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Unitarisation of Anomalous Couplings in Vector Boson Scattering

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MOTIVATION

Hope for interesting features to show up in the near future:

➡ **LHC Run 2**

INTRODUCTION

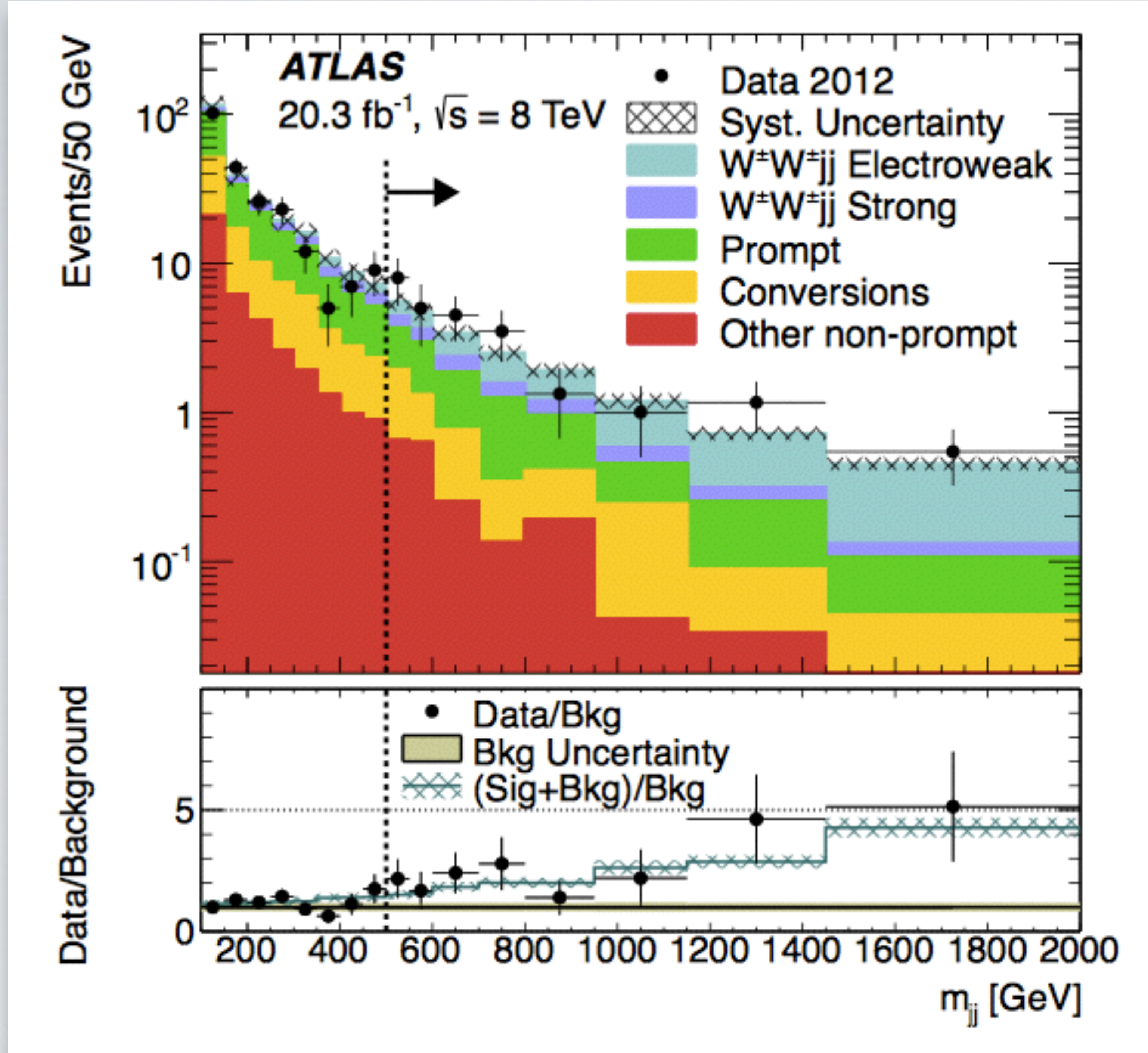
- Interactions among vector bosons predicted in the SM
- Coupling structure determined by the Glashow-Weinberg-Salam model of weak interactions
- Measurement has only begun recently
  - Room for new physics

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

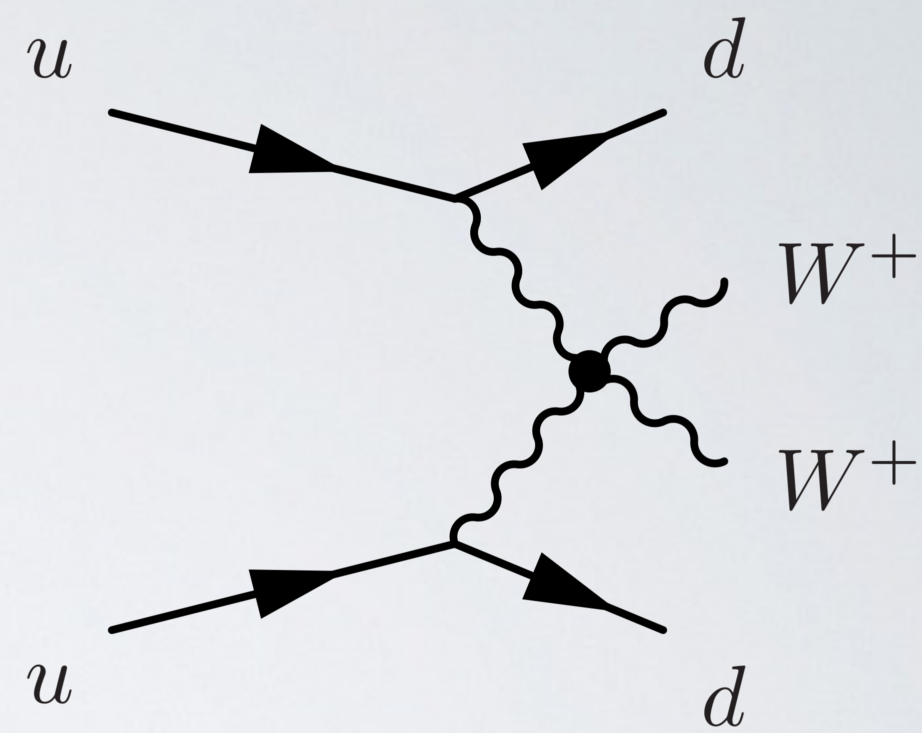
$$\begin{aligned} W^a_{\mu\nu} =& \mathrm{i}g(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu \\ &+ g\epsilon_{abc} W^b_\mu W^c_\nu) \end{aligned}$$

$$B_{\mu\nu} = \mathrm{i}g(\partial_\mu B_\nu - \partial_\nu B_\mu)$$

VECTOR BOSON SCATTERING



arXiv:1405.6241 [hep-ex]



First results for electroweak production of same sign W-pairs from ATLAS

➡ Vector Boson Scattering

NEW PHYSICS

At least two ways to describe new physics effects:

- Propose specific new physics model like SUSY
  - new particles
  - new symmetries
- Problem: relatively long way from physics model to explicitly modeling e.g. deviations in vector boson couplings

- Use model independent approach: effective field theory (EFT) or anomalous couplings
  - SM particle content with modified couplings

ANOMALOUS COUPLINGS

- **Lagrangian approach** (a little outdated)**:** constructed to contain all possible Lorentz-structures

$$\mathcal{L}^{VVV'V'} = c_0^{WW} W_\mu^+ W^{-\mu} W_\nu^+ W^{-\nu} + c_1^{WW} W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} + c_0^{WZ} W_\mu^+ Z^\mu W_\nu^- Z^\nu + c_1^{WZ} W_\mu^+ W^{-\mu} Z_\nu Z^\nu + c^{ZZ} (Z_\mu Z^\mu)^2 .$$

➡ Not necessarily gauge invariant

➡ Could add arbitrary number of terms with derivatives accompanied by factor m_V^{-1}

- Standard Model values:

$$c_{0,\mathrm{SM}}^{WW} = -c_{1,\mathrm{SM}}^{WW} = \frac{2}{\cos^2\theta_W} c_{0,\mathrm{SM}}^{WZ} = -\frac{2}{\cos^2\theta_W} c_{0,\mathrm{SM}}^{WZ} = g^2, \qquad c_{\mathrm{SM}}^{ZZ} = 0,$$

EFFECTIVE FIELD THEORY

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{f_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_k \frac{f_k}{\Lambda^4}\mathcal{O}_k^{(8)}$$

- Use model independent approach: effective field theory (EFT) or anomalous couplings
    - SM particle content with modified couplings
- Extend SM by adding higher dimensional operators

- Capture any physics beyond SM in accessible energy range
- Balance energy dimension by inserting appropriate mass scale Λ (scale of New Physics)
- Dim. 8 operators as tools to model potential deviations of quartic gauge couplings from their SM values

DIM. 8 OPERATORS

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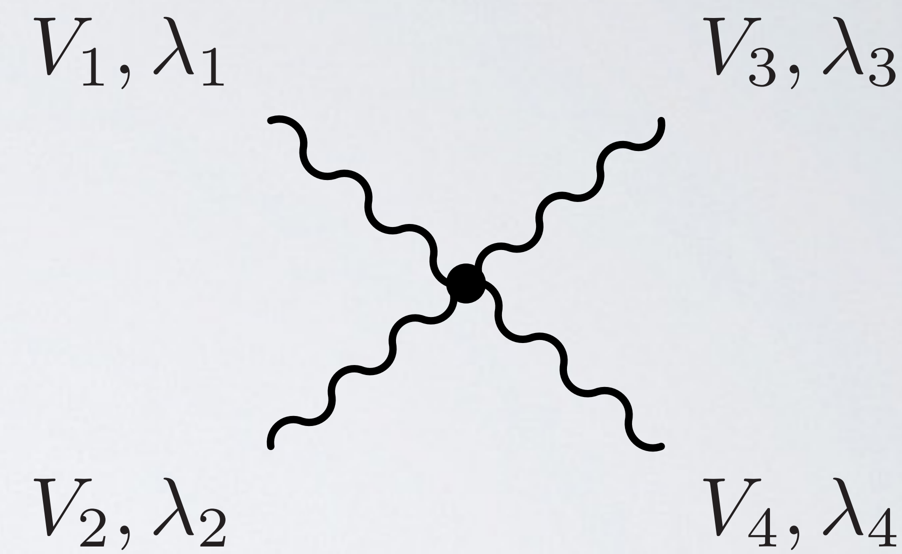
- 3 classes of dim. 8 operators:

$$\mathcal{O}_{\mathrm{S},0} = [(D_\mu \Phi)^\dagger (D_\nu \Phi)] \times [(D^\mu \Phi)^\dagger (D^\nu \Phi)]$$
$$\mathcal{O}_{\mathrm{M},0} = \mathrm{Tr}[W_{\mu\nu} W^{\mu\nu}] \times [(D_\beta \Phi)^\dagger (D^\beta \Phi)]$$
$$\mathcal{O}_{\mathrm{T},0} = \mathrm{Tr}[W_{\mu\nu} W^{\mu\nu}] \times \mathrm{Tr}[W_{\alpha\beta} W^{\alpha\beta}]$$



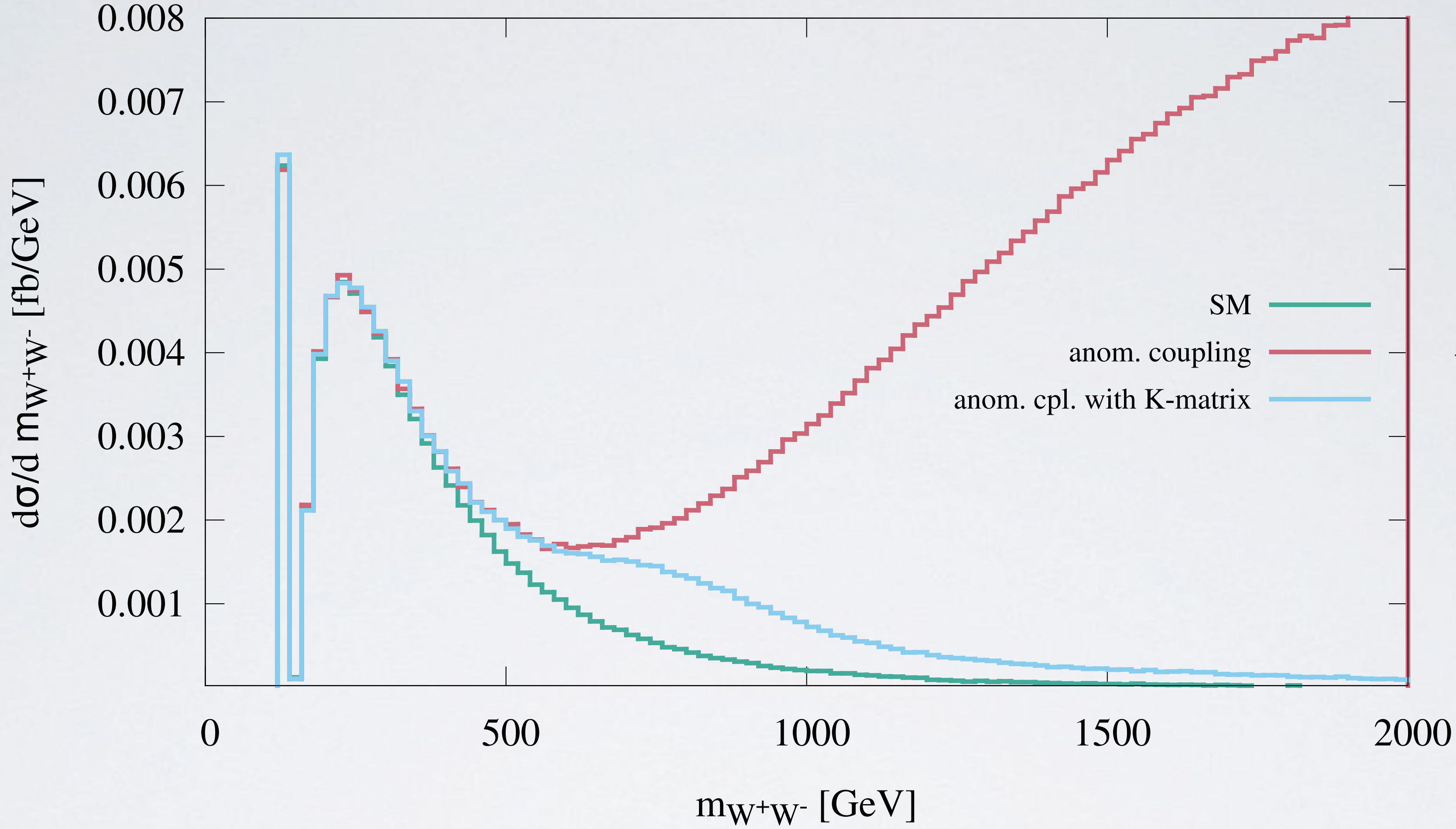
- 18 dim. 8 operators in total

- All are effecting vector boson 4-vertices in different ways

- Problem: Different combinations of polarizations lead to unphysically large cross sections

$$\epsilon_L^\mu(k) = \frac{k^\mu}{m} + \mathcal{O}\left(\frac{m}{E}\right)$$

WHY UNITARISATION?



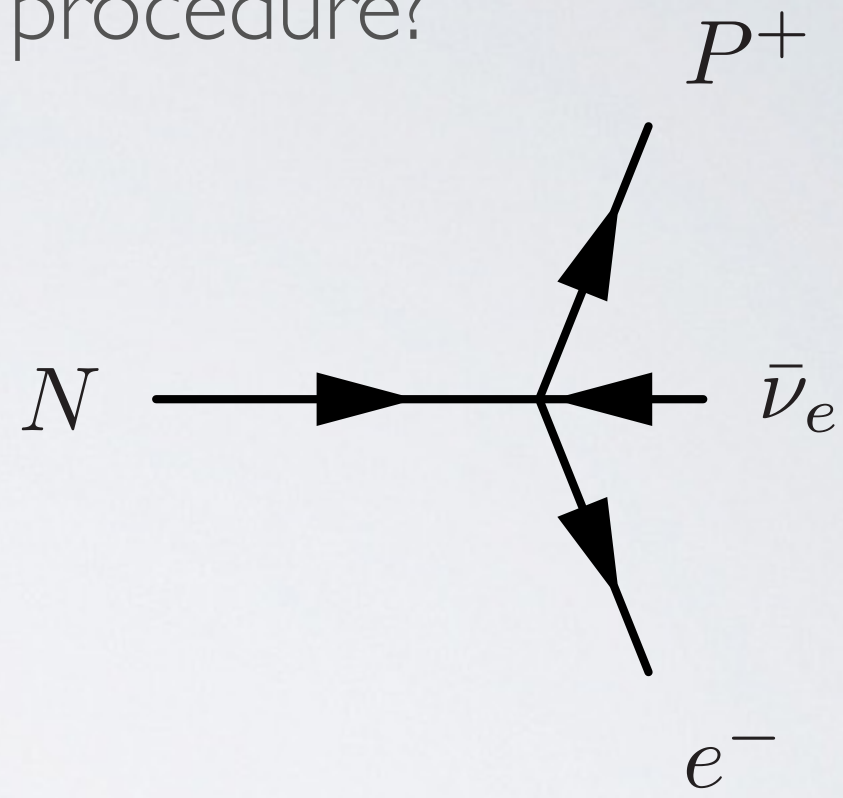
$$\frac{f_{S,0}}{\Lambda^4} = \frac{f_{S,1}}{\Lambda^4} = 100\ \text{TeV}^{-4}$$

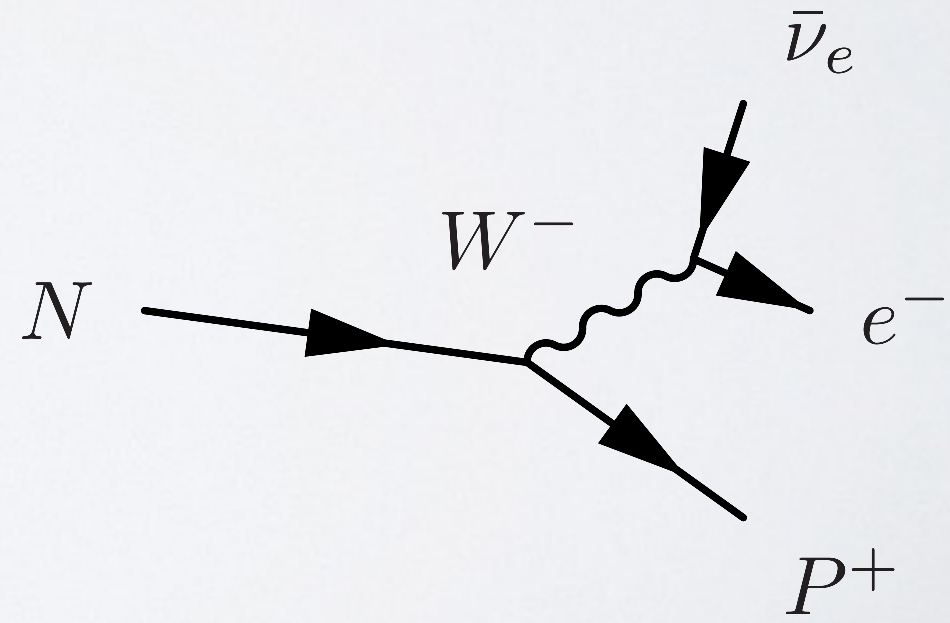
- Want to study deviations from the SM in an accessible energy range, e.g. at the LHC
- All effective operators break tree- level unitarity for high c.o.m. energy
  - Unphysically large cross- sections
  - When comparing predictions to data, all couplings would need to be zero

UNITARISATION

Is this a reasonable procedure?

- Historic example: Fermi's 4-point- interaction (1933):
  - effective 4-vertex leads to rise of the amplitude with s
  - Insertion of W- propagator unitarizes the amplitude and gives correct low energy behavior





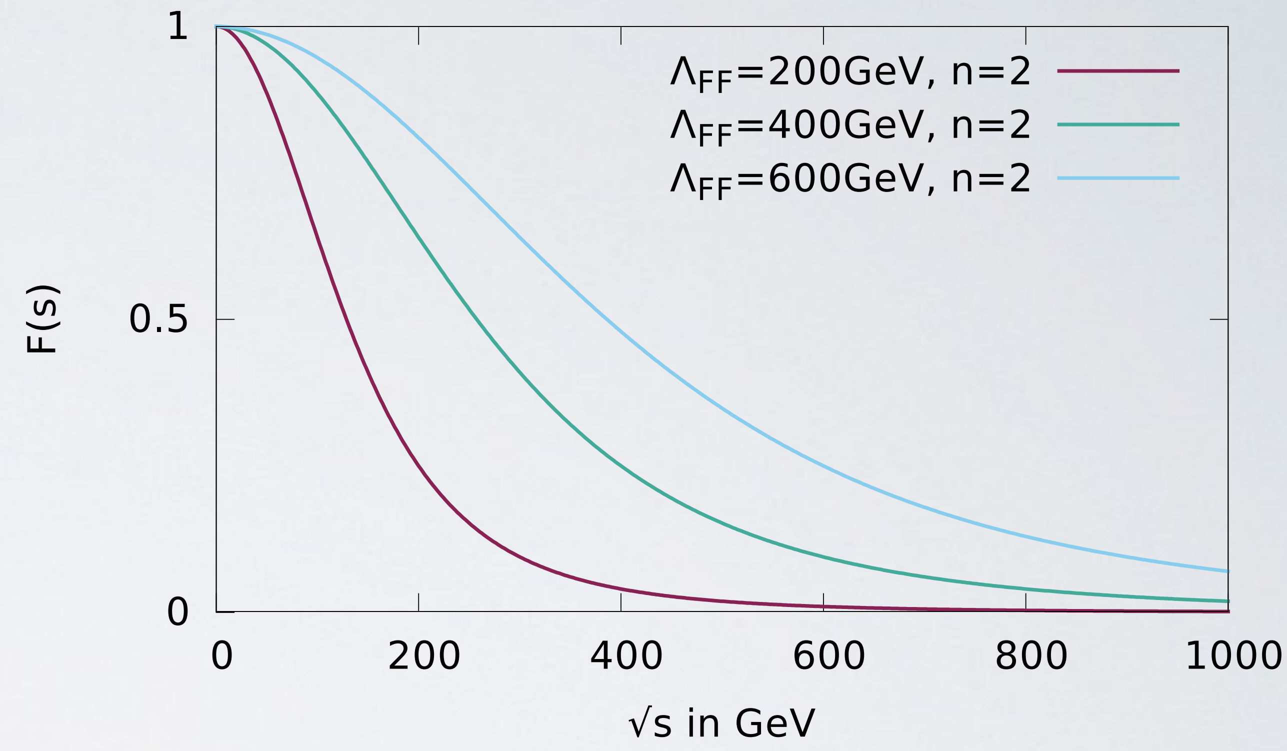
FORM FACTORS

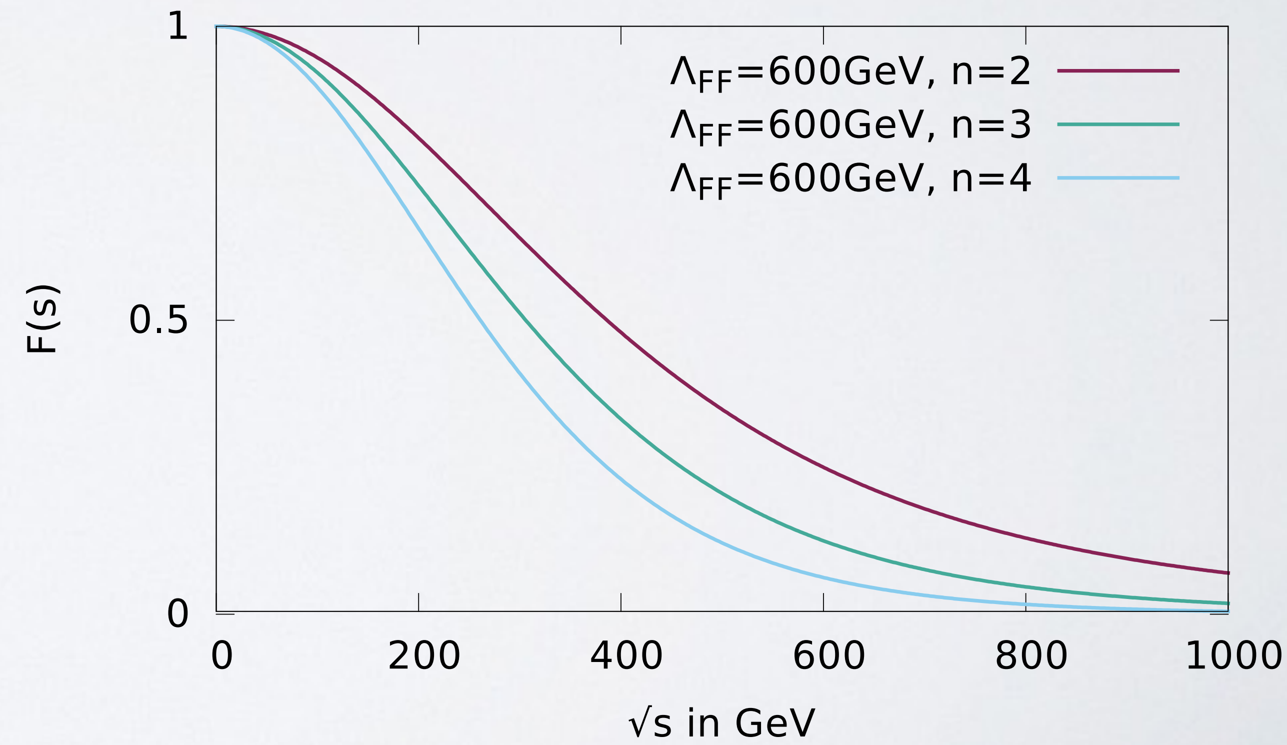
- Multiply amplitudes by factor resembling a propagator:

$$\mathcal{A} \to \mathcal{A} \times \frac{1}{\left(1 + \frac{s}{\Lambda_{\mathrm{FF}}^2}\right)^n}$$

- Choice of form factor lacks strong physical motivation

- One has to deal with two input-parameters depending on the operator structure and coupling strength





UNITARITY CONSIDERATIONS

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- Unitarity of Scattering operator S leads to condition for eigenvalues of transition operator T:

$$SS^\dagger = I$$
$$\Leftrightarrow (I + 2\mathrm{i}T)(I + 2\mathrm{i}T)^\dagger = I$$

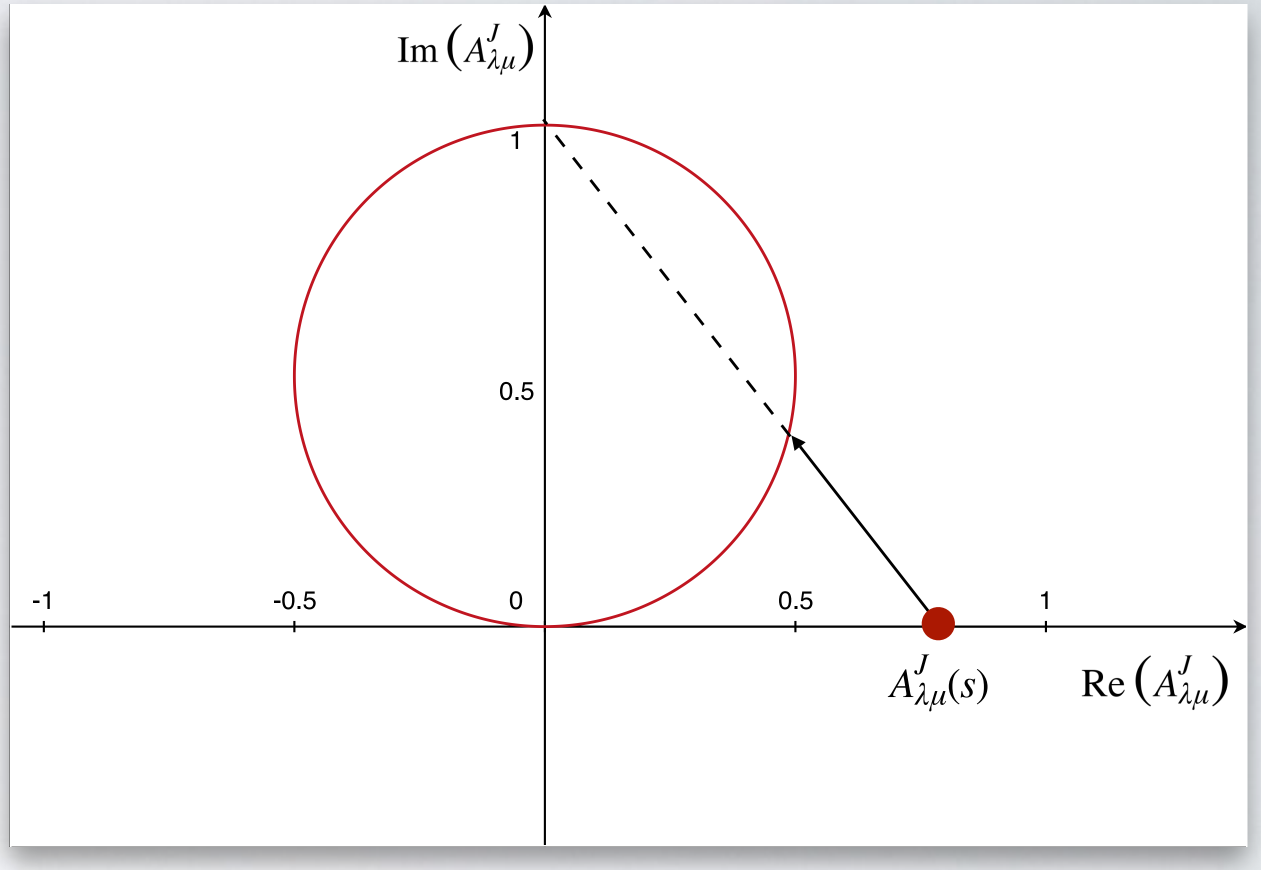
  - Argand-circle condition: $\left|t_j - \frac{\mathrm{i}}{2}\right| = \frac{1}{2}$

- T can be expressed in terms of the so called K-matrix:

$$T = \frac{K}{I - \mathrm{i}K}$$

- Perturbatively, K can be expressed in terms of partial wave amplitudes

$$A^J_{\lambda\mu}(s) = \int \mathrm{d}(\cos\theta)\ \mathcal{A}_{V_1V_2\to V_3V_4}(s,\theta)\ d^J_{\lambda\mu}(\theta)$$

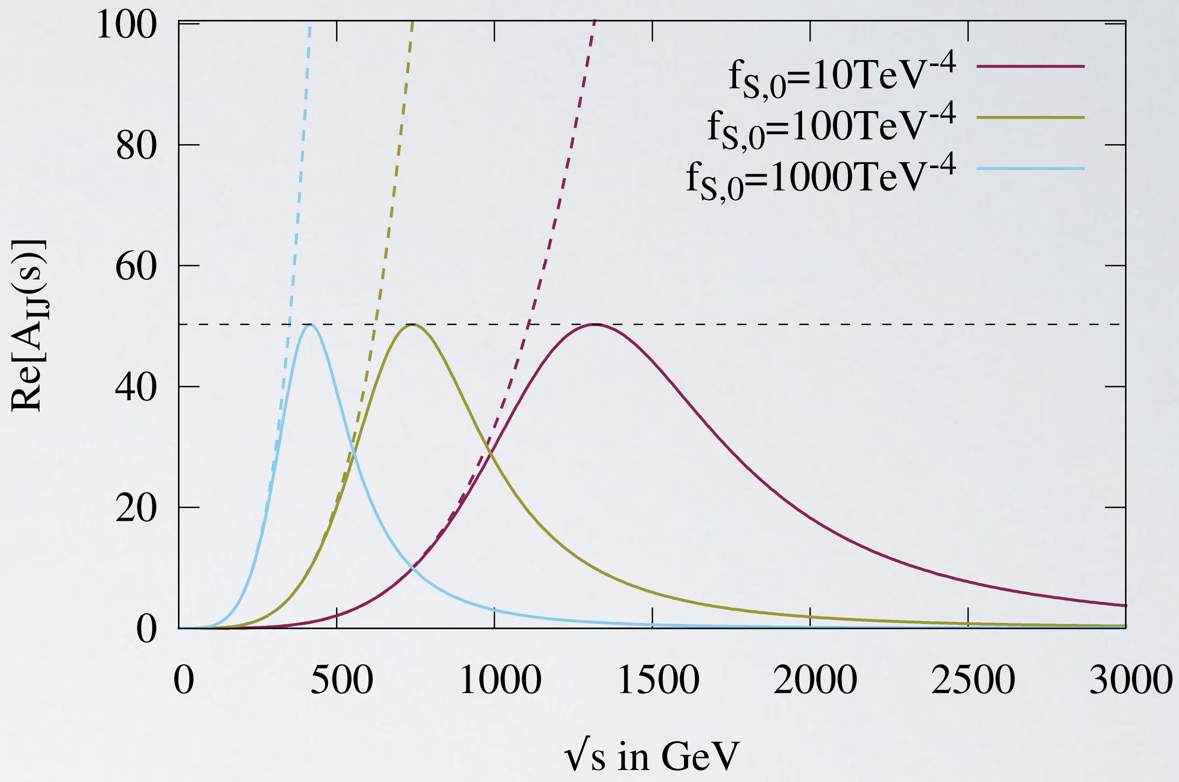


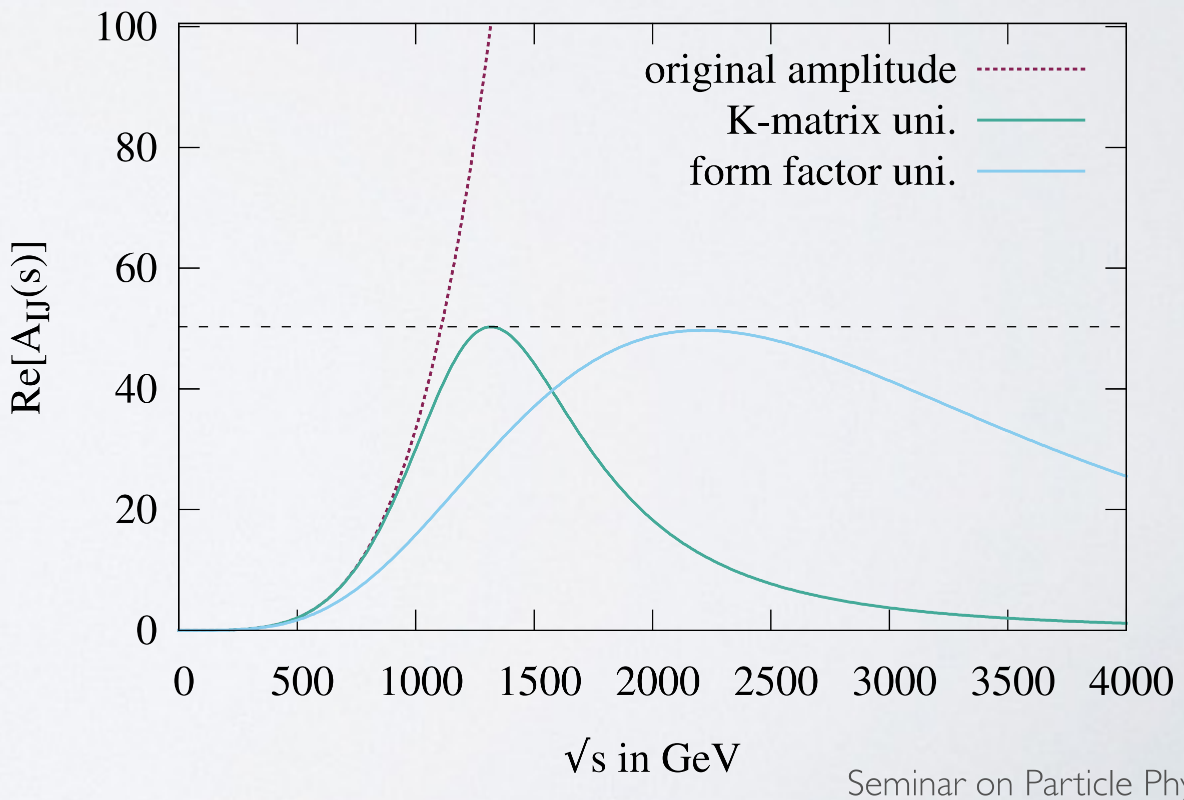
  - Unitarized partial wave amplitudes:

$$\boxed{\hat{A}^J_{\lambda\mu} = \frac{A^J_{\lambda\mu}}{1 - \mathrm{i}A^J_{\lambda\mu}/32\pi}}$$

K-MATRIX-UNITARISATION

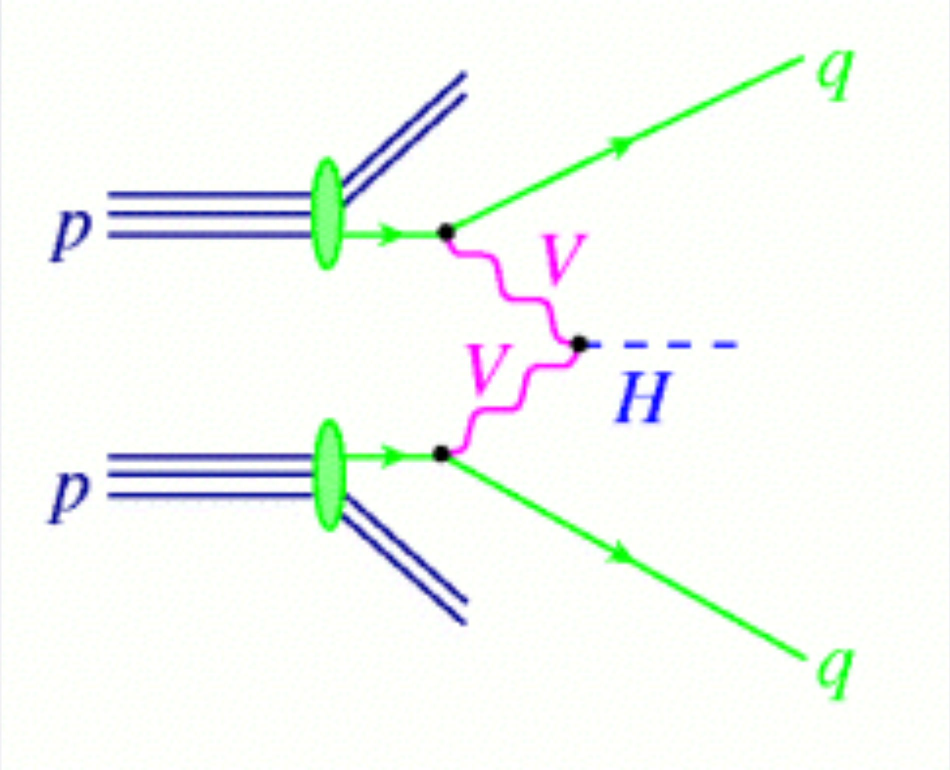
- Partial waves include dependence on couplings
  - Dynamic unitarisation for any size of coupling
- Saturation at high energies while keeping original low energy behavior
- Worked out for the operators $\mathcal{L}_{S,i}$





VBFNLO

- Parton level Monte Carlo program for NLO QCD simulation of
  - Vector boson fusion
  - Double and triple vector boson production in hadronic collisions
  - Double vector boson production in association with a hadronic jet
- Includes Higgs and vector boson decays with full spin correlations and all off-shell effects.
- Anomalous Couplings available for most processes
- Efficient by reusing electroweak part of diagrams in terms of leptonic tensors



https://www.itp.kit.edu/~vbfnloweb

arXiv:1404.3940 [hep-ph]

VBFNLO / WHIZARD COMPARISON

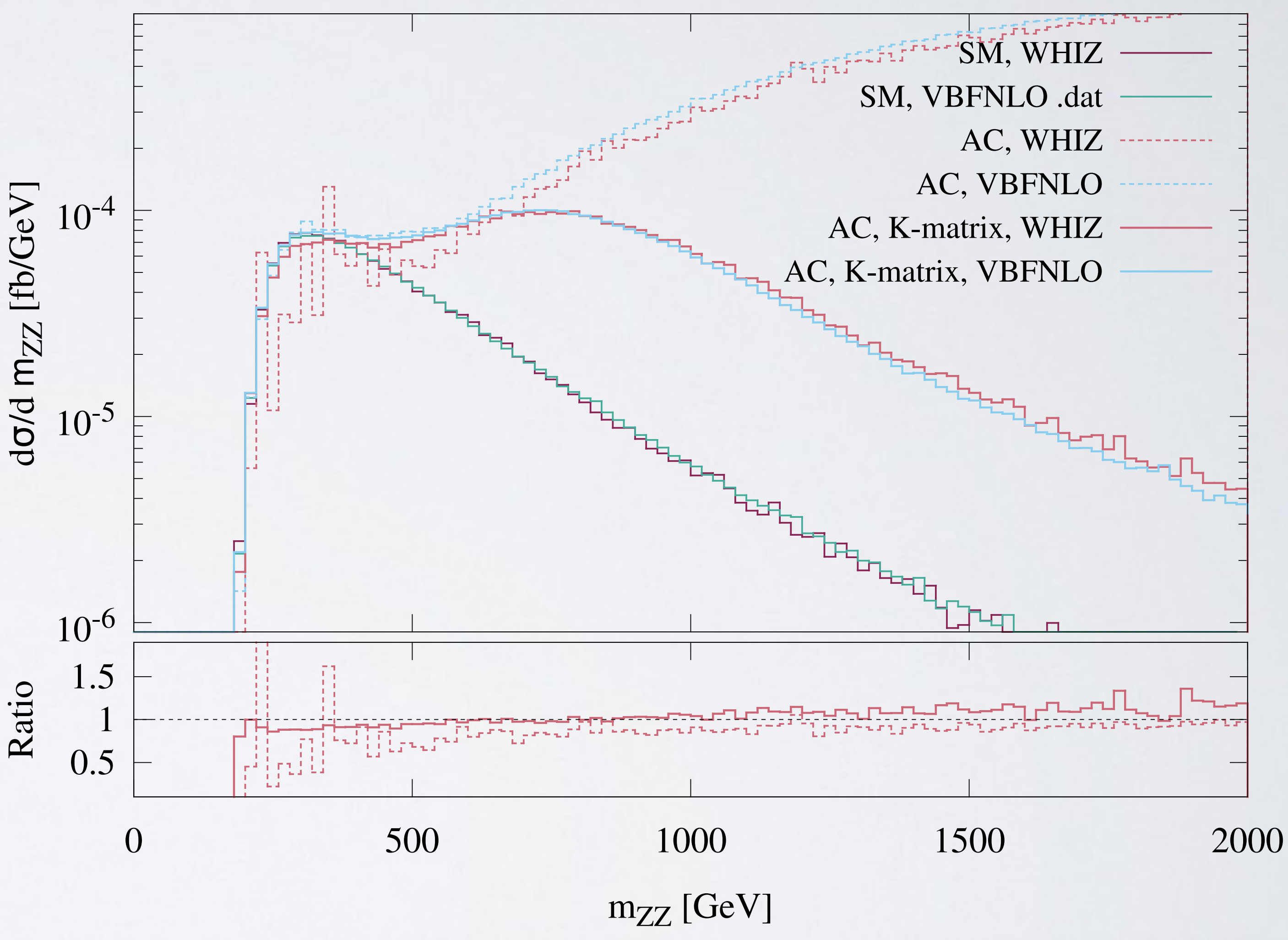
- K-Matrix unitarisation implemented in **VBFNLO** (Parton level Monte Carlo program @ NLO QCD) for $\mathcal{L}_{S,i}$

- Comparable to implementation in WHIZARD

  ➡ Qualitative agreement found for invariant mass distributions

  ➡ Agreement at per mill level for total cross sections

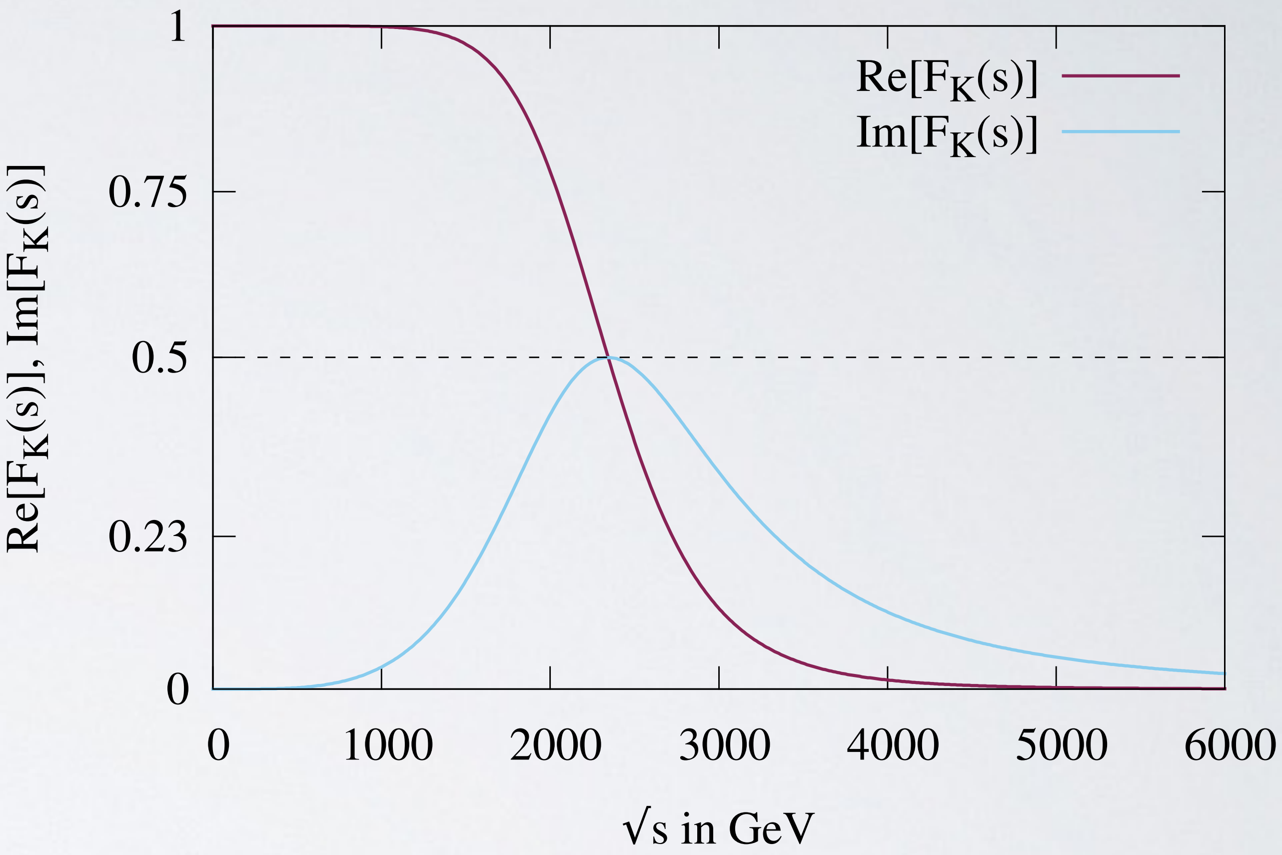
$$\frac{f_{S,0}}{\Lambda^4} = \frac{f_{S,1}}{\Lambda^4} = 100\ \text{TeV}^{-4}$$



K-MATRIX-LIKE FORM FACTOR

Need different treatment for the other operators

- No diagonalizing basis for S-matrix available
- More than one important contribution in helicity space
- Translation to off-shell implementation difficult



➡ Use **K-matrix-like form factors**:

$$\mathcal{F}_K(s) = \frac{\hat{A}^J_{\lambda\mu}}{A^J_{\lambda\mu}} = \frac{1}{1 - \mathrm{i}A^J_{\lambda\mu}} = \frac{1}{1 - \mathrm{i}\frac{s^2}{\Lambda_K^4}}.$$

MATHEMATICA FRAMEWORK

Framework for calculating analytic expressions of partial wave amplitudes:

- Start from an arbitrary operator set, i.e. Lagrangian
- Generate analytic expressions for amplitudes by inserting explicit momenta and helicity eigenvectors
- Get partial waves for all of the 9x9 possible helicity configurations
- Determine leading contributions and use them for unitarisation

$$J=0: \begin{pmatrix} 0 & -\frac{m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 \\ 0 & -\frac{m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 \\ \frac{c_w^2 m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$J=1: \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$J=2: \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_w^2 m_W^2}{5\sqrt{6}v^2}\bar{f}_{M,1}s^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{m_W^2}{5\sqrt{6}v^2}\bar{f}_{M,1}s^2 & 0 & 0 \end{pmatrix},$$

CONCLUSION

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- Use a model independent way to describe New Physics: Effective Field Theory
- Need unitarisation scheme to suppress unphysical behavior for high c.o.m. energy
- K-matrix unitarisation implemented in VBFNLO for $\mathcal{L}_{S,i}$ and form factors for other operators
- Proposition of K-matrix-like form factors for other operators
- Development of Mathematica framework for calculating partial wave amplitudes of arbitrary dim. 8 operator set
- Simulation in VBFNLO including NLO QCD corrections
- New features available in **VBFNLO 3.0beta:** **https://www.itp.kit.edu/~vbfnloweb**

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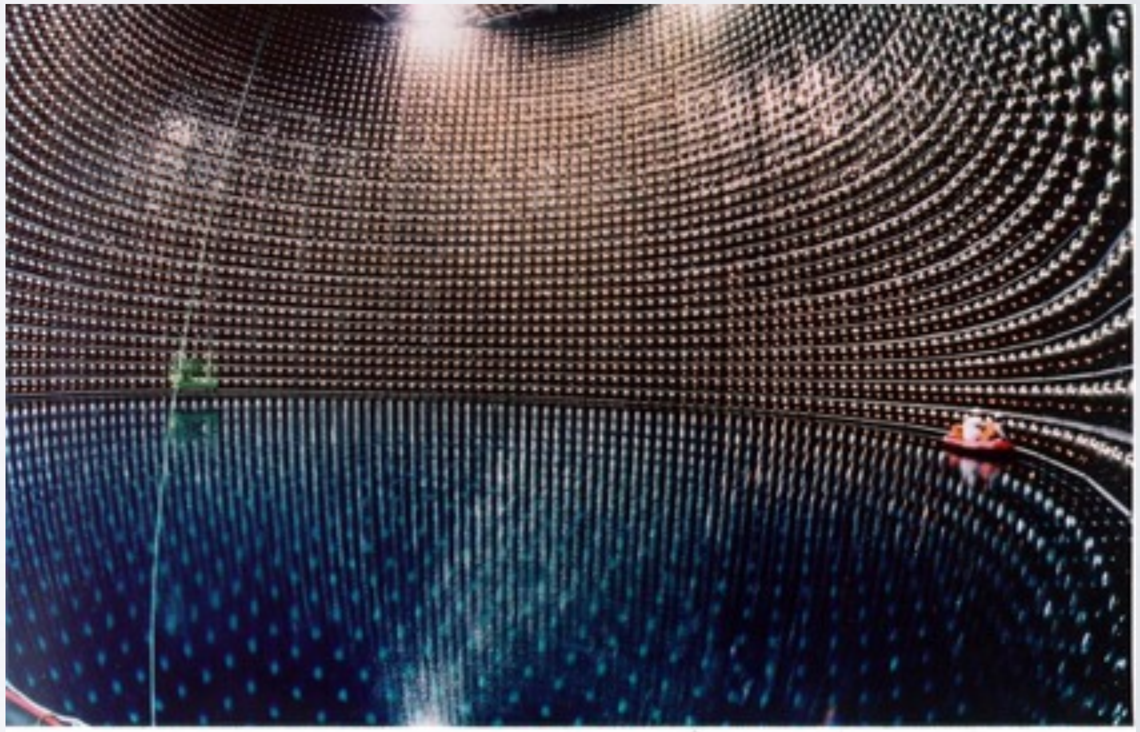
ONE-LOOP FERMION MASS CORRECTIONS AND FLAVOR SYMMETRIES

Maximilian Löschner
Advisor: Walter Grimus

Seminar on Particle Physics
10 Nov. 2015

INTRODUCTION

- Measurement of ν-oscillations:
  - ν's are massive and have non-vanishing mass differences
- Possibly far reaching implications of mass generating mechanisms:
  - Lepton number violation, leptogenesis, baryon asymmetry
  - Composition and origin of dark matter



SOME OPEN QUESTIONS

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Experiment

1. Value of the CP-violating phase in the mixing matrix
2. Normal or inverted mass hierarchy
3. Absolute mass scale of the lightest neutrinos
4. Dirac or Majorana nature

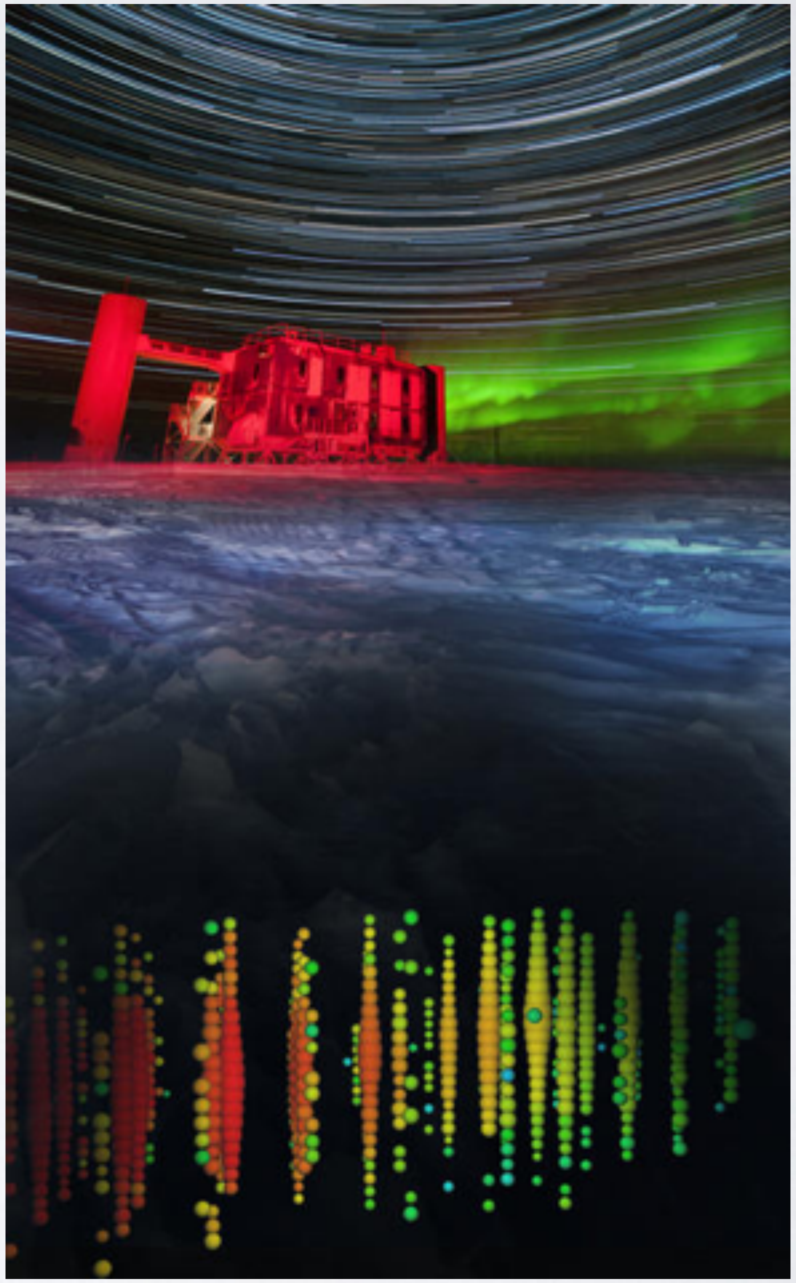
Theory

1. Smallness of ν masses
2. Strong hierarchy in mass spectra of charged leptons
3. Mild hierarchy in ν spectrum
4. One small and two large mixing angles in lepton mixing matrix

MASS MECHANISMS

- In order to add mass terms to SM Lagrangian, necessarily need to introduce new particles
  - At least three right-handed ν's for gauge invariant Yukawa mass terms
- ν masses are at least 10^6 times smaller then electron mass
  - $y \lesssim 10^{-11}$
  - seems unnaturally small

$$\mathcal{L}_{\text{Yuk},\nu} = y(\bar{\nu}_L \bar{\phi}^0 - \bar{l}_L \phi^-)\nu_R$$

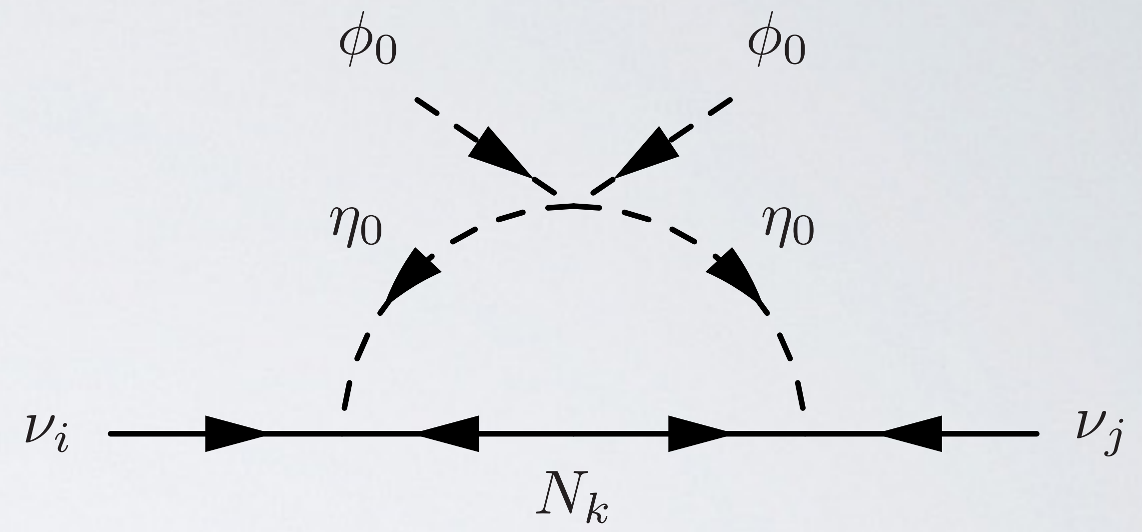


RADIATIVE GENERATION

Example: **Scotogenic Model**

[arXiv:1408.4785]

- Extend SM by three right-handed ν's and second scalar doublet
- Impose exact Z_2 - symmetry: all SM particles even, new particles odd
  - $y(\nu\phi^0 - l\phi^+)N$ forbidden
  - $y(\nu\eta^0 - l\eta^+)N$ allowed



$$N_i\,; \quad (\eta^+, \eta^0) \text{ with } \langle \eta^0 \rangle = 0$$

- Vanishing VEV of second scalar doublet
  - No tree level Dirac mass
  - Radiative Majorana mass
- Possible dark matter candidate

OUTLOOK

- Determine one-loop mass corrections in general framework, investigate stability of tree level masses and influence of (discrete) flavor symmetries

$$\mathcal{L}_{\text{toy}} = i\overline{\chi}_L \gamma_\mu \partial^\mu \chi_L + \left(\frac{1}{2} y \chi_L^T C^{-1} \chi_L \phi + \text{h.c.} \right) + \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

  - First step: Generalization of a simple toy model with arbitrary # of fermion and scalar fields imposing Z_2 - symmetry

- In the end: apply to specific models like the *scotogenic model* and produce numerical results for corrections to masses and mixing angles

„There is nothing new to be discovered in physics now.
All that remains is more and more precise measurement."
- Lord Kelvin, 1900

THANKS!

Backup Slides

DIM. 6 VS. DIM. 8

	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
$\mathcal{O}_{WWW}$	X	X					X	X	X	X
$\mathcal{O}_{W}$	X	X	X	X	X		X	X	X	
$\mathcal{O}_{B}$	X	X		X	X					
$\mathcal{O}_{\Phi d}$			X	X						
$\mathcal{O}_{\Phi W}$			X	X	X	X				
$\mathcal{O}_{\Phi B}$				X	X	X				
$\mathcal{O}_{\tilde{W}WW}$	X	X					X	X	X	X
$\mathcal{O}_{\tilde{W}}$	X	X	X	X	X					
$\mathcal{O}_{\tilde{W}W}$			X	X	X	X				
$\mathcal{O}_{\tilde{B}B}$				X	X	X				

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0},\ \mathcal{O}_{S,1}$	X	X	X						
$\mathcal{O}_{M,0},\ \mathcal{O}_{M,1},\mathcal{O}_{M,6},\mathcal{O}_{M,7}$	X	X	X	X	X	X	X		
$\mathcal{O}_{M,2},\ \mathcal{O}_{M,3},\mathcal{O}_{M,4},\mathcal{O}_{M,5}$		X	X	X	X	X	X		
$\mathcal{O}_{T,0},\ \mathcal{O}_{T,1},\mathcal{O}_{T,2}$	X	X	X	X	X	X	X	X	X
$\mathcal{O}_{T,5},\ \mathcal{O}_{T,6},\mathcal{O}_{T,7}$		X	X	X	X	X	X	X	X
$\mathcal{O}_{T,8},\ \mathcal{O}_{T,9}$			X			X	X	X	X

POLARIZATIONS

$$\mathcal{A}_{S,1} = 2m_W^2 m_Z^2 \frac{f_{S,1}}{\Lambda^4}\epsilon_1\cdot\epsilon_2\epsilon_3^*\cdot\epsilon_4^*,$$

$$\mathcal{A}_{M,2} = \frac{4m_W^2 m_Z^2 \sin^4\theta_W}{v^2}\frac{f_{M,2}}{\Lambda^4}\epsilon_1.\epsilon_2\left(k_3.k_4\epsilon_3^*.\epsilon_4^* - \epsilon_3^*.k_4\epsilon_4^*.k_3\right),$$

$$\mathcal{A}_{T,0} = \frac{128m_W^2 m_Z^2 \cos^4\theta_W}{v^4}\frac{f_{T,0}}{\Lambda^4}\left(k_1\cdot k_2\epsilon_1\cdot\epsilon_2 - \epsilon_1\cdot k_2\epsilon_2\cdot k_1\right)\left(k_3\cdot k_4\epsilon_3^*\cdot\epsilon_4^* - \epsilon_3^*\cdot k_4\epsilon_4^*\cdot k_3\right)$$

- Each operator has its „fingerprint“ of which polarization states are the most important
- For a 2 to 2- process one has $3^4 = 81$ possible combinations
  - Number of independent amplitudes can be reduced using C-, P- and Bose symmetry

POLARIZATIONS

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Reduction of polarization matrix (for WW to ZZ):

W- interchange

Z- interchange

− − − −	− − −0	− − −+	− − 0−	− − 00	− − 0+	− − +−	− − +0	− − ++
−0 − −	−0 − 0	−0 − +	−00−	−000	−00+	−0 + −	−0 + 0	−0 + +
− + − −	− + −0	− + −+	− + 0−	− + 00	− + 0+	− + +−	− + +0	− + ++
0 − − −	0 − −0	0 − −+	0 − 0−	0 − 00	0 − 0+	0 − +−	0 − +0	0 − ++
00 − −	00 − 0	00 − +	000−	0000	000+	00 + −	00 + 0	00 + +
0 + − −	0 + −0	0 + −+	0 + 0−	0 + 00	0 + 0+	0 + +−	0 + +0	0 + ++
+ − − −	+ − −0	+ − −+	+ − 0−	+ − 00	+ − 0+	+ − +−	+ − +0	+ − ++
+0 − −	+0 − 0	+0 − +	+00−	+000	+00+	+0 + −	+0 + 0	+0 + +
+ + − −	+ + −0	+ + −+	+ + 0−	+ + 00	+ + 0+	+ + +−	+ + +0	+ + ++

Parity

POLARIZATIONS

Final polarization matrix for WW to ZZ:

$$\begin{pmatrix} --\,-- & --\,00 & --\,-0 & --\,-+ \\ ++\,-- & ++\,00 & ++\,-0 & ++\,-+ \\ 00\,-- & 0000 & 00-0 & 00-+ \\ -0\,-- & -000 & -0-0 & -0-+ \\ 0+\,-- & 0+00 & 0+-0 & 0+-+ \\ -+\,-- & -+00 & -+-0 & -+-+ \end{pmatrix}$$

Corresponding helicity differences:

$$\begin{pmatrix} 0\;0 & 0\;0 & 0\text{-}1 & 0\text{-}2 \\ 0\;0 & 0\;0 & 0\text{-}1 & 0\text{-}2 \\ 0\;0 & 0\;0 & 0\text{-}1 & 0\text{-}2 \\ \text{-}1\;0 & \text{-}1\;0 & \text{-}1\text{-}1 & \text{-}1\text{-}2 \\ \text{-}1\;0 & \text{-}1\;0 & \text{-}1\text{-}1 & \text{-}1\text{-}2 \\ \text{-}2\;0 & \text{-}2\;0 & \text{-}2\text{-}1 & \text{-}2\text{-}2 \end{pmatrix}$$

Result of this framework:
Partial waves that could be used as input for K-matrix like form factors

$$J=0: \begin{pmatrix} 0 & -\frac{m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 \\ 0 & -\frac{m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 \\ \frac{c_w^2 m_W^2}{2v^2}\bar{f}_{M,1}s^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$J=1: \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$J=2: \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_w^2 m_W^2}{5\sqrt{6}v^2}\bar{f}_{M,1}s^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{m_W^2}{5\sqrt{6}v^2}\bar{f}_{M,1}s^2 & 0 & 0 \end{pmatrix},$$

K-MATRIX

- Unitarity considerations lead to definition of inverse K-operator:

$$K^{-1} \equiv T^{-1} + \mathrm{i}I$$

- Then T can be expressed as

$$T = \frac{K}{I - \mathrm{i}K}$$

- If perturbative expansion of T exists then:

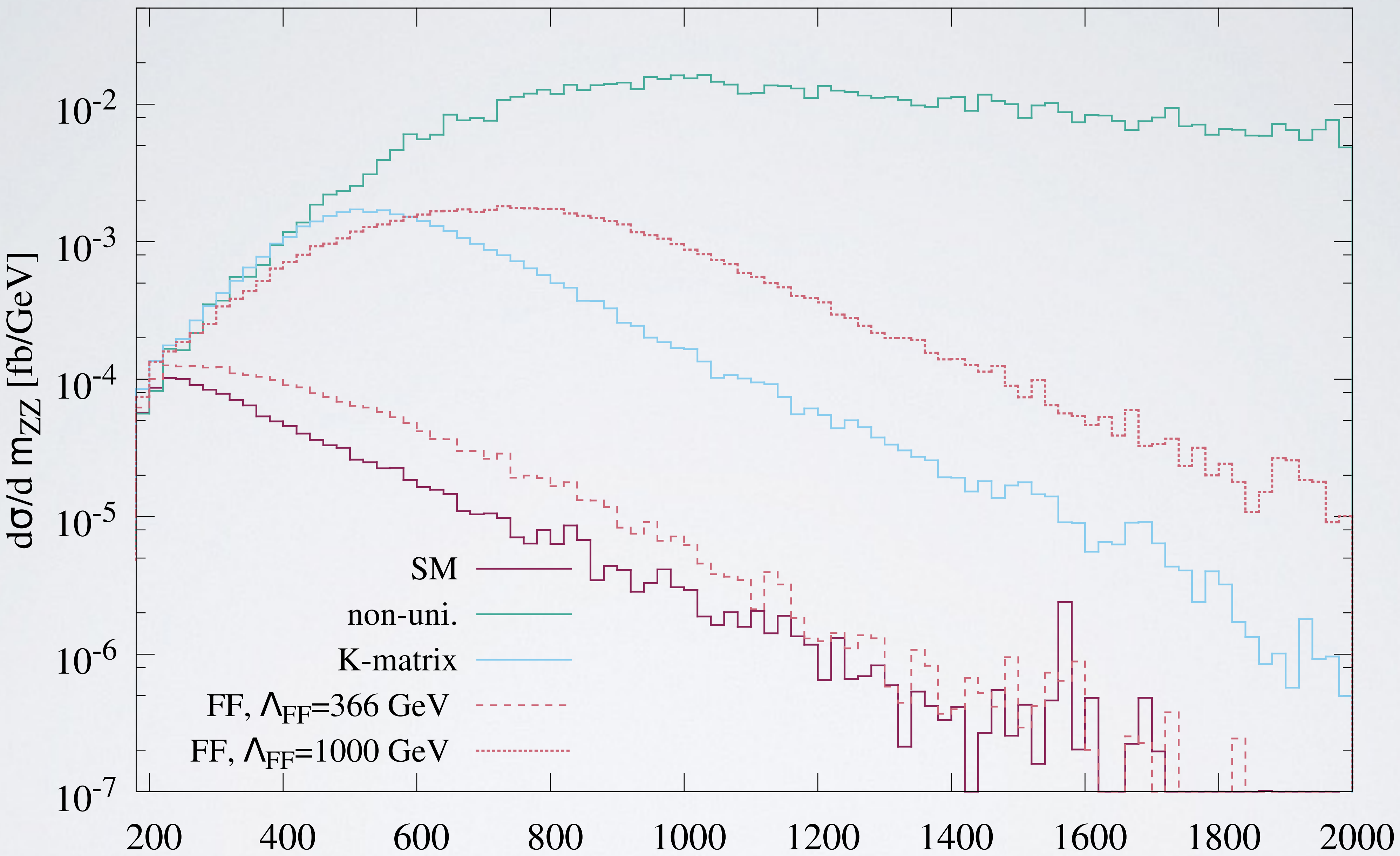
$$K^{(1)} = T^{(1)}$$

ANOMALOUS COUPLINGS

- By expanding arbitrary set of EFT operators in terms of the fields, relations to anomalous couplings can easily be found:

$$c_i^{VV'} = c_{i,\mathrm{SM}}^{VV'} + g^2 \Delta c_i^{VV'} .$$

K-MATRIX VS. FORM FACTOR



WHIZARD COMPARISON

process	SM		no K-Matrix		K-Matrix	
	VBFNLO	WHIZ	VBFNLO	WHIZ	VBFNLO	WHIZ
W^+W^+	1.3102(4)	1.311(1)	51.49(2)	51.54(4)	2.452(1)	2.466(2)
W^+W^-	0.9019(7)	0.902(2)	24.594(6)	21.52(4)	1.530(1)	1.455(4)
W^+Z	0.1473(1)	0.1480(3)	2.633(1)	2.637(3)	0.2413(2)	0.2426(5)
ZZ	0.02840(3)	0.0284(1)	3.141(2)	3.142(6)	0.08301(6)	0.0829(2)

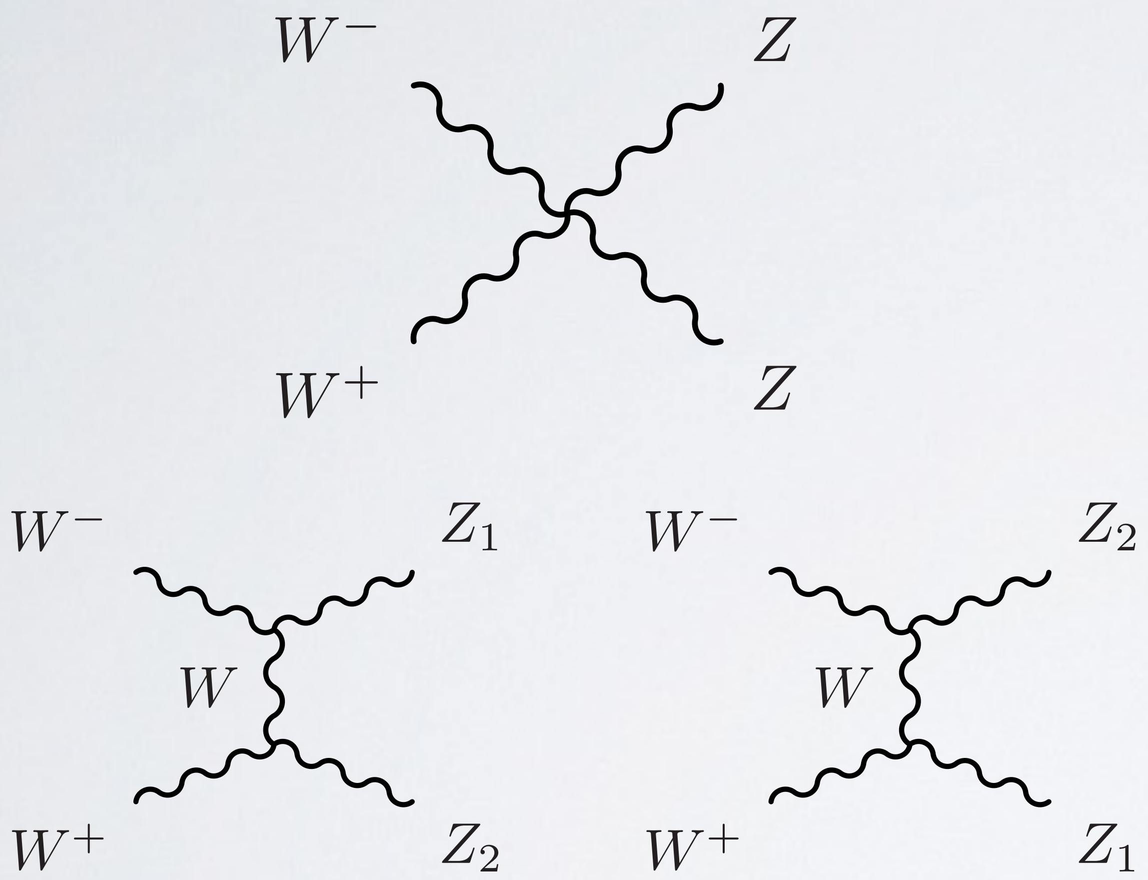
Agreement in terms of total cross sections (surprisingly) very good, except for W+W-

➡ Differences in operator sets show up

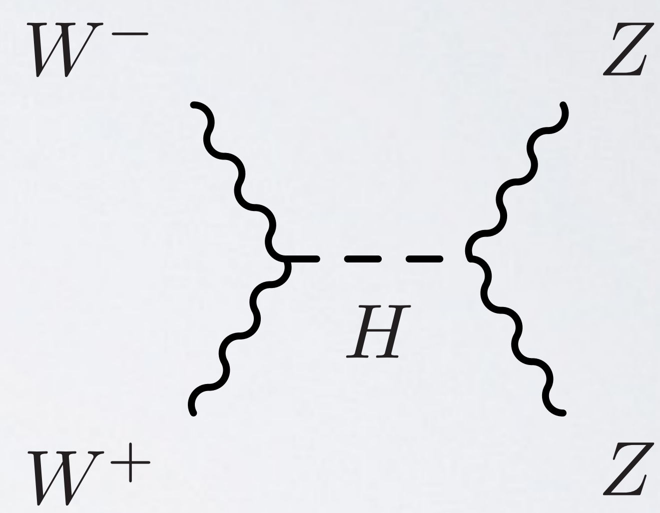
➡ Can be cured by introducing new operator

UNITARITY AND HIGGS

Need to take care that cancellations between Higgs- and vector-boson- sector don't get spoiled in BSM- Higgs- models



$$\sim \frac{s}{v^2} + \mathcal{O}(1)$$



$$\sim -\frac{s^2}{v^2(s - m_H^2)} + \mathcal{O}(1)$$

A CLOSER LOOK

$$\mathcal{O}_{\mathrm{S},0} = [(D_\mu\Phi)^\dagger(D_\nu\Phi)] \times [(D^\mu\Phi)^\dagger(D^\nu\Phi)]$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$D_\mu = \begin{pmatrix} \partial_\mu + \frac{\mathrm{i}}{2}Z_\mu(g\mathrm{c}_W + g'\mathrm{s}_W) + \mathrm{i}eA_\mu & \frac{\mathrm{i}g}{\sqrt{2}}W_\mu^+ \\ \frac{\mathrm{i}g}{\sqrt{2}}W_\mu^- & \partial_\mu + \frac{\mathrm{i}}{2}Z_\mu\sqrt{g^2+g'^2} \end{pmatrix}$$

$$W_{\mu\nu} = \frac{\mathrm{i}g}{2}\tau^i(\partial_\mu W_\nu^i - \partial_\nu W_\mu^i) + g\epsilon_{ijk}W_\mu^j W_\nu^k$$

$$\begin{aligned} \Rightarrow \mathcal{O}_{\mathrm{S},0} = \quad & m_W^4 W^+\cdot W^+ W^-\cdot W^- \\ & + m_W^2 m_Z^2 W^+\cdot W^- Z\cdot Z \\ & + m_Z^4 Z\cdot Z Z\cdot Z \end{aligned}$$