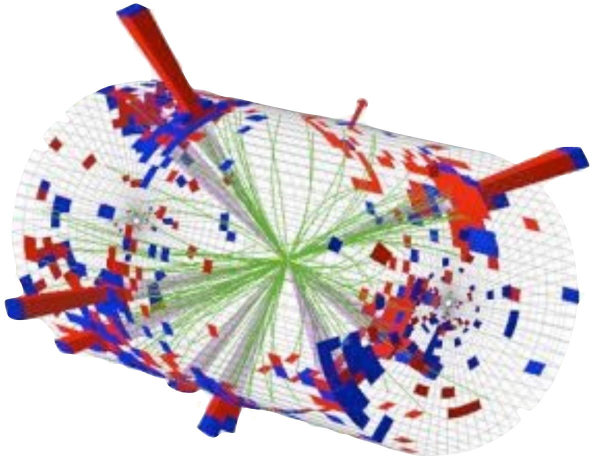

UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

An Effective Theory for Jet Processes

based on 1508.06645 with Neubert, Rothen and Shao

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Seminar at University of Wien, March 8, 2016

Overview

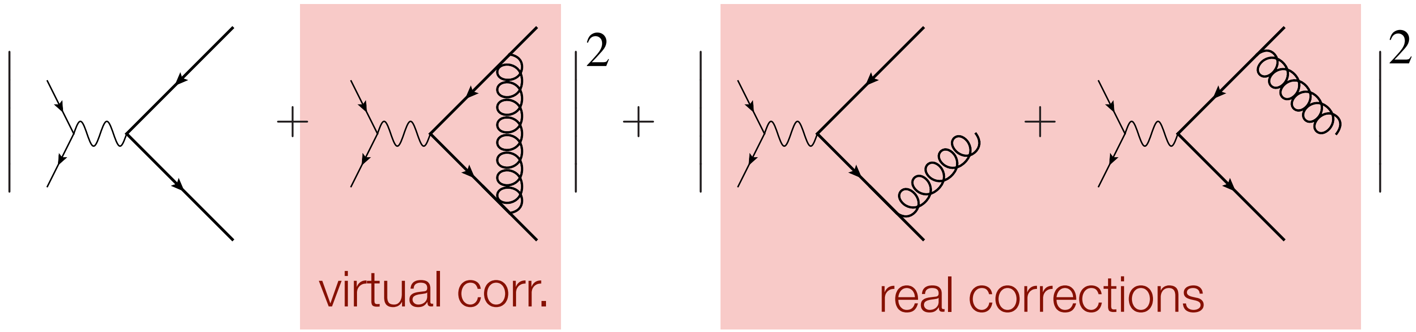
- Introduction
  - Jet cross sections
  - Soft-collinear factorization
- Factorization and resummation for jets
  - Small angle soft radiation
  - Factorization for jet cross sections
  - RG evolution and resummation
- Appendices…

Jet Cross Sections

NLO result for total hadronic cross section

$e^+e^- \to q\bar{q}$ $\qquad$ $e^+e^- \to q\bar{q}g$

$\sigma_{\text{tot}} =$



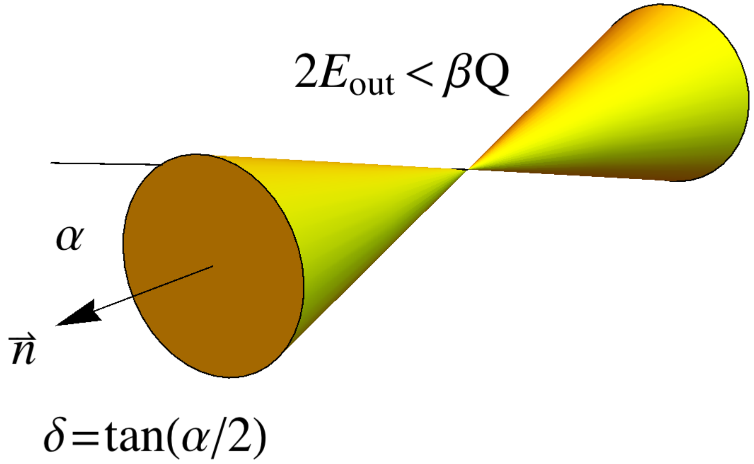
$$= \sigma_0 \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

Real and virtual corrections suffer from soft and collinear infrared divergences, e.g.

$$\sigma_{\text{virtual}} = \sigma_0 \frac{2\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2} \right)^{\epsilon} \left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 16 + \frac{7\pi^2}{3} \right)$$

in d=4-2ε. Divergences cancel in the sum!

Sterman-Weinberg jets '77

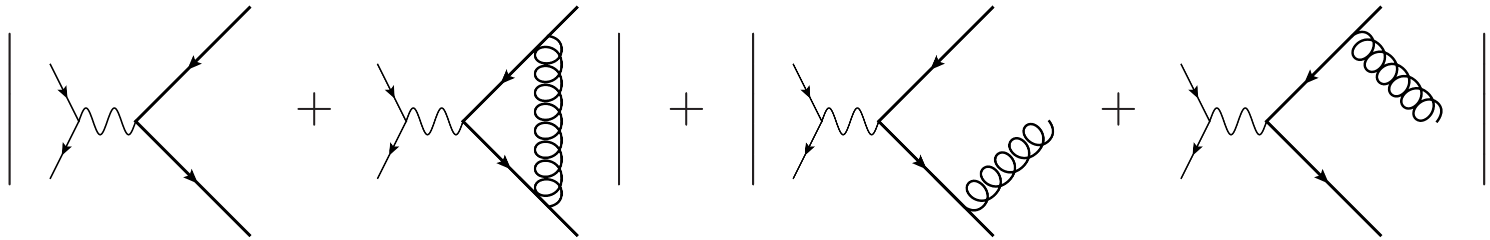


Original definition of a two-jet cross section in e^+e^- collisions with cms energy Q. Two parameters

- Cone angle parameter δ
- Energy fraction outside the cone β

Infrared safe: σ(δ, β) includes for soft and collinear radiation.

NLO result for jet cross section

$\sigma(\delta, \beta) =$ 

$$= \sigma_0 \left\{ 1 + \frac{\alpha_s(\mu)}{3\pi} \left[-16 \ln \delta \ln \beta - 12 \ln \delta + 10 - \frac{4\pi^2}{3} + \mathcal{O}(\delta, \beta) \right] \right\}$$

Sterman & Weinberg '77

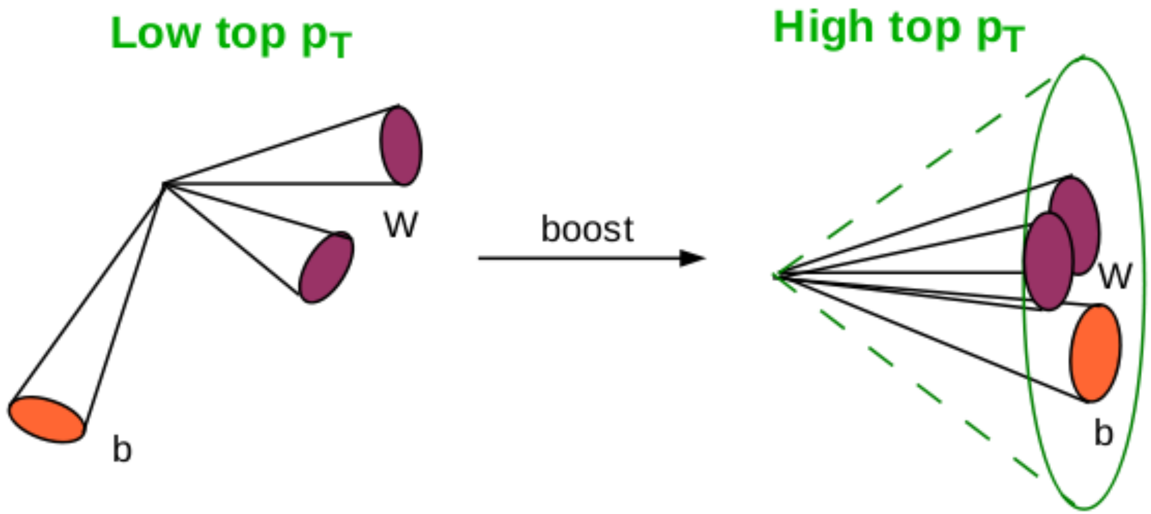
IR finite, but problems for small β and/or δ:

1) Large logarithms can compensate α_s suppression: fixed-order perturbation theory becomes unreliable.

2) Value of μ ? $\mu = Q$, $Q\,\beta$, $Q\,\delta$, $Q\,\beta\,\delta$?

Why narrow jets?

Analysis of jet substructure can provide important information.



Jet substructure studies are currently based on parton shower. Would be important to be able to obtain systematically improvable predictions

- Need higher-log resummation for narrow jets. (For LL results, see Dasgupta et al. '15, '16)

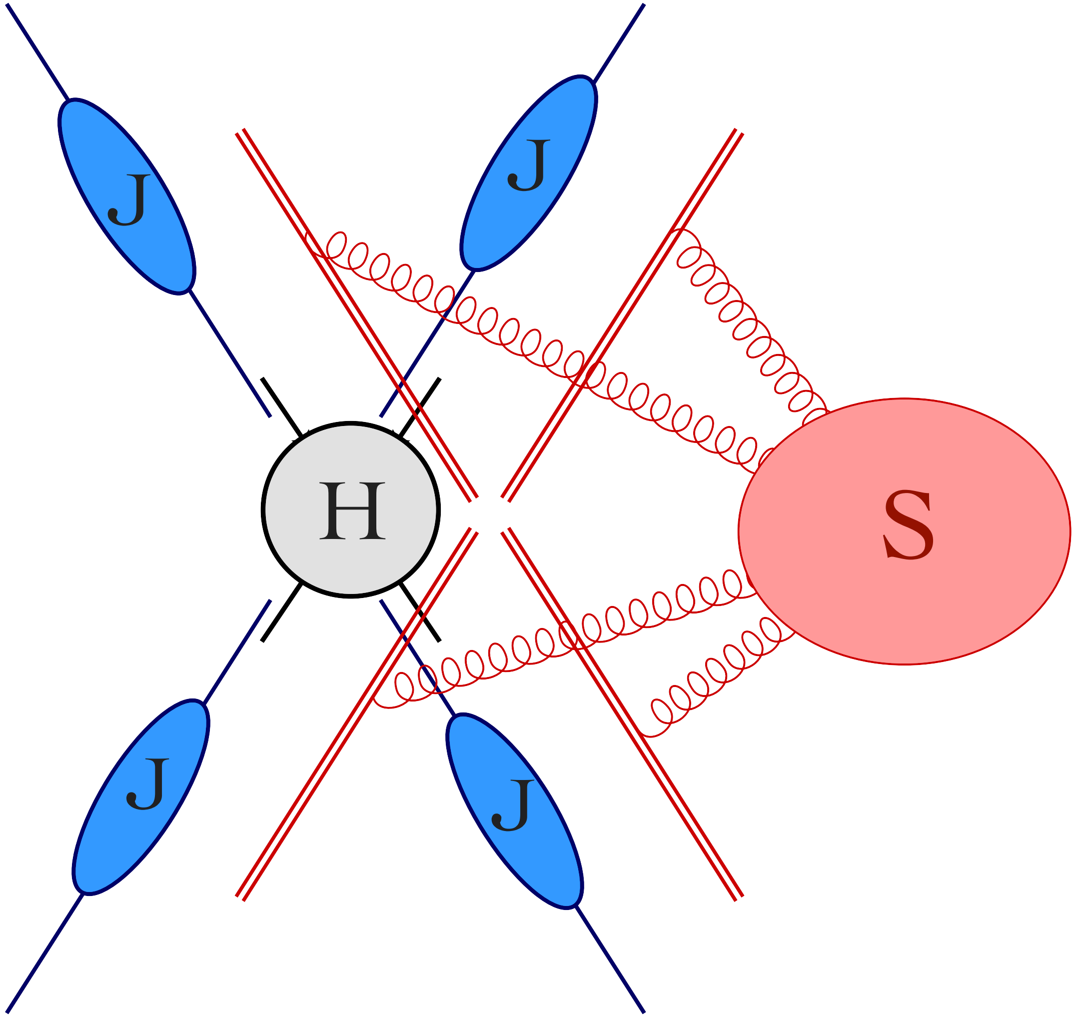
An EFT for jet processes

In the following, we will describe an effective theory which

- separates the contributions associated with different scales ("factorization")
- allows one to resum large logarithms of scale ratios ("resummation")

Despite the fact that jet cross sections are the most important class of collider observables, factorization and resummation beyond the leading logarithms has not been achieved earlier.

- Higher-log resummation was available only for global variables such as thrust, broadening, C-parameter, …



Soft-collinear factorization

QCD made simple(r)

There are two limits where the perturbative expressions for the scattering of quarks and gluons simplify considerably

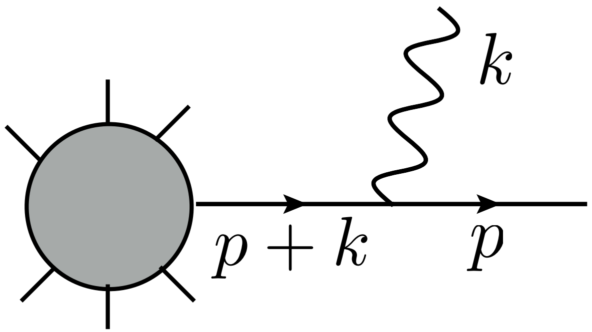
- Collinear limit, where multiple particles move in a similar direction.
- Soft limit, in which particles with small energy and momentum are emitted.

At the same time the cross sections are enhanced in these regions.

- Large logarithms $\ln(\beta)$ and $\ln(\delta)$ in SW cross section arise from soft and collinear emissions!

Soft limit

When particles with small energy and momentum are emitted, the amplitudes simplify:



$$\bar{u}(p)\not{\varepsilon}(k,\lambda)\,\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\cdots$$

$$\approx \boxed{\frac{p\cdot\varepsilon(k,\lambda)}{p\cdot k}}\,\bar{u}(p)\ldots$$

Soft emission factors from the rest of the amplitude.

Denominator $p\cdot k = E\,\omega\,(1-\cos\theta)$ leads to logarithmic enhancements at small energy and small angle.

Wilson lines

Multiple emissions can be obtained from

$$S_i = \mathbf{P} \exp \left[ig \int_0^\infty ds\, n_i \cdot A_s^a(sn_i) \boldsymbol{T}_i^a \right]$$

$n_i^\mu = p_i^\mu / E$ is a vector in the direction of the energetic particle, and $\boldsymbol{T}_i^a$ is its color charge. $\mathbf{P}$ indicates that the color matrices are path ordered.

Emissions are only sensitive to the total charge $\boldsymbol{T}_i$ of the object they radiate off. Also, the emission of soft quarks is suppressed compared to gluon emission.

Wilson line and eikonal interaction

Consider one-gluon matrix element of Wilson line

$$\langle k, \lambda, b| \boldsymbol{S}_i |0\rangle = i g_s \boldsymbol{T}^a \int_0^\infty ds \, \langle k, \lambda, b| n_i \cdot A^a(s n_i)|0\rangle + \mathcal{O}(g_s^2)$$

$$= i g_s \boldsymbol{T}^a \int_0^\infty ds \, e^{i s n_i \cdot k} \langle k, \lambda, b| n_i \cdot A^a_\mu(0)|0\rangle$$

$$= i g_s \boldsymbol{T}^b n_i \cdot \varepsilon(k,\lambda) \frac{e^{i s n_i \cdot k}}{i n_i \cdot k}\Bigg|_0^\infty$$

need small imaginary part n · k ≡ n · k + iε

$$= -g_s \boldsymbol{T}^b \frac{n_i \cdot \varepsilon(k,\lambda)}{n_i \cdot k} = \boxed{-g_s \boldsymbol{T}^b \frac{p_i \cdot \varepsilon(k,\lambda)}{p_i \cdot k}}$$

eikonal interaction

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

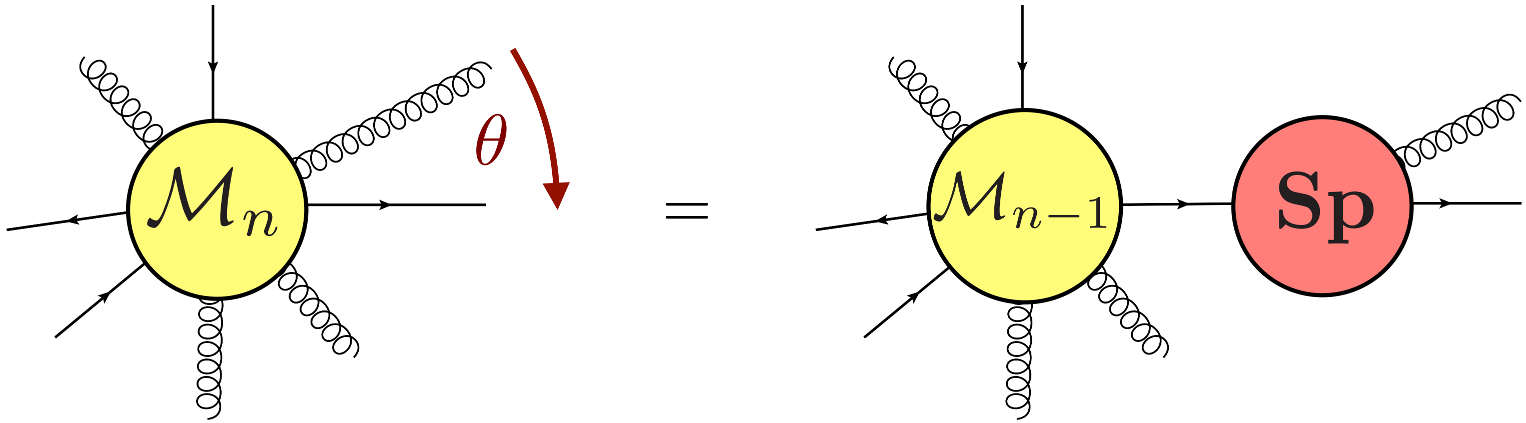
$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles / jets (color matrices)

hard scattering amplitude with *m* particles (vector in color space)

If one considers a jet of several (nearly) collinear energetic particles, their soft radiation is described by a single Wilson line with the total color charge.

Collinear factorization

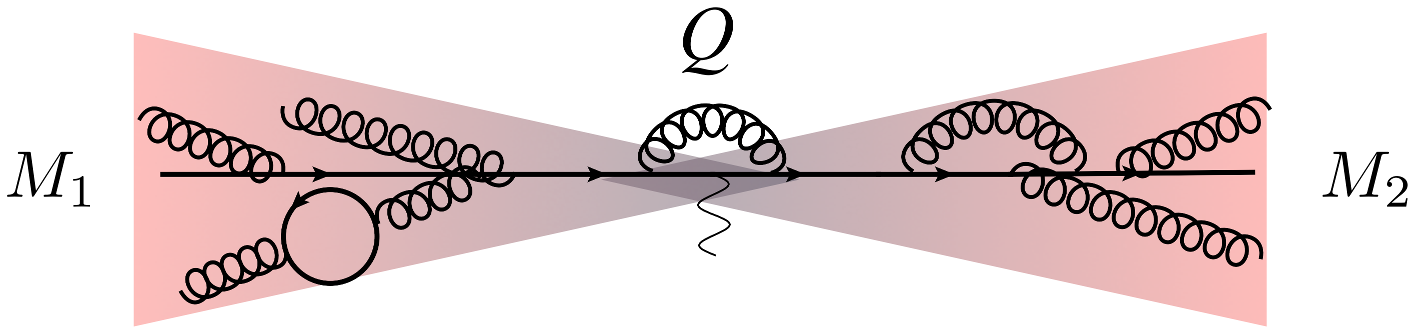


In the limit $\theta \longrightarrow 0$, where the partons become collinear, the n-parton amplitude factorizes into a product of an (n-1)-parton amplitude times a splitting amplitude $\mathbf{Sp}$. Similarly for several collinear partons.

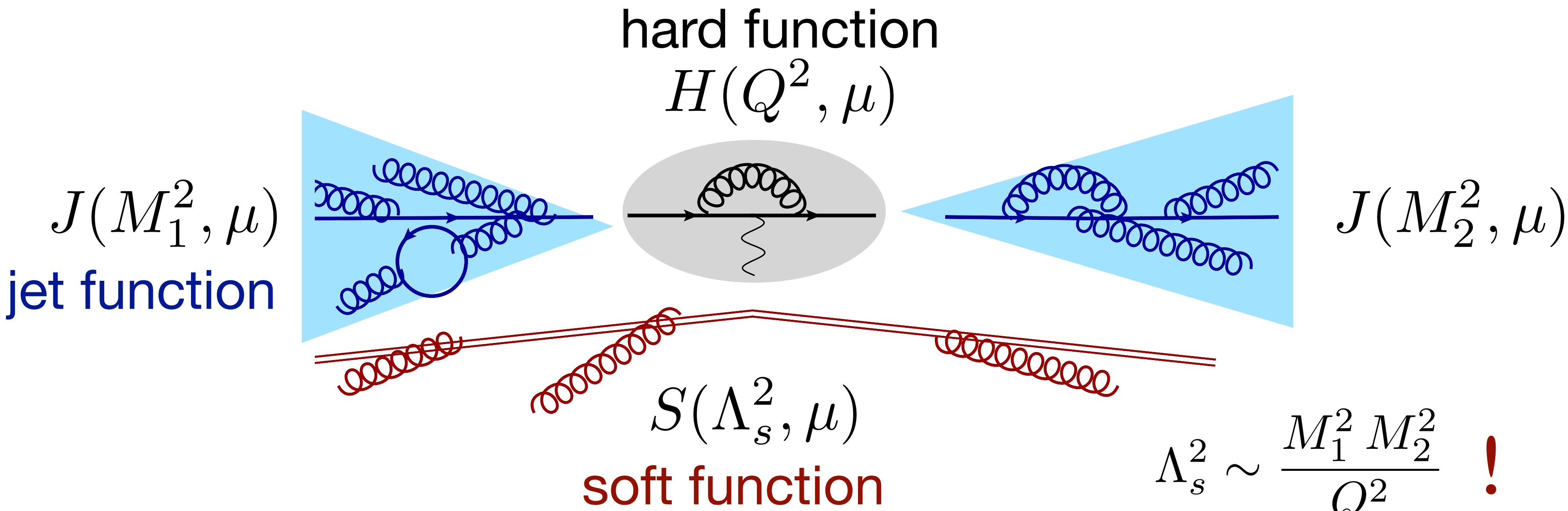
Leading contribution to the squared amplitude does not involve interference with the other particles!

…but see Almelid, Duhr, Gardi 1507.00047!

Soft-Collinear Factorization: 2-jet case



For $M_1 \sim M_2 \ll Q$ the cross section factorizes:



$$\Lambda_s^2 \sim \frac{M_1^2\, M_2^2}{Q^2} \quad !$$

Soft-Collinear Effective Theory

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke, Diehl et al. 2002; ...

In collider processes, we have an *interplay of three momentum regions*

Hard } **high-energy**

Collinear

Soft } **low-energy part**

Correspondingly, EFT for such processes has two low-energy modes:

collinear fields describing the energetic partons propagating in each direction of large energy, and

soft fields which mediate long range interactions among them.

Diagrammatic Factorization

The simple structure of soft and collinear emissions forms the basis of the classic factorization proofs, which were obtained by analyzing Feynman diagrams.

Collins, Soper, Sterman 80's ...

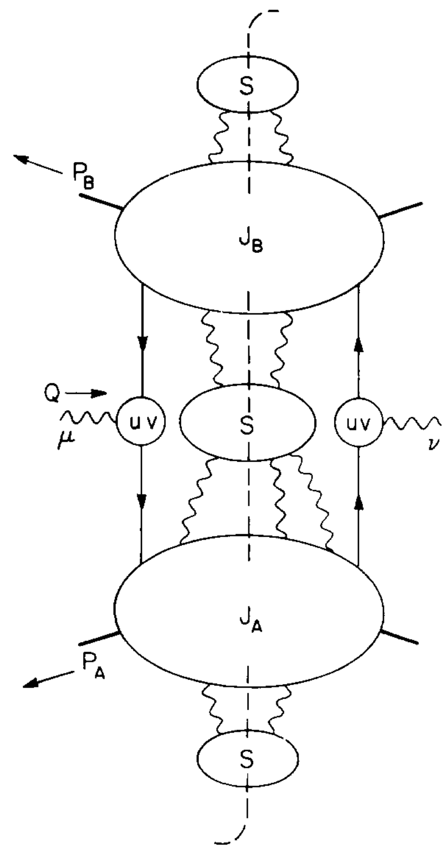
Advantages of the the SCET approach:

Simpler to exploit gauge invariance on the Lagrangian level

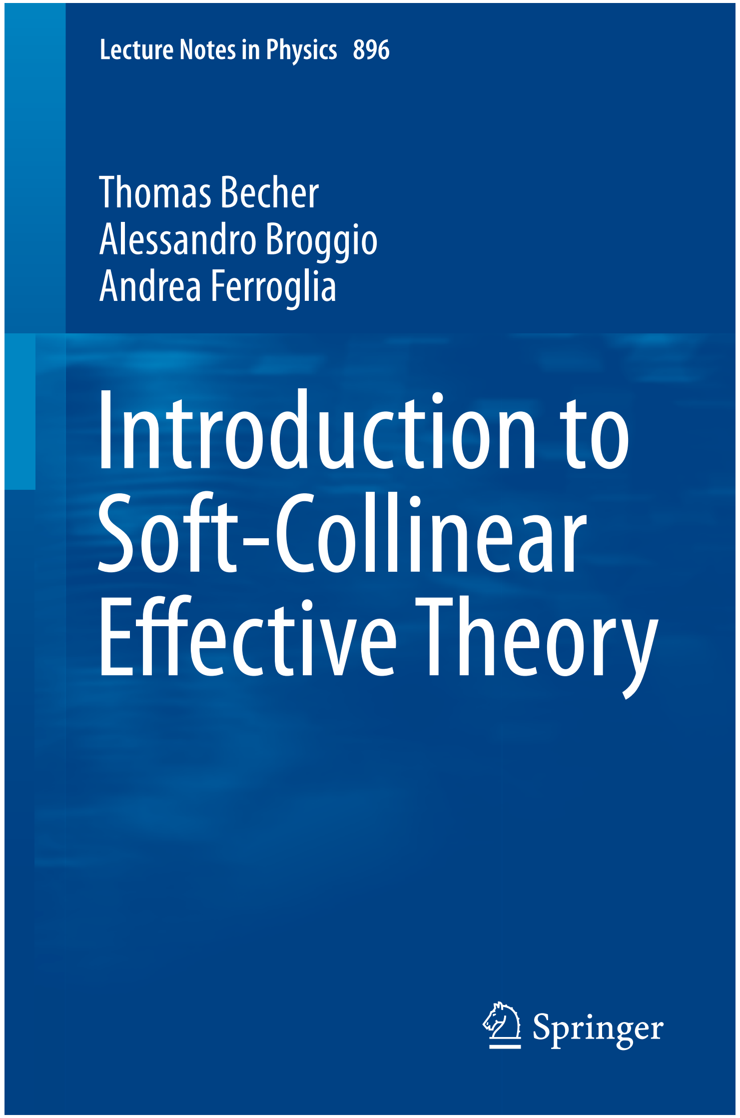
Operator definitions for the soft and collinear contributions

Resummation with renormalization group

Can include power corrections



Collins and Soper '81

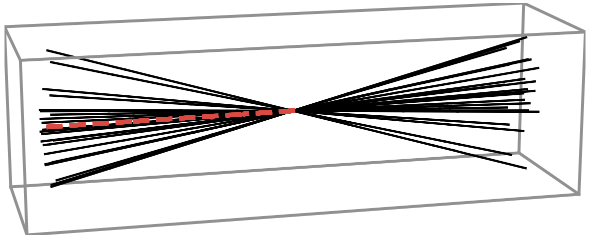


arXiv:1410.1892

See also Iain Stewart's <u>edX</u> EFT online course.

Resummation for Thrust

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



$$1 - T \approx \frac{M_1^2 + M_2^2}{Q^2}$$

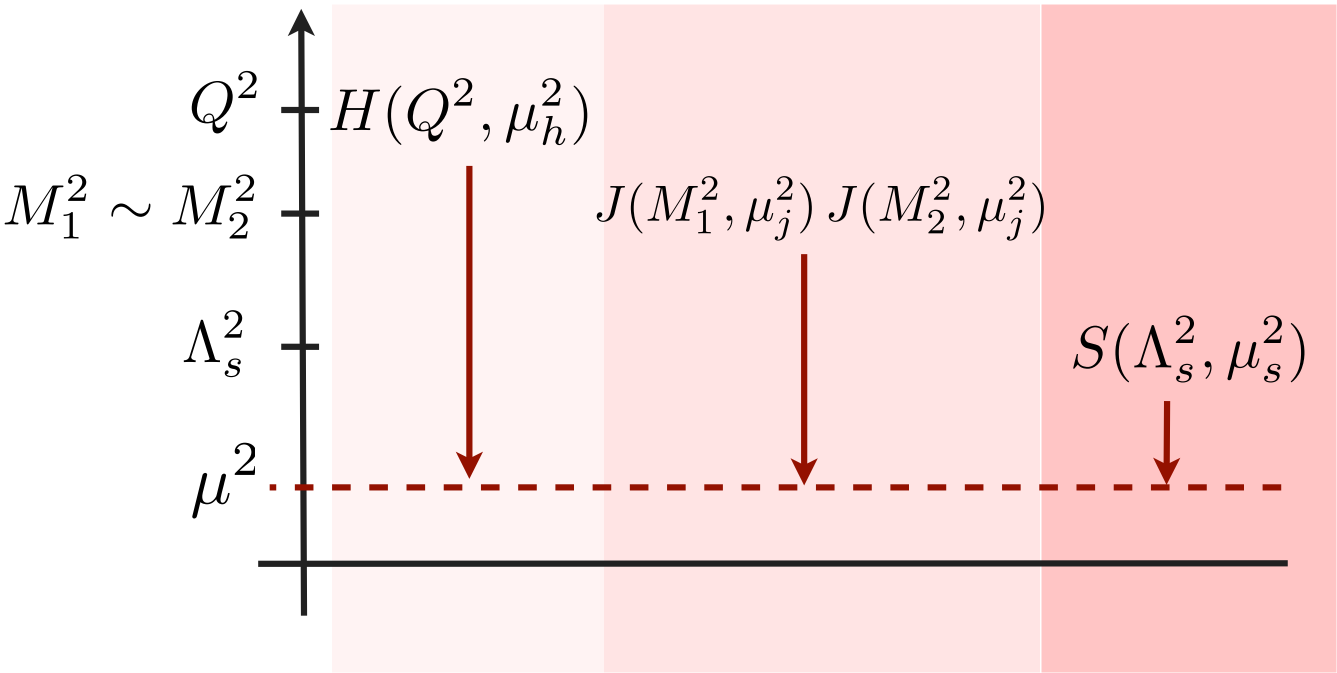
- The perturbative result for the thrust distribution contains logarithms $\alpha_s^n \ln^{2n} \tau$, where $\tau = 1\text{-}T$.
    - Near the end-point $\tau \to 0$ the logarithmic terms dominate.
- Using SCET one can derive the factorization theorem

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T\!\left(\tau Q - \frac{M_1^2 + M_2^2}{Q}, \mu\right)$$

Q^2	$\gg$	$M_1^2 \sim M_2^2 \sim \tau Q^2$	$\gg$	$\tau^2 Q^2$
hard		collinear		soft

Resummation by RG evolution

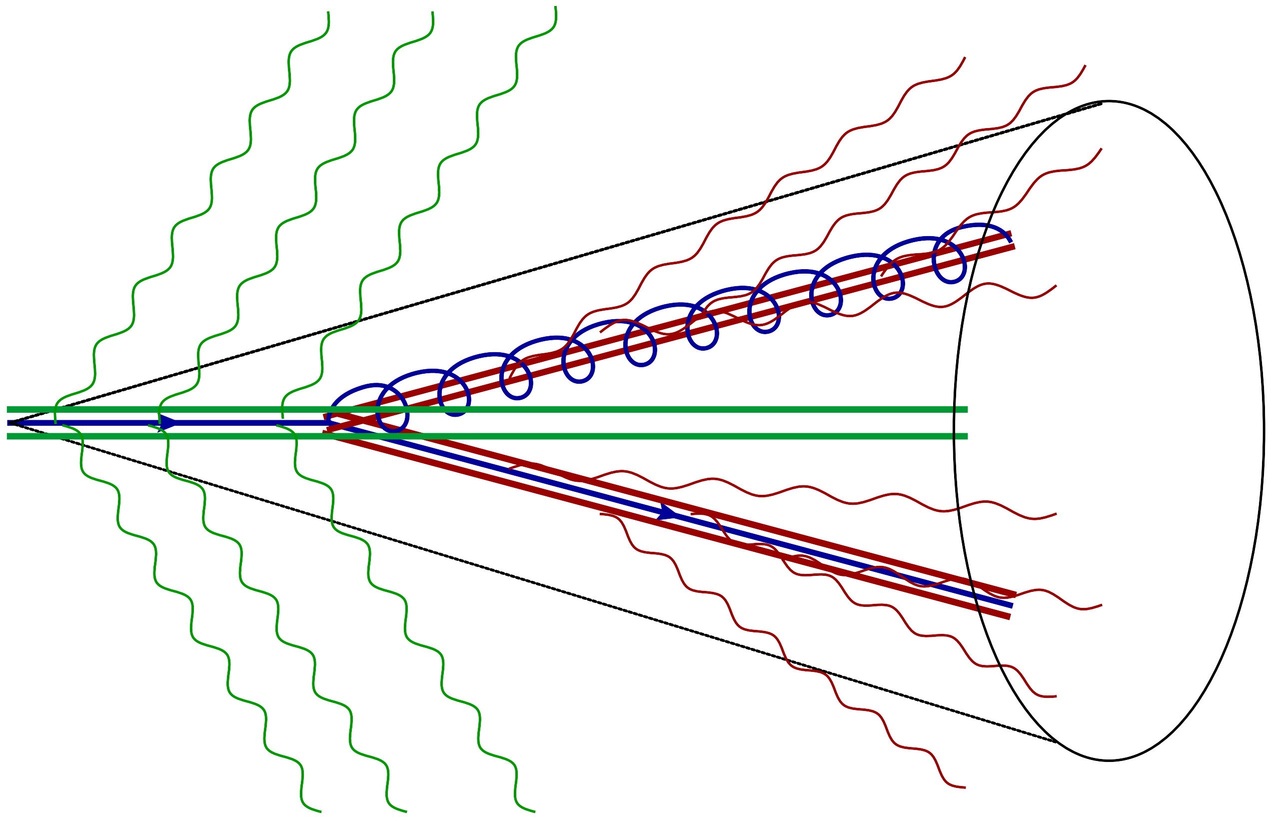
Evaluate each part at its characteristic scale, evolve to common reference scale μ



Each contribution is evaluated at its natural scale. No large perturbative logarithms.

→N³LL resummation for thrust TB, Schwartz '08.

Precision determination of α_s Abbate et al. '11.



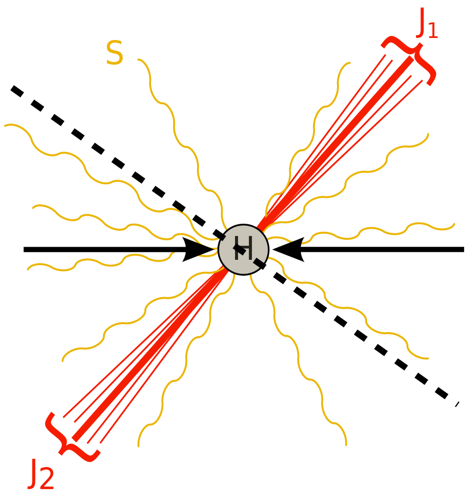
From SCET to J_{et} $E_{ffective}$ T_{heory}

factorization and resummation for jet processes

TB, Neubert, Rothen, Shao, arXiv:1508.06645

Non-global logarithms

Dasgupta, Salam '01



$$\frac{d\sigma}{dM} \stackrel{?}{=} H \cdot J_1 \otimes J_2 \otimes S$$

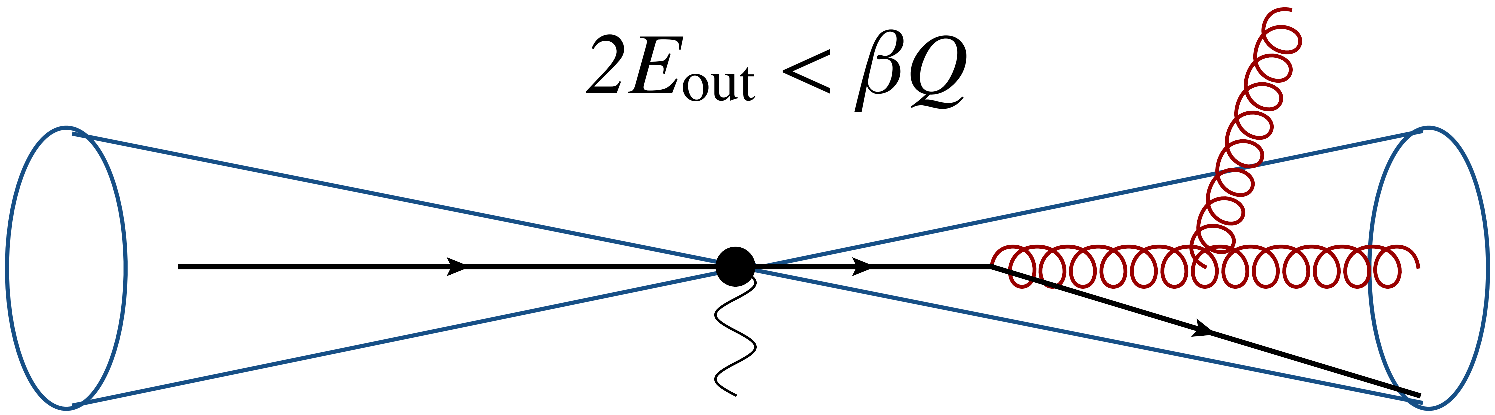
Consider hemisphere jet masses M_1 and M_2 in $e^+e^- \to 2$ jets. Factorization and resummation works for

$$Q^2\tau \approx M_{tot}{}^2 = M_1{}^2 + M_2{}^2 \text{ or } M_h = \max(M_1, M_2)$$

but fails for the non-global observables

$$M = M_1 \text{ or } M_l = \min(M_1, M_2)$$

Non-global, because it only probes one hemisphere.



Jet observables are non-global because they are insensitive to emissions inside the jets.

Dasgupta and Salam ‘02 extracted the leading non-global logarithm; arises from gluons inside jet radiating back out.

$$\propto \alpha_s^2 C_F C_A \pi^2 \ln^2 \beta$$

These types of logarithms do not exponentiate.

LL resummation

- The leading logarithms arise from configurations in which the emitted gluons are **strongly ordered**

$$E_1 \gg E_2 \gg E_3 \gg \ldots \gg E_m$$

- Multi-gluon emission amplitudes become extremely simple in this limit, especially at large N_c

$$\left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 = \left|\langle p_1 \cdots p_m \left| Y_a^\dagger Y_b \right| 0\rangle\right|^2 = N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)\,(p_1 \cdot p_2) \cdots (p_m \cdot p_b)}$$

- Using their structure Banfi, Marchesini, Smye '02 derived an integral equation for resummation of **leading logs at large N_c**: BMS equation.

Non-global logarithms

A lot of recent work on these types of logarithms

- Resummation of leading logs beyond large N_c Weigert '03, Hatta, Ueda '13 + Hagiwara '15; Caron-Huot '15.

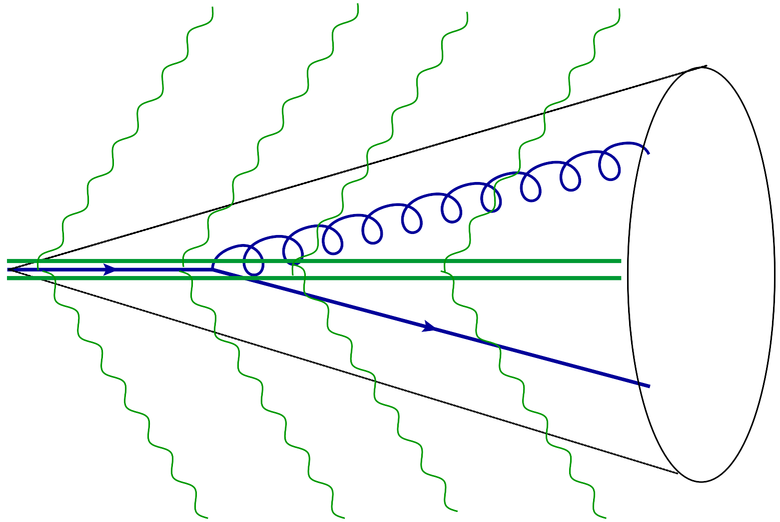
- Fixed-order results: 2 loops for $S(\omega_L,\omega_R)$. Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11; with jet-cone Kelley, Schwartz, Schabinger and Zhu '11; von Manteuffel, Schabinger and Zhu '13, leading non-global log up to 5 loops by solving BMS equation Schwartz, Zhu '14, 5 loops and arbitrary N_c Delenda, Khelifa-Kerfa '15

- Approximate resummation of such logs, based on resummation for observables with n soft subjets. Larkoski, Moult and Neill '15

A systematic factorization of non-global observables was missing.

Soft factorization revisited

As discussed, large-angle soft radiation only sees total charge. Identical to radiation of a single particle flying in the jet direction. Described by **Wilson line along jet direction**.



We will now see that this picture **breaks down for non-global observables due to the relevance of small angle soft radiation**!

Soft emission from a jet

Consider again the emission of single soft a gluon from energetic particles with momenta p_i inside a narrow jet:

$$\sum_i Q_i \frac{p_i \cdot \varepsilon}{p_i \cdot k} = Q_{\mathrm{tot}} \frac{n \cdot \varepsilon}{n \cdot k} + \ldots$$

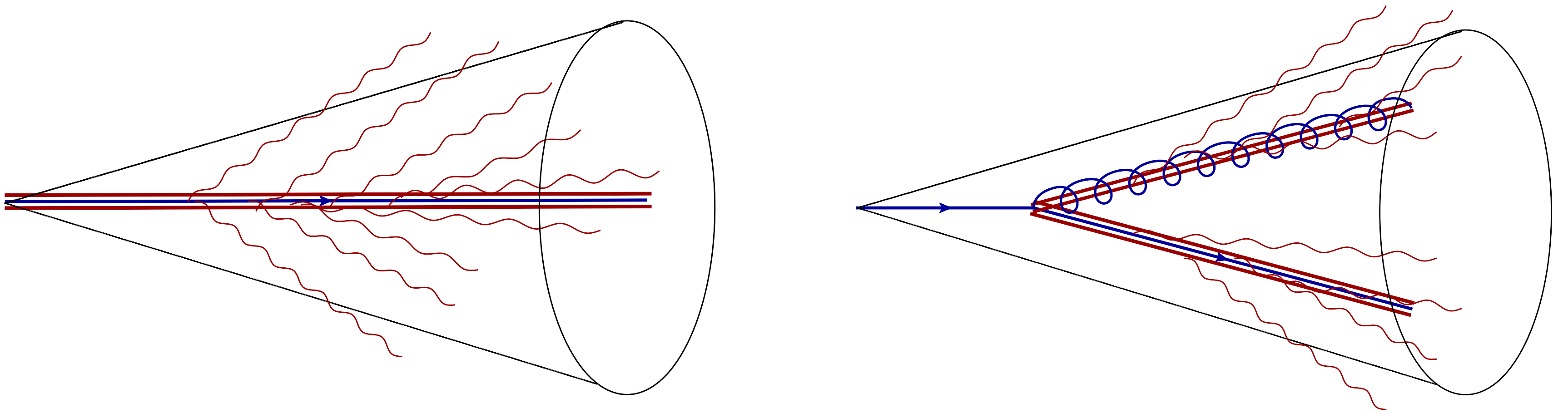
Approximation: $p_i^\mu \approx E_i \, n^\mu$

This approximation breaks down when the soft emission has a small angle, i.e. when $k^\mu \approx \omega \, n^\mu$!

Small region of phase space, but it turns out that it gives a leading contribution to jet rates!

Coft factorization

TB, Neubert, Rothen, Shao,1508.06645



For cone-jet processes with narrow cones, small angle soft radiation becomes relevant

- collinear *and* soft ("coft")
- resolves individual collinear partons: operators with multiple Wilson lines

Momentum modes for jet processes

TB, Neubert, Rothen, Shao,1508.06645; Chien, Hornig and Lee 1509.04287

	Region	Energy	Angle	Inv. Mass
standard SCET	Hard	Q	1	Q
	Collinear	Q	δ	$Q\delta$
	Soft	βQ	1	βQ
new	Coft	βQ	δ	$\beta\delta Q$

Full jet cross section is recovered after adding the contributions from all regions ("method of regions")

- Additional coft mode has very low characteristic scale $\beta\delta Q$! Jets are less perturbative than they seem!
- Effective field theory has additional "coft" degree of freedom.

Momentum modes again (for experts)

Split momenta into light-cone components

$$p^\mu = p_+ \frac{n^\mu}{2} + p_- \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

Scaling of the momentum components ($\beta \sim \delta^2$)

$$\begin{array}{rl} & \quad\;\; (\; p_+ \;,\; p_- \;,\; p_\perp \;) \\ \text{collinear:} & p_c \sim Q\,(\; 1 \;,\; \delta^2 \;,\; \delta \;) \\ \text{soft:} & p_s \sim Q\,(\; \beta \;,\; \beta \;,\; \beta \;) \\ \text{coft:} & p_t \sim \beta Q\,(\; 1 \;,\; \delta^2 \;,\; \delta \;) \end{array}$$

Note: every component of coft mode is smaller than the corresponding collinear one. Different than $\mathrm{SCET_I}$, $\mathrm{SCET_{II}}$, $\mathrm{SCET_{1.5}}$, $\mathrm{SCET_n}$,$\mathrm{SCET_+}$, …

Method of region expansion

To isolate the different contributions, one expands the amplitudes as well as the phase-space constraints in each momentum region.

- Generic soft mode has O(1) angle: after expansion, it is always outside the jet.
- Collinear mode has large energy $E \gg \beta Q$. Can never go outside the jet.
- Coft mode can be inside or outside, but its contribution to the momentum inside the jet is negligible.

Expansion is performed on the integrand level: the full result is obtained after combining the contributions from the different regions.

Checks at one and two loops

hard	$\Delta\sigma_h = \frac{\alpha_s C_F}{4\pi}\sigma_0 \left(\frac{\mu}{Q}\right)^{2\epsilon}\left(-\frac{4}{\epsilon^2}-\frac{6}{\epsilon}+\frac{7\pi^2}{3}-16\right)$
collinear	$\Delta\sigma_{c+\bar{c}} = \frac{\alpha_s C_F}{4\pi}\sigma_0 \left(\frac{\mu}{Q\delta}\right)^{2\epsilon}\left(\frac{4}{\epsilon^2}+\frac{6}{\epsilon}+c_0\right)$
soft	$\Delta\sigma_s = \frac{\alpha_s C_F}{4\pi}\sigma_0 \left(\frac{\mu}{Q\beta}\right)^{2\epsilon}\left(\frac{4}{\epsilon^2}-\pi^2\right)$
coft	$\Delta\sigma_{t+\bar{t}} = \frac{\alpha_s C_F}{4\pi}\sigma_0 \left(\frac{\mu}{Q\delta\beta}\right)^{2\epsilon}\left(-\frac{4}{\epsilon^2}+\frac{\pi^2}{3}\right),$

$$\Delta\sigma^{\text{tot}} = \frac{\alpha_s C_F}{4\pi}\sigma_0 \left(-16\ln\delta\ln\beta + 12\ln\delta + c_0 + \frac{5\pi^2}{3} - 16\right)$$

Constant c_0 depends on definition of jet axis:

$c_0 = -3\pi^2 + 26,$ (Sterman-Weinberg)

$c_0 = -5\pi^2/3 + 14 + 12\ln 2$ (thrust axis)

Have repeated the same check at two-loop order and checked against numerical result from Event 2 generator

Sample ingredient at O(α_s^2)

Coft function with two Wilson lines

$$
\begin{aligned}
\left\langle \widetilde{\boldsymbol{\mathcal{U}}}_1(Q\delta\tau,\epsilon)\right\rangle &= 1 + \frac{\alpha_0 C_F}{4\pi}\, e^{-2\epsilon L}\left(-\frac{2}{\epsilon^2} - \frac{\pi^2}{2} - \frac{14\zeta_3}{3}\,\epsilon - \frac{7\pi^4}{48}\,\epsilon^2\right) \\
&\quad + \left(\frac{\alpha_0}{4\pi}\right)^2 e^{-4\epsilon L}\left(C_F^2 V_F + C_F C_A V_A + C_F T_F n_f V_f\right),
\end{aligned}
$$

with $L = \ln\frac{Q\delta\tau}{\mu}$ (τ is the Laplace conjugate of β) and

$$
\begin{aligned}
V_F &= \frac{2}{\epsilon^4} + \frac{\pi^2}{\epsilon^2} + \frac{28\zeta_3}{3\epsilon} + \frac{5\pi^4}{12}\,, \\
V_A &= -\frac{11}{6\epsilon^3} - \frac{1}{\epsilon^2}\left(\frac{67}{18} + \frac{\pi^2}{6}\right) + \frac{1}{\epsilon}\left(-\frac{211}{27} - \frac{11\pi^2}{36} + 3\zeta_3\right) - \frac{836}{81} - \frac{1139\pi^2}{108} - \frac{341\zeta_3}{9} + \frac{31\pi^4}{90}\,, \\
V_f &= \frac{2}{3\epsilon^3} + \frac{10}{9\epsilon^2} + \frac{1}{\epsilon}\left(\frac{74}{27} + \frac{\pi^2}{9}\right) - \frac{374}{81} + \frac{109\pi^2}{27} + \frac{124\zeta_3}{9}\,.
\end{aligned}
$$

Can be extracted using two-loop results for hemisphere soft function Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11: Take energy in one hemisphere to ∞!

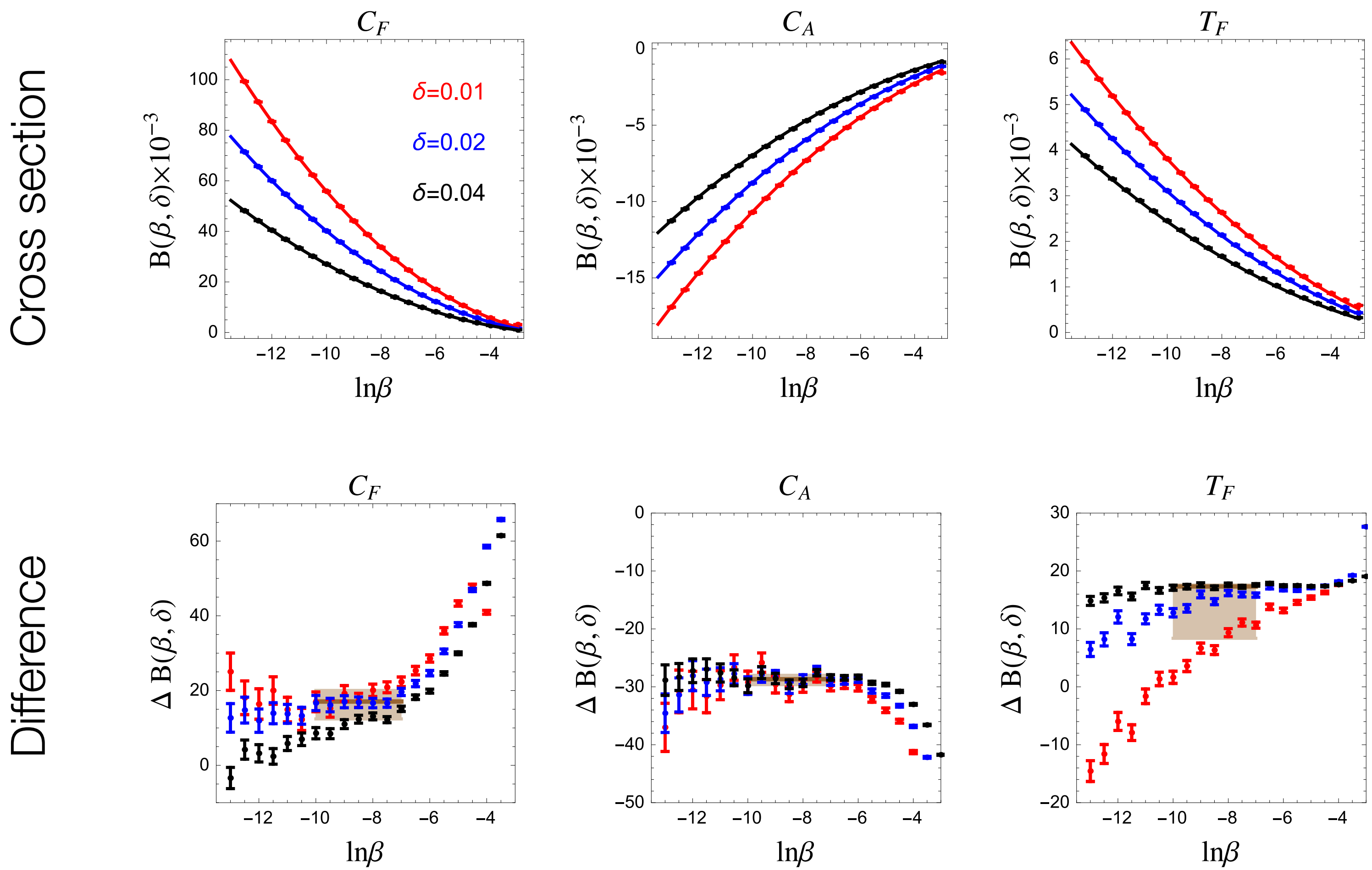
Note the $1/\varepsilon^n$ divergences!

Two-loop result

$$\frac{\sigma(\beta)}{\sigma_0} = 1 + \frac{\alpha_s}{2\pi} A(\beta,\delta) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\beta,\delta) + \dots$$

$$\begin{aligned}
B(\beta,\delta) = C_F^2 \Bigg[& \left(32\ln^2\beta + 48\ln\beta + 18 - \frac{16\pi^2}{3}\right)\ln^2\delta + \left(-2 + 10\zeta_3 - 12\ln^2 2 + 4\ln 2\right)\ln\beta \\
& + \left((8 - 48\ln 2)\ln\beta + \frac{9}{2} + 2\pi^2 - 24\zeta_3 - 36\ln 2\right)\ln\delta + c_2^F\Bigg] \\
+ C_F C_A \Bigg[& \left(\frac{44\ln\beta}{3} + 11\right)\ln^2\delta - \frac{2\pi^2}{3}\ln^2\beta + \left(\frac{8}{3} - \frac{31\pi^2}{18} - 4\zeta_3 - 6\ln^2 2 - 4\ln 2\right)\ln\beta \\
& + \left(\frac{44\ln^2\beta}{3} + \left(-\frac{268}{9} + \frac{4\pi^2}{3}\right)\ln\beta - \frac{57}{2} + 12\zeta_3 - 22\ln 2\right)\ln\delta + c_2^A\Bigg] \\
+ C_F T_F n_f \Bigg[& \left(-\frac{16\ln\beta}{3} - 4\right)\ln^2\delta + \left(-\frac{16}{3}\ln^2\beta + \frac{80\ln\beta}{9} + 10 + 8\ln 2\right)\ln\delta + \left(-\frac{4}{3} + \frac{4\pi^2}{9}\right)\ln\beta + c_2^f\Bigg]
\end{aligned}$$

- $1/\varepsilon^4$, $1/\varepsilon^3$, $1/\varepsilon^2$, $1/\varepsilon$ divergences have cancelled!
- Two-loop constants c_2^F, c_2^A, c_2^f, unknown. (Could be obtained from two-loop collinear result.)



Data points from **Event2 NLO generator**, solid lines are our prediction. Difference yields unknown constants

$$c_2^F = 17.1^{+3.0}_{-4.7}\,, \qquad c_2^A = -28.7^{+0.7}_{-1.0}\,, \qquad c_2^f = 17.3^{+0.3}_{-9.0}$$

Note: Event2 suffers from numerical instability in n_f channel

Factorization for two-jet cross section

TB, Neubert, Rothen, Shao, arXiv:1508.06645

Laplace space
$\tau \leftrightarrow \beta$

color trace

integration over angles

$$\widetilde{\sigma}(\tau) = \sigma_0\, H(Q)\, \widetilde{S}(Q\tau) \left[\sum_{m=1}^{\infty} \left\langle \mathcal{J}_m(Q\delta) \otimes \widetilde{\mathcal{U}}_m(Q\delta\tau) \right\rangle \right]^2$$

Coft functions with m Wilson lines

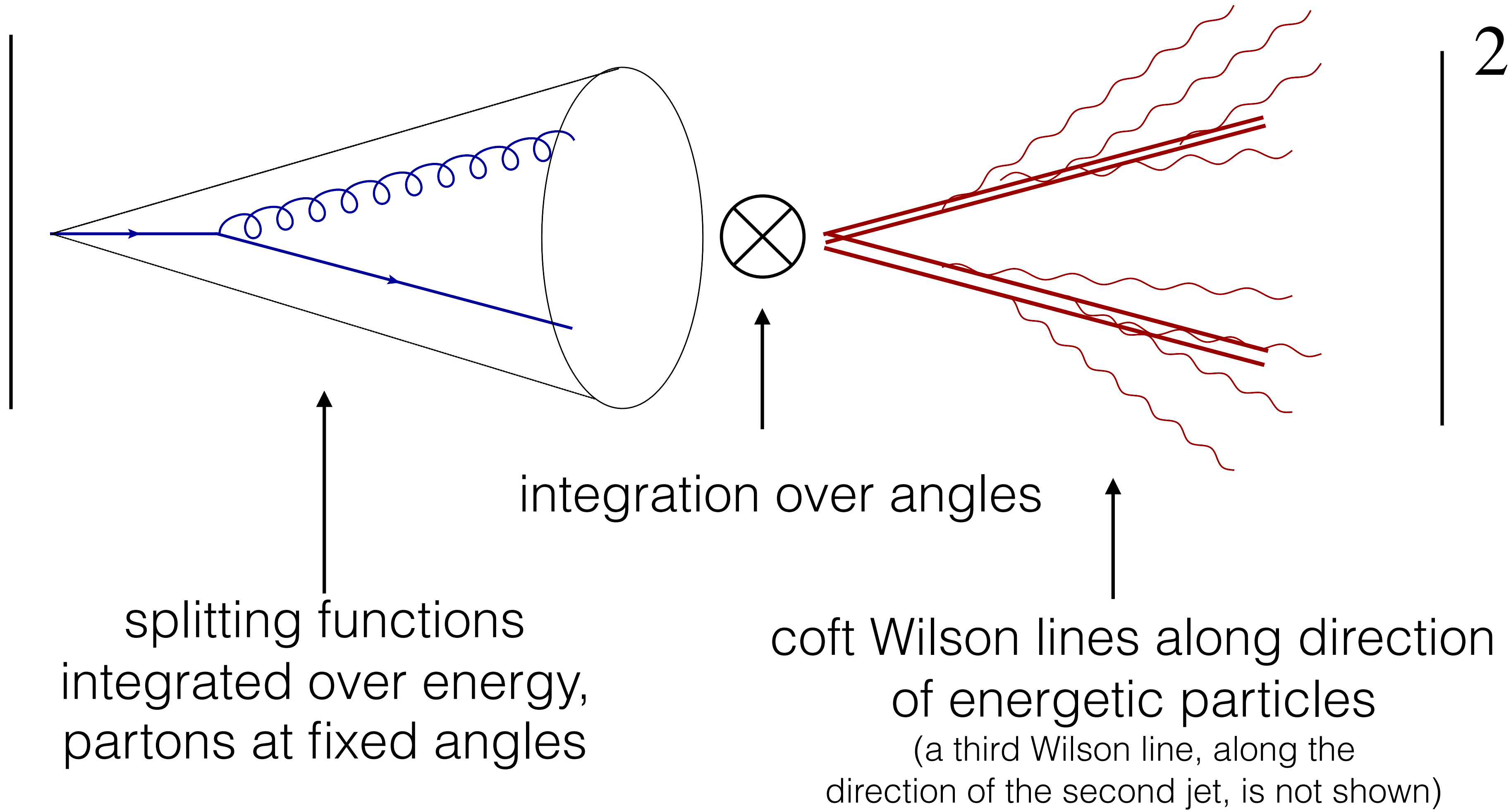
Soft function

Hard function

Jet functions with m partons at fixed direction

First all-order factorization theorem for non-global observable. Achieves full scale separation!

$$\left\langle \boldsymbol{\mathcal{J}}_2(Q\delta) \otimes \widetilde{\boldsymbol{\mathcal{U}}}_2(Q\delta\tau) \right\rangle =$$



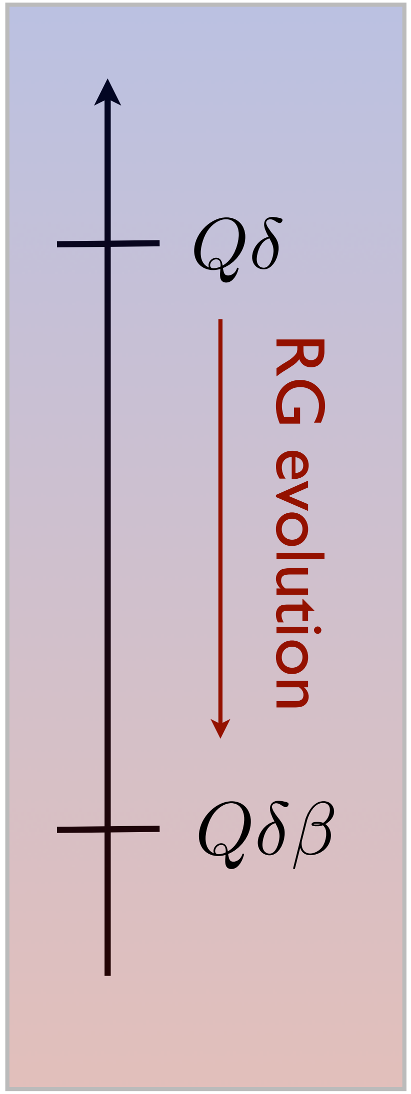
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations, e.g.

$$\frac{d}{d\ln\mu}\mathcal{J}_m(Q\delta,\mu) = \sum_k \mathcal{J}_k(Q\delta,\mu)\,\mathbf{\Gamma}^J_{km}$$

1. Compute $\mathcal{J}_m$ at a their characteristic high scale $\mu_h \sim Q\delta$
2. Evolve $\mathcal{J}_m$ to the scale of low energy physics $\mu_l \sim Q\delta\beta$

Avoids large logarithms $\alpha_s{}^n \ln^n(\beta)$ of scale ratios which can spoil convergence of perturbation theory.



(N)LL resummation

Need tree-level matrix elements

$$\boldsymbol{\mathcal{U}}_m = \mathbf{1} + \mathcal{O}(\alpha_s) \;\; ; \;\; \boldsymbol{\mathcal{J}}_1 = \mathbf{1} \quad , \;\; \boldsymbol{\mathcal{J}}_m \sim \alpha_s^{m-1}$$

and one-loop anomalous dimensions

$$\boldsymbol{\Gamma}^J = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_1 & \boldsymbol{R}_1 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_4 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

Formal solution: $\boldsymbol{\mathcal{J}}_m(\mu_l) = \boldsymbol{\mathcal{J}}_m(\mu_h) \boldsymbol{U}_{km}(\mu_h, \mu_l)$ with

$$\boldsymbol{U}(\mu_h, \mu_l) = \mathbf{P} \exp\left[\int_{\alpha_s(\mu_l)}^{\alpha_s(\mu_h)} d\alpha \, \frac{\boldsymbol{\Gamma}^J(\alpha)}{\beta(\alpha)} \right]$$

Fixed-order expansion

Challenging to solve RG explicitly, but order-by-order structure is similar to parton shower

$$
\begin{aligned}
\alpha_s &: \boldsymbol{R}_1 + \boldsymbol{V}_1 \,, \\
\alpha_s^2 &: \boldsymbol{R}_1(\boldsymbol{R}_2 + \boldsymbol{V}_2) + \boldsymbol{V}_1(\boldsymbol{R}_1 + \boldsymbol{V}_1) \,, \\
\alpha_s^3 &: \boldsymbol{R}_1\big[\boldsymbol{R}_2(\boldsymbol{R}_3 + \boldsymbol{V}_3) + \boldsymbol{V}_2(\boldsymbol{R}_2 + \boldsymbol{V}_2)\big] \\
&\quad + \boldsymbol{V}_1\big[\boldsymbol{R}_1(\boldsymbol{R}_2 + \boldsymbol{V}_2) + \boldsymbol{V}_1(\boldsymbol{R}_1 + \boldsymbol{V}_1)\big] \,.
\end{aligned}
$$

- Reproduces results from BMS equation in large N_C limit
- Our RG has close connection to functional RG by Caron-Huot ‘15

Summary and Outlook

- We have, for the first time, derived a factorization theorem for a non-global observable.
- Based on an EFT, which includes a new "coft" (collinear+soft) momentum mode.
- RG evolution in this EFT can be used to systematically resum large logarithms, also beyond LL and large N_c.
  - Will need to develop numerical techniques to solve the associated RG equations.
- Numerous possible applications: jet structure, jet substructure, jet vetoes, …