Non-Leptonic Multibody $B$ Decays in QCD Factorization: First Results

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Levels of complexity in $B$ decays

- Purely leptonic $f_B$
- Inclusive semileptonic: Heavy Quark Expansion (HQE)
- Inclusive Nonleptonic (Lifetimes, Mixing): HQE
- Exclusive semileptonic: $F^{B \rightarrow M}(q^2)$
- Inclusive FCNC $b \rightarrow s l l$ and $b \rightarrow s \gamma$: (HQE + ...)
- Exclusive FCNC $b \rightarrow s l l$ and $b \rightarrow s \gamma$: $F^{B \rightarrow M}(q^2) + ...$
- Two-Body Non-leptonic: QCD Factorization (QCD-F)
- Multi-Body Non-Leptonic: ???

Make Use of the fact that $\alpha_s(m_b) \ll 1$
Standard theoretical machinery

- Effective Hamiltonian

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu)O_i(\mu) \]

- \( C_i \): Wilson Coefficients: short distance, \( \alpha_s(M_W) \)
- \( O_i \): Local operators: Long distance physics
- \( \mu \): renormalization point
- Decay amplitudes:

\[ A(B \rightarrow f) = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) \langle f | O_i(\mu) | B \rangle \]

- How to compute the operator matrix elements?
\[ \langle f | O_i(\mu) | B \rangle \text{ still contains the large scale } m_b \]

- There are contributions which can be calculated perturbatively: \( \alpha_s(m_b) \)
- Factorization of these perturbative contributions
- Suitable definition of (universal) non-perturbative quantities
- OPE and Effective Field Theories
\[ \langle M_\bar{n}M_n | O_i | B \rangle = \langle M_\bar{n} | \bar{h}_\nu \Gamma \xi_\bar{n} | B \rangle \times \int dz \ T_i(z) \langle M_n | \bar{\chi}_n(zn)\Gamma' \chi_n(0) | 0 \rangle \]

\[ \sim F^{B \to M} \ T_i \otimes \phi_M \]

(Beneke, Buchalla, Neubert, Sachrajda, Bauer, Pirjol, Rothstein, Stewart, ...)
Established methodology for two body decays
Anatomy of $B \to D\pi$ and $B \to \pi\pi$ is understood
Phenomenology works
Indications of the presence of subleading terms
... but the two body decays are only a small fraction of the total non-leptonic width!
Clear need for a QCD-based description of multi-body decays
Three-body non-leptonics

Kinematics: $p_B \rightarrow p_1 + p_2 + p_3$

- Two independent kinematical variables $p_i^2 = 0$

$$s_{ij}^2 = (p_i + p_j)^2 \quad s_{12} + s_{13} + s_{23} = M_B^2$$

Historically:

- "Isobar" Model:
- Description via pseudo two-particle decays:

$$(B \rightarrow M M_1 M_2) = (B \rightarrow M^* M \quad \text{and} \quad M^* \rightarrow M_1 M_2)$$

- sum over all possibilities for $M^*$, including $\Gamma(M^*)$
- possibly add a flat “non-resonant” background!
(sketch borrowed from J. Virto)
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances

\[ f_0, \rho \ldots \]

\[ B, D \rightarrow \pi \]

modulus

\[ s [\text{GeV}^2] \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

15

10

5

0

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

180

90

0

\[ \kappa, K^* \ldots \]

\[ B, D \rightarrow K \]

phase \[ [^\circ] \]

\[ s [\text{GeV}^2] \]
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances

\[ B, D \rightarrow f_0, \rho \ldots \rightarrow \pi \]
\[ B, D \rightarrow \kappa, K^* \ldots \rightarrow K \]

...so what’s not to like?

- some resonances don’t look like Breit–Wigners at all!
- use exact scattering phase shifts instead
Study the Dalitz Distribution:
Specifically for $B^+ \to \pi^+\pi^-\pi^+$

Dalitz Plot is symmetric:

\[ s_{12} = s_{+-}^{low} \quad s_{23} = s_{+-}^{high} \]

\[ s_{12} = s_{++} \]

(Plot from LHCb arXiv:1408.5373)
Split the Dalitz Plot into Regions:

- **Region 1:** "Mercedes Star"
  \[ s_{++} \sim s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/3 \]

- **Region 2:** Collinear Decay Products
  - **Region 2a:** \((\pi^+\pi^+)_{\text{coll}}\) recoil against \(\pi^-\)
    \[ s_{++} \sim 0, \quad s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/2 \]
  - **Region 2b:** \((\pi^+\pi^-)_{\text{coll}}\) recoil against \(\pi^+\)
    \[ s_{+-}^{\text{low}} \sim 0, \quad s_{++} \sim s_{+-}^{\text{high}} \sim 1/2 \]

- **Region 3:** Soft Decay Products
  - **Region 3a:** Soft \(\pi^+\)
    \[ s_{++} \sim s_{+-}^{\text{low}} \sim 0, \quad s_{+-}^{\text{high}} \sim 1 \]
  - **Region 3b:** Soft \(\pi^-\)
    \[ s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 0, \quad s_{++} \sim 1 \]
Introduction

Three-body non-leptonics

Checks and Applications

Preliminaries

Regions

New Non-Perturbative Input

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Non-Leptonic Multibody B Decays in QCD Factorization
Region 1: The Center

- Three “disconnected” collinear directions: $n_1, n_2, n_3$

\[
\langle \pi^- n_1 \pi^+ n_2 \pi^- n_3 | O_i | B \rangle = \langle \pi^- n_3 | \bar{d} n_3 \Gamma h_v | B \rangle \\
\times \int dudv \, T_i(u, v) \langle \pi^- n_1 | \bar{d} n_1 (\bar{u}) \Gamma u n_1 (u) | 0 \rangle \langle \pi^+ n_2 | \bar{u} n_2 (\bar{v}) \Gamma d n_2 (v) | 0 \rangle \\
\sim F_{B \rightarrow \pi}^{B \rightarrow \pi} T_i \otimes \phi_{\pi} \otimes \phi_{\pi}
\]

(Figure borrowed from J. Virto)
\[
\frac{1}{m_b^2} \text{ and } \alpha_s \text{ supressed with respect to a two body decay}
\]

At leading order / leading power / leading twist all convolutions are finite

\[\rightarrow \text{ factorization:}\]

\[B \xrightarrow{F_{B \to \pi}} expr \quad T^I \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \]

\[+\]

\[B \xrightarrow{} expr \quad \Phi_B \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \quad \Phi_\pi \xrightarrow{\pi} \Phi_\pi \]

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Non-Leptonic Multibody B Decays in QCD Factorization
Extrapolation to collinear $\pi^-\pi^-$

- There are no resonances in this channel
- No infrared / collinear problems expected
- Perturbative result expected to be regular: No "soft" propagators

\[
\frac{d\Gamma}{ds_{--} ds_{++}} \approx 0.84 \Gamma_0 f_+ \left( \frac{m_B^2}{2} \right)^2 + \mathcal{O}(s_{--})
\]
Extrapolation to collinear $\pi^+\pi^-$

- There are resonances in this channel: $\rho$, $\omega$, ...
- Perturbative result expected to be IR singular
- "soft" propagators

\[
\frac{d\Gamma}{ds_{+-} ds_{--}} \simeq \frac{0.38}{s_{+-}} \Gamma_0 f_+(0)^2 + \text{regular}
\]
Region 2b: new non-perturbative input

- Factorization breaks down in the resonance regions
- New, nonperturbative input is needed
- Three-body decay resembles two-body decay

Operators are the same as in two-body decays ...
... but the final states are different

\[
\langle \pi^- \pi^+ \pi^- | O_i | B \rangle = \\
\langle \pi^- | h_v \Gamma \xi_n | B \rangle \times \int dz \ T_1(z) \langle \pi^- \pi^+ | \bar{\chi}_n(z\bar{n}) \Gamma' \chi_n(0) | 0 \rangle \\
+ \langle \pi^- \pi^+ | \bar{h}_v \Gamma \xi_n | B \rangle \times \int dz \ T_2(z) \langle \pi^- | \bar{\chi}_n(zn) \Gamma' \chi_n(0) | 0 \rangle \\
\sim F^{B \to \pi} T_1 \otimes \phi_{\pi\pi} + F^{B \to \pi\pi} T_2 \otimes \phi_{\pi}
\]

- Two-Pion light-cone distribution (Polyakov, Diehl, Gousset, ...)
- Generalized (soft) Form factor (Feldmann, Khofjamirian, van Dyk, ThM ...)

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Factorization formula similar to the two-body case
Two-Pion Light Cone Distribution

- Definition: \( s = (k_1 + k_2)^2, \ k_1 = \zeta k_{12}, \ k_2 = \bar{\zeta} k_{12} \)

\[
\phi_{\pi\pi}^q(z, \zeta, s) = \int \frac{dx^-}{2\pi} \ e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2) | \bar{q}(x^- n_-) n_+ q(0) | 0 \rangle
\]

- Normalization from the local limit:

\[
\int dz \ \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1) F_\pi(s) \quad \text{(pion time-like FF)}
\]

- \( F_\pi(s) \): Data (BaBar) + Theory \( (\chi^{PT}, \text{Asymptotics}...) \)

- \( z \) and \( \zeta \) dependence asymptotically known
Timelike Pion Form Factor known from Data

(Hanhart, Kubis, ...)

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Non-Leptonic Multibody B Decays in QCD Factorization
Generalized (soft) Form factor

- Relevant Form factor:

\[ \langle \pi^+(k_1)\pi^-(k_2)|\bar{u}k_3P_{L,R}b|B^-(p)\rangle = \mp \frac{m_\pi}{2} F_t(\zeta, s) \]

- \( F_t(\zeta, s) \) can be related to the two-pion light-cone distribution via a Light-Cone Sum Rule (Khodjamirian, Hambrock)

\[
F_t(\zeta, s) = \frac{m_b^2}{\sqrt{2}\hat{f}_B m_\pi} \int_{u_0}^1 \frac{du}{u} \exp\left[ \frac{(1 + s\bar{u})m_B^2}{M^2} - \frac{m_b^2}{uM^2} \right] \phi_{\pi\pi}(u, \zeta, s)
\]
Merging the Regions ...

- The starting point is the large-\(m_b\) limit
- Do the regions match properly?
- Is \(m_b\) large enough?
- Is there a central region for \(m_b \sim 5\) GeV?
Introduction
Three-body non-leptonics
Checks and Applications

$m_B \approx 20\text{GeV}$

$m_B \approx 15\text{GeV}$

$m_B \approx 10\text{GeV}$

$m_B \approx 5\text{GeV}$

Differential Branching Fraction $\times 10^6$

$S_{+-}$

$S_{+-}$

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Non-Leptonic Multibody $B$ Decays in QCD Factorization
Probably there is no perturbatively calculable central region for realistic $m_b$

For realistic $m_b$ the Dalitz plot consists only of edges

Three-body decays become quasi two-body

The factorization formula is the one known from the two-body decays with new non-perturbative input

These are just first results, many more checks need to be performed.
First Application: $B \rightarrow \rho \pi$

- Amplitude near $s_{+-} \ll m_b^2$
  \[ A \sim \frac{G_F}{\sqrt{2}} [4m_B^2 f_0(s_{+-})(2\zeta - 1)F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4)F_t(\zeta, s_{+-})] \]

- Definition of the $\rho$: Integration around the $\rho$ mass:
  \[ BR(B^- \rightarrow \rho \pi^-) \sim \int_0^1 ds_{++} \int_{s_{\rho}^-}^{s_{\rho}^+} ds_{+-} \frac{\tau_B m_B |A|^2}{32(2\pi)^3} \]
with $s^\pm_\rho = (m_\rho \pm n\Gamma_\rho)^2 / m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6}$ \hspace{1cm} (n = 0.5)

$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6}$ \hspace{1cm} (n = 1)

$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6}$ \hspace{1cm} (n = 1.5)

$BR(B^+ \to \rho\pi^+)_\text{EXP} = (8.3 \pm 1.2) \cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_\text{QCD} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$
CP Violation Studies

- Three-Body Decays contain more information than two-body:
- Compare the Dalitz Plots of $B^+$ vs. $B^-$
- Bin-wise asymmetry

$$\Delta(i) = \frac{N(i) - \bar{N}(i)}{N(i) + \bar{N}(i)}$$

- Miranda Procedure: Use the significance (well suited for “noisy” data)

$$S_{CP}(i) = \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}}$$

= more robust probe of CP violation

(Bediga, Bigi, et al. ...)

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CP Violation is distributed over the Dalitz plot

Strong phases depend on the bin:
eg. Breit Wigner

\[
\text{Im BW}(s) = \frac{m_R \Gamma_R}{(m_R^2 - s)^2 + m_R^2 \Gamma_R^2}
\]

Expample: \(B \rightarrow K \pi \pi\):
Interferences between different channels
eg \(B \rightarrow K^* \pi \rightarrow K \pi \pi\) and \(B \rightarrow K \rho \rightarrow K \pi \pi\)
Recent LHCb Results

- LHCb analysis of $B^+ \to h^+ h^- h'^+$ decays using full LHC Run 1 data sample
- Much larger samples (20-40x) than $B$-factories
- CP asymmetries measured in full phase space:

  $A_{CP}(B^\pm \to K^\pm \pi^+ \pi^-) = +0.025 \pm 0.004 \pm 0.004 \pm 0.007$,

  $A_{CP}(B^\pm \to K^\pm K^+ K^-) = -0.036 \pm 0.004 \pm 0.002 \pm 0.007$,

  $A_{CP}(B^\pm \to \pi^\pm \pi^+ \pi^-) = +0.058 \pm 0.008 \pm 0.009 \pm 0.007$,

  $A_{CP}(B^\pm \to \pi^\pm K^+ K^-) = -0.123 \pm 0.017 \pm 0.012 \pm 0.007$. 
Huge CP Asymmetries in some regions of phase space

Needs a full amplitude analysis, including the phases

Experimental input needed (like the time like pion form factor)

QCD Factorization may provided a tool which eventually may be better than what was done for the two body decays!
Summary

- Multi-body decays ...
  - ... are abundant
  - ... contain important information in their kinematic distributions
  - ... are theoretically most complex
- QCD based Ansatz: QCD factorization
- No perturbative central region, even for $B$ decays
- Quasi two-body with new non-perturbative input
We started with $B \rightarrow \pi\pi\pi$: Heavy to light

Different (but known) technology for $B \rightarrow D\pi\pi$ or even $B \rightarrow D\bar{D}\pi$

From LHCb: Multi-body $\Lambda_b$ decays

In $\Lambda_b \rightarrow \Lambda X$: Polarization Analysis for the $\Lambda$ → more CP violating observables involving the $\Lambda$ spin

... and much more

More work needed to establish QCD-Factorization!