Heavy Jet Mass with Massive Quarks

Christopher Lepenik

in collaboration with
Vicent Mateu and André Hoang

University Vienna - Seminar on particle physics

19.1.2016

universität
wien

Outline

Motivation

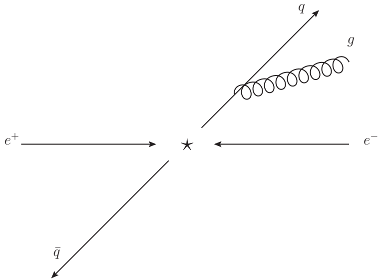
- Jet properties are essential observables for high-precision determination of α_s.
- Observables to quantify the geometric shape of a jet are called "Event Shapes". Examples: Thrust, C-parameter, Heavy Jet Mass (HJM).
- Aim of this thesis: Clarify the role of primary massive quarks in HJM-distributions.

Introduction - Jet Formation

- We considered the process $e^+e^- \to$ hadrons.
- At NLO a gluon is radiated - suppressed by $\alpha_s(Q)$, but:

$$\text{splitting prob.} \sim \frac{\alpha_s}{E_g^2(1-\cos^2\theta)} + \mathcal{O}\left(\hat{m}^2\right).$$

$$(\hat{m} := m/Q)$$

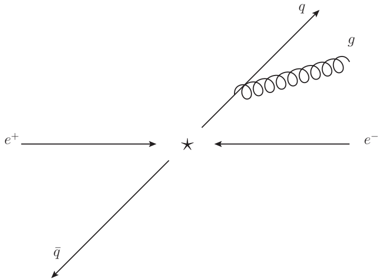


Introduction - Jet Formation

- We considered the process $e^+e^- \to$ hadrons.
- At NLO a gluon is radiated - suppressed by $\alpha_s(Q)$, but:

$$\text{splitting prob.} \sim \frac{\alpha_s}{E_g^2(1-\cos^2\theta)} + \mathcal{O}\left(\hat{m}^2\right).$$

$\to$ Soft enhancement

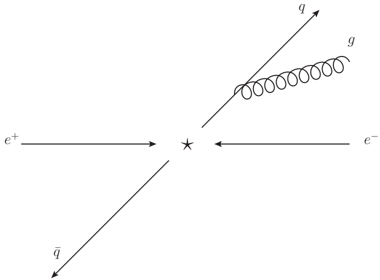


Introduction - Jet Formation

- We considered the process $e^+e^- \to$ hadrons.
- At NLO a gluon is radiated - suppressed by $\alpha_s(Q)$, but:

$$\text{splitting prob.} \sim \frac{\alpha_s}{E_g^2(1-\cos^2\theta)} + \mathcal{O}\left(\hat{m}^2\right).$$

$\to$ Collinear enhancement



Introduction - Jet Formation

- $\Rightarrow$ Essentially there is a Jet of collinear particles and soft radiation.
- Multi-scale problem - relevant scales:
  - Hard scale: Scale of hard interaction μ_H,
  - Jet scale: Scale of the jet μ_J,
  - Soft scale: Scale of soft radiation μ_S,
  - Non-perturbative scale Λ_{QCD}.
- Jet properties strongly affected by $\alpha_s \rightarrow \alpha_s$-determination via jet observables $\rightarrow$ event shapes.

Introduction - Event Shapes and the Heavy Jet Mass

- A very popular event shape is thrust:

$$\tau := 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}.$$

- Dijet limit for $\tau \sim \tau_{\min}$ (2-jet event shape) - spherical event for $\tau_{\max}$.
- We want to split the event into two hemispheres $\mathcal{H}_l$, $\mathcal{H}_r$ - take plane orthogonal to the maximizing vector $\vec{n}$ - "thrust axis".

Introduction - Event Shapes and the Heavy Jet Mass

- Use hemispheres to define more event shapes:
  - Hemisphere masses

$$\rho_i := \frac{\left(\sum_{j\in\mathcal{H}_i} p_j\right)^2}{Q^2}, \quad (i = l, r).$$

  - Heavy Jet Mass

$$\rho := \max\{\rho_l, \rho_r\}.$$

- Three massless particles:

$$\rho_{\min} = 0,$$
$$\rho_{\max} = \frac{1}{3}.$$

- Massive particles: $\rho_{\min,\max}$ depends on $\hat{m}$.

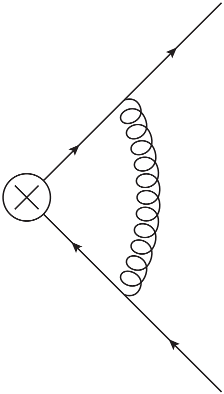
Heavy Jet Mass in QCD

- First step to get HJM-distribution:
  Calculate fixed order differential cross section w.r.t. HJM $\frac{\mathrm{d}\sigma}{\mathrm{d}\rho}$ at $\mathcal{O}(\alpha_s)$.
- Total cross section:

$$\sigma = \sum_X \int \mathrm{d}\Pi_X \, (2\pi)^d \delta^{(d)}(q - P_X) \sum_{i=a,v} L^i_{\mu\nu} \left\langle 0 \middle| \mathcal{J}_i^{\dagger\,\mu} \middle| X \right\rangle \left\langle X \middle| \mathcal{J}_i^{\nu} \middle| 0 \right\rangle$$

with current $\mathcal{J}_i^\mu = \bar{\psi}\Gamma_i^\mu \psi$ ($\Gamma_v^\mu = \gamma^\mu$, $\Gamma_a^\mu = \gamma^\mu\gamma^5$) and $L^i_{\mu\nu}$ the leptonic tensor.

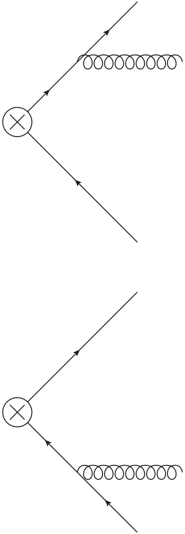
Heavy Jet Mass in QCD - Virtual Part



- From now on: $\bar{\rho} := \rho - \rho_{\min}$.
- Virtual diagram is tedious but straight forward.
- Clear that $\frac{\mathrm{d}\sigma^i_{\text{virt}}}{\mathrm{d}\rho} \sim \delta\left(\bar{\rho}\right) \Rightarrow$ total cross section is sufficient.

$$\frac{1}{\sigma_0^v}\frac{\mathrm{d}\sigma^v_{\text{virt}}}{\mathrm{d}\rho} = \delta(\bar{\rho})\Bigg[\frac{\sigma^v_{\text{Born}}}{\sigma_0^v}$$
$$+\frac{C_F\alpha_s}{4\pi}\left((v^2-3)\left(2v+(1+v^2)\log\frac{1-v}{1+v}\right)\frac{1}{\varepsilon}+\mathcal{O}(\varepsilon^0)\right)$$
$$+\mathcal{O}(\alpha_s^2)\Bigg],$$
$$v = \sqrt{1-4\hat{m}^2}.$$

Heavy Jet Mass in QCD - Radiative Part



- First step: Compute $\frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}x_1\,\mathrm{d}x_2}$ in d dimensions with $x_i = \frac{2p_i^0}{Q}$.
- Next: Project onto HJM.
- Technical problem:
  - Full computation in $d = 4 - 2\varepsilon$ dimensions is extremely hard.
  - Computation in $d = 4$ dimensions much simpler but contains no information about the point $\bar{\rho} = 0$.

Heavy Jet Mass in QCD - Radiative Part

- Solution:
  - Compute in $d = 4$ dimensions $\rightarrow$ get result for $\bar{\rho} > 0$,
  - compute the contribution at $\bar{\rho} = 0$ in $d = 4 - 2\varepsilon$ dimensions in the soft limit,
  - deduce full result.

Heavy Jet Mass in QCD - Radiative Part

- $d = 4$ result $\rightarrow$ threshold expansion $(0 < \bar{\rho} \ll 1)$

$$\frac{1}{\sigma_0^i}\frac{\mathrm{d}\sigma_{\mathrm{rad}}^{i,d=4}}{\mathrm{d}\rho} = \frac{C_F\alpha_s(\mu)}{4\pi}\left[f_+^i\frac{1}{\bar{\rho}} + \mathcal{O}\left(\bar{\rho}^0\right)\right] + \mathcal{O}(\alpha_s^2).$$

  - Result valid in the region $\bar{\rho} > 0$, not integrable.
  - At the end we will replace $1/\bar{\rho}$ with a distribution which looks the same for $\bar{\rho} > 0$ and is integrable - the "plus distribution":

$$\frac{1}{\bar{\rho}} \quad \rightarrow \quad \left[\frac{\Theta(\bar{\rho})}{\bar{\rho}}\right]_+ := \lim_{\varepsilon\to 0}\left[\Theta(\bar{\rho}-\varepsilon)\frac{1}{\bar{\rho}} + \log(\varepsilon)\delta\left(\bar{\rho}-\varepsilon\right)\right].$$

Heavy Jet Mass in QCD - Radiative Part

- Integral:

$$\int_0^\Delta \mathrm{d}x \left[\frac{\Theta(x)}{x}\right]_+ = \log \Delta.$$

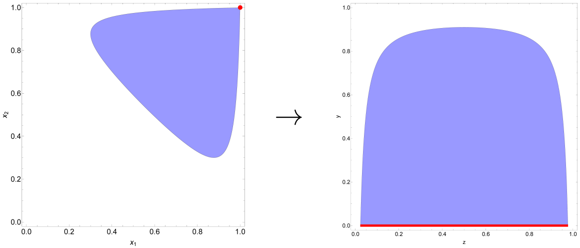
- Background:

$$\frac{\Theta(x)}{x^{1+a\varepsilon}} = -\frac{1}{a\varepsilon}\delta(x) + \left[\frac{\Theta(x)}{x}\right]_+ + \mathcal{O}(\varepsilon).$$

Heavy Jet Mass in QCD - Radiative Part

- $d = 4 - 2\varepsilon$ calculation in the soft limit - what do we look for?
  - Structure that is non-zero at $\bar{\rho} = 0$,
  - must be integrable,
  - divergence in the virtual contribution must be canceled,
  - $\Rightarrow$ δ-function!

Heavy Jet Mass in QCD - Radiative Part



- First step: Change of coordinates to make soft limit simpler - $x_1 = 1 - zy$, $x_2 = 1 - y(1-z)$. Soft limit: $y \ll 1$.
- Use cumulant $\Sigma^i_{\rho,\text{rad}}(\rho_c) := \int\limits_{\rho_\text{min}}^{\rho_c} \mathrm{d}\rho \, \frac{\mathrm{d}\sigma}{\mathrm{d}\rho}$: Desired δ-coefficient is the constant term in the cumulant.

Heavy Jet Mass in QCD - Radiative Part

- Soft limit:

$$\Sigma^i_{\rho,\mathrm{rad}}(\rho_{\min} + \Delta) = \int_{\rho_{\min}}^{\rho_{\min}+\Delta} \mathrm{d}\rho \int \mathrm{dPS}_3^{(d)}(y,z)\, \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} \delta(\rho - \rho(y,z))$$

$$= \int \mathrm{dPS}_3^{(d)}(y,z)\, \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} \Theta(\Delta + \rho_{\min} - \rho(y,z)).$$

  - $\Delta \ll 1$,
  - $\rho_{\min} - \rho(y,z) = -y \left.\frac{\mathrm{d}\rho(y,z)}{\mathrm{d}y}\right|_{y=0} + \mathcal{O}(y^2)$,
  - $\mathrm{dPS}_3^{(d)}(y,z) = \mathrm{dPS}_{3,\mathrm{soft}}^{(d)}(y,z) +$ [higher orders in y],
  - $\frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} = \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad,soft}}}{\mathrm{d}y\,\mathrm{d}z} + \mathcal{O}(y^0)$.

Heavy Jet Mass in QCD - Radiative Part

- Soft limit:

$$\Sigma^i_{\rho,\mathrm{rad}}(\rho_{\min}+\Delta) = \int_{\rho_{\min}}^{\rho_{\min}+\Delta} \mathrm{d}\rho \int \mathrm{dPS}_3^{(d)}(y,z)\, \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} \delta(\rho-\rho(y,z))$$

$$= \int \mathrm{dPS}_3^{(d)}(y,z)\, \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} \Theta(\Delta+\rho_{\min}-\rho(y,z)).$$

  - $\Delta \ll 1$,
  - $\rho_{\min} - \rho(y,z) = -y \left.\frac{\mathrm{d}\rho(y,z)}{\mathrm{d}y}\right|_{y=0} + \mathcal{O}(y^2)$,
  - $\mathrm{dPS}_3^{(d)}(y,z) = \mathrm{dPS}_{3,\mathrm{soft}}^{(d)}(y,z) +$ [higher orders in y],
  - $\frac{\mathrm{d}^2\sigma^i_{\mathrm{rad}}}{\mathrm{d}y\,\mathrm{d}z} = \frac{\mathrm{d}^2\sigma^i_{\mathrm{rad,soft}}}{\mathrm{d}y\,\mathrm{d}z} + \mathcal{O}(y^0)$.

Heavy Jet Mass in QCD - Radiative Part

- After taking the limit and doing the integral one gets the desired coefficient via

$$\frac{1}{\sigma_0^i}\Sigma_{\rho,\text{rad}}^{i,\text{soft}}(\rho_{\min}+\Delta)=\frac{C_F\alpha_s}{4\pi}\left[f_{\delta,\text{rad}}^i+\mathcal{O}\left(\log\Delta\right)\right]+\mathcal{O}(\alpha_s^2).$$

- Obtain full analytic δ-coefficient: $f_\delta^i=f_{\delta,\text{rad}}^i+f_{\delta,\text{virt}}^i$.
- Divergences cancel!

$$f_{\delta,\text{rad}}^v=-(v^2-3)\left(2v+(1+v^2)\log\frac{1-v}{1+v}\right)\frac{1}{\varepsilon}+\mathcal{O}(\varepsilon^0).$$

Heavy Jet Mass in QCD - Putting Together

- Complete analytic form of QCD cross section:

$$\frac{1}{\sigma_0^i}\frac{\mathrm{d}\sigma^i}{\mathrm{d}\rho} = f_0^i\delta(\bar{\rho}) + \frac{C_F\alpha_s}{4\pi}\left[f_\delta^i\delta(\bar{\rho}) + f_+^i\left[\frac{\Theta(\bar{\rho})}{\bar{\rho}}\right]_+ + [\text{non-distr.}]\right] + \mathcal{O}(\alpha_s^2).$$

- Cross check:

$$\sigma_{\mathrm{tot}}^i = \int \mathrm{d}\rho\frac{\mathrm{d}\sigma^i}{\mathrm{d}\rho} \quad \checkmark$$

Heavy Jet Mass in QCD - Radiative Part

$$\frac{C_F \alpha_s}{4\pi}\left[f_\delta^i \delta(\bar\rho) + f_+^i \left[\frac{\Theta(\bar\rho)}{\bar\rho}\right]_+ + [\text{non-distr.}]\right]$$

- How do the coefficients look like? Like this! $(v = \sqrt{1-4\hat m^2})$

$$\begin{aligned} f_\delta^v =& -(3+10v^2-3v^4)\log\frac{1-v}{1+v} + (v^2-3)\Bigg[2v(1-v) - 2v\log\frac{(1-v)^3(1+v)}{16} \\ &+ (1+v^2)\left[-\frac{\pi^2}{6} - 2\log\frac{1-v}{1+v}\log\frac{2v^2}{1+v} + 2\mathrm{Li}_2\frac{v+1}{v-1} - 4\mathrm{Li}_2\frac{1-v}{1+v}\right]\Bigg], \\ f_+^v =& 2(3-v^2)\left(-2v + (1+v^2)\log\frac{1+v}{1-v}\right). \end{aligned}$$

Heavy Jet Mass in QCD - Large Logarithms

- Problem: Large logarithms at small $\bar{\rho}$ spoil perturbative expansion!
- Cumulant in the limit $\hat{m} \sim \bar{\rho} \ll 1$:

$$\frac{1}{\sigma_0^i}\Sigma_\rho^i(\rho) = 1 + \frac{C_F\alpha_s}{4\pi}\left[-6\log\bar{\rho} - 4\log^2\bar{\rho} + \mathcal{O}(\bar{\rho}^0, \hat{m}^0)\right] + \mathcal{O}(\alpha_s^2).$$

- Origin: Multi-scale problem - logarithms of ratios of appearing scales - large scale hierarchies near the dijet region:
  - Hard $\sim Q$,
  - Jet $\sim Q\sqrt{\bar{\rho}}$,
  - Soft $\sim Q\bar{\rho}$.
- Need for resummation.

Heavy Jet Mass in QCD - Log Counting

- Reorganize expansion by considering $\alpha_s \log \bar{\rho} \sim \mathcal{O}(1)$.
- Counting more clear in exponent of cumulant:

$$\log \Sigma \sim \log \bar{\rho} \sum_{i=0} (\alpha_s \log \bar{\rho})^{i+1} + \sum_{i=0} (\alpha_s \log \bar{\rho})^{i+1} + \alpha_s \sum_{i=0} (\alpha_s \log \bar{\rho})^{i} + \dots .$$

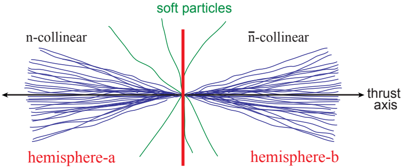
- Terms are called LL, NLL, NNLL, ...
- Resummation via factorization and renormalization group equations (RGE) $\rightarrow$ SCET.

Soft Collinear Effective Theory (SCET) - Introduction

- SCET is an effective field theory of QCD constructed for situations with collinear and soft degrees of freedom which corresponds to our dijet-configuration.
- The SCET cross section can be factorized into factors corresponding to only one of the characteristic scales.
- First step: Light cone coordinates → define $n^\mu = (1,0,0,-1)$, $\bar{n}^\mu = (1,0,0,1)$.
- Express vectors in the form

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu \quad \leftrightarrow \quad (p^+, p^-, p_\perp) = (\bar{n} \cdot p, n \cdot p, |\vec{p}_\perp|).$$

Soft Collinear Effective Theory (SCET) - Introduction



[Fleming et al., 2008]

- Start with QCD Lagrangian - split QCD fields into the new d.o.f: $\psi = \psi_n + \psi_{\bar{n}} + \psi_s$.
- Expand Lagrangian in small power counting parameter $1 \gg \lambda \sim \bar{\rho}$ (dijet limit).

Soft Collinear Effective Theory (SCET) - Introduction

- Resulting Lagrangian:

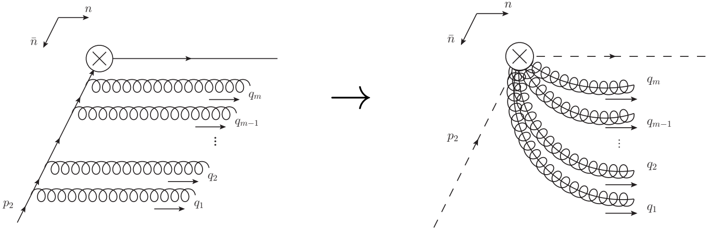
$$\mathcal{L}_{SCET} = \bar{\psi}_s(i\not{D}_s - m)\psi_s - \frac{1}{4}\left(F_s^{\mu\nu,A}\right)^2 - \frac{1}{4}\left(F_n^{\mu\nu,A}\right)^2 + \bar{\xi}_n \frac{\not{\bar{n}}}{2} iD_+ \xi_n + \bar{\xi}_n (i\not{D}_\perp^n - m)\frac{1}{iD_-^n}(i\not{D}_\perp^n + m)\frac{\not{\bar{n}}}{2}\xi_n + [\bar{n}\text{-terms}].$$

- $D_+ = \partial_+ - ig_s A_{n,+} - ig_s A_{s,+}$ contains soft gluon field.
- Relevant modes:

Mode	$p^\mu = (+,-,\perp)$	Fields
Hard	$Q(1,1,1)$	-
n-collinear	$Q(\lambda^2,1,\lambda)$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
Soft	$Q(\lambda^2,\lambda^2,\lambda^2)$	A_s^μ

Soft Collinear Effective Theory (SCET) - Introduction

- Last step: Integrate out far off-shell modes (e.g. $p_n + p_{\bar{n}}$) $\rightarrow$ get collinear Wilson lines $W_{n,\bar{n}}$:



- A collinear Wilson line contains an arbitrary number of collinear gluons.
- Strucure on the right hand side: $\bar{\xi}_n W_n \Gamma_i^{\mu} \xi_{\bar{n}}$.
- Is needed to preserve collinear gauge invariance.

Soft Collinear Effective Theory (SCET) - Factorization

- SCET cross section: Use SCET fields and restrict final states to dijet situation:

$$\sigma = \sum_X^{\text{res}} \int \mathrm{d}\Pi_X \, (2\pi)^d \delta^{(d)}(q - P_X) \sum_{i=a,v} L^i_{\mu\nu} \left\langle 0 \middle| J_i^{\dagger\,\mu} \middle| X \right\rangle \left\langle X \middle| J_i^{\nu} \middle| 0 \right\rangle$$
$$+ \,[\text{non-singular}].$$

- Trick to decouple collinear and soft fields: Apply field redefinition $\xi_n \to Y_n \xi_n^{(0)}$, $A_n \to Y_n A_n^{(0)} Y_n^\dagger$ with soft Wilson lines Y_n.

Soft Collinear Effective Theory (SCET) - Factorization

- Resulting current:

$$\mathcal{J}_i^\mu = \bar{\psi}\Gamma_i^\mu\psi \quad \to \quad J_i^\mu = \int \mathrm{d}\omega\,\mathrm{d}\bar{\omega}\, C(\omega,\bar{\omega})\bar{\xi}^{(0)}_{n,\omega} W_n Y_n^\dagger \Gamma_i^\mu Y_{\bar{n}} W_{\bar{n}}^\dagger \xi^{(0)}_{\bar{n},\bar{\omega}}.$$

- Containing
  - Collinear Wilson lines W_n from integrating out the off-shell modes and to preserve collinear gauge invariance,
  - Soft Wilson lines Y_n from collinear-soft decoupling,
  - the matching coefficient C.

Soft Collinear Effective Theory (SCET) - Factorization

- With this setup one can proof factorization for hemisphere masses and heavy jet mass ($M_i^2 = m^2 + s_i^2$):

$$\frac{1}{\sigma_0}\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2} = H(Q,\mu)\int \mathrm{d}l^+\,\mathrm{d}l^-\; J_n(s_a - Ql^+, m, \mu) J_{\bar{n}}(s_b - Ql^-, m, \mu) \times S_{\mathrm{hemi}}(l^+, l^-, \mu, m) + [\text{non-singular}],$$

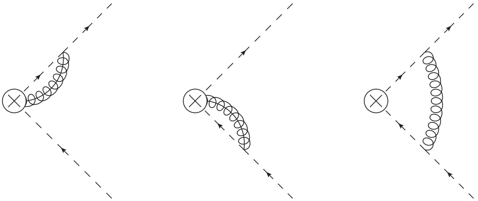
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} = 2Q^2 \int_0^{Q^2\bar{\rho}} \mathrm{d}s_a\; \frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2}\bigg|_{s_b = Q^2\bar{\rho}}.$$

- Achieved: Each factor of the formula describes the dynamics at a different scale.

Soft Collinear Effective Theory (SCET) - Factorization

$$H(Q,\mu)\int \mathrm{d}l^+\,\mathrm{d}l^-\,J_n(s_a-Ql^+,m,\mu)J_{\bar{n}}(s_b-Ql^-,m,\mu)S_{\mathrm{hemi}}(l^+,l^-,\mu)$$

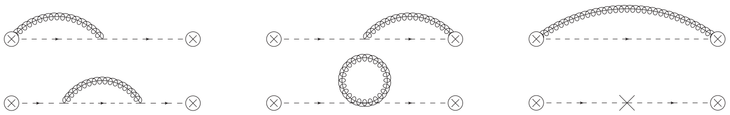
- The hard function describes the dynamics of the hard scale $\rightarrow$ hard process given by SCET matching coefficient.
- Universal for all e^+e^- event-shapes.
- No mass-effects of primary particles.
- Appearing logs: $\log\frac{Q^2}{\mu^2}$.



Soft Collinear Effective Theory (SCET) - Factorization

$$H(Q,\mu)\int \mathrm{d}l^+\,\mathrm{d}l^-\, J_n(s_a - Ql^+, m, \mu) J_{\bar{n}}(s_b - Ql^-, m, \mu) S_{\text{hemi}}(l^+, l^-, \mu)$$

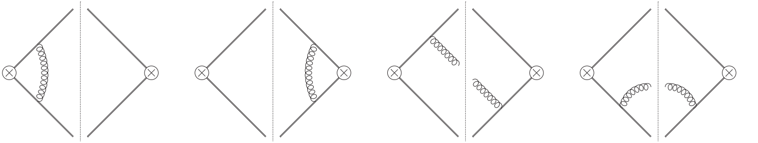
- The jet function describes the dynamics at the jet scale $\rightarrow$ dynamics of collinear quarks and gluons within the jet(s).
- Mass corrections from primary particles.
- $\mathcal{O}(\alpha_s)$ diagrams: coll. quarks and gluons, coll. Wilson lines.
- Appearing logs (after ρ projection): $\log\frac{Q^2\bar{\rho}}{\mu^2}$, $\log\frac{Q^2(\bar{\rho}+\hat{m}^2)}{\mu^2}$ $\rightarrow$ roughly the same for small $\hat{m}$.



Soft Collinear Effective Theory (SCET) - Factorization

$$H(Q,\mu)\int \mathrm{d}l^+\,\mathrm{d}l^-\, J_n(s_a-Ql^+,m,\mu)J_{\bar{n}}(s_b-Ql^-,m,\mu)S_{\text{hemi}}(l^+,l^-,\mu)$$

- The soft function describes the dynamics at the soft scale $\rightarrow$ dynamics of soft radiation and cross talk between jets.
- $\Rightarrow$ Gets non-perturbative corrections.
- No mass-corrections of primary particles.
- $\mathcal{O}(\alpha_s)$ diagrams: Soft gluons, soft Wilson lines.
- Appearing log: $\log\frac{Q\bar{\rho}}{\mu}$.



...

Soft Collinear Effective Theory (SCET) - Factorization

- Non-perturbative corrections:
  - In general given by a non-perturbative shape function convoluted with partonic distribution.
  - Attention: In general two-dimensional. Implementation must be done at the hemisphere mass level! $\rightarrow$ ρ-projection afterwards.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\rho} \sim \int \mathrm{d}\rho' \, \frac{\mathrm{d}\sigma_{\text{part}}}{\mathrm{d}\rho}(\rho') S_{\text{mod}}(\rho - \rho') \quad \text{Not possible!}$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a\,\mathrm{d}M_b}(s_a, s_b) = \int \mathrm{d}s_a'\,\mathrm{d}s_b' \, \frac{\mathrm{d}^2\sigma_{\text{part}}}{\mathrm{d}M_a\,\mathrm{d}M_b}(s_a', s_b') S_{\text{mod}}(s_a - s_a', s_b - s_b')$$

  - Leads to a shift of the cross section in the tail region.

Soft Collinear Effective Theory (SCET) - Factorization

- Non-singular part:
  - SCET reproduces singular part of cross-section.
  - Non-singular contributions become important in the far tail.
  - Include non-singular part in final result:

$$\frac{\mathrm{d}\sigma_{\mathrm{ns}}^{i}}{\mathrm{d}\rho} = \frac{\mathrm{d}\sigma_{\mathrm{QCD}}^{i}}{\mathrm{d}\rho} - \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}\rho}$$

Soft Collinear Effective Theory (SCET) - Large Log Resummation

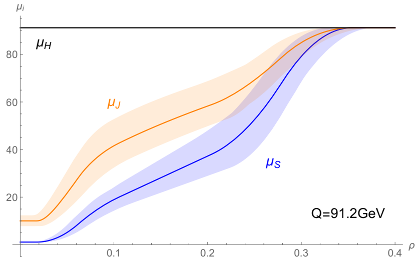
- Let's solve the large log problem!
- Choose scales to minimize log in every factor and evolve to common scale by using the appropriate RGE.
- Non-trivial anomalous dimensions:
  - $\gamma_H(Q,\mu) = \Gamma_H^{\text{cusp}}[\alpha_s] \log \frac{Q^2}{\mu^2} + \gamma_H[\alpha_s]$,
  - $\tilde{\gamma}_F(y,\mu) = \Gamma_F^{\text{cusp}}[\alpha_s] \log(iy\mu) + \gamma_F[\alpha_s]$ with $F = J, S$.
  - Cusp-terms responsible for resumming double logs, non-cusp-terms for resumming single logs.

Soft Collinear Effective Theory (SCET) - Large Log Resummation

- To use a different scale in every cross section factor use evolution kernels obtained from solving the RGE. Write
  - $H(Q,\mu) = H(Q,\mu_H)U_H(Q,\mu_H,\mu)$,
  - $\tilde{F}(y,\mu) = \tilde{U}_F(y,\mu,\mu_F)\tilde{F}(y,\mu_F)$.

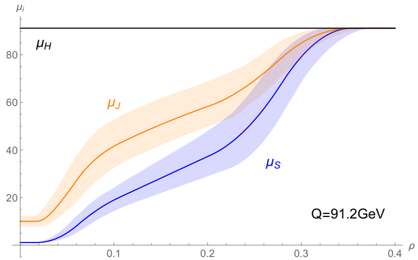
Soft Collinear Effective Theory (SCET) - Large Log Resummation

- Use profile functions to describe the characteristic scales along the whole distribution:
  - Peak (non-pert.): $\mu_H \sim Q$, $\mu_J \sim \sqrt{\Lambda_{\rm QCD} Q}$, $\mu_S \gtrsim \Lambda_{\rm QCD}$,
  - tail (resum.): $\mu_H \sim Q$, $\mu_J \sim Q\sqrt{\bar{\rho}}$, $\mu_S \sim Q\bar{\rho}$,
  - far-tail (fixed-order): $\mu_H \sim \mu_J \sim \mu_S \sim Q$.
- Profiles for HJM with massive particles are still a work in progress.



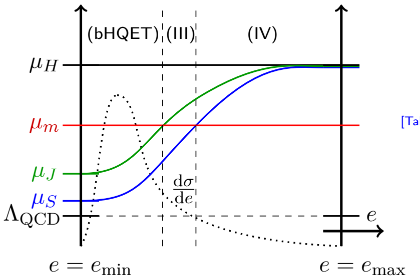
Soft Collinear Effective Theory (SCET) - Large Log Resummation

- Theoretical errors can be estimated through profile variations.
- Convergence can be tested via $\mathrm{N^kLL}$ vs. $\mathrm{N^{k+1}LL}$ comparisons.



Soft Collinear Effective Theory (SCET) - Large Log Resummation

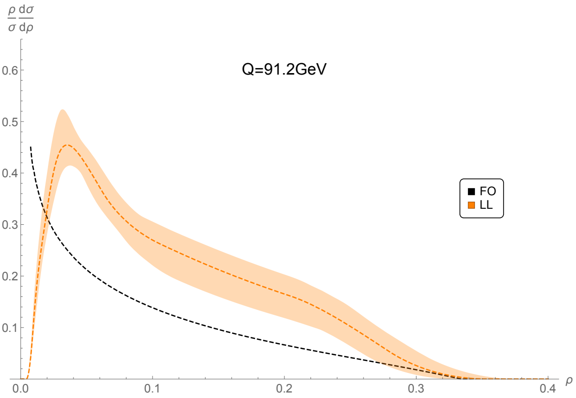
- In general a more sophisticated setup is needed → scenarios (current description only valid in scenario IV).
- In scenario III the evolution of the cross section factors is modified.
- In the bHQET region "boosted heavy quark effective theory" must be applied to resum additional large logs.



[Talk by M. Preisser (2015)]

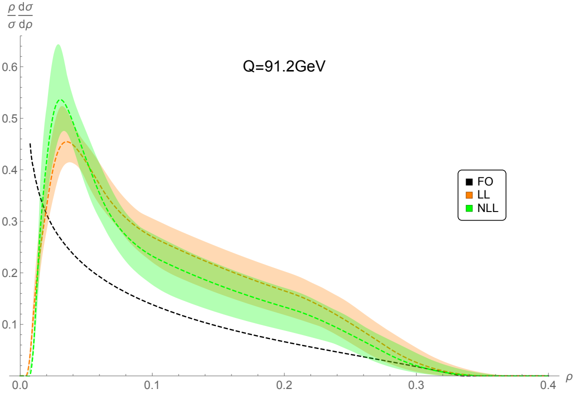
Results - Convergence

- Convergence of single flavour cross section ($b\bar{b}$) at $\bar{m}_b(\bar{m}_b) = 4.2\text{GeV}$:



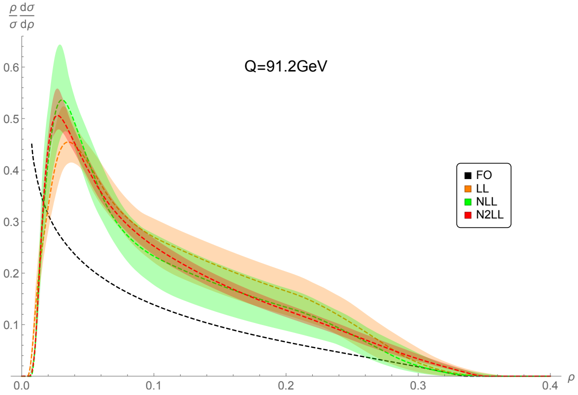
Results - Convergence

- Convergence of single flavour cross section ($b\bar{b}$) at $\bar{m}_b(\bar{m}_b) = 4.2\text{GeV}$:



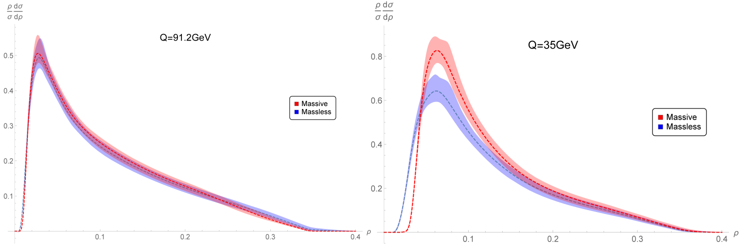
Results - Convergence

- Convergence of single flavour cross section ($b\bar{b}$) at $\bar{m}_b(\bar{m}_b) = 4.2\mathrm{GeV}$:



Results - Mass effects

- Mass effect in single flavour production:



- Larger effect for larger $\hat{m}$.

Summary and Outlook

- Achievements:
  - Obtained analytic expressions for the heavy jet mass cross-section in SCET and QCD with primary massive quarks.
  - Starting point for a more general consideration of mass effects in the HJM-distribution.
- Next steps:
  - Generalize results to describe the remaining scenarios.
  - Effect of $b\bar{b}$ expected to be small in all flavor production, but should be implemented as correction for α_s extraction from data (LEP, ect.).
  - Generalize setup for top quark production (implement width and instability effects).