From gauge fields to physical particles

Hadrons as relativistic bound states of confined quarks and glue

Reinhard Alkofer

Institute of Physics, University Graz

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A prelude: The Higgs boson and gauge invariance

CERN 2012: The Higgs boson has been measured!

The ‘textbook‘ Higgs is a gauge-dependent state and thus unphysical!?!

Gauge invariance broken? No! (Elitzur‘s theorem)

The custodial (global!) SU(2) symmetry of the SM is broken:
Goldstone bosons become elements of the elementary BRST quartets!

Gauge-invariant states are necessarily composite!

Relation between gauge-invariant and gauge-dependent states:
Fröhlich, Morchia, Strocchi, PLB 97 (1980), NPB 190 (1981)
Physical H, W and Z are gauge-invariant H-H, H-W and H-Z bound states with same mass as elementary fields in unitary gauge.

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Outline

Motivation: Why Functional Approaches to QCD?

Where to look for the **nucleon in QCD?**

Free propagation of lowest three-quark bound state:

Six-quark Green function!

Calculating it requires either

- to employ a lattice (*i.e.*, give up Poincaré invariance)
- to use Monte-Carlo algorithms (*i.e.*, use a statistical method)
- to run programs on supercomputers

or

- to fix a gauge (*i.e.*, sacrifice gauge invariance)
- to truncate equations in a way which is verified *á posteriori*

Method 1:

Excellent results for hadron properties & insight into hadron structure!

Method 2:

Relation of observables to confinement, DχSB, axial anomaly, ...

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QCD correlation functions contribute to the understanding of

- ★ confinement of gluons, quarks, and colored composites.
- ★ DχSB, *i.e.*, generation of quark masses and chirality-changing quark-gluon interactions.
- ★ $U_A(1)$ anomaly and topological properties.

Within **Functional Methods**
(Exact Renorm. Group, Dyson-Schwinger eqs., *n*PI methods, . . .):
Input into hadron phenomenology via **QCD bound state eqs.**.

- Bethe-Salpeter equations for **mesons**
  form factors, decays, reactions, ...
- covariant Faddeev equations for **baryons**
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- in principle ab initio ☺
- perturbation theory included ☺
- some elementary Green's functions quite well-known ☺
- in other gauges complicated . . . ☹
- truncations for numerical solutions necessary ☹

State-of-the-art:

- **3-particle-irreducible** truncation with dynamical propagators and three-point functions for mesons.
- 2PI **rainbow-ladder truncation** (= dressed gluon exchange) with dynamical propagators for baryons.

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Basics of Covariant Gauge Theory

Gauge theory: Unphysical degrees of freedom!

QED: Physical states obey Lorentz condition.

$$\partial_\mu A^\mu |\Psi\rangle = 0 \quad (\mathrm{Gupta-Bleuler}).$$

$\Rightarrow$ In symmetric phase:
Two physical massless photons in physical state space.

Time-like photon (i.e. negative norm state!) cancels longitudinal photon in S–matrix elements!

Guaranteed on an algebraic level!

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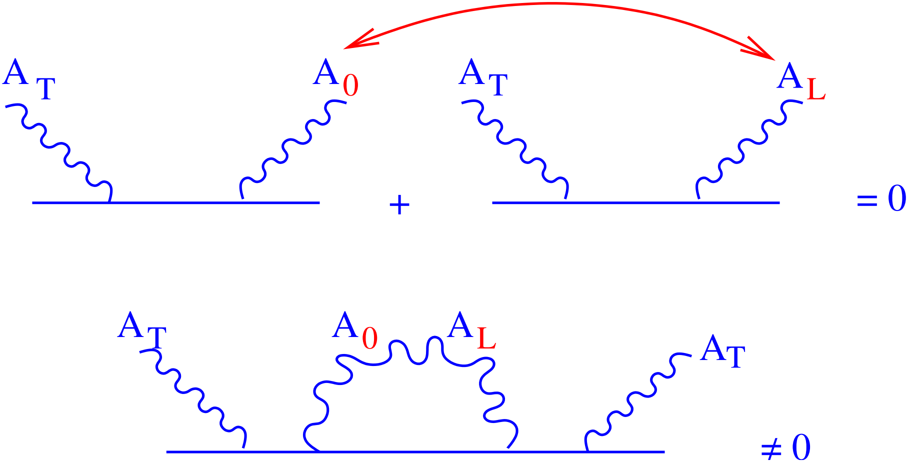
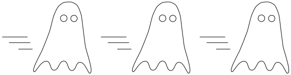
Covariant Gauge Theory

In S-Matrix

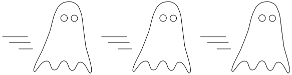


Quantum Yang-Mills theory:

Selfinteraction of gluons:
transverse gluons scatter into longitudinal ones and vice versa!



⇒ Faddeev–Popov ghosts = anticomm. scalar fields.



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Important in quantum fluct., but no associated particles!

Global ghost field as 'gauge parameter':
BRST symmetry of the gauge-fixed action!

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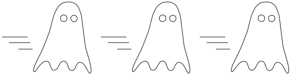

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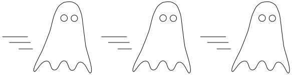

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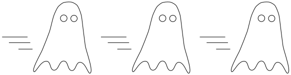

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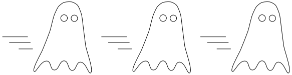

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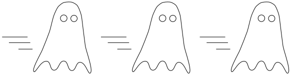

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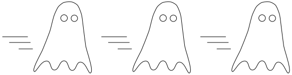

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Covariant Gauge Theory

Symmetry of the gauge-fixed generating functional:

$$\delta_B A^a_\mu = D^{ab}_\mu c^b \lambda \,, \qquad \delta_B q = -igt^a c^a q \lambda \,,$$
$$\delta_B c^a = -\frac{g}{2} f^{abc} c^b c^c \lambda \,, \qquad \delta_B \bar{c}^a = \frac{1}{\xi} \partial_\mu A^a_\mu \lambda \,,$$

Becchi–Rouet–Stora & Tyutin (BRST), 1975

- Parameter $\lambda \in$ Grassmann algebra of the ghost fields
- λ carries ghost number $N_{\text{FP}} = -1$
- Via Noether theorem: BRST charge operator Q_B
- generates ghost # graded algebra $\delta_B \Phi = \{iQ_B, \Phi\}$

Covariant Gauge Theory

BRST algebra: $Q_B^2 = 0$, $[iQ_c, Q_B] = Q_B$,

- complete in indefinite metric state space $\mathcal{V}$.
- generates ghost # graded $\delta_B \Phi = \{iQ_B, \Phi\}$.
- $\mathcal{L}_{GF} = \delta_B \left(\bar{c}\left(\partial_\mu A^\mu + \frac{\alpha}{2} B\right)\right)$ BRST exact.

Positive definite subspace $\mathcal{V}_{\text{pos}} = \text{Ker}(Q_B)$
(*i.e.* all states $|\psi\rangle \in \mathcal{V}$ with $Q_B|\psi\rangle = 0$)
contains $\text{Im}Q_B$ (*i.e.* all states $Q_B|\phi\rangle$),
c.f. exterior derivative in differential geometry.

Hilbert space: cohomology $\mathcal{H} = \frac{\text{Ker}Q_B}{\text{Im}Q_B} \simeq \mathcal{V}_s$ BRST singlet
longitudinal & timelike gluons, ghosts : BRST quartet

(c.f. Gupta–Bleuler mechanism in QED)

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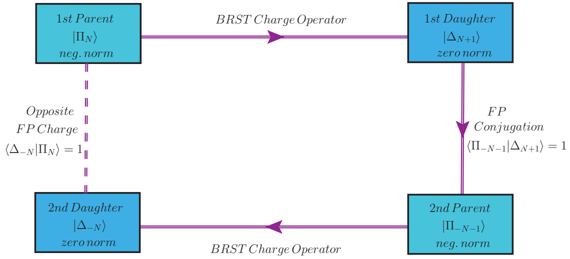
Covariant Gauge Theory

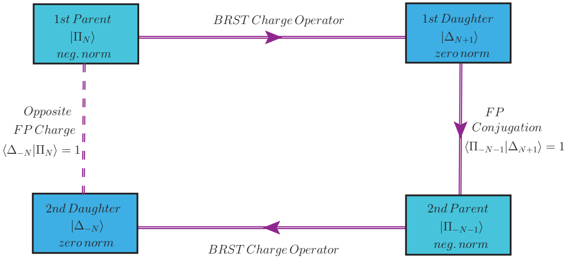
Perturbative BRST quartet of time-like gluons:



$\|\Pi_0\rangle$	lin. combination of time-like and long. gluon
$\|\Delta_1\rangle$	ghost
$\|\Pi_{-1}\rangle$	antighost
$\|\Delta_0\rangle$	lin. combination of time-like and long. gluon

Kugo–Ojima confinement

Non-perturbative BRST quartets of transverse gluons, resp., quarks:



$\|\Pi_0\rangle$	transverse gluons	quarks
$\|\Delta_1\rangle$	gluon-ghost bound states	quark-ghost bound states
$\|\Pi_{-1}\rangle$	gluon-antighost bound states	quark-antighost bound states
$\|\Delta_0\rangle$	gluon-ghost-antigh./gluonic b.s.	quark-gh.-antigh./quark-gluon

N. Alkofer and R.A., Phys. Lett. B **702** (2011) 158 [arXiv:1102.2753 [hep-th]];
PoS **FACESQCD** (2011) 043 [arXiv:1102.3119 [hep-th]].

Kugo–Ojima confinement criterion

$\Rightarrow$ Physical states are BRST singlets!
(BRST cohomology: Hilbert space $\mathcal{H} = \frac{\mathrm{Ker}\, Q_{BRST}}{\mathrm{Im}\, Q_{BRST}}$.)

Time–like and longitudinal gluons (BRST quartet) removed from asymptotic states as in QED, but:

Transverse gluons and quarks also BRST quartets, i.e. confined, if **ghost propagator is highly infrared singular!**
($\Rightarrow$ **Kugo–Ojima confinement criterion**)

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Kugo–Ojima confinement criterion

Realization of Confinement depends on global gauge structure:
Globally conserved current ($\partial^\mu J^a_\mu = 0$)

$$J^a_\mu = \partial^\nu F^a_{\mu\nu} + \{Q_B, D^{ab}_\mu \bar{c}^b\}$$

with charge $$Q^a = G^a \quad + \quad N^a.$$

QED: MASSLESS PHOTON states in both terms.
Two different combinations yield:
unbroken global charge $\tilde{Q}^a = G^a + \xi N^a$.
spont. broken displacements (photons as Goldstone bosons).

No massless gauge bosons in $\partial^\nu F^a_{\mu\nu}$: $G^a \equiv 0$.
(QCD, e.w. Higgs phase, ...)

Kugo–Ojima confinement criterion

QCD: Unbroken global charge

$$Q^a = N^a = \{Q_B, \int d^3x D_0^{ab} \bar{c}^b\}$$

well–defined in $\mathcal{V}$.
With $D_\mu^{ab} \bar{c}^b(x) \stackrel{x^0 \to \pm\infty}{\longrightarrow} (\delta^{ab} + u^{ab}) \partial_\mu \bar{\gamma}^b + \dots$

$\Rightarrow$ Kugo-Ojima Confinement Criterion:

$$u^{ab}(0) = -\delta^{ab}$$

where

$$\int dx e^{ip(x-y)} \langle 0 | T \, D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y) | 0 \rangle =: \; (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2),$$

If fulfilled: **Physical States $\equiv$ BRST singlets $\equiv$ color singlets!**

Kugo–Ojima confinement criterion

<u>In Landau gauge:</u>

Ghost propagator more sing. than simple pole

$\Downarrow$

Kugo-Ojima criterion

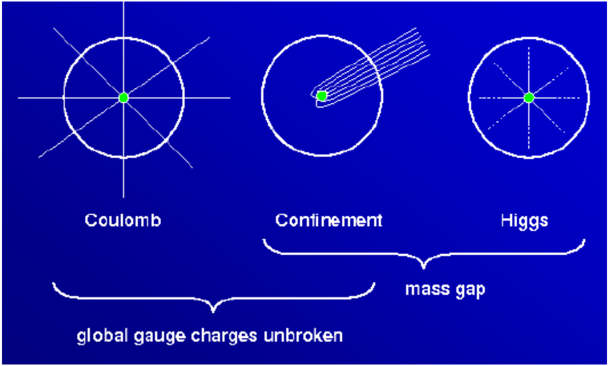
T. Kugo, hep-th/9511033, Int. Symp. “BRS Symmetry”, Kyoto.

Coulomb, Confinement, & Higgs phase

Confinement vs. Higgs mechanism?

No gauge-invariant order parameter!

A possible definition of **confinement** in the presence of fundamental charges†:



cf., V. Mader et al., Eur. Phys. J. **C74** (2014) 2881 [arXiv:1309.0497]; and refs. therein

†Wilson loop gives only a clear criterion in the absence of quarks!

Confinement and Higgs mechanism

If BRST or equivariant BRST symmetry:
Infrared saturation of Quantum E.o.M. of gauge boson propagator discriminates phases of gauge theories!

Coulomb:	massless gauge boson
Confinement:	unphysical current
Higgs:	physical current

Saturating part of current is given by:

$\tilde{j}_\mu(x) = j_\mu^{U(1)} - i\partial_\mu b$ linear covariant Abelian U(1) ,

$\tilde{j}^a_\mu(x) = j_\mu^{LCG\,a} - is(D_\mu\bar{c})^a$ in LCG (Kugo-Ojima scenario),

$\tilde{j}^a_\mu(x) = j_\mu^{GLCG\,a} - is_\alpha(D_\mu\bar{c})^a$ in ghost-antighost sym. GLCG ,

$\tilde{j}_\mu(x) = j_\mu^{MAG} - i\partial_\mu b$ SU(2) in Maximally-Abelian Gauge ,

$\tilde{j}^a_k(x) = j_k^{C\,a} - is(D_k\bar{c})^a$ spatial components in Coulomb gauge ,

$\tilde{j}^a_\mu(x) = j_\mu^{GZ\,a} - is\chi^a_\mu$ in Gribov-Zwanziger theory .

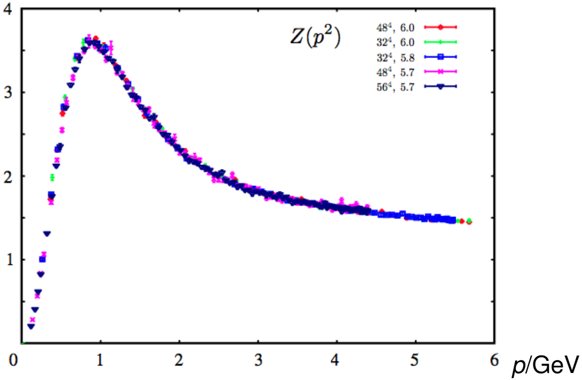
V. Mader et al., Eur. Phys. J. **C74** (2014) 2881 [arXiv:1309.0497]

1. Motivation: Why Functional Approaches to QCD?
2. Basics of Covariant Gauge Theory
3. QCD Green functions in Landau gauge
   - Gluon, Ghost and Quark Propagators
   - Three-point vertex functions
4. Relativistic Three-Fermion Bound State Equations
   - Structure of Baryonic Bound State Amplitudes
   - Quark Propagator and Rainbow Truncation
   - Interaction Kernels and Rainbow-Ladder Truncation
   - Coupling of E.M. Current and Quark-Photon Vertex
   - Some Selected Results
5. Summary and Outlook

Gluon Propagator

pure Yang-Mills, $T = 0$
Landau gauge Gluon Ren. Fct. $D^{tr}_{\text{Gluon}} = \mathbf{Z(p^2)}/p^2$

A. Sternbeck *et al.*, PoS LAT2006, 76



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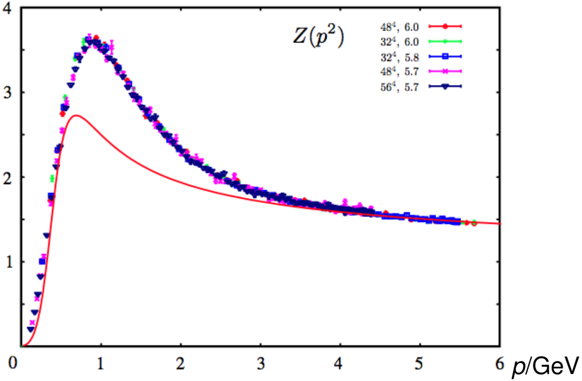

— L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. 79 (1997) 3591

Dyson-Schwinger eqs. (DSEs)

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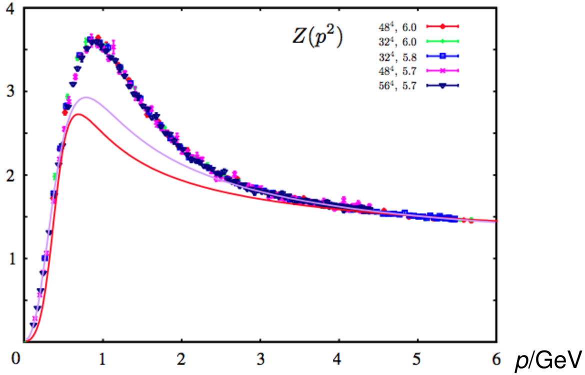

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— C. S. Fischer, R.A., Phys. Lett. **B536** (2002) 177;

C. Lerche, L. von Smekal, Phys. Rev. **D65** (2002) 125006.

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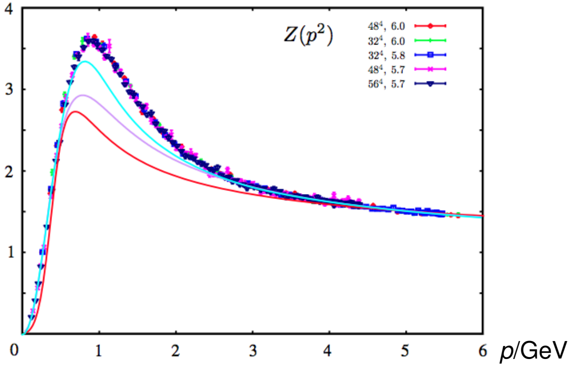

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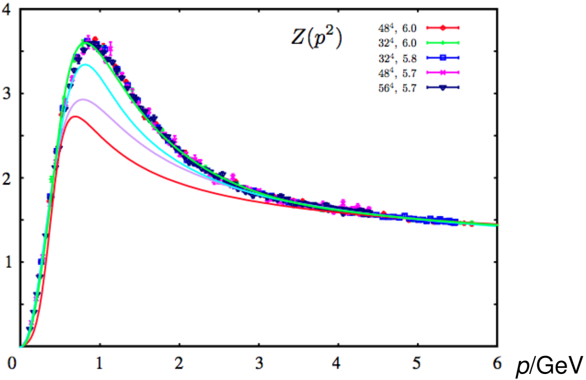
— J.M.Pawlowski, D.Litim, S.Nedelko, L.v.Smekal, PRL**93** (2004) 152002
Exact Renormalization Group (ERG) eqs.

Gluon Propagator



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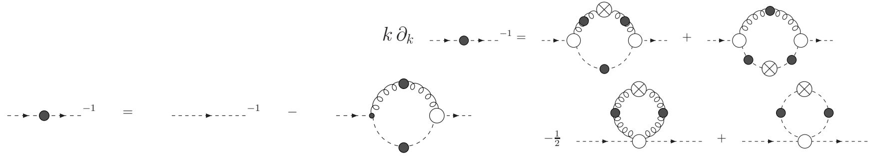
— C.S. Fischer, A. Maas, J.M. Pawlowski, Ann. Phys. **324** (2009) 2408;
L. Fister, PhD thesis

combination of ERG,DSE & 2PI!

Use DSEs **and** FRG eqs:

→ Two different towers of equations for Green functions
E.g. ghost propagator



MATHEMATICA-based derivation of functional equations:

R. A., M. Q. Huber, K. Schwenzer, Comp. Phys. Comm. **180** (2009) 965;
M. Q. Huber and J. Braun, Comp. Phys. Comm. **183** (2012) 2441 .

Infrared Exponents for Gluons and Ghosts:

Apply asymptotic expansion to all primitively divergent Green functions,
[R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.]
use DSEs **and** ERGEs:
→ Two different towers of equations for Green functions

IR-Analysis of whole tower of equations ⇒

Unique scaling solution

+ an one-parameter family of solutions with IR trivial Green functions.
[C.S. Fischer and J.M. Pawlowski, PRD **80** (2009) 025023]

Scaling vs. decoupling solution:

- Lattice calculations for gluon propagator: decoupling solutions.
  [A. Cucchieri et al., many others; but also: A. Sternbeck et al.]
- Scaling solution respects, decoupling solutions break BRST.
- IR behaviour might depend on non-pert. completion of gauge.
  [A. Maas, Phys. Lett. **B689** (2010) 107.]
- Similar (identical) analytical structure for gluon propagator.

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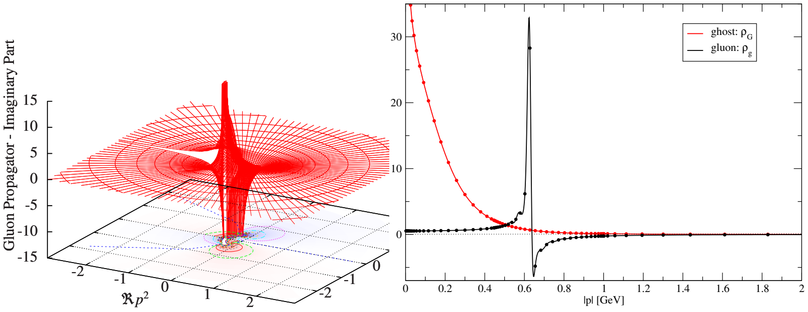
Analytical structure for gluon propagator

Gluon propagator is positivity violating, cut along time-like half-axis.

R.A., W. Detmold, C.S. Fischer and P. Maris, PR**D70** (2004) 014014

Imaginary part of gluon propagator:



S. Strauss, C. S. Fischer, C. Kellermann, Phys. Rev. Lett. **109** (2012) 252001 [arXiv:1208.6239]

Infrared Exponents for Gluons and Ghosts

Scaling solution:
n external ghost & antighost legs and m external gluon legs
(one external scale p^2; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

Scaling / decoupling solution:

- Ghost propagator IR divergent (Kugo-Ojima for scaling)
- Gluon propagator IR suppressed (Gribov-Zwanziger)
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent / finite

Note:
Same dichotomy of solutions in Coulomb gauge (variational / ERG)
and in maximally Abelian gauge (DSE / ERG).

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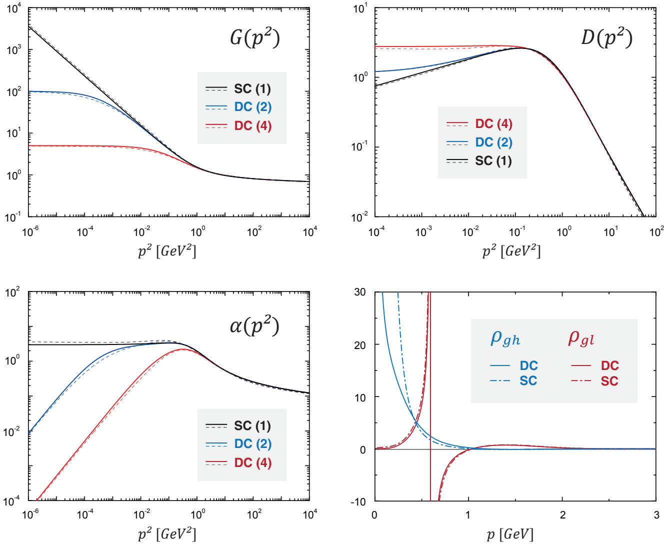
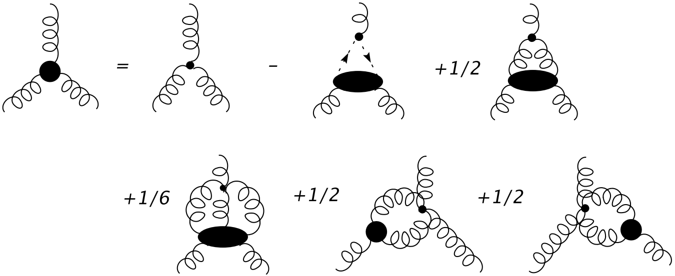
Gluon and Ghost Propagators



DSE for three-gluon vertex:

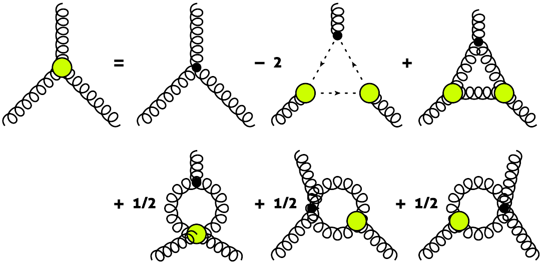
[G. Eichmann, R. Williams, M. Vujinovic, RA, Phys. Rev. D89 (2014) 105014]



Three-gluon vertex

DSE for three-gluon vertex (truncated):

[G. Eichmann, R. Williams, M. Vujinovic, RA, Phys. Rev. D89 (2014) 105014]



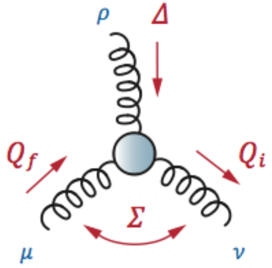
- 14 tensor elements → 4 fully transverse ones
- apply cyclic perm. w.r.t. external legs → bose symmetry
- Is there a zero in component projected on tree-level tensor?
  (*cf.*, A. Maas)

Three-gluon vertex

Applications:

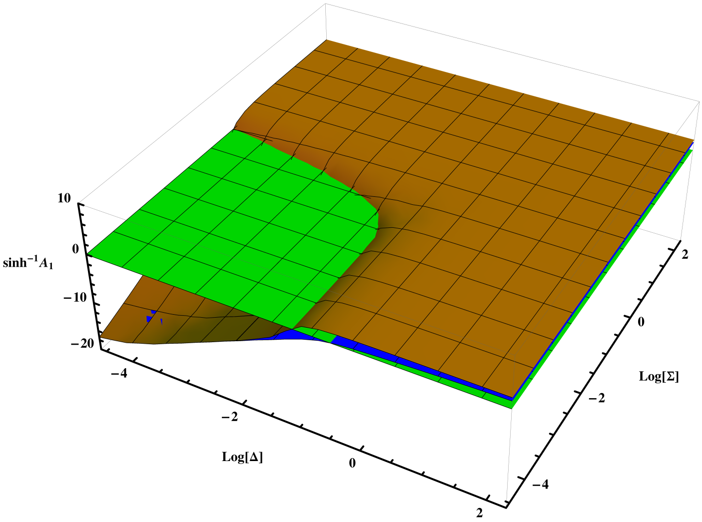
- quark-gluon vertex (and consequently unquenching)
- conformal window
- symmetry preserving Bethe-Salpeter kernel
- irreducible three-body forces in the baryon

Kinematics:



Three-gluon vertex

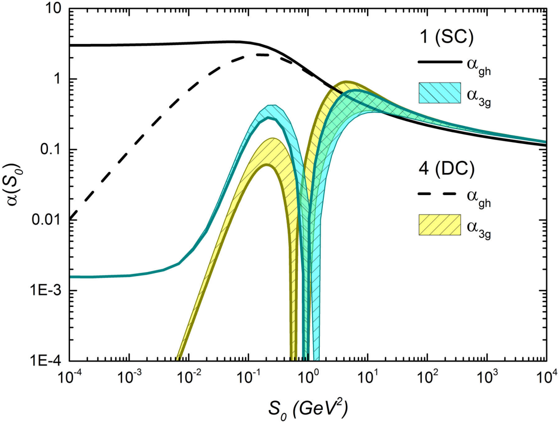
$\Gamma^{(1)\mu\nu\rho}_{3g,TTT}$



Green: zero; Orange: ghost triangle; Blue: ghost + swordfish.

Three-gluon vertex

Running coupling:



Coupling quarks: quark prop. and quark-gluon vertex

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys. **324** (2009) 106;
C.S. Fischer and R. Williams, Phys. Rev. Lett. 103 (2009) 122001;
A. Windisch, M. Hopfer, G. Eichmann, RA, in preparation.

Chiral symmetry dynamically or explicitely broken:

- quark propagator infrared finite:

$$S(p) = \frac{\not p - M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2)$$

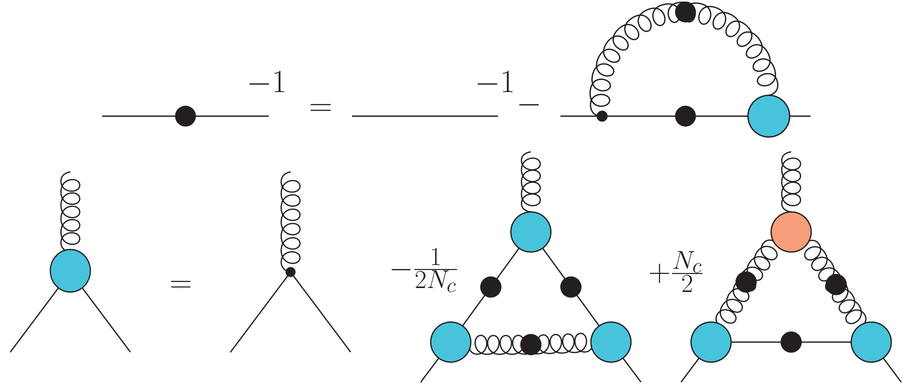
AND

- quark-gluon vertex
  incl. dynamically generated χSB tensors structures:

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \quad G^2_\mu = \hat{p}_\mu \,, \quad G^3_\mu = \ldots$$

Coupling quarks: quark prop. and quark-gluon vertex

DSEs for quark propagator and quark-gluon vertex via 3PI action:

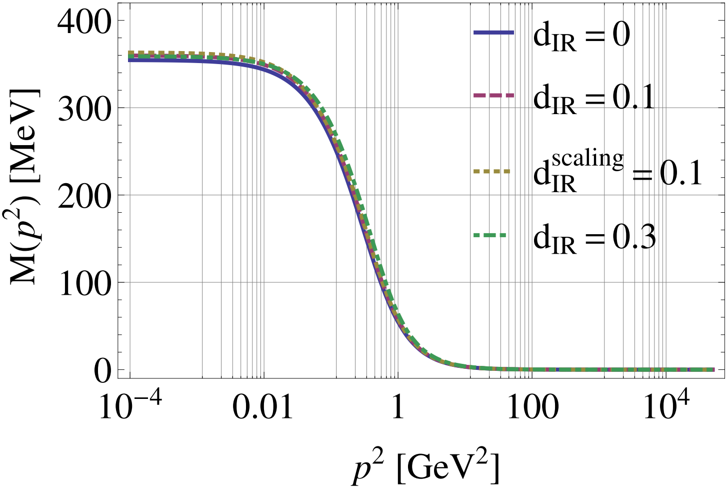


hadron physics

IR behaviour
confinement

Coupling quarks to gluons: quark propagator

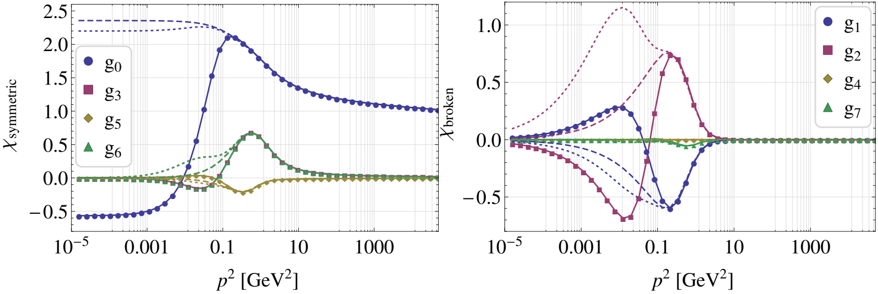
Quark mass function with calculated QGV and modeled 3gV:



Coupling quarks to gluons: quark-gluon vertex

Eight transverse tensor structures,
e.g., at symm. momenta $x = p_1^2 = p_2^2 = p_3^2$:

Significant IR enhancement!

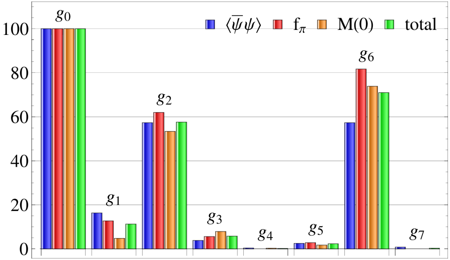


The zero in the three-gluon vertex causes zero in the tree-level structure of quark-gluon vertex!

DχSB in QGV!!!

Coupling quarks to gluons: quark-gluon vertex

Relative importance:



dynamically generated **scalar**-type coupling important!

Summary on Landau gauge 2- and 3-point-functions

For a small number of light flavours:*

- IR enhanced ghost propagator
- IR suppressed gluon propagator
- IR decoupled quark propagator & dynamically generated mass
- ghost-gluon vertex close to tree-level
- zero in three-gluon vertex
- infrared enhanced quark-gluon vertex &
  dynamically generated chirality-changing interactions

NB: Four-gluon vertex currently under investigation $\rightarrow$
primitively divergent Landau gauge QCD Green functions known!

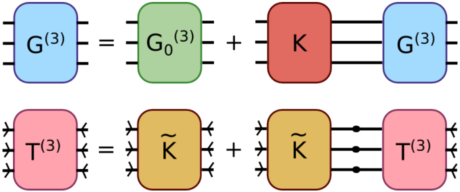
Higher order Green functions are finite upon upon renormalization of prim. dvgcs. & fulfill (multi-)linear equations.

---

*Increase N_f: 2nd order phase transition to conformal window! Propagators and 3-point-functions change significantly!

cf., M. Hopfer, C. Fischer, RA, JHEP **1411** (2014) 035 [arXiv:1405.7031]

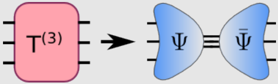
Dyson-Schwinger eq. for 6-point fct. $\Longrightarrow$ 3-body bound state eq.:



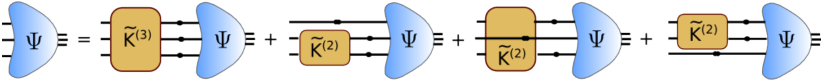
BOUND STATE:

Pole in $G^{(3)}$
or (equiv.) for $P^2 = -M_B^2$
Pole in $T^{(3)}$

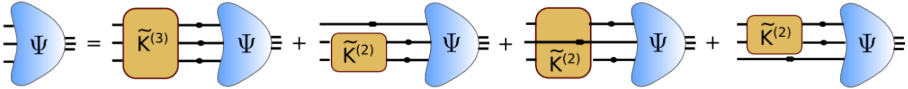
bound state amplitudes:



covariant 3-body bound state eq. (*cf.*, Bethe-Salpeter for 2-body BS)**:**



3-body bound state eq.:



NB: With 3-particle-irreducible interactions $\tilde{K}^{(3)}$ neglected:
Poincaré-covariant Faddeev equation.

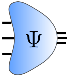
Elements needed for bound state equation:

- Tensor structures (color, flavor, Lorentz / Dirac) of the BS ampl.
- Full quark propagators for *complex* arguments
- Interaction kernels $K_{2,3}$

Needed for coupling to e.m. current:

- Full quark-photon vertex

Structure of baryonic bound state amplitudes



$\equiv \sim \langle 0|q_\alpha q_\beta q_\gamma|B_\mathcal{I}\rangle \propto \Psi_{\alpha\beta\delta\mathcal{I}}$ (with multi-indices $\alpha = \{x, D, c, f, \ldots\}$)

and $\mathcal{I}$ baryon (multi-)index $\Longrightarrow$ baryon quantum numbers

C. Carimalo, J. Math. Phys. **34** (1993) 4930.

Comparison to mesonic BS amplitudes $\langle 0|q_\alpha \bar{q}_\beta|M_\mathcal{I}\rangle \propto \Phi_{\alpha\beta\mathcal{I}}$:

- scalar and pseudoscalar mesons: 4 tensor structures each
- vector and axialvector mesons: 12 tensor struct. each, 8 transv.
- tensor and higher spin mesons: 8 transverse struct. each

C. H. Llewellyn-Smith, Annals Phys. **53** (1969) 521.

Relativistic three-fermion bound state equations

Requirements for (baryonic) bound state amplitudes:

- positive energy (for fermionic bound states)
- well-defined parity
- irreducible representation of the Poincaré group otherwise.

Possible and recommended:

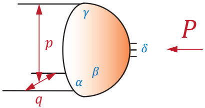
Complete orthogonal Dirac tensor basis
s.t. partial-wave composition in rest frame.

Relativistic three-fermion bound state equations

s	l	T_{ij}	
$1/2$	0	$\mathbb{1}\otimes\mathbb{1}$	**s waves (8)**
$1/2$	0	$\gamma^\mu_T\otimes\gamma^\mu_T$	
$1/2$	1	$\mathbb{1}\otimes\frac{1}{2}[\not p,\not q]$	**p waves (36)**
$1/2$	1	$\mathbb{1}\otimes\not p$	
$1/2$	1	$\mathbb{1}\otimes\not q$	
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\frac{1}{2}[\not p,\not q]$	
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\not p$	
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\not q$	
$3/2$	1	$3(\not p\otimes\not q-\not q\otimes\not p)-\gamma^\mu_T\otimes\gamma^\mu_T[\not p,\not q]$	
$3/2$	1	$3\,\not p\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not p$	
$3/2$	1	$3\,\not q\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not q$	
$3/2$	2	$3\,\not p\otimes\not p-\gamma^\mu_T\otimes\gamma^\mu_T$	**d waves (20)**
$3/2$	2	$\not p\otimes\not p+2\,\not q\otimes\not q-\gamma^\mu_T\otimes\gamma^\mu_T$	
$3/2$	2	$\not p\otimes\not q+\not q\otimes\not p$	
$3/2$	2	$\not q\otimes[\not q,\not p]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\not p]$	
$3/2$	2	$\not p\otimes[\not p,\not q]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\not q]$	

$$\chi(x_1,x_2,x_3)=\langle 0|\,T\,\psi(x_1)\,\psi(x_2)\,\psi(x_3)\,|N\rangle$$

Momentum space:
Jacobi coordinates p, q, P
⇒ 5 Lorentz invariants
⇒ 64 Dirac basis elements



$$\chi(p,q,P)=\sum_k \underbrace{f_k(p^2,\,q^2,\,p\cdot q,\,p\cdot P,\,q\cdot P)}_{\text{Momentum}}\;\underbrace{\tau^k_{\alpha\beta\gamma\delta}(p,q,P)}_{\text{Dirac}}\otimes\text{Flavor}\otimes\text{Color}$$

Complete, orthogonal **Dirac tensor basis**
(partial-wave decomposition in nucleon rest frame):
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

$$T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$
$$(\gamma_5\otimes\gamma_5)\,T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$

$$(A\otimes B)_{\alpha\beta\gamma\delta}=A_{\alpha\beta}\,B_{\gamma\delta}$$

Relativistic three-fermion bound state equations

Facts about the decomposition:

- Independent of any truncation of the bound state equation.
- Only Poincaré covariance and parity invariance exploited.
- It includes all possible internal spin and orbital angular momenta.
- For positive-parity, positive-energy (particle) baryons it consists of

spin-$\frac{1}{2}$ particle: 64 elements

	# elements
s-wave	8
p-wave	36
d-wave	20

G. Eichmann et al., PRL 104 (2010) 201601

spin-$\frac{3}{2}$ particle: 128 elements

s-wave	4
p-wave	36
d-wave	60
f-wave	28

H. Sanchis Alepuz et al. PRD 84 (2011) 096003

Relativistic three-fermion bound state equations

Antisymmetry of the nucleon amplitude under quark exchange:

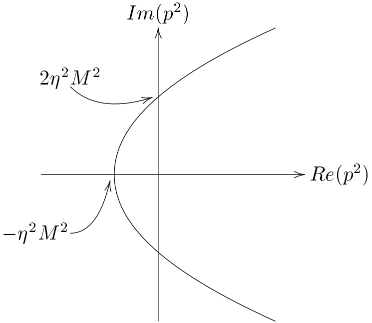
$$\Psi(p,q,P) = \Big\{ \underbrace{\psi_1(p,q,P)}_{M_A}\ \underbrace{\text{Flavor}_1}_{M_A} + \underbrace{\psi_2(p,q,P)}_{M_S}\ \underbrace{\text{Flavor}_2}_{M_S} \Big\}\ \underbrace{\text{Color}}_{A}$$

	M_A	M_S
Proton:	(ud)u	$\sqrt{2}$ [uu]d − [ud]u
Neutron:	(ud)d	[ud]d − $\sqrt{2}$ [dd]u

Flavor states decouple if the Faddeev kernel is flavor-independent (e.g. rainbow-ladder) ⇒ 2 degenerate solutions of the equation:

$$M_A \sim S^{+}_{11}, \ldots \qquad M_S \sim A^{+}_{11}, \ldots$$

Quark Propagator and Rainbow Truncation



In bound state eqs.:

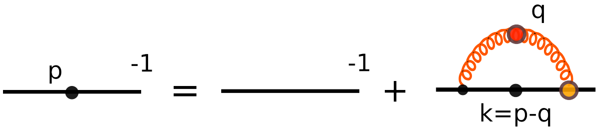
- Knowledge of the quark propagator inside parabolic region required.
- $\eta \geq 1/2$ for mesons and $\eta \geq 1/3$ for baryons.
- For ground states no singularities in parabolic region.

- Lattice: Values for real $p^2 \geq 0$ only.
- Dyson-Schwinger / ERG eqs.: complex p^2 accessible.†

†Beyond singularities: A. Windisch et al., arXiv:1304.3642; in preparation.

Quark Propagator and Rainbow Truncation

Dyson-Schwinger eq. for Quark Propagator:



$$S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k) \Gamma^\nu(k,p;q) D_{\mu\nu}(q)$$

$$D_{\mu\nu}(q)\Gamma_\nu(k,p;q) : \quad \begin{cases} D_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \frac{Z(q^2)}{q^2} \\ \Gamma_\nu(k,p;q) = \gamma_\nu Z_{1f} + \Lambda_\nu(k,p;q) \end{cases}$$

Quark Propagator and Rainbow Truncation

Rainbow truncation

Projection onto tree-level tensor γ_μ, restrict momentum dependence

$$\gamma_\nu Z_{1f} + \Lambda_\nu(k,p;q=p-k) \to \left(Z_{1f} + \Lambda(q^2)\right)\gamma_\nu$$

$$Z_{1f}\frac{g^2}{4\pi}D_{\mu\nu}(q)\Gamma_\nu(k,p;q) \to \begin{cases} Z_{1f}\frac{g^2}{4\pi}T_{\mu\nu}(q)\frac{Z(q^2)}{q^2}\left(Z_{1f}+\Lambda(q^2)\right)\gamma_\nu \\ =: Z_2^2 T_{\mu\nu}(q)\frac{\alpha_{eff}(q^2)}{q^2}\gamma_\nu \end{cases}$$

Interaction Kernels and Rainbow-Ladder Truncation

- Truncation of the quark-gluon vertex in the quark DSE.
- The BSE interaction kernel must be truncated accordingly.
- **Physical requirement: Chiral symmetry**, here axial WT id., $\{\gamma^5\Sigma(-p_-) + \Sigma(p_+)\gamma^5\}_{\alpha\beta} = -\int K^{q\bar{q}}_{\alpha\gamma\delta\beta}\{\gamma^5 S(-p_-) + S(p_+)\gamma^5\}$, which relates quark DSE and $q\bar{q}$ (meson) BSE kernel.

Ladder truncation

$q\bar{q}$ kernel compatible with rainbow truncation and axial WT id.:

$$K^{q\bar{q}} = 4\pi Z_2^2 \frac{\alpha_{eff}(q^2)}{q^2} T_{\mu\nu}(q)\gamma^\mu \otimes \gamma^\nu$$

Together constitute the DSE/BSE **Rainbow-Ladder truncation.**

Note: the truncation can and should be systematically improved!

Interaction Kernels and Rainbow-Ladder Truncation

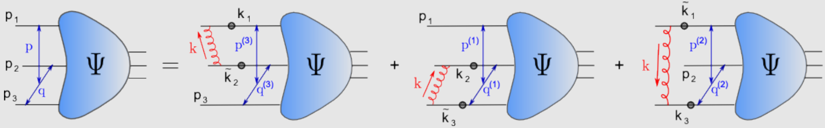
Rainbow-Ladder truncated three-body BSE:

- Previous studies used successfully the quark-diquark ansatz (reduction to a two-body problem).
- pNRQCD: 3-body contribution $\sim$ 25 MeV for heavy baryons.

> Supported by this, **the three-body irreducible kernel $K^{(3)}$ is neglected** (Faddeev approximation).

- Quark-quark interaction $K^{(2)}$: **same as quark-antiquark truncated kernel.** (!Different color factor!)

Rainbow-Ladder truncated covariant Faddeev equation



Interaction Kernels and Rainbow-Ladder Truncation

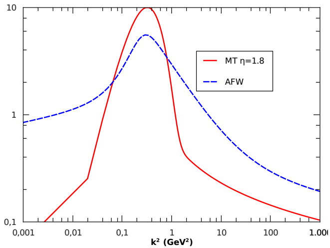
Effective interaction:

Maris-Tandy model (Maris & Tandy PRC60 1999)

$\alpha(k^2) = \alpha_{IR}(k^2; \Lambda, \eta) + \alpha_{UV}(k^2)$

- Purely phenomenological model.
- Λ fitted to f_π.
- Ground-state pseudoscalar properties *almost* insensitive to η around 1.8

Describes very succesfully hadron properties.



DSE motivated model (R.A.,C.S. Fischer,R. Williams EPJ A38 2008)

$\alpha(k^2; \Lambda_S, \Lambda_B, \Lambda_{IR}, \Lambda_{YM})$

- DSE-based in the deep IR.
- Designed to give correct masses of π, ρ and η' *($U_A(1)$ anomaly!)*.
- 4 energy scales! Fitted to π, K and η'.

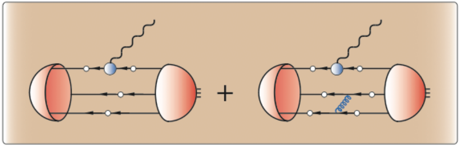
Note: The resulting qq-interaction is chirality-conserving, flavour-blind and current-quark mass independent.

Beyond Rainbow-Ladder

- "Corrections beyond-RL" refers to corrections to the effective coupling but also to additional structures beyond vector-vector interaction.
- They can induce a different momentum dependence of the interaction.
- They can also **induce a quark-mass and quark-flavour dependence of the interaction**
- Question: how important are beyond-RL effects?

Coupling of E.M. Current and Quark-Photon Vertex

Electromagnetic current in the three-body approach:



by "gauging of equations"
M. Oettel, M. Pichowsky and L. von Smekal, Eur. Phys. J. A **8** (2000) 251 [nucl-th/9909082].

Impulse appr.	+	Coupling to spectator q	+	Coupling to 2-q kernel not present in RL appr.	+	Coupling to 3-q kernel not present in Faddeev appr.

Additional Input: Quark-Photon Vertex

Coupling of E.M. Current and Quark-Photon Vertex

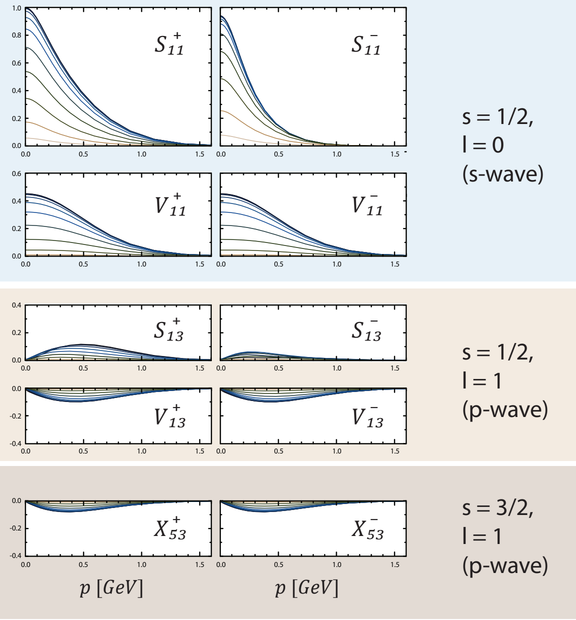
Quark-Photon Vertex:

- Vector WT id. determines vertex up to purely transverse parts: "Longitudinal" part (Ball-Chiu vertex) completely specified by dressed quark propagator.
- Can be straightforwardly calculated in Rainbow-Ladder appr.:
  - important for renormalizibility (Curtis-Pennington term),
  - anomalous magnetic moment,
  - contains ρ meson pole!

The latter property is important to obtain the correct physics!

All elements specified to calculate baryon amplitudes and properties:
Use computer with sufficient RAM ($\sim$ tens of GB) and run for a few hours . . .

Some Selected Results



s = 1/2,
l = 0
(s-wave)

s = 1/2,
l = 1
(p-wave)

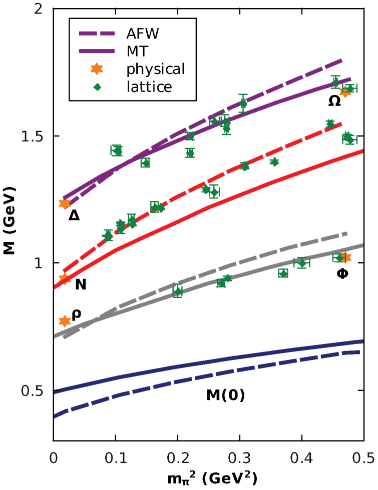
s = 3/2,
l = 1
(p-wave)

Dominant covariants (M_A solution)

$$\sim f_i^{00}(p^2, q^2, 0)$$

q [GeV]
0.00
0.05
0.09
0.15
0.22
0.32
0.46
0.63
0.86
1.17

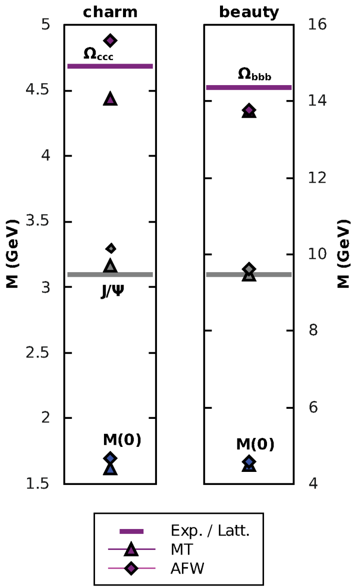
Some Selected Results



PoS QNP2012 (2012) 112

- Both models designed to reproduce correctly DχSB and pion properties within RL. **They capture beyond-RL effects at this quark-mass.**
- This behaviour extends to other light states (ρ, N, Δ), one gets a good description.
- Both interactions similar at intermediate momentum region: $\sim$ **0.5 − 1 GeV is the relevant momentum region for DχSB & ground-state hadron props.**
- Slight differences at larger current masses, however, **qualitative model indep.**

Some Selected Results



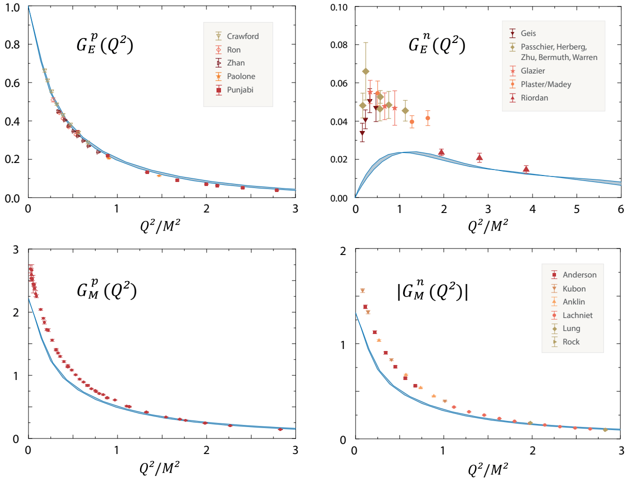
- The trend of both models is maintained for c-quarks, but unexpectedly, **for b-quarks both models exactly agree**.
- Υ-mass is very well reproduced, but not so much Ω_{bbb}: **Effect of 3-body interactions?**
- To make precise statements, we should fit the models to the heavy sector (where corrections to RL should be suppressed) and study the evolution to light quarks. **Remember: models capture beyond-RL effects at u/d-quark mass.**

Nucleon electromagnetic form factors

Nucleon em. FFs
vs. momentum transfer
Eichmann, PRD 84 (2011)

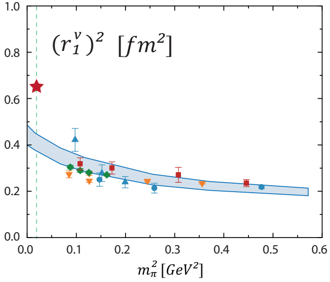
- Good agreement with recent **data** at large Q^2
- Good agreement with **lattice** at large quark masses
- **Missing pion cloud** below ~2 GeV^2, in chiral region

~ **nucleon quark core** without pion effects



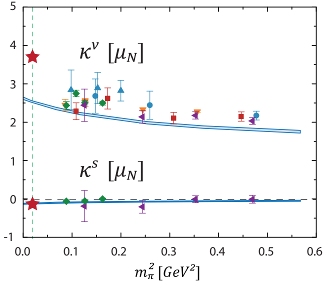
Nucleon electromagnetic form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



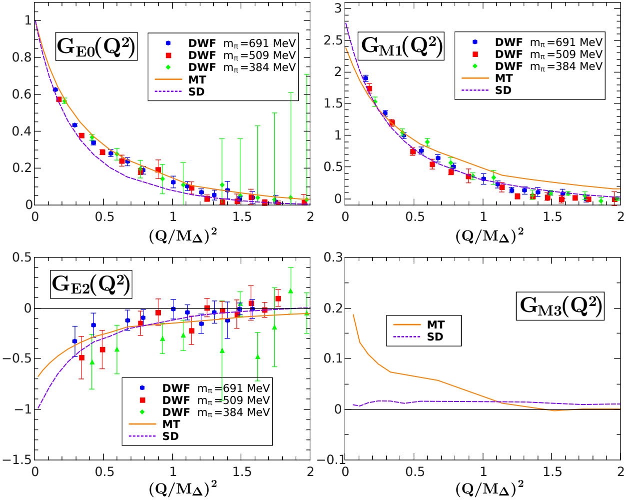
Eichmann, PRD 84 (2011)

- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

Δ electromagnetic form factors

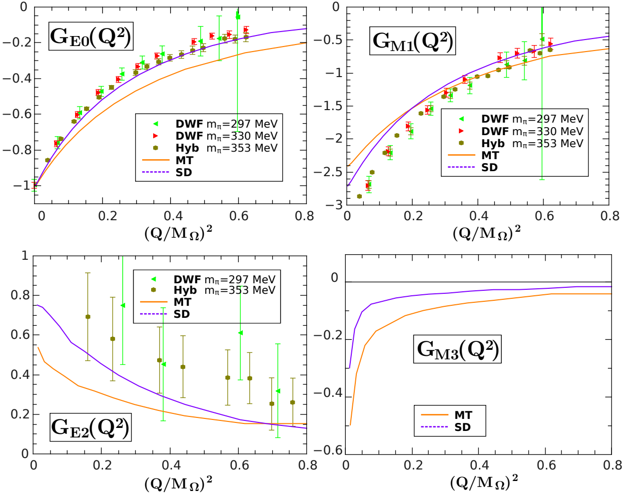
H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].



- G_{E2} and G_{M3}: Deviation from sphericity!
- Important: Difference to quark-diquark model in G_{E2} and G_{M3}.
- Large G_{E2} for small Q^2!
- “Small” G_{M3} is a prediction!

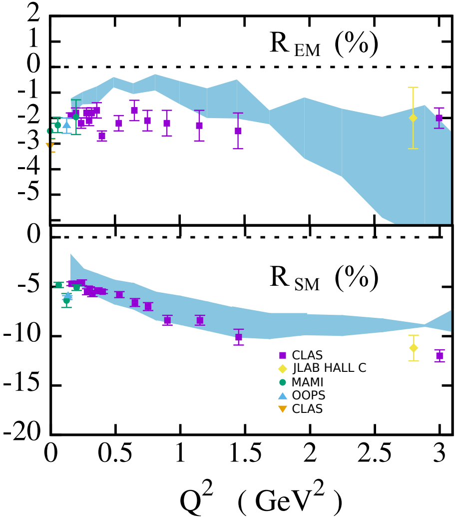
Ω electromagnetic form factors

H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].



- Again deviation from sphericity!
- Only weak quark mass dependence!

$\Delta \to N\gamma$ electromagnetic transition form factors



H. Sanchis-Alepuz *et al.*, in preparation

Slight deviation from corresponding results in diquark-quark model!

Good agreement with experimental results.

Deviation from sphericity is an effect from sub-leading components required by Poincaré invariance, *i.e.*, relativistic physics!

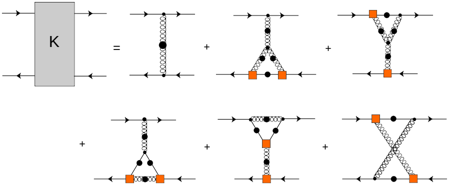
Summary

Hadrons from QCD bound state equations:

- Fundamental (*i.e.*, quark and gluon) Landau gauge Green functions well enough determined for hadron phenomenology!
- QCD bound state equations:
  Unified approach to mesons and baryons feasible!
- So far (shown):
  In rainbow-ladder appr. meson observables
  and N / Δ / Ω masses and (e.m., axial, . . .) form factors.
- So far (not shown):
  Behaviour of propagators in conformal window, *i.e.*, for $N_f > N_f^{\rm crit}$.
  M. Hopfer, C. Fischer, RA, JHEP **1411** (2014) 035 [arXiv:1405.7031]
  Meson properties beyond rainbow-ladder approximation.
  M. Vujinovic, R. Williams, arXiv:1411.7619; M. Mitter et al., arXiv:1411.7978;
  R. Williams et al., arXiv:1512.00455; M. Vujinovic, RA, in preparation

Outlook

SU(2) gauge theory with two massless fund. fermions:



J^{PC}	NA, 1PI	NA + AB, 1PI	NA, 3PI	NA + AB, 3PI
0^{-+}	0	0	0	0
0^{++}	1.39(3)	1.22(2)	1.33(3)	1.25(2)
1^{--}	2.27(5)	2.00(4)	2.37(5)	1.99(4)
1^{++}	2.87(5)	2.65(5)	3.09(6)	2.67(5)

QCD with two light flavours:

	RL	2PI-3L	3PI-3L	PDG
0^{+} (π)	0.14	0.14	0.14	0.14
0^{++} (σ)	0.64	0.52	1.1(1)	0.48(8) or 1.3
$1(\rho)$	0.74	0.77	0.74	0.78
$1^{++}(a1)$	0.97	0.96	1.3(1)	1.23(4)
1^{+} (b1)	0.85	1.1	1.3(1)	1.23
f_π	0.092	0.103	0.105	0.092

Outlook

- In rainbow-ladder appr. 2-photon processes as,
  e.g., nucleon Compton scattering.
- Dynamical hadronization incl. dressed vertex functions in the Exact Renormalization Group approach.
- Technicolour theories:
  Bound states for near-conformal gauge theories
  as, *e.g.*, decay width of techni-ρ-meson ($\rightarrow$ signal at LHC?).
- Uncharged (technicolour) bound states as candidates for **Dark Matter**?