_{\tilde g}^{\mathrm{DRED}} = \frac{\alpha_s}{4\pi}\,(-4\,C_A) + \frac{\alpha_e}{4\pi}\,(2 N_F T_R)$

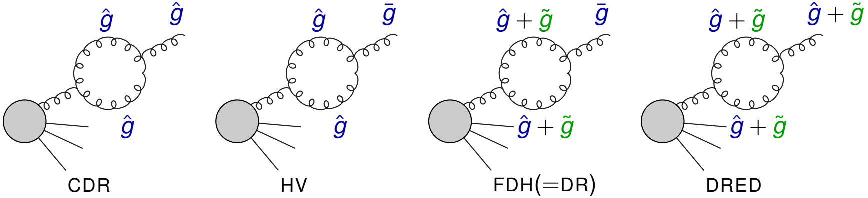
FDH: Three constants γ_{cusp}, γ_{q}, γ_{g} describe all IR divergences

additional virtual state $\tilde g$: *values* of γ's change

[Kunszt, Signer, Trocsanyi '94]
[Catani, Seymour, Trocsanyi '97]

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

[Signer, DS '08]



$\gamma_g^{\text{CDR}} = \frac{\alpha_s}{4\pi}\left[-\frac{11}{3}C_A + \frac{2}{3}N_F\right]$

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...

$\gamma_{\tilde{g}}^{\text{DRED}} = \frac{\alpha_s}{4\pi}\,(-4\,C_A) + \frac{\alpha_e}{4\pi}\,(2N_F T_R)$

DRED: Four constants needed: γ_{cusp}, γ_{q}, $\gamma_{\hat{g}}$, $\gamma_{\tilde{g}}$

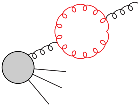
additional external $\tilde{g}$: additional $\gamma_{\tilde{g}}$ [Signer, DS '08; Broggio,Gnendiger,Signer,DS,Visconti]

split $g = \hat{g} + \tilde{g}$ required to understand factorization
(solves "problem" of [Beenakker, Kuijf, v Neerven, Smith '88][v Neerven, Smith '04])

Apply in practice to calculation of $\mathcal{M}^{\text{1-loop}}$

Direct computation of $\mathcal{M}^{\text{1-loop}}$

understand IR structure:



$$\mathcal{M}^{\text{HV}}_{\text{1-loop}}(\ldots g \ldots) =$$

$$\mathcal{M}^{\text{FDH}}_{\text{1-loop}}(\ldots g \ldots) =$$

$$\mathcal{M}^{\text{DRED}}_{\text{1-loop}}(\ldots g \ldots) =$$

Apply in practice to calculation of $\mathcal{M}^{\text{1-loop}}$

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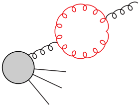

$\Downarrow$

$$\mathcal{M}^{\text{HV}}_{\text{1-loop}}(\ldots g \ldots) =$$

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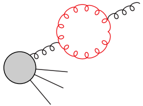


$\Downarrow$

$$\mathcal{M}^{\text{HV}}_{\text{1-loop}}(\dots g \dots) = \dots + \frac{\gamma_g^{\text{HV}}}{2\epsilon}\mathcal{M}^{\text{HV}}_{\text{tree}}(\dots g \dots)$$

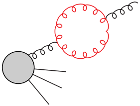
$$\mathcal{M}^{\text{FDH}}_{\text{1-loop}}(\dots g \dots) = \dots + \frac{\gamma_g^{\text{FDH}}}{2\epsilon}\mathcal{M}^{\text{FDH}}_{\text{tree}}(\dots g \dots)$$

$$\mathcal{M}^{\text{DRED}}_{\text{1-loop}}(\dots g \dots) = \dots + \frac{\gamma_{\hat{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\dots \hat{g} \dots) + \frac{\gamma_{\tilde{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\dots \tilde{g} \dots$$

Apply in practice to calculation of $\mathcal{M}^{\text{1-loop}}$

Direct computation of $\mathcal{M}^{\text{1-loop}}$

understand IR structure:



$\Downarrow$

$$\mathcal{M}^{\text{HV}}_{\text{1-loop}}(\ldots g \ldots) = \ \ldots + \frac{\gamma_g^{\text{HV}}}{2\epsilon}\mathcal{M}^{\text{HV}}_{\text{tree}}(\ldots g \ldots)$$

$$\mathcal{M}^{\text{FDH}}_{\text{1-loop}}(\ldots g \ldots) = \ \ldots + \frac{\gamma_g^{\text{FDH}}}{2\epsilon}\mathcal{M}^{\text{FDH}}_{\text{tree}}(\ldots g \ldots)$$

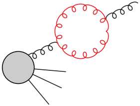
$$\mathcal{M}^{\text{DRED}}_{\text{1-loop}}(\ldots g \ldots) = \ \ldots + \frac{\gamma_{\hat{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\ldots \hat{g} \ldots) + \frac{\gamma_{\tilde{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\ldots \tilde{g} \ldots$$

"Factorization problem" of van Neerven, Smith, et al was:

expected r.h.s.$= \ \ldots + \frac{\gamma_g^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\ldots g \ldots)$

Apply in practice to calculation of $\mathcal{M}^{\text{1-loop}}$

Direct computation of $\mathcal{M}^{\text{1-loop}}$



understand IR structure:

⇓

$$\mathcal{M}^{\text{HV}}_{\text{1-loop}}(\ldots g \ldots) = \ldots + \frac{\gamma_g^{\text{HV}}}{2\epsilon}\mathcal{M}^{\text{HV}}_{\text{tree}}(\ldots g \ldots)$$

$$\mathcal{M}^{\text{FDH}}_{\text{1-loop}}(\ldots g \ldots) = \ldots + \frac{\gamma_g^{\text{FDH}}}{2\epsilon}\mathcal{M}^{\text{FDH}}_{\text{tree}}(\ldots g \ldots)$$

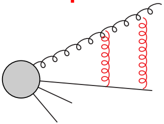
$$\mathcal{M}^{\text{DRED}}_{\text{1-loop}}(\ldots g \ldots) = \ldots + \frac{\gamma_{\hat{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\ldots \hat{g} \ldots) + \frac{\gamma_{\tilde{g}}^{\text{DRED}}}{2\epsilon}\mathcal{M}^{\text{DRED}}_{\text{tree}}(\ldots \tilde{g} \ldots$$

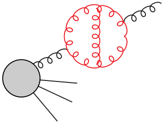
Remainder: finite, regularization-independent $\mathcal{M}_{\text{fin}}$
⇒ can simply translate between schemes

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- It works similarly
  - PhD theses C. Gnendiger, A. Visconti
  - [Gnendiger, Signer, DS '14][Broggio, Gnendiger, Signer, DS, Visconti PLB '15, JHEP '15]

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





Key insight: All IR divs $\longrightarrow$ factor $\mathbf{Z}$, $\mathbf{Z}^{-1}|\mathcal{A}\rangle$ =fin.

[Gardi, Magnea '09]
[Becher, Neubert '09]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

2nd insight [valid at least up to 2-loop order]

$$\Gamma = \sum_{(ij)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2}\, \gamma^{\text{cusp}}\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i$$

Changes in FDH and DRED: Same structure …but the γ's change and everything depends on α_s, α_e, $\alpha_{4\epsilon}$

Is DRED supersymmetric???

Slavnov-Taylor id. and Quantum Action Principle

Slavnov-Taylor identities: desired identities for Green functions

$$0 = \delta_{\rm sym}\langle T\phi_1\ldots\phi_n\rangle = \langle T(\delta_{\rm sym}\phi_1)\ldots\phi_n\rangle + \ldots$$

Regularized Quantum Action Principle

$$i\,\delta_{\rm sym}\langle T\phi_1\ldots\phi_n\rangle^{\rm reg} = \langle T\phi_1\ldots\phi_n\Delta\rangle^{\rm reg}, \quad \Delta = \int d^Dx\,\delta_{\rm sym}\mathcal{L}^{\rm reg}$$

Proofs for particular regularizations:

BPHZ [Lowenstein et al '71]
DREG [Breitenlohner, Maison '77]
DRED [DS '05]

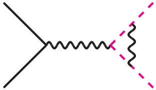
$\rightsquigarrow$ can check whether STI is valid on regularized level

Further explicit checks of SUSY in DRED?

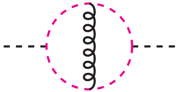
Status: many SUSY identities checked in DRED:

[Capper,Jones,van Nieuwenhuizen'80] [Martin, Vaughn '93] [Jack, Jones, North '96] [Beenakker,Höpker,Zerwas'96]
[Hollik,Kraus,DS'99] [Hollik,DS'01] [Fischer,Hollik,Roth,DS'03] [Harlander,Kant,Mihaila,Steinhauser'07]

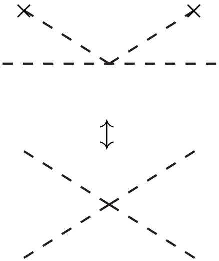
- sufficient for many SUSY processes
  $\Rightarrow$ multiplicative renormalization o.k.
  $\Rightarrow$ no SUSY-restoring counterterms



- but not all identities have been checked
  e.g. two-loop Higgs mass calculations:
  $\Rightarrow$ SUSY-restoring counterterms required?



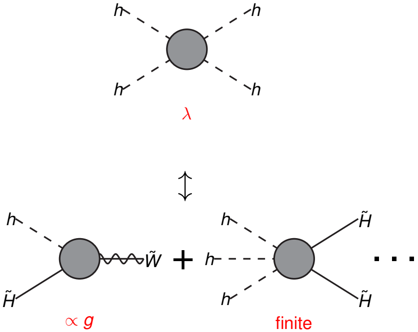
Higgs boson mass and quartic coupling



Higgs mass

- M_h governed by quartic Higgs self coupling λ
- $\lambda \propto g^2$ in MSSM

Quartic coupling and SUSY

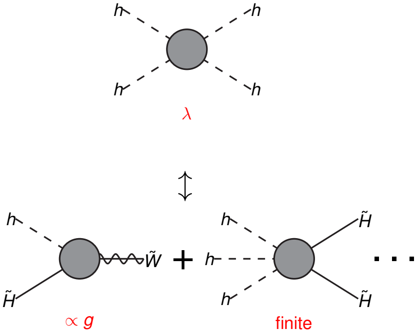


Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- Needs to be verified

$$0 \stackrel{?}{=} \delta_{\mathrm{SUSY}} \langle hhh\tilde{H} \rangle$$

Quartic coupling and SUSY

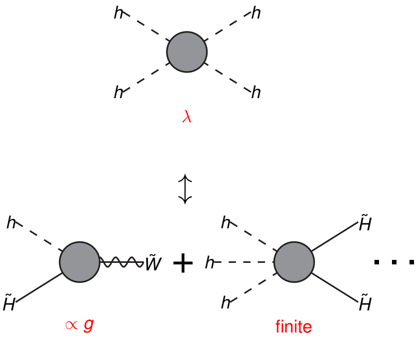


Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- Needs to be verified

explicit: $0 = \dfrac{\delta^5 S(\Gamma)}{\delta\phi_a \delta\phi_b \delta\phi_c \delta\tilde{H} \delta\bar{\epsilon}} = \Gamma_{\tilde{H} Y_{\phi_i} \bar{\epsilon}} \Gamma_{\phi_a \phi_b \phi_c \phi_i} + \Gamma_{\phi_a \phi_b Y_\lambda \epsilon} \Gamma_{\phi_c \tilde{H} \lambda} + \dots$

Quartic coupling and SUSY



$$0 \stackrel{?}{=} \delta_{\mathrm{SUSY}}\langle hhh\tilde{H}\rangle$$

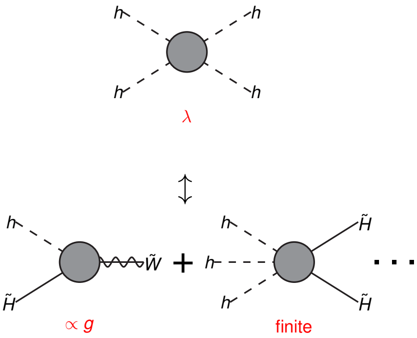
Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- Needs to be verified

If verified:

- Usual, multiplicative renormalization o.k.
- otherwise, SUSY-restoring counterterms would have to be added

Quartic coupling and SUSY



Slavnov-Taylor identity

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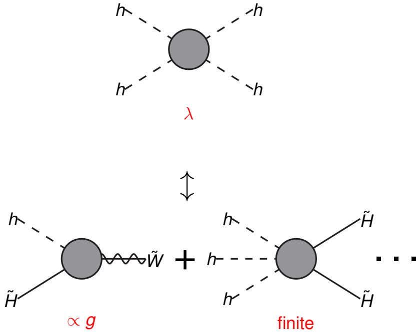
Strategy:

- Use quantum action principle in DRED [DS '05]

$$0 \stackrel{?}{=} \delta_{\mathrm{SUSY}}\langle hhh\tilde{H}\rangle \equiv \langle \Delta hhh\tilde{H}\rangle$$

Quartic coupling and SUSY



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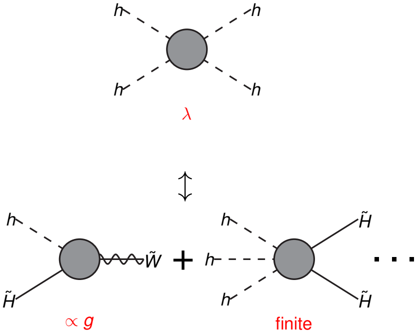
Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- Needs to be verified

Strategy:

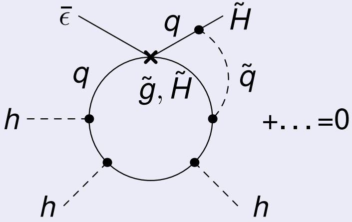
- Use quantum action principle in DRED [DS '05]
- $\delta_{\rm SUSY}\langle hhh\tilde{H}\rangle = \langle \Delta hhh\tilde{H}\rangle$ where $\Delta = \delta_{\rm SUSY}\int d^Dx\mathcal{L}$

Quartic coupling and SUSY

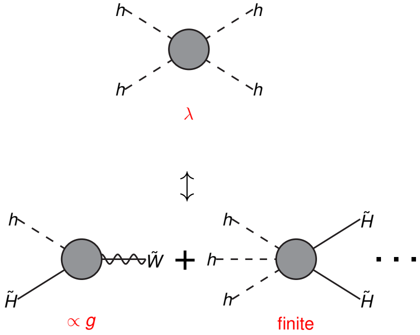


Old check: STI at 2-loop
$\mathcal{O}(\alpha_{t,b}^2, \alpha_{t,b}\alpha_s)$ [Hollik, DS '05]

$\langle \Delta hhh\tilde{H}\rangle = 0 \Leftrightarrow$

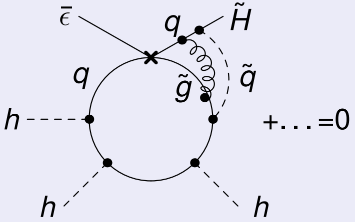


Quartic coupling and SUSY



New check: 3-loop STI at $\mathcal{O}(\alpha_t\alpha_s^2)$ [DS, Unger, preliminary]

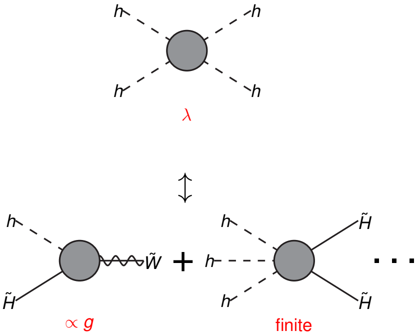
$\langle \Delta hhh\tilde{H}\rangle = 0 \Leftrightarrow$



Ingredients:

- structure of possible SUSY-restoring c.t. $\Rightarrow$ can set $p_{h_i} = 0$
- can simplify closed fermion loop to at most three γ-matrices
- recipe of [DS '05] then proves result

Quartic coupling and SUSY



Outlook:

- check complete two-loop, three-loop (?)
- check further, relevant STIs

Outline

1. Regularization: DREG, FDH, DRED: UV, IR

2. Renormalization of VEVs
   - Why interesting? Definitions of VEV
   - Essential point: relation to gauge fixing
   - Investigate in detail: extended BRS/STI
   - Results and insights

3. Conclusions

Renormalization: which counterterms are necessary?

Other questions: physical meaning of renormalized theory, unitarity/symmetries of physical S-matrix, absense of anomalies. . .

Higgs vacuum

What is the VEV v and its physical properties?

Renormalization of VEVs

Higgs/spontaneously broken gauge invariance:

$$\phi \to \phi + v$$

such that $\langle \phi \rangle = 0$, i.e. tadpoles vanish
Need to renormalize:

$$\phi \to \sqrt{Z}\phi, \quad v \to v + \delta v$$

Motivation

δv appears in/is needed for:

- δv in loop calculations:

$$M_W = gv/2, \qquad \rightarrow \delta M_W = \delta g v/2 + g\delta v/2,$$
$$\tan\beta = \frac{v_u}{v_d}, \qquad \rightarrow \delta\tan\beta = \left(\frac{\delta v_u}{v_u} - \frac{\delta v_d}{v_d}\right)\tan\beta$$

- β_v, $\beta_{\tan\beta}$ in spectrum generators (Softsusy, SPheno, FlexibleSUSY, Sarah)

Aim: better understanding of δv, general computation

Could get δv from e.g. $\delta M_W \rightarrow$ no insight.

Details and questions

Most generic renormalization transformation:

$$(\phi + v) \to \sqrt{Z}\phi + v + \delta v$$

$$\text{or} \quad (\phi + v) \to \sqrt{Z}(\phi + v + \delta\bar{v})$$

Ultimately δv is important for $\delta \tan\beta$, β functions, etc.

$\delta\bar{v}$ characterizes to what extent v renormalizes differently from ϕ.

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Questions:

1. When/why $\delta\bar{v} \neq 0$?
2. Properties of $\delta\bar{v}$?
3. β_v and applications.

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3. β_v and applications.

Idea:

- $\delta\bar{v} = v\delta\hat{Z}$
- compute $\delta\hat{Z}$ (using STI)

Current status for RGE coefficients

needed by SUSY spectrum generators (Spheno, Softsusy, SuseFlav, FlexibleSUSY, Sarah)

Model	β(phys. parameter)	γ(fields)
$\forall$ gauge theory	✓[Machacek,Vaughn '83, Luo et al '03]	✓
$\forall$ SUSY model	✓[Martin, Vaughn; Jack, Jones; Yamada '93]	✓ partially

Note in SUSY: $\gamma(\text{ scalar in WZ gauge+Landau or } R_\xi \text{ gauge}) \neq \gamma(\text{ superfield}) \stackrel{?}{=} \gamma(\text{ light cone gauge})$

Model	$\beta_v^{(1)}$	$\beta_v^{(2)}$
MSSM	✓[Chankowski Nucl.Phys. B423]	✓[Yamada 94] $O(g^2 Y^2)$
E_6SSM	✓[Athron, DS, Voigt '12]	✗
$\forall$ gauge theory	?	✗
$\forall$ SUSY model	?	✗

Here: fill the gaps

Meaning of running v, alternative treatment

- Fix renormalization scale μ, renormalize in $\overline{\text{MS}}/\overline{\text{DR}}$-scheme
- adjust v such that tadpoles $\langle\phi\rangle = 0$

$\Rightarrow$ v= minimum of renormalized effective scalar potential at scale μ

- Change μ
- change parameters, including v, according to β functions
- all Green functions unchanged, including $\langle\phi\rangle = 0$

$\Rightarrow$ Minimum v of renormalized effective scalar potential is μ-dependent and gauge dependent $\Rightarrow$ not an observable

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Very different treatment of v possible,

e.g. [Jegerlehner, Kalmykov, Kniehl '13][Bednyakov, Pikelner, Velizhanin '13]:

- always define v_{bare} =Minimum of bare scalar potential
- then v_{bare}=abbreviation of combination of bare parameters
- In this scheme, $\delta v, \delta M_W, \delta \tan\beta$=gauge independent, but tadpoles are divergent (physical quantities unchanged)

Motivation

Questions:

1. When/why $\delta\bar{v} \neq 0$?
2. Properties of $\delta\bar{v}$?
3. β_v and applications.

Idea:

- $\delta\bar{v} = v\delta\hat{Z}$
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Personal motivation/more detailed questions:

Motivation

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Idea:

- $\delta\bar{v} = v\delta\hat{Z}$
- compute $\delta\hat{Z}$ (using STI)

Personal motivation/more detailed questions:

2001: A: "MSSM: $\frac{\delta\bar{v}_u}{v_u} - \frac{\delta\bar{v}_d}{v_d}$ =finite!"
B: "Why?"
A: ???

Motivation

Questions:

1. When/why $\delta\bar{v} \neq 0$?
2. Properties of $\delta\bar{v}$?
3. β_v and applications.

Idea:

- $\delta\bar{v} = v\delta\hat{Z}$
- compute $\delta\hat{Z}$ (using STI)

Personal motivation/more detailed questions:

2001: A: "MSSM: $\frac{\delta\bar{v}_u}{v_u} - \frac{\delta\bar{v}_d}{v_d}$ =finite!"
B: "Why?"
A: ???

SUSY non-renormalization theorems? only at one-loop? Later: also true at $\mathcal{O}(\alpha_s\alpha_{\text{top}})$ [Rzehak]

Motivation

Questions:

1. When/why $\delta\bar{v} \neq 0$?
2. Properties of $\delta\bar{v}$?
3. β_v and applications.

Idea:

- $\delta\bar{v} = v\delta\hat{Z}$
- compute $\delta\hat{Z}$ (using STI)

Personal motivation/more detailed questions:

2011: A: " Also true in 2HDM!"

Motivation

Questions:

1. When/why $\delta\bar{v} \neq 0$?
2. Properties of $\delta\bar{v}$?
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2011: C: "Why $\delta\bar{v} \neq 0$ at all?"

Motivation

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- $\delta\bar{v} = v\delta\hat{Z}$
- compute $\delta\hat{Z}$ (using STI)

Personal motivation/more detailed questions:

2011: C: "Why $\delta\bar{v} \neq 0$ at all?"
Answer: global gauge invariance broken by R_ξ gauge

$\rightarrow$ idea: take this seriously!

Idea as usual:

- get most general possible divergence from Slavnov-Taylor identity
- use BRS invariance
- Trick: extend BRS invariance for additional insight

QCD running in 5 loops

Konstantin Chetyrkin (KIT)

in collaboration to

Karlsruhe Institute of Technology

$\beta_{\rm QCD}$ **is expressed completely through Z-factors appearing in the (renormalized) QCD Lagrangian**

$$\begin{aligned}\mathcal{L}_R^{QCD} &= -\frac{1}{4} Z_3 \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2} g\, Z_1^{3g} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) \left(A_\mu \times A_\nu\right)^a - \frac{1}{4} g^2\, Z_1^{4g} \left(A_\mu \times A_\nu\right)^2 \\ &+ Z_3^c\, \partial_\nu \bar{c} \left(\partial_\nu c\right) + g\, Z_1^{ccg}\, \partial^\mu \bar{c} \left(A_\mu \times c\right) + Z_2\, \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi - Z_{\bar{\psi}\psi} m_f\, \bar{\psi}\psi + g Z_1^{\psi\psi g}\, \bar{\psi} A\!\!\!/ \psi\end{aligned}$$

Minimal (and simplest) set of Z-factors to compute β: $\boxed{Z_3, Z_3^c, Z_1^{ccg}}$ ← **used STI**

Let us concentrate on Z_1^{ccg} and consider vertex function

Abelian Higgs model = ϕ^4 + QED

$$\mathcal{L} = |D^\mu\phi|^2 - m^2|\phi+v|^2 - \lambda|\phi+v|^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\text{fix,gh}}$$

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi}F^2$$
$$F = \partial^\mu A_\mu - \xi ev(2\,\text{Im}\phi)$$

global transformation:

$$\delta_{\text{rigid}}\phi(x) = i\alpha(\phi(x)+v),$$

but $\quad \delta_{\text{rigid}}\mathcal{L} \neq 0$

Symmetry broken by $\mathcal{L}_{\text{fix}}$, **cannot conclude** $\delta_{\text{rigid}}\mathcal{L}_{\text{bare}} = 0$, hence

$$\phi + v \to \sqrt{Z_\phi}(\phi + v + \delta\bar{v})$$

is allowed and expected to be required!

Influence of global gauge invariance in a nutshell

When does $\delta\bar{v}$ appear?

$$\begin{aligned} \text{global gauge invariance} \quad &\Rightarrow \quad \delta\bar{v} = 0 \\ \text{no global gauge invariance} \quad &\Rightarrow \quad \delta\bar{v} \neq 0 \end{aligned}$$

R_ξ gauge fixing:

$$F = \partial^\mu A_\mu - \xi e v (2\,\mathrm{Im}\phi)$$

R_ξ breaks global gauge invariance for $\xi \neq 0 \quad \Rightarrow \quad \delta\bar{v} \neq 0$.

Investigation of δv in a nutshell

Problem: R_ξ breaks global gauge invariance for $\xi \neq 0 \Rightarrow \delta\bar{v} \neq 0$.

Trick: Keep global gauge invariance in intermediate steps! [Kraus,Sibold 95]

also [Kraus '97] [Hollik,Kraus,Roth,Rupp,Sibold,DS '02]

Introduce background field $\hat{\phi}(x)$, only at the end: $\hat{\phi}(x) = \hat{v}$ =const

$$\phi \to \phi_{\text{eff}} := \phi + \hat{\phi}$$

where $\hat{\phi}$ has same gauge transformation as ϕ.
Modified R_ξ gauge fixing:

$$F = \partial^\mu A_\mu + ie\xi(\hat{\phi}^\dagger\phi - \phi^\dagger\hat{\phi})$$

- global gauge invariance! $\Rightarrow \delta\bar{v} = 0$
- renormalization $\phi_{\text{eff}} \to \sqrt{Z}(\phi + \sqrt{\hat{Z}}\hat{\phi})$
- $\delta\bar{v}_{\text{eff}} = \hat{v}\delta\hat{Z}$, easy to compute using STI

Abelian Higgs model with background field $\phi_{\text{eff}} = \phi + \hat{\phi}$

$$\mathcal{L} = |D^\mu \phi_{\text{eff}}|^2 - m^2|\phi_{\text{eff}}|^2 - \lambda|\phi_{\text{eff}}|^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

$$\begin{aligned}
\mathcal{L}_{\text{fix,gh}} &= s\left[\bar{c}\left(\frac{\xi}{2}B + F\right)\right], \quad s\bar{c} = B, sB = 0 \\
F &= \partial^\mu A_\mu + ie\xi(\hat{\phi}^\dagger\phi - \phi^\dagger\hat{\phi}) \\
\mathcal{L}_{\text{ext}} &= K_\phi s\phi + K_{\phi^\dagger} s\phi^\dagger,
\end{aligned}$$

This $\mathcal{L}$ reproduces standard-$\mathcal{L}$ for $\hat{\phi} = \hat{v}$, $\hat{q} = 0$ and is invariant under:

Abelian Higgs model with background field $\phi_{\text{eff}} = \phi + \hat{\phi}$

$$\mathcal{L} = |D^\mu \phi_{\text{eff}}|^2 - m^2|\phi_{\text{eff}}|^2 - \lambda|\phi_{\text{eff}}|^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

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This $\mathcal{L}$ reproduces standard-$\mathcal{L}$ for $\hat{\phi} = \hat{v}$, $\hat{q} = 0$ and is invariant under:

global symmetry:

$$\delta_{\text{rigid}}\phi = i\alpha\phi$$

$$\delta_{\text{rigid}}\hat{\phi} = i\alpha\hat{\phi}$$

Abelian Higgs model with background field $\phi_{\text{eff}} = \phi + \hat{\phi}$

$$\mathcal{L} = |D^\mu \phi_{\text{eff}}|^2 - m^2 |\phi_{\text{eff}}|^2 - \lambda |\phi_{\text{eff}}|^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

$$\mathcal{L}_{\text{fix,gh}} = s\left[\bar{c}\left(\frac{\xi}{2}B + F\right)\right], \quad s\bar{c} = B, sB = 0$$

$$F = \partial^\mu A_\mu + ie\xi(\hat{\phi}^\dagger \phi - \phi^\dagger \hat{\phi})$$

$$\mathcal{L}_{\text{ext}} = K_\phi s\phi + K_{\phi^\dagger} s\phi^\dagger,$$

This $\mathcal{L}$ reproduces standard-$\mathcal{L}$ for $\hat{\phi} = \hat{v}$, $\hat{q} = 0$ and is invariant under:

BRS invariance:

$$s\phi_{\text{eff}} = c\delta_{\text{gauge}}\phi_{\text{eff}} = -iec\phi_{\text{eff}}$$

$$s\text{Rest} = \text{standard}$$

$$s^2 = 0$$

Abelian Higgs model with background field

Secondary trick: BRS transformation $\to$ control $\hat{\phi}$

$$s\hat{\phi} = \hat{q}, s\hat{q} = 0$$

this means that

$$s\phi_{\text{eff}} = -iec\phi_{\text{eff}}$$

requires:

$$s\phi = -iec\phi_{\text{eff}} - \hat{q}$$

hence:

$$\mathcal{L}_{\text{ext}} = \ldots + K_\phi \left(-iec(\phi + \hat{\phi}) - \hat{q}\right)$$

Abelian Higgs model with background field

As before: most general divergence structure, most general $\mathcal{L}_{\text{bare}}$ from

$$\delta_{\text{rigid}}\mathcal{L} = 0 \quad \Rightarrow \quad \delta_{\text{rigid}}\mathcal{L}_{\text{bare}} = 0$$

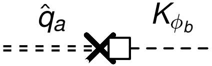
$$s\mathcal{L} = 0 \quad \Rightarrow \quad \int \frac{\delta\Gamma_{\text{bare}}}{\delta K_\varphi}\frac{\delta\Gamma_{\text{bare}}}{\delta\varphi} = 0$$

Result: $\mathcal{L}_{\text{bare}}$ generated by the renormalization transformation

$$\phi_{\text{eff}} = \phi + \hat{\phi} \to \sqrt{Z}\left(\phi + \sqrt{\hat{Z}}\hat{\phi}\right)$$

$$\hat{q} \to \sqrt{Z}\sqrt{\hat{Z}}\hat{q}$$

Crucial: $\mathcal{L}_{\text{bare}} = \ldots + K_\phi\left(-ie_{\text{bare}}c_{\text{bare}}(\phi + \sqrt{\hat{Z}}\hat{\phi}) - \sqrt{\hat{Z}}\hat{q}\right)$



$$= -\frac{i}{2}\delta\hat{Z}$$

Abelian Higgs model with background field

Compare with standard approach:

$$\phi + v \to \sqrt{Z}(\phi + v + \delta\bar{v})$$

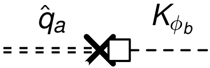
Here, for $\hat{\phi} = \hat{v}$, $\phi_{\text{eff}} = \phi + \hat{v} \Rightarrow$

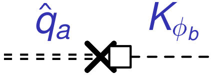
$$\phi + \hat{v} \to \sqrt{Z}(\phi + \sqrt{\hat{Z}}\hat{v})$$

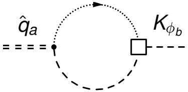
hence

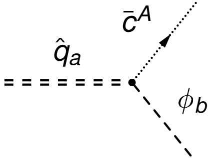
$$v + \delta\bar{v}\big|_{\text{standard}} = \sqrt{\hat{Z}}\hat{v}\Big|_{\text{here}}$$

What have we gained?

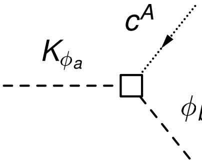
- $\delta\bar{v} \leftrightarrow$ dimensionless field renormalization constant
- $\hat{Z}$ can be directly obtained from 

Understanding of $\delta\bar{v}$ from  $= -\frac{i}{2}\delta\hat{Z}$

Very few Feynman rules for  with well-defined origin



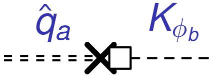
$= \xi e$

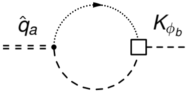


$= e$

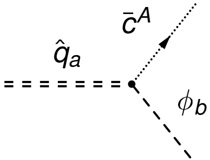
[breaking of global invariance$\propto \xi e(\hat{\phi}^\dagger \phi - \phi^\dagger \hat{\phi})$]

[BRS transform$\propto e$]

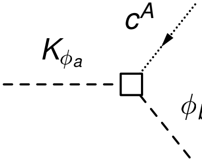
Understanding of $\delta\bar{v}$ from  $= -\frac{i}{2}\delta\hat{Z}$

Very few Feynman rules for  with well-defined origin

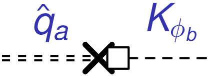
$$\mathcal{L}_{\text{ext}} = K_\phi \, s\phi + \cdots$$
$$= K_\phi \left(-iec(\phi + \hat{\phi})\right) - K_\phi \, \hat{q} + \cdots$$

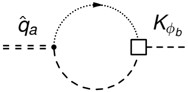


$= \xi e$



$= e$

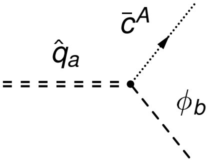
Understanding of $\delta\bar{v}$ from  $= -\frac{i}{2}\delta\hat{Z}$

Very few Feynman rules for  with well-defined origin

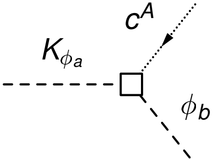
$$F = \partial^\mu A_\mu + ie\xi(\hat{\phi}^\dagger\phi - \phi^\dagger\hat{\phi})$$

$$\mathcal{L}_{\text{fix,gh}} = s\left[\bar{c}\,(F + \xi B/2)\right]$$

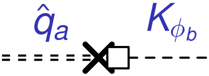
$$= -\bar{c}ie\xi(\hat{q}^\dagger\phi - \phi^\dagger\hat{q}) + \dots$$

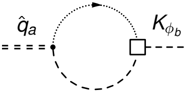


$= \xi e$

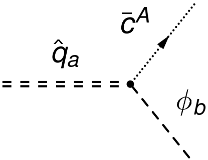


$= e$

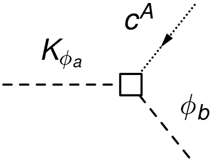
Understanding of $\delta\bar{v}$ from  $= -\frac{i}{2}\delta\hat{Z}$

Very few Feynman rules for  with well-defined origin

Feynman rules $\propto \hat{q}$ only from $\hat{\phi}$ in gauge fixing, vanish for $\xi = 0$, can depend only on gauge couplings (also in general models!)



$= \xi e$



$= e$

General model

Scalar, spinor, vector fields ϕ_a, ψ_p, V^A with Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{inv}}|_{\phi\to\phi_{\text{eff}}} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

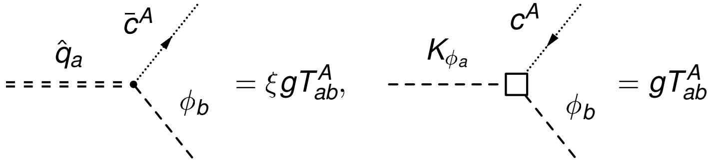
with

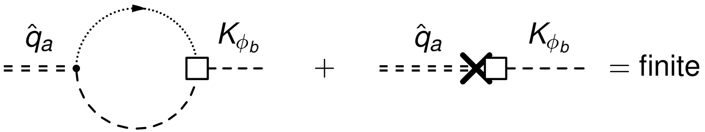
$$\begin{aligned}
\mathcal{L}_{\text{inv}} &= -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \frac{1}{2} (D_\mu \phi)_a (D^\mu \phi)_a + i \psi^\alpha_p \sigma^\mu_{\alpha\dot\alpha} \left(D^\dagger_\mu \bar\psi^{\dot\alpha} \right)_p \\
&\quad - \frac{1}{2!} m^2_{ab} \phi_a \phi_b - \frac{1}{3!} h_{abc} \phi_a \phi_b \phi_c - \frac{1}{4!} \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \\
&\quad - \frac{1}{2} \left[(m_f)_{pq} \psi^\alpha_p \psi_{q\alpha} + \text{h.c.} \right] - \frac{1}{2} \left[Y^a_{pq} \psi^\alpha_p \psi_{q\alpha} \phi_a + \text{h.c.} \right] \\
\mathcal{L}_{\text{fix,gh}} &= s \left[\bar c^A \left(F^A + \xi B^A/2 \right) \right] \\
\mathcal{L}_{\text{ext}} &= K_{\phi_a} s \phi_a + K_{V^A_\mu} s V^A_\mu + K_{c^A} s c^A + [K_{\psi_p} s \psi_p + \text{h.c.}]
\end{aligned}$$

General, modified R_ξ gauge fixing:

$$F^A = \partial^\mu V^A_\mu + i g \xi (\hat\phi)_a T^A_{ab} \phi_b$$

Calculation of $\delta\hat{Z}$ – 1 Loop



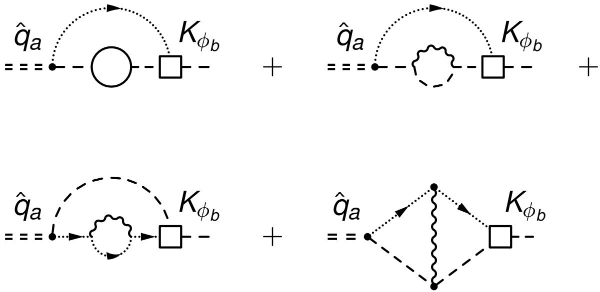


$\Rightarrow$

$$(4\pi)^2 \delta\hat{Z}^{(1)} = 2g^2 \xi C^2(S) \frac{1}{\epsilon}$$

vanishes for $\xi = 0$, only depends on squared gauge couplings

Calculation of $\delta\hat{Z}$ – 2 Loop

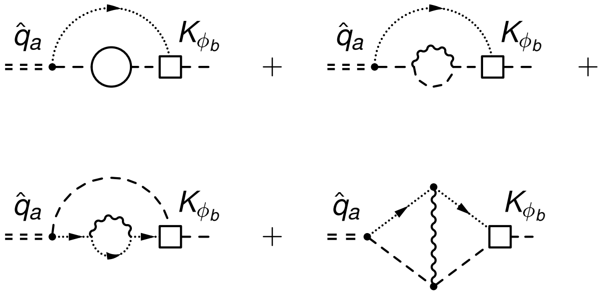


$\Rightarrow$

$$(4\pi)^4 \delta\hat{Z}^{(2)} = g^2 \xi C^2(S) Y^2(S) \left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \right) + O(g^4)$$

no Y^4, no λ terms (in contrast to δZ)

Calculation of $\delta\hat{Z}$ – 2 Loop



$\Rightarrow$

$$(4\pi)^4 \delta\hat{Z}^{(2)} = g^2 \xi C^2(\mathrm{S}) Y^2(\mathrm{S}) \left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right) + O(g^4)$$

no Y^4, no λ terms (in contrast to δZ)

Note alternative:

$$\frac{\delta m_t}{m_t} = \frac{\delta \quad t}{t} + \frac{\delta Z}{2} + \frac{\delta \hat{Z}}{2}$$

Results – β_v

$$v + \delta\bar{v} = \sqrt{\hat{Z}}\hat{v} \qquad \rightarrow \hat{\gamma}(S)$$

$$v + \delta v = \sqrt{Z}\sqrt{\hat{Z}}\hat{v} \qquad \rightarrow \gamma(S) + \hat{\gamma}(S)$$

$\Rightarrow$

$$\beta_v = [\gamma(S) + \hat{\gamma}(S)]\, v$$

$$\hat{\gamma}^{(1)}(\mathrm{S}) = \frac{\xi}{(4\pi)^2} 2g^2 C^2(\mathrm{S})$$

$$\hat{\gamma}^{(2)}(\mathrm{S}) = \frac{\xi}{(4\pi)^4}\left\{ g^4 \left[2\,(1+\xi)\, C^2(\mathrm{S})C^2(\mathrm{S}) + \frac{7-\xi}{2} C_2(\mathrm{G})C^2(\mathrm{S})\right] - 2g^2 C^2(\mathrm{S})\, Y^2(\mathrm{S}) \right\}$$

can now be implemented into spectrum generator (generators) Sarah [Staub], FlexibleSUSY

Results – $\beta_{\tan\beta}$ in the MSSM

$$\tan\beta = \frac{v_u}{v_d} \quad \Rightarrow \quad \frac{\beta_{\tan\beta}}{\tan\beta} = \gamma_u - \gamma_d + \hat{\gamma}_u - \hat{\gamma}_d$$

MSSM:

$$\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta} = \gamma^{(1)}_{uu} - \gamma^{(1)}_{dd} \qquad \leftarrow \text{cancellation of } \hat{\gamma} \text{ terms}$$

$$\frac{\beta^{(2)}_{\tan\beta}}{\tan\beta} = \gamma^{(2)}_{uu} - \gamma^{(2)}_{dd} + \frac{\xi}{(4\pi)^2}\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta}$$

[Yamada 02]

Understanding:

- $\frac{\delta\bar{v}_u}{v_u} - \frac{\delta\bar{v}_d}{v_d}$ =finite $\Leftrightarrow \hat{\gamma}_u - \hat{\gamma}_d = 0$
- Only true at one-loop because squared gauge couplings are equal
- similar in 2HDM, NMSSM

Results – $\beta_{\tan\beta}$ in the E_6SSM

E_6SSM: extra $U(1)_N$ gauge symmetry

$$\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta} = \gamma^{(1)}_{uu} - \gamma^{(1)}_{dd} + \frac{\xi}{(4\pi)^2} 2g_N^2 \left[\left(\frac{N_{H_u}}{2}\right)^2 - \left(\frac{N_{H_d}}{2}\right)^2 \right]$$ [Athron, DS, Voigt '12]

- $\frac{\delta\bar{v}_u}{v_u} - \frac{\delta\bar{v}_d}{v_d}$ =divergent $\Leftrightarrow \hat{\gamma}_u = \hat{\gamma}_d \neq 0$
- Already one-loop difference due to different $U(1)_N$-charges

General method of extended BRS/STI

Here: used $\hat{\phi}$, $s\hat{\phi} = \hat{q}$ to obtain information on $\delta\bar{v}$

Other possibilities

- $s\xi \neq 0 \Rightarrow \partial_\xi(\ldots)$ calculable [Kraus,Häussling '95][Grassi,Gambino '00][Freitas, DS '02]
- $se \neq 0 \Rightarrow \delta e^{\text{div}}$ calculable [Flume,Kraus; Kraus; Kraus, DS '01]

Outline

Conclusions

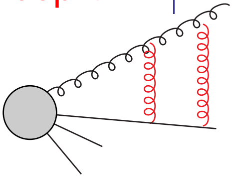
DREG, HV, FDH, DRED: all consistent schemes

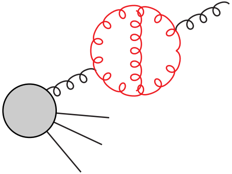
- old puzzles resolved
- DRED, FDH = DREG with additional ϵ-scalar parton
- IR structure and renormalization in FDH, DRED understood
- practical transition rules
- DRED supersymmetric in many (not all?) important cases

Divergence structure of δv (example of extended BRS/STI)

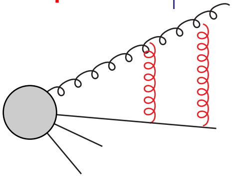
- if tadpoles=0: VEV is gauge- and μ-depedent quantity
- $\delta \bar{v}$ from gauge fixing, $\propto$ squared gauge couplings and $\propto \xi$
- $\rightarrow \delta \hat{Z}_{H_u} - \delta \hat{Z}_{H_d}$ =finite in MSSM at 1-loop accidentally
- 2-loop β functions, γ^{SUSY} complete

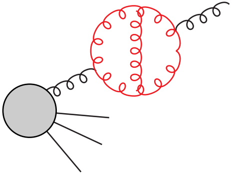
1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





Key insight: All IR divs $\longrightarrow$ factor **Z**, $\mathbf{Z}^{-1}|\mathcal{A}\rangle$ =fin.

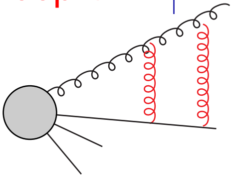
[Gardi, Magnea '09]
[Becher, Neubert '09]

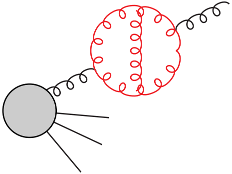
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

2nd insight [valid at least up to 2-loop order]

$$\Gamma = \sum_{(ij)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2}\, \gamma^{\text{cusp}}\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





Key insight: All IR divs $\longrightarrow$ factor $\mathbf{Z}$, $\mathbf{Z}^{-1}|\mathcal{A}\rangle$ =fin.

[Gardi, Magnea '09]
[Becher, Neubert '09]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

2nd insight [valid at least up to 2-loop order] μ-dependence of Γ in α_s and $\ln\mu$:

$$\Gamma = \left(\frac{\alpha_s}{4\pi}\right)^n \underset{\substack{\uparrow \\ \gamma^{\text{cusp}},\,\gamma^i}}{\Gamma_n} = \left(\frac{\alpha_s}{4\pi}\right)^n \left[\underset{\substack{\uparrow \\ \gamma^{\text{cusp}}}}{\Gamma'_n} \ln\mu + \Gamma''_n \right]$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- Integrate $\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$, use $\frac{d}{d\ln\mu^2}\alpha_s(\mu) = -\epsilon\alpha_s(\mu) + \beta^s_{20}\frac{\alpha_s^2}{4\pi} + \ldots$

1-loop CDR	1-loop FDH,DRED
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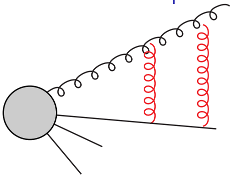
Result:

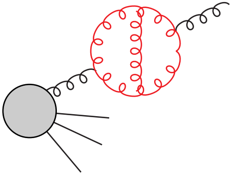
$$\ln\mathbf{Z} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\Gamma'_1}{4\epsilon^2} + \frac{\Gamma_1}{2\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\beta_{20}\Gamma'_1}{16\epsilon^3} + \frac{\Gamma'_2 - 4\beta_{20}\Gamma_1}{16\epsilon^2} + \frac{\Gamma_2}{4\epsilon}\right) + \ldots$$

Same three constants γ^{cusp}, γ^{q}, γ^{g} describe all IR divergences

1-loop agrees with previous result, at 2-loop also β needed

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED





Changes in FDH and DRED: Same structure …

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

…but the γ's change and everything depends on α_s, α_e, $\alpha_{4\epsilon}$

$$\Gamma = \sum_{(ij)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2}\, \gamma^{\text{cusp}}\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- Integrate $\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$, use β^s, β^e, etc

$$
\begin{aligned}
\ln \mathbf{Z}^{\text{FDH}}_{\text{2-loop}} = & \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_{20}\Gamma'_{10}}{16\epsilon^3} + \frac{\Gamma'_{20} - 4\beta_{20}\Gamma_{10}}{16\epsilon^2} + \frac{\Gamma_{20}}{4\epsilon}\right) \\
& + \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) \left(-\frac{3\beta^e_{11}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{11} - 4\beta^e_{11}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{11}}{4\epsilon}\right) \\
& + \left(\frac{\alpha_e}{4\pi}\right)^2 \left(-\frac{3\beta^e_{02}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{02} - 4\beta^e_{02}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{02}}{4\epsilon}\right) + \mathcal{O}(\alpha_{4\epsilon}, \alpha^3).
\end{aligned}
$$

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- Integrate $\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$, use β^s, β^e, etc

$$\ln \mathbf{Z}^{\text{FDH}}_{\text{2-loop}} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_{20}\Gamma'_{10}}{16\epsilon^3} + \frac{\Gamma'_{20} - 4\beta_{20}\Gamma_{10}}{16\epsilon^2} + \frac{\Gamma_{20}}{4\epsilon}\right)$$
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FDH: three constants γ^{cusp}, γ^{q}, γ^{g} = CDR-values $+\mathcal{O}(N_\epsilon)$

- need β^e, α_e-dependence separately
- Leads to translation rules between CDR, HV, FDH

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- Integrate $\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$, use β^s, β^e, etc

$$\ln \mathbf{Z}^{\text{FDH}}_{\text{2-loop}} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_{20}\Gamma'_{10}}{16\epsilon^3} + \frac{\Gamma'_{20} - 4\beta_{20}\Gamma_{10}}{16\epsilon^2} + \frac{\Gamma_{20}}{4\epsilon}\right)$$
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FDH: three constants γ^{cusp}, γ^{q}, γ^{g} = CDR-values $+\mathcal{O}(N_\epsilon)$

- Logic and result fully compatible with [Kilgore '12]
- Differences to [Kilgore '12]: slightly different γ^i, other sample processes

1-loop CDR	1-loop FDH,DRED
2-loop CDR	2-loop FDH,DRED

- Integrate $\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$, use β^s, β^e, etc

$$\ln \mathbf{Z}^{\mathrm{FDH}}_{\text{2-loop}} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_{20}\Gamma'_{10}}{16\epsilon^3} + \frac{\Gamma'_{20} - 4\beta_{20}\Gamma_{10}}{16\epsilon^2} + \frac{\Gamma_{20}}{4\epsilon}\right)$$
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DRED: again same structure but four constants γ^{cusp}, γ^{q}, $\gamma^{\hat{g}}$, $\gamma^{\tilde{g}}$

- turns out to depend even on $\alpha_{4\epsilon}$

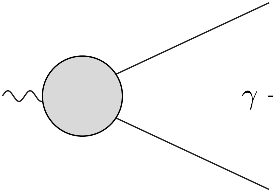
IR structure in all four schemes

$\Updownarrow$

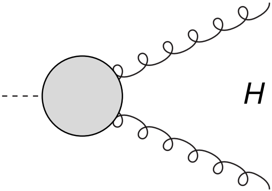
common structure, depends on anomalous dimensions γ and β functions of all couplings

Allows transition rules between schemes, once all γ's are known

Two simple processes/form factors

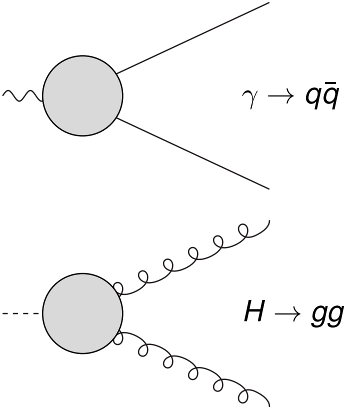


$\gamma \to q\bar{q}$



$H \to gg$

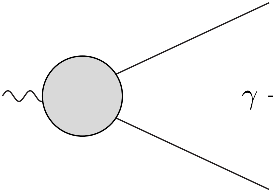
Two simple processes/form factors



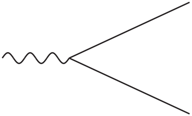
- IR structure, transition rules
- determine γ's
- study concrete calculation/renormalization

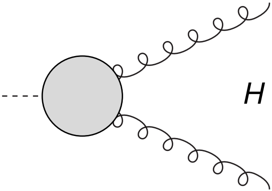
Two simple processes/form factors

Simple kinematics



$\gamma \to q\bar{q} \;=\; F^q(s)$





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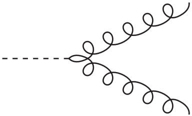
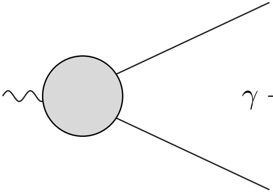


Two simple processes/form factors

Simple kinematics



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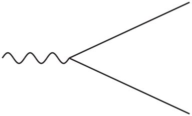
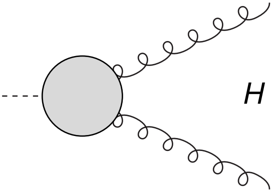
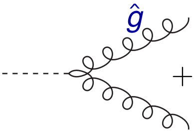
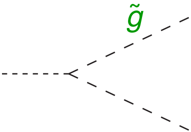




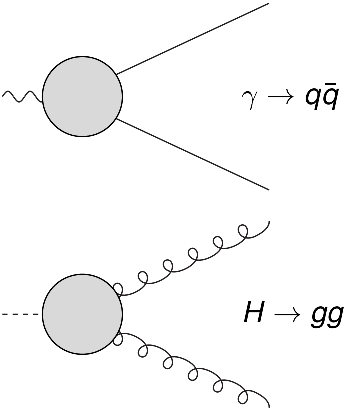
$H \to gg \;=\; F^{\hat{g}}(s)$



$+F^{\tilde{g}}(s)$



Two simple processes/form factors

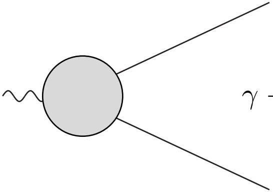


Simple colour/IR structure:

$$\Gamma' = -2C_{F,A}\gamma^{\text{cusp}}$$

$$\Gamma = 2\gamma^{i}(i = q, g, \hat{g}, \tilde{g})$$

Two simple processes/form factors



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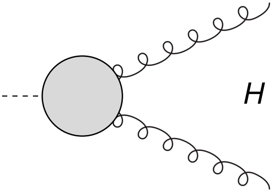


$H \to gg$

Simple colour/IR structure:

$$\Gamma' = -2C_{F,A}\gamma^{\text{cusp}}$$
$$\Gamma = 2\gamma^{i}(i = q, g, \hat{g}, \tilde{g})$$

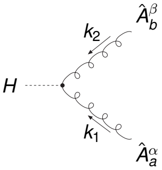
$$\mathbf{Z}^{-1}F = \text{fin.}$$
$$\ln F - \ln \mathbf{Z} = \text{fin.}$$
$$\left[F^{2\text{L}} - \frac{1}{2}(F^{1\text{L}})^2\right] - \ln \mathbf{Z}\Big|^{2\text{L}} = \text{fin.}$$

Example: Hgg in FDH/DRED. Starting point: gauge invariant $\mathcal{L}_{\text{eff}}$

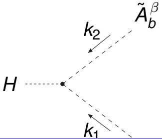
$$\mathcal{L}_{\text{eff}} = \lambda H O_1 + \lambda_\epsilon H \tilde{O}_1 + \ldots$$

with

$$O_1 = -\frac{1}{4} \hat{F}^{\mu\nu} \hat{F}_{\mu\nu}, \qquad \tilde{O}_1 = -\frac{1}{2} \left(\hat{D}^\mu \tilde{A}^\nu\right) \left(\hat{D}_\mu \tilde{A}_\nu\right).$$

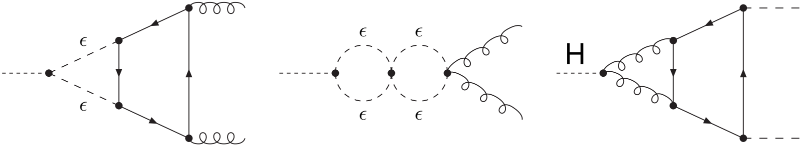


$$= \quad i\lambda \left[(k_1 \cdot k_2)\, \hat{g}^{\alpha\beta} - k_1^\beta\, k_2^\alpha \right] \delta^{ab}$$



$$= \quad i\lambda_\epsilon \left[(k_1 \cdot k_2)\, \tilde{g}^{\alpha\beta} \right] \delta^{ab}$$

Need two-loop diagrams and counterterm diagrams such as



- all couplings λ, λ_ϵ, α_s, α_e, $\alpha_{4\epsilon}$ appear
- Need also: all one-loop renormalization constants for these couplings
- Need also: two-loop $\delta\lambda$, $\delta\lambda_\epsilon$

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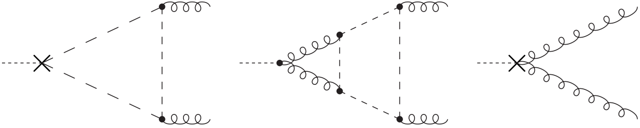

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Problem: renormalization of $\mathcal{L}_{\text{eff}}$, λ, λ_ϵ

Situation in CDR, HV:

- $\frac{\lambda+\delta\lambda}{\lambda} = 1 + \alpha_s \frac{\partial}{\partial\alpha_s} \ln Z_{\alpha_s}$ [Spiridonov '84, Chetyrkin, Kniehl, Steinhauser '97]
- known from literature, no dedicated computation required

Situation in FDH, DRED:

Problem: renormalization of $\mathcal{L}_{\text{eff}}$, λ, λ_ϵ

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- Reason: full basis of local operators $O_{1\ldots5}$ can all be written as $\frac{\partial}{\partial \text{fields,parameters}} \mathcal{L}_{\text{QCD}}$

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Situation in FDH, DRED:

- e.g. $\Box \tilde{A}^\mu \tilde{A}_\mu$ exists in operator basis
- has no counterpart in $\mathcal{L}_{\text{QCD}}$ $\Rightarrow$ Spiridonov's method fails

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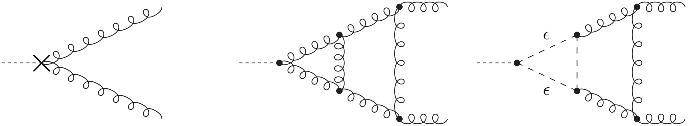
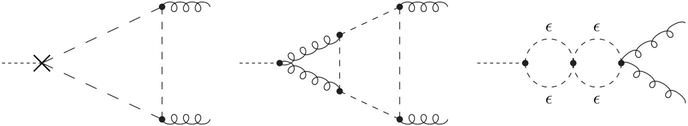
- e.g. $\Box \tilde{A}^\mu \tilde{A}_\mu$ exists in operator basis
- has no counterpart in $\mathcal{L}_{\text{QCD}}$ $\Rightarrow$ Spiridonov's method fails
- way out: determine operator mixing renormalization constants from explicit off-shell calculations (one- and two-loop)
- off-shell means: non-gauge invariant operators also needed

Hgg in FDH: Sample results



$$\delta\lambda^{(2)} = \lambda\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big(\frac{(\beta^s_{20})^2}{\epsilon^2} - \frac{\beta^s_{30}}{\epsilon}\Big) + \Big(\lambda + \lambda_\epsilon\Big)\Big(\frac{\alpha_s}{4\pi}\Big)\Big(\frac{\alpha_e}{4\pi}\Big)\Big(-\frac{\beta^s_{21}}{2\epsilon}\Big)$$

[Spiridonov '84] applicable, but mixing occurs



$$\delta\lambda^{(1)}_\epsilon = \lambda_\epsilon\Big[\Big(\frac{\alpha_s}{4\pi}\Big)\Big(-\frac{3C_A}{\epsilon}\Big) + \Big(\frac{\alpha_e}{4\pi}\Big)\frac{N_F}{\epsilon} + \Big(\frac{\alpha_{4\epsilon}}{4\pi}\Big)C_A\Big(\frac{-1+N_\epsilon}{\epsilon}\Big)\Big]$$

explicit calculation

Hgg in FDH: Result for $\bar{G}^{(2)} = F^{g,2\mathrm{L}} - \frac{1}{2}(F^{g,1\mathrm{L}})^2$

$$\bar{G}^{(2)}(\alpha_s,\alpha_e) = G^{(2),\mathrm{CDR}}(\alpha_s) \quad + \left(\frac{\alpha_s}{4\pi}\right)^2 \; N_\epsilon \; \left\{ C_A^2 \left[-\frac{1}{4\epsilon^3} + \frac{-\frac{7}{18}+\frac{N_\epsilon}{72}}{\epsilon^2} + \frac{\frac{49}{27}-\frac{\pi^2}{72}}{\epsilon} \right] + C_A N_F \frac{1}{9\epsilon^2} \right\}$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon} \right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

- IR prediction was: $\left[G^{(2)} - \ln \mathbf{Z}\big|^{2\mathrm{L}}\right]$=fin. for both FDH and CDR

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$$+ \left(\tfrac{\alpha_s}{4\pi}\right)\left(\tfrac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon} \right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

- IR prediction was: $\left[G^{(2)} - \ln \mathbf{Z}\right|^{2\text{L}}\Big]$=fin. for both FDH and CDR
- This is valid!
- can read off e.g. $\bar{\gamma}^g_{20} = \gamma^g_{20} + N_\epsilon\left(\frac{98}{27} - \frac{\pi^2}{36}\right) C_A^2$

Hgg in FDH: Result for $\bar{G}^{(2)} = F^{g,2\text{L}} - \frac{1}{2}(F^{g,1\text{L}})^2$

$$\bar{G}^{(2)}(\alpha_s,\alpha_e) = G^{(2),\text{CDR}}(\alpha_s) \quad + \left(\tfrac{\alpha_s}{4\pi}\right)^2 \; N_\epsilon \;\left\{ C_A^2\left[-\frac{1}{4\epsilon^3} + \frac{-\frac{7}{18}+\frac{N_\epsilon}{72}}{\epsilon^2} + \frac{\frac{49}{27}-\frac{\pi^2}{72}}{\epsilon}\right] + C_A N_F \frac{1}{9\epsilon^2}\right\}$$
$$+ \left(\tfrac{\alpha_s}{4\pi}\right)\left(\tfrac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon}\right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

- IR prediction was: $\left[G^{(2)} - \ln \mathbf{Z}|^{2\text{L}}\right]$=fin. for both FDH and CDR
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- after subtracting IR divs, the difference is $\mathcal{O}\left(N_\epsilon \epsilon^0\right)$
- $\Rightarrow$ translation rules

Hgg in FDH: Result for $\bar{G}^{(2)} = F^{g,2L} - \frac{1}{2}(F^{g,1L})^2$

$$\bar{G}^{(2)}(\alpha_s,\alpha_e) = G^{(2),\text{CDR}}(\alpha_s) \quad + \left(\frac{\alpha_s}{4\pi}\right)^2 N_\epsilon \left\{ C_A^2\left[-\frac{1}{4\epsilon^3} + \frac{-\frac{7}{18}+\frac{N_\epsilon}{72}}{\epsilon^2} + \frac{\frac{49}{27}-\frac{\pi^2}{72}}{\epsilon}\right] + C_A N_F \frac{1}{9\epsilon^2}\right\}$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon}\right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

- IR prediction was: $\left[G^{(2)} - \ln \mathbf{Z}|^{2L}\right]$=fin. for both FDH and CDR
- This is valid!
- can read off e.g. $\bar{\gamma}^g_{20} = \gamma^g_{20} + N_\epsilon \left(\frac{98}{27} - \frac{\pi^2}{36}\right) C_A^2$
- after subtracting IR divs, the difference is $\mathcal{O}\left(N_\epsilon \epsilon^0\right)$
- $\Rightarrow$ translation rules
- Similarly, obtain all two-loop F^q, F^g, γ's in all schemes

$$\gamma_{\text{cusp}}^{\text{FDH,DRED}}, \gamma_q^{\text{FDH,DRED}}, \gamma_g^{\text{FDH}}, \gamma_{\hat{g}}^{\text{DRED}}, \gamma_{\tilde{g}}^{\text{DRED}}$$