Lecture 3

Building Blocks $\quad \chi_n = W_n^\dagger \xi_n$

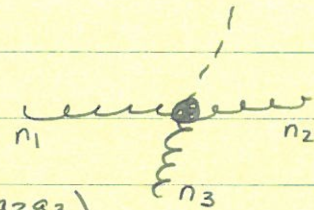
$$T^a \mathcal{B}_{n\perp}^{\mu a} = \frac{1}{g}\left[W_n^\dagger \, i D_{n\perp}^{\mu} W_n\right], \qquad \mathcal{B}_{n\perp}^{\mu a} = \frac{\bar n_\nu}{i\bar n\cdot\partial_n}\, i G_n^{B\nu\mu}\, \mathcal{W}_n^{BA}$$

↑ adjoint Wilson line

add to Lecture 1 Picture

$$\left.\begin{matrix} P_\perp^\mu \\ q_s\,,\ A_s^\mu \end{matrix}\right\} \text{often suppressed}$$

eg $pp \to H + 1$-jet, remove top eg.



Ampl

- $\mathcal{B}^{a_1\mu_1}_{n_1\perp}\,\mathcal{B}^{a_2\mu_2}_{n_2\perp}\,\mathcal{B}^{a_3\mu_3}_{n_3\perp}\, H\, \underbrace{T_{\mu_1\mu_2\mu_3}}_{\text{how many}}\, (if^{a_1a_2a_3})$

↑ do $d^{a_1a_2a_3}$ by charge conjugation

[• $\mathcal{B}^{a_1\mu_1}_{\perp n_1}\,\bar\chi^{\alpha}_{n_2}\,\Gamma^{\mu_2}\chi^{\beta}_{n_3}\, H\, T^{\Gamma}_{\mu_1\mu_2}\, T^{a_1}_{\alpha\beta}$]

Counting Easy with] helicity basis

$$\mathcal{B}^a_{n\pm} \equiv -\epsilon^{\mu}_{\mp}(n,\bar n)\,\mathcal{B}^{\perp}_{n\mu}, \qquad \epsilon_\pm(n,\bar n) = \frac{1}{\sqrt2}(0,1,\mp i,0)$$

[Similar $\quad J^{\alpha\beta}_{n_1n_2\pm} \propto \epsilon^{\mu}_{\mp}(n_1,n_2)\,\bar\chi^{\alpha}_{n_1\pm}\gamma_\mu \underbrace{\chi^{\beta}_{n_2\pm}}_{\left(\frac{1\pm\gamma_5}{2}\right)\chi_n}$]

Allowed

$\mathcal{B}\mathcal{B}\mathcal{B}H$		$\mathcal{B}JH$	
$+\,+\,+$		$+\,+$	
$+\,+\,-$		$+\,-$	
$-\,-\,+$	c's Fixed by Parity	$-\,+$	fixed by charge conj.
$-\,-\,-$		$-\,-$	

4 non-trivial Wilson coefficients C for SCET

[see arXiv: 1508.02397 for more on helicity ops in SCET]

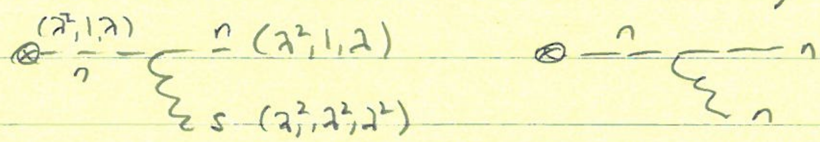
$\boxed{\text{SCET } \mathcal{L}}$ ($\alpha = 2$ case $\quad p_s^\mu \sim Q\lambda^2$)

For isolated purely n-collinear or purely (ultra)soft we have full QCD $\mathcal{L}$ for each sector

soft: nothing to expand $\qquad$ n-collinear: boost everything $\overset{+\ -\ \perp}{(\lambda^2, 1, \lambda)} \to \overset{+\ -\ \perp}{(\lambda, \lambda, \lambda)}$ same

Nontrivial interactions btwn sectors

$\mathcal{L}^{(0)}$ note ⊙ usoft & n-collinear interact leaving ~ on-shell



⊙ hard interactions produce ξ_n satisfying $\not{n}\,\xi_n = 0$
[hard interaction breaks boost arguement]

$$\psi = \Big(\frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4}\Big)\psi = \underset{\lambda}{\xi_n} + \varphi_n \qquad [\text{decompose } \mathcal{L}_{QCD}]$$

$$\mathcal{L}_{QCD} = \bar\psi\, i\not{D}\,\psi = \bar\xi_n \frac{\bar{\not{n}}}{2}\, in\cdot D\, \xi_n + \bar\varphi_n \frac{\not{n}}{2}\, i\bar n\cdot D\, \varphi_n + \bar\xi_n\, i\not{D}_\perp \varphi_n + \bar\varphi_n\, i\not{D}_\perp \xi_n$$

e.o.m $\delta/\delta\bar\varphi_n \Rightarrow$

$$\varphi_{\bar n} = \underset{\sim\lambda^2\text{-small}}{\frac{1}{i\bar n\cdot D}}\, \underset{\lambda}{i\not{D}_\perp}\, \frac{\bar{\not{n}}}{2}\, \underset{\lambda}{\xi_n}$$

smaller than ξ_n for hard production

$$\mathcal{L}_{QCD} = \bar\xi_n \Big(in\cdot D + i\not{D}_\perp \frac{1}{i\bar n\cdot D} i\not{D}_\perp\Big) \frac{\bar{\not{n}}}{2}\, \xi_n \qquad [\text{still QCD}]$$

Expand:
- couple only to ξ_n in path integral $J\xi_n$
- $in\cdot D = \underset{\lambda^2}{in\cdot\partial} + \underset{\lambda^2}{g n\cdot A_n} + \underset{\lambda^2}{g n\cdot A_s}$

$\quad iD_\perp = \underset{\lambda}{i\partial_n^\perp} + \underset{\lambda}{gA_{n\perp}} + \cdots \qquad A_{us}^\perp \ll A_{n\perp}$, $\ i\partial_{us}^\perp \ll i\partial_n^\perp$

$\quad i\bar n\cdot D = \underset{\lambda^0}{i\bar n\cdot\partial_n} + \underset{\lambda^0}{g\bar n\cdot A_n} + \text{---} \qquad$ etc

multipole expansion

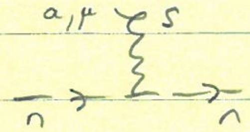
$$\mathcal{L}_{n\xi}^{(0)} = \bar\xi_n \Big(in\cdot D + i\not{D}_{n\perp} \frac{1}{i\bar n\cdot D_n} i\not{D}_{n\perp}\Big) \frac{\bar{\not{n}}}{2}\, \xi_n$$

for gluons $\quad \mathcal{L}_{ng}^{(0)} = \mathcal{L}_{ng}^{(0)}[n\cdot D, D_{n\perp}, \bar n\cdot D_n]$ too $+ \Big[\begin{matrix}\text{gauge fixing} \\ + \\ \text{ghosts}\end{matrix}\Big]$

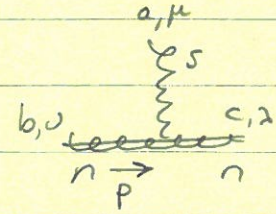
$$\mathcal{L}^{(0)}_{SCET_I} = \mathcal{L}^{(0)}_{us} + \sum_n \left(\mathcal{L}^{(0)}_{nq} + \mathcal{L}^{(0)}_{ng} \right) + \mathcal{L}^{(0)}_{Glauber}$$

must cancel for factorization (Lecture #4)

Mixed Feyn Rules



$= i g T^a \frac{\bar{n}\!\!\!/}{2} n^\mu$

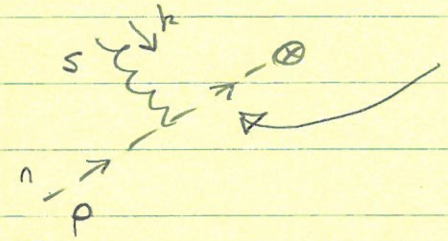


$= g f^{abc} n^\mu \bar{n}\cdot p_n g^{\nu\lambda}$

[Feyn. Gauge for collin]

softs have eikonal coupling $\propto n_\mu$ to collinears

too

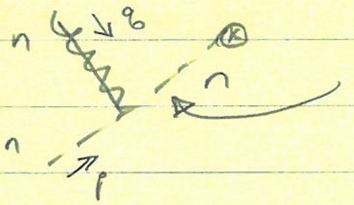


$$\propto \frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot(k+p) + p_\perp^2 + i0} = \frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot k + p^2 + i0} = \frac{1}{n\cdot k + i0}$$

on-shell $p^2 = 0$

eikonal propagator

[softs do not change p_n^+, $\bar{n}\cdot p_n$
neither soft or collinear can change direction n]



$$\propto \frac{\bar{n}\cdot(p+q)}{(p+q)^2 + i0}$$

Ultrasoft - Collinear Factorization

put $n\cdot A_s$ into Soft Wilson lines

$$Y_n(x) = P \exp\left(i g \int_{-\infty}^{0} ds\, n\cdot A_s(x+ns) \right)$$

$[n\cdot D_s Y_n] = 0$

$Y_n^\dagger Y_n = 1 = Y_n Y_n^\dagger$

by using field redefinition : $\xi_n(x) = Y_n(x)\, \xi_n'(x)$

$A_n(x) = Y_n(x)\, A_n'(x)\, Y_n^\dagger(x)$

$W_n = \sum_{\text{perms}} \exp\left(\frac{-g}{i\bar{n}\cdot\partial_n} \bar{n}\cdot A_n \right) \to Y_n W_n' Y_n^\dagger$ [same for ghost]

$\chi_n \to Y_n \chi_n'$,

$\mathcal{B}_{n\perp} \to Y_n \mathcal{B}_{n\perp}' Y_n^\dagger$

$$\mathcal{L}_{nq}^{(0)} = \bar{\xi}_n' \frac{\bar{n}\!\!\!/}{2}\left[Y^\dagger\, i n\cdot D_s\, Y + Y^\dagger \left(Y g\, n\cdot A_n' Y^\dagger \right) Y + \cdots \right] \xi_n'$$

$$= \bar{\xi}_n' \frac{\bar{n}\!\!\!/}{2}\left[i n\cdot\partial + g\, n\cdot A_n' + i D\!\!\!\!/_{n\perp}' \frac{1}{i\bar{n}\cdot D_n'}\, i D\!\!\!\!/_{n\perp}' \right] \xi_n'$$

Same for $\mathcal{L}_{ng}^{(0)}$, so purely n-collinear

More on Homework

Reappear in currents

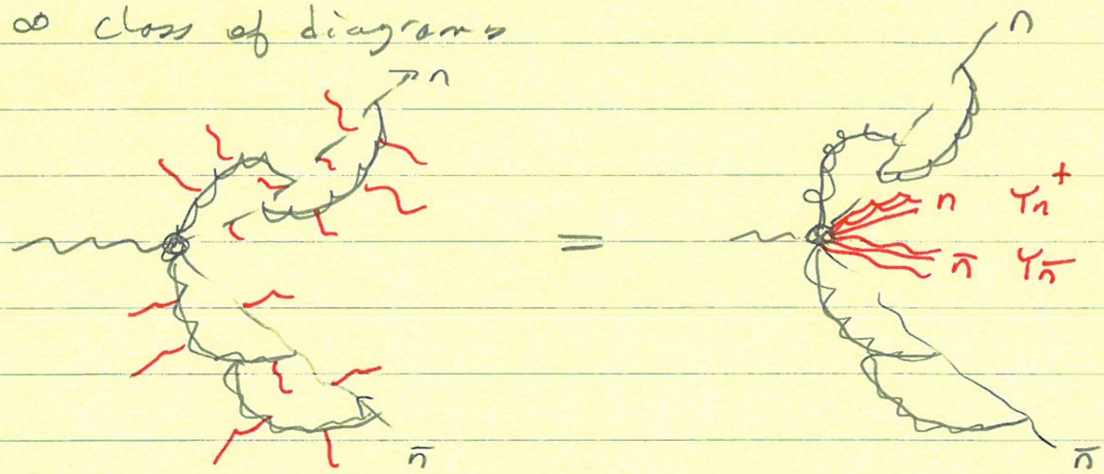
eg 1 $\quad (\bar{\chi}_n \Gamma \chi_{\bar{n}}) \to \bar{\chi}_n' \left(Y_n^\dagger Y_{\bar{n}} \right) \Gamma \chi_{\bar{n}}'$

(n-collin) (soft) ($\bar{n}$-collin)

factored up to global color & spin indices

eg 2 $\quad (\bar{\chi}_n \Gamma \chi_n) \to \bar{\chi}_n' \cancel{Y_n^\dagger}\, \Gamma\, \cancel{Y_n}\, \chi_n'$ cancel here

Sums ∞ class of diagrams



[Similar things happen in $SCET_{II}$]

Renormalization Group Evolution (& Matching)

UV renormalization of SCET

$\frac{1}{\epsilon}$ divergences in dim. reg

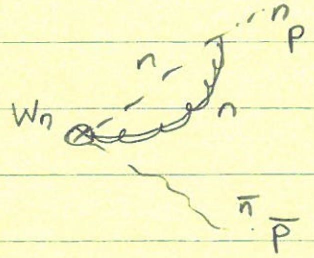
$d = 4 - 2\epsilon$

compare renormalized QCD & SCET to extract C's (not covered here)

Example use Feyn. Gauge $p_c^2 \neq 0$ IR regulator

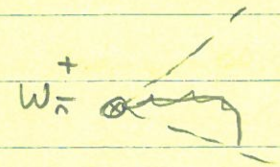
$e^+e^- \to$ dijets $\qquad \bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}} = (\bar{\xi}_n W_n)\gamma_\perp^\mu (W_{\bar{n}}^\dagger \xi_{\bar{n}})$

$C_F = 4/3$



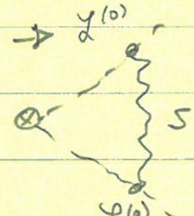
$$= \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\left(\frac{\cancel{-p^2}}{\mu^2}\right) + \cdots\right]$$

from $\int \frac{d^dk\ \bar{n}\cdot(k+p)}{\bar{n}\cdot k\,(k+p)^2 k^2}$ see SCET notes



$= \text{same } p^2 \to \bar{p}^2$

pre-field redef'n $\to Y^{(0)}$



$$= \frac{\alpha_s C_F}{4\pi}\left[\frac{-2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{(\cancel{-p^2})(-\bar{p}^2)}{\mu^2(-Q^2)}\right) + \cdots\right]$$

$$\int \frac{d^dk\ n\cdot\bar{n}}{\left(n\cdot k + \frac{p^2}{Q}\right)\left(\bar{n}\cdot k + \frac{\bar{p}^2}{Q}\right)k^2}$$

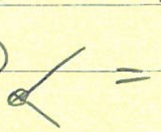
$\bar{n}\cdot p = n\cdot\bar{p} = Q$

$$= -\frac{\alpha_s C_F}{4\pi}\,\frac{1}{\epsilon}$$

Sum: $\frac{1}{\epsilon}\ln(-p^2)$ & $\frac{1}{\epsilon}\ln(-\bar{p}^2)$ cancel out†, $\ln^2(-p^2), \ln(-p^2), \ldots$ agree with QCD

$$\text{Sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \cdots\right], \text{ so}$$

$(Z_C - 1)$



$$= \frac{-\alpha_s C_F}{4\pi}\left[\ '' \ + \ '' \ + \ '' \ \right]$$ is counterterm diagram

from $\mathcal{L}^{hard} = \underbrace{Z_C\, C(\mu)}_{C^{bare}}\, \mathcal{O} = C(\mu)\,\mathcal{O} + (Z_C - 1)\, C(\mu)\,\mathcal{O}$

$$0 = \mu\frac{d}{d\mu} C^{bare} = \mu\frac{d}{d\mu}\left[Z_C(\mu,\epsilon)\, C(\mu)\right] = \left[\mu\frac{d}{d\mu} Z_C\right] C + Z_C\left[\mu\frac{d}{d\mu} C\right]$$

$$\mu\frac{d}{d\mu} C(\mu) = \underbrace{\left[- Z_C^{-1}(\mu,\epsilon)\, \mu\frac{d}{d\mu} Z_C(\mu,\epsilon)\right]}_{\equiv\, \gamma_C} C(\mu)$$

$\mathcal{O}(\alpha_s)$: $Z_C^{-1} \to 1$, use $\mu\frac{d}{d\mu}\alpha_s = -2\epsilon\,\alpha_s + \mathcal{O}(\beta_0\,\alpha_s^2)$

$$\left[\begin{aligned} \mu\frac{d}{d\mu} Z_C &= \frac{C_F}{4\pi}\alpha_s(-2\epsilon)\left(\cancel{-\frac{2}{\epsilon^2}} - \frac{2}{\epsilon}\ln\frac{\mu^2}{-Q^2} - \frac{3}{\epsilon}\right) \\ &\quad + \frac{C_F}{4\pi}\alpha_s\left(\cancel{-4/\epsilon}\right) \end{aligned}\right]$$

$\Big\{$ $\alpha_s^{bare} = \mu^{2\epsilon} Z_{\alpha_s}\, \alpha_s(\mu)$ implies this

from $\mu\frac{d}{d\mu}\ln\mu^2 = 2$

$$\gamma_C = -\frac{\alpha_s(\mu)}{4\pi}\left[4 C_F \ln\frac{\mu^2}{-Q^2} + 6 C_F\right] \qquad \text{finite}$$

Squared Amplitude gives $H = |C(Q,\mu)|^2$ <u>hard function</u>

$$\begin{aligned} \mu\frac{d}{d\mu} H(Q,\mu) &= (\gamma_C + \gamma_C^*)\, H(Q,\mu) \\ &= -\frac{\alpha_s(\mu)}{2\pi}\left[8 C_F \ln\frac{\mu}{Q} + 6 C_F\right] H(Q,\mu) \end{aligned}$$

$$\Rightarrow \quad H(Q,\mu_1) = \exp\left[-\#\,\alpha_s \ln^2\!\left(\frac{\mu_1}{Q}\right) + \cdots\right] H(Q,\mu_0)$$

$U(\mu_1,\mu_0)$

frozen α_s result

running coupling & bndry condition terms, rewrite $-\frac{\#}{\alpha_s(\mu_0)} f\left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right) + \cdots$

$\mu_0 \sim Q$ bndry condition

Sudakov Form Factor

$\boxed{\text{Details on Hmwk}}$

$\Rightarrow$ $\bar{\chi}_n \Gamma \chi_{\bar{n}}$ SCET operator restricts radiation

Coeffient $|C|^2$ encodes no emission probability for $\mu < Q$

Factorization Examples

$$\mathcal{L}^{(0)} = \mathcal{L}_s^{(0)} + \sum_n \mathcal{L}_n^{(0)}$$

decoupled

$$\mathcal{O}^{(0)} = \prod_n O_n^{(0)} \quad \text{decoupled}$$

Decouple Measurement

$$e = e_s + \sum_n e_n$$

eg. $e^+e^- \to$ dijets , hemisphere masses $M_a^2 \mid M_b^2$

$$J^\mu = C \, \bar{\chi}_n \gamma_\perp^\mu (Y_n^+ Y_{\bar{n}}) \chi_{\bar{n}}$$

$$\frac{d\sigma}{dM_a^2 dM_b^2} = \cdots = \sigma_0 |C|^2 \int ds_n ds_{\bar{n}} d\ell^+ d\ell^- \, \delta(M_a^2 - s_n - Q\ell^+) \, \delta(M_b^2 - s_{\bar{n}} - Q\ell^-)$$

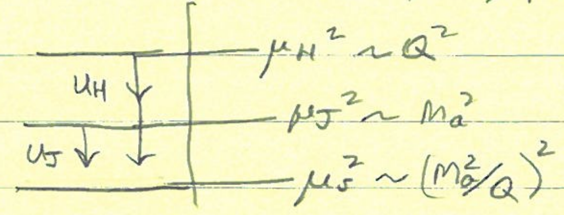
$$* \, \mathrm{Im}\Big[\text{----} \langle 0 | T \, \underbrace{\bar{\chi}_{n,Q}(0) \, \Gamma \, \chi_n(x) | 0 \rangle}_{\text{jet function } J(s_n)} \text{---} \Big]$$

$$* \, \mathrm{Im}\Big[\text{-----} \langle 0 | T \, \underbrace{\bar{\chi}_{\bar{n},Q}(0) \, \Gamma \, \chi_{\bar{n}}(y) | 0 \rangle}_{J(s_{\bar{n}})} \text{---} \Big]$$

$$* \sum_{X_s} \underbrace{\text{-------}}_{\text{soft function } S(\ell^+, \ell^-)} \langle 0 | Y_n^+ Y_{\bar{n}} | X_s \rangle \langle X_s | Y_{\bar{n}}^+ Y_n | 0 \rangle \Big]$$

$$= \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^- \, J(M_a^2 - Q\ell^+, \mu) \, J(M_b^2 - Q\ell^-, \mu) \, S(\ell^+, \ell^-, \mu)$$

Sum logs $\alpha_s^k \ln^j\left(\frac{M_{a,b}^2}{Q^2}\right)$



$$\left.\begin{array}{l} H(Q, \mu_H) \\ J(s, \mu_J) \\ S(\ell^+, \ell^-, \mu_s) \end{array}\right\} \text{ no large logs}$$

RGE sums large logs L

$$\int_0^{m_{cut}} d\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \cdots \quad \} \text{ LL series}$$

$$+ \alpha_s L + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \cdots \quad \} \text{ NLL series}$$

$$+ \alpha_s + \cdots \quad \cdots$$

$$\vdots \qquad \vdots$$

final eg. Inclusive Higgs Production

$pp \to H + \text{any } X$

$$\sigma(m_H^2) \;\propto\; \sum_X \langle pp|\, \underbrace{G^2 H}_{\text{after int. out top}} \,|H+X\rangle\langle H+X|\, G^2 H\,|pp\rangle$$

$= \ldots$

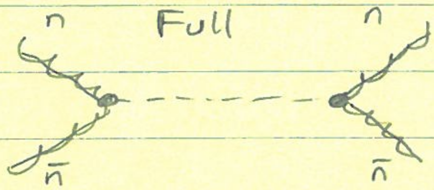
$$= \ldots \int d\omega\, d\bar\omega \,\langle p_n|\, \mathcal{B}_{n\perp}^{\mu}\, \delta(\omega - i\bar n\cdot\partial_n)\, \mathcal{B}_{n\perp\mu}\,|p_n\rangle$$
$$\langle p_{\bar n}|\, \mathcal{B}_{\bar n\perp}^{\mu}\, \delta(\bar\omega - i n\cdot\partial_{\bar n})\, \mathcal{B}_{\bar n\perp\mu}\,|p_{\bar n}\rangle$$
$$H_{\text{incl}}(\omega, \bar\omega)$$

$\mathcal{O}(\lambda^0)$ momentum exchange with hard interaction (picture below)

Y's cancel since measurement is hard, doesn't probe them

$\mathcal{Y}_n^T \mathcal{Y}_n = \mathbb{1}$ for adjoint

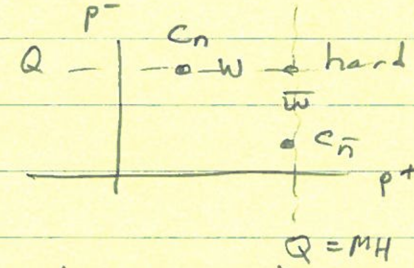
Full



$\longrightarrow$

SCET

H from matching at $\mu \sim Q$



$$\sigma(m_H) = \sigma_0 \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\, f(x_1,\mu)\, f(x_2,\mu)\, H_{\text{incl}}(x_1, x_2, m_H, \mu)$$
$$* \,\delta(E_{cm}^2 x_1 x_2 - m_H^2)$$