From gauge fields to physical particles

Hadrons as relativistic bound states of confined quarks and glue

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Vienna, April 12, 2016
A prelude: The Higgs boson and gauge invariance

CERN 2012: The Higgs boson has been measured!

The ‘textbook’ Higgs is a gauge-dependent state and thus unphysical!?!?

Gauge invariance broken? No! (Elitzur‘s theorem)

The custodial (global!) SU(2) symmetry of the SM is broken:
Goldstone bosons become elements of the elementary BRST quartets!

Gauge-invariant states are necessarily composite!

Relation between gauge-invariant and gauge-dependent states:
Physical $H$, $W$ and $Z$ are gauge-invariant $H-H$, $H-W$ and $H-Z$ bound states with same mass as elementary fields in unitary gauge.
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Outline

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2 Basics of Covariant Gauge Theory
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Motivation: Why Functional Approaches to QCD?

Where to look for the **nucleon in QCD?**

Free propagation of lowest three-quark bound state: **Six-quark Green function!**

Calculating it requires either

- to employ a lattice (*i.e.*, give up Poincaré invariance)
- to use Monte-Carlo algorithms (*i.e.*, use a statistical method)
- to run programs on supercomputers

or

- to fix a gauge (*i.e.*, sacrifice gauge invariance)
- to truncate equations in a way which is verified *à posteriori*

**Method 1:**
Excellent results for hadron properties & insight into hadron structure!

**Method 2:**
Relation of observables to confinement, $D_{\chi}$SB, axial anomaly, ...
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QCD correlation functions contribute to the understanding of

★ confinement of gluons, quarks, and colored composites.
★ $D\chi_{SB}$, i.e., generation of quark masses and chirality-changing quark-gluon interactions.
★ $U_A(1)$ anomaly and topological properties.

Within Functional Methods
(Exact Renorm. Group, Dyson-Schwinger eqs., $n$PI methods, ...):
Input into hadron phenomenology via QCD bound state eqs.

- Bethe-Salpeter equations for mesons
  form factors, decays, reactions, ...
- covariant Faddeev equations for baryons
  nucleon form factors, Compton scattering, meson production, ...
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Functional approaches to Landau gauge QCD:

- in principle ab initio
- perturbation theory included
- some elementary Green’s functions quite well-known
- in other gauges complicated . . .
- truncations for numerical solutions necessary

State-of-the-art:

- 3-particle-irreducible truncation with dynamical propagators and three-point functions for mesons.
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Gauge theory: **Unphysical degrees of freedom!**

**QED:** Physical states obey Lorentz condition.

\[ \partial_\mu A^\mu |\Psi\rangle = 0 \quad (\text{Gupta – Bleuler}). \]

\( \Rightarrow \) In symmetric phase:

Two physical massless photons in physical state space.

Time-like photon (i.e. negative norm state!) cancels longitudinal photon in \( S \)-matrix elements!

**Guaranteed on an algebraic level!**
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Covariant Gauge Theory

In S-Matrix

\[ A_T + A_0 + A_T = 0 \]

\[ A_T + A_0 + A_L = \neq 0 \]
Quantum Yang-Mills theory:

Selfinteraction of gluons:
transverse gluons scatter into longitudinal ones and vice versa!

⇒ Faddeev–Popov ghosts = anticomm. scalar fields.

Ghosts are unphysical
(anti-commuting scalar)
Yang–Mills degrees of freedom!
Important in quantum fluct., but no associated particles!

Global ghost field as ‘gauge parameter‘:
BRST symmetry of the gauge-fixed action!
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Covariant Gauge Theory

Symmetry of the gauge-fixed generating functional:

\[ \delta_B A^a_{\mu} = D^{ab}_{\mu} c^b \lambda , \quad \delta_B q = -igt^a c^a q \lambda , \]
\[ \delta_B c^a = -\frac{g}{2} f^{abc} c^b c^c \lambda , \quad \delta_B \bar{c}^a = \frac{1}{\xi} \partial_\mu A^a_{\mu} \lambda , \]

Becchi–Rouet–Stora & Tyutin (BRST), 1975

- Parameter \( \lambda \in \) Grassmann algebra of the ghost fields
- \( \lambda \) carries ghost number \( N_{FP} = -1 \)
- Via Noether theorem: BRST charge operator \( Q_B \)
- Generates ghost # graded algebra \( \delta_B \Phi = \{ iQ_B, \Phi \} \)
Covariant Gauge Theory

BRST algebra: \( Q_B^2 = 0, [iQ_c, Q_B] = Q_B, \)
- complete in indefinite metric state space \( \mathcal{V} \).
- generates ghost # graded \( \delta_B \Phi = \{iQ_B, \Phi\} \).
- \( L_{GF} = \delta_B \left( \bar{c} \left( \partial_\mu A^\mu + \frac{\alpha}{2} B \right) \right) \) BRST exact.

Positive definite subspace \( \mathcal{V}_{pos} = \text{Ker}(Q_B) \)
(i.e. all states \( |\psi\rangle \in \mathcal{V} \) with \( Q_B|\psi\rangle = 0 \))
contains \( \text{Im}Q_B \) (i.e. all states \( Q_B|\phi\rangle \)),
c.f. exterior derivative in differential geometry.

Hilbert space: cohomology \( \mathcal{H} = \frac{\text{Ker}Q_B}{\text{Im}Q_B} \cong \mathcal{V}_s \) BRST singlet
longitudinal & timelike gluons, ghosts : BRST quartet
(c.f. Gupta–Bleuler mechanism in QED)
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Perturbative BRST quartet of time-like gluons:

|$\Pi_0$⟩ lin. combination of time-like and long. gluon
|$\Delta_1$⟩ ghost
|$\Pi_{-1}$⟩ antighost
|$\Delta_0$⟩ lin. combination of time-like and long. gluon
Kugo–Ojima confinement

Non-perturbative BRST quartets of transverse gluons, resp., quarks:

\[
\begin{align*}
|\Pi_0\rangle & \quad \text{transverse gluons} \\
|\Delta_1\rangle & \quad \text{gluon-ghost bound states} \\
|\Pi_{-1}\rangle & \quad \text{gluon-antighost bound states} \\
|\Delta_0\rangle & \quad \text{gluon-ghost-antigh./gluonic b.s.} \\
\end{align*}
\]

Kugo–Ojima confinement criterion

⇒ Physical states are BRST singlets!
   (BRST cohomology: Hilbert space $\mathcal{H} = \frac{\text{Ker} Q_{\text{BRST}}}{\text{Im} Q_{\text{BRST}}}$.)

Time–like and longitudinal gluons (BRST quartet) removed from asymptotic states as in QED, but:

Transverse gluons and quarks also BRST quartets, i.e. confined, if ghost propagator is highly infrared singular!
   (⇒ Kugo–Ojima confinement criterion)
Kugo–Ojima confinement criterion

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Kugo–Ojima confinement criterion

Realization of Confinement depends on global gauge structure:
Globally conserved current \( (\partial^\mu J^a_\mu = 0) \)

\[
J^a_\mu = \partial^\nu F^a_{\mu\nu} + \{Q_B, D^{ab}_\mu \bar{c}^b\}
\]

with charge

\[
Q^a = G^a + N^a.
\]

**QED:** MASSLESS PHOTON states in both terms.
Two different combinations yield:
unbroken global charge \( \tilde{Q}^a = G^a + \xi N^a \).
spont. broken displacements (photons as Goldstone bosons).

No massless gauge bosons in \( \partial^\nu F^a_{\mu\nu} : G^a \equiv 0 \).
(QCD, e.w. Higgs phase, ...)

R. Alkofer (Graz)
**Kugo–Ojima confinement criterion**

**QCD:** Unbroken global charge

\[
Q^a = N^a = \{ Q_B, \int d^3 x D_0^{ab} \bar{c}^b \}
\]

well-defined in \( \mathcal{V} \).

With \( D^{ab \bar{c}b}(x) x^0 \to \pm \infty \),

\[
(\delta^{ab} + u^{ab}) \partial_\mu \bar{\gamma}^b + \ldots
\]

⇒ Kugo-Ojima Confinement Criterion:

\[
u^{ab}(0) = -\delta^{ab}
\]

where

\[
\int dx e^{ip(x-y)} \langle 0 | T D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y) | 0 \rangle
\]

\[
=: (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2),
\]

If fulfilled: Physical States \( \equiv \) BRST singlets \( \equiv \) color singlets!
Kugo–Ojima confinement criterion

In Landau gauge:

Ghost propagator more sing. than simple pole

\[ \downarrow \]

Kugo-Ojima criterion

Confinement vs. Higgs mechanism?

No gauge-invariant order parameter!

A possible definition of **confinement** in the presence of fundamental charges†:

\[ \text{Coulomb} \quad \text{Confinement} \quad \text{Higgs} \]

- **Coulomb**
  - Global gauge charges unbroken

- **Confinement**
  - Mass gap

†Wilson loop gives only a clear criterion in the absence of quarks!

Confinement and Higgs mechanism

If BRST or equivariant BRST symmetry:

Infrared saturation of Quantum E.o.M. of gauge boson propagator discriminates phases of gauge theories!

- **Coulomb:** massless gauge boson
- **Confinement:** unphysical current
- **Higgs:** physical current

Saturating part of current is given by:

\[
\tilde{j}_\mu(x) = j^{U(1)}_\mu - i\partial_\mu b
\]

\[
\tilde{j}_a^a(x) = j^{LCG}_\mu - is(D_\mu \bar{c})^a
\]

\[
\tilde{j}_a^a(x) = j^{GLCG}_\mu - is_\alpha(D_\mu \bar{c})^a
\]

\[
\tilde{j}_\mu(x) = j^{MAG}_\mu - i\partial_\mu b
\]

\[
\tilde{j}_k^a(x) = j^C_k - is(D_k \bar{c})^a
\]

\[
\tilde{j}_a^a(x) = j^{GZ}_\mu - is\chi_\mu^a
\]

Motivation: Why Functional Approaches to QCD?

Basics of Covariant Gauge Theory

QCD Green functions in Landau gauge
- Gluon, Ghost and Quark Propagators
- Three-point vertex functions

Relativistic Three-Fermion Bound State Equations
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Summary and Outlook
pure Yang-Mills, $T = 0$
Landau gauge Gluon Ren. Fct. $D_{\text{Gluon}}^{tr} = Z(p^2)/p^2$

A. Sternbeck et al., PoS LAT2006, 76
A. Sternbeck *et al.*, PoS LAT2006, 76

![Graph showing gluon propagator $Z(p^2)$ vs. $p$/GeV.](image)


**Dyson-Schwinger eqs. (DSEs)**
Gluon Propagator

A. Sternbeck et al., PoS LAT2006, 76


Gluon Propagator

A. Sternbeck et al., PoS LAT2006, 76

\begin{figure}
\centering
\includegraphics[width=\textwidth]{diagram.png}
\caption{\(Z(p^2)\) vs. \(p/\text{GeV}\).
\(48^4, 6.0\), 32^4, 6.0, 32^4, 5.8, 48^4, 5.7, 56^4, 5.7\)}
\end{figure}


**Exact Renormalization Group (ERG) eqs.**
Gluon Propagator

$Z(p^2)$

  L. Fister, PhD thesis

**combination of ERG,DSE & 2PI!**
Infrared Exponents for Gluons and Ghosts

Use DSEs and FRG eqs:

→ Two different towers of equations for Green functions
E.g. ghost propagator

\[ k \partial_k \rightarrow \mathbf{1} = \mathbf{1} - \frac{1}{2} \mathbf{1} \]

MATHEMATICA-based derivation of functional equations:


use DSEs and ERGEs:

→ Two different towers of equations for Green functions

IR-Analysis of whole tower of equations ⇒

Unique scaling solution


Scaling vs. decoupling solution:

- Lattice calculations for gluon propagator: decoupling solutions. [A. Cucchieri et al., many others; but also: A. Sternbeck et al.]
- Scaling solution respects, decoupling solutions break BRST.
- Similar (identical) analytical structure for gluon propagator.
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Unique scaling solution
+ an one-parameter family of solutions with IR trivial Green functions.

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Analytical structure for gluon propagator

Gluon propagator is positivity violating, cut along time-like half-axis.

Imaginary part of gluon propagator:

Infrared Exponents for Gluons and Ghosts

Scaling solution:

\( n \) external ghost & antighost legs and \( m \) external gluon legs (one external scale \( p^2 \); solves DSEs and STIs):

\[
\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}
\]

Scaling / decoupling solution:

- Ghost propagator IR divergent (Kugo-Ojima for scaling)
- Gluon propagator IR suppressed (Gribov-Zwanziger)
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent / finite

Note:

Same dichotomy of solutions in Coulomb gauge (variational / ERG) and in maximally Abelian gauge (DSE / ERG).
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Gluon and Ghost Propagators

\[ G(p^2) \]

\[ D(p^2) \]

\[ \alpha(p^2) \]

\[ \rho_{gh} \]

\[ \rho_{gl} \]
Three-gluon vertex

DSE for three-gluon vertex:

\[ G. \text{Eichmann, R. \text{Williams, M. Vujinovic, RA, Phys. Rev. D89 (2014) 105014} } \]

\[ = \quad - \quad +\frac{1}{2} \]

\[ +\frac{1}{6} \quad +\frac{1}{2} \quad +\frac{1}{2} \]
Three-gluon vertex

DSE for three-gluon vertex (truncated):

\[ \begin{align*}
G. \text{Eichmann}, R. \text{Williams}, M. \text{Vujinovic}, RA, \text{Phys. Rev. D89 (2014) 105014} \]

\[ = - \frac{1}{2} + \frac{1}{2} \]

14 tensor elements \(\rightarrow\) 4 fully transverse ones

apply cyclic perm. w.r.t. external legs \(\rightarrow\) bose symmetry

Is there a zero in component projected on tree-level tensor? (cf., A. Maas)
Three-gluon vertex

Applications:
- quark-gluon vertex (and consequently unquenching)
- conformal window
- symmetry preserving Bethe-Salpeter kernel
- irreducible three-body forces in the baryon

Kinematics:
Three-gluon vertex

\[ \Gamma^{(1)\mu\nu\rho}_{3g,TTT} \]

Green: zero; Orange: ghost triangle; Blue: ghost + swordfish.
Three-gluon vertex

Running coupling:

![Graph showing running coupling](image)

- **1 (SC)**
  - $\alpha_{gh}$
  - $\alpha_{3g}$
- **4 (DC)**
  - $\alpha_{gh}$
  - $\alpha_{3g}$
Coupling quarks: quark prop. and quark-gluon vertex


Chiral symmetry dynamically or explicitly broken:

- quark propagator infrared finite:

\[ S(p) = \frac{\not{p} - M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \]

AND

- quark-gluon vertex incl. dynamically generated \( \chi \)SB tensors structures:

\[ \Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu, \quad G^1_\mu = \gamma_\mu, \quad G^2_\mu = \hat{p}_\mu, \quad G^3_\mu = \ldots \]
Coupling quarks: quark prop. and quark-gluon vertex

DSEs for quark propagator and quark-gluon vertex via 3PI action:

\[-1 = -1\]

\[-\frac{1}{2N_c} = +\frac{N_c}{2}\]

hadron physics

IR behaviour confinement
Coupling quarks to gluons: quark propagator

Quark mass function with calculated QGV and modeled 3gV:

![Graph showing quark mass function with different values of \( d_{IR} \).]
Coupling quarks to gluons: quark-gluon vertex

Eight transverse tensor structures, e.g., at symm. momenta $x = p_1^2 = p_2^2 = p_3^2$:

Significant IR enhancement!

The zero in the three-gluon vertex causes zero in the tree-level structure of quark-gluon vertex!

$D\chi_{SB}$ in QGV!!!
Relative importance:

dynamically generated \textbf{scalar}-


type coupling important!
Summary on Landau gauge 2- and 3-point-functions

For a small number of light flavours:
- IR enhanced ghost propagator
- IR suppressed gluon propagator
- IR decoupled quark propagator & dynamically generated mass
- ghost-gluon vertex close to tree-level
- zero in three-gluon vertex
- infrared enhanced quark-gluon vertex & dynamically generated chirality-changing interactions

NB: Four-gluon vertex currently under investigation → primitively divergent Landau gauge QCD Green functions known!

Higher order Green functions are finite upon upon renormalization of prim. dvgcs. & fulfill (multi-)linear equations.

*Increase $N_f$: 2nd order phase transition to conformal window! Propagators and 3-point-functions change significantly!

Motivation: Why Functional Approaches to QCD?

Basics of Covariant Gauge Theory

QCD Green functions in Landau gauge
- Gluon, Ghost and Quark Propagators
- Three-point vertex functions

Relativistic Three-Fermion Bound State Equations
- Structure of Baryonic Bound State Amplitudes
- Quark Propagator and Rainbow Truncation
- Interaction Kernels and Rainbow-Ladder Truncation
- Coupling of E.M. Current and Quark-Photon Vertex
- Some Selected Results

Summary and Outlook
Dyson-Schwinger eq. for 6-point fct. \(\implies 3\)-body bound state eq.:

\[
G^{(3)} = G_0^{(3)} + K G^{(3)}
\]

\[
T^{(3)} = \tilde{K} + \tilde{K} T^{(3)}
\]

**BOUND STATE:**
Pole in \(G^{(3)}\)

or (equiv.) for \(P^2 = -M_B^2\)

Pole in \(T^{(3)}\)

**bound state amplitudes:**

**covariant 3-body bound state eq.** (cf., Bethe-Salpeter for 2-body BS):

\[
\Psi \equiv \tilde{K}^{(3)} \Psi \equiv \tilde{K}^{(2)} + \tilde{K}^{(2)} \Psi \equiv + \tilde{K}^{(2)} \Psi \equiv + \tilde{K}^{(2)} \Psi
\]
Relativistic three-fermion bound state equations

3-body bound state eq.:

\[ \Psi = \tilde{K}^{(3)} \Psi + \tilde{K}^{(2)} \Psi + \tilde{K}^{(2)} \Psi + \tilde{K}^{(2)} \Psi \]

NB: With 3-particle-irreducible interactions \( \tilde{K}^{(3)} \) neglected:

Poincaré-covariant Faddeev equation.

**Elements needed for bound state equation:**

- Tensor structures (color, flavor, Lorentz / Dirac) of the BS ampl.
- Full quark propagators for *complex* arguments
- Interaction kernels \( K_{2,3} \)

**Needed for coupling to e.m. current:**
- Full quark-photon vertex
Relativistic three-fermion bound state equations

Structure of baryonic bound state amplitudes

\[ \Psi \equiv \sim \langle 0 | q_\alpha q_\beta q_\gamma | B_\mathcal{I} \rangle \propto \psi_{\alpha\beta\delta\mathcal{I}} \] (with multi-indices \( \alpha = \{ x, D, c, f, \ldots \} \))

and \( \mathcal{I} \) baryon (multi-)index \( \Rightarrow \) baryon quantum numbers


Comparison to mesonic BS amplitudes \( \langle 0 | q_\alpha \bar{q}_\beta | M_\mathcal{I} \rangle \propto \Phi_{\alpha\beta\mathcal{I}} : \)

- scalar and pseudoscalar mesons: 4 tensor structures each
- vector and axialvector mesons: 12 tensor struct. each, 8 transv.
- tensor and higher spin mesons: 8 transverse struct. each

Relativistic three-fermion bound state equations

Requirements for (baryonic) bound state amplitudes:

- positive energy (for fermionic bound states)
- well-defined parity
- irreducible representation of the Poincaré group otherwise.

Possible and recommended:

Complete orthogonal Dirac tensor basis
s.t. partial-wave composition in rest frame.
### Relativistic three-fermion bound state equations

<table>
<thead>
<tr>
<th>$s$</th>
<th>$l$</th>
<th>$T_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>0</td>
<td>$1 \otimes 1$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>0</td>
<td>$\gamma_7^\mu \otimes \gamma_T^\mu$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>1</td>
<td>$1 \otimes \frac{1}{2} [\hat{p}, \hat{q}]$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>1</td>
<td>$1 \otimes \hat{p}$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>1</td>
<td>$1 \otimes \hat{q}$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>1</td>
<td>$\gamma_7^\mu \otimes \gamma_T^\mu \frac{1}{2} [\hat{p}, \hat{q}]$</td>
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<td>$1/2$</td>
<td>1</td>
<td>$\gamma_7^\mu \otimes \gamma_T^\mu \hat{p}$</td>
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<tr>
<td>$1/2$</td>
<td>1</td>
<td>$\gamma_7^\mu \otimes \gamma_T^\mu \hat{q}$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>1</td>
<td>$3 (\hat{p} \otimes \hat{q} - \hat{q} \otimes \hat{p}) - \gamma_T^\mu \otimes \gamma_T^\mu [\hat{p}, \hat{q}]$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>1</td>
<td>$3 \hat{p} \otimes 1 - \gamma_T^\mu \otimes \gamma_T^\mu \hat{p}$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>1</td>
<td>$3 \hat{q} \otimes 1 - \gamma_T^\mu \otimes \gamma_T^\mu \hat{q}$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$3 \hat{p} \otimes \hat{p} - \gamma_T^\mu \otimes \gamma_T^\mu$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$\hat{p} \otimes \hat{p} + 2 \hat{q} \otimes \hat{q} - \gamma_T^\mu \otimes \gamma_T^\mu$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$\hat{p} \otimes \hat{q} + \hat{q} \otimes \hat{p}$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$\hat{q} \otimes [\hat{q}, \hat{p}] - \frac{1}{2} \gamma_T^\mu \otimes [\gamma_T^\mu, \hat{p}]$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$\hat{p} \otimes [\hat{p}, \hat{q}] - \frac{1}{2} \gamma_T^\mu \otimes [\gamma_T^\mu, \hat{q}]$</td>
</tr>
</tbody>
</table>

\[ \chi(x_1, x_2, x_3) = \langle 0 | T \psi(x_1) \psi(x_2) \psi(x_3) | N \rangle \]

**Momentum space:**

Jacobi coordinates $p, q, P$

$\Rightarrow$ 5 Lorentz invariants

$\Rightarrow$ 64 Dirac basis elements

\[ \chi(p, q, P) = \sum_k f_k(p^2, q^2, p \cdot q, p \cdot P, q \cdot P) \]

Complete, orthogonal Dirac tensor basis

(partial-wave decomposition in nucleon rest frame):

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

\[ T_{ij} (\Lambda_+ \gamma_5 C \otimes \Lambda_+) \]

\[ (\gamma_5 \otimes \gamma_5) T_{ij} (\Lambda_+ \gamma_5 C \otimes \Lambda_+) \]

\[ (A \otimes B)_{\alpha \beta \gamma \delta} = A_{\alpha \beta} B_{\gamma \delta} \]
Facts about the decomposition:

- Independent of any truncation of the bound state equation.
- Only Poincaré covariance and parity invariance exploited.
- It includes all possible internal spin and orbital angular momenta.
- For positive-parity, positive-energy (particle) baryons it consists of

\[
\text{spin-} \frac{1}{2} \text{ particle: 64 elements}
\]

<table>
<thead>
<tr>
<th># elements</th>
<th>s-wave</th>
<th>p-wave</th>
<th>d-wave</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>36</td>
<td>20</td>
</tr>
</tbody>
</table>

G. Eichmann et al., PRL 104 (2010) 201601

\[
\text{spin-} \frac{3}{2} \text{ particle: 128 elements}
\]

<table>
<thead>
<tr>
<th># elements</th>
<th>s-wave</th>
<th>p-wave</th>
<th>d-wave</th>
<th>f-wave</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>36</td>
<td>60</td>
<td>28</td>
</tr>
</tbody>
</table>

H. Sanchis Alepuz et al. PRD 84 (2011) 096003
Antisymmetry of the nucleon amplitude under quark exchange:

\[
\Psi(p, q, P) = \left\{ \begin{array}{c}
\psi_1(p, q, P) \quad \text{Flavor}_1 \\
\psi_2(p, q, P) \quad \text{Flavor}_2
\end{array} \right\} \quad \text{Color}
\]

\[
M_A \quad M_A \quad M_S \quad M_S \quad A
\]

Proton: (ud)u \quad \sqrt{2} \ [uu]d - [ud]u
Neutron: (ud)d \quad [ud]d - \sqrt{2} \ [dd]u

Flavor states decouple if the Faddeev kernel is flavor-independent (e.g. rainbow-ladder) \Rightarrow 2 \text{ degenerate solutions of the equation:}

\[
M_A \sim S_{11}^+, \ldots \quad M_S \sim A_{11}^+, \ldots
\]
In bound state eqs.:

- Knowledge of the quark propagator inside parabolic region required.
- $\eta \geq 1/2$ for mesons and $\eta \geq 1/3$ for baryons.
- For ground states no singularities in parabolic region.

- Lattice: Values for real $p^2 \geq 0$ only.
- Dyson-Schwinger / ERG eqs.: complex $p^2$ accessible.†

†Beyond singularities: A. Windisch et al., arXiv:1304.3642; in preparation.
Dyson-Schwinger eq. for Quark Propagator:

\[
S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu S(k) \Gamma^\nu(k, p; q) D_{\mu\nu}(q)
\]

\[
D_{\mu\nu}(q) \Gamma^\nu(k, p; q) : \begin{cases} 
D_{\mu\nu}(q) = \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z(q^2)}{q^2} \\
\Gamma^\nu(k, p; q) = \gamma^\nu Z_{1f} + \Lambda^\nu(k, p; q)
\end{cases}
\]
Rainbow truncation

Projection onto tree-level tensor $\gamma_\mu$, restrict momentum dependence

$$\gamma_\nu Z_1 f + \Lambda_\nu(k, p; q = p - k) \rightarrow \left(Z_1 f + \Lambda(q^2)\right) \gamma_\nu$$

$$Z_1 \frac{g^2}{4\pi} D_{\mu\nu}(q) \Gamma_\nu(k, p; q) \rightarrow \begin{cases} Z_1 \frac{g^2}{4\pi} T_{\mu\nu}(q) \frac{Z(q^2)}{q^2} \left(Z_1 f + \Lambda(q^2)\right) \gamma_\nu \\ =: Z_2 \frac{g^2}{4\pi} T_{\mu\nu}(q) \frac{\alpha_{\text{eff}}(q^2)}{q^2} \gamma_\nu \end{cases}$$
Interaction Kernels and Rainbow-Ladder Truncation

- **Truncation** of the quark-gluon vertex in the quark DSE.
- The BSE interaction kernel must be truncated accordingly.
- **Physical requirement**: **Chiral symmetry**, here axial WT id.,
  \[
  \{\gamma^5 \Sigma(-p_-) + \Sigma(p_+)^5\}_{\alpha\beta} = -\int K^q\bar{q} \{\gamma^5 S(-p_-) + S(p_+)^5\},
  \]
  which relates quark DSE and $q\bar{q}$ (meson) BSE kernel.

**Ladder truncation**

$q\bar{q}$ kernel compatible with rainbow truncation and axial WT id.:

\[
K^q\bar{q} = 4\pi Z_2^2 \frac{\alpha_{\text{eff}}(q^2)}{q^2} T_{\mu\nu}(q) \gamma^\mu \otimes \gamma^\nu
\]

Together constitute the DSE/BSE **Rainbow-Ladder truncation**.

Note: the truncation can and should be systematically improved!
Interaction Kernels and Rainbow-Ladder Truncation

Rainbow-Ladder truncated three-body BSE:

- Previous studies used successfully the quark-diquark ansatz (reduction to a two-body problem).
- \( pNRQCD \): 3-body contribution \( \sim 25 \text{ MeV} \) for heavy baryons.

Supported by this, the **three-body irreducible kernel** \( K^{(3)} \) is neglected (Faddeev approximation).

- Quark-quark interaction \( K^{(2)} \): **same as quark-antiquark truncated kernel**. (!Different color factor!)

Rainbow-Ladder truncated **covariant Faddeev equation**
Interactive Kernels and Rainbow-Ladder Truncation

Effective interaction:

Maris-Tandy model \((\text{Maris} \& \text{Tandy} \ PRC60 \ 1999)\)

\[ \alpha(k^2) = \alpha_{IR}(k^2; \Lambda, \eta) + \alpha_{UV}(k^2) \]

- Purely phenomenological model.
- \(\Lambda\) fitted to \(f_\pi\).
- Ground-state pseudoscalar properties \textit{almost} insensitive to \(\eta\) around 1.8

Describes very successfully hadron properties.

DSE motivated model \((\text{R.A.}, \ C.S. \ Fischer, R. \ Williams \ EPJ \ A38 \ 2008)\)

\[ \alpha(k^2; \Lambda_S, \Lambda_B, \Lambda_{IR}, \Lambda_{YM}) \]

- DSE-based in the deep IR.
- Designed to give correct masses of \(\pi, \rho\) and \(\eta'\) \((U_A(1) \text{ anomaly!})\).
- 4 energy scales! Fitted to \(\pi, K\) and \(\eta'\).
Note: The resulting qq-interaction is chirality-conserving, flavour-blind and current-quark mass independent.

Beyond Rainbow-Ladder

- “Corrections beyond-RL” refers to corrections to the effective coupling but also to additional structures beyond vector-vector interaction.
- They can induce a different momentum dependence of the interaction.
- They can also induce a quark-mass and quark-flavour dependence of the interaction.
- Question: how important are beyond-RL effects?
Coupling of E.M. Current and Quark-Photon Vertex

Electromagnetic current in the three-body approach:

by “gauging of equations”

Impulse appr. + Coupling to spectator q + Coupling to 2-q kernel not present in RL appr. + Coupling to 3-q kernel not present in Faddeev appr.

Additional Input: Quark-Photon Vertex
Coupling of E.M. Current and Quark-Photon Vertex

Quark-Photon Vertex:

- Vector WT id. determines vertex up to purely transverse parts: “Longitudinal” part (Ball-Chiu vertex) completely specified by dressed quark propagator.
- Can be straightforwardly calculated in Rainbow-Ladder appr.:
  - important for renormalizibility (Curtis-Pennington term),
  - anomalous magnetic moment,
  - contains $\rho$ meson pole!

The latter property is important to obtain the correct physics!

All elements specified to calculate baryon amplitudes and properties:
Use computer with sufficient RAM ($\sim$ tens of GB) and run for a few hours ...
Some Selected Results

<table>
<thead>
<tr>
<th>$S_{11}^+$</th>
<th>$S_{11}^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{11}^+$</td>
<td>$V_{11}^-$</td>
</tr>
<tr>
<td>$S_{13}^+$</td>
<td>$S_{13}^-$</td>
</tr>
<tr>
<td>$V_{13}^+$</td>
<td>$V_{13}^-$</td>
</tr>
<tr>
<td>$X_{53}^+$</td>
<td>$X_{53}^-$</td>
</tr>
</tbody>
</table>

Dominant covariants ($M_A$ solution)

$s = 1/2, \ l = 0$
(s-wave)

$s = 1/2, \ l = 1$
(p-wave)

$s = 3/2, \ l = 1$
(p-wave)

$q [GeV]$

| 0.00 | 0.05 | 0.09 | 0.15 | 0.22 | 0.32 | 0.46 | 0.63 | 0.86 | 1.17 |

$\sim f_i^{00} (p^2, q^2, 0)$
Some Selected Results

- Both models designed to reproduce correctly $D\chi_{SB}$ and pion properties within RL. They capture beyond-RL effects at this quark-mass.
- This behaviour extends to other light states ($\rho$, $N$, $\Delta$), one gets a good description.
- Both interactions similar at intermediate momentum region: $\sim 0.5 - 1$ GeV is the relevant momentum region for $D\chi_{SB} &$ ground-state hadron props.
- Slight differences at larger current masses, however, qualitative model indep.

PoS QNP2012 (2012) 112
The trend of both models is maintained for c-quarks, but unexpectedly, for b-quarks both models exactly agree.

ϒ-mass is very well reproduced, but not so much Ω_{bbb}: Effect of 3-body interactions?

To make precise statements, we should fit the models to the heavy sector (where corrections to RL should be suppressed) and study the evolution to light quarks. Remember: models capture beyond-RL effects at u/d-quark mass.
Some Selected Results

Nucleon electromagnetic form factors

Nucleon em. FFs vs. momentum transfer
Eichmann, PRD 84 (2011)

- Good agreement with recent data at large $Q^2$
- Good agreement with lattice at large quark masses
- Missing pion cloud below $\sim 2$ GeV$^2$, in chiral region

~ nucleon quark core without pion effects
Nucleon electromagnetic form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius

\[(r_1^v)^2 \, [fm^2]\]

- Pion-cloud effects missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)

\[\kappa^v \, [\mu_N]\]

\[\kappa^s \, [\mu_N]\]

- But: pion-cloud cancels in \(\kappa^s \Leftrightarrow\) quark core
  
  Exp: \(\kappa^s = -0.12\)
  
  Calc: \(\kappa^s = -0.12(1)\)
Some Selected Results

\[ \Delta \text{electromagnetic form factors} \]


- \( G_{E2} \) and \( G_{M3} \): Deviation from sphericity!
- Important: Difference to quark-diquark model in \( G_{E2} \) and \( G_{M3} \).
- Large \( G_{E2} \) for small \( Q^2 \)!
- “Small” \( G_{M3} \) is a prediction!
Some Selected Results

Ω electromagnetic form factors


- Again deviation from sphericity!
- Only weak quark mass dependence!
Some Selected Results

\[ \Delta \to N\gamma \] electromagnetic transition form factors

H. Sanchis-Alepuz et al., in preparation

Slight deviation from corresponding results in diquark-quark model!

Good agreement with experimental results.

Deviation from sphericity is an effect from sub-leading components required by Poincaré invariance, i.e., relativistic physics!
Summary

Hadrons from QCD bound state equations:

- Fundamental (i.e., quark and gluon) Landau gauge Green functions well enough determined for hadron phenomenology!
- QCD bound state equations: Unified approach to mesons and baryons feasible!
- So far (shown): In rainbow-ladder appr. meson observables and $N / \Delta / \Omega$ masses and (e.m., axial, . . .) form factors.
- So far (not shown): Behaviour of propagators in conformal window, i.e., for $N_f > N_f^{\text{crit}}$.

Meson properties beyond rainbow-ladder approximation.
M. Vujinovic, R. Williams, arXiv:1411.7619; M. Mitter et al., arXiv:1411.7978;
R. Williams et al., arXiv:1512.00455; M. Vujinovic, RA, in preparation
SU(2) gauge theory with two massless fund. fermions:

\[ K = i T^a + \frac{1}{2} \lambda^{ab} \partial_\mu T_{ab} \partial^\mu \]

QCD with two light flavours:

<table>
<thead>
<tr>
<th></th>
<th>RL</th>
<th>2PI-3L</th>
<th>3PI-3L</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^+ (\pi))</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>(0^{++} (\sigma))</td>
<td>0.64</td>
<td>0.52</td>
<td>1.1(1)</td>
<td>0.48(8) or 1.3</td>
</tr>
<tr>
<td>(1 (\rho))</td>
<td>0.74</td>
<td>0.77</td>
<td>0.74</td>
<td>0.78</td>
</tr>
<tr>
<td>(1^{++} (a1))</td>
<td>0.97</td>
<td>0.96</td>
<td>1.3(1)</td>
<td>1.23(4)</td>
</tr>
<tr>
<td>(1^+ (b1))</td>
<td>0.85</td>
<td>1.1</td>
<td>1.3(1)</td>
<td>1.23</td>
</tr>
<tr>
<td>(f_\pi)</td>
<td>0.092</td>
<td>0.103</td>
<td>0.105</td>
<td>0.092</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(J^{PC})</th>
<th>NA, 1PI</th>
<th>NA + AB, 1PI</th>
<th>NA, 3PI</th>
<th>NA + AB, 3PI</th>
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</thead>
<tbody>
<tr>
<td>0^{--}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0^{++}</td>
<td>1.39(3)</td>
<td>1.22(2)</td>
<td>1.33(3)</td>
<td>1.25(2)</td>
</tr>
<tr>
<td>1^{--}</td>
<td>2.27(5)</td>
<td>2.00(4)</td>
<td>2.37(5)</td>
<td>1.99(4)</td>
</tr>
<tr>
<td>1^{++}</td>
<td>2.87(5)</td>
<td>2.65(5)</td>
<td>3.09(6)</td>
<td>2.67(5)</td>
</tr>
</tbody>
</table>
Outlook

- In rainbow-ladder appr. 2-photon processes as, e.g., nucleon Compton scattering.
- Dynamical hadronization incl. dressed vertex functions in the Exact Renormalization Group approach.
- Technicolour theories: Bound states for near-conformal gauge theories as, e.g., decay width of techni-$\rho$-meson ($\rightarrow$ signal at LHC?).
- Uncharged (technicolour) bound states as candidates for Dark Matter?