Scattering Amplitudes and Precision Simulations for the LHC

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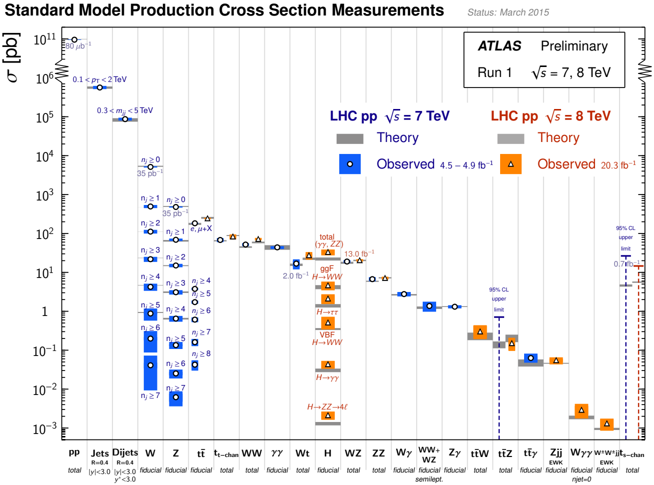
Success of LHC Run 1 (+2)

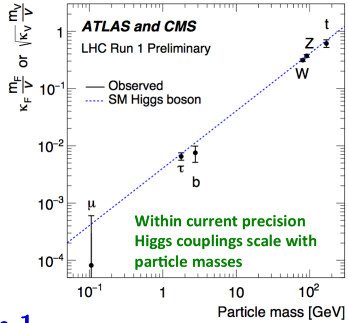


Standard Model Production Cross Section Measurements

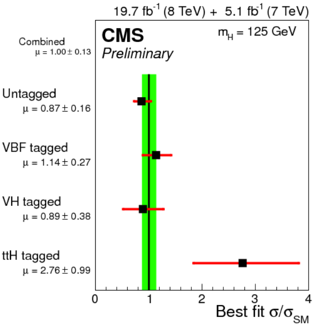
Status: March 2015

Data–theory consistency from milli-barn to femto-barn range

Success of Standard Model (Higgs discovery)



Within current precision Higgs couplings scale with particle masses



Run 1

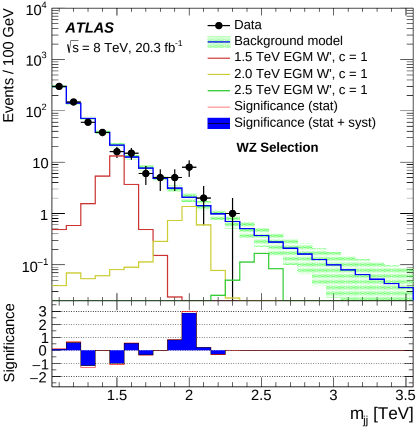
- SM promoted to realistic description of EW symmetry breaking (*at present* precision level and energy scales)
- M_H measurement $\Rightarrow$ instead of disproving the SM, Run1 has turned it into a fully predictive theory – at the quantum level!

Run2

- can falsify or verify SM with more stringent tests at higher precision and higher energy

Multi-TeV searches

2.5σ diboson anomaly at $M_{VV} \sim 2$ TeV (not confirmed at 13 TeV)

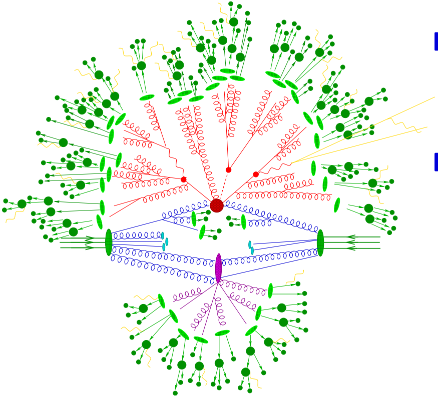


- nontrivial multijet final states (often with MET and/or leptons)
- sophisticated selection strategies (e.g. boosted jets)
- requires higher-order calculations at TeV energies (EW Sudakov logs,. . .)

TH precision crucial for direct/indirect BSM sensitivity and interpretation of discoveries

Theoretical simulations of LHC collisions

$$\boxed{d\sigma \;=\; d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma^{\mathrm{EW}}_{\mathrm{NLO}} + \alpha_S^2\, d\sigma_{\mathrm{NNLO}} \;+\; \ldots}$$



High-energy scattering

- NLO QCD+EW and NNLO "revolutions"

Parton-shower MC simulations

- matching to (N)NLO matrix elements
- multijet merging at NLO

More and more general and widely applicable algorithms

NLO QCD calculations and NLO revolution

Born, virtual and real $2 \to n$ contributions ($|\mathcal{M}|^2$, flux factor and PDFs implicit)

$$\sigma_n^{\mathrm{NLO}} = \int \mathrm{d}\Phi_n \mathcal{B}(\Phi_n) + \int \mathrm{d}\Phi_n \mathcal{V}(\Phi_n) + \int \mathrm{d}\Phi_{n+1} \mathcal{R}(\Phi_{n+1})$$

- UV renormalisation ⇒ reduction of μ_R dependence
- soft/collinear cancellations+PDF renormalisation ⇒ reduction of μ_F dependence

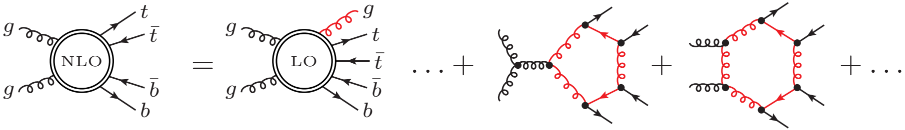
General "solution to NLO problem" exists since 1970s (tensor reduction) and 1990s (subtraction methods).

NLO revolution: $2 \to 4\,(5,6)$ **processes**

> *"the barrier that has existed for 15 years to NLO computations for more than 5 particles has been broken, allowing NLO computations for process of a complexity that matches that of LHC events. This is the most important development in theoretical particle physics of the past few years."* [M. Peskin 2011]

One-loop multi-leg methods and tools

First 6-particle NLO steps: $pp \to t\bar{t}b\bar{b}$ [Bredenstein, Denner, Dittmaier, S.P. '09]



10^3 diagrams $\Rightarrow$ 100 MB algebraic expression

3 years

Solutions to one-loop multi-leg bottleneck

- radically new approaches: on-shell method, OPP reduction, ...
- automated 1-loop algorithms (`CutTools`, `BlackHat`, `Collier`, `GoSam`, `HELAC 1-loop`, `MadLoop`, `NGluon`, `OpenLoops`, `Recola`, `Samurai`, ...)
- vast range of multi-particle NLO predictions at LHC ($pp \to 5j$, $W+5j$, $Z+4j$, $H+3j$, $WWjj$, $WZjj$, $\gamma\gamma+3j$, $W\gamma\gamma j$, $WWb\bar{b}$, $b\bar{b}b\bar{b}$, $t\bar{t}b\bar{b}$, $t\bar{t}jj$, $t\bar{t}t\bar{t}$, ...)

Flexibility and efficiency of best methods and tools is great but still insufficient

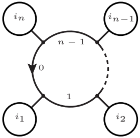
This talk

1. Scattering Amplitudes with OpenLoops
2. (N)NLO QCD at parton level
3. Matching and Multi-jet Merging at NLO QCD
4. NLO EW corrections

OpenLoops method [Cascioli, Maierhöfer, S.P. '11]

Strategy

- handle all process-dependent one-loop ingredients via tree-like algorithm
- hybrid "tree–loop" approach ⇒ very high speed and flexibility [Van Hameren '09]
- diagrammatic representation

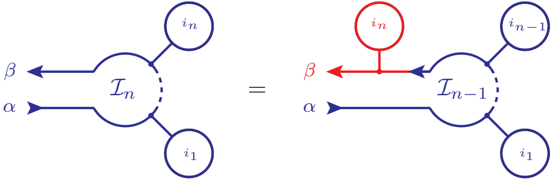


$$= \int \frac{\mathrm{d}^D q\, \mathcal{N}(\mathcal{I}_n; q)}{D_0 D_1 \dots D_{n-1}} = \underbrace{\sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n)}_{\text{numerical recursion [OpenLoops]}} \underbrace{\int \frac{\mathrm{d}^D q\, q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{\text{tensor integrals [Denner, Dittmaier]}}$$

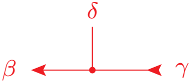
OpenLoops recursion [Cascioli, Maierhöfer, S.P '11]

Recursive merging of q-*dependent* trees

$$\sum_{r=0}^{n} \mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\; q^{\mu_1}\ldots q^{\mu_r} =$$



Interaction terms depend only on $\mathcal{L}_{\text{int}}$ $\Rightarrow$ *automation!*



$$= \quad Y^{\beta}_{\gamma\delta} + Z^{\beta}_{\nu;\gamma\delta}\; q^{\nu}$$

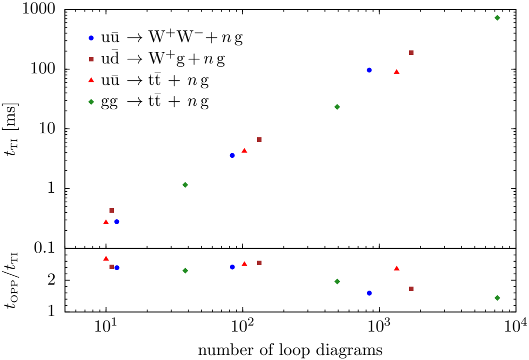
Recursion for polynomial coefficients $\Rightarrow$ *very high speed!*

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n) = \Big[Y^{\beta}_{\gamma\delta}\; \mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\; \mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) \Big]\; w^{\delta}(i_n)$$

OpenLoops performance for $2 \to 2, 3, 4$ processes

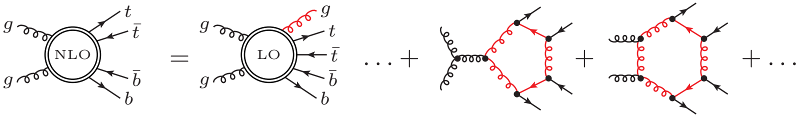
Orders of magnitude improvements for multi-particle amplitudes

- $\mathcal{O}(10^2–10^3)$ in code generation (code size and time for generation+compilation)
- $\mathcal{O}\left(10^2\right)$ in speed of amplitudes (wrt original OPP automation)



$\Rightarrow$ **large scale applicability at the technical frontier**

OpenLoops 1.0 [Cascioli, Lindert, Maierhöfer, S.P. '14]



Automated generator of NLO QCD matrix elements (>30'000 lines of code)

- public library with more than 100 LHC processes at `openloops.hepforge.org`

Interface to multi-purpose Monte Carlo programs

- Munich [Kallweit] ⇒ very powerful (N)NLO parton level MC
- Sherpa [Höche, Krauss, Schönherr, Siegert et al.] ⇒ NLO matching and merging
- Powheg [Nason, Oleari et al.]
- Herwig [Gieseke, Plätzer wt al.]
- Geneva [Alioli, Bauer, Tackmann et al.]
- Whizard [Kilian, Ohl, Reuter et al.]

Completely automated NLO simulations for any $2 \rightarrow 2, 3, 4$ SM processes at LHC

State-of-the-art applications in Top, EW and Higgs physics

NLO QCD+EW

- S–MC@NLO for $pp \to t\bar{t}b\bar{b}$ with $m_b > 0$ [Cascioli, Maierhöfer, Moretti, S.P., Siegert, arXiv:1309.5912]
- NLO for $pp \to W^+W^-b\bar{b}$ with $m_b > 0$ [Cascioli, Kallweit, Maierhöfer, S.P., arXiv:1312.0546]
- NLO QCD+EW for $W + 1, 2, 3$ jets [Kallweit, Lindert, Maierhöfer, S.P., Schönherr, arXiv:1412.5157]
- NLO QCD+EW for $\ell\ell/\ell\nu/\nu\nu + 0, 1, 2$ jets [Kallweit, Lindert, Maierhöfer, S.P., Schönherr, arXiv:1511.08692]

NLO merging

- MEPS@NLO for $\ell\ell\nu\nu$+0,1 jets, [Cascioli, Höche, Krauss, Maierhöfer, S.P., Siegert, arXiv:1309.0500]
- (1-loop)^2 merging for $pp \to HH$+0,1 jets, [Maierhöfer, Papaefstathiou, arXiv:1401.0007]
- MEPS@NLO for WWW+0,1 jets, [Höche, Krauss, S.P., Schönherr, Thompson arXiv:1403.7516]
- MEPS@NLO for $t\bar{t}$+0,1,2 jets, [Höche, Krauss, Maierhöfer, S.P., Schönherr, Siegert arXiv:1402.6293]

NNLO QCD

- $pp \to \gamma Z$ and γW [Grazzini, Kallweit, Rathlev, Torre, arXiv:1309.7000; arXiv:1504.01330]
- $q\bar{q} \to t\bar{t}$ [Abelof, Gehrmann–de Ridder, Maierhöfer, S.P., arXiv:1404.6493]
- $pp \to ZZ$ [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, S.P., Rathlev, Tancredi, Weihs, arXiv:1405.2219]
- $pp \to W^+W^-$ [Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, S.P., Rathlev, Tancredi arXiv:1408.5243]

Outline

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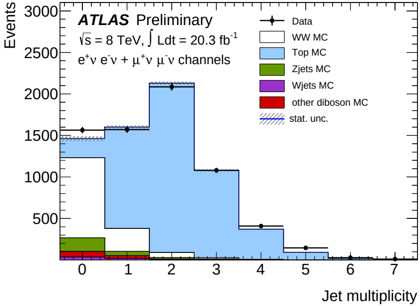
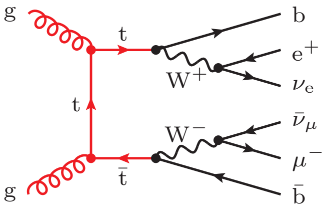
4. NLO EW corrections

$pp \to t\bar{t} \to W^+W^-b\bar{b}$

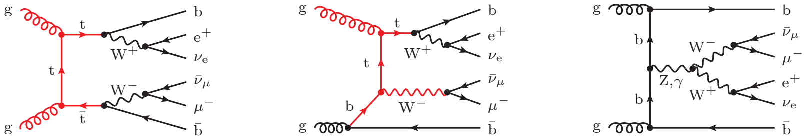




Vast top-physics program at LHC

- SM benchmark and omnipresent Higgs- and BSM-background
- 3 decades of precision calculations
- **full description of production×decay crucial**: jet veto, m_t-measurements, . . .

$W^+W^-b\bar{b}$ production at NLO [Cascioli,Kallweit,Maieröfer,S.P. '13]

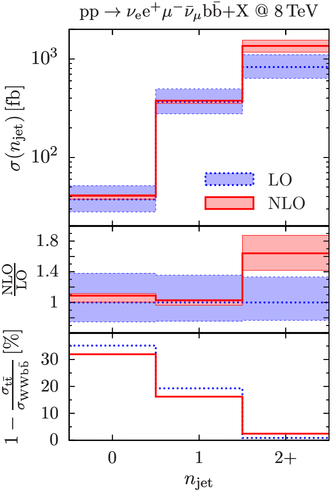


First unified description of $t\bar{t}$+Wt production and decay at NLO

- **full set of $2 \to 6$ diagrams** $(\Gamma_t > 0)$ and **full b-quark phase space** $(m_b > 0)$
- multi-particle, multi-scale $(\Gamma_t, m_b, \ldots, m_{t\bar{t}})$ simulation with $\mathcal{O}\left(10^3\right)$ loop diagrams

done with Munich+OpenLoops

First $W^+W^-b\bar{b}$ NLO predictions for $N_{\text{jet}} = 0, 1$



Jet veto and jet bins

- key to suppress top backgrounds in $H \to W^+W^-$ and many other analyses

Excellent perturbative convergence

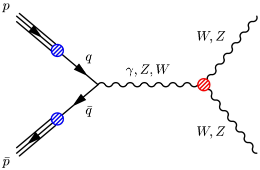
- small NLO correction and reduction of scale uncertainty from 40% to $< 10\%$

Single-top and other $\mathcal{O}(\Gamma_t/m_t)$ effects

- from 1% to 30–40% with jet veto

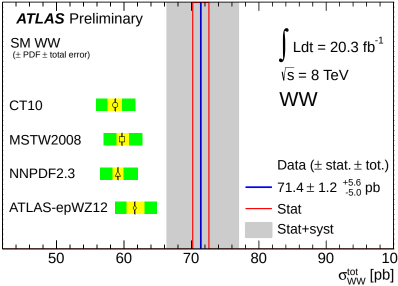
$W^+W^-b\bar{b}$ crucial for accurate simulation of top-production and decay

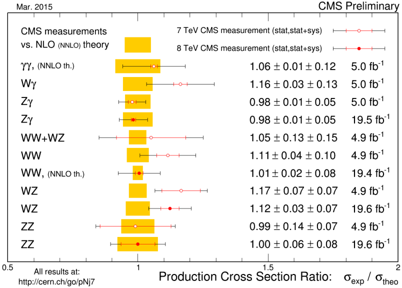
Diboson production at LHC



- test SU(2)×U(1) gauge structure
- interplay with $H \to VV$
- BSM searches, . . .

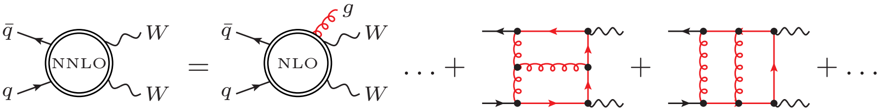
Some tensions between NLO QCD and Run1 data





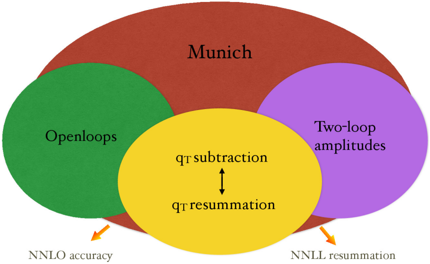
$$\sim 2.5\,\sigma \text{ (20\%) excess in } \sigma^{\text{ATLAS}}_{W^+W^-} \simeq 3 \times \sigma(H \to W^* W)$$

Diboson production at NNLO QCD



Flexible NNLO+NNLL framework based on q_T-subtraction [Catani, Grazzini '06]

Kallweit



Munich

Openloops

Two-loop amplitudes

q_T subtraction ↕ q_T resummation

NNLO accuracy

NNLL resummation

Cascioli
Maierhöfer
Lindert
S.P.

Gehrmann, Tancredi
von Manteuffel, Weihs;
Caola, Henn, Melnikov
Smirnov, Smirnov

Grazzini, Kallweit, Rathlev, Wiesemann

⇒ **predictions for $Z\gamma, W\gamma$ at NNLO and ZZ, WW at NNLO+NNLL** [2013–15]

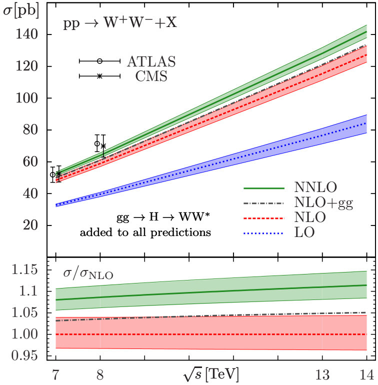
NNLO predictions for $pp \to W^+W^-$ [Gehrmann et al. '14]

Unexpectedly large QCD corrections

- $+58\%$ NLO and $+12\%$ NNLO at 14 TeV
- well beyond expected size from scale uncertainties and $gg \to W^+W^-$ ($+4\%$)

Residual scale uncertainty

- 3% NNLO scale variation
- consistent with 2% higher-order correction to $gg \to W^+W^-$ [Melnikov et al., 1511.08617]



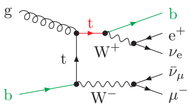
Comparison with ATLAS and CMS data

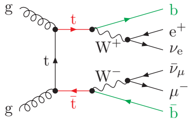
- NNLO reduces significance of excess in 8 TeV ATLAS measurement and agrees well with published 8 TeV result by CMS

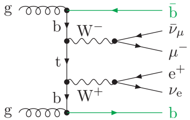
Theoretical definition(s) of top-free W^+W^- production

Huge Wt and $t\bar{t}$ contamination from $\overbrace{W^+W^-b}^{+40\%\ \mathrm{NLO}}$ and $\overbrace{W^+W^-b\bar{b}}^{+400\%\ \mathrm{NNLO}}$

- intimately connected with W^+W^- through $g \to b\bar{b}$ singularities
- top subtraction tricky and not unique ⇒ theoretical ambiguity in $\sigma_{WW}^{\mathrm{(N)NLO}}$!







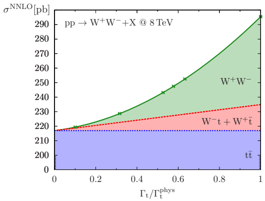
Definition A: **veto b-quark emissions in 4F scheme** $(m_b > 0)$

- ⇒ $\ln(m_b/M_W)$ terms might jeopardize NNLO accuracy!

Definition B: **top-resonance fit in 5F-scheme** $(m_b = 0)$

$$\lim_{\xi_t \to 0} \sigma_{\mathrm{full}}^{\mathrm{5F}}(\xi_t \Gamma_t) = \xi_t^{-2} \left[\sigma_{t\bar{t}}^{\mathrm{5F}} + \xi_t\, \sigma_{Wt}^{\mathrm{5F}} + \xi_t^2\, \sigma_{W^+W^-}^{\mathrm{5F}} \right]$$

⇒ for inclusive $\sigma_{WW}^{\mathrm{NNLO}}$ only 1–2% ambiguity (A vs B)



Relevant issue for percent-precision tests of W^+W^- physics! . . . Relation to $\sigma_{WW}^{\mathrm{EXP}}$?

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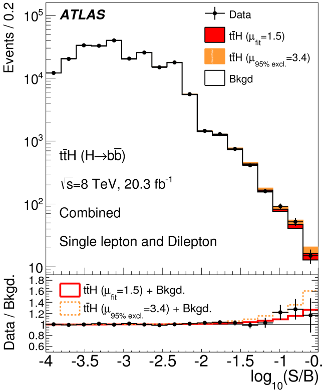
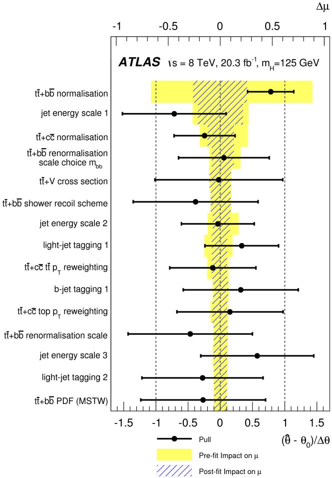
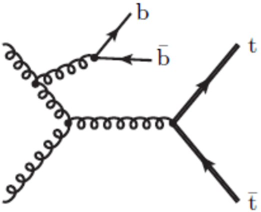
$t\bar{t}H$ searches in the dominant $H \to b\bar{b}$ channel

- $\sim 3'000\ t\bar{t}H$ events but only 3–4$\times\sigma_{\rm SM}$ exclusion at Run1
- heavy background contamination with large theory uncertainty
- requires nontrivial $t\bar{t}b\bar{b}$ and $t\bar{t}+2\,$jet simulations





Irreducible $t\bar{t}b\bar{b}$ QCD background at NLO



NLO $t\bar{t}b\bar{b}$ [Bredenstein et al '09/'10; Bevilacqua et al '09];

- $t\bar{t}b\bar{b}$ dominates $t\bar{t}H(b\bar{b})$ systematics
- NLO reduces uncertainty from 80% to 20–30%

NLO+PS $t\bar{t}b\bar{b}$ 5F scheme ($m_b = 0$) with POWHEL [Garzelli et al '13/'14]

- $t\bar{t}b\bar{b}$ **NLO MEs cannot describe collinear $g \to b\bar{b}$ splittings**

⇒ *inclusive* $t\bar{t}$+b-jets simulation requires parton shower in collinear $b\bar{b}$ region

⇒ **NLO merging** $t\bar{t} + 0, 1, 2$ **jets** (see later)

NLO+PS $t\bar{t}b\bar{b}$ 4F scheme ($m_b > 0$) with SHERPA+OPENLOOPS [Cascioli et al '13]

- $t\bar{t}b\bar{b}$ **NLO MEs cover full b-quark phase space**

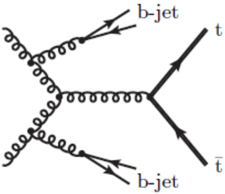
⇒ inclusive NLO accurate $t\bar{t}$+b-jets simulation possible

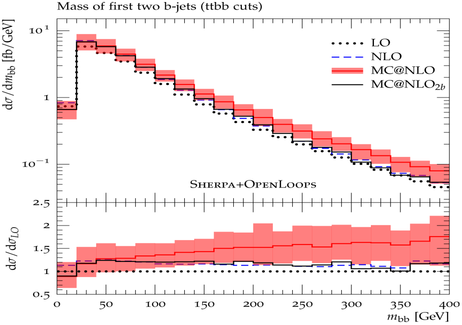
S–MC@NLO $t\bar{t}b\bar{b}$ 4F scheme [Cascioli et al '13]

Good perturbative stability but unexpected MC@NLO enhancement

	ttb	$ttbb$	$ttbb\,(m_{bb} > 100)$
$\sigma_{\mathrm{LO}}[\mathrm{fb}]$	$2644^{+71\%+14\%}_{-38\%-11\%}$	$463.3^{+66\%+15\%}_{-36\%-12\%}$	$123.4^{+63\%+17\%}_{-35\%-13\%}$
$\sigma_{\mathrm{NLO}}[\mathrm{fb}]$	$3296^{+34\%+5.6\%}_{-25\%-4.2\%}$	$560^{+29\%+5.4\%}_{-24\%-4.8\%}$	$141.8^{+26\%+6.5\%}_{-22\%-4.6\%}$
$\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$	1.25	1.21	1.15
$\sigma_{\mathrm{MC@NLO}}[\mathrm{fb}]$	$3313^{+32\%+3.9\%}_{-25\%-2.9\%}$	$600^{+24\%+2.0\%}_{-22\%-2.1\%}$	$181^{+20\%+8.1\%}_{-20\%-6.0\%}$
$\sigma_{\mathrm{MC@NLO}}/\sigma_{\mathrm{NLO}}$	1.01	1.07	1.28

Large enhancement (∼30%) in Higgs region from double $g \to b\bar{b}$ splittings





matching, shower and 4F/5F systematics remain to be understood!

$t\bar{t}+$ multijet background and merging at NLO

NLO $t\bar{t}+2$ jets [Bevilacqua, Czakon, Papadopoulos, Worek '10/'11]

- reduces uncertainty from 80% to 15%
- experiments need inclusive particle-level simulation with $t\bar{t}+0,1,2$ jets at NLO

MEPS@NLO merging [Höche, Krauss, Schönherr, Siegert '12]

0-jet	NLO+PS $t\bar{t}$
1-jet	NLO+PS $t\bar{t}+1\,\mathrm{j}$
...	...
$\geq n$ jets	NLO+PS $t\bar{t}+n\,\mathrm{j}$

- NLO and log accuracy for $0,1,\ldots n$ jets
- separated via k_T-algo at merging scale Q_cut
- smooth PS–MEs transition ↔ MEs with PS-like scale and Sudakov FFs

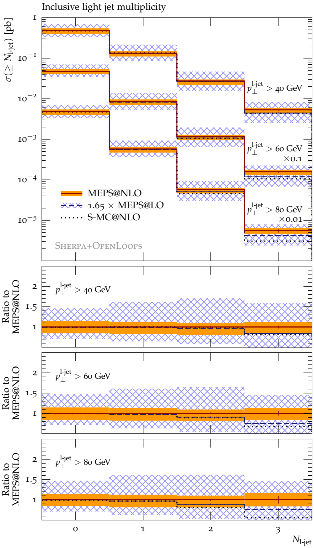
[see also FxFx, UNLOPS, GENEVA, MINLO]

NLO merging for $t\bar{t}+0,1$ jets

- FxFx with MadGraph5/aMC@NLO [Frederix, Frixione '12]
- MEPS@NLO with Sherpa+GoSam [Höche et al '13]

MEPS@NLO for $t\bar{t} + 0, 1, 2$ jets (Sherpa+OpenLoops)

[Höche, Krauss, Maierhöfer, S. P. , Schönherr, Siegert '14]



Consistency with LO merging and NLO+PS

- decent (10–20%) mutual agreement

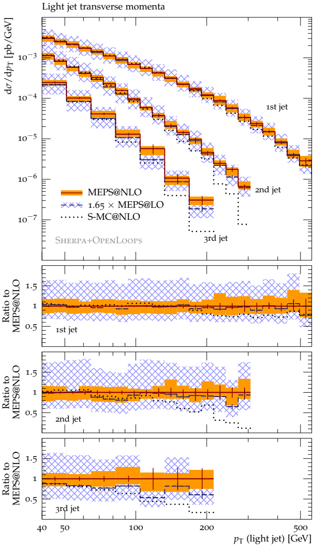
Reduction of μ_R, μ_F, μ_Q variations

$N_{\text{light-jet}} \geq$	0	1	2
LO	48%	65%	80%
NLO	17%	18%	19%

More realistic uncertainties when multijet emission described by matrix elements instead of parton shower!

MEPS@NLO for $t\bar{t} + 0, 1, 2$ jets (SHERPA+OPENLOOPS) II

[Höche, Krauss, Maierhöfer, S. P. , Schönherr, Siegert '14]



Consistency with LO merging and NLO+PS

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Reduction of μ_R, μ_F, μ_Q variations

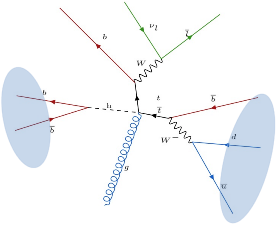
$N_{\text{light-jet}} \geq$	0	1	2
LO	48%	65%	80%
NLO	17%	18%	19%

Differential distributions

- similarly mild scale dependence
- small shape corrections

⇒ **Precision for omnipresent $t\bar{t}$ +multijet background**

Boosted $t\bar{t}H(b\bar{b})$ analysis [Plehn, Salam, Spannowsky '10]



Original strategy

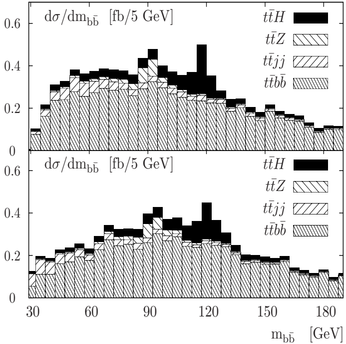
1. Two $p_T > 200$ GeV fat jets ($t \to bjj$ and $H \to b\bar{b}$)
2. identify $t \to bjj$ with top tagger
3. identify $H \to b\bar{b}$ with substructure and 2 b-tags
4. 3rd b-tag for $t \to b\ell\nu$

Significance in $|m_{b\bar{b}} - m_H| < 10$ GeV window

- strong $t\bar{t}+$ jets suppression

⇒ $t\bar{t}b\bar{b}$ dominated background

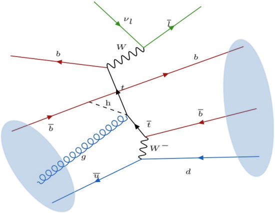
- $S/\sqrt{B} \simeq 4\sigma$ with 100 fb^{-1}
- $S/B = 35\%$ (decent systematics)



New boosted $t\bar{t}H(b\bar{b})$ analysis [Moretti, Petrov, S.P., Spannowsky, ArXiv:1510.08468]

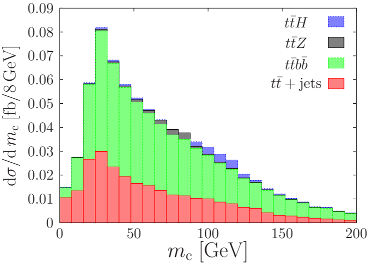
(A) Update of original analysis

- HEPTopTagger [Plehn et al. '10]
- more conservative b-tagging
- LO → NLO simulations of $t\bar{t}b\bar{b}$ and $t\bar{t}+$ multijets



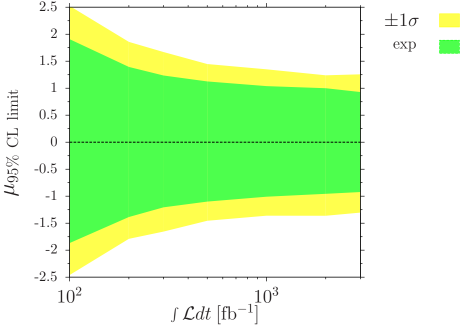
Higher background in Higgs signal region

- $S/B = 13\%$ only!
- only 17% pure $H \to b\bar{b}$ jets
- large $t\bar{t}+$ jets contamination due to sizable (7%) probability that b-quark escapes top tagger



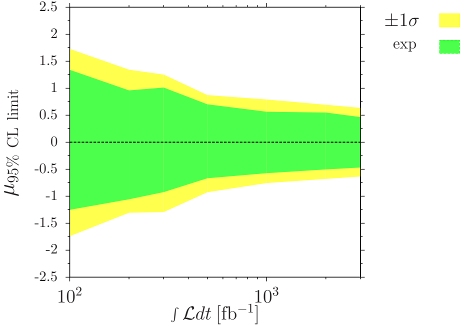
Expected $\sigma_{t\bar{t}H}$ sensitivity [Moretti, Petrov, S.P., Spannowsky, ArXiv:1510.08468]

95% CL limits on $\Delta\sigma_{t\bar{t}H}/\sigma_{t\bar{t}H}$ for $\Delta B/B = 15\%$ (or $\sim 1/\sqrt{\mathcal{L}}$ above 300 fb^{-1})



(A) Update of original analysis

- $|\Delta\sigma/\sigma| \lesssim 100\%$ (50%) at 3 ab^{-1}



(B) Adding regions with one fat jet

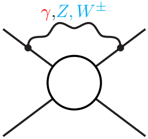
- $|\Delta\sigma/\sigma| \lesssim 50\%$ (25%) at 3 ab^{-1}

$t\bar{t}+X$ background systematics dominates above $\mathcal{O}\left(100\,\mathrm{fb}^{-1}\right)$

Outline

1. Scattering Amplitudes with OpenLoops
2. (N)NLO QCD at parton level
3. Matching and Multi-jet Merging at NLO QCD
4. NLO EW corrections

EW Sudakov logarithms at $Q \sim \mathrm{TeV} \gg M_W$



Soft/collinear logarithms from *virtual* EW bosons

- order $\alpha_w \ln^2(Q^2/M_W^2) \sim 25\% \gg \alpha_S$ in any TeV scale observable!
- analogies with IR QCD effects and EW symmetry breaking subtleties

Universality and factorisation [Denner,S.P. '01]

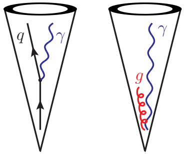
$$\delta\mathcal{M}^{1-\mathrm{loop}}_{\mathrm{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^{\pm}} I^{a}(k)I^{\bar{a}}(l)\ln^2\frac{\hat{s}_{kl}}{M^2} + \gamma^{\mathrm{ew}}(k)\ln\frac{\hat{s}}{M^2}\right\}\mathcal{M}_0$$

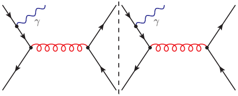
- depend on external EW charges (anomalous dimensions) and kinematic details
- large negative EW corrections exceed NLO QCD uncertainties at $Q^2 \gg M_W^2$

⇒ EW corrections crucial for SM tests and BSM searches at TeV scale

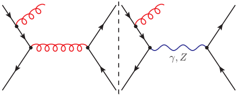
Nontrivial NLO EW features (wrt NLO QCD)

- protons and jets $\supset g, q, \gamma$ (photon–jet separation subtle)
- Subtle QCD–EW interplay, e.g. NLO EW emissions of photons and QCD-partons



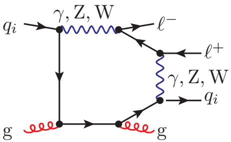


$\alpha \times \alpha_S^n \alpha^m$

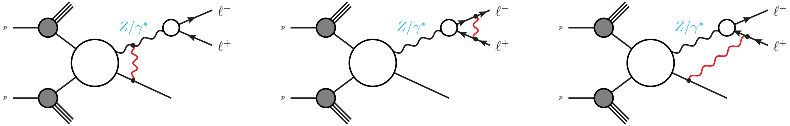


$\alpha_S \times \alpha_S^{n-1} \alpha^{m+1}$

- more involved than NLO QCD: virtual corrections involve massive particles (γ, Z, W, H, b, t) and tend to dominate



- nontrivial $V \to$ lepton decays: final-state interactions and non-fact effects



NLO EW automation

Technical tour de force

- implementation of loop recursion, UV+R_2 CTs, Catani-Seymour subtraction, general $\mathcal{O}\left(\alpha_S^n\alpha^m\right)$ bookkeeping at NLO, complex masses scheme,. . .

First automated tools and multi-particle applications (2014–15)

Tools	first results	
RECOLA+COLLIER	$pp \to \ell^+\ell^- jj$	[arXiv:1411.0916]
OPENLOOPS+ MUNICH/SHERPA	$pp \to W + 1, 2, 3$ jets	[arXiv:1412.5156]
	$pp \to \ell\ell/\ell\nu/\nu\nu + 0, 1, 2$ jets	[arXiv:1511.08692]
MADGRAPH5_AMC@NLO	$pp \to t\bar{t} + V$	[arXiv:1504.03446]
GOSAM+ MADDIPOLE	$pp \to W + 2$ jets	[arXiv:1507.08579]

Full NLO QCD+EW automation [Kallweit,Lindert,Maierhöfer,S.P.,Schönherr '14]

- Loop amplitudes: OPENLOOPS [Cascioli et al. '13] and COLLIER [Denner et al. '14]
- Monte Carlo: MUNICH [Kallweit] or SHERPA [Hoeche et al.]

$pp \to W + 1, 2, 3$ jets at NLO QCD+EW

[Kallweit,Lindert,Maierhöfer,S.P.,Schönherr '14]

Technical motivation

- highest # of jets with on-shell W ⇒ study tool performance for $n_{\text{jets}} = 1, 2, 3$

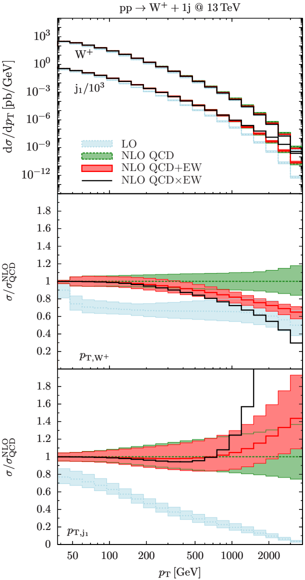
$W + 3$ jets	# QCD trees	# EW trees	# QCD 1-loop	# EW 1-loop
$u_i \bar{d}_i \to W^+ q_j \bar{q}_j g$	12	33	352	1042
$u_i \bar{d}_i \to W^+ q_i \bar{q}_i g$	24	66	704	2084
$u_i \bar{d}_i \to W^+ ggg$	54	-	2043	2616

  - many flavour combinations & crossings ⇒ unconceivable w.o. automation
  - NLO EW more complex but less CPU expensive than NLO EW!

Pheno importance of $pp \to V$+multijets

- precision tests of QCD theory and tools
- crucial for TeV scale searches with leptons+jets+MET
- large EW corrections in Sudakov app. [Chiesa et al. '13] untested in multijet regime
- $V + 1$ jet production pathologic at NLO QCD+EW

NLO QCD+EW corrections to $pp \to W + 1\,\text{jet}$

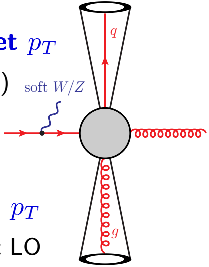


Large NLO corrections at high $p_{T,W}$

- $+100\%$ (QCD) $-$ 20–35% (EW)
- large EW$\times$QCD uncertainty!

Giant NLO corrections at high jet p_T

- $+1000\%$ (QCD) $+$ 10–50% (EW)
- huge uncertainties!

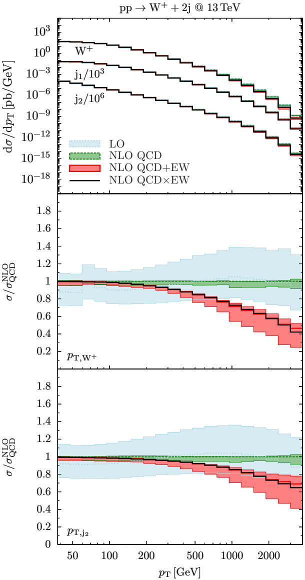


Problem of $pp \to W+$ jet at high p_T

- NLO dominated by $W + 2$ jets at LO

$\Rightarrow$ W+**multijets NLO QCD+EW mandatory!!**

NLO QCD+EW corrections to $pp \to W + 2, 3$ jets



Stable NLO QCD behaviour

- small and almost p_T independent
- $\lesssim 10\%$ scale dependence at NLO

Large negative EW effects (resummation desirable)

- -30–60% at $p_{T,W}$ =1–4 TeV
- -15–25% at $p_{T,j}$ =1–4 TeV

Take home message

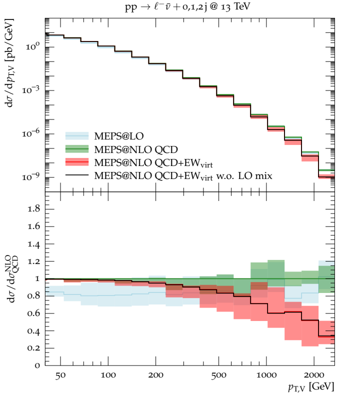
- NLO QCD+EW for $W + 2, 3$ jets under control
- next: NLO QCD+EW merging of $W + 1, 2, 3$ jets

⇒ reliable prediction for *inclusive* $W+$jet production

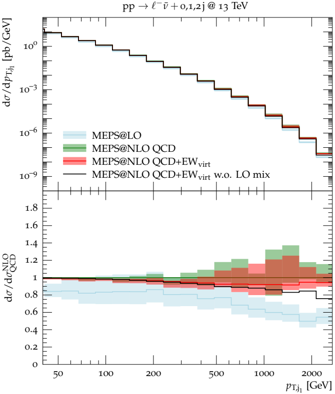
$V + 0, 1, 2$ jets with off-shell $W \to \ell\nu$ and $Z/\gamma^* \to \ell\ell/\nu\nu$

[Kallweit, Lindert, Maierhöfer, S.P., Schönherr, arXiv:1511.08692]

NLO QCD+EW$_{\mathrm{virt}}$ multi-jet merging ⇒ inclusive $V + 1$ jet observables stable!



At $p_{T,V} \sim$ TeV EW effects enhanced through (soft) multi-jet contributions



At $p_{T,\mathrm{jet}} \sim$ TeV EW effects cancel due to (hard) multi-jet contributions

Multi-jet NLO QCD+EW effects crucial at the TeV scale

Concluding remarks

Automation of (N)NLO QCD+EW simulations

- high potential to improve sensitivity of many SM tests and BSM searches at LHC

Powerful tools (but can't provide precision & physics insights if used as black boxes!)

- technically and physically highly involved simulations (many particles, many scales, resonances, process interferences, EW–QCD interplay, . . .)
- precision will require thorough understanding of physics and uncertainties

The NLO problem is solved (?)

- fundamental problems solved in the '70s –'90s (reduction, IR subtraction)
- modern algorithms more automated and widely applicable
- for complex LHC simulations still serious efficiency bottlenecks and lacking physics+precision (chain decays at NLO, IR behaviour for NNLO subtraction, NLO EW matching, interplay of shower with resonances and multi-scale matrix elements,. . .)

⇒ the NLO business is still (and should remain!) work in progress

Backup slides

Structure of NLO Calculations

Born, virtual and real $2 \to n$ contributions ($|\mathcal{M}|^2$, flux factor and PDFs implicit)

$$\sigma_n^{\mathrm{NLO}} \quad = \quad \int \mathrm{d}\Phi_n \mathcal{B}(\Phi_n) + \int \mathrm{d}\Phi_n \mathcal{V}(\Phi_n) + \int \mathrm{d}\Phi_{n+1} \mathcal{R}(\Phi_{n+1})$$

- UV renormalisation ⇒ reduction of μ_R dependence
- soft/collinear cancellations+PDF renormalisation ⇒ reduction of μ_F dependence

Dipole subtraction method [Catani, Seymour '96; Catani, Dittmaier, Seymour, Trocsanyi '99]

- factorisation and universality of IR (sof/collinear) singularities

$$\mathcal{R}(\Phi_{n+1}) \quad \longrightarrow \quad \mathcal{B}(\Phi_n) \otimes \mathcal{S}(\Phi_1) \qquad\qquad \mathcal{I} = \int \mathrm{d}\Phi_1 \mathcal{S}(\Phi_1) \quad \text{analytically}$$

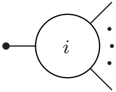
- NLO formula suitable for numerical integration

$$\sigma_n^{\mathrm{NLO}} \quad = \quad \int \mathrm{d}\Phi_n \mathcal{B}(\Phi_n) + \int \mathrm{d}\Phi_n \Big[\mathcal{V}(\Phi_n) + \mathcal{B}(\Phi_n) \otimes \mathcal{I} \Big] + \int \mathrm{d}\Phi_{n+1} \Big[\mathcal{R}(\Phi_{n+1}) - \mathcal{B}(\Phi_n$$

Tree recursion

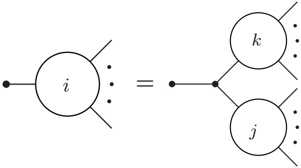
Colour-stripped tree diagrams are built numerically in terms of sub-trees

$w^{\beta}(i) =$



$\beta \leftrightarrow$ off-shell line spin

and recursively merged by attaching vertices and propagators

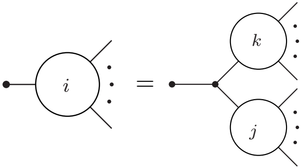


$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

(sub-tree $=$ individual topology with off-shell line $\neq$ off-shell current)

Completely generic and automatic

- **flexible** (only $\mathcal{L}_{\mathrm{int}}$ dependent)
- **fast** (many diagrams share *common sub-trees*)
- **efficient colour bookkeeping** (colour factorisation and algebraic reduction)

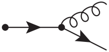


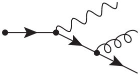
$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

sub-tree $=$ individual topology with off-shell line $\neq$ off-shell current

Example

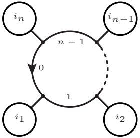
$w_\alpha(1) \quad =$  $= \bar{u}_\alpha(p_1,\lambda_1)$ $\qquad w_\mu(2) =$  $= \epsilon^*_\mu(p_2,\lambda_2)$

$w_\beta(12) \quad =$  $= \dfrac{g_s\,[(\not{p}_{12}+m)\gamma^\mu]_{\alpha\beta}}{p_{12}^2 - m^2}\, w_\alpha(1)\, w_\mu(2)$ $\qquad w_\nu(3) =$  $= \epsilon^*_\nu(p_3,\lambda_3)$

$w_\gamma(123) \quad =$  $= \dfrac{e\,[(\not{p}_{123}+m)\gamma^\nu(1-\gamma_5)]_{\beta\gamma}}{2\sqrt{2}s_{\mathrm{w}}(p_{123}^2 - m^2)}\, w_\beta(12)\, w_\nu(3)$ etc.

Recursion terminates when full set of diagram can be obtained via sub-diagram merging

Colour-stripped loop diagrams



$$= \int \frac{\mathrm{d}^D q\, \mathcal{N}(\mathcal{I}_n;q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1\dots\mu_r}(\mathcal{I}_n) \underbrace{\int \frac{\mathrm{d}^D q\; q^{\mu_1}\dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{\text{tensor integral}}$$

`OpenLoops` computes *symmetrised $\mathcal{N}_{\mu_1\dots\mu_r}(\mathcal{I}_n)$ coefficients*

tensor-rank	R	0	1	2	3	4	5	6	7
# coeff. per diagram	$\binom{R+4}{4}$	1	5	15	35	70	126	210	310

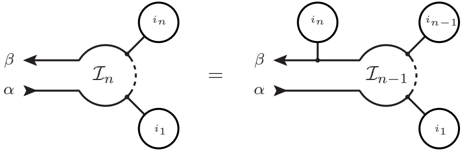
$\underbrace{\hspace{5em}}_{\text{6 particles}}$

and applies **two alternative** methods for the **reduction to scalar integrals:**

(A) **Tensor-integral reduction** [Denner/Dittmaier '05]

(B) **OPP reduction** [Ossola, Papadopolous, Pittau '07] based on numerical evaluation of $\mathcal{N}(\mathcal{I}_n;q) = \sum \mathcal{N}_{\mu_1\dots\mu_r}(\mathcal{I}_n)\, q^{\mu_1}\dots q^{\mu_r}$ at multiple q-values (**strong speed-up!**)

One-loop amplitudes with conventional tree generators



Tree generators for "usual" OPP-input $\mathcal{N}(\mathcal{I}_n; q)$

Cut-open loops can be built by recursively attaching external sub-trees

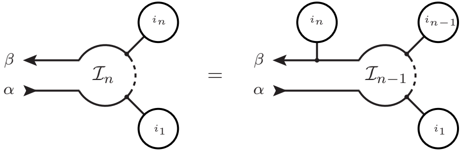
$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) = X^{\beta}_{\gamma\delta}(\mathcal{I}_n, i_n, \mathcal{I}_{n-1})\, \mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1}; q)\, w^{\delta}(i_n)$$

like in **conventional tree generators**

- one-loop automation in `Helac-NLO` (off-shell recursion) and `MadLoop` (diagrams)
- CPU expensive OPP reduction (multiple-q evaluations) since *tree algorithms conceived for fixed momenta*

Nature of loop amplitudes requires loop-momentum *functional* dependence!

OpenLoops recursion [Cascioli, Maierhöfer, S.P '11]



OpenLoops recursion for $\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)$

Handle building blocks of recursion as *polynomials* in the loop momentum q

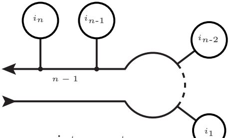
$$\underbrace{\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q)}_{\sum_{r=0}^{n}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\,q^{\mu_1}\ldots q^{\mu_r}} = \underbrace{X^{\beta}_{\gamma\delta}(\mathcal{I}_n,i_n,\mathcal{I}_{n-1})}_{Y^{\beta}_{\gamma\delta}+q^{\nu}\,Z^{\beta}_{\nu;\gamma\delta}}\ \underbrace{\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)}_{\sum_{r=0}^{n-1}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\,q^{\mu_1}\ldots q^{\mu_r}}\ w^{\delta}(i_n)$$

and construct polynomial coefficients with "open loops recursion"

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)=\left[Y^{\beta}_{\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})+Z^{\beta}_{\mu_1;\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\right]\ w^{\delta}(i_n)$$

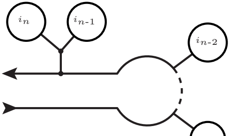
Parent-child relations

Pinch relations



n-point parent

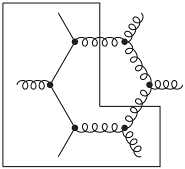
$\mathcal{I}_{n-2}$ open loop
$\Longleftarrow$



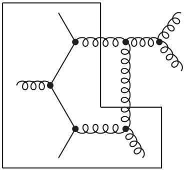
$(n-1)$-point child

n-point loop diagrams constructued from pre-computed $(n-1)$-point child diagrams

Example



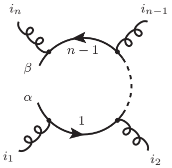
6-point parent



5-point child

Example of OpenLoops recursion: fermion loop

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q) \quad =$$



$$= \; g_s\left[(\not{q} + \not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma} \; \mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q) \; \varepsilon^{*}_{\nu}(p_n,\lambda_n)$$

- n-point open-loop coefficients of rank $r = 0, 1, \ldots, n$

$$\mathcal{N}^{\beta}_{;\alpha}(\mathcal{I}_n) \quad = \quad g_s\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1}) \; \varepsilon^{*}_{\nu}(p_n,\lambda_n)$$

$$\mathcal{N}^{\beta}_{\mu_1;\alpha}(\mathcal{I}_n) \quad = \quad g_s\left\{\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{\mu_1;\alpha}(\mathcal{I}_{n-1}) + \left[\gamma_{\mu_1}\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1})\right] \; \varepsilon^{*}_{\nu}(p_n,$$

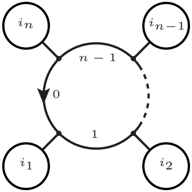
etc.

- initial condition for 0-point rank-0 open loop

$$\mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_0) \quad = \quad \delta^{\gamma}_{\alpha}$$

- rank, i.e. complexity, increases with n ⇒ symmetrised $\mu_1 \ldots \mu_r$ components!
- bookkeeping of tensor components fully automated

R_2 rational terms

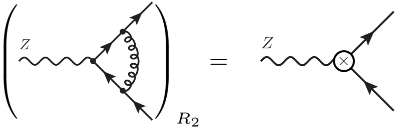


$$= \sum_{r=0}^{R} \underbrace{\mathcal{N}_{\mu_1\ldots\mu_r}(\mathcal{I}_n)}_{\text{in } D=4} \int \frac{\mathrm{d}^D q \; q^{\mu_1}\ldots q^{\mu_r}}{D_0 D_1 \ldots D_{n-1}}$$

Extra rational terms from $3 < \mu_1, \ldots, \mu_r \leq D-1$ coefficient components

$$R_2 = \sum_{\mu_1\ldots\mu_r=0}^{D-1} \mathcal{N}_{\mu_1\ldots\mu_r}\Big|_{D=4-2\varepsilon} T_{\mathrm{UV}}^{\mu_1\ldots\mu_r} \quad - \quad \sum_{\mu_1\ldots\mu_r=0}^{3} \mathcal{N}_{\mu_1\ldots\mu_r}\Big|_{D=4} T_{\mathrm{UV}}^{\mu_1\ldots\mu_r}$$

From catalogue of 2-, 3- and 4-point 1PI diagrams (depends only on model)



$$= -\frac{g_s^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\gamma^\mu(g_{\mathrm{V}}^Z - g_{\mathrm{A}}^Z\gamma_5) \qquad \text{etc.}$$

[Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09–'11; Shao, Zhang, Chao '11]

Flexibility and automation of OpenLoops generator

Process	size [MB]	t_{code} [s]
$u\bar{u} \to t\bar{t}$	0.1	2.2
$u\bar{u} \to W^+W^-$	0.1	7.2
$u\bar{d} \to W^+g$	0.1	4.2
$gg \to t\bar{t}$	0.2	5.4
$u\bar{u} \to t\bar{t}g$	0.4	12.8
$u\bar{u} \to W^+W^-g$	0.4	39.8
$u\bar{d} \to W^+gg$	0.5	22.9
$gg \to t\bar{t}g$	1.2	52.9
$u\bar{u} \to t\bar{t}gg$	3.6 $(200)^*$	236 $(\sim 10^6)^*$
$u\bar{u} \to W^+W^-gg$	2.5 $(1000)^*$	381.7 $(\sim 10^6)^*$
$u\bar{d} \to W^+ggg$	4.2	366.2
$gg \to t\bar{t}gg$	16.0	3005

$^*pp \to t\bar{t}b\bar{b}$ & $WWb\bar{b}$ (Bredenstein, Denner, Dittmaier, Kallweit, S.P. '09–'11)

Fast code generation/compilation

- few seconds to minutes
- $\mathcal{O}(10^3)$ speed-up in $2 \to 4$

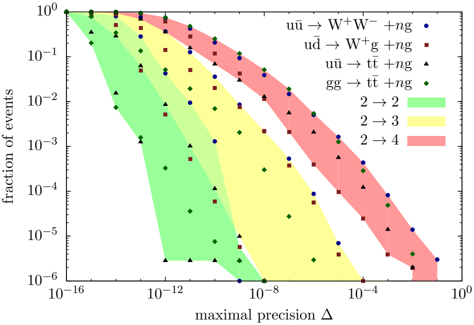
Compact code

- 100 kB to few MB object files
- $\mathcal{O}(10^2$–$10^3)$ compression in $2 \to 4$

large-scale applicability!

Numerical stability with **tensor reduction** in double precision

Stability Δ in samples of 10^6 points ($\sqrt{\hat{s}} = 1\,\mathrm{TeV}$, $p_\mathrm{T} > 50\,\mathrm{GeV}$, $\Delta R_{ij} > 0.5$)



Average number of correct digits

- 11-15

Cross section accuracy

- depends on tails
- stability issues grow with n_part

$2 \to 4$ processes very stable

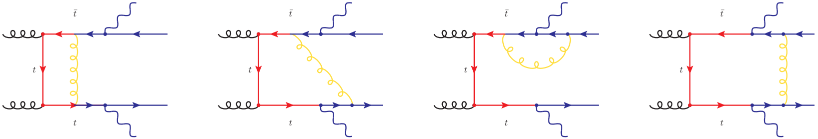
- $\lesssim 0.01\%$ prob. that $\Delta_S < 10^{-3}$
- thanks to Gram-determinant expansions in `Collier`!

Real-life NLO applications

- $\mathcal{O}(10^{-4})$ unstable points in most challenging $2 \to 4$ calculations considered so far
- can be monitored and safely suppressed thanks to **online instability-trigger**

Finite-width effects vs NWA [Cascioli, Maierhöfer,Kallweit, S.P. '13]

Examples of factorisable and non-factorisable 1-loop diagrams

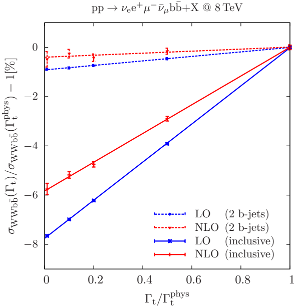


Separation of narrow- and finite-top-width parts

- via numerical $\Gamma_t \to 0$ extrapolation

$$\lim_{\xi_t \to 0} \mathrm{d}\sigma_{W^+W^-b\bar{b}}(\xi_t \Gamma_t) = \xi_t^{-2} \left[\mathrm{d}\sigma_{t\bar{t}} + \xi_t \, \mathrm{d}\sigma_{\mathrm{FtW}}\right]$$

⇒ permille-level convergence demonstrates nontrivial cancellation of soft-gluon $\ln(\Gamma_t/m_t)$ singularities



$\sigma_{t\bar{t}}$ = on-shell $t\bar{t}$ production × decay

σ_{FtW} = $\mathcal{O}(\Gamma_t/m_t)$ effects dominated by Wt + interference + off-shell $t\bar{t}$ +...

= 6–8% of $\sigma_{\text{inclusive}}$ (cf. sub-percent effect with $t\bar{t}$ cuts!)

$\sigma_{W^+W^-b\bar{b}}$ in Jet Bins I [Cascioli, Maierhöfer,Kallweit, S.P. '13]

Generic-Jet Bins: complete cross section and finite-top-width (FtW) effects

	μ_0	σ[fb]	σ_0[fb]	σ_1[fb]	σ_{2+}[fb]
LO	μ_{WWbb}	$1232^{+34\%}_{-24\%}$	$37^{+38\%}_{-25\%}$	$367^{+36\%}_{-24\%}$	$828^{+33\%}_{-23\%}$
NLO	μ_{WWbb}	$1777^{+10\%}_{-12\%}$	$41^{+3\%}_{-8\%}$	$377^{+1\%}_{-6\%}$	$1359^{+14\%}_{-14\%}$
K	μ_{WWbb}	1.44	1.09	1.03	1.64
LO	m_t	$1317^{+35\%}_{-24\%}$	$35^{+37\%}_{-25\%}$	$373^{+36\%}_{-24\%}$	$909^{+35\%}_{-24\%}$
NLO	m_t	$1817^{+8\%}_{-11\%}$	$40^{+4\%}_{-8\%}$	$372^{+1\%}_{-8\%}$	$1405^{+13\%}_{-13\%}$
K	m_t	1.38	1.14	1.00	1.55
	μ_0	σ^{FtW}[fb]	σ_0^{FtW}[fb]	σ_1^{FtW}[fb]	$\sigma_{2+}^{\mathrm{FtW}}$[fb]
LO	μ_{WWbb}	$91^{+41\%}_{-27\%}$	$13^{+42\%}_{-27\%}$	$71^{+40\%}_{-27\%}$	$7^{+45\%}_{-29\%}$
NLO	μ_{WWbb}	$107^{+6\%}_{-11\%}$	$13^{+1\%}_{-7\%}$	$61^{+2\%}_{-16\%}$	$33^{+51\%}_{-31\%}$
K	μ_{WWbb}	1.18	0.99	0.86	4.70
LO	m_t	$63^{+36\%}_{-25\%}$	$8^{+36\%}_{-25\%}$	$49^{+36\%}_{-24\%}$	$6^{+46\%}_{-29\%}$
NLO	m_t	$100^{+17\%}_{-16\%}$	$13^{+14\%}_{-14\%}$	$65^{+9\%}_{-12\%}$	$23^{+42\%}_{-28\%}$
K	m_t	1.58	1.47	1.32	3.89

$\sigma_{W^+W^-b\bar{b}}$ in Jet Bins II [Cascioli, Maierhöfer,Kallweit, S.P. '13]

b-Jet Bins: complete cross section and finite-top-width (FtW) effects

	μ_0	$\sigma\,[\mathrm{fb}]$	$\sigma_0\,[\mathrm{fb}]$	$\sigma_1\,[\mathrm{fb}]$	$\sigma_{2+}\,[\mathrm{fb}]$
LO	μ_{WWbb}	$1232^{+34\%}_{-24\%}$	$37^{+38\%}_{-25\%}$	$367^{+36\%}_{-24\%}$	$828^{+33\%}_{-23\%}$
NLO	μ_{WWbb}	$1777^{+10\%}_{-12\%}$	$65^{+20\%}_{-17\%}$	$571^{+14\%}_{-14\%}$	$1140^{+7\%}_{-10\%}$
K	μ_{WWbb}	1.44	1.73	1.56	1.38
LO	m_t	$1317^{+35\%}_{-24\%}$	$35^{+37\%}_{-25\%}$	$373^{+36\%}_{-24\%}$	$909^{+35\%}_{-24\%}$
NLO	m_t	$1817^{+8\%}_{-11\%}$	$63^{+20\%}_{-17\%}$	$584^{+14\%}_{-14\%}$	$1170^{+5\%}_{-9\%}$
K	m_t	1.38	1.80	1.56	1.29
	μ_0	$\sigma^{\mathrm{FtW}}\,[\mathrm{fb}]$	$\sigma_0^{\mathrm{FtW}}\,[\mathrm{fb}]$	$\sigma_1^{\mathrm{FtW}}\,[\mathrm{fb}]$	$\sigma_{2+}^{\mathrm{FtW}}\,[\mathrm{fb}]$
LO	μ_{WWbb}	$91^{+41\%}_{-27\%}$	$13^{+42\%}_{-27\%}$	$71^{+40\%}_{-27\%}$	$7^{+45\%}_{-29\%}$
NLO	μ_{WWbb}	$107^{+6\%}_{-11\%}$	$20^{+18\%}_{-17\%}$	$82^{+4\%}_{-10\%}$	$5^{+2\%}_{-10\%}$
K	μ_{WWbb}	1.18	1.49	1.16	0.77
LO	m_t	$63^{+36\%}_{-25\%}$	$8^{+36\%}_{-25\%}$	$49^{+36\%}_{-24\%}$	$6^{+46\%}_{-29\%}$
NLO	m_t	$100^{+17\%}_{-16\%}$	$16^{+22\%}_{-18\%}$	$77^{+16\%}_{-15\%}$	$6^{+12\%}_{-16\%}$
K	m_t	1.58	1.89	1.58	1.10

$WWb\bar{b}$ cross section in jet bins [Cascioli, Maierhöfer,Kallweit, S.P. '13]

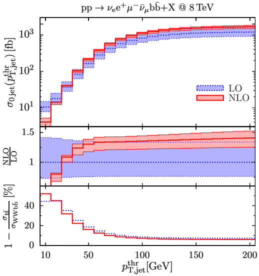
Jet-Veto and Binning Effects

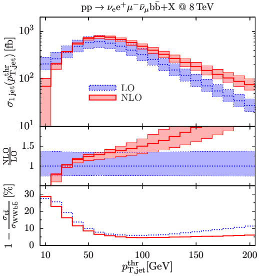
0-jet bin vs p_{T}-veto

- smooth inclusive limit at large p_{T} and very strong p_{T} sensitivity below $50\,\mathrm{GeV}$:
  - FtW effects increase up to 50%
  - K-factor falls very fast
- at low p_{T} IR singularity calls for NLO+PS matching
- typical veto $p_{\mathrm{T}} \sim 30\,\mathrm{GeV}$ yields 98% suppression and still decent NLO stability ($K \sim 1$)

1-jet bin vs p_{T} threshold

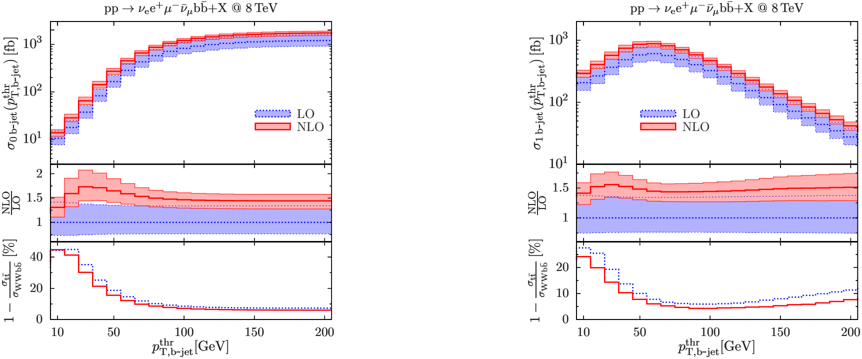
- low p_{T} behaviour driven by veto on 2nd jet and analogous to 0-jet case
- high p_{T} region driven by 1st jet and NLO radiation dominates over b-jets from $W^+W^-b\bar{b}$





$WWbb$ cross section in b-jet bins

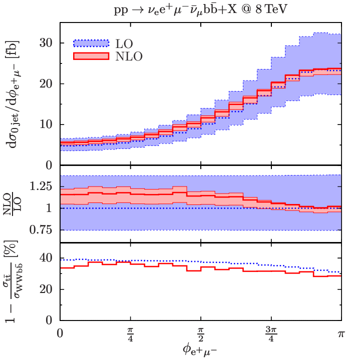
[Cascioli, Maierhöfer,Kallweit, S.P. '13

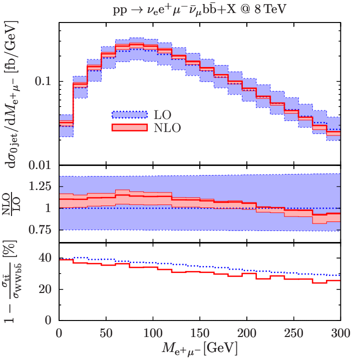


- NLO radiation doesn't change b-jet multiplicity ⇒ rather stable K-factor and uncertainties
- single-top and off-shell effects still enhanced at small b-jet p_T

In general: nontrivial interplay of NLO and off-shell/single-top effects

Top background to 0-jet bin of $H \to W^+W^-$ analysis





NLO distributions in key variables for $H \to W^+W^-$ measurement

- better than 10% accuracy and stable shape
- $\mathcal{O}(\Gamma_t/M_t)$ contributions around 25–40%

$\Rightarrow$ requires full $WWb\bar{b}$ NLO simulation!

NLO+PS for $W^+W^-b\bar{b}$ (conceptual and technical issues)

Need of NLO+PS matching

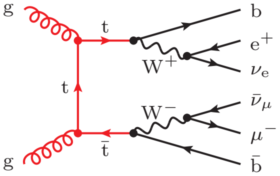
- NLO precision in the context of fully exclusive simulations for experimental analysis
- describes higher-order resummation effects in the shower aproximation and, possibly, related uncertainties (both should be small!)

NLO+PS matching for a process with intermediate resonances

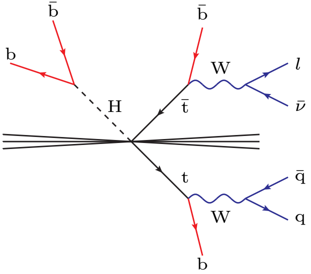
- matrix elements provide NLO accurate description of "Breit-Wigner" top-distributions (with off-shell effects,. . .)
- crucial for precision observables sensitive to shape of top resonance (kinematic m_t measurements!), edges of on-shell $t\bar{t}$ phase space, single-top Wt contributions, . . .

Nontrivial conceptual and technical (open) issue

- recoil of standard shower emissions off $W^+W^-b\bar{b}$ final states induce arbitrary kinematic distortions of m_{Wb}
- potentially very strong distortions of Breit-Wigner shape (formally of order $\alpha_S^2 m_t/\Gamma_t \sim 1$!)
- requires yet unknown technique for matching PS to off-shell resonances at NLO



Theory priorities in $t\bar{t}H$ searches



Key priority is precision for backgrounds

- various multi-particle processes: $t\bar{t}+$ jets, $t\bar{t}V+$ jets, $t\bar{t}\gamma\gamma$, $VV+$ jets
- NLO automation crucial but $2 \to 4$ CPU intensive

NLO matching & merging crucial

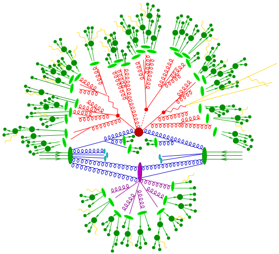
- *various* new methods (FxFx, MEPS@NLO, MINLO, UNLOPS, GENEVA, MINLO,. . .)
- *various* automated tools support NLO precision for signal and most backgrounds: MG5_aMC@NLO, Sherpa+OpenLoops/GoSam, Powheg/Powhel

Theory uncertainty estimates nontrivial

- still limited experience in NLO matching+merging framework
- sophisticated analyses (profile likelihood, MEM, background reweighting, . . .)

Parton Showers in a Nutshell

High-energy n-parton final state $\Rightarrow$ realistic multi-parton/hadron event



Chain of **ordered** emissions $\mu_Q > t_1 > t_2 > \cdots > t_{\mathrm{IR}}$

$$\mathrm{d}\sigma_n \simeq \mathrm{d}\sigma_{n-1}\frac{\alpha_s}{2\pi}\frac{\mathrm{d}t_n}{t_n}\mathrm{d}z\,\mathrm{d}\phi P(z,\phi) \qquad \frac{\mathrm{d}t}{t} = \frac{\mathrm{d}k_{\mathrm{T}}^2}{k_{\mathrm{T}}^2}$$

Sudakov FF resums no-emission probability ($\mathcal{V}$-like term)

$$\Delta(\mu_Q^2, t_0) = \exp\left\{-\frac{\alpha_s}{2\pi}\int_{t_{\mathrm{IR}}}^{\mu_Q^2}\frac{\mathrm{d}t}{t}\int \mathrm{d}z\,\mathrm{d}\phi P(z,\phi),\right\}$$

resummation scale $\mu_Q^2 \sim \hat{s}$ and IR cut-off $t_{\mathrm{IR}} \sim 1\,\mathrm{GeV}$

First emission master formula

$$\sigma_n^{\mathrm{LO+PS}} = \int \mathrm{d}\Phi_n \mathcal{B}(\Phi_n)\left\{\Delta(\mu_Q^2, t_{\mathrm{IR}}) + \int_{t_0}^{\mu_Q^2}\frac{\alpha_s}{2\pi}\frac{\mathrm{d}t_1}{t_1}\int \mathrm{d}z\,\mathrm{d}\phi P(z,\phi)\Delta(\mu_Q^2, t_1)\right\}$$

- unitarity leaves inclusive LO normalisation and uncertainty unchanged
- emissions iterated with $\mu_Q^2 \to t_1 \to t_2 \to \ldots$
- resummation of large logarithms in exclusive oservables (jet vetoes, etc.)

Sherpa Formulation of MC@NLO Matching

Matching NLO calculations to parton showers

- NLO accuracy + shower resummation w.o. double counting of 1^{st} emission
- achieved in MC@NLO [Frixione, Webber '02] by using shower kernels as NLO subtraction terms

Sherpa shower ideally suited: dipole subtraction terms as splitting kernels

$$\frac{\alpha_s}{2\pi}\,\frac{\mathrm{d}t}{t}\,\mathrm{d}z\,\mathrm{d}\phi P(z,\phi) \longrightarrow \theta(\mu_Q - t)\mathcal{S}(\Phi_1)\mathrm{d}\Phi_1 \qquad\qquad t = t(\Phi_1)$$

Sherpa's MC@NLO master formula [Höche, Krauss, Schönherr, Siegert '11]

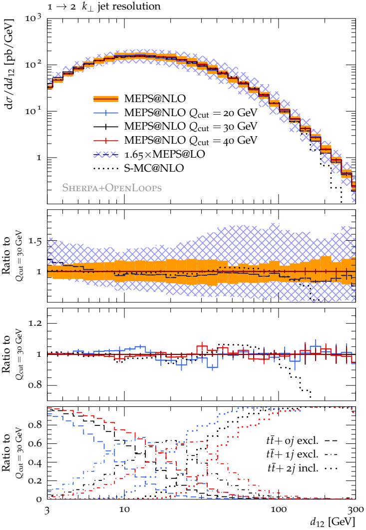
$$\sigma_n^{\mathrm{MC@NLO}} = \int \mathrm{d}\Phi_n \Big[\mathcal{B}(\Phi_n) + \mathcal{V}(\Phi_n) + \mathcal{B}(\Phi_n) \otimes \mathcal{I}\Big] \Big\{\Delta(\mu_Q^2, t_{\mathrm{IR}}) + \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathcal{S}(\Phi_1)\,\Delta(\mu_Q^2, t)\Big\}$$

$$+ \quad \int \mathrm{d}\Phi_{n+1} \Big[\mathcal{R}(\Phi_{n+1}) - \mathcal{B}(\Phi_n) \otimes \mathcal{S}(\Phi_1)\Big]$$

- shower resummation effectively acts starting from $\mathcal{O}(\alpha_s^2)$, and iterated emissions yield fully realistic events
- inclusive observables with n $(n+1)$ particles preserve NLO (LO) accuracy

MEPS@NLO for $t\bar{t} + 0, 1, 2$ jets (SHERPA+OPENLOOPS)

[Höche, Krauss, Maierhöfer, S. P. , Schönherr, Siegert '14]



Small merging scale choice

- $Q_{\rm cut} = 30$ GeV such that exp. resolved jets are described by MEs

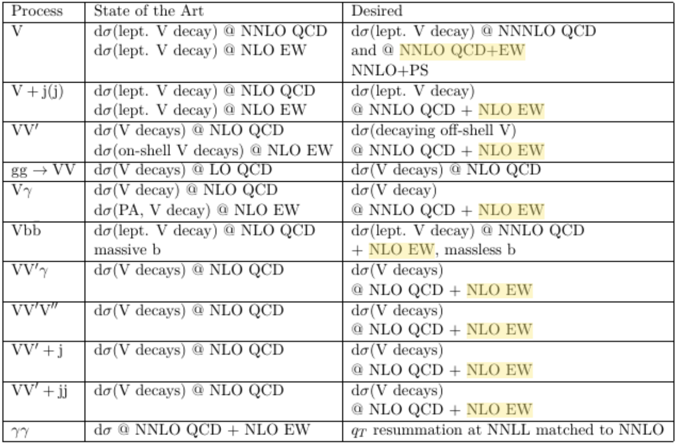
Merging scale uncertainty

- $Q_{\rm cut} = 30 \pm 10$ GeV

$\Rightarrow$ $\ll 10\%$ dependence

does not spoil $t\bar{t} + 0, 1, 2$ jets NLO precision

Les Houches priority list for $pp \to V(V')+$ jets



Process	State of the Art	Desired
V	dσ(lept. V decay) @ NNLO QCD dσ(lept. V decay) @ NLO EW	dσ(lept. V decay) @ NNNLO QCD and @ NNLO QCD+EW NNLO+PS
V + j(j)	dσ(lept. V decay) @ NLO QCD dσ(lept. V decay) @ NLO EW	dσ(lept. V decay) @ NNLO QCD + NLO EW
VV′	dσ(V decays) @ NLO QCD dσ(on-shell V decays) @ NLO EW	dσ(decaying off-shell V) @ NNLO QCD + NLO EW
gg → VV	dσ(V decays) @ LO QCD	dσ(V decays) @ NLO QCD
Vγ	dσ(V decay) @ NLO QCD dσ(PA, V decay) @ NLO EW	dσ(V decay) @ NNLO QCD + NLO EW
Vbb	dσ(lept. V decay) @ NLO QCD massive b	dσ(lept. V decay) @ NNLO QCD + NLO EW, massless b
VV′γ	dσ(V decays) @ NLO QCD	dσ(V decays) @ NLO QCD + NLO EW
VV′V″	dσ(V decays) @ NLO QCD	dσ(V decays) @ NLO QCD + NLO EW
VV′ + j	dσ(V decays) @ NLO QCD	dσ(V decays) @ NLO QCD + NLO EW
VV′ + jj	dσ(V decays) @ NLO QCD	dσ(V decays) @ NLO QCD + NLO EW
$\gamma\gamma$	dσ @ NNLO QCD + NLO EW	q_T resummation at NNLL matched to NNLO

Table 3: Wishlist part 3 – Electroweak Gauge Bosons (V = W, Z)

EW Sudakov effects at 1 TeV

Typical size at 1-loop

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{LL}} \simeq -\frac{4\alpha}{\pi s_{\mathrm{w}}^2}\ln^2\left(\frac{1\,\mathrm{TeV}}{M_W}\right) \simeq -26.4\% \qquad \left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{NLL}} \simeq +\frac{6\alpha}{\pi s_{\mathrm{w}}^2}\ln\left(\frac{1\,\mathrm{TeV}}{M_W}\right) \simeq +15.6\%$$

Typical size at 2-loops [Bauer, Becher, Ciafaloni, Comelli, Denner, Fadin, Jantzen, Kühn, Lipatov, Manohar Martin, Melles, Penin, S.P., Smirnov, . . .]

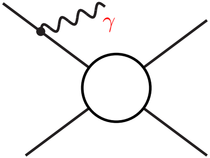
$$\left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\mathrm{LL}} \simeq +\frac{8\alpha^2}{\pi^2 s_{\mathrm{w}}^4}\ln^4\left(\frac{1\,\mathrm{TeV}}{M_W}\right) \simeq 3.5\% \qquad \left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\mathrm{NLL}} \simeq -\frac{24\alpha^2}{\pi^2 s_{\mathrm{w}}^4}\ln^3\left(\frac{1\,\mathrm{TeV}}{M_W}\right) \simeq -4.1\%$$

Bottom line

- ⇒ Large negative EW corrections exceed NLO QCD uncertainties at $Q^2 \gg M_W^2$
- ⇒ systematic inclusion of EW effects important for any search at the TeV scale

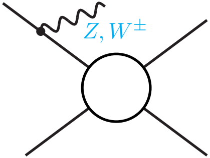
Electroweak bremsstrahlung

Real photon emission



- mandatory since soft/collinear γ unresolved
- complete cancellation of QED singularities

Real Z, W emission [Ciafaloni,Comelli]



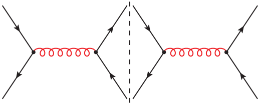
- **inclusive emission**: only partial $\ln(\hat{s}/M_W)$ cancellation

↔ free SU(2) charges, collinear IS logs, kinematic $M_{Z,W}$ effects

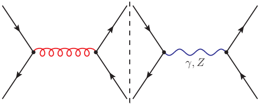
- **typical experimental cuts**: modest $\ln(\hat{s}/M_W)$ cancellation (strongly dependent on process and analysis)
- **bottom line**: needs to be considered but can be regarded as separate (tree-level) process

Nontrivial QCD-EW interferences for $q\bar{q} \rightarrow q\bar{q} + \ldots$

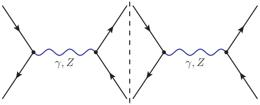
LO contributions of order α^m, α^{m+1},. . . , α^{m+k}



QCD

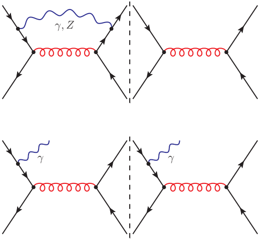


mixed

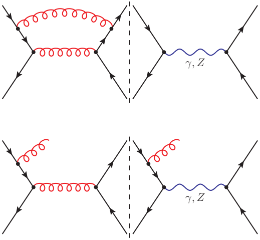


EW

NLO EW corrections of order α^{m+1} (nontrivial bookkeeping)



“standard’



“mixed”

$\ldots + \mathcal{O}\left(\alpha^{m+k+1}\right)$

⇒ EW corrections can involve emissions of photons and QCD-partons

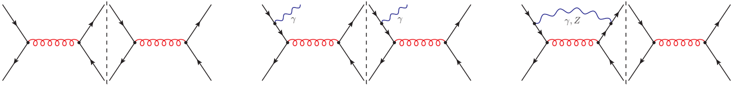
Nontrivial QCD-EW interplay in $pp \to X + \geq 2\,\text{jets}$

$q\bar{q} \to q\bar{q} + \ldots$ cross sections receive various Born contributions

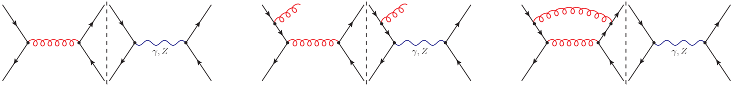
$$\underbrace{\mathcal{O}\left(\alpha_S^n \alpha^m\right)}_{\text{“QCD”}} + \underbrace{\mathcal{O}\left(\alpha_S^{n-1} \alpha^{m+1}\right)}_{\text{“EW}-\text{QCD interf.”}} + \cdots + \underbrace{\mathcal{O}\left(\alpha_S^{n-k} \alpha^{m+k}\right)}_{\text{“EW”}}$$

$\mathcal{O}\left(\alpha_S^n \alpha^{m+1}\right)$ NLO EW corrections to leading QCD Born, e.g. in $q\bar{q} \to q\bar{q}$

- EW corrections × QCD Born



- QCD corrections × EW–QCD interference

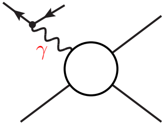


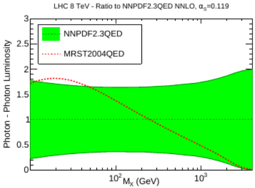
In practice

- only full $\mathcal{O}\left(\alpha_S^n \alpha^{m+1}\right)$ IR finite ⇒ nontrivial bookkeeping (automated)
- $\mathcal{O}\left(\alpha\right)$ corrections can involve emissions of photons and QCD-partons
- protons and jets $\supset g, q, \gamma$

Photons in the initial state

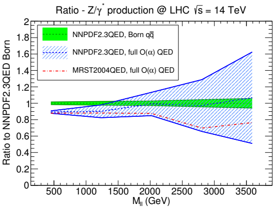
Factorisation of $q \to q\gamma$ singularities $\Rightarrow$ QED PDFs with photon

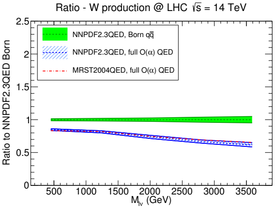


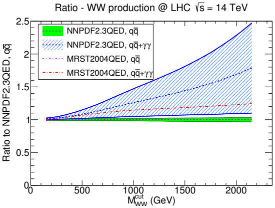


- LO QED evolution
- γ-fit to DIS+DY data (NNPDF)
- $\mathcal{O}\left(50\%\right)$ γ-uncertainty

Very large γ-induced effects with $\mathcal{O}\left(100\%\right)$ uncertainty in TeV region







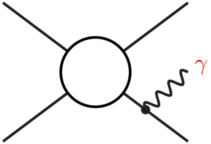
Wanted: - NLO QED PDFs

- new fit of γ-PDF with accurate high-energy data & theory [Boughezal et al.'14]

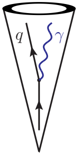
Photons (and jets) in the final state

Cancellation of FS photon singularities

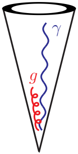
- requires IR subtraction method [Catani,Dittmaier,Seymour, Trocsanyi; Frixione, Kunszt, Signer]
- photon emission off quarks renders IR safe jet definition nontrivial at NLO EW



Option A: Democratic jet-algorithm approach (jets $\equiv$ photons)



collinear $q \to q\gamma$ singularities cancelled clustering q, g, γ on same footing



soft gluon singularities $\leftrightarrow$ hard photons inside jets: cancelled in jet-production (NLO EW) + γ-production (NLO QCD)

Option B: Separation of jets from photons ($E_\gamma/E_{\text{jet}} < z_{\text{thr}}$ inside jets)

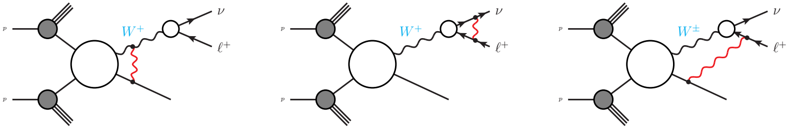
- $q \to q\gamma$ singularity must be absorbed into fragmentation function

$\Rightarrow$ requires careful theoretical *and experimental* treatment of photon–jet interplay

Decays of Z/W bosons

Leptonic Z and W decays are notrivial at NLO EW (in contrast to NLO QCD)

- NLO EW corrections to production×resonance×decay + non-fact corrections



Option A: complex mass scheme [Denner, Dittmaier]

- exact NLO description (always desirable)
- high complexity corresponding to total number of particles *after* decays

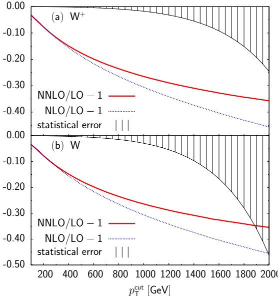
Option B: narrow-width approximation (production×decay)

- simpler but applicability to V+multijets limited to certain $\mathcal{O}\left(\alpha_S^n \alpha^{m+1}\right)$ (see later)
- captures all large $\ln(\hat{s}/M_W^2)$ effects (present only in production sub-process)
- typical uncertainty $\lesssim$ 1–3% (apart form $\gamma^*/Z^* \to \ell^+\ell^-$ at small $m_{\ell\ell}$)

EW corrections to $pp \to V + 1\,$jet

Very large EW corrections to $pp \to Z/W + 1\,$jet

- NLO (electro)weak [Maina, Ross, Moretti '04;Kühn, Kulesza, S.P.,Schulze '04–'07]
- EW Sudakov logs beyond NLO [Kühn, Kulesza, S.P.,Schulze '04–'07; Becher, Garcia i Tormo '13]
- NLO QCD+EW with off-shell Z/W decays [Denner,Dittmaier,Kasprzik,Muck '09–'11]



Complexity and efficiency of $pp \to W^+ + n$ jets $(n \le 3)$

	$pp \to W + n$ jets @LO				$pp \to W + n$ jets @NLO				
	$\alpha_s^n \alpha$	$\alpha_s^{n-1}\alpha^2$	$\alpha_s^{n-2}\alpha^3$	$\alpha_s^{n-3}\alpha^4$	$\alpha_s^{n+1}\alpha$	$\alpha_s^n\alpha^2$	$\alpha_s^{n-1}\alpha^3$	$\alpha_s^{n-2}\alpha^4$	$\alpha_s^{n-3}\alpha^5$
$u_i \bar{d}_i \to W + ng$	×	-	-	-	×	×	-	-	-
$u_i \bar{d}_i \to W + q\bar{q} + (n-2)g$	×	×	×	-	×	×	(×)	(×)	-
$\gamma u_i \to d_i W + (n-1)g$	-	×	-	-	-	-	-	-	-
$\gamma u_i \to d_i W + q\bar{q} + (n-3)g$	-	×	×	×	-	-	-	-	-
$\gamma\gamma \to \bar{u}_i d_i W + (n-2)g$	-	-	×	-	-	-	-	-	-
$u_i \bar{d}_i \to W + (n+1)g$	-	-	-	-	×	-	-	-	-
$u_i \bar{d}_i \to W + q\bar{q} + (n-1)g$	-	-	-	-	×	×	(×)	-	-
$u_i \bar{d}_i \to W + q\bar{q}q'\bar{q}' + (n-3)g$	-	-	-	-	×	×	(×)	(×)	(×)
$u_i \bar{d}_i \to W + ng + \gamma$	-	-	-	-	-	×	-	-	-
$u_i \bar{d}_i \to W + q\bar{q} + (n-2)g + \gamma$	-	-	-	-	-	×	(×)	(×)	(×)

× (×) = (not) included in 1412.5156

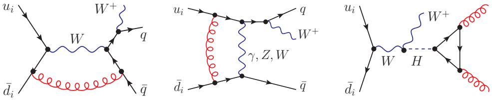
Ingredients of order $\alpha_S^{n+1}\alpha + \alpha_S^n\alpha^2$ calculation

- very many crossings and flavour combinations $(u_i, d_i, q, q' \in \{u, d, c, s, b\})$
- 2000–3000 virtual EW diagrams/channel: more complex than QCD but faster

"Pseudo resonances" in QCD×EW interferences

(IR EW singularities tricky. . .)

- external W stable $(\Gamma_W = 0)$ but small $\Gamma_{\text{reg}} \to 0$ for s-channel t, W, Z, H prop.



$$\times \text{ QCD Born} \Rightarrow \frac{Q^2 - M^2}{(Q^2 - M^2)^2 + \Gamma_{\text{reg}}^2 M^2}$$

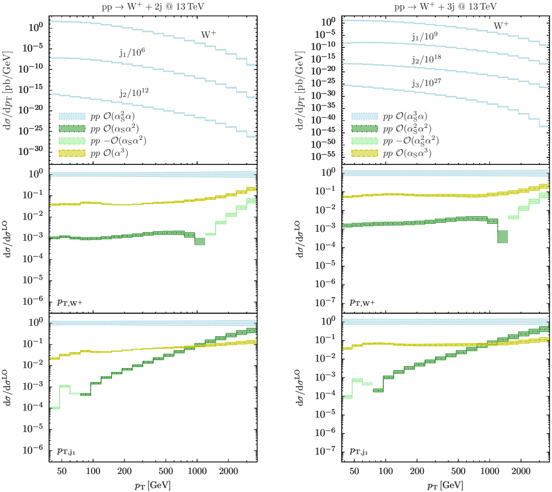
NLO QCD vs EW complexity in $pp \to W + 1, 2, 3$ jets

Number of diagrams in $pp \to W + 1, 2, 3$ jets (in parenthesis: $q = u_i, d_i$ case)

Channel	QCD trees	EW trees	QCD 1-loop	EW 1-loop
$u_i\bar{d}_i \to W^+g$	2	-	11	32
$u_i\bar{d}_i \to W^+q\bar{q}$	2 (4)	7 (14)	33 (66)	105 (210)
$u_i\bar{d}_i \to W^+gg$	8	-	150	266
$u_i\bar{d}_i \to W^+q\bar{q}g$	12 (24)	33 (66)	352 (704)	1042 (2084)
$u_i\bar{d}_i \to W^+ggg$	54	-	2043	2616

- moderate growth of complexity wrt NLO QCD (up to 3× more loop diagrams)
- 1-loop QCD and EW similarly fast ⇒ 0.1% stat precision for $W + 1, 2, 3$ jets at NLO QCD+EW costs 13,210,6300 CPU h (dominated by NLO QCD!)

LO EW–QCD interplay in $pp \to W^+ + 2, 3$ jets at 13 TeV



“QCD cuts” throughout

- $p_T > 30\,\text{GeV}$, $\eta < 4.5$

⇒ QCD dominates

EW contributions

(WV, VBF, single-t)

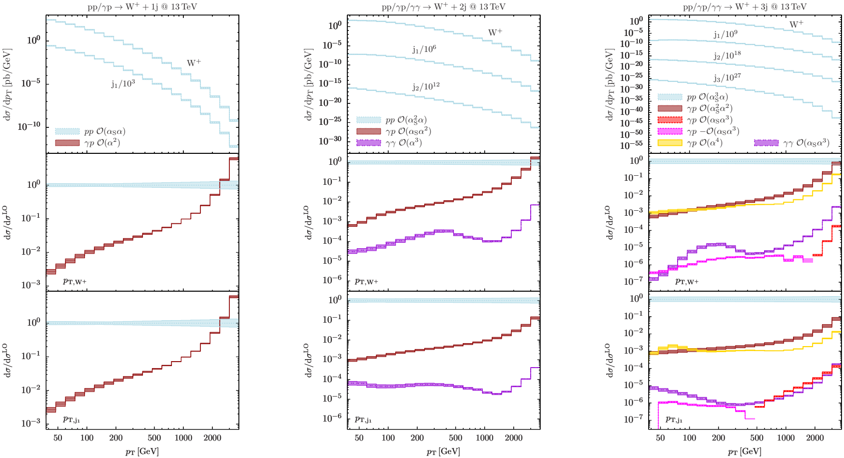
- 3–6% in σ_{int}
- 10–20% at 1–4 TeV

EW–QCD interference

- $\mathcal{O}\left(10^{-3}\right)$ in σ_{int}
- 10–50% at 1–4 TeV (dominant!)

⇒ **nontrivial QCD–EW interplay at the TeV scale** (with V+jets “QCD cuts”)

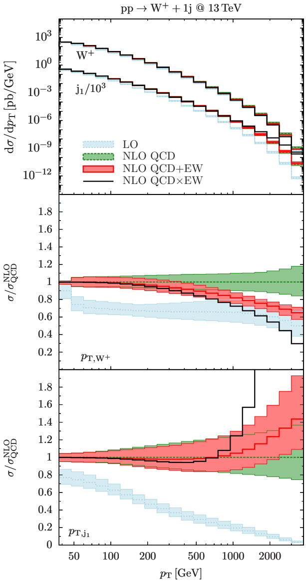
LO γ-induced contributions in $pp \to W^+ + 1, 2, 3\,$jets



Single-γ contributions

- from $\mathcal{O}\left(10^{-3}\right)$ in $\sigma_{\rm int}$ to 5–100% at $p_{T,W}$ =1–4 TeV!
- driven by γ-PDF (NNPDF2.3 QED) at large x (huge γ-PDF uncertainty. . .)

NLO QCD+EW corrections to $pp \to W^+ + 1\,$jet



Inclusive $\sigma(pp \to W + 1, 2, 3\,\text{jets})$ ($p_{T,j} > 30$ GeV)

- $\lesssim 1\%$ EW correction

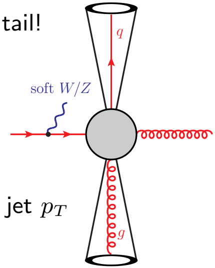
W-boson p_T (Sudakov behaviour)

- $+100\%$ QCD correction in the tail
- -20–35% EW correction at 1–4 TeV

Jet p_T (pathologic behaviour!)

- factor-10 QCD correction in the tail!
- *positive* 10–50% EW correction

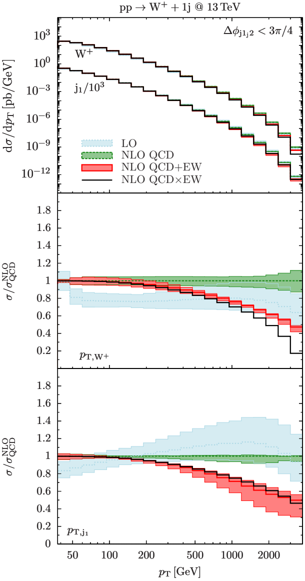
  (QCD-EW real emission!)



Origin of dramatic instability

- huge di-jet contributions at high jet p_T

Same observables with “dijet-veto cut” $\phi_{jj} < \frac{3}{4\pi}$



QCD corrections

- moderate at high $p_{T,\text{jet}}$

EW corrections

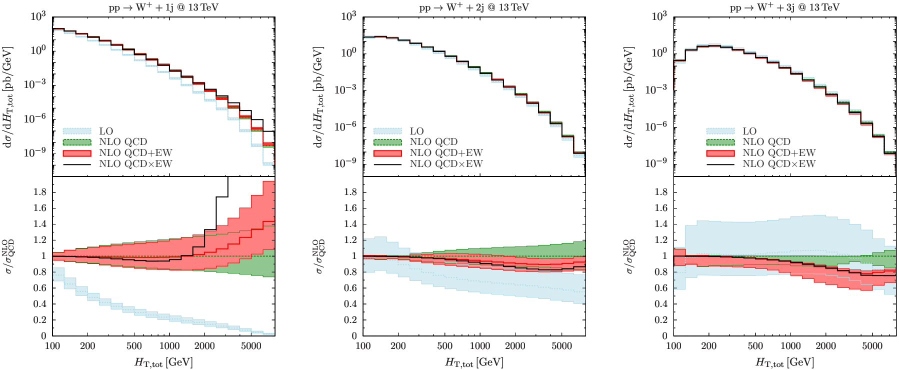
- Sudakov behaviour in both tails
- -20–50% at 1–4 TeV (more pronounced)

Bottom line

- $W + 1$ jet at NLO ok for *exclusive* case
- *inclusive* case requires $W + 2$ jets at NLO

$\Rightarrow$ **strong motivation for V+multijets!**

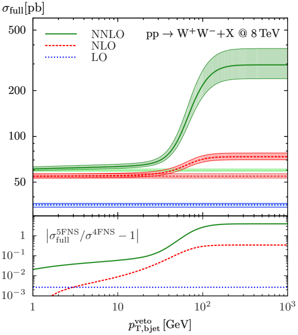
NLO corrections to $H_{T,\text{tot}}$ in $pp \to W^+ + 1, 2, 3$ jets

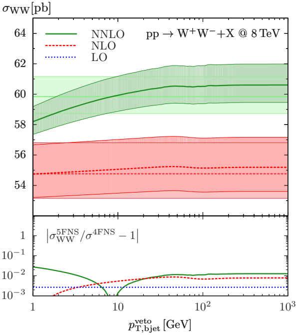


- NLO QCD in $H_{T,\text{tot}}$ tail well behaved only starting from $W + 3$ jets (calls for NLO multi-jet merging)
- only -20% EW corrections at very high $H_{T,\text{tot}}$ (more if also $p_{T,W}$ is high!)

$pp \to W^+W^-$ at NNLO vs jet vero in the 5F scheme

Top resonances, $g \to b\bar{b}$ singularities and b-jet veto ($p_\mathrm{T} < p_{\mathrm{T},bjet}^{\mathrm{veto}}$)





Full 5F cross section vs 4F

- top contamination huge at large $p_{\mathrm{T},bjet}^{\mathrm{veto}}$ and 10% at $10\,\mathrm{GeV}$, where sensitivity to singularity shows up
- no "robust" W^+W^- definition

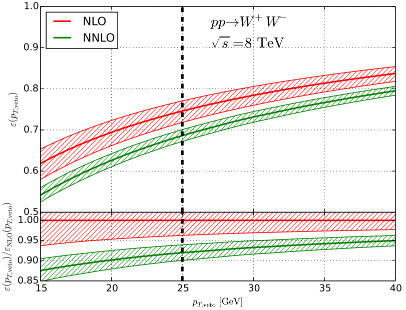
Top-free 5F cross section vs 4F

- very stable top subtraction at $p_{\mathrm{T},bjet}^{\mathrm{veto}} > 10\,\mathrm{GeV}$
- 1% agreement with 4FNS

$\Rightarrow$ **NNLO prediction solid!**

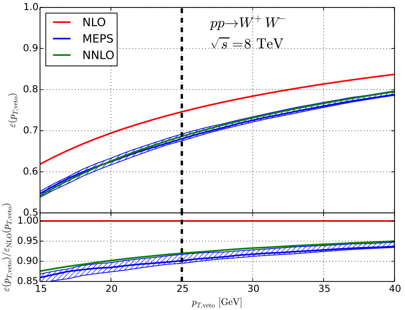
Jet-veto efficiency for $pp \rightarrow W^+W^-$ [Grazzini et al. (preliminary)]

NNLO vs NLO



- fiducial region of ATLAS (CMS) measurement involves jet veto at $p_T = 25(30)$ GeV
- NNLO correction of -8% wrt NLO
- NNLO seems consistent with Powheg

NNLO vs MEPS@NLO (Sherpa)



- MEPS@NLO ⇒ 1st emission at NLO + LLs + particle level
- quite stable wrt scale variations
- consistent with NNLO