The anomalous magnetic moments of the light (charged) leptons ($e^{\pm}$, $\mu^{\pm}$): precision tests of the standard model

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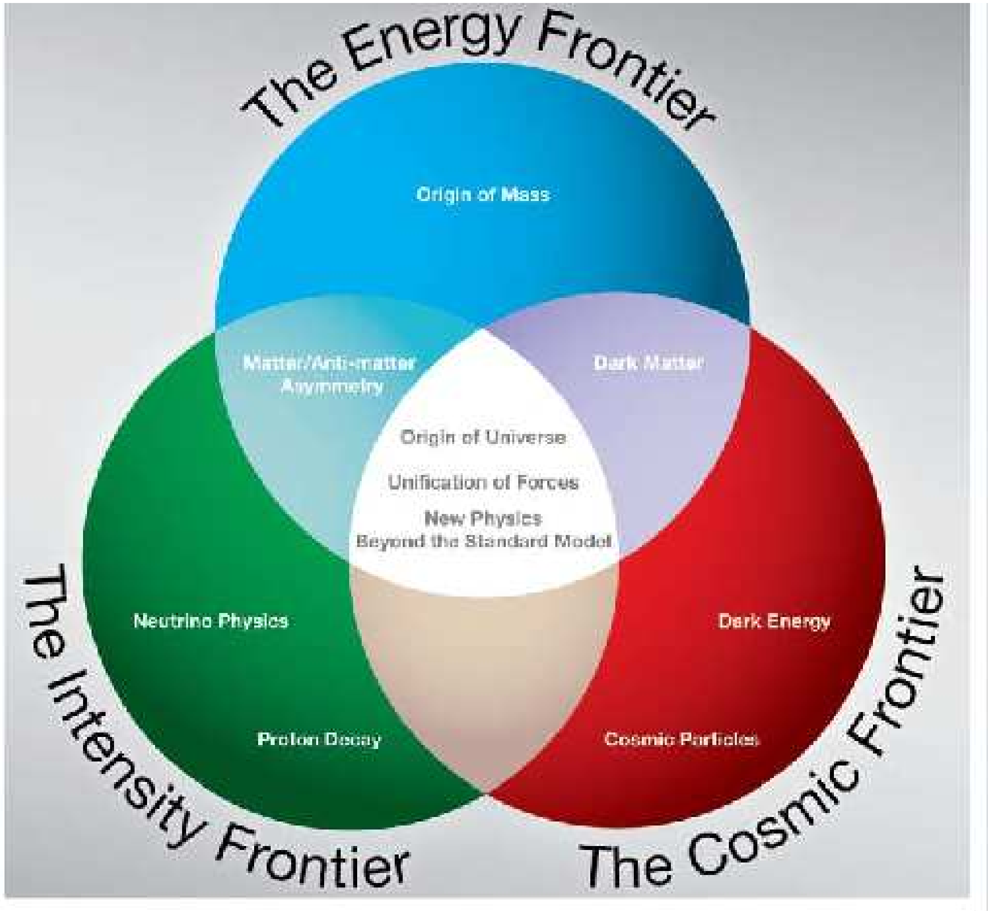
OUTLINE

- Introduction - Motivation - Context
- Experimental aspects
- Theory

- QED contributions
- Weak contributions
- Strong interactions

- Conclusions - Perspectives for the future

General context: looking for new physics by exploring the “three frontiers”



In this presentation, the focus will be on the intensity (or precision) frontier, illustrated trough the anomalous magnetic moments of the electron and the muon

Leptonic sector of the three-family standard model

- charged leptons: $\ell = e^{\pm}, \mu^{\pm}, \tau^{\pm}$

$$q = \pm 1 \text{ (charge)} \qquad s = \frac{1}{2} \text{ (spin)}$$

differ only in their masses

$$\begin{aligned} m_e &= 0.510\,998\,910(13) \text{ MeV} \\ m_\mu &= 105.658\,367(4) \text{ MeV} \\ m_\tau &= 1\,776.82(16) \text{ MeV} \end{aligned}$$

- neutral leptons: $\nu_\ell = \nu_e, \nu_\mu, \nu_\tau$

$$q = 0 \qquad s = \frac{1}{2}$$

Response of a charged lepton to an external (and static) electromagnetic field

$$\langle \ell; p'|\mathcal{J}_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p',p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\Big[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\Big]\mathrm{u}(p)$$

(uses only the conservation of the electromanetic current $\mathcal{J}_\rho$, $k_\mu \equiv p'_\mu - p_\mu$)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow$ $\not{P}$, $\not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/e_\ell$

$F_4(k^2) \rightarrow$ $\not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\; G_M(k^2) = F_1(k^2) + F_2(k^2)$$

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops $\longrightarrow$ calculable!

Response of a charged lepton to an external (and static) electromagnetic field

For a relativistic, point-like spin 1/2 particle, described by the Dirac equation with the minimal coupling prescription, one has

$$i\hbar\,\frac{\partial\psi}{\partial t} = \left[c\boldsymbol{\alpha}\cdot\left(-i\hbar\boldsymbol{\nabla} - \frac{e_\ell}{c}\boldsymbol{\mathcal{A}}\right) + \beta m_\ell c^2 + e_\ell\mathcal{A}_0\right]\psi$$

In the non relativistic limit, this reduces to the Pauli equation for the two-component spinor φ describing the large components of the Dirac spinor ψ,

$$i\hbar\,\frac{\partial\varphi}{\partial t} = \left[\frac{(-i\hbar\boldsymbol{\nabla} - (e_\ell/c)\boldsymbol{\mathcal{A}})^2}{2m_\ell} - \underbrace{\frac{e_\ell\hbar}{2m_\ell c}\boldsymbol{\sigma}\cdot\mathsf{B}}_{\boldsymbol{\mu}_\ell\cdot\mathsf{B}} + e_\ell\mathcal{A}_0\right]\varphi$$

with

$$\boldsymbol{\mu}_\ell = g_\ell\left(\frac{e_\ell}{2m_\ell c}\right)\mathsf{S}\,,\quad \mathsf{S} = \hbar\,\frac{\boldsymbol{\sigma}}{2}\,,\quad g_\ell^{\mathrm{Dirac}} = 2$$

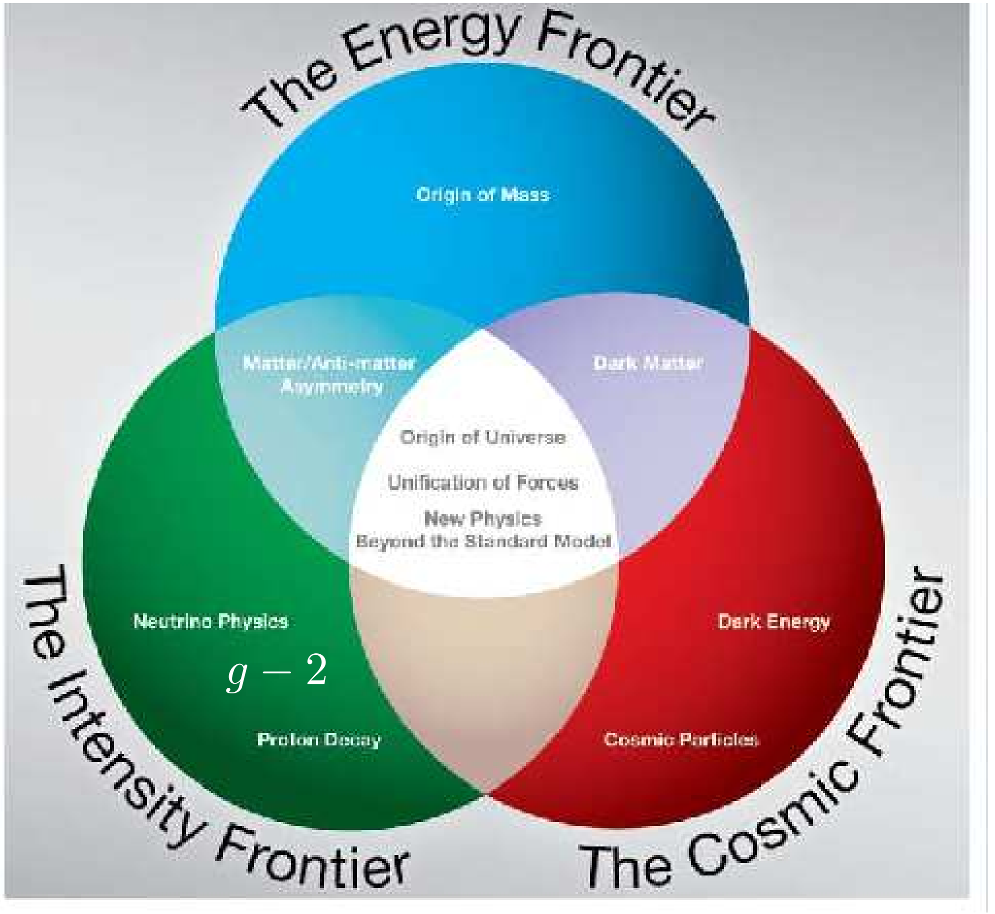
At tree level, $g_\ell = g_\ell^{\mathrm{Dirac}} \equiv 2$.

The *anomalous* magnetic moment is induced at loop level:

$$a_\ell \equiv \frac{g_\ell - g_\ell^{\mathrm{Dirac}}}{g_\ell^{\mathrm{Dirac}}} (\equiv F_2(0))$$

a_ℓ probes all the degrees of freedom of the standard model, *and possibly beyond...*

... provided one can measure it with enough precision !



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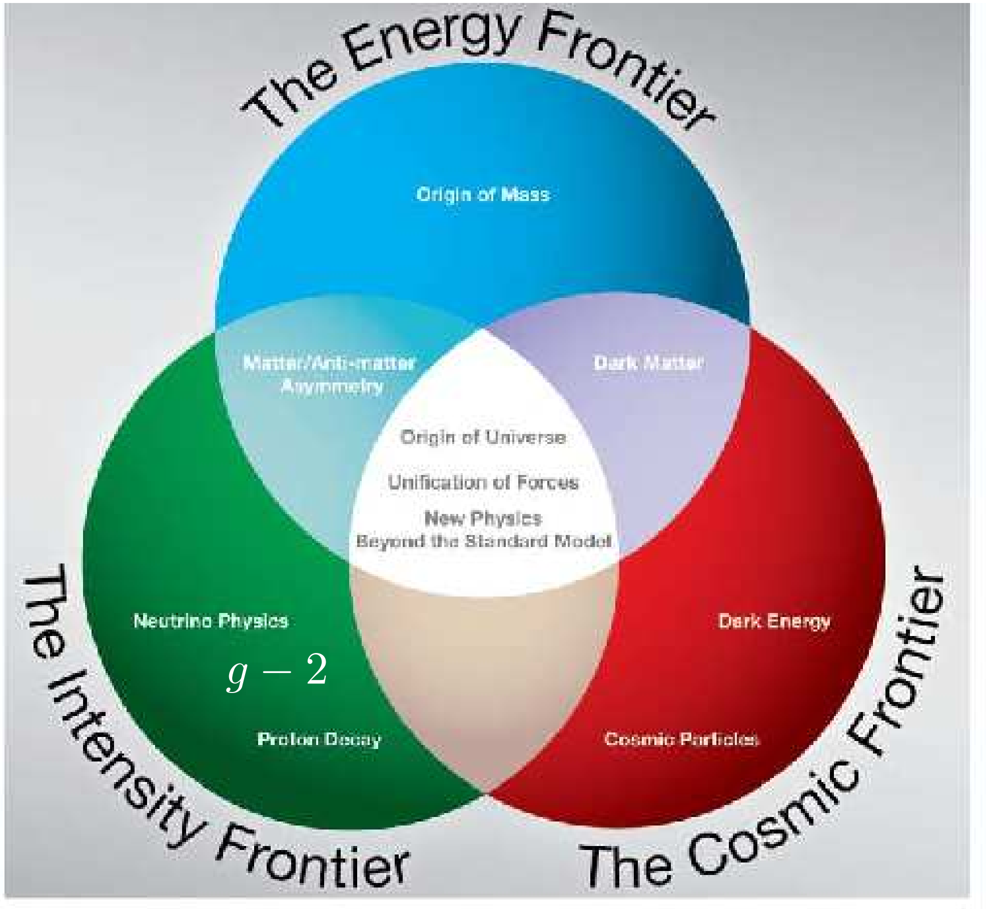


And predict its value in the standard model with a comparable accuracy...

Experimental aspects

- The electron case

1947: hf splitting in H and D (0.2% discrepancy with the value $g_e^{\rm Dirac} = 2$)

[J. Nafe, E. B. Nelson, I. I. Rabi, Phys. Rev. 71, 914 (1947)]

1958: first direct measurement of g_e for *free* electrons

[H. G. Dehmelt, Phys. Rev. 109, 381 (1958)]

1968 $\longrightarrow$ 1987: Penning trap type experiments $\longrightarrow$ single trapped electron (geonium)

$$a_{e^-}^{exp} = 1\,159\,652\,188.4(4.3) \cdot 10^{-12} \quad [3.7\,\mathrm{ppb}]$$
$$a_{e^+}^{exp} = 1\,159\,652\,187.9(4.3) \cdot 10^{-12} \quad [3.7\,\mathrm{ppb}]$$

[R.S. van Dyck Jr. et al., PRL 59, 26 (1987)]

g_{e^-}/g_{e^+} = 1 + (0.5$\pm$2.1)$\times 10^{-12}$ probes CPT invariance

$\left(\rightarrow |M_{K^0} - M_{\overline{K}^0}|/M_{K^0} \leq 10^{-18}\right.$ (90% CL)$\left.\right)$

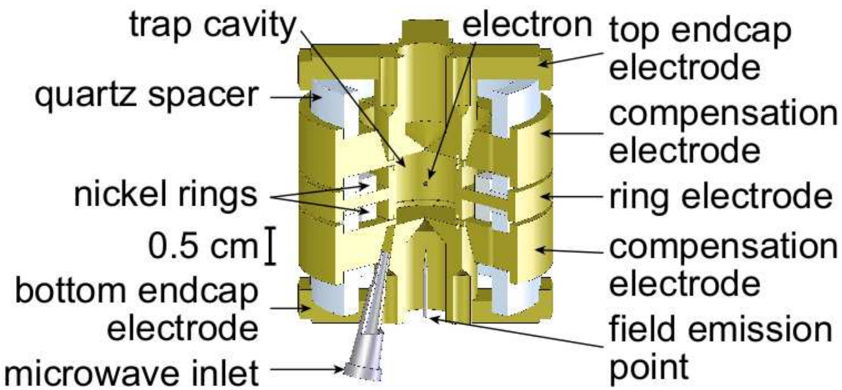
New series of high precision measurements conducted by the Harvard group (G. Gabrielse et al.)

$$a_e^{exp} = 1\,159\,652\,180.85(0.76) \cdot 10^{-12} \quad [0.66\,\mathrm{ppb}]$$

[Odom et al., PRL 97, 030801 (2006)]

$$a_e^{exp} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\,\mathrm{ppb}]$$

[D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)]



- The muon case

Note: $\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\, s$

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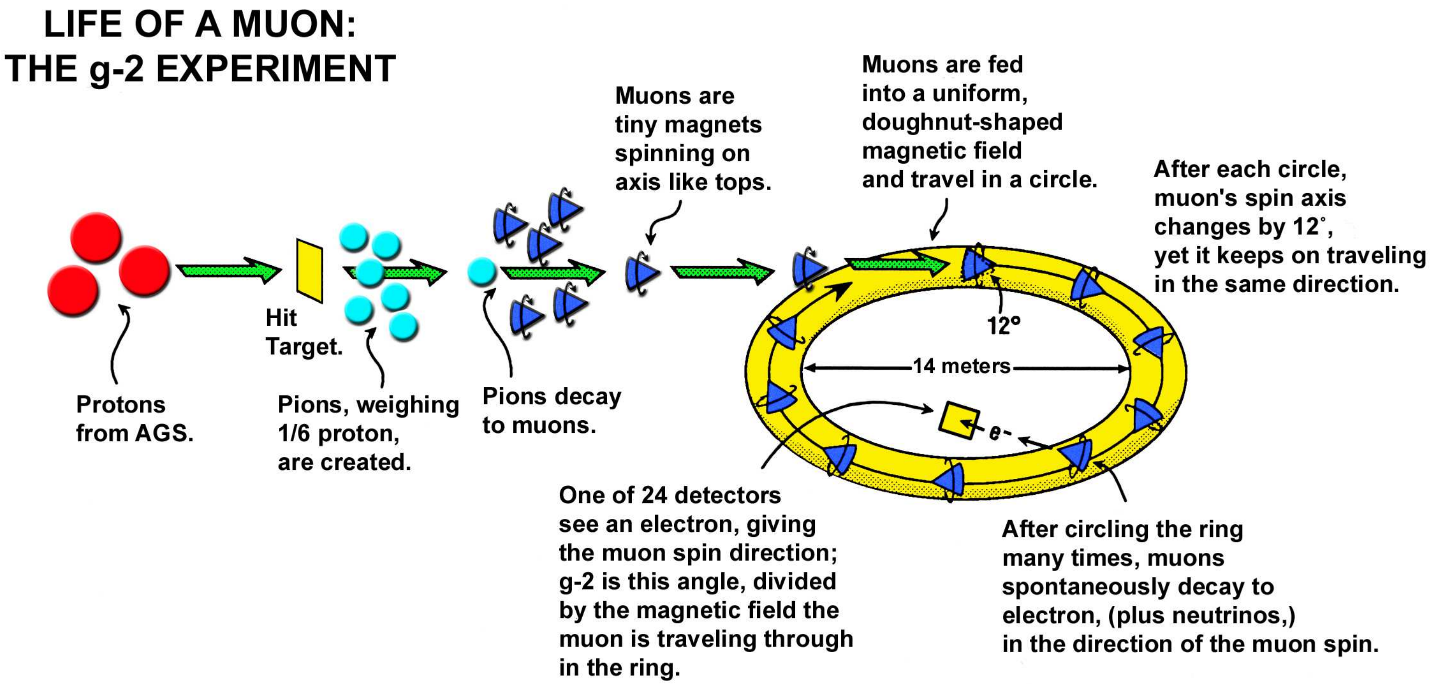
- muon storage ring experiment (CERN & BNL)



- muon storage ring experiment $[\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0]$

$$\vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B} + \vec{E}]$$

- $\gamma \sim 29.3$ [electrostatic focusing will not affect the spin] [Muon g-2 Coll., H. N. Brown et al., (2001)]

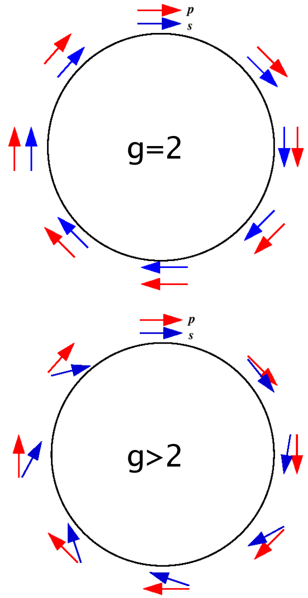
- $|d_\mu| < 1.9 \cdot 10^{-19}$ [95% CL] [Muon g-2 Coll., G. W. Brown et al., Phys. Rev. D 80 (2009)]

Large advantage to using a storage ring

$$\omega_s = g\frac{eB}{2mc}$$

$$\omega_c = \frac{eB}{mc}$$

$$\begin{aligned}\omega_a &= \omega_s - \omega_c,\\ &= \frac{eB}{mc}\left(\frac{g}{2} - 1\right),\\ &= \frac{eB}{mc}\frac{g-2}{2},\\ &= a_\mu\frac{eB}{mc},\end{aligned}$$



Measures a_μ directly

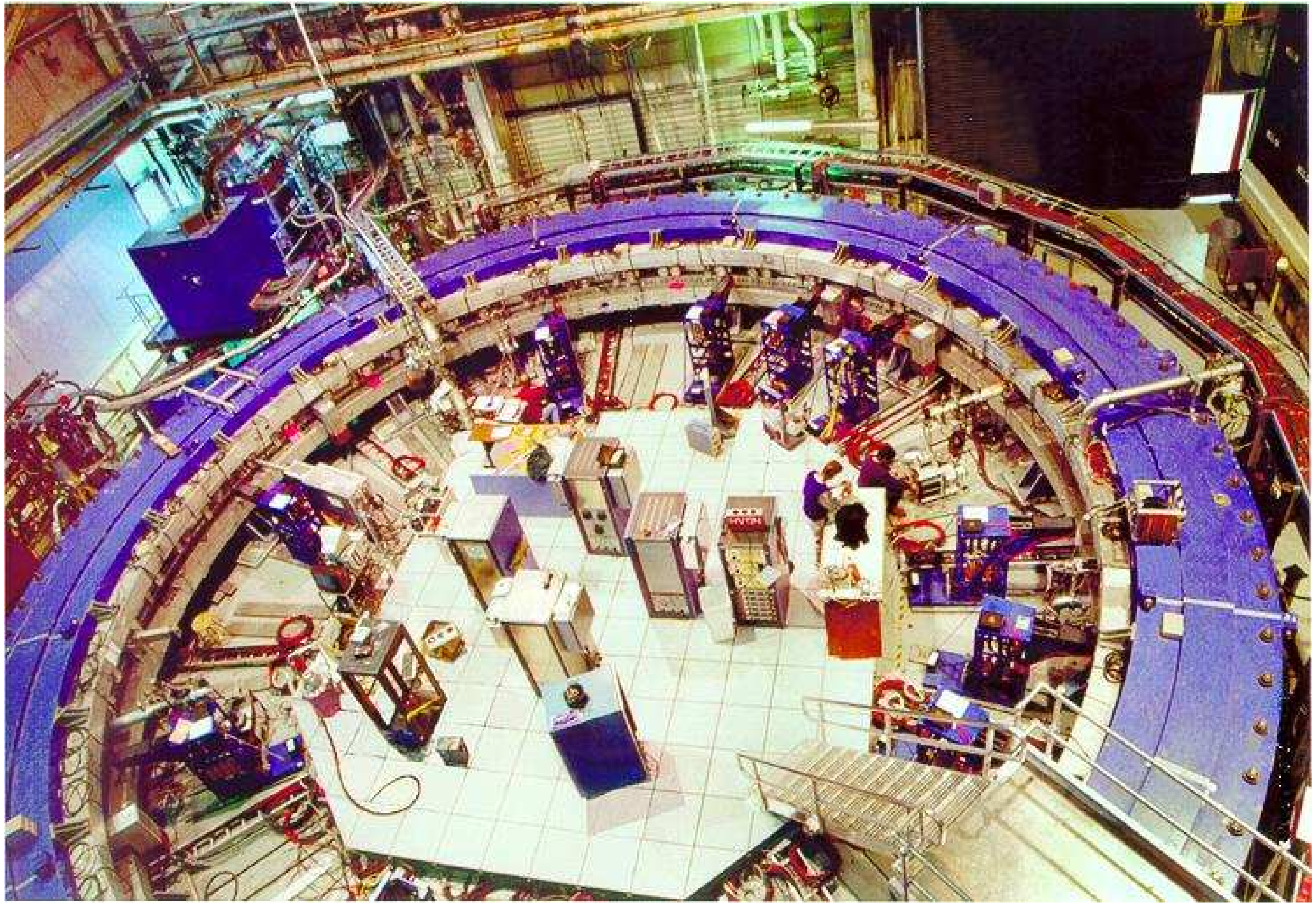
Factor of 800 in precision for free over experiments measuring g

All recent experiments and proposals CERN II/III, BNL, FNAL, & J-PARC rely on this trick

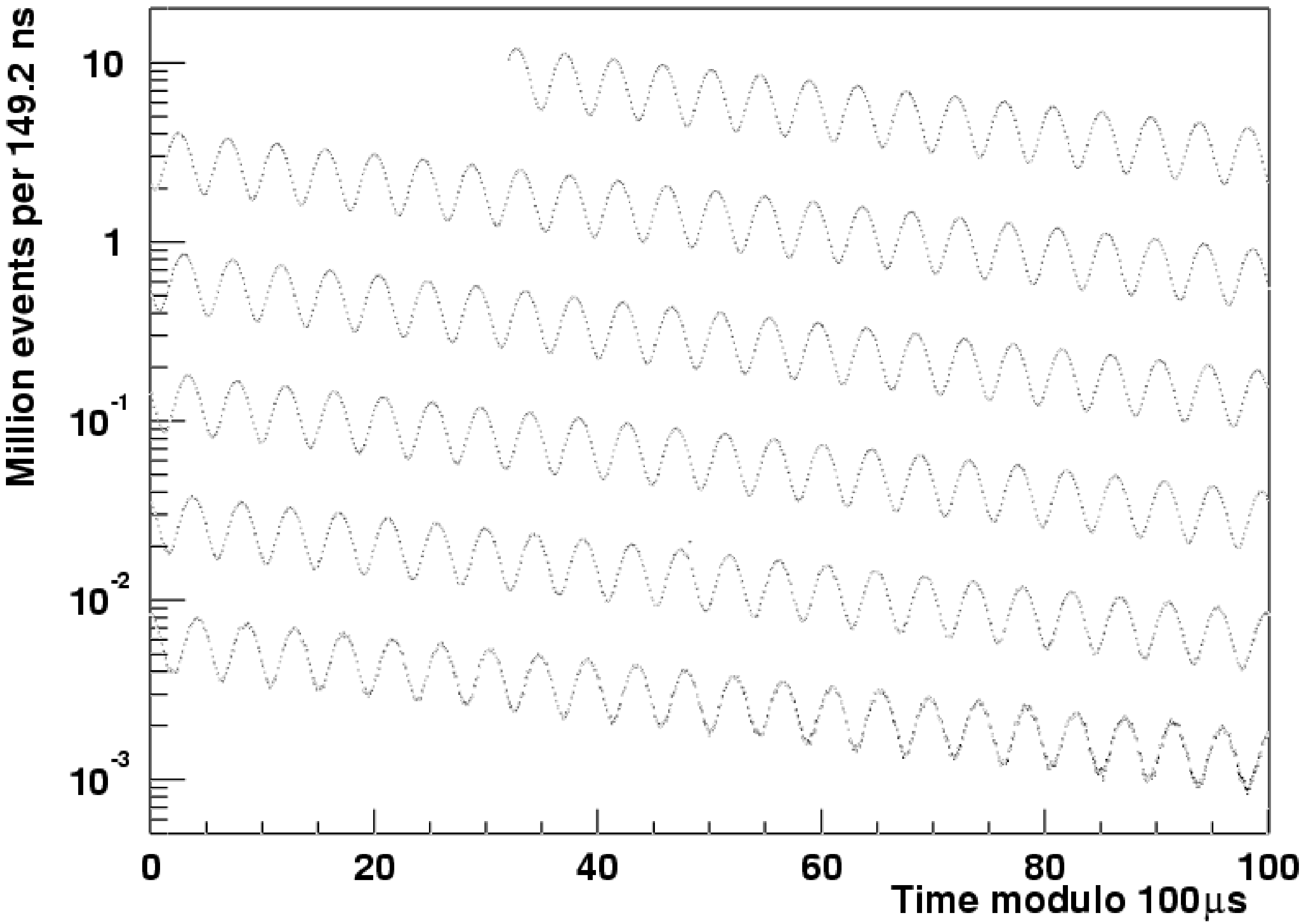
Table 1: Summary of a_μ results from CERN and BNL, showing the evolution of experimental precision over time. The average is obtained from the 1999, 2000 and 2001 data sets only.

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72

$$a_\mu^{exp} = 11\,659\,209.1(5.4)(3.3) \cdot 10^{-10} \quad [0.54\,\mathrm{ppm}]$$



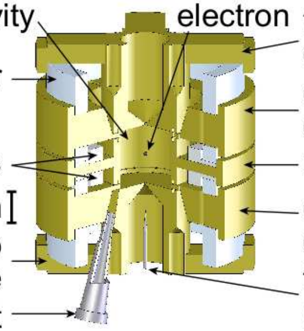
BNL-E821



Million events per 149.2 ns
10
1
10^{-1}
10^{-2}
10^{-3}
0
20
40
60
80
100
Time modulo 100μs

BNL-E821

Experimentally measured to very high precision:



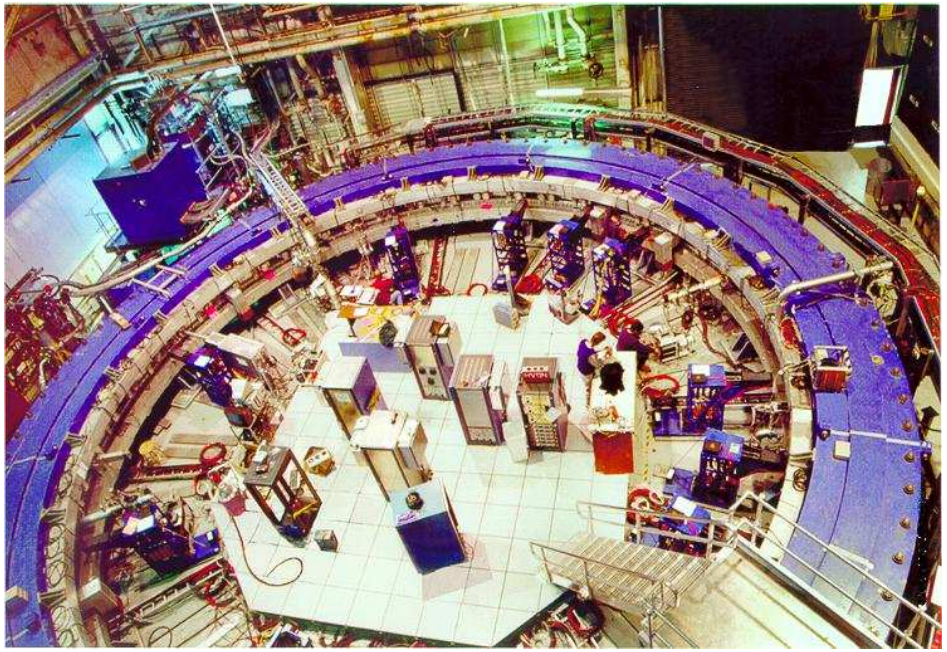
$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13} \text{ [0.24ppb]}$$

[D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)]

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3,\ p \sim 3.094 \text{ GeV/c}$$



$$a_\mu^{\text{exp}} = 11\,659\,209.1(6.3) \cdot 10^{-10}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \text{ [0.54ppm]}$$

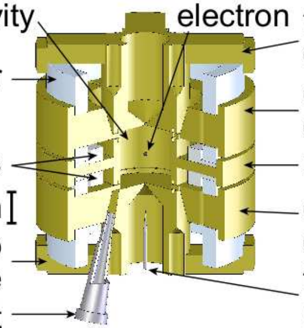
[G. W. Bennett et al, Phys Rev D 73, 072003 (2006)]

$$a_\mu^{\text{exp}} = \frac{\mathcal{R}}{\lambda_{\text{exp}} - \mathcal{R}}, \quad \mathcal{R} \equiv \frac{\omega_a}{\omega_p}, \quad \lambda = \frac{\mu_\mu}{\mu_p}$$

$$\mathcal{R}_{\text{E821}} = 0.003\,707\,206\,3(20) \qquad \lambda_{\text{exp}} = 3.183\,345\,107(84)$$

[P. J. Mohr et al, Rev. Mod. Phys. 84, 1527 (2012)]

Experimentally measured to very high precision:



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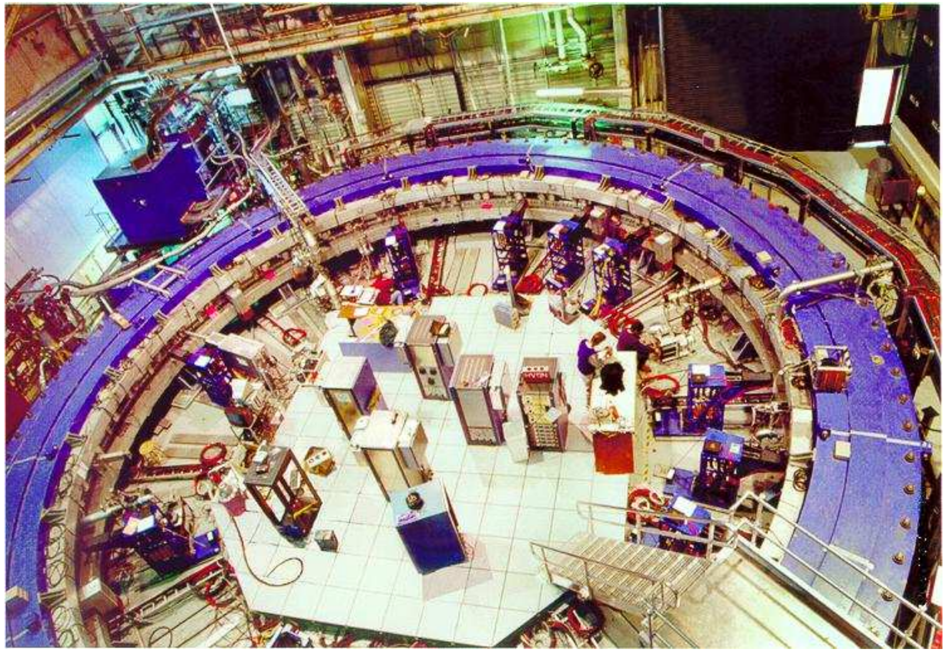

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Are the SM calculations able to achieve a comparable level of accuracy ? General structure $a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{had}} + a_\ell^{\text{weak}}$

The tau case

$$\tau_\tau = (290.6 \pm 1.1) \times 10^{-15}\,\mathrm{s}$$

- $e^+e^- \to \tau^+\tau^-\gamma$

$-0.052 < a_\tau^{exp} < +0.058$ (L3, 1998, 95% CL)

[Phys. Lett. B 434, 169 (1998)]

$-0.068 < a_\tau^{exp} < +0.065$ (OPAL, 1998, 95% CL)

[Phys. Lett. B 431, 188 (1998)]

- $e^+e^- \to e^+e^-\tau^+\tau^-$

$-0.052 < a_\tau^{exp} < +0.013$ (DELPHI, 2004, 95% CL) $\quad a_\tau^{exp} = -0.018(17)$

[Eur. Phys. J. C 35, 159 (2004)]

- Reanalysis of experiments $\longrightarrow\ -0.007 < a_\tau^{exp} < +0.005$

[G. A. Gonzalez-Sprinberg, A. Santamaria, J. Vidal, Nucl. Phys. B 582, 3 (2000) [hep-ph/0002203]

- $\tau^- \to \ell^-\nu_\tau\bar{\nu}_\ell\gamma$ at B factories $\longrightarrow\ \Delta a_\tau \sim 0.012$

[S. Eidelman, D. Epifanov, M. Fael, L. Mercolli, M. Passera, arXiv:1601.07987 [hep-ph]]

- theory: $a_\tau = 117721(5) \cdot 10^{-8}$ [42 ppm]

[S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)]
[S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)]

Theory

Useful guidelines:

- Within the framework of a renormalizable quantum field theory, $F_2(k^2)$, $F_3(k^2)$ and $F_4(k^2)$ can only arise through loop corrections. These loop contributions have to be finite and calculable, since the possible counterterms correspond to non renormalizable interactions

- a_ℓ is dimensionless: the contributions from loops involving only photons and the lepton ℓ (one-flavour QED) are mass independent and thus universal

- Massive degrees of freedom with $M \gg m_\ell$ contribute to a_ℓ through powers of m_ℓ^2/M^2 times logarithms (decoupling)

- Light degrees of freedom with $m \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

Theory I: QED (a_e and a_μ)

- QED contributions : loops with only photons and leptons

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

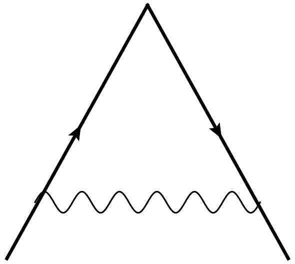
$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

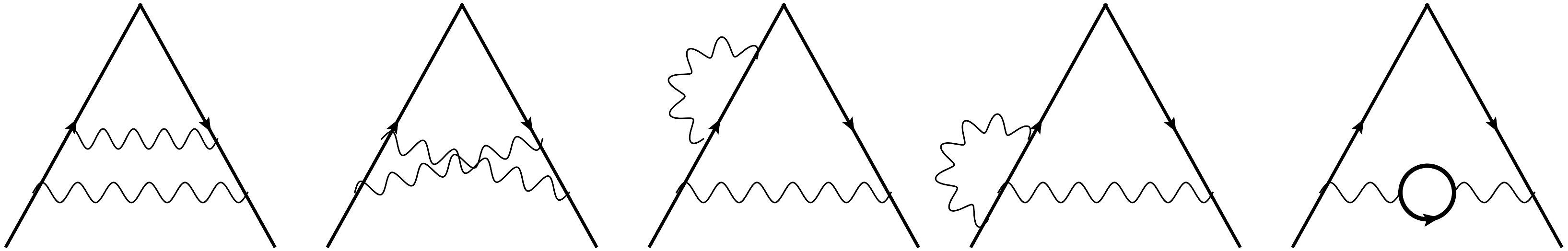
a_ℓ is finite (no renormalization needed) and dimensionless

Analytic expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$, $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ known



$$A_1^{(2)} = \frac{1}{2}$$

[J. Schwinger, Phys. Rev. 73, 416L (1948)]



$$A_1^{(4)} = \frac{3}{4}\zeta(3) - \frac{\pi^2}{2}\ln 2 + \frac{\pi^2}{12} + \frac{197}{144} = -0.328\,478\,965\,579\,193...$$

[C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)]

[A. Petermann, Helv. Phys. Acta 30, 407 (1957)]

Analytic expressions for $A_1^{(2)}, A_1^{(4)}, A_1^{(6)}, A_2^{(4)}, A_2^{(6)}, A_3^{(6)}$ known

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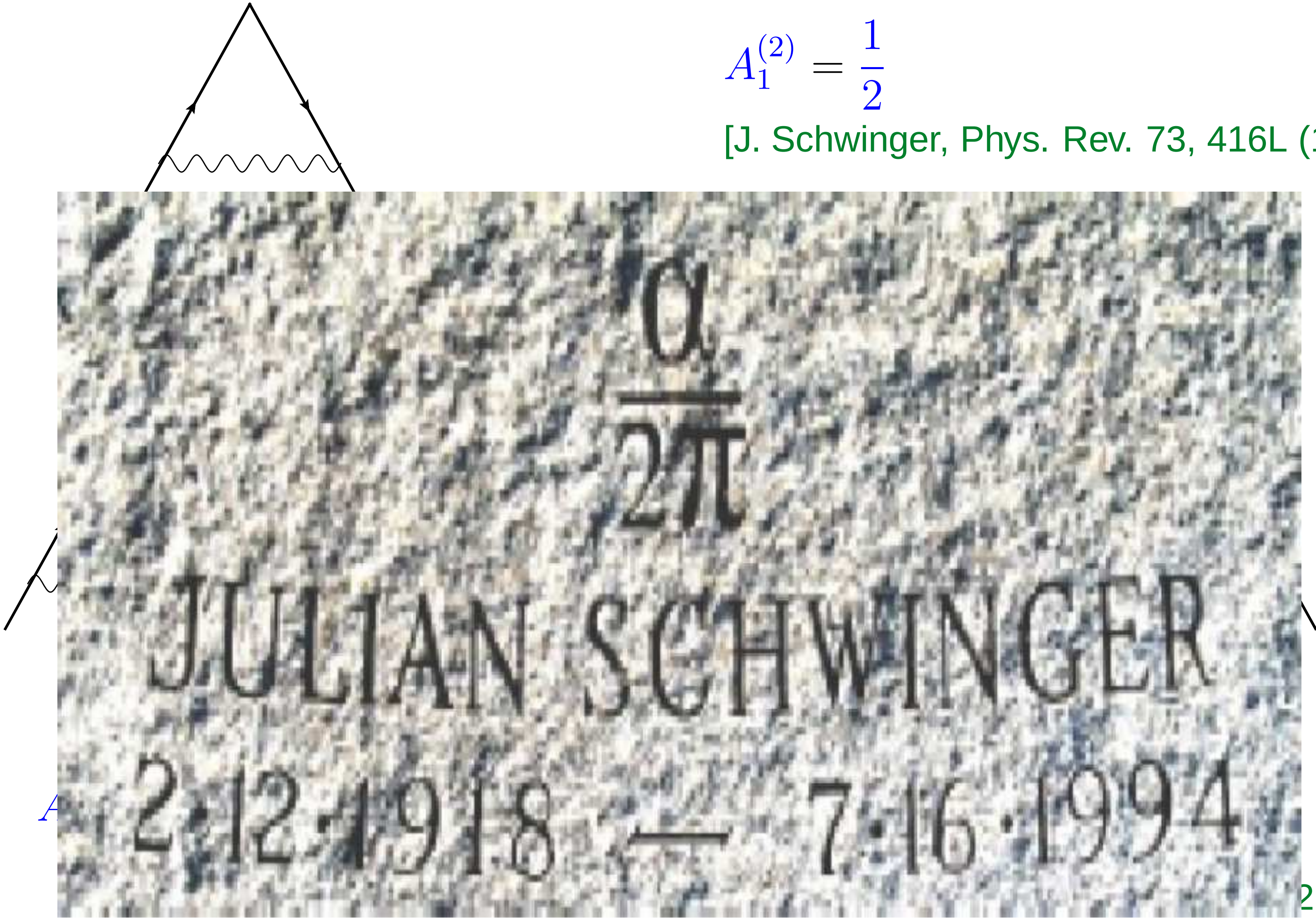


26 (1958)]

[A. Petermann, Helv. Phys. Acta 30, 407 (1957)]

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt \sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{t}{m_\ell^2}}$$

[H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)]
[A. Petermann, Phys. Rev. 105, 1931 (1955)]
[H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)]
[M. Passera, Phys. Rev. D 75, 013002 (2007)]

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{3}\ln\left(\frac{m_\ell}{m_{\ell'}}\right) - \frac{25}{36} + \frac{\pi^2}{4}\frac{m_{\ell'}}{m_\ell} - 4\left(\frac{m_{\ell'}}{m_\ell}\right)^2 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad +3\left(\frac{m_{\ell'}}{m_\ell}\right)^2 + \mathcal{O}\left[\left(\frac{m_{\ell'}}{m_\ell}\right)^3\right], \; m_\ell \gg m_{\ell'} \end{aligned}$$

[M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)]

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,312\,0(83)$$

$$m_\mu/m_e = 206.768\,2843(52)$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, arXiv:1203.5425v1[physics.atom-ph]

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} dt \sqrt{1-\frac{4m_{\ell'}^2}{t}}\,\frac{t+2m_{\ell'}^2}{t^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\dfrac{t}{m_\ell^2}}$$

[H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)]
[A. Petermann, Phys. Rev. 105, 1931 (1955)]
[H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)]
[M. Passera, Phys. Rev. D 75, 013002 (2007)]

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{45}\left(\frac{m_\ell}{m_{\ell'}}\right)^2 + \frac{1}{70}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &+\frac{9}{19600}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 + \mathcal{O}\left[\left(\frac{m_\ell}{m_{\ell'}}\right)^3 \ln\left(\frac{m_\ell}{m_{\ell'}}\right)\right], \quad m_{\ell'} \gg m_\ell \end{aligned}$$

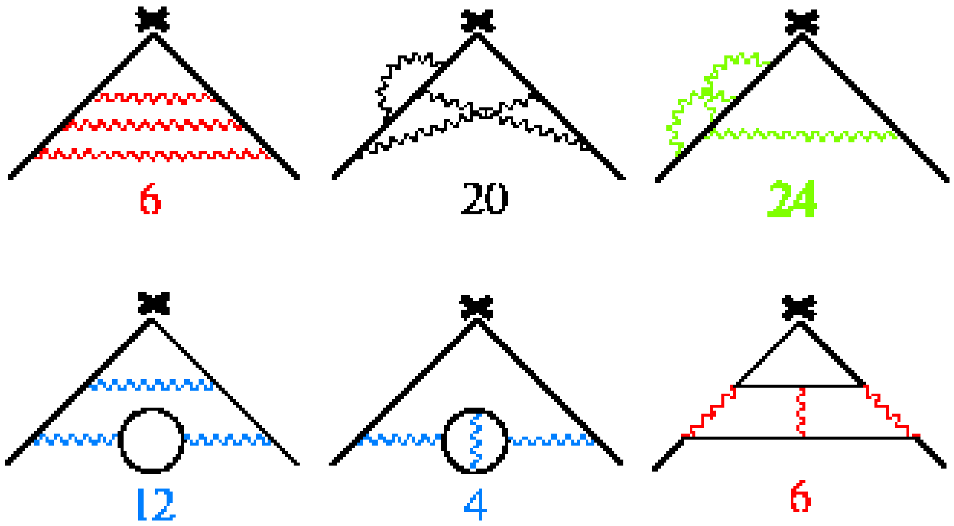
[B.E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1965)]
[M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)]

$$\begin{aligned} A_2^{(4)}(m_e/m_\mu) &= 5.197\,386\,67(26)\cdot 10^{-7} \\ A_2^{(4)}(m_e/m_\tau) &= 1.837\,98(34)\cdot 10^{-9} \\ A_2^{(4)}(m_\mu/m_\tau) &= 7.8079(15)\cdot 10^{-5} \end{aligned}$$

$$m_\mu/m_\tau = 5.946\,49(54)\cdot 10^{-2} \qquad m_e/m_\tau = 2.875\,92(26)\cdot 10^{-4}$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, arXiv:1203.5425v1[physics.atom-ph]

order $(\alpha/\pi)^3$: 72 diagrams



$$A_1^{(6)} = \frac{87}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(a_4 + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4$$
$$+\frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} \qquad \left[a_p = \sum_1^\infty 1/(2^n n^p)\right]$$

[S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)]
[S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)]

$$A_1^{(6)} = 1.181\,241\,456\ldots$$

numerical evaluations: $A_1^{(6)}(\text{num}) = 1.181\,259(40)\ldots$

[T. Kinoshita, Phys. Rev. Lett. 75, 4728 (1995)]

order $(\alpha/\pi)^4$: 891 diagrams

only some diagrams are known analytically $\longrightarrow$

Automated generation of diagrams

Systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

Analytic evaluation at $\mathcal{O}(\alpha^4)$? $\longrightarrow$ [A. Kataev, Phys. Rev. D 86, 013019 (2012)]
[A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)]
[A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015); arXiv:1602.02785 [hep-ph]]

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373) \cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118) \cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28) \cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

order $(\alpha/\pi)^4$: 891 diagrams

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Automated generation of diagrams

Systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

Analytic evaluation at $\mathcal{O}(\alpha^4)$? $\longrightarrow$ [A. Kataev, Phys. Rev. D 86, 013019 (2012)]

[A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)]

[A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015); arXiv:1602.02785 [hep-ph]]

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

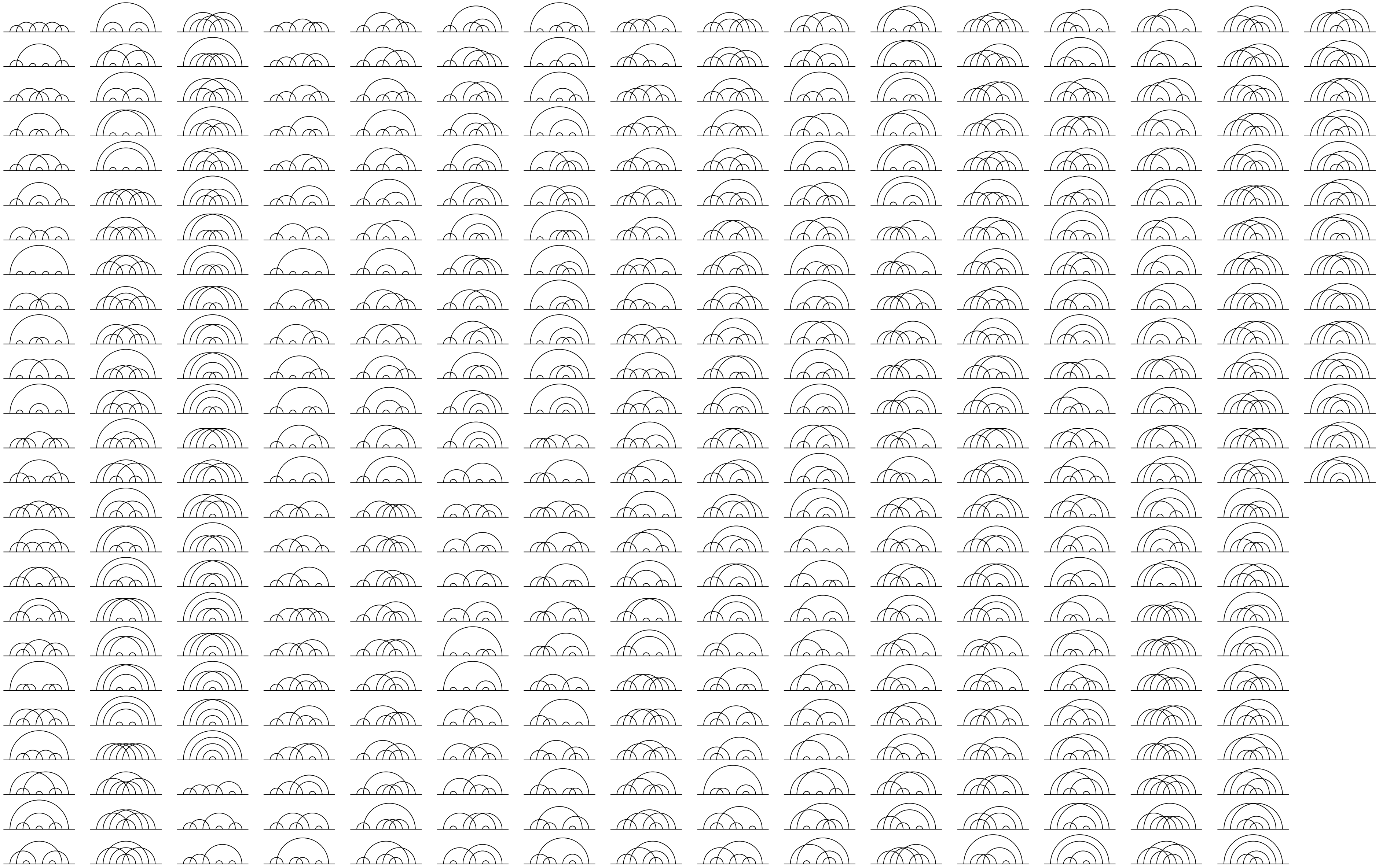
Five of these subsets are known analytically

[S. Laporta, Phys. Lett. B 328, 522 (1994)]

[J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)]

Complete numerical results have been published

[T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012); Phys. Rev. D 91, 033006 (2015)]



[T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)]

order $(\alpha/\pi)^4$: 891 diagrams

only some diagrams are known analytically $\longrightarrow$

numerical evaluation of Feynman-parametrized loop integrals

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]} \\
&= -1.557(70) && \text{[Kinoshita (1995)]} \\
&= -1.4092(384) && \text{[Kinoshita (1997)]} \\
&= -1.5098(384) && \text{[Kinoshita (2001)]} \\
&= -1.7366(60) && \text{[Kinoshita (2005)]} \\
&= -1.7260(50) && \text{[Kinoshita (2005)]} \\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]} \\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \leftarrow \\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]} \\
&= -1.91298(84) && \text{[Aoyama et al., Phys. Rev. D 91, 033006 (2015)]}
\end{aligned}
$$

$$a_\ell{}^{\mathrm{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.912\,07(84)$	$130.877\,3(61)$
$C_\ell^{(10)}$	$7.795(336)$	$751.92(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$$a_\ell^{\mathrm{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

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$\Delta C_e^{(8)} \cdot (\alpha/\pi)^4 \sim 0.2 \cdot 10^{-13}$ $\quad \Delta C_e^{(10)} \cdot (\alpha/\pi)^5 \sim 0.2 \cdot 10^{-13}$ $\quad \Delta a_e^{\mathrm{exp}} = 2.8 \cdot 10^{-13}$

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$(\alpha/\pi)^n$	$2.32\ldots \cdot 10^{-3}$	$5.39\ldots \cdot 10^{-6}$	$1.25\ldots \cdot 10^{-8}$	$2.91\ldots \cdot 10^{-11}$	$6.76\ldots \cdot 10^{-14}$

$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13}$ $\quad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$

$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13}$ $\quad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.6 \cdot 10^{-13}$ $\quad \Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$

$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9}$ $\quad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$

QED prediction ?

$\longrightarrow$ requires an input for the fine structure constant α that matches the experimental accuracy on a_e

$$\frac{\Delta a_e}{a_e} = 0.24\text{ppb} \rightarrow \frac{\Delta\alpha}{\alpha} \sim 0.24\text{ppb} \rightarrow \Delta\alpha \lesssim 2 \cdot 10^{-12}$$

- quantum Hall effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \qquad [19.7\text{ppb}]$$

[P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)]

- atomic recoil velocity through photon absorption

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{M_{\text{atom}}}{m_e} \cdot \frac{h}{M_{\text{atom}}}$$

$$\frac{\Delta R_\infty}{R_\infty} = 5 \cdot 10^{-12} \qquad \Delta\left(\frac{M_{\text{Rb}}}{m_e}\right) = 4.4 \cdot 10^{-10}$$

$$\alpha^{-1}[Cs\,02] = 137.036\,0001(11) \qquad [7.7\text{ppb}]$$

[A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)]

$$\alpha^{-1}[Rb\,06] = 137.035\,998\,84(91) \qquad [6.7\text{ppb}]$$

[P. Cladé et al, Phys. Rev. A 74, 052109 (2006)]

$$\alpha^{-1}[Rb\,08] = 137.035\,999\,45(62) \qquad [4.6\text{ppb}]$$

[M. Cadoret et al, Phys. Rev. Lett. 101, 230801 (2008)]

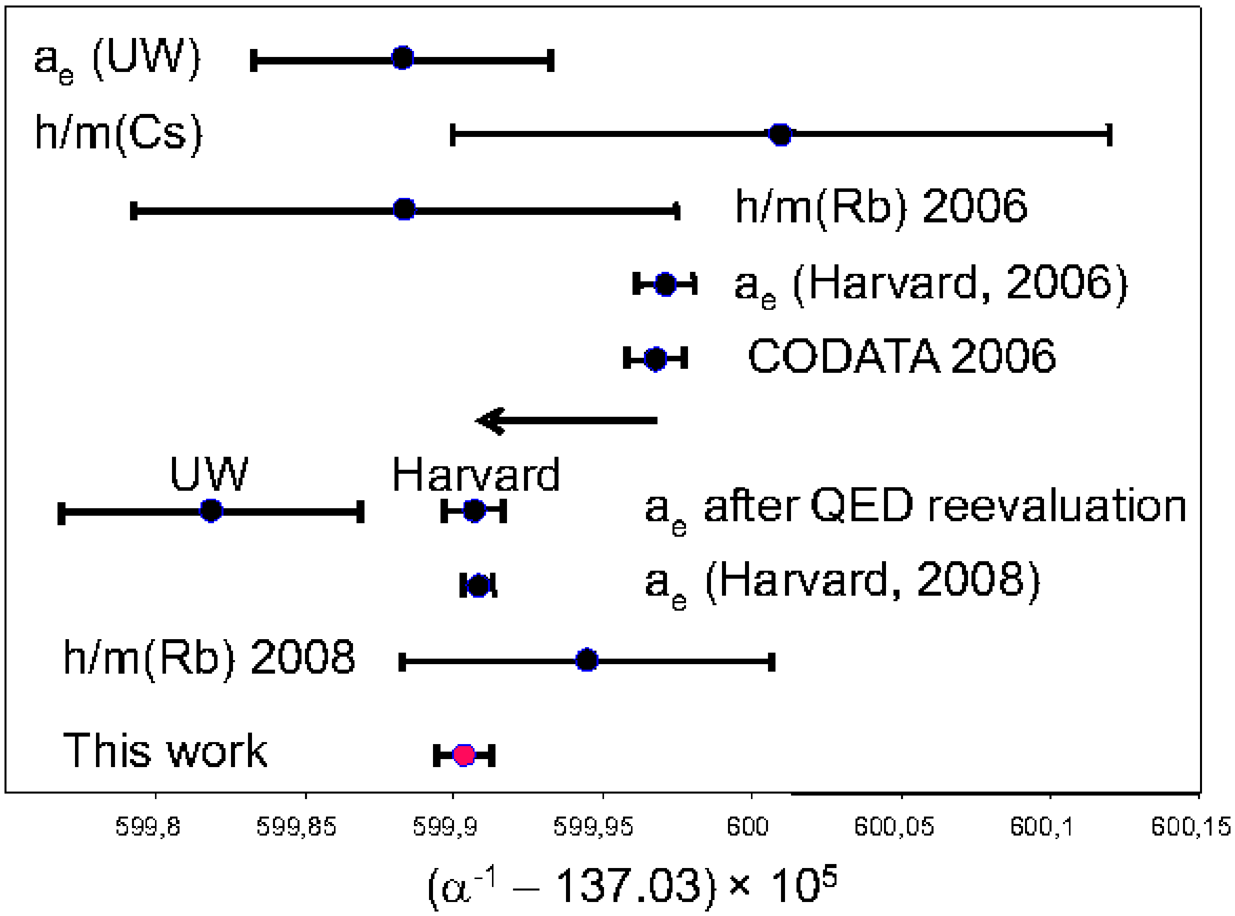
$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\text{ppb}]$$

[R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)]

$$a_e^{\rm QED} = 1\,159\,652\,179.908(25)_{\alpha^4}(23)_{\alpha^5}(763)_{\alpha(Rb11)} \cdot 10^{-12} \qquad a_e^{\rm exp} - a_e^{\rm QED} = 0.822(813)\cdot 10^{-12}$$

$$\alpha[a_e(HV\,08)] = 137.035\,999\,157\,0(29)_{\alpha^4}(27)_{\alpha^5}(18)_{\rm had}(331)_{\rm exp} \qquad [0.25\text{ppb}]$$

[Aoyama et al., Phys. Rev. D 91 033006 (2015)]



[R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)]

$$a_\mu^{\rm QED}(Rb) = 1\,165\,847\,189.51(9)_{\rm mass}(19)_{\alpha^4}(7)_{\alpha^5}(77)_{\alpha(Rb11)} \cdot 10^{-12}$$

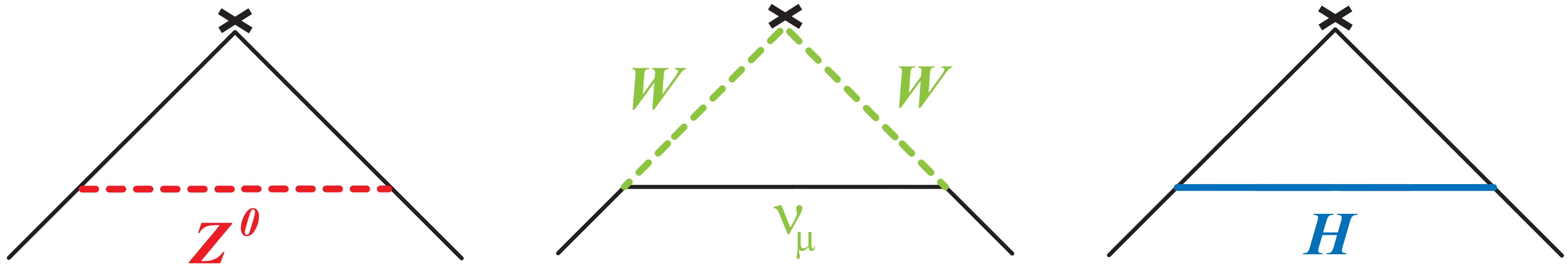
$$a_\mu^{\rm QED}(a_e) = 1\,165\,847\,188.46(9)_{\rm mass}(19)_{\alpha^4}(7)_{\alpha^5}(30)_{\alpha(a_e)} \cdot 10^{-12}$$

[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]

$$a_\mu^{\rm exp} - a_\mu^{\rm QED} = 737.0(6.3) \cdot 10^{-10}$$

Theory II: Weak interactions

• Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$
$$= 19.48\times10^{-10}$$

[W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)]

[G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)]

[R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)]

[I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)]

[M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)]

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

[A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)]

Two-loop fermionic contributions

[A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)]

[M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)]

$$a_\mu^{\text{weak}} = (154 \pm 1) \cdot 10^{-11}$$
$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Recent update: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

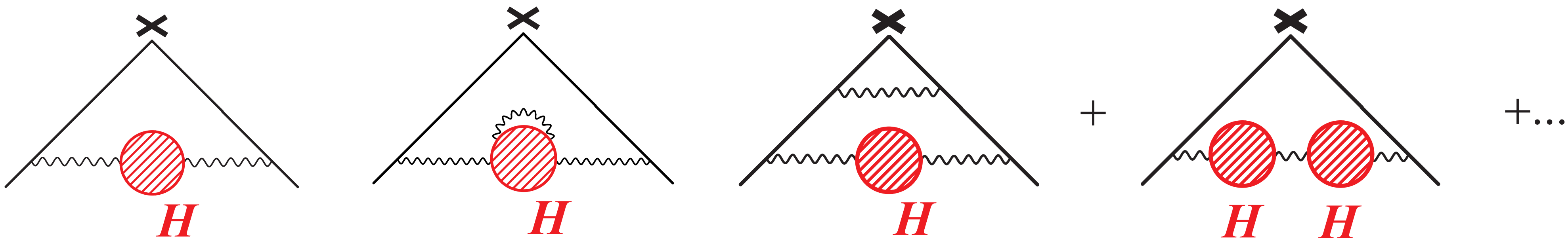
[C. Gnendinger et al., Phys. Rev. D 88, 053005 (2013)]

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathsf{weak}} = 721.65(6.38) \cdot 10^{-10}$$

Theory III: Strong interactions

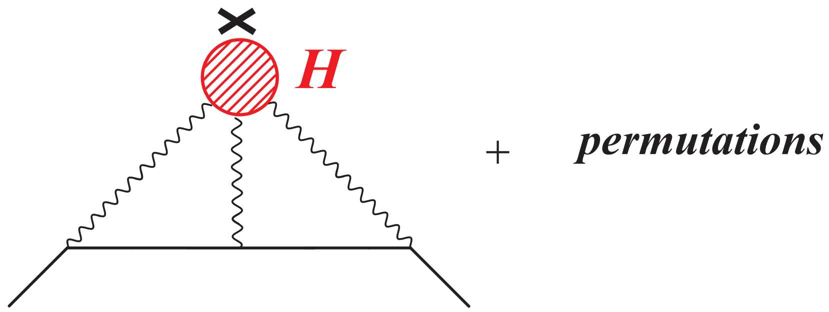
- Hadronic contributions : quark and gluon loops

$$a_\ell^{\text{had}} = a_\ell^{\text{HVP-LO}} + a_\ell^{\text{HVP-HO}} + a_\ell^{\text{HLxL}}$$



Lowest-order hadronic vacuum polarization (HVP-LO)

Higher-order hadronic vacuum polarization (HVP-HO)



Hadronic light-by-light scattering (HLxL)

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

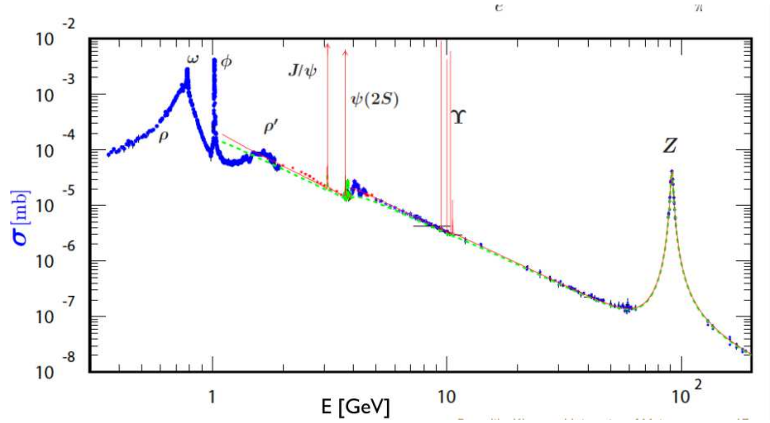
[C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)]
[L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)]
[M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)]

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

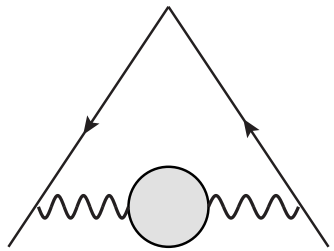
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

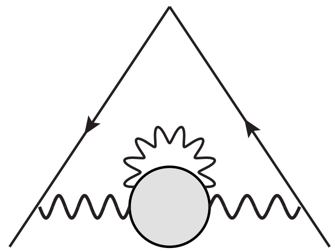
Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$





- Some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons

- some radiative exclusive modes, e.g. $\pi^0\gamma$

$$a_\mu^{\pi^0\gamma}(600\,\mathrm{MeV} - 1030\,\mathrm{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

- The two most recent determinations are in good agreement (being based on the same data sets, this should not be a surprise) and give a relative precision of 0.6%

Latest (published) results

$$a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$ [M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)]

$$a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-LO}} = 1.866(11) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013)]

$$a_\mu^{\text{HVP-NLO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^3 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K^{(2)}(t) R^{\text{had}}(t)$$

[J. Calmet, S. Narison, M. Perrottet, E. de Rafael, Phys. Lett. B 61, 283 (1976)]

[B. Krause, Phys. Lett. B 390, 392 (1997)]

$$a_\mu^{\text{HVP-NLO}} = -9.84 \pm 0.07 \cdot 10^{-10}$$ [K. Hagiwara et al., J. Phys. G 38, 085003 (2011)]

$$a_e^{\text{HVP-NLO}} = -0.2234(14) \cdot 10^{-12}$$ [D. Nomura, T. Teubner, Nucl. Phys. B 867, 236 (2013)]

$$a_\mu^{\text{HVP-NNLO}} = 1.24 \pm 0.01 \cdot 10^{-10}$$ [A. Kurz et al., Phys. Lett. B 734, 144 (2014)]

$$a_e^{\text{HVP-NNLO}} = 0.028(1) \cdot 10^{-12}$$

Hadronic vacuum polarization

- Some tension between, for instance, the high-precision data collected in the region of the ρ resonance by BaBar and KLOE/KLOE-2

Experiment	$a_\mu^{\rm HVP-LO\,2\pi}(600-900\ {\rm MeV})$
BaBar	376.7(2.0)(1.9)
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)

- These tensions need to be resolved in order to achieve higher precision

$\longrightarrow$ new data (KLOE-2, BaBar, VEPP-2000, BESIII,...)

Experiment	$a_\mu^{\rm HVP-LO\,2\pi}(600-900\ {\rm MeV})$
BESIII	368.2(2.5)(3.3) [BESIII Coll., Phys. Lett. B 753, 629 (2016)]

Hadronic vacuum polarization

- Update including new data available since 2011

$$
\begin{aligned}
a_\mu^{\text{HVP-LO}} &= 686.99 \pm 4.21 \cdot 10^{-10} \\
a_\mu^{\text{HVP-NLO}} &= -9.934 \pm 0.091 \cdot 10^{-10} \\
a_\mu^{\text{HVP-NNLO}} &= 1.226 \pm 0.012 \cdot 10^{-10}
\end{aligned}
$$

$$
\begin{aligned}
a_e^{\text{HVP-LO}} &= 1.8464(121) \cdot 10^{-12} \\
a_e^{\text{HVP-NLO}} &= -0.2210(14) \cdot 10^{-12} \\
a_e^{\text{HVP-NNLO}} &= 0.0279(2) \cdot 10^{-12}
\end{aligned}
$$

[F. Jegerlehner, arXiv:1511.04473 [hep-ph]]

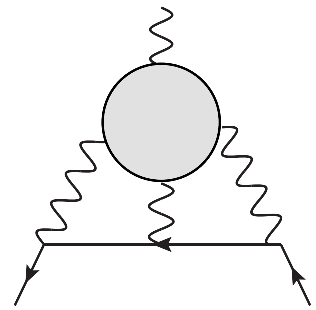
- Possibility to extract HVP from Bhabha scattering?

[C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)]

- Alternative for the (near?) future: Lattice QCD

Hadronic light-by-light: the really complicated thing

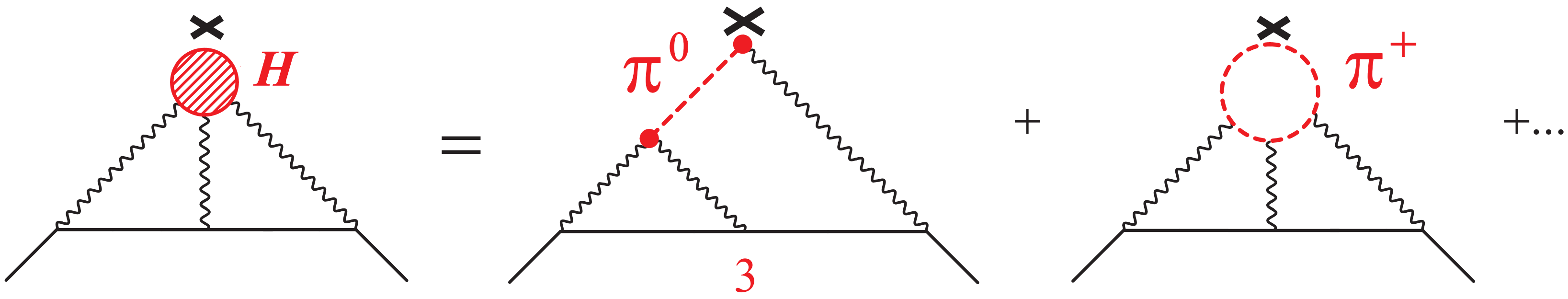
- occurs at order $\mathcal{O}(\alpha^3)$
- not related, as a whole, to an experimental observable...

? $\longrightarrow$ 

- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

- Many identifiable contributions...



Hadronic light-by-light: the really complicated thing

- Need some organizing principle: ChPT, large-N_c (turns out to be most relevant in practice) [E. de Rafael, Phys. Lett. B 322, 239 (1994)]

$$a_\mu^{\rm HLxL} = N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa\right] + \mathcal{O}(N_c^0)$$

[M. Knecht, A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

[M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)]

M. J. Ramsey-Musolf, M. B. Wise, Phys. Rev. Lett. 89, 041601 (2002)]

[J. Prades, E. de Rafael, A. Vainshtein, Glasgow White Paper (2008)]

- Impose QCD short-distance properties [K. Melnikov, A. Vainshtein, Phys. Rev. D, 113006 (2004)]

- Only two (so far) attemps at a “complete”, but model-dependent calculation...

$$a_\mu^{\text{HLxL}} = +(8.3 \pm 3.2) \cdot 10^{-10}$$

[J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)]

$$a_\mu^{\text{HLxL}} = +(89.6 \pm 15.4) \cdot 10^{-11}$$

[M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)]

[M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)]

...after the sign change [M.K. and A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

Hadronic light-by-light: the really complicated thing

Recent (partial) reevaluations

$$a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$ [J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306]

"best estimate"

$$a_\mu^{\text{HLxL}} = (11.5 \pm 4.0) \cdot 10^{-10}$$ [A. Nyffeler, Phys. Rev. D 79, 073012 (2009)]

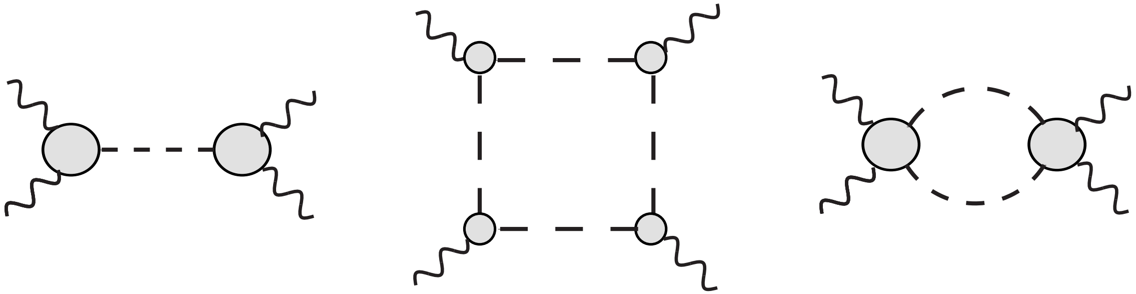
more conservative estimate

$$a_e^{\text{HLxL}} = (0.035 \pm 0.010) \cdot 10^{-12}$$ [J. Prades, E. de Rafael, A. Vainshtein, in *Lepton Dipole Moments*]

Hadronic light-by-light: the really complicated thing

- More recently: dispersive approaches

- for $\Pi_{\mu\nu\rho\sigma}$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

[G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); arXiv:1506.01386 [hep-ph]]

Needs input from data (transition form factors,...)

[G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)]

[A. Nyffeler, arXiv:1602.03398 [hep-ph]]

Main unanswered issues:

- how will short-distance constraints be imposed?
- how will Π^{residual} be estimated? Cf. axial vectors (leading in large-N_c) $\to 3\pi$ channel

- for $F_2^{\text{HLxL}}(k^2)$

only pion pole with VMD form factor (two-loop graph) reconstructed this way so far

[V. Pauk and M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014) [arXiv:1409.0819 [hep-ph]]]

Summary - Conclusions

Perspectives for the future

- The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$
\begin{aligned}
a_e^{\text{exp}} &= 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}] \\
a_\mu^{\text{exp}} &= 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]
\end{aligned}
$$

- The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}]$$
$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]$$

- In the case of the electron, there is good agreement between theory and experiment:

$$a_e^{\text{exp}} - a_e^{\text{SM}} = (-0.91 \pm 0.82) \cdot 10^{-12}$$

- The anomalous magnetic moments of the electron and of the muon are among the most precisely measured observables of the standard model

$$
\begin{aligned}
a_e^{\text{exp}} &= 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\text{ppb}] \\
a_\mu^{\text{exp}} &= 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\text{ppm}]
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$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim 30 \cdot 10^{-10} \;\sim\; 2 \cdot a_\mu^{\text{weak}} \;\sim\; a_\mu^{\text{QED}}(\alpha^4)$$

$$a_\mu^{\text{HVP-LO}} = (692.3 \pm 4.2) \cdot 10^{-10} \qquad a_\mu^{\text{HVP-HO}} = (-8.60 \pm 0.07) \cdot 10^{-10}$$

$$a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$

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Higher order QED effects?

$$A_2^{(12)}(m_\mu/m_e) \sim A_2^{(6)}(m_\mu/m_e;\text{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \sim 0.6 \cdot 10^4 \longrightarrow \delta a_\mu^{\text{QED}} \sim 1 \cdot 10^{-12}$$

Higher order QCD effects?

$$a_\mu^{\text{HVP-NNLO}} = (1.24 \pm 0.01) \cdot 10^{-10} \qquad a_\mu^{\text{HLxL-HO}} \sim (0.3 \pm 0.2) \cdot 10^{-10}$$

[A. Kurz et al., Phys. Lett. B 734, 144 (2014)] [G. Colangelo et al., Phys. Lett. B 735, 90 (2014)]

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Manifestation of BSM degrees of freedom?

$\longrightarrow$ many proposals, situation remains inconclusive

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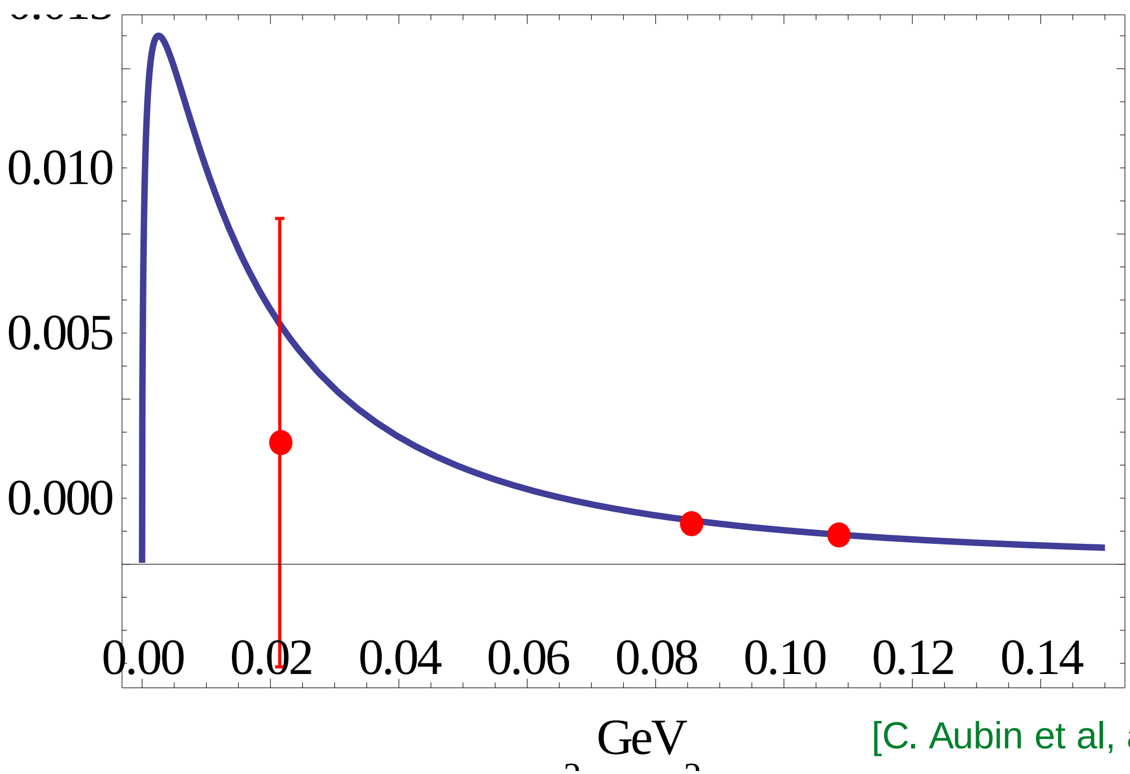
- Emergence of a pattern?

Observable	Tension wrt SM	Limited by
$B \to D^{(*)}\tau\nu / B \to D^{(*)}\ell\nu, \ell = e, \mu$	3.9σ	experiment
$(g-2)_\mu$	$\gtrsim 3.5\sigma$	exp. and th.
$B^0 \to K^{*0}\mu\mu$ ang. dist., BR	3.4σ	exp. and th.
$B_s \to \phi\mu\mu$ BR	3.0σ	experiment
Dimuon CP asymmetry	3.0σ	experiment
V_{ub} exclusive vs. inclusive	3.0σ	exp. and th.
$B^+ \to K^+\mu\mu / B^+ \to K^+ ee$	2.6σ	experiment
$h \to \tau\mu$	2.4σ	experiment

[cf. O. Leroy, talk at Lake Louise Winter Institute, Feb. 2016]

Can lattice QCD contribute?



[C. Aubin et al, arXiv:1311.5504 [hep-lat]]

Strategies are being devised

- twisted boundary conditions [C. Aubin, T. Blum, M. Golterman and S. Peris, arXiv:1307.4701 [hep-lat]]

- time moments and Padé approximants [B. Chakraborty et al, arXiv:1403.1778 [hep-lat]]

[M. Golterman, K. Maltman and S. Peris, arXiv:1405.2389 [hep-lat]]

- Moments of the MB transform [E. de Rafael, Phys. Lett. B 736, 522 (2014)]

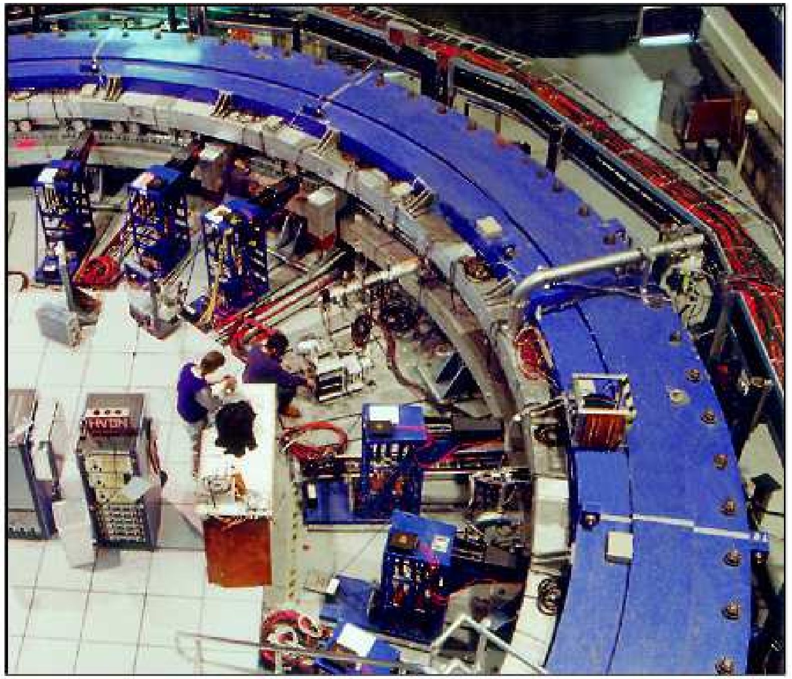
First results also for HLxL...

[T. Blum, S. Chowdhury, M. Hayakawa, T. Izubuchi, Phys. Rev. Lett. 114, 012001 (2015)]

- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL and at J-PARC

Present experimental status and the new project at FNAL

David Hertzog
University of Illinois at Urbana-Champaign



Muon g-2 and EDM in the LHC era - Feb. 2010


EMMERT

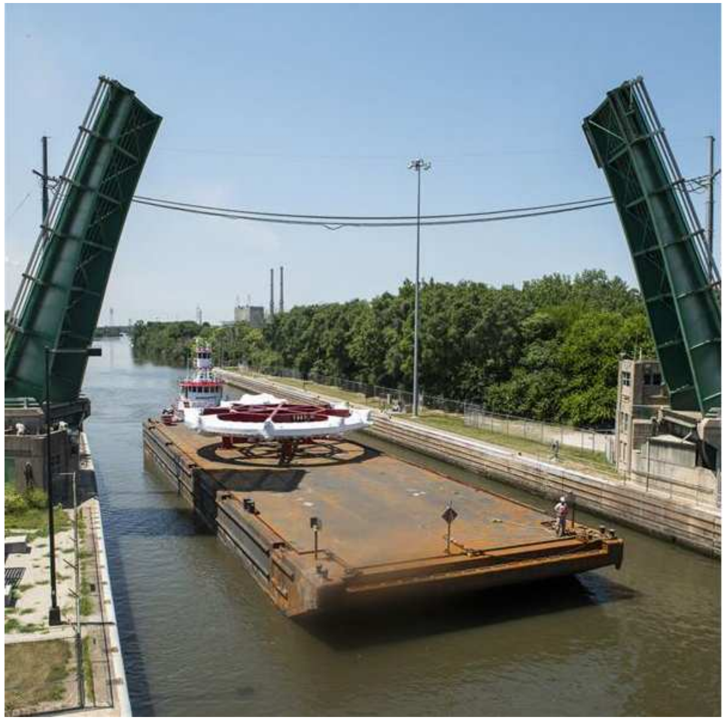
5 nights sheltering from
Storm near Norfolk
Virginia.

Tennessee-Tombigbee Waterway
Mississippi, Illinois and
Des Plaines rivers.

25th June - July 20th

S. Maxfield, PhiPsi 2013


MISS KATIE




OVERSIZE LOAD
EMMERT


EMMERT

S. Maxfield, PhiPsi 2013

Magic vs "New Magic"

- **Complimentary!**

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

BNL/Fermilab Approach

$$a_\mu - \frac{1}{\gamma^2 - 1} = 0$$

$$\eta \approx 0$$

$$\gamma_{\text{magic}} = 29.3$$

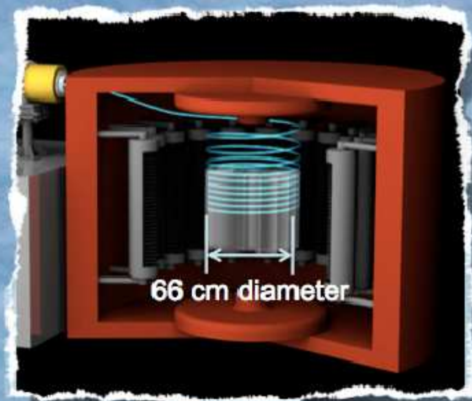
$$p_{\text{magic}} = 3.09\ \text{GeV}/c$$

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

J-PARC Approach

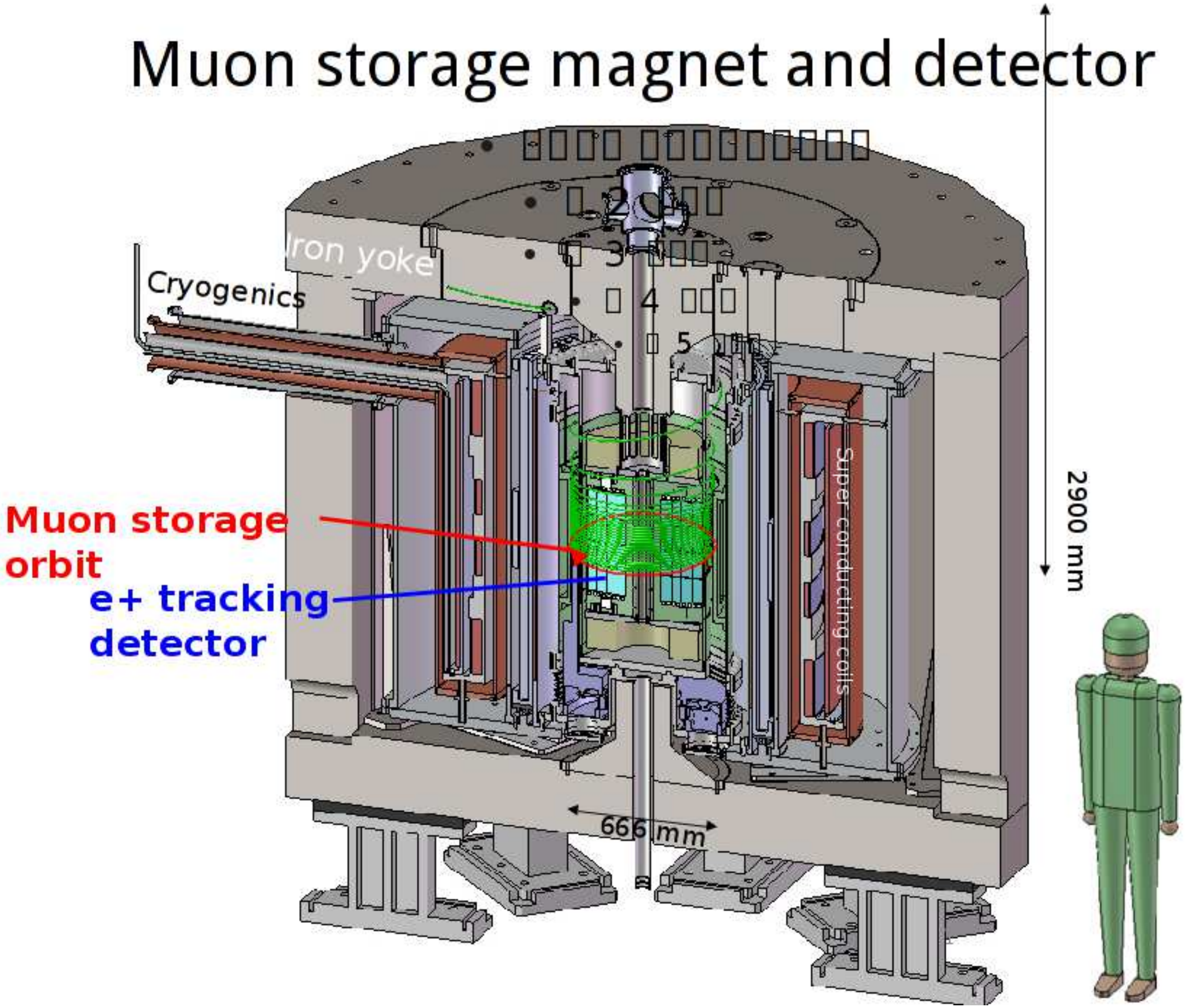
$$\vec{E} = 0$$

$$\vec{\omega} = \vec{\omega}_a + \vec{\omega}_\eta$$



N. Saito, LPNHE Workshop Paris, May 2012

Muon storage magnet and detector



- Two new experiments, with the aim of reducing the experimental uncertainty by a factor of 4, are being prepared, at FNAL and at J-PARC (first results expected in ~ 2 years)

- New high-precision data from VEPP-2000, BESIII,...

- a_e is expected to be about $(m_\mu/m_e)^2 \sim 40000$ times less sensitive to BSM effects that a_μ. But it is known with much better (~ 2300) precision... Possibilities to observe BSM effects through a_e

[G. F. Giudice, P. Paradisi, M. Passera, JHEP 1211, 113 (2012)]

- Needs improvements on the determinations of a_e [from 0.24ppb to 0.06ppb], of R_∞, $m_e/m_{\rm u}$,...

$$\alpha^2 = \frac{2R_\infty}{c}\frac{M_{\rm at}}{m_{\rm u}}\frac{m_{\rm u}}{m_e}\frac{h}{M_{\rm at}}$$

that are within reach on a timescale similar to the one of the new $(g-2)_\mu$ experiments.

[F. Terranova, G. M. Tino, Phys. Rev. A 89, 052118 (2014)]

Thanks for your attention!