Variable Flavor Number Scheme for Final State Jets in Deep Inelastic Scattering

Daniel Samitz

in collaboration with Andre Hoang and Piotr Pietrulewicz

universität wien

$\int dk \prod$ Doktoratskolleg Particles and Interactions

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Outline

1. Motivation
2. Soft-Collinear Effective Theory
3. DIS in the OPE Region $1 - x \sim 1$
4. VFNS in the OPE Region
5. DIS in the Endpoint Region $1 - x \ll 1$
6. VFNS in the Endpoint Region
7. Outlook and Conclusions

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Motivation

- Deep inelastic scattering (DIS) is a benchmark process for the extraction of parton distribution functions (PDFs)
- high precision in theoretical predictions is important for reliable fits
- a framework for the inclusion of mass effects from heavy quarks with arbitrary scaling of the mass with respect to other scales is needed
- in the endpoint region (one single jet in the final state) additional large logarithms can weaken convergence of the perturbation series
- conceptual issues about factorization in the endpoint region

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Soft-Collinear Effective Theory

Bauer, Fleming, Pirjol, Stewart (2001)

- define two light-like vectors:

$$n^\mu = (1,0,0,-1) \qquad \bar{n}^\mu = (1,0,0,1)$$
$$n^2 = \bar{n}^2 = 0 \qquad n \cdot \bar{n} = 2$$

- every four-vector can be decomposed into light-cone components

$$p^\mu = \underbrace{\bar{n}\cdot p}_{p^-}\,\frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+}\,\frac{\bar{n}^\mu}{2} + p_\perp^\mu$$
$$p^2 = p^+ p^- + p_\perp^2$$

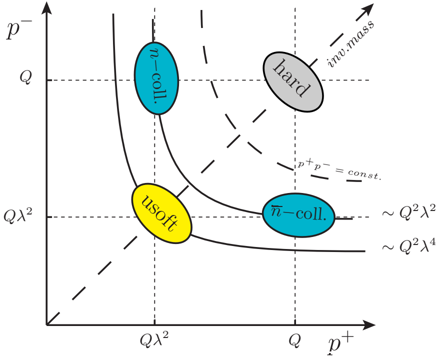
- classify momenta with respect to the scaling of their light-cone components ($\lambda \ll 1$)

	$(p^+, p^-, p^\perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll.	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
$\bar{n}$-coll.	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Soft-Collinear Effective Theory

Bauer, Fleming, Pirjol, Stewart (2001)

integrate out hard (far off-shell) modes to get effective field theory with only collinear quarks and gluons (ξ_n, A_n^μ) and usoft gluons (A_{us}^μ) as fluctuating fields→ SCET



effective theory to describe highly boosted, collimated objects and low-energetic radiation between them → Jets

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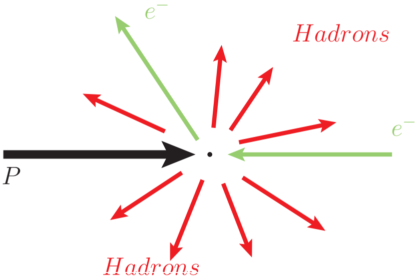
Deep Inelastic Scattering

$$P^\mu = \frac{Q}{x}\frac{\bar{n}^\mu}{2} \qquad \text{(Breit frame)}$$

$$q^\mu = Q\frac{n^\mu}{2} - Q\frac{\bar{n}^\mu}{2}$$

$$P_X^\mu = Q\frac{n^\mu}{2} + \frac{Q(1-x)}{x}\frac{\bar{n}^\mu}{2}$$



$$x = \frac{-q^2}{2\,P\cdot q} = \frac{1}{1+\frac{P_X^2}{Q^2}}$$

$$y = \frac{Q^2}{s\,x}$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left((1-y)^2+1\right)F_1(x,Q^2) + \frac{1-y}{x}F_L(x,Q^2)\right]$$

$$F^{\mu\nu} = \frac{1}{4\pi}\int \mathrm{d}^4z\,\mathrm{e}^{iqz}\langle P|J^\mu(z)J^\nu(0)|P\rangle$$

Factorization

$$P^2 \sim \Lambda^2_{\mathrm{QCD}}$$
$$-q^2 = Q^2$$
$$P_X^2 = \frac{Q^2(1-x)}{x} \sim Q^2$$

at the scale Q^2 match QCD onto effective theory containing only non-pert. $\bar{n}$-collinear modes

$$F_1(x, Q^2) = \sum_i \int_x^1 \frac{\mathrm{d}\xi}{\xi} H_i\left(\frac{x}{\xi}, Q, \mu\right) f_{i/P}(\xi, \mu)$$

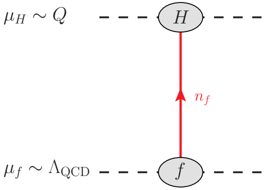
$$\chi_{\bar{n}}(x) = W^\dagger_{\bar{n}} \xi_{\bar{n}}(x)$$
$$W_{\bar{n}}(x) = \mathrm{P}\exp\left[\mathrm{ig}\int_{-\infty}^{\mathrm{x}} \mathrm{ds}\, \mathrm{n}\cdot \mathrm{A}_{\bar{\mathrm{n}}}(\mathrm{sn})\right]$$

$$f_{q/P}(\xi) = \langle P|\bar{\chi}^{(q)}_{\bar{n}}(0)\frac{\not{n}}{2}\left[\delta\left(P^+\xi - \mathcal{P}^+\right)\chi^{(q)}_{\bar{n}}(0)\right]|P\rangle$$

$$f_{g/P}(\xi) = -\frac{P^+\xi}{T_f g^2}\langle P|\mathrm{Tr}\left[W^\dagger_{\bar{n}}[(\mathcal{P}^\mu_\perp + gA^\mu_{\bar{n}\perp})W_{\bar{n}}][\delta(P^+\xi - \mathcal{P}^+)W^\dagger_{\bar{n}}[(\mathcal{P}^\perp_\mu + gA^\perp_{\bar{n}\mu})W_{\bar{n}}]]\right]|P\rangle$$

DGLAP Evolution

matching and renormalization does not depend on IR physics
→ can be calculated perturbatively with partonic initial sates

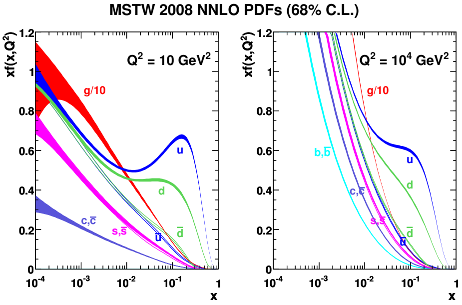


$$f_i^{\mathrm{bare}} = \sum_j Z_{ij}^{\overline{\mathrm{MS}}} \otimes f_j$$

DGLAP evolution for PDFs

Dokshitzer (1977), Gribov & Lipatov (1972), Altarelli & Parisi (1977)

$$f_i(x,\mu) = \sum_j \int_x^1 \frac{\mathrm{d}\xi}{\xi} U_{f,ij}\left(\frac{x}{\xi},\mu,\mu_0\right) f_j(\xi,\mu_0)$$



A.D.Martin, W.J.Stirling, R.S.Thorne, G.Watt, *Eur.Phys.J.* **C63**, 189 (2009)

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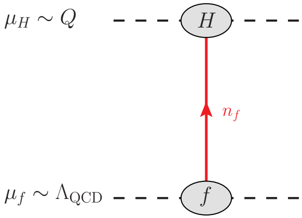
Mass Effects: Fixed Flavor Number Scheme (FFNS)

add one massive flavor in the calculation

$$H_i(\mu_H) \otimes U^{n_f}_{f,ij}(\mu_H, \mu_f) \otimes f_{j/P}(\mu_f)$$

$$H_i = H^{n_f}_{i,m=0} + H_{i,m}$$



$$H_{i,m} \xrightarrow[m^2 \gg Q^2]{} 0$$

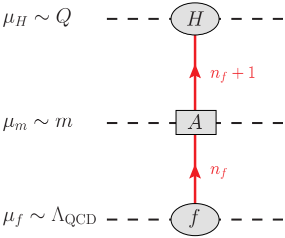
$$H_{i,m} \xrightarrow[m^2 \ll Q^2]{} \sim \log \frac{m^2}{Q^2}$$

$$H_{g,m}(x, m, Q) \to -\frac{\alpha_s(\mu_m)}{2\pi} \left[P^{(0)}_{qg}(x) \left(\ln \frac{m^2}{Q^2} - \ln \left(\frac{1-x}{x} \right) \right) + (1-2x)^2 \right] + \mathcal{O}(\alpha_s^2)$$

decoupling of very heavy particles, BUT..
... no resummation of logs including the mass as a new scale

Mass Effects: Variable Flavor Number Scheme (VFNS)

have to include effects of the massive flavor also in the running of the PDF



$$\tilde{H}_i(\mu_H) \otimes U^{n_f+1}_{f,ij}(\mu_H, \mu_m) \otimes A_{jk}(\mu_m) \otimes U^{n_f}_{f,kl}(\mu_m, \mu_f) \otimes f_{l/P}(\mu_f)$$

$$\tilde{H} = H^{n_f}_{m=0} + \tilde{H}_m$$

$$A_{Qg}(x, m, \mu_m) = -\frac{\alpha_s(\mu_m)}{2\pi} P^{(0)}_{qg}(x) \ln \frac{m^2}{\mu_m^2} + \mathcal{O}(\alpha_s^2)$$

$$\tilde{H}_m \xrightarrow[m^2 \ll Q^2]{} H_{m=0}$$

the additional flavor in $U^{n_f+1}_f(\mu_H, \mu_m)$ resumes the logs of $\log \frac{\mu_m^2}{\mu_H^2} \sim \log \frac{m^2}{Q^2}$

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Kinematics

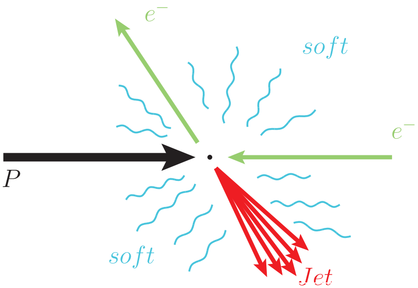
- $P_X^\mu \sim Q\frac{n^\mu}{2} + Q(1-x)\frac{\bar{n}^\mu}{2}$

  n-collinear scaling for $(1-x) \ll 1$
  $\Rightarrow$ only n-coll. and soft modes allowed in the final state

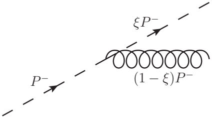
- invariant mass of the hadronic final state much smaller than the hard scale

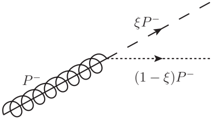
$$P_X^2 \sim Q^2(1-x) \ll Q^2$$

- $\Rightarrow$ additional resummation of $\log\frac{P_X^2}{Q^2} \sim \log(1-x)$ needed



PDF Evolution





no flavor mixing in the PDF evolution (all off-diagonal elements in the DGLAP mixing subleading)

$$U_{f,ij}(z) = U_{f,ii}(z)\delta_{ij} + \mathcal{O}\Big((1-z)^0\Big)$$

$$f_i(z,\mu) = \sum_j \int_z^1 \frac{\mathrm{d}\xi}{\xi} U_{f,ij}\left(\frac{z}{\xi},\mu,\mu_f\right) f_j(\xi,\mu_f)$$

$$\Rightarrow \phi_q(1-z,\mu) = \int_0^{1-z} \mathrm{d}z'\, U_\phi(1-z-z',\mu,\mu_\phi)\phi_q(z',\mu_\phi)$$

$$\phi_q(1-z) = f_q(z) + \mathcal{O}(1-z) \qquad U_\phi(1-z) = U_{f,qq}^{z\to 1}(z)$$

Factorization

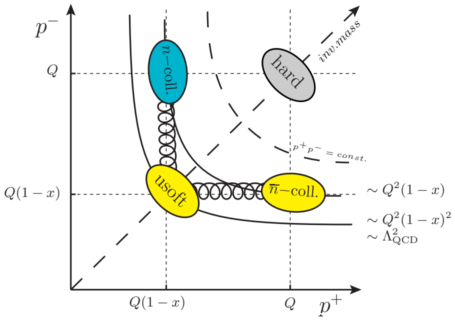
- three different scales in the process
  - hard interaction: Q
  - incoming proton, $\bar{n}$-coll.: Λ_{QCD}
  - outgoing jet, n-coll.: $Q\sqrt{1-x}$

- need to integrate out different modes at their respective scales

- go through a multi-step matching procedure between different effective theories

Factorization

- $J^\mu \to C(Q,\mu)\bar{\chi}_{\bar{n}}\gamma^\mu\chi_n$



$H(Q,\mu) = |C(Q,\mu)|^2$

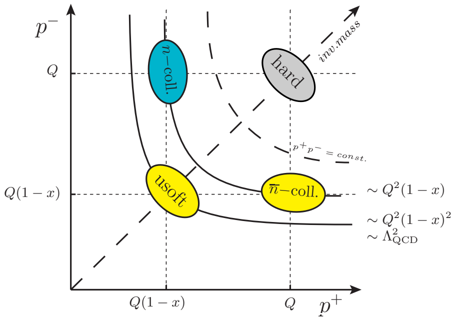
SCET I

Factorization

- $J^\mu \rightarrow C(Q,\mu)\bar{\chi}_{\bar{n}}\gamma^\mu\chi_n$

- $\chi_n \rightarrow Y_n\chi_n^{(0)}$
  $\chi_{\bar{n}} \rightarrow Y_{\bar{n}}\chi_{\bar{n}}^{(0)}$

$H(Q,\mu) = |C(Q,\mu)|^2$



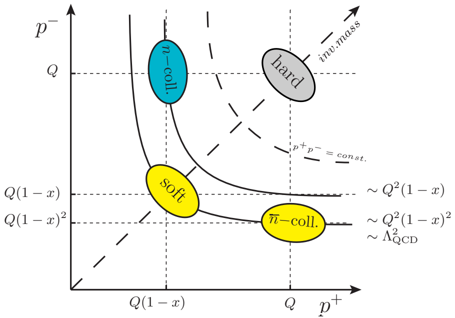
SCET I

Factorization

- $J^\mu \to C(Q,\mu)\bar{\chi}_{\bar{n}}\gamma^\mu\chi_n$

- $\chi_n \to Y_n\chi_n^{(0)}$
  $\chi_{\bar{n}} \to Y_{\bar{n}}\chi_{\bar{n}}^{(0)}$

- $Y_n \to S_n$
  $Y_{\bar{n}} \to S_{\bar{n}}$



SCET II

$$H(Q,\mu) = |C(Q,\mu)|^2$$

$$J(Qr_n^+,\mu) = \frac{-1}{2\pi N_c Q}$$
$$\times\,\mathrm{Im}\left[i\int d^4z\, e^{ir_n\cdot z}\langle 0|\mathrm{T}\left\{\bar{\chi}^{(0)}_{n,Q}(0)\frac{\bar{\not{n}}}{2}\chi_n^{(0)}(z)\right\}|0\rangle\right]$$

$$\phi_{q/P}(1-z,\mu) = \langle P|\bar{\chi}^{(0)}_{\bar{n}}S^\dagger_{\bar{n}}S_n\frac{\not{n}}{2}\delta(Q(1-z)-n\cdot\mathcal{P})S_n^\dagger S_{\bar{n}}\chi^{(0)}_{\bar{n},Q}|P\rangle$$

Factorization

- $J^\mu \to C(Q,\mu)\bar{\chi}_{\bar{n}}\gamma^\mu\chi_n$
- $\chi_n \to Y_n\chi_n^{(0)}$
  $\chi_{\bar{n}} \to Y_{\bar{n}}\chi_{\bar{n}}^{(0)}$
- $Y_n \to S_n$
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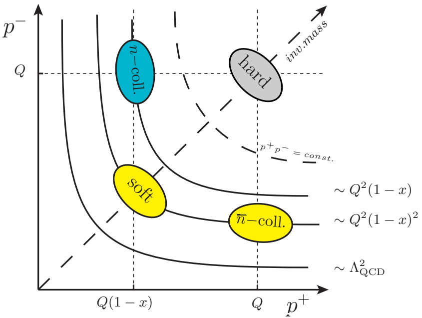


$$H(Q,\mu) = |C(Q,\mu)|^2$$

$$J(Qr_n^+,\mu) = \frac{-1}{2\pi N_c Q} \times \mathrm{Im}\left[i\int d^4z\, e^{ir_n\cdot z}\langle 0|\mathrm{T}\left\{\bar{\chi}_{n,Q}^{(0)}(0)\frac{\bar{n}\!\!\!/}{2}\chi_n^{(0)}(z)\right\}|0\rangle\right]$$

SCET II

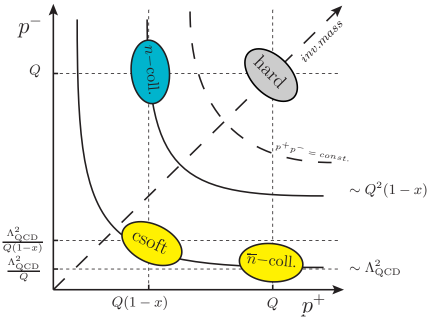
$$\phi_{q/P}(1-z,\mu) = \langle P|\bar{\chi}_{\bar{n}}^{(0)}S_{\bar{n}}^\dagger S_n\frac{n\!\!\!/}{2}\delta(Q(1-z)-n\cdot\mathcal{P})S_n^\dagger S_{\bar{n}}\chi_{\bar{n},Q}^{(0)}|P\rangle$$

Factorization

- $J^\mu \to C(Q,\mu)\bar{\chi}_{\bar{n}}\gamma^\mu\chi_n$
- $\chi_n \to Y_n\chi_n^{(0)}$
  $\chi_{\bar{n}} \to Y_{\bar{n}}\chi_{\bar{n}}^{(0)}$
- $Y_n \to S_n$
  $Y_{\bar{n}} \to S_{\bar{n}}$
- $S_n \to V_{\bar{n}}$
  $S_{\bar{n}} \to X_{\bar{n}}$

$$H(Q,\mu) = |C(Q,\mu)|^2$$

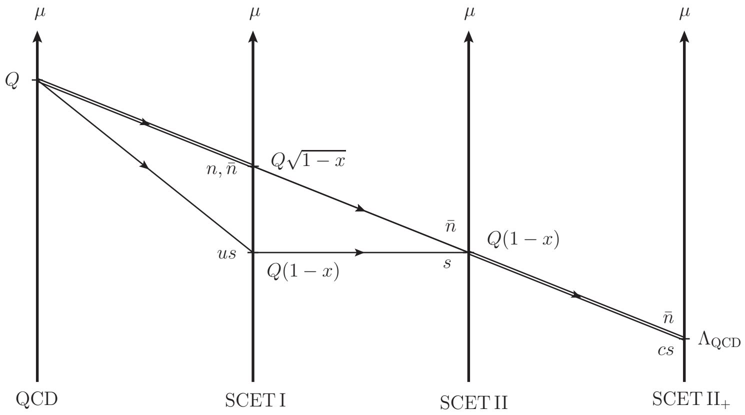
$$J(Qr_n^+,\mu) = \frac{-1}{2\pi N_c Q} \times \mathrm{Im}\left[i\int d^4z\, e^{ir_n\cdot z}\langle 0|\mathrm{T}\left\{\bar{\chi}_{n,Q}^{(0)}(0)\frac{\bar{n}\!\!\!/}{2}\chi_n^{(0)}(z)\right\}|0\rangle\right]$$



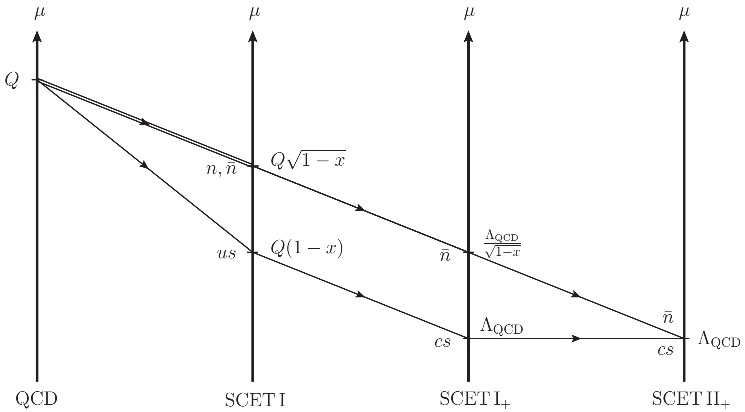
SCET II_+

$$\phi_{q/P}(1-z,\mu) = \langle P|\bar{\chi}_{\bar{n}}^{(0)}X_{\bar{n}}^\dagger V_{\bar{n}}\frac{n\!\!\!/}{2}\delta(Q(1-z)-n\cdot\mathcal{P})V_{\bar{n}}^\dagger X_{\bar{n}}\chi_{\bar{n},Q}^{(0)}|P\rangle$$

Multi-step Matching



Multi-step Matching



Factorization Theorem

$$F_1(x, Q^2) = \sum_{i=q,\bar{q}} \frac{e_i^2}{2} H^{(n_f)}(Q, \mu) \times \int \mathrm{d}s\, J^{(n_f)}(s, \mu)\, \phi_{i/P}^{(n_f)}\left(1 - x - \frac{s}{Q^2}, \mu\right)$$

$$F_L(x, Q^2) = 0$$

$$H^{(n_f)}(Q, \mu) = U_H^{(n_f)}(Q, \mu_H, \mu) \times H^{(n_f)}(Q, \mu_H) \qquad \mu_H \sim Q$$

$$J^{(n_f)}(s, \mu) = \int \mathrm{d}s'\ U_J^{(n_f)}(s - s', \mu, \mu_J)\, J^{(n_f)}(s', \mu_J) \qquad \mu_J \sim Q\sqrt{1-x}$$

$$\phi_{i/P}^{(n_f)}(1 - z, \mu) = \int \mathrm{d}z'\ U_\phi^{(n_f)}(1 - z - z', \mu, \mu_\phi)\, \phi_{i/P}^{(n_f)}(z', \mu_\phi) \qquad \mu_\phi \sim \Lambda_{\mathrm{QCD}}$$

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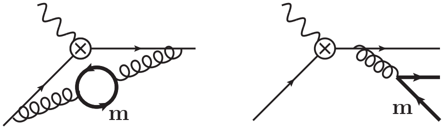
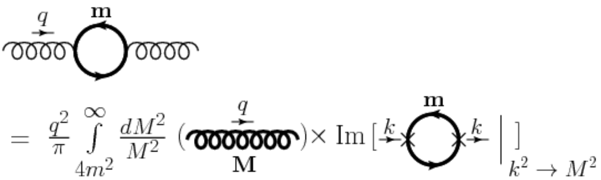
Secondary Massive Quarks

no flavor mixing in PDF evolution → only light quarks enter hard interaction
⇒ effects of heavy quarks only via secondary radiation



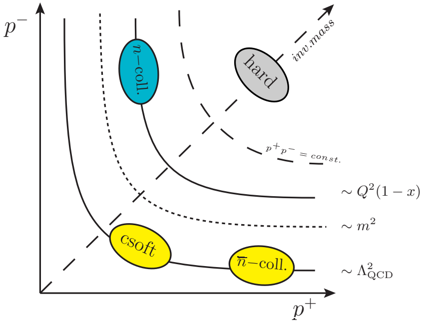
can be obtained from 1-loop calculation with massive gluon and dispersion relation



VFNS for the PDF

$$\phi^{(n_f)}(1-z,m,\mu) = \begin{cases} \phi^{(n_l+1)}(1-z,m,\mu) & \text{for } \mu > \mu_m \\ \phi^{(n_l)}(1-z,\mu) & \text{for } \mu < \mu_m \end{cases}$$

- at the scale μ_m change from a scheme with (n_l) light flavors to a scheme with (n_l+1) flavors
- leads to a threshold correction at the matching scale, that relates the PDF in the two different schemes with each other.



$$\mathcal{M}^+_\phi(1-z,m,\mu_m) = \int \mathrm{d}z'\, \phi^{(n_l+1)}(z'-z,m,\mu_m)\, \left(\phi^{(n_l)}\right)^{-1}(1-z',m,\mu_m)$$

Fixed Order Threshold Correction for the PDF

fixed order threshold correction at two loops
$\nu_\phi \sim Q(1-x),\ \tilde{\ell} = \ell/\nu_\phi,\ L_m = \ln \frac{m^2}{\mu_m^2}$:

$$\frac{\nu_\phi}{Q}\,\mathcal{M}_\phi^{+(2)}\left(\frac{\ell}{Q}, m, Q, \mu_m, \nu_\phi\right)\bigg|_{\mathrm{FO}} = \frac{\alpha_s^2 C_F T_F}{(4\pi)^2}$$
$$\times \left\{\delta(\tilde{\ell})\left[\left(\frac{8}{3}L_m^2 + \frac{80}{9}L_m + \frac{224}{27}\right)\ln\left(\frac{\nu_\phi}{Q}\right)\right.\right.$$
$$\left. + 2L_m^2 + \left(\frac{2}{3} + \frac{8\pi^2}{9}\right)L_m + \frac{73}{18} + \frac{20\pi^2}{27} - \frac{8}{3}\zeta_3\right]$$
$$\left. + \left[\frac{\theta(\tilde{\ell})}{\tilde{\ell}}\right]_+ \left[\frac{8}{3}L_m^2 + \frac{80}{9}L_m + \frac{224}{27}\right]\right\}$$

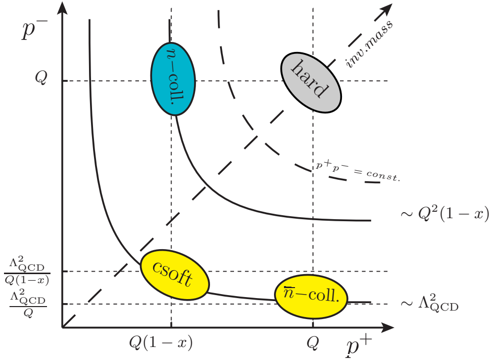
remaining large logarithm in fixed order expansion

$$\ln\left(\frac{\nu_\phi}{Q}\right) \sim \ln(1-x)$$

Rapidity Logarithms

- $\bar{n}$-coll. and (c)soft modes in the PDF separated in rapidity

$$e^{Y_{\bar{n}}} \sim \frac{Q}{\Lambda_{\rm QCD}} \qquad e^{Y_{S_c}} \sim \frac{Q(1-x)}{\Lambda_{\rm QCD}}$$



Rapidity Logarithms

- $\bar{n}$-coll. and (c)soft modes in the PDF separated in rapidity

$$e^{Y_{\bar{n}}} \sim \frac{Q}{\Lambda_{\mathrm{QCD}}} \qquad e^{Y_{S_c}} \sim \frac{Q(1-x)}{\Lambda_{\mathrm{QCD}}}$$

- factorize PDF into collinear and (c)soft function: $\phi_{q/P} = S_c \otimes g_{q/P}$
- each single function contains rapidity divergences in loop integrals for $k^+ \to 0$ (as $\frac{1}{\eta}$ poles with a rapidity regulator η) that cancel in full PDF...
- ...but because of the different rapidity scales a logarithm remains
- can be resummed by using rapidity RGE

$$S^{\mathrm{bare}} = Z_S \otimes S$$

Z_S contains all $\frac{1}{\epsilon}$ (UV) and $\frac{1}{\eta}$ (rapidity) divergences

$$S(\mu) \to S(\mu, \nu)$$

- introduces running with respect to rapidity scale ν

$$S(\mu, \nu) = U_{S,\nu}(\nu, \nu_0) \otimes S(\mu, \nu_0)$$

Parton Distribution Functions for $\Lambda_{\rm QCD} \sim Q(1-x) \ll Q$

$\lambda = \frac{\Lambda_{\rm QCD}}{Q}$

$$\phi_{q/P}(1-z,\mu) = \langle P|\bar{\chi}_{\bar{n}} S_{\bar{n}}^{\dagger} S_n \frac{\not{n}}{2} \delta(Q(1-z) - n\cdot\mathcal{P}) S_n^{\dagger} S_{\bar{n}} \chi_{\bar{n},Q}|P\rangle$$
$$= Q \int \mathrm{d}\ell \, g_{q/P}(Q(1-z)-\ell,\mu)\, S(\ell,\mu)$$

collinear function: $g_{q/P}(\ell,\mu) = \langle P|\bar{\chi}_{\bar{n}} \frac{\not{n}}{2} \chi_{\bar{n},Q}|P\rangle\, \delta(\ell)$ $\qquad p_{\bar{n}} = (p_{\bar{n}}^{+}, p_{\bar{n}}^{-}, p_{\bar{n}}^{\perp}) \sim Q(1,\lambda^2,\lambda)$

soft function: $S(\ell,\mu) = \frac{1}{N_c} \langle 0|S_{\bar{n}}^{\dagger} S_n\, \delta(\ell - n\cdot\mathcal{P})\, S_n^{\dagger} S_{\bar{n}}|0\rangle$

soft Wilson lines:

$$S_n^{\dagger}(x) = \mathrm{P}\exp\left[ig\int_0^{\infty} \mathrm{d}s\, n\cdot A_s(sn^{\mu}+x^{\mu})\right]$$
$$S_{\bar{n}}(x) = \mathrm{P}\exp\left[ig\int_{-\infty}^{0} \mathrm{d}s\, \bar{n}\cdot A_s(s\bar{n}^{\mu}+x^{\mu})\right]$$

$p_s = (p_s^{+}, p_s^{-}, p_s^{\perp}) \sim Q(\lambda,\lambda,\lambda)$

Parton Distribution Functions for $\Lambda_{\rm QCD} \lesssim Q(1-x) \ll Q$

$$\lambda = \frac{\Lambda_{\rm QCD}}{Q}$$

$$\phi_{q/P}(1-z,\mu) = \langle P|\bar\chi_{\bar n} X^\dagger_{\bar n} V_{\bar n} \frac{\not{n}}{2} \delta(Q(1-z) - n\cdot\mathcal{P}) V^\dagger_{\bar n} X_{\bar n} \chi_{\bar n,Q}|P\rangle$$

$$= Q\int \mathrm{d}\ell\, g_{q/P}(Q(1-z)-\ell,\mu)\, S_c(\ell,\mu)$$

collinear function: $g_{q/P}(\ell,\mu) = \langle P|\bar\chi_{\bar n}\frac{\not{n}}{2}\chi_{\bar n,Q}|P\rangle\,\delta(\ell)$ $\quad p_{\bar n} = (p^+_{\bar n}, p^-_{\bar n}, p^\perp_{\bar n}) \sim Q(1,\lambda^2,\lambda)$

csoft function: $S_c(\ell,\mu) = \frac{1}{N_c}\langle 0|X^\dagger_{\bar n} V_{\bar n}\,\delta(\ell - n\cdot\mathcal{P})\, V^\dagger_{\bar n} X_{\bar n}|0\rangle$

csoft Wilson lines:

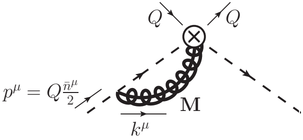
$$V^\dagger_{\bar n}(x) = \mathrm{P}\exp\left[ig\int_0^\infty \mathrm{d}s\, n\cdot A_{cs}(sn^\mu + x^\mu)\right]$$

$$X_{\bar n}(x) = \mathrm{P}\exp\left[ig\int_{-\infty}^0 \mathrm{d}s\, \bar n\cdot A_{cs}(s\bar n^\mu + x^\mu)\right]$$

$$p_{cs} = (p^+_{cs}, p^-_{cs}, p^\perp_{cs}) \sim Q(\lambda\,\kappa, \tfrac{\lambda}{\kappa}, \lambda)$$

$$\kappa = \frac{Q(1-x)}{\Lambda_{\rm QCD}}$$

Collinear Function $g_{q/q}$



$$L_M = \ln\left(\frac{M^2}{\mu^2}\right), \; \nu_\phi \sim Q(1-x), \; \tilde{\ell} = \ell/\nu_\phi$$

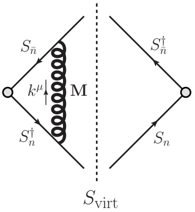
$$g^{(1)}_{q/q}(\ell, M, Q, \mu, \nu) = \frac{\alpha_s C_F}{4\pi}\,\delta(\ell) \times \left\{ -4 L_M \ln\left(\frac{\nu}{Q}\right) - 3L_M + \frac{9}{2} - \frac{2\pi^2}{3} \right\}$$

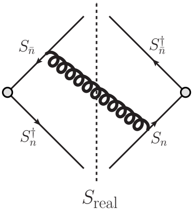
$$\nu_\phi\, Z^{(1)}_g = \frac{\alpha_s C_F}{4\pi}\,\delta(\tilde{\ell}) \left\{ \frac{4}{\eta}\left[\frac{1}{\epsilon} - L_M + \mathcal{O}(\epsilon)\right] + \frac{1}{\epsilon}\left[4 \ln\left(\frac{\nu}{Q}\right) + 3\right] \right\}$$

$$\gamma_{\nu,g} = -\frac{\alpha_s C_F}{4\pi}\, 4L_M$$

$$g_{q/q}(\nu) = \exp\left[\gamma_{\nu,\mathrm{g}} \ln\left(\frac{\nu_\mathrm{g}}{\nu}\right)\right] \mathrm{g}_{\mathrm{q/q}}(\nu_\mathrm{g}) \qquad \nu_g \sim Q$$

Soft Function S





$$L_M = \ln\left(\frac{M^2}{\mu^2}\right),\ \nu_\phi \sim Q(1-x),\ \tilde{\ell} = \ell/\nu_\phi$$

$$\nu_\phi S^{(1)}(\ell, M, \mu, \nu) = \frac{\alpha_s C_F}{4\pi} \times \left\{ 4L_M \ln\left(\frac{\nu}{\nu_\phi}\right)\delta(\tilde{\ell}) - 4L_M \left[\frac{\Theta(\tilde{\ell})}{\tilde{\ell}}\right]_+ \right\}$$

$$\nu_\phi\, Z_S^{(1)} = \frac{\alpha_s C_F}{4\pi}\left\{\delta(\tilde{\ell})\left(-\frac{4}{\eta}\left[\frac{1}{\epsilon} - L_M + \mathcal{O}(\epsilon)\right] - \frac{4}{\epsilon}\ln\left(\frac{\nu}{\nu_\phi}\right)\right) + \frac{4}{\epsilon}\left[\frac{\Theta(\tilde{\ell})}{\tilde{\ell}}\right]_+\right\}$$

$$\gamma_{\nu,S} = \frac{\alpha_s C_F}{4\pi} 4L_M = -\gamma_{\nu,g}$$

$$S(\nu) = \exp\left[\gamma_{\nu,\mathrm{S}} \ln\left(\frac{\nu_\mathrm{S}}{\nu}\right)\right] \mathrm{S}(\nu_\mathrm{S}) \qquad \nu_S \sim \nu_\phi$$

Resummed Threshold Correction for the PDF

$$\mathcal{M}_\phi^+ = \exp\left[\gamma_{\mathcal{M}} \ln\left(\frac{\nu_\mathrm{S}}{\nu_\mathrm{g}}\right)\right] \mathcal{M}_\mathrm{S}^+(\nu_\mathrm{S}) \otimes \mathcal{M}_\mathrm{g}^+(\nu_\mathrm{g})$$

$$\gamma_{\mathcal{M}} = \frac{\alpha_s^2 C_F T_F}{(4\pi)^2}\left(\frac{8}{3}L_m^2 + \frac{80}{9}L_m + \frac{224}{27}\right) + \mathcal{O}(\alpha_s^3)$$

$$\mu_m = m,\ \nu_g = Q,\ \nu_S = \nu_\phi = Q(1-x)$$

$$\nu_\phi \mathcal{M}_\phi^+(\frac{\ell}{Q}, m, Q) = \exp\left[\frac{\alpha_\mathrm{s}^2 \mathrm{C_F T_F}}{(4\pi)^2}\,\frac{224}{27}\,\ln(1-\mathrm{x})\right]$$
$$\times \left\{\delta(\tilde{\ell}) + \frac{\alpha_s^2 C_F T_F}{(4\pi)^2}\left[\delta(\tilde{\ell})\left(\frac{20\pi^2}{27} + \frac{73}{18} - \frac{8}{3}\zeta_3\right) + \frac{224}{27}\left[\frac{\Theta(\tilde{\ell})}{\tilde{\ell}}\right]_+\right]\right\}$$

Threshold Correction for the PDF with N^3LL Resummation

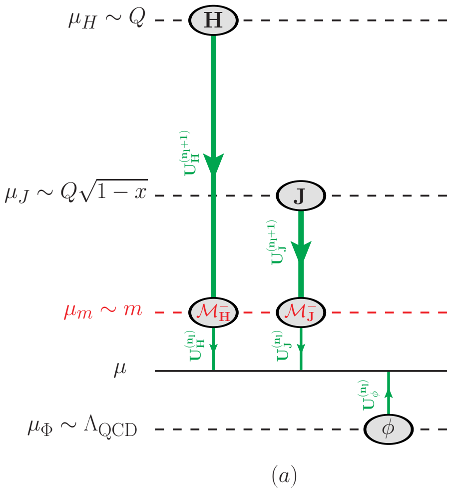
N^3LL resummation ($\alpha_s \ln \sim 1$, all terms up to "α_s^2")
$\Rightarrow$ α_s^2, $\alpha_s^2 \ln$, $\alpha_s^3 \ln$, $\alpha_s^4 \ln^2$

$$\frac{\nu_S}{Q}\,\mathcal{M}_\phi^+\!\left(\frac{\ell}{Q}, m, Q, \mu_m, \nu_g, \nu_S\right) = \delta(\tilde\ell) + \left[\frac{\left(\alpha_s^{(n_l+1)}\right)^2}{(4\pi)^2}\,\delta(\tilde\ell)\,\ln\left(\frac{\nu_S}{\nu_g}\right)\,\mathcal{M}^{(2)}_{\phi,\ln}(m,\mu_m)\right]_{\mathcal{O}(\alpha_s)}$$
$$+\left[\frac{\left(\alpha_s^{(n_l+1)}\right)^2}{(4\pi)^2}\left(\delta(\tilde\ell)\,\mathcal{M}^{(2)}_{\phi,1}(Q,m,\mu_m,\nu_\phi,\nu_g) + \left[\frac{\theta(\tilde\ell)}{\tilde\ell}\right]_+\mathcal{M}^{(2)}_{\phi,\ln}(m,\mu_m)\right)\right.$$
$$+\frac{\left(\alpha_s^{(n_l+1)}\right)^3}{(4\pi)^3}\,\delta(\tilde\ell)\,\ln\left(\frac{\nu_S}{\nu_g}\right)\,\mathcal{M}^{(3)}_{\phi,\ln}(m,\mu_m)$$
$$\left.+\frac{\left(\alpha_s^{(n_l+1)}\right)^4}{(4\pi)^4}\,\delta(\tilde\ell)\,\ln^2\left(\frac{\nu_S}{\nu_g}\right)\,\mathcal{M}^{(4)}_{\phi,\ln^2}(m,\mu_m)\right]_{\mathcal{O}(\alpha_s^2)} + \mathcal{O}(\alpha_s^3)$$

$\mathcal{M}^{(4)}_{\phi,\ln^2}$ from exponentiation with 2-loop anomalous dimension
$\mathcal{M}^{(3)}_{\phi,\ln}$ from full 3-loop calculation in the OPE region

J.Aiblinger et al., *Nucl.Phys.* **B886**, 733 (2014)

Consistency Relations



Consistency Relations

physical form factor independent of the choice of the renormalization scale μ
$\Rightarrow$ consistency relations between the evolution kernels and threshold corrections of the components in the factorization theorem

$$U_{\phi}^{(n_f)}(1-z,\mu_0,\mu) = Q^2\, U_H^{(n_f)}(Q,\mu_0,\mu)\, U_J^{(n_f)}(Q^2(1-z),\mu,\mu_0)$$

$$\mathcal{M}_{\phi}^{+}(1-z,m,\mu) = Q^2 \mathcal{M}_H^{-}(Q,m,\mu)\, \mathcal{M}_J^{-}(Q^2(1-z),m,\mu)$$

Threshold Corrections

missing α_s^3 terms $\mathcal{M}^{(3)}_{H,\ln}$ and $\mathcal{M}^{(3)}_{J,\ln}$ can be obtained from $\mathcal{M}^{(3)}_{\phi,\ln}$ via consistency relations

$\nu_g \sim Q,\ \nu_S \sim Q(1-x),\ \nu_m \sim m$

$$\mathcal{M}^{(3)}_{\phi,\ln} \ln\left(\frac{\nu_S}{\nu_g}\right) = \mathcal{M}^{(3)}_{H,\ln} \ln\left(\frac{\nu_m^2}{\nu_g^2}\right) + \mathcal{M}^{(3)}_{J,\ln} \ln\left(\frac{\nu_g \nu_S}{\nu_m^2}\right)$$
$$= \mathcal{M}^{(3)}_{H,\ln} \left[\ln\left(\frac{\nu_S}{\nu_g}\right) - \ln\left(\frac{\nu_g \nu_S}{\nu_m^2}\right)\right] + \mathcal{M}^{(3)}_{J,\ln} \ln\left(\frac{\nu_g \nu_S}{\nu_m^2}\right)$$

$\mathcal{M}^{(3)}_{\phi,\ln} = \mathcal{M}^{(3)}_{H,\ln}$

$\mathcal{M}^{(3)}_{J,\ln} = \mathcal{M}^{(3)}_{H,\ln}$

Outline

1. Motivation
2. Soft-Collinear Effective Theory
3. DIS in the OPE Region $1 - x \sim 1$
4. VFNS in the OPE Region
5. DIS in the Endpoint Region $1 - x \ll 1$
6. VFNS in the Endpoint Region
7. Outlook and Conclusions

Outlook

- PDFs are strongly suppressed for $x \to 1$
  $\Rightarrow$ not much data for the endpoint region available

- to focus on jet production away from the endpoint region consider 1-jettiness

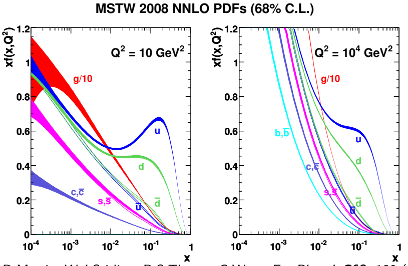
$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min \{ q_B \cdot p_i , q_J \cdot p_i \}$$

  for $\tau_1 \ll 1$: two jets in the final state, resummation of $\ln \tau_1$ needed

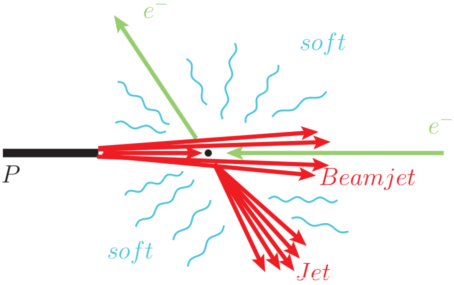
- include initial state radiation in the factorization theorem

$$H \times J \otimes S \otimes \mathcal{I} \hat{\otimes} f$$

  Stewart, Tackmann, Waalewijn (2010)



A.D.Martin, W.J.Stirling, R.S.Thorne, G.Watt, *Eur.Phys.J.* **C63**, 189 (2009)



Conclusions

- derived a factorization theorem for DIS in the endpoint region $1 - x \ll 1$ valid for $\Lambda_{\mathrm{QCD}} \lesssim Q(1-x)$
- set up a VFNS for DIS in the endpoint region for secondary massive quarks
- studied resummation of rapidity logarithms $\ln(1-x)$ in the PDF threshold correction $\mathcal{M}_\phi$
- achieved N^3LL resummation of rapidity logarithms in $\mathcal{M}_\phi$
- with consistency relations extended this to $\mathcal{M}_H$ and $\mathcal{M}_J$

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Thank you for your attention!