Beyond Infrared-safe Jet Observables

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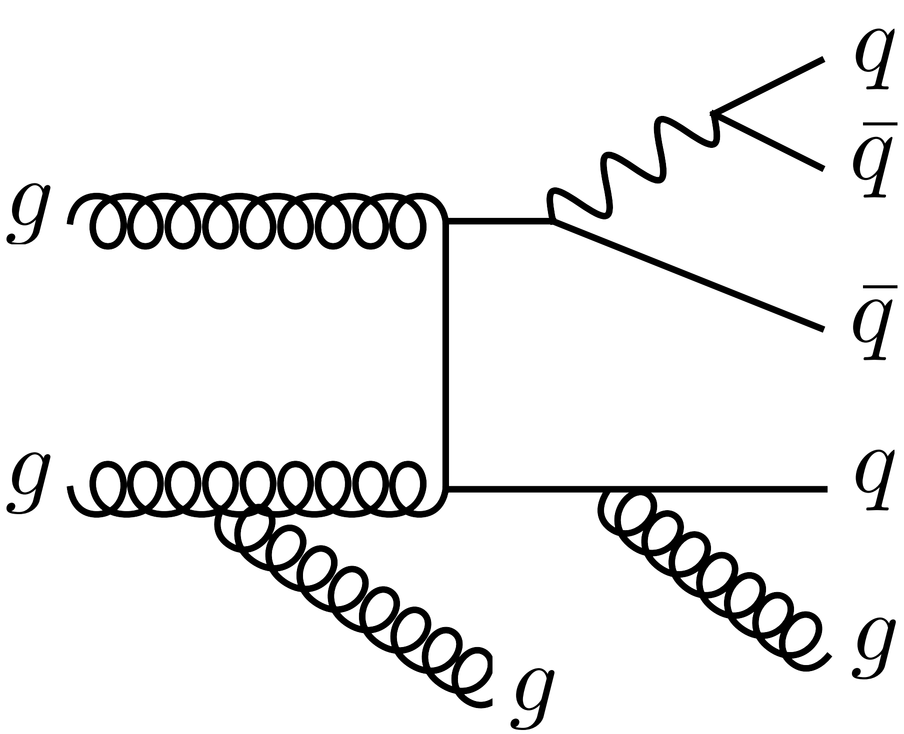
Outline

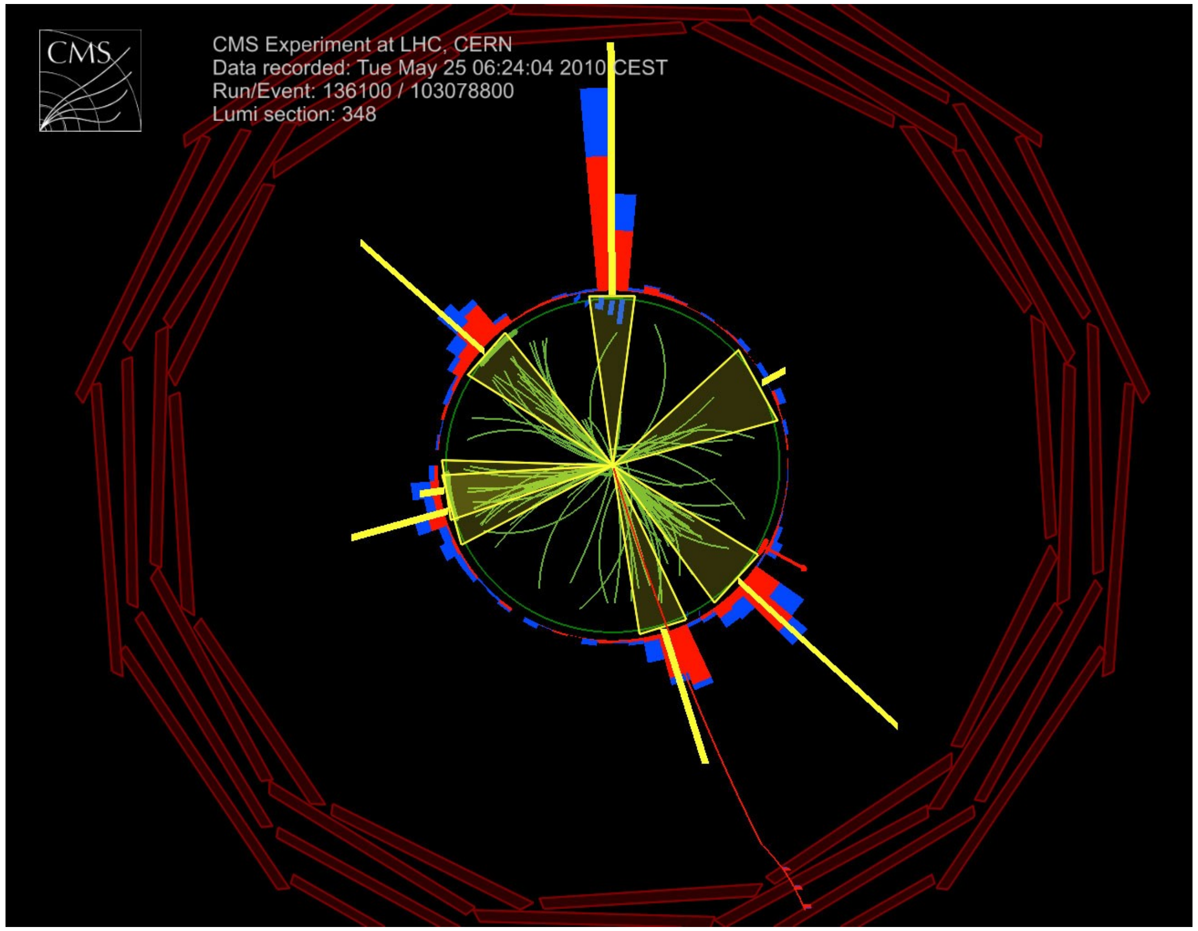
- Introduction to Jets
- Infrared Safety
- Jet Charge
- Ratio of Two Angularities
- Generalized Angularities
- Out-of-jet Hadron Multiplicity
- Conclusions

Introduction

What is a Jet?

Energetic quarks and gluons radiate and hadronize → Produce jets of hadrons

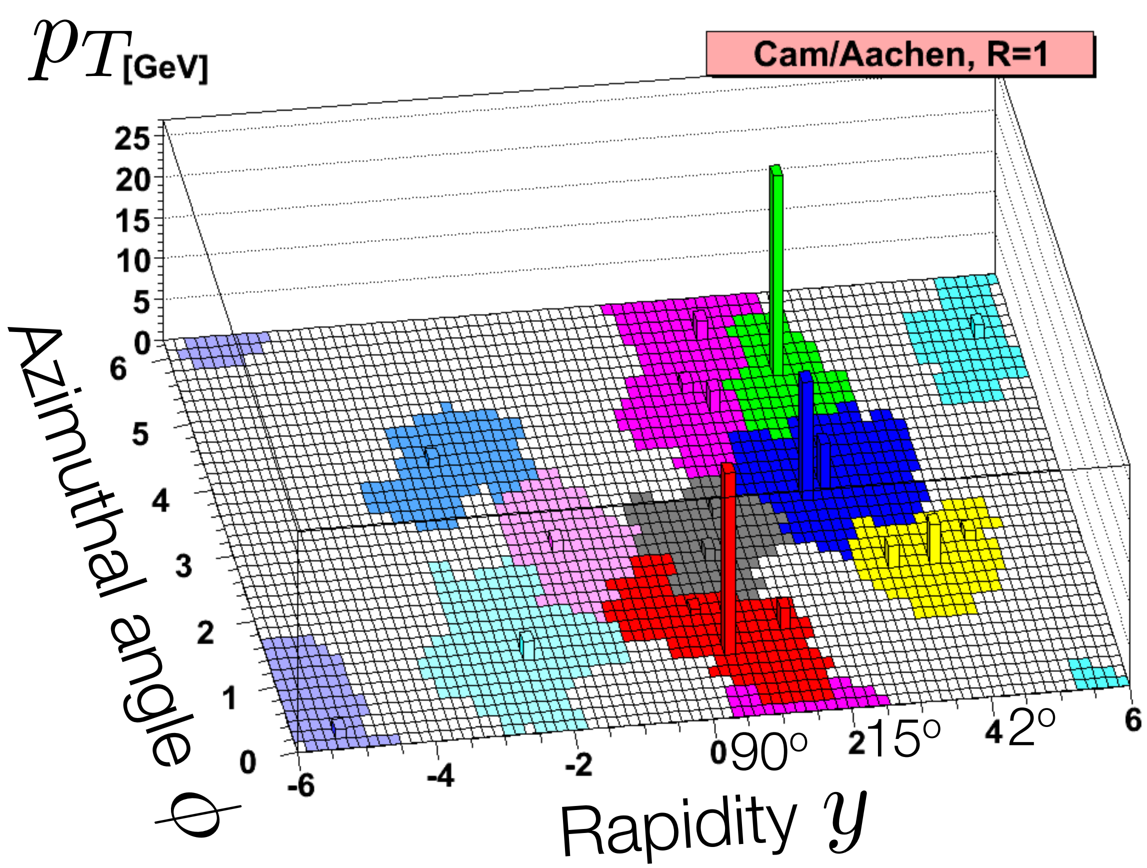




Jet Algorithms

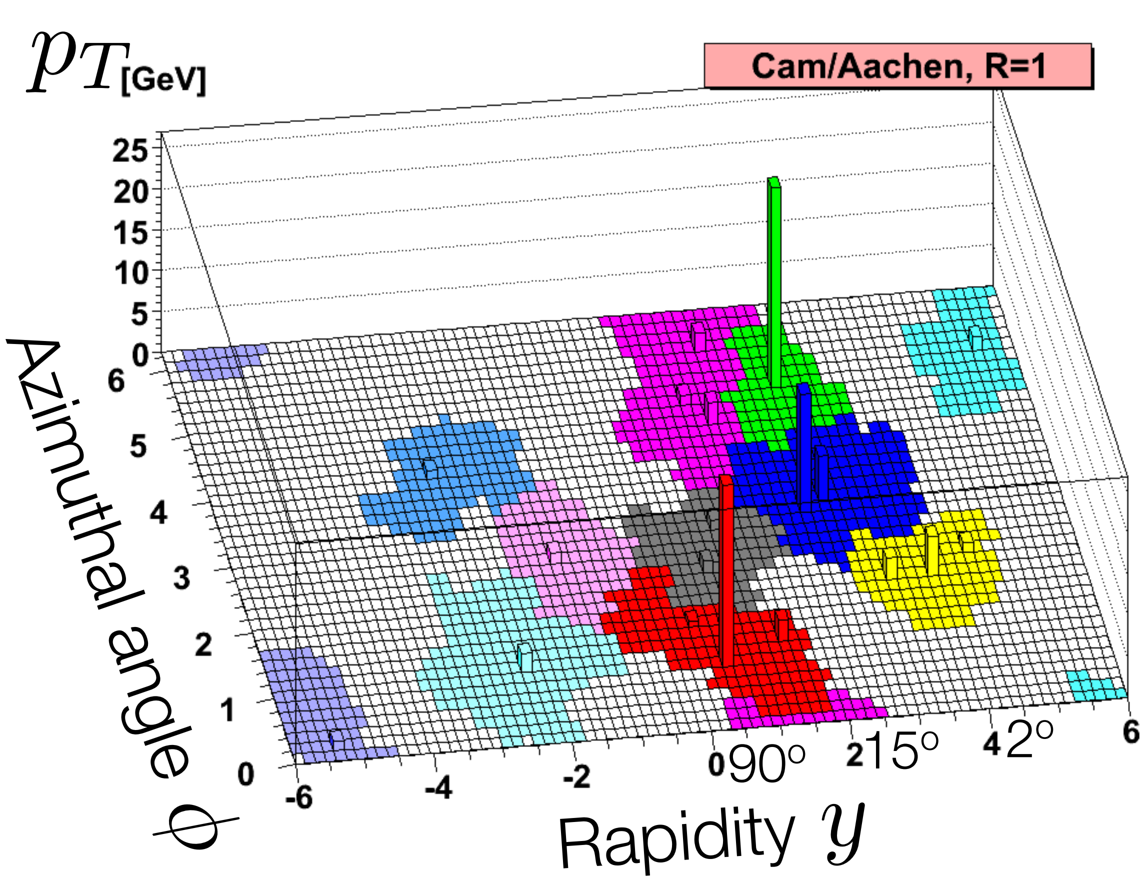
- Repeatedly cluster nearest "particles" $p_i, p_j \to p_i + p_j$
- Cut off by jet "radius" R

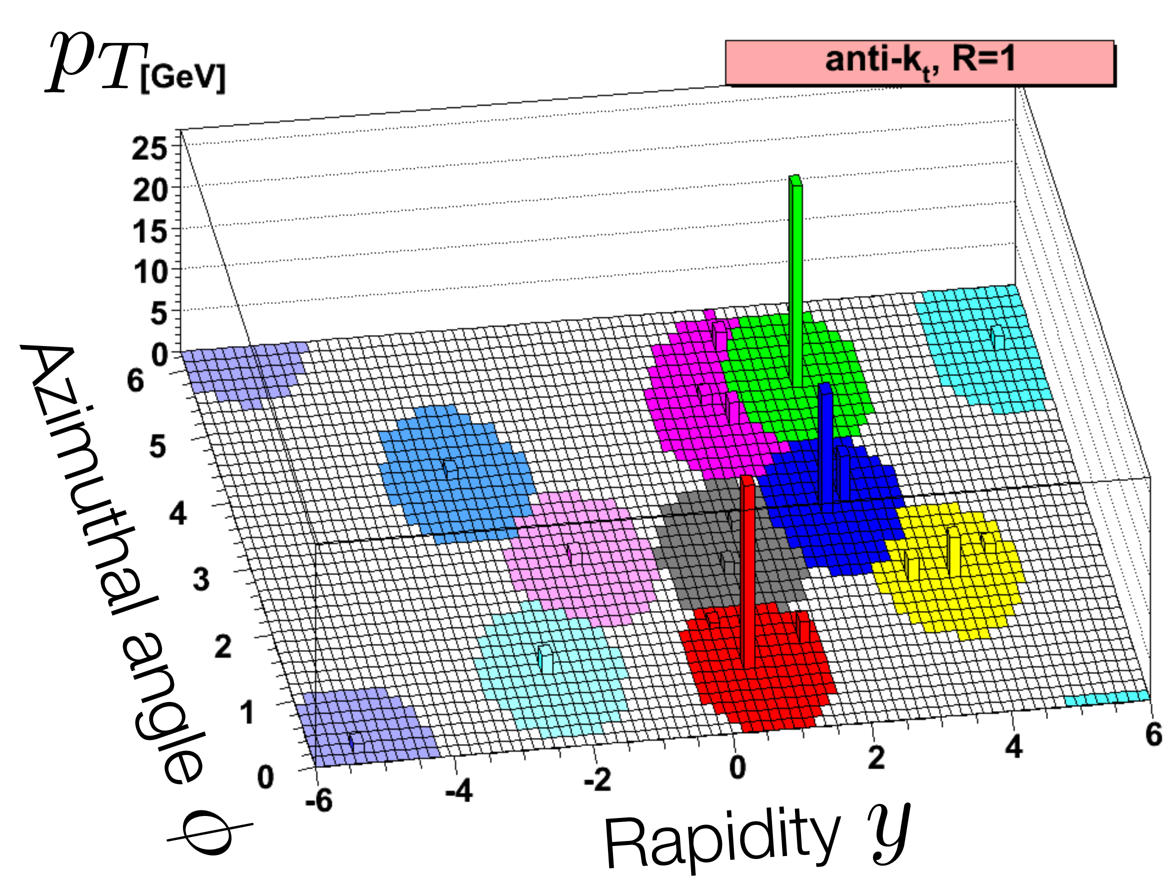
$$\text{distance} = (\Delta y)^2 + (\Delta \phi)^2$$



Jet Algorithms

- Repeatedly cluster nearest "particles" $p_i, p_j \to p_i + p_j$
- Cut off by jet "radius" R
- Default at LHC: anti-k_T (Cacciari, Salam, Soyez)

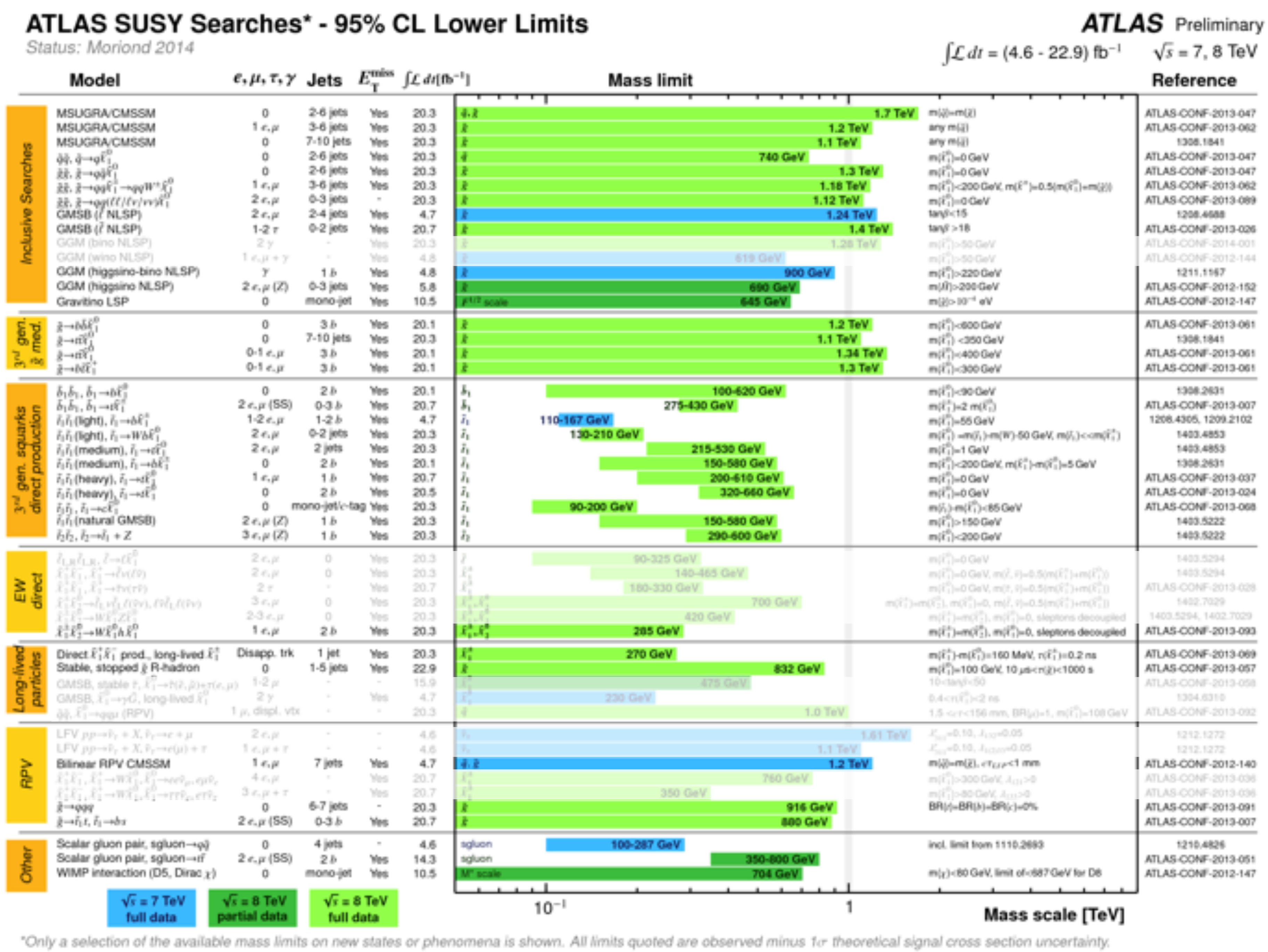




(arXiv:0802.1189)

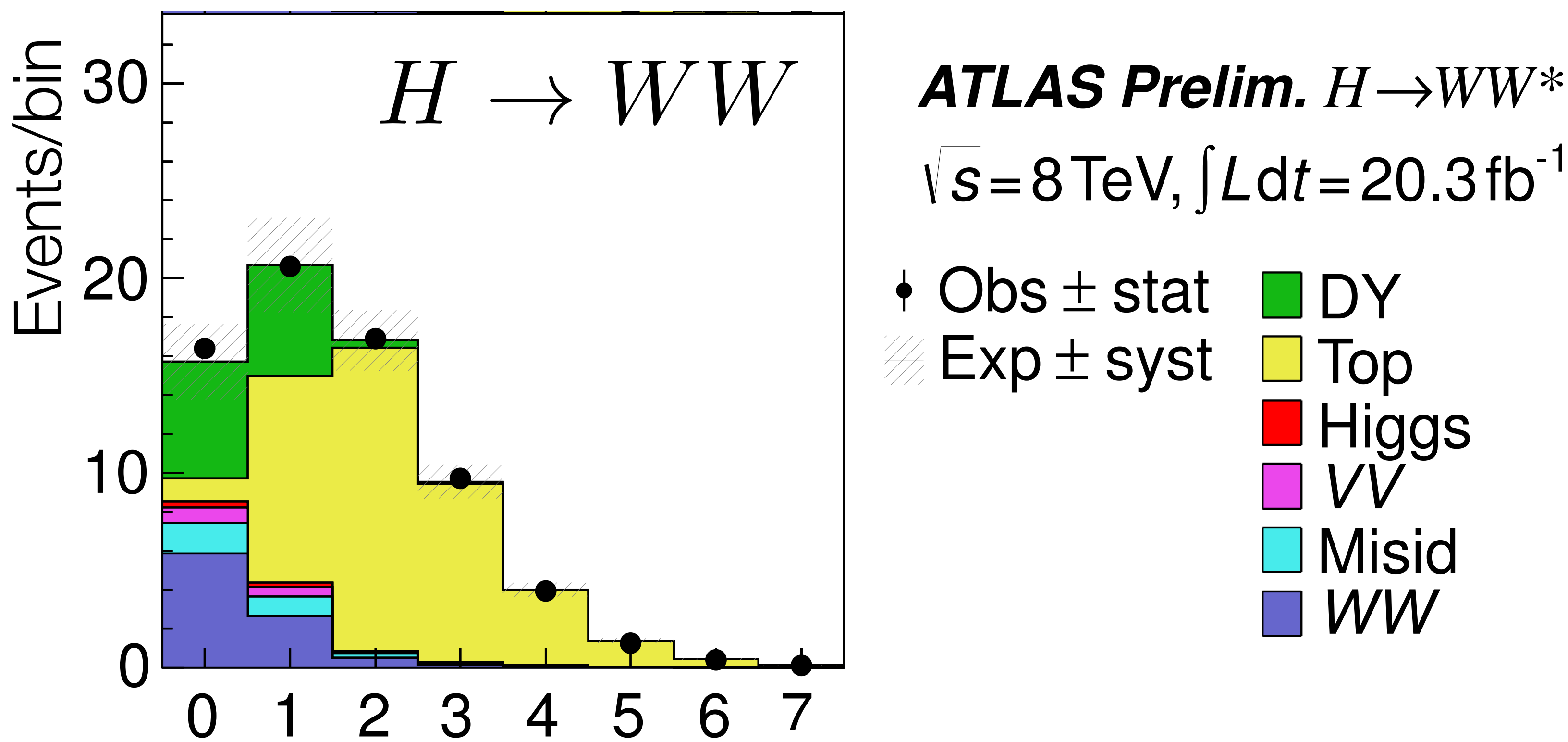
Jets at the LHC

- Most measurements involve jets as signal or background



Jet Cross Sections

- Bin by jet multiplicity to improve background rejection



(ATLAS-CONF-2014-060)

- Large logarithms lead to large theory uncertainties

no jets above this p_T

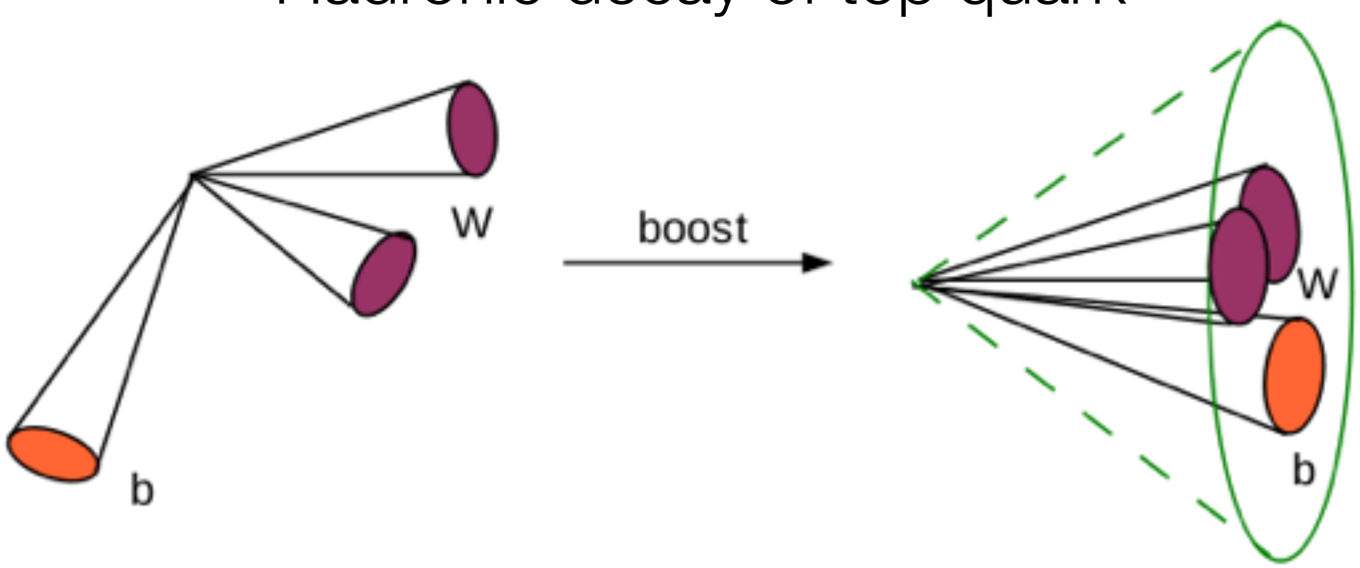
$$\sigma(H + 0 \text{ jets}) \propto 1 - \frac{6\alpha_s}{\pi} \ln^2 \frac{p_T^{\text{cut}}}{m_H} + \ldots$$

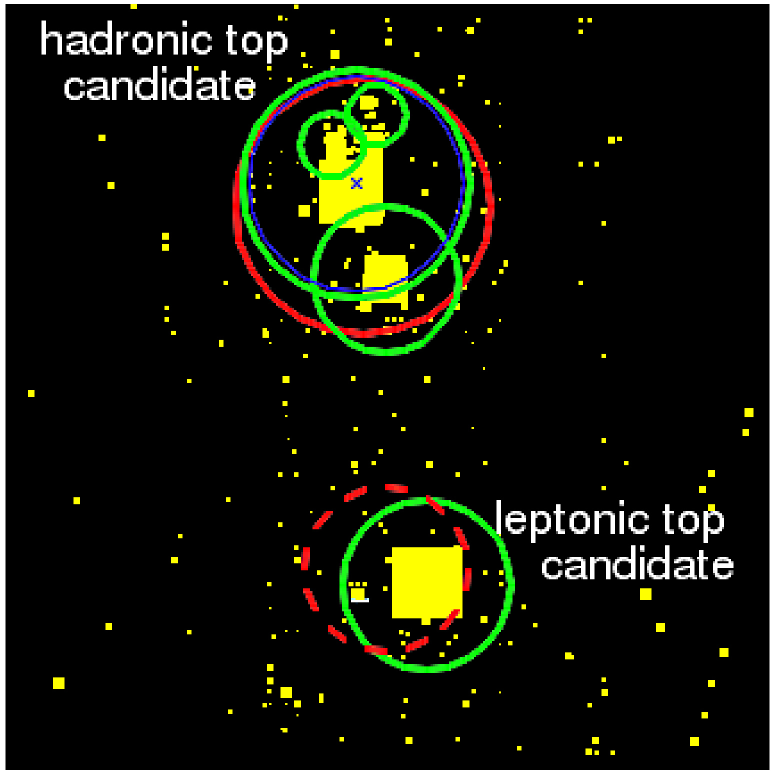
(Berger, Marcantonini, Stewart, Tackmann, WW; Banfi, Monni, Salam, Zanderighi; Becher, Neubert, Rothen; Stewart, Tackmann, Walsh, Zuberi; Liu, Petriello; …)

Jet Substructure for Boosted Objects

- New heavy particles could produce boosted top, W, Higgs
  → decay products lie within one "fat" jet
- Distinguish from QCD jets using jet substructure
- Avoids combinatorial background

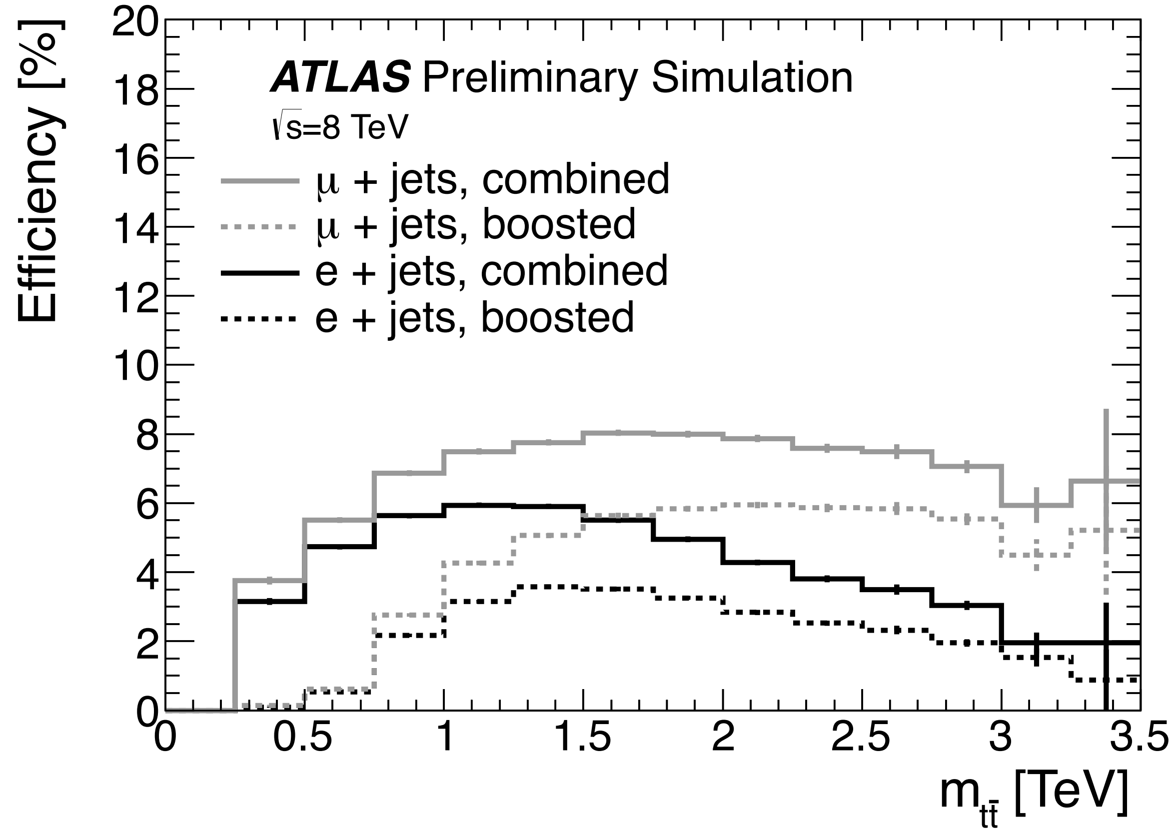
Hadronic decay of top quark

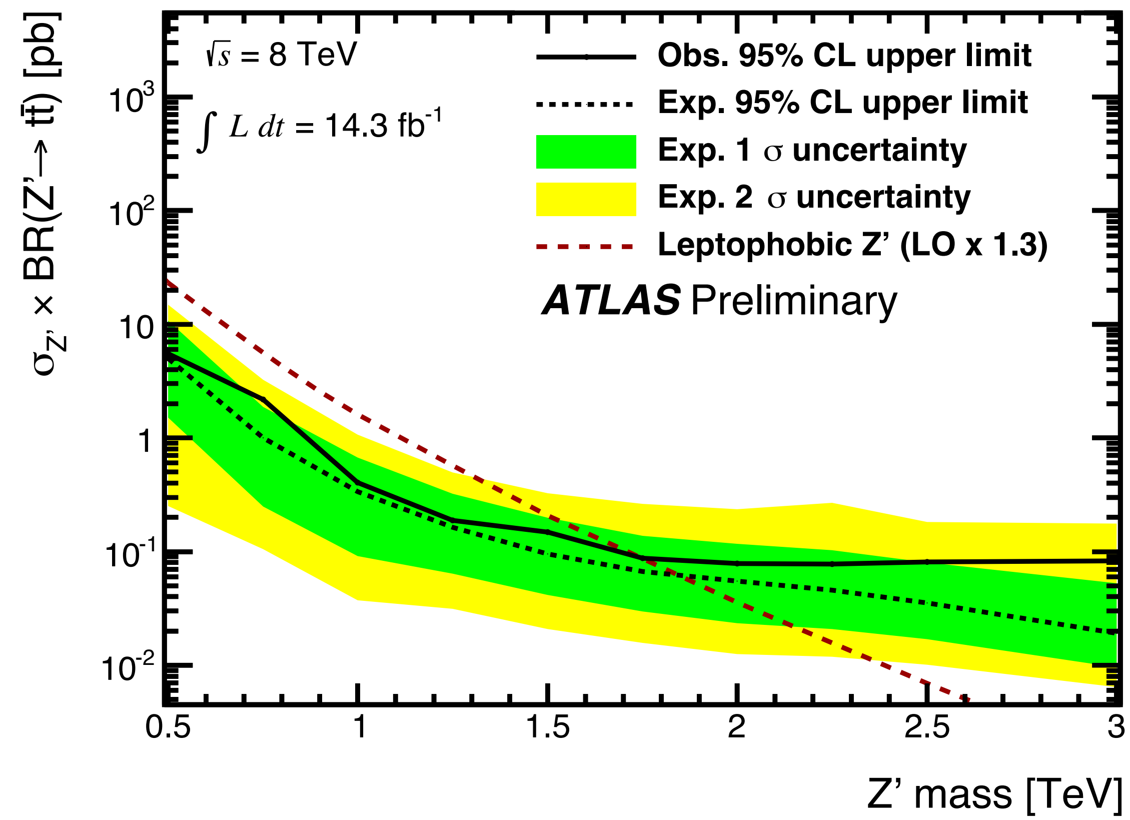




(ATLAS-CONF-2013-052)

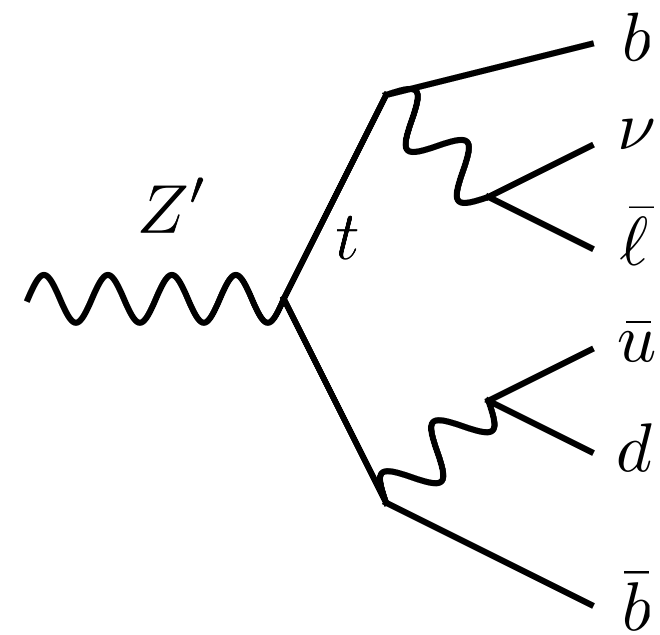
Top Tagging in $Z' \to t\bar{t}$





(ATLAS-CONF-2013-052)

- One leptonic and one hadronic top
- Boosted analysis crucial for large $m_{Z'}$

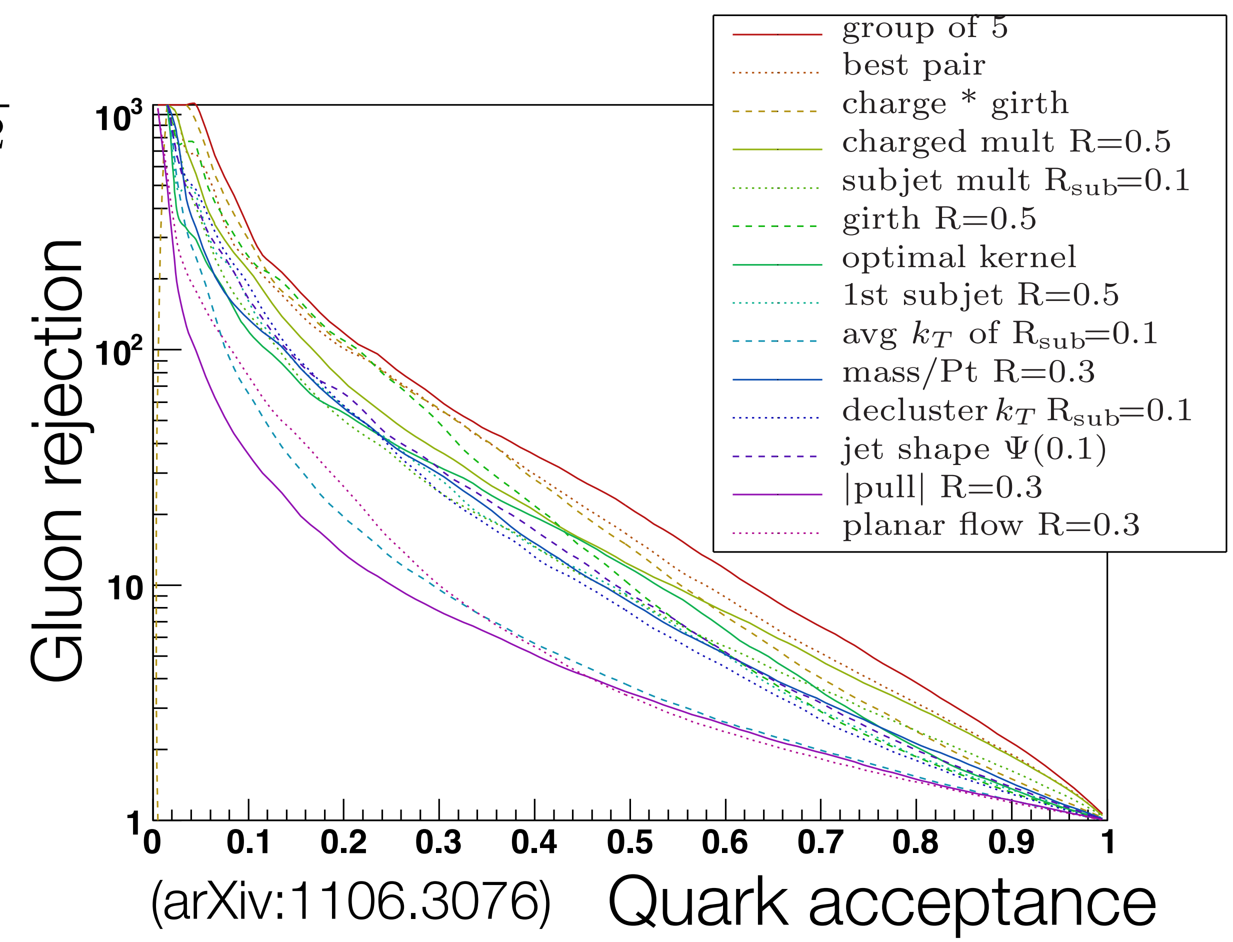


Jet Substructure for Quark/Gluon Discrimination

- New physics often more quarks than QCD backgrounds
- Extensive Pythia study (Gallicchio, Schwartz)
  - Charged hadron multiplicity and jet “girth” are good

$$\text{girth} = \sum_{i \in \text{jet}} \frac{p_T^i}{p_T^J} \sqrt{(y_i - y_J)^2 + (\phi_i - \phi_J)^2}$$

  - More variables only give marginal improvement

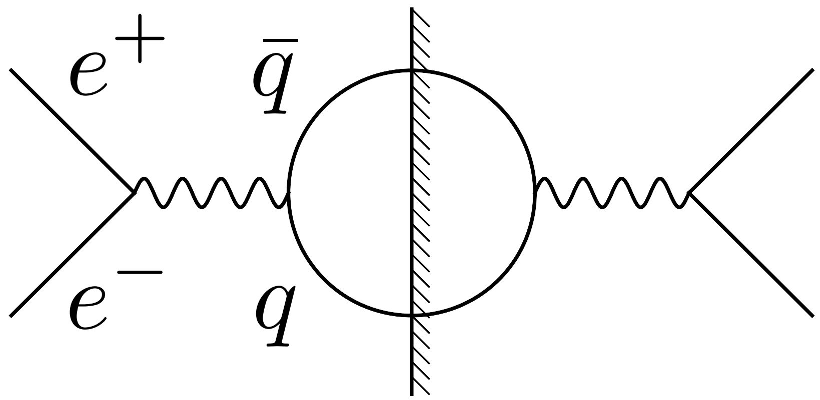


(arXiv:1106.3076)

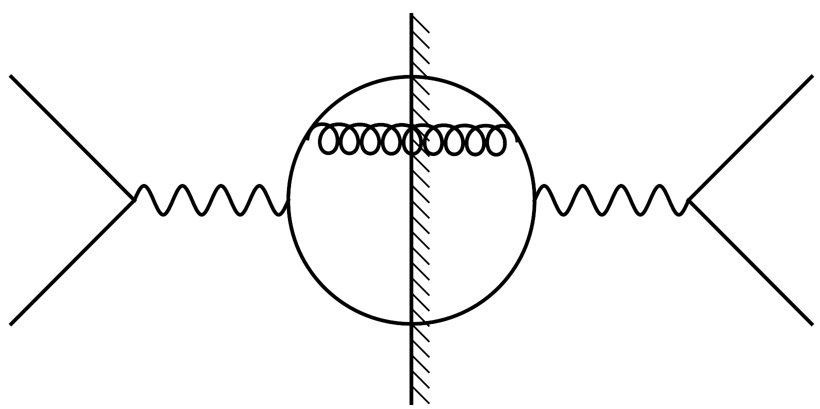
Infrared Safety

Soft and Collinear Divergences

- Tree:



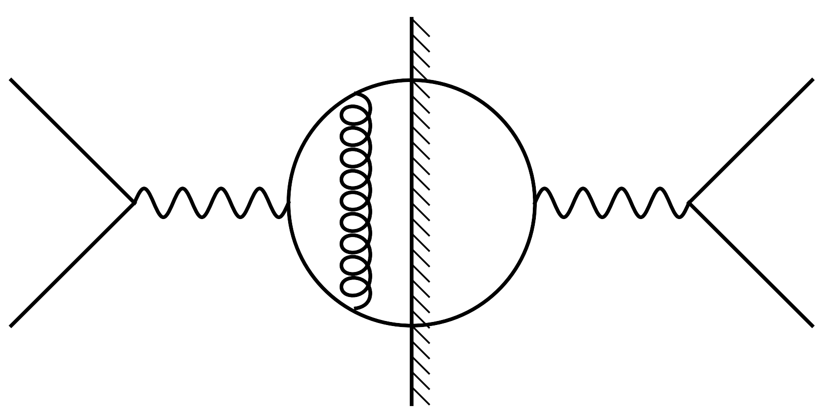
- Real: divergences from phase-space integration



$$\sigma_{\rm R} \sim \mu^{2\epsilon} \int \frac{dE}{E^{1+2\epsilon}} \, \frac{d\theta}{\theta^{1+2\epsilon}}$$

soft collinear

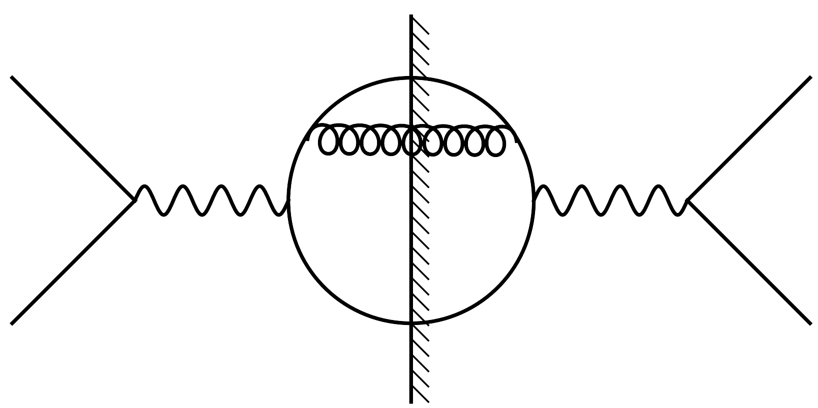
- Virtual: divergences from loop integration

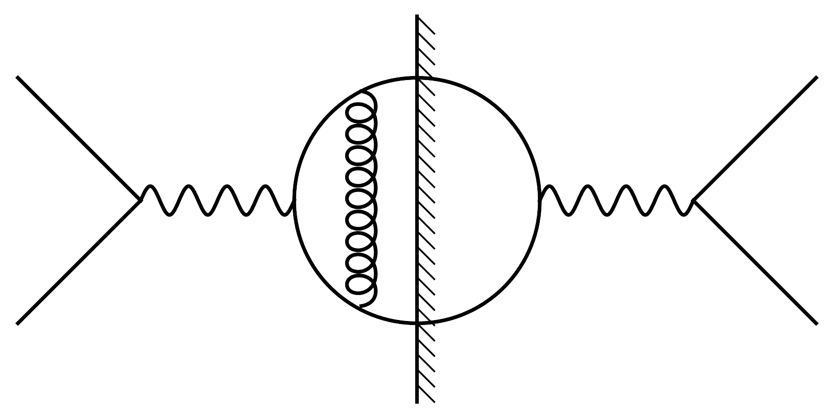


$$\sigma_{\rm V} \sim \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} + \ldots \right]$$

Infrared Safety

- IR divergences cancel between real and virtual corrections for IR safe measurements (Kinoshita, Lee, Nauenberg)
- IR safe: measure same value as for virtual when E or $\theta \to 0$
- E.g. quark energy E_q is not IR safe in $\theta \to 0$ limit





$$+\infty\, \delta(\theta)\, \delta(E_q - Q/2 - E) \qquad -\infty\, \delta(\theta)\, \delta(E_q - Q/2)$$

- IR safety has historically been an issue for jet algorithms

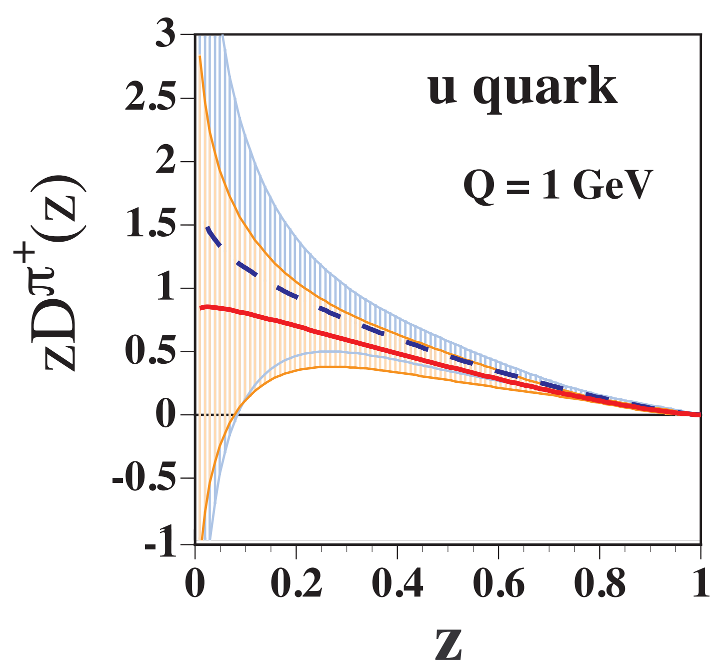
Hadronization Effects

- IR divergences are cut off by Λ_{QCD} in QCD
- IR unsafe → not (fully) calculable in perturbation theory
- One example: the energy fraction z of a hadron in e^+e^-

$$\frac{d\sigma_h}{dz} = \sum_{i=q,\bar{q},g} \int \frac{dx}{x} \frac{d\hat{\sigma}_i}{dx} D_i^h\left(\frac{z}{x}\right)$$

$$e^+e^- \to i(x) + \ldots$$

$$i(x) \to h(z) + \ldots$$



(hep-ph/0702250)

- Fragmentation function D is nonperturbative but process independent, "absorbing" divergences (Collins, Soper; …)

Motivation for IR Unsafe Jet Observables

- Experimentalists use them, e.g. in quark/gluon discrimination
- Questions for theorists:
  - Does this allow for powerful new observables?
  - How can we calculate them?

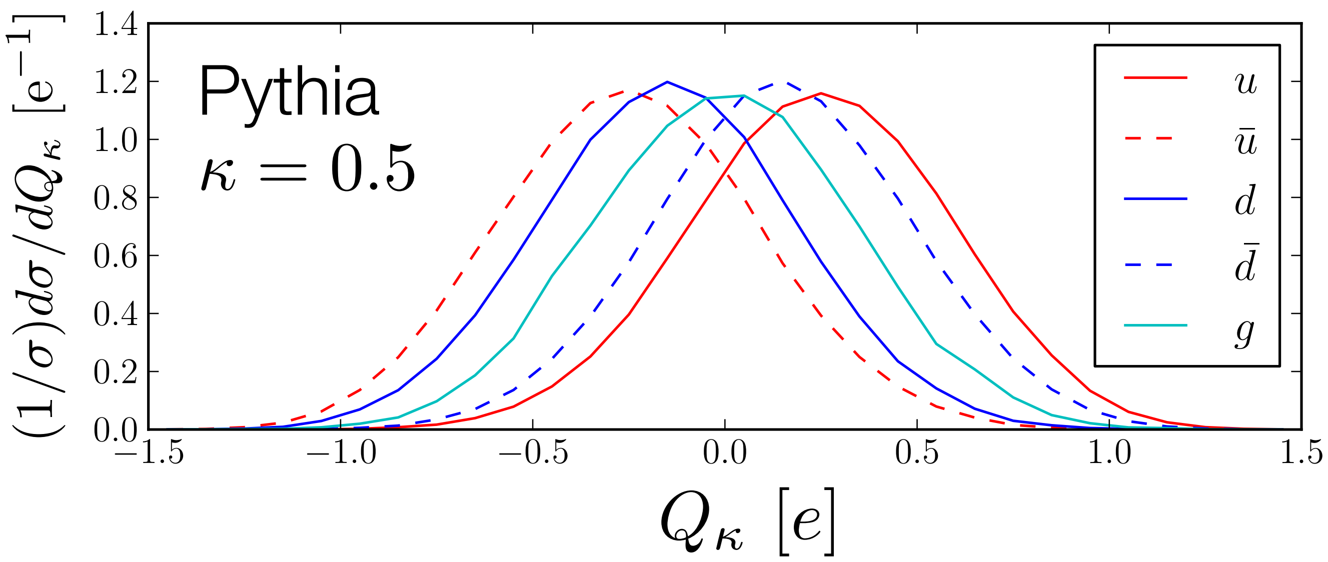
Jet Charge

Krohn, Lin, Schwartz, WW (arXiv:1209.2421)
WW (arXiv:1209.3091)

Defining Jet Charge

$$Q_\kappa = \sum_{i \in \text{jet}} Q_i \left(\frac{p_T^i}{p_T^J} \right)^\kappa$$

(Feynman, Field)

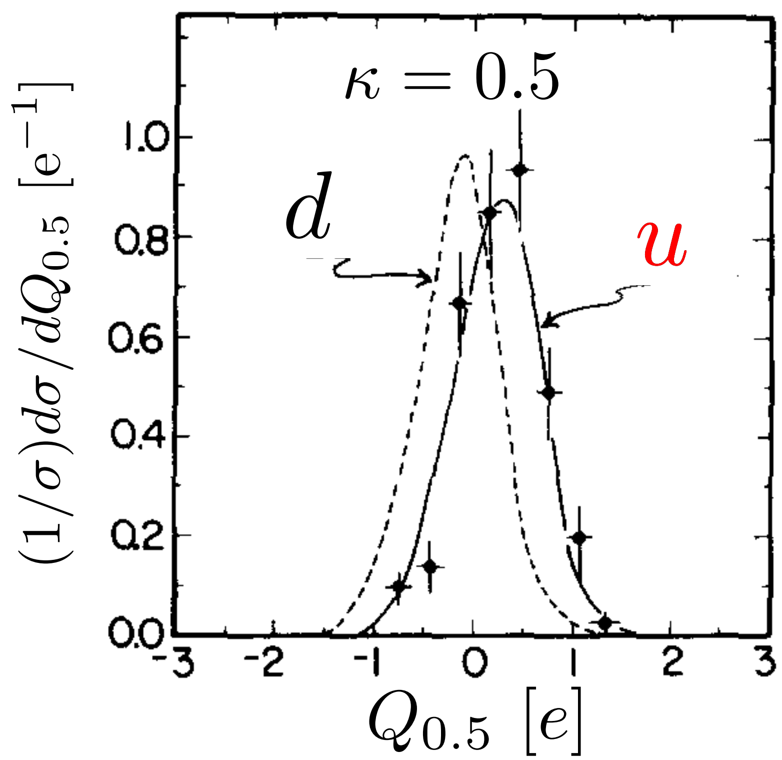


- If κ too small: sensitive to soft hadrons ➞ contamination
- If κ too large: only sensitive to most energetic hadron ➞ need more statistics

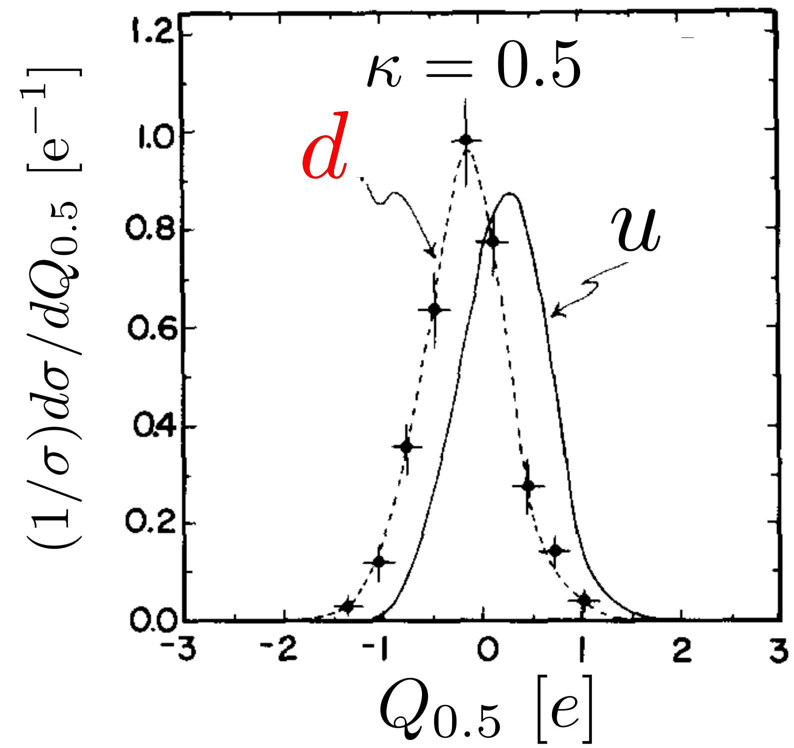
Historical Applications

- Test parton model

(Nucl. Phys. B184, 13 (1981))



$$\nu_\mu p \rightarrow \mu^- u X$$



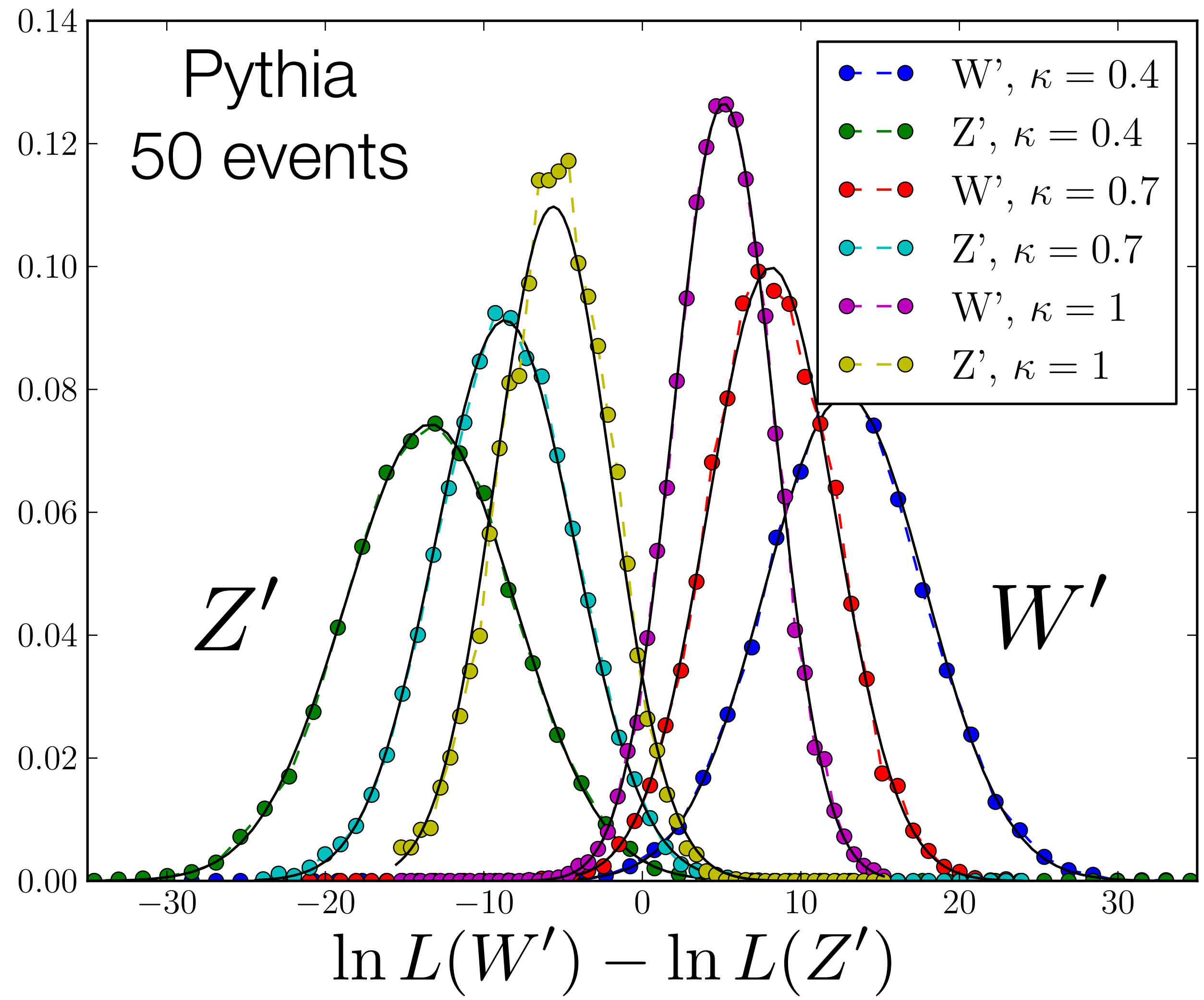
$$\bar{\nu}_\mu p \rightarrow \mu^+ d X$$

- Jet charge at LEP:
  - Forward-backward charge asymmetry (AMY (1990),…)
  - $B^0 \leftrightarrow \overline{B^0}$ mixing (ALEPH (1992), …)

Possible LHC application: W' vs. Z'

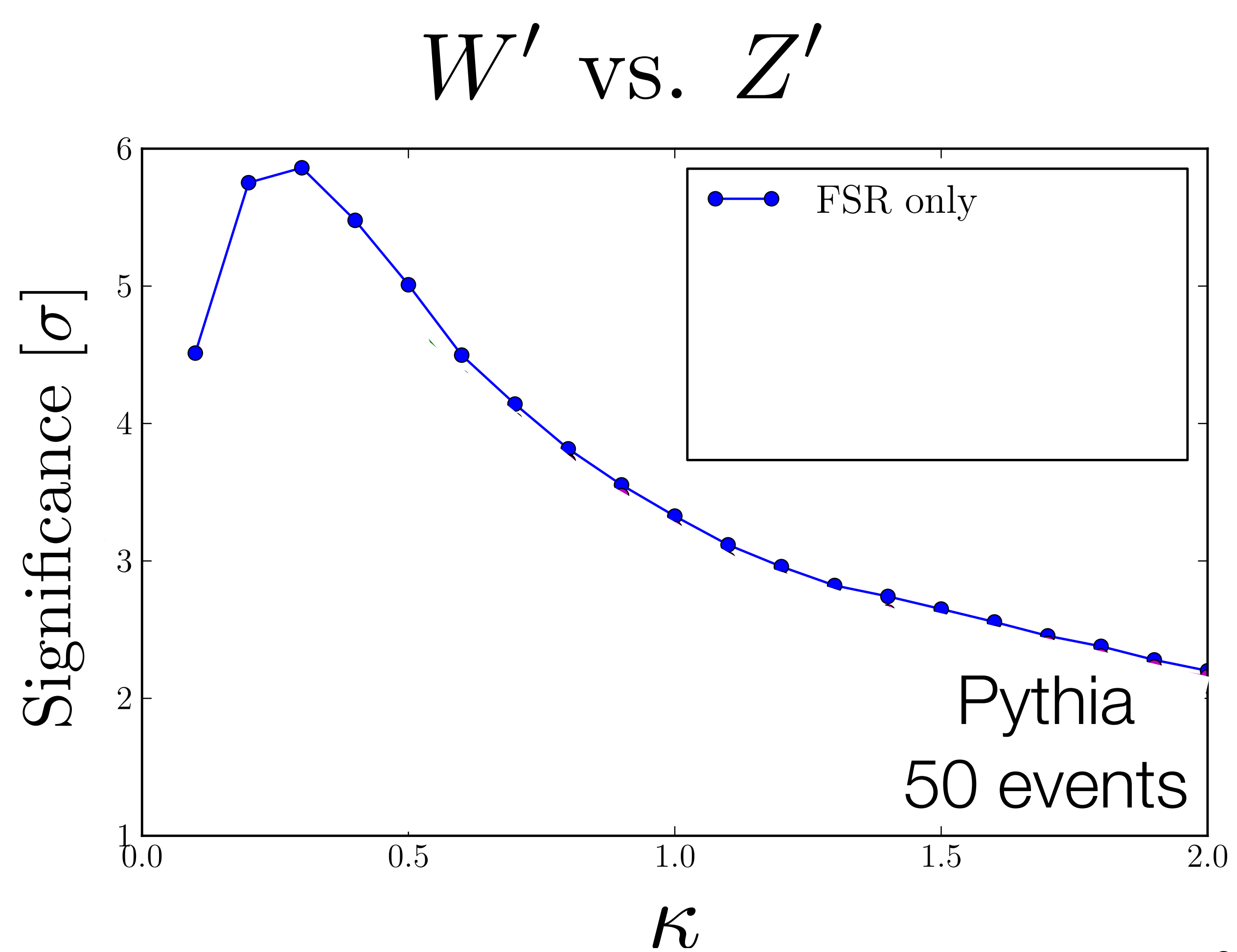
- Leptophobic W' or Z' with 1 TeV mass
- 2-dim. likelihood discriminant based on both jet charges

$$Z' \to u\bar{u}$$
$$Z' \to d\bar{d}$$
vs.
$$W' \to u\bar{d}$$
$$W' \to d\bar{u}$$



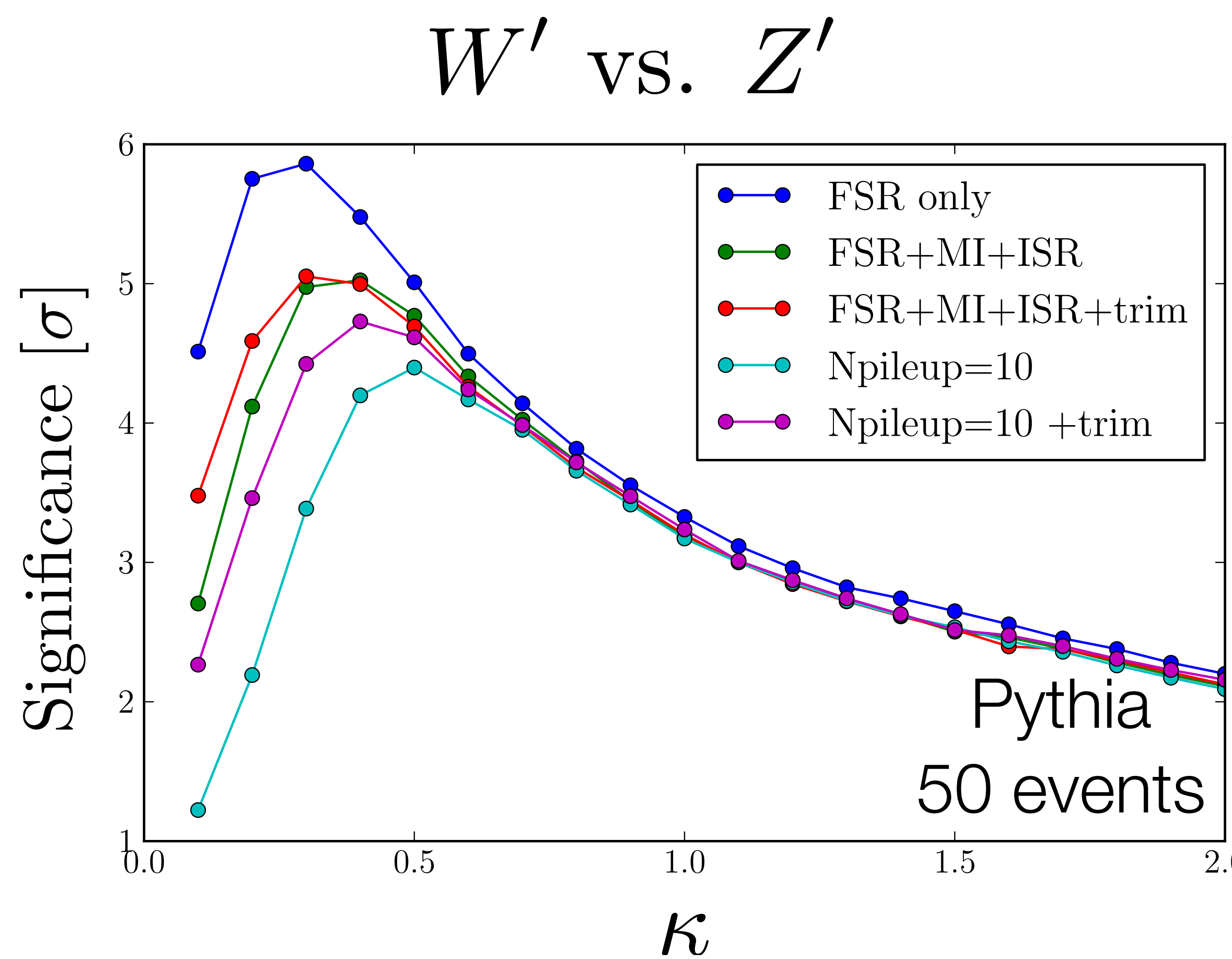
LHC Challenges

- Trade off between soft contamination and statistics
- We did not include: backgrounds, detector effects, …



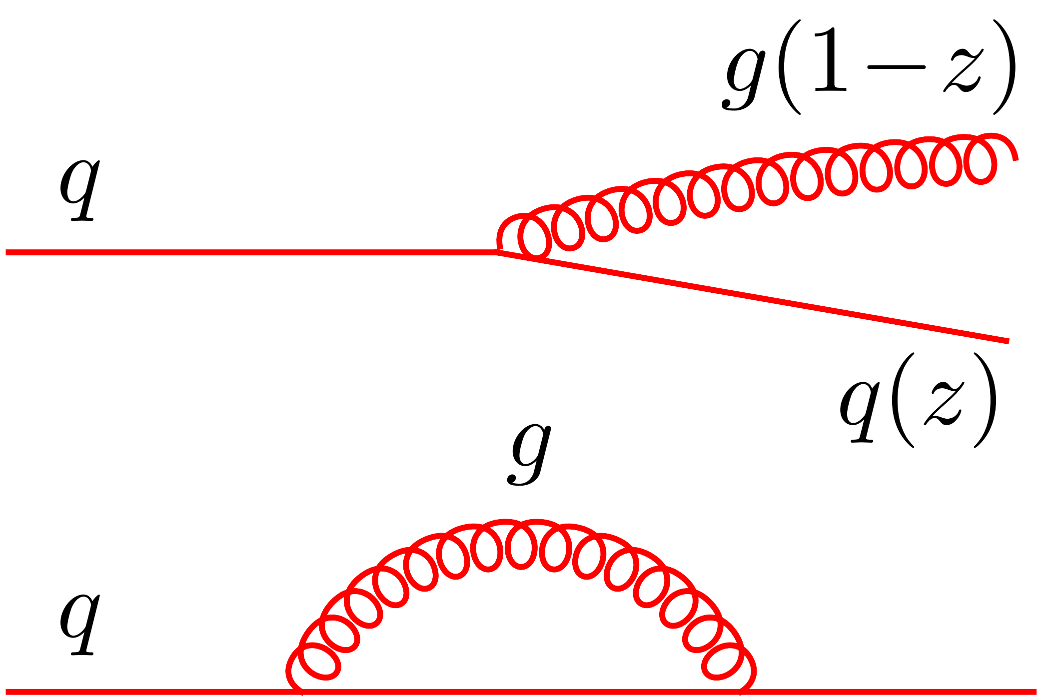
LHC Challenges

- Trade off between soft contamination and statistics
- We did not include: backgrounds, detector effects, …
- Various sources of contamination:
    - Initial-State Radiation
    - Multiparton Interactions
    - Pile-up
- All soft → increase κ



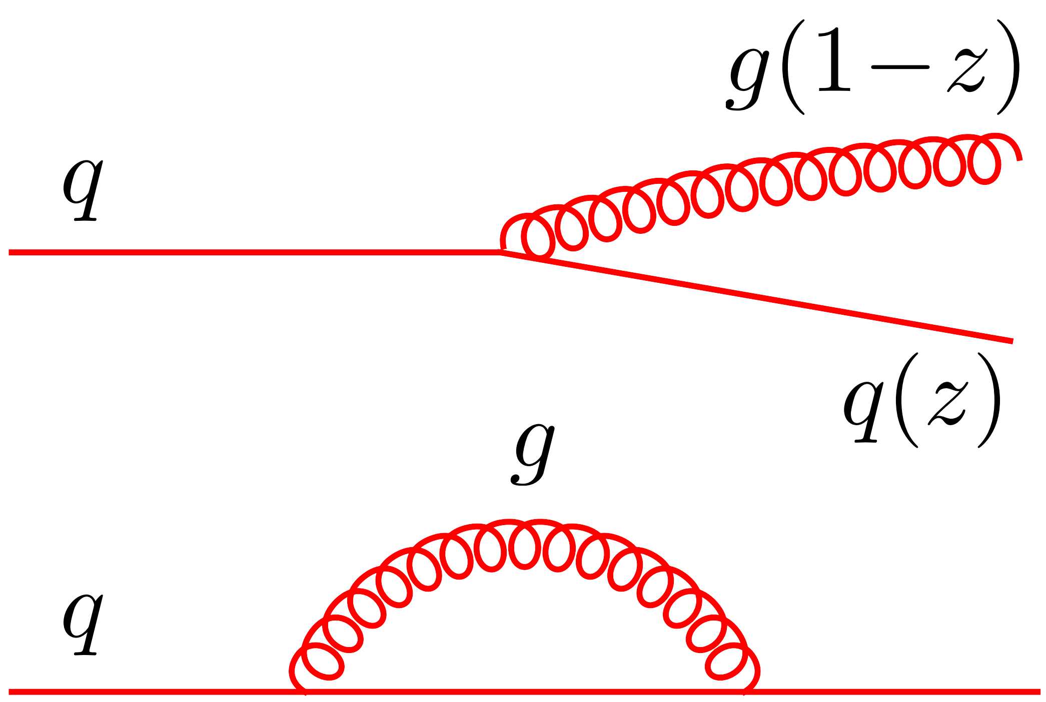
Jet Charge Not Infrared Safe

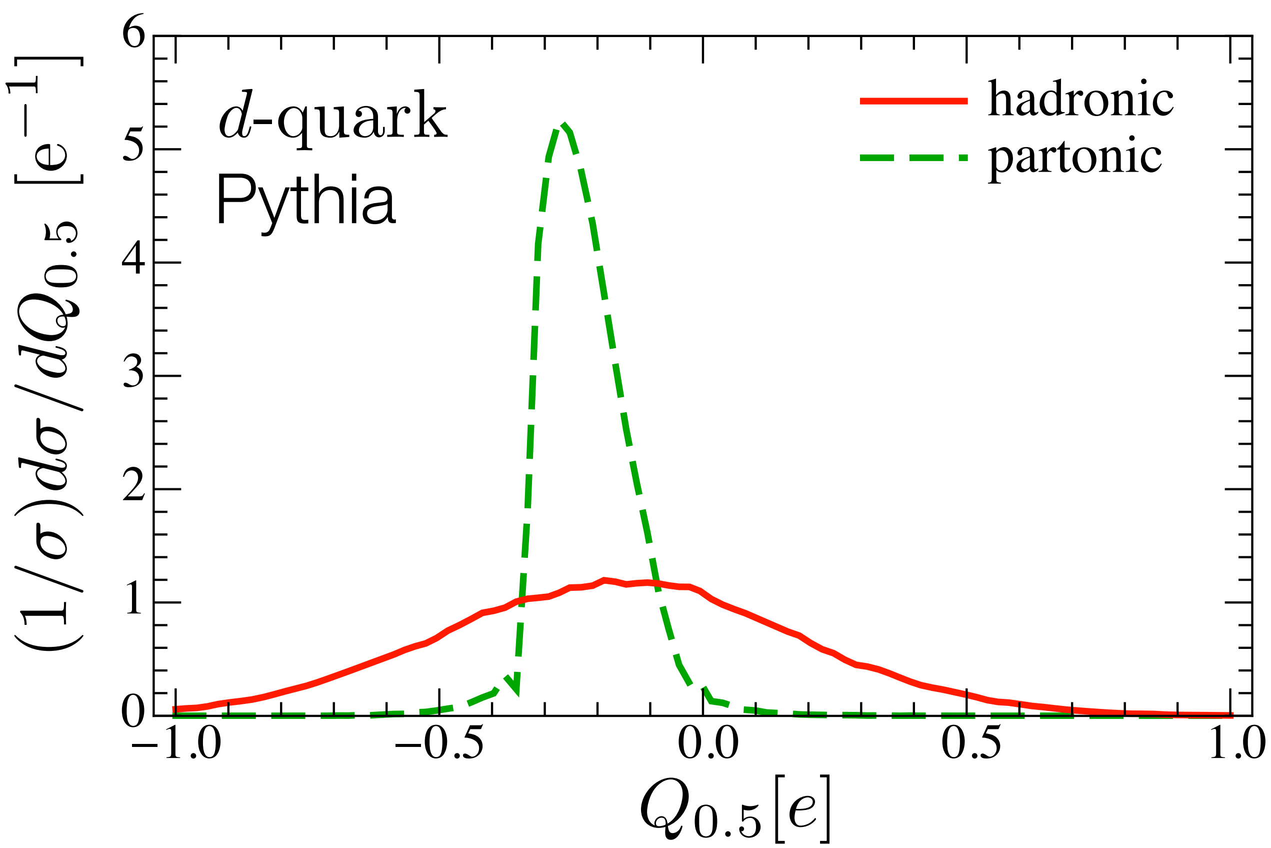
- Consider $q \to qg$ in collinear limit
- $Q_q z^\kappa \neq Q_q$ ➡ divergences don't cancel between real/virtual



Jet Charge Not Infrared Safe

- Consider $q \to qg$ in collinear limit
- $Q_q z^\kappa \neq Q_q \rightarrow$ divergences don't cancel between real/virtual
- Jet charge only defined for hadrons





Average Jet Charge Calculation

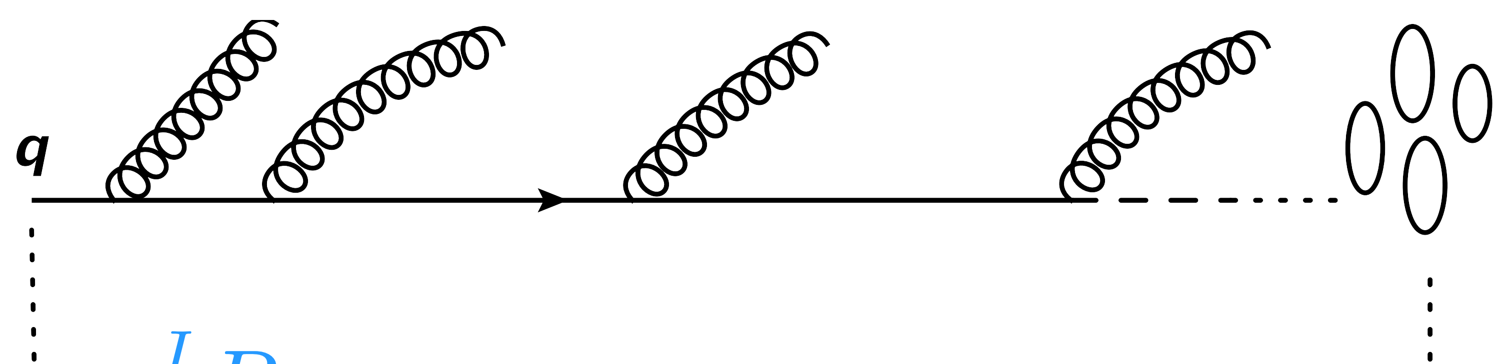
$$\langle Q_\kappa \rangle = \underbrace{\sum_h \int dz}_{\text{hadron } h} \ \underbrace{Q_h z^\kappa}_{\text{charge}} \ \underbrace{\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h\in\text{jet}}}{dz}}_{\text{weight}}$$

- At LO, weight = fragmentation function $D_q^h(z, \underbrace{\mu \sim p_T^J R}_{\text{Jet scale}})$

Average Jet Charge Calculation

$$\langle Q_\kappa \rangle = \underbrace{\sum_h \int dz}_{\text{hadron } h} \; \underbrace{Q_h z^\kappa}_{\text{charge}} \; \underbrace{\frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h\in\text{jet}}}{dz}}_{\text{weight}}$$

- At LO, weight = fragmentation function $D_q^h(z, \mu \sim p_T^J R)$ (Jet scale)
- Calculate p_T^J, R dependence from evolution to $\mu \sim \Lambda_{\text{QCD}}$
- $D_q^h(z, \mu \sim \Lambda_{\text{QCD}})$ describes hadronization

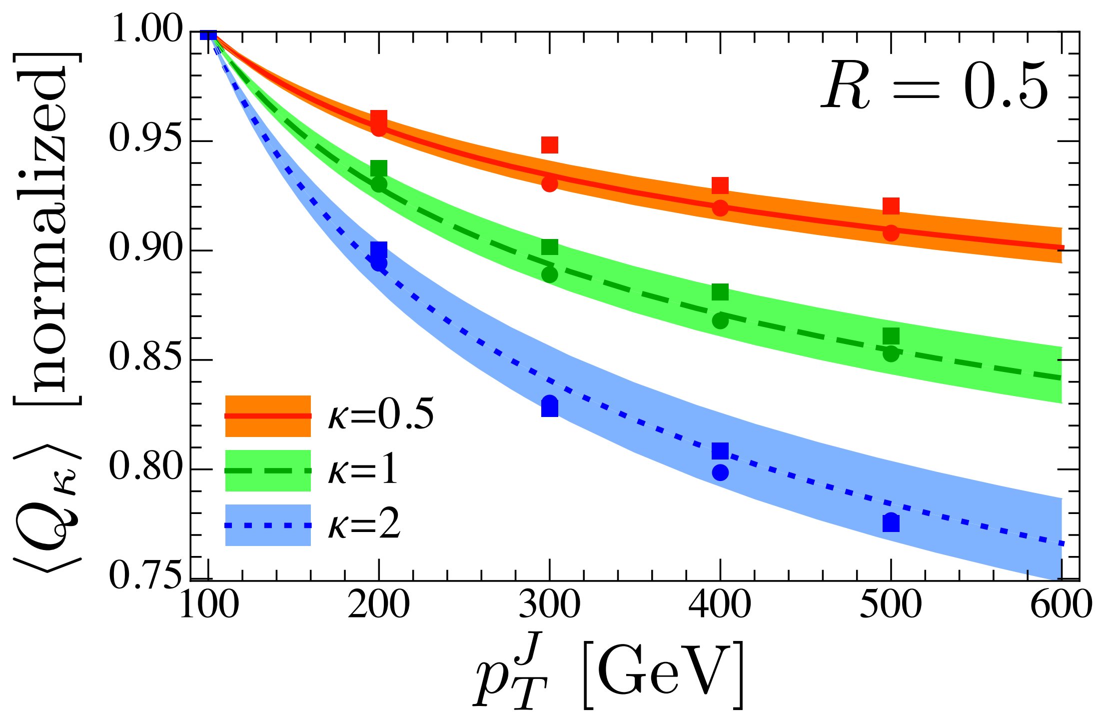


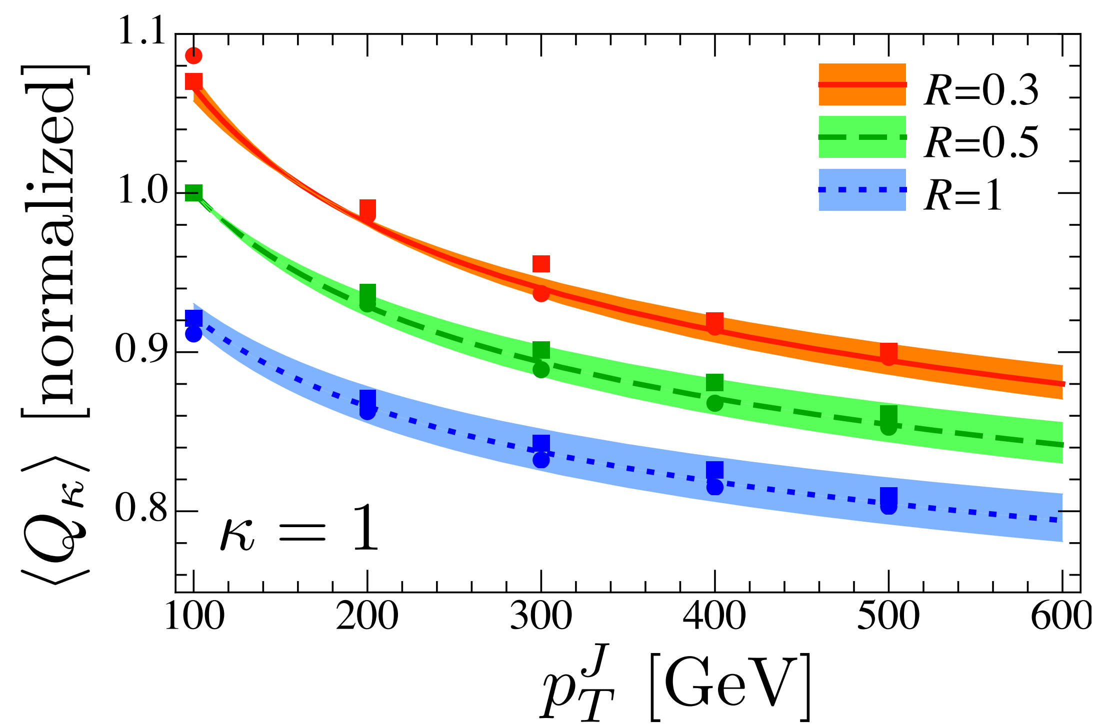
RG Evolution vs. Pythia's Parton Shower

$$\langle Q_\kappa(p_T^J R, \text{flavor})\rangle = \underbrace{\text{perturbative}}_{\text{perturbative splitting + evolution}}(\kappa, p_T^J R) \times \text{hadronization}(\kappa, \text{flavor})$$

- Normalize average jet charge: $\dfrac{\langle Q_\kappa(p_T^J R)\rangle}{\langle Q_\kappa(50\ \text{GeV})\rangle}$

→ Hadronization (and flavor dependence) drops out





✓ Good agreement

Fragmentation Functions vs. Pythia's Hadronization

- Average jet charge at $p_T^J = 100\ \mathrm{GeV},\ R = 0.5$

κ	u-quark			d-quark		
	PYTHIA	DSS	AKK08	PYTHIA	DSS	AKK08
0.5	0.271	0.237	0.221	-0.162	-0.184	-0.062
1	0.144	0.122	0.134	-0.078	-0.088	-0.046
2	0.055	0.046	0.064	-0.027	-0.030	-0.027

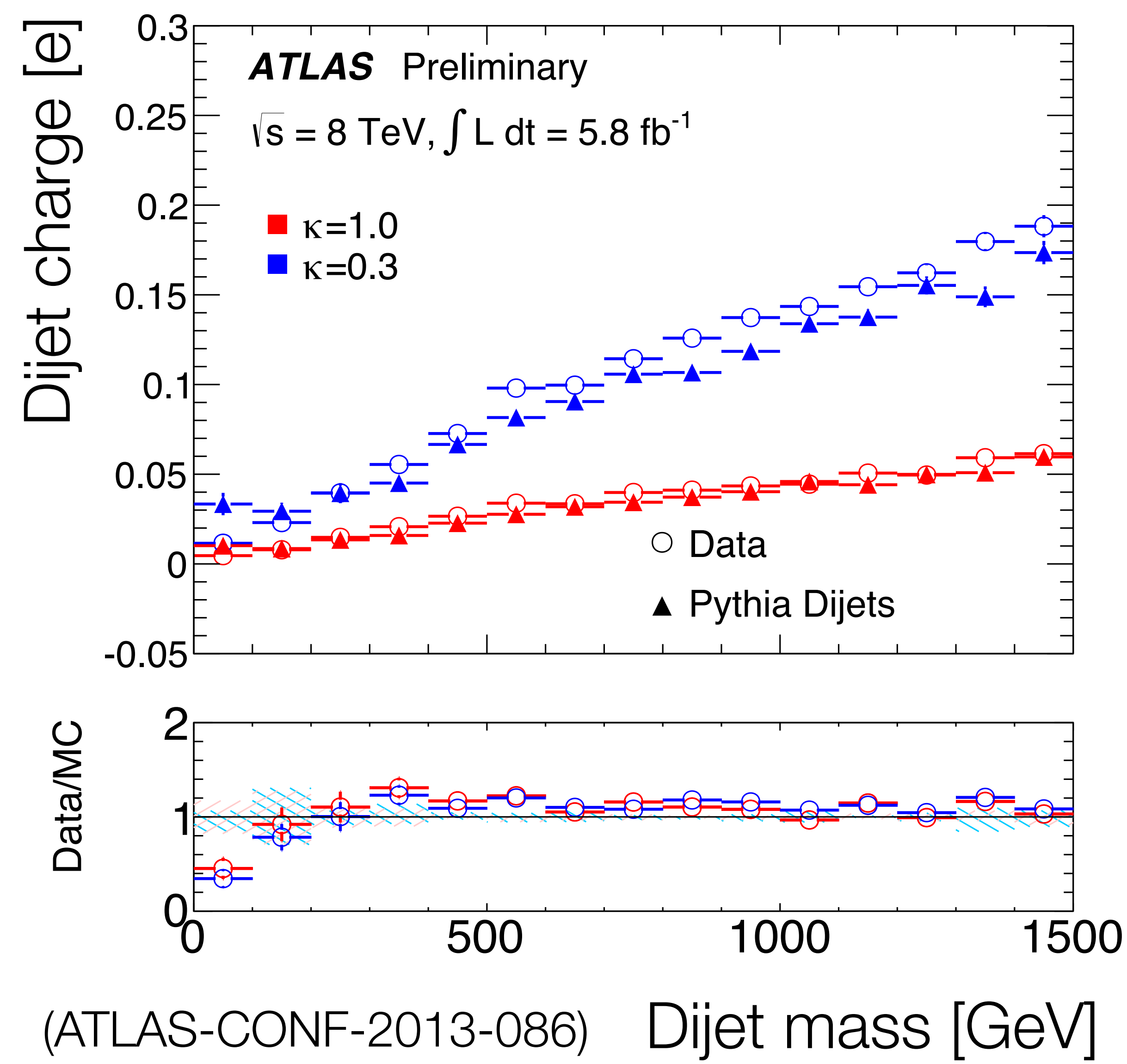
(DSS = De Florian, Sassot, Stratmann, AKK08 = Albino, Kniehl, Kramer)

✓ Pythia consistent with fragmentation functions

- Large uncertainties as we need $D_q^{h^+} - D_q^{h^-} = D_q^{h^+} - D_{\bar{q}}^{h^+}$
  Most fragmentation data is e^+e^- giving $D_q^{h^+} + D_{\bar{q}}^{h^+}$

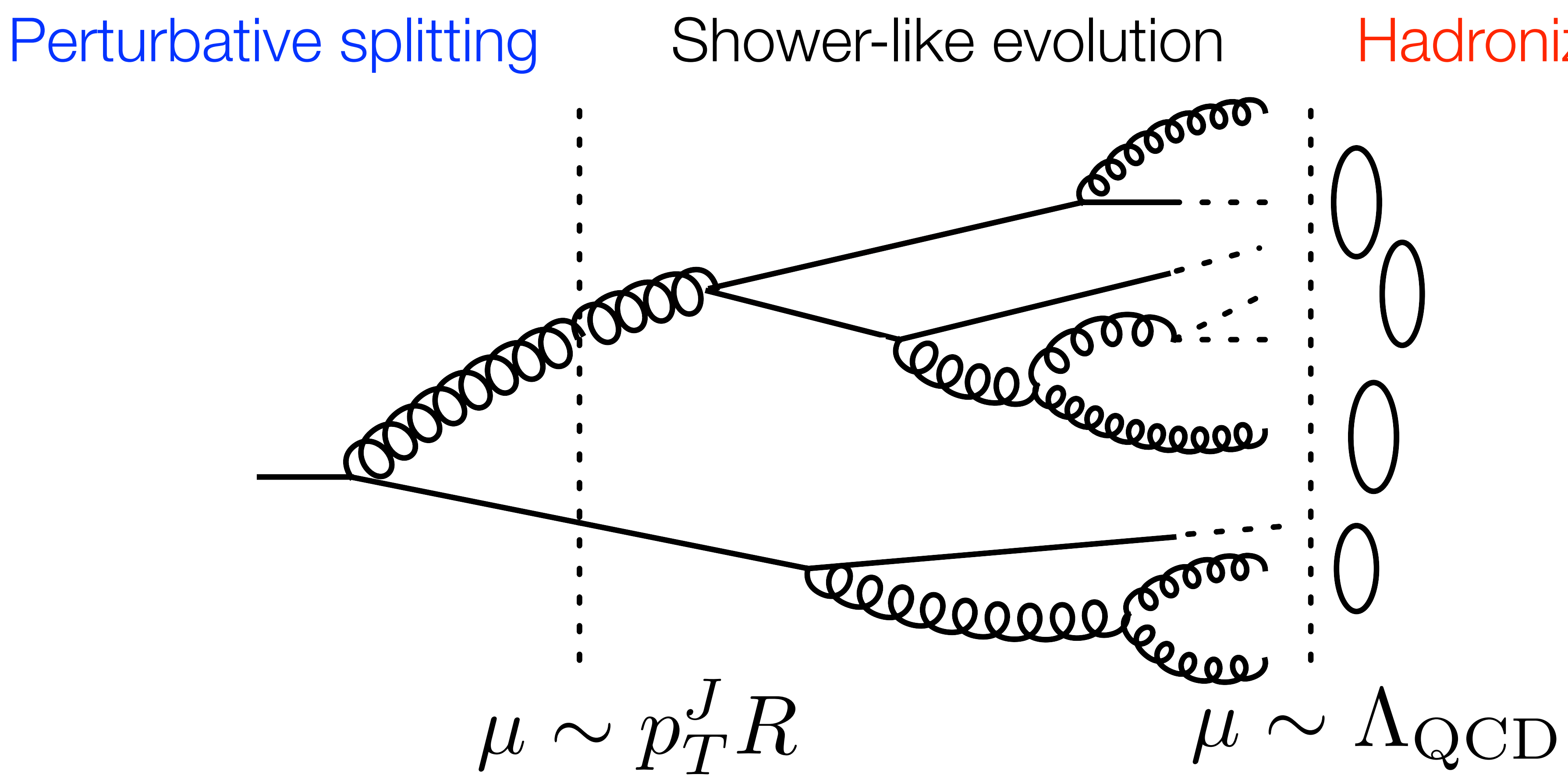
Average Dijet Charge at the LHC

- Depends on proton structure and scattering process
- Pure QCD measurement of valence structure of proton
- Study of scale violation effect is ongoing



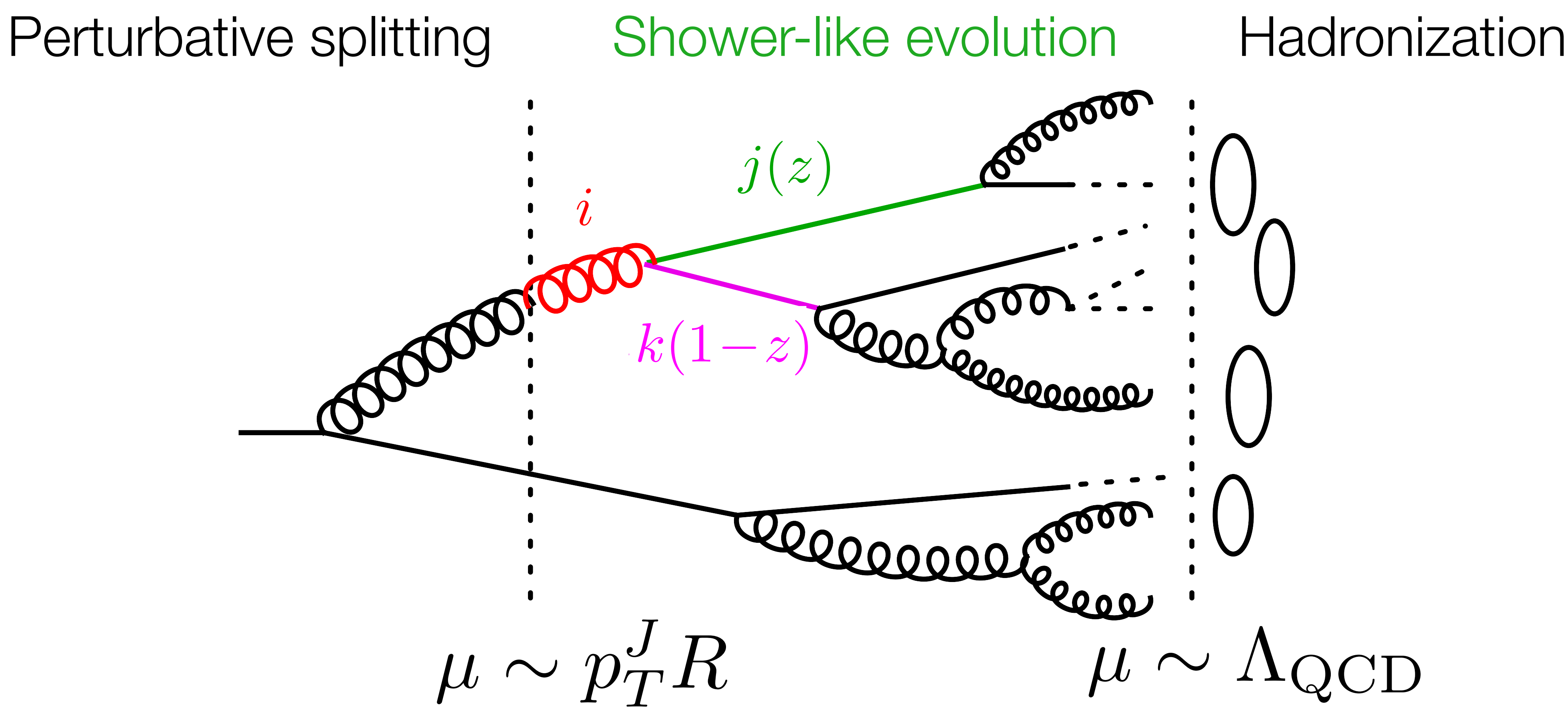
(ATLAS-CONF-2013-086)

Full Jet Charge Distribution



- Perturbative splitting reduces μ-dependence (Jain, Procura, WW)
- Hadronization depends on full charge distribution $D_i(Q_\kappa, \mu)$
  - Moments related to multi-hadron fragmentation functions

Full Jet Charge Distribution



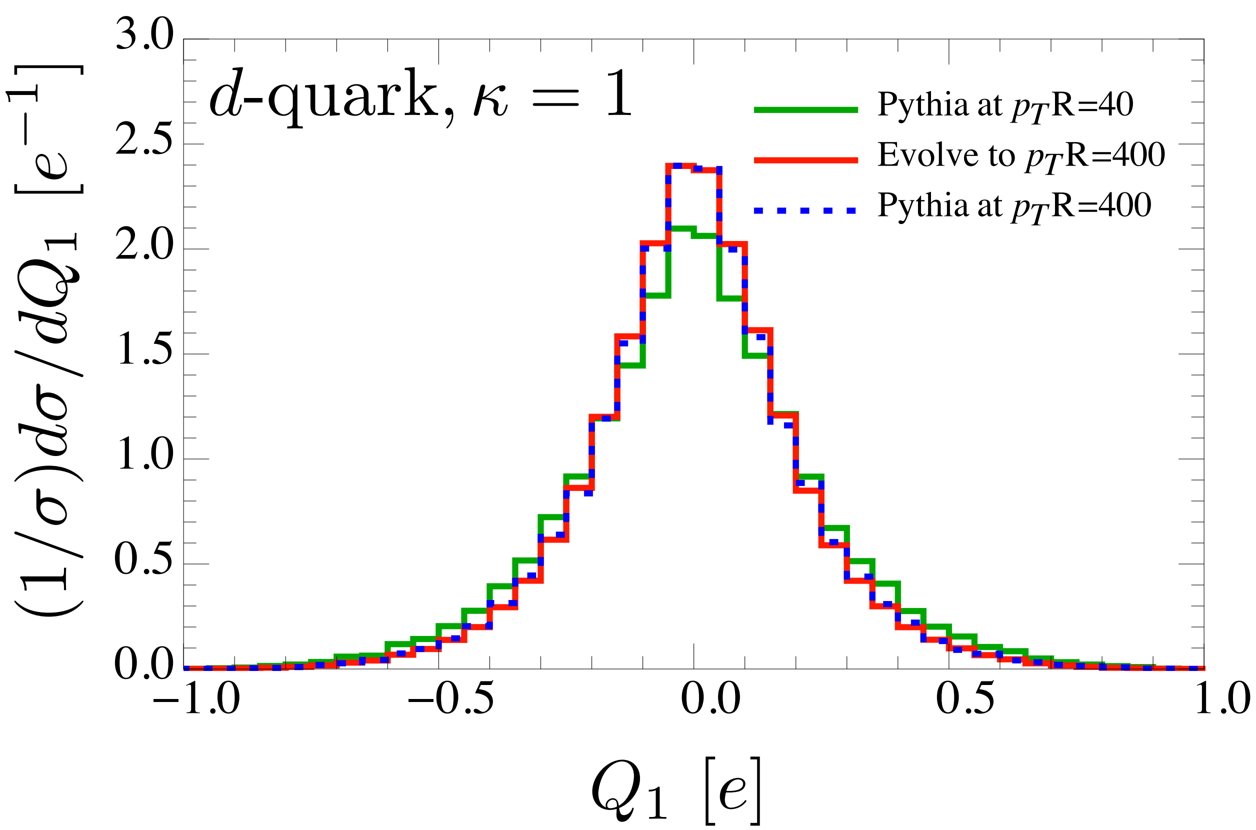
- RGE:

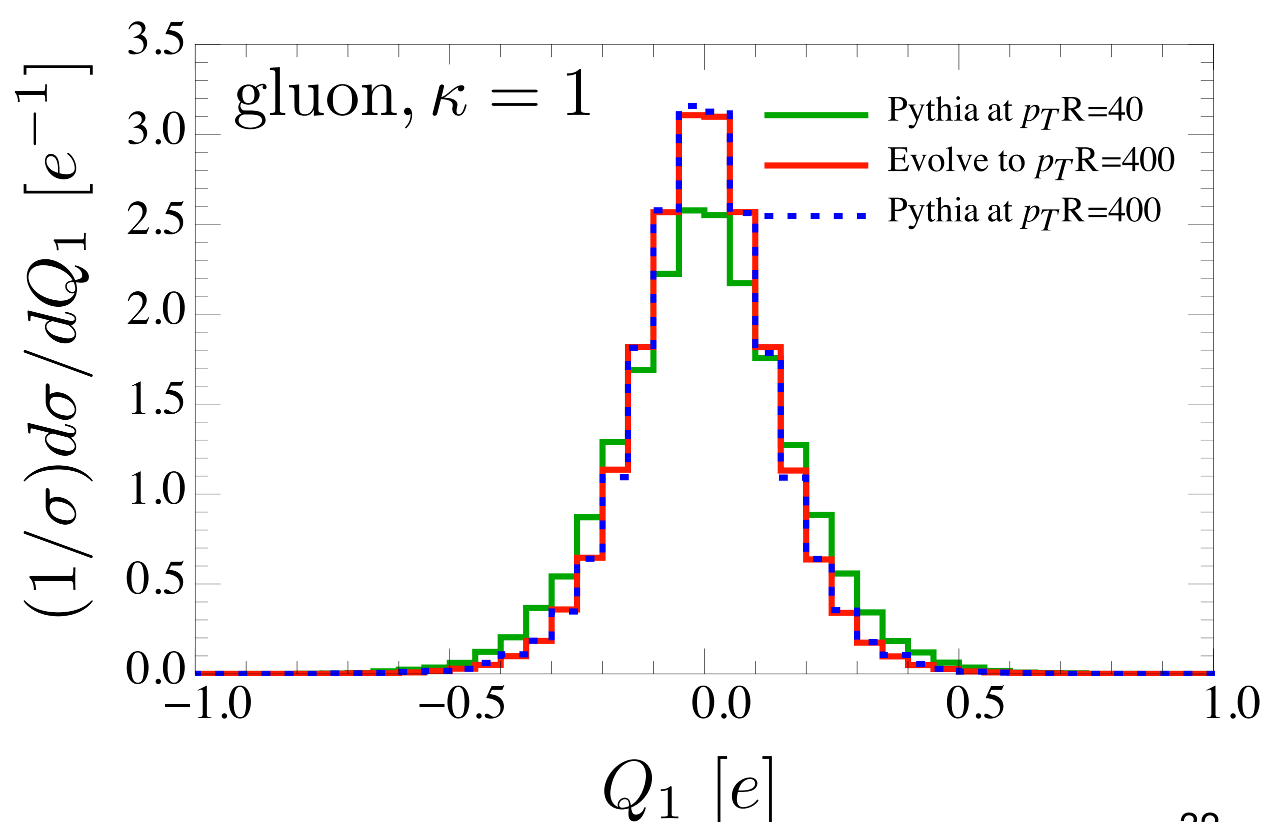
$$\mu \frac{d}{d\mu} D_i(Q_\kappa, \mu) = \overbrace{\sum_j \int dz \, \frac{\alpha_s}{2\pi} P_{ji}(z)}^{\text{Splitting probability}} \; \overbrace{\int dQ_\kappa^a \, D_j(Q_\kappa^a, \mu) \int dQ_\kappa^b \, D_k(Q_\kappa^b, \mu)}^{\text{Sample over distributions of branches}}$$

$$\times \; \underbrace{\delta[Q_\kappa - z^\kappa Q_\kappa^a - (1-z)^\kappa Q_\kappa^b]}_{\text{Charge is (weighted) sum of branches}}$$

RG Evolution vs. Pythia's Parton Shower

- ✓ Use Pythia as input and evolve ➞ good agreement
- Can go to higher orders, which involves $1 \to n$ splittings
- Distribution changes more slowly than fragmentation functions or parton distribution functions





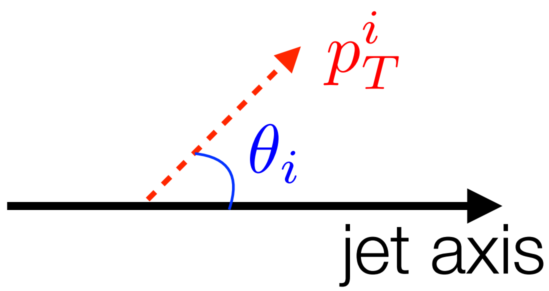
Ratio of Two Angularities

Thaler, Larkoski (arXiv:1307.1699)
Larkoski, Moult, Neill (arXiv:1401.4458)
Procura, WW, Zeune (arXiv:1410.6483)

Ratios of Observables are Not IR Safe

- Angularities e_α probe the radial energy distribution in jet

$$e_\alpha = \sum_{i \in \text{jet}} \frac{p_T^i}{P_T^J} \left(\frac{\theta_i}{R} \right)^\alpha$$



(Berger, Kucs, Sterman; Almeida et al.)

- e_α is IR safe: $p_T^i \theta_i^\alpha + p_T^j \theta_j^\alpha \overset{\text{coll.}}{\to} (p_T^i + p_T^j)\theta_i^\alpha$

Ratios of Observables are Not IR Safe

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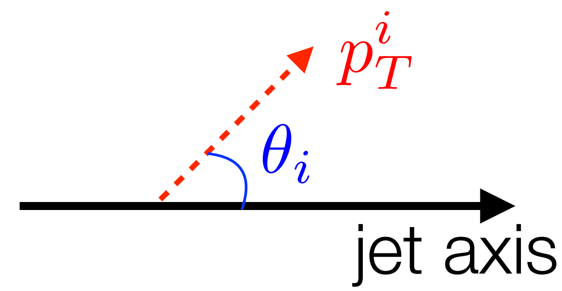
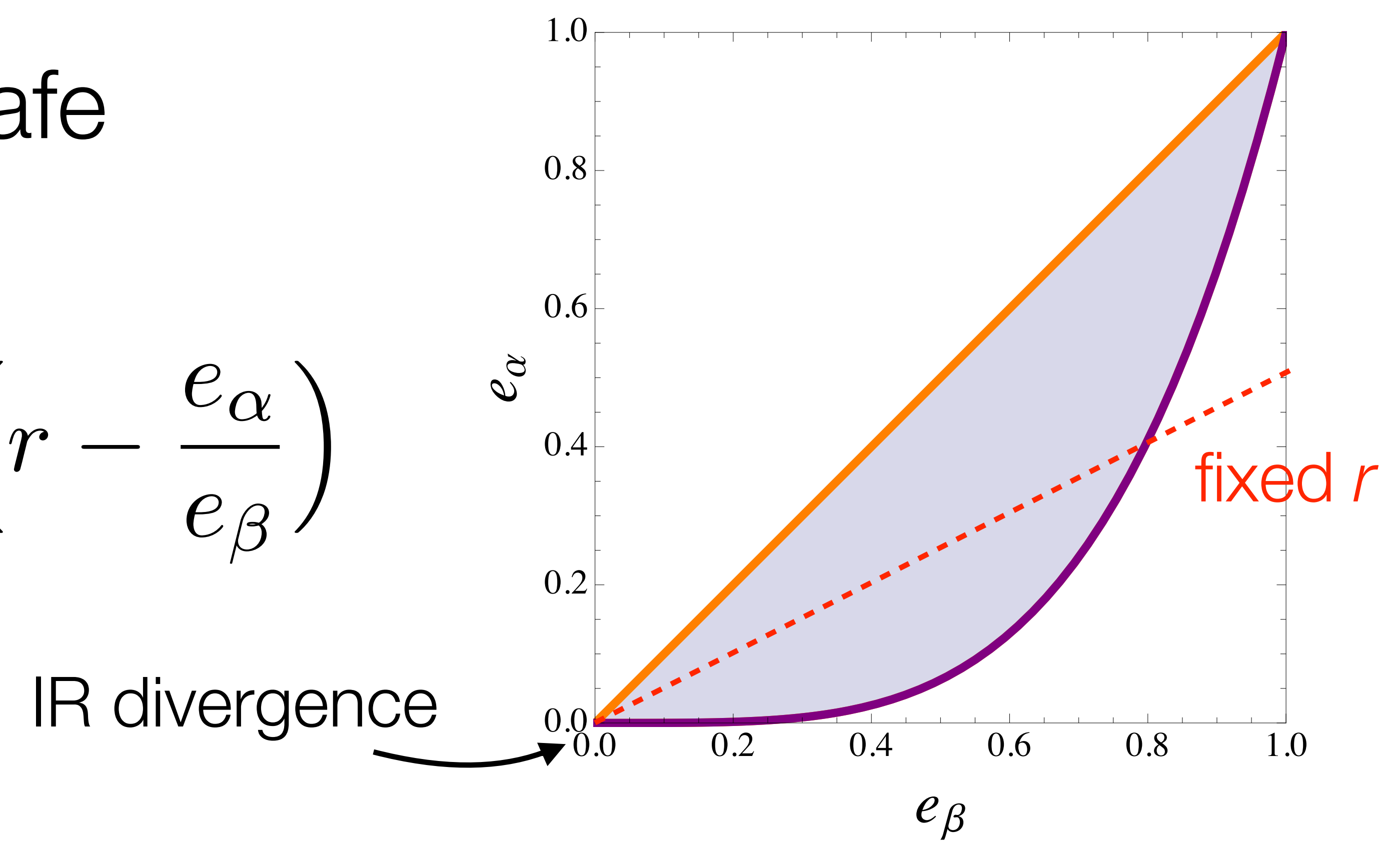


(Berger, Kucs, Sterman; Almeida et al.)

- e_α is IR safe: $p_T^i \theta_i^\alpha + p_T^j \theta_j^\alpha \xrightarrow{\text{coll.}} (p_T^i + p_T^j)\theta_i^\alpha$

- Ratio $r = e_\alpha / e_\beta$ is not IR safe
(Soyez, Salam, Kim, Dutta, Cacciari)

$$\frac{d\sigma}{dr} = \int de_\alpha \, de_\beta \, \frac{d^2\sigma}{de_\alpha \, de_\beta} \, \delta\!\left(r - \frac{e_\alpha}{e_\beta} \right)$$



Ratios of Observables are Sudakov Safe

- Resummation of $\ln e_\alpha, \ln e_\beta$ is required when $e_\alpha, e_\beta \ll 1$
- IR divergence is Sudakov suppressed (Larkoski, Thaler)

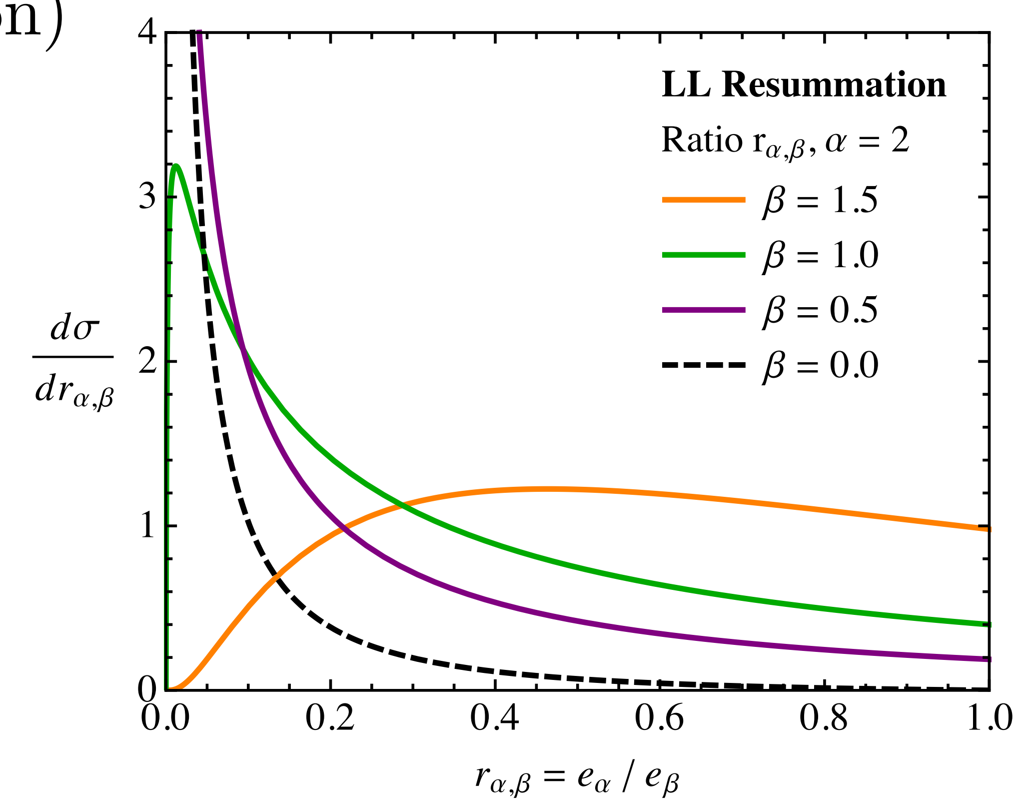
$$\frac{d^2\sigma_i^{\mathrm{LL}}}{de_\alpha\, de_\beta} = \frac{\partial}{\partial e_\alpha}\frac{\partial}{\partial e_\beta}\exp\left[-\frac{\alpha_s C_i}{\pi}\left(\frac{1}{\beta}\ln^2 e_\beta + \frac{1}{\alpha-\beta}\ln^2\frac{e_\alpha}{e_\beta}\right)\right]$$

$$C_i = C_F\,(\text{quark}), C_A\,(\text{gluon})$$

- No valid expansion in α_s

$$\frac{d\sigma}{dr} = \int de_\alpha\, de_\beta\, \frac{d^2\sigma}{de_\alpha\, de_\beta}\,\delta\!\left(r - \frac{e_\alpha}{e_\beta}\right)$$

$$= \sqrt{\alpha_s}\,\frac{\sqrt{C_F\beta}}{\alpha-\beta}\,\frac{1}{r} + \mathcal{O}(\alpha_s)$$



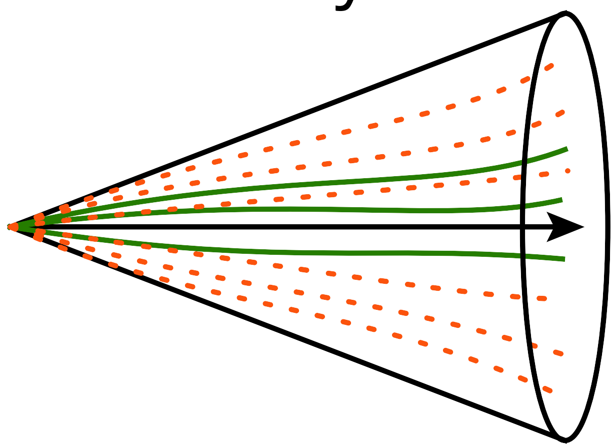
(arXiv:1307.1699)

Two Angularities Beyond Leading Log

- Degrees of freedom in Soft-Collinear Effective Theory

Mode:	Scaling $(-,+,\perp)$	Scale
collinear	$p_T^J(1, e_\beta^{2/\beta}, e_\beta^{1/\beta})$	$p_T^J e_\beta^{1/\beta}$
soft	$p_T^J(e_\alpha, e_\alpha, e_\alpha)$	$p_T^J e_\alpha$

$$\alpha > \beta$$



- Leads to two factorization theorems (Larkoski, Moult, Neill)

$$\frac{d^2\sigma_i}{de_\alpha\, de_\beta} \overset{e_\alpha \sim e_\beta}{=} H_i(p_T^J, \mu) \int de'_\beta\, J_i(e_\beta - e'_\beta, \mu)\, S_i(e_\alpha, e'_\beta, \mu)$$

$$\frac{d^2\sigma_i}{de_\alpha\, de_\beta} \overset{e_\alpha^\beta \sim e_\beta^\alpha}{=} H_i(p_T^J, \mu) \int de'_\alpha\, J_i(e'_\alpha, e_\beta, \mu)\, S_i(e_\alpha - e'_\alpha, \mu)$$

- Convolution structure follows from measurement:

$$e_\alpha\, p_T^J R = \sum_{c\in\text{jet}} p_T^c \theta_c^\alpha + \sum_{s\in\text{jet}} p_T^s \theta_s^\alpha \sim p_T^J \times e_\beta^{\alpha/\beta} + e_\alpha\, p_T^J \times 1$$

power suppressed unless $e_\alpha^\beta \sim e_\beta^\alpha$

Resummation for the Phase-Space Boundaries

- H, J and S calculated from their field-theoretic definitions
  (Lee, Hornig, Ovanesyan; Ellis et al.; Larkoski, Moult, Neill)

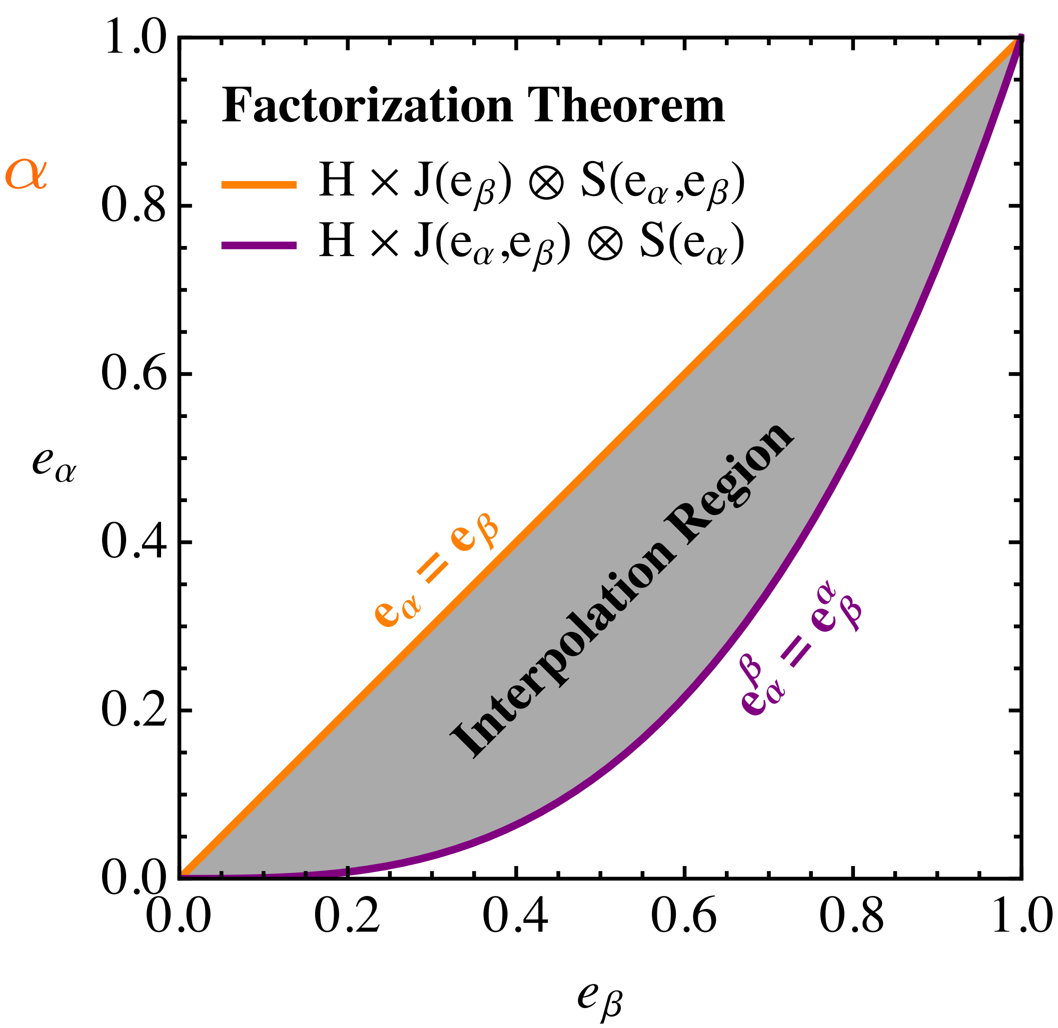
- Each depends on one scale

$$\mu_H \sim p_T^J, \ \mu_J \sim p_T^J\, e_\beta^{1/\beta}, \ \mu_S \sim p_T^J\, e_\alpha$$

$\rightarrow$ sum logs with RG evolution

- Factorization theorems correspond to phase-space boundaries

- Conjecture for NLL interpolation
  (Larkoski, Moult, Neill)



(arXiv:1401.4458)

Factorization in Intermediate Regime

- Power expansion for $e_\beta^{\alpha/\beta} \ll e_\alpha \ll e_\beta$ gives

$$\frac{d^2\sigma_i}{de_\alpha\, de_\beta} = H_i(p_T^J,\mu) J_i(e_\beta,\mu)\, S_i(e_\alpha,\mu) \quad \rightarrow \quad \text{inconsistent!}$$

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- Collinear-soft mode missing (Procura, WW, Zeune)

Mode:	Scaling $(-,+,\perp)$	Scale
collinear	$p_T^J(1, e_\beta^{2/\beta}, e_\beta^{1/\beta})$	$p_T^J e_\beta^{1/\beta}$
collinear-soft	$p_T^J\Big(e_\alpha^{\frac{-\beta}{\alpha-\beta}}\, e_\beta^{\frac{\alpha}{\alpha-\beta}}, e_\alpha^{\frac{2-\beta}{\alpha-\beta}}\, e_\beta^{\frac{\alpha-2}{\alpha-\beta}}, e_\alpha^{\frac{1-\beta}{\alpha-\beta}}\, e_\beta^{\frac{\alpha-1}{\alpha-\beta}}\Big)$	$p_T^J e_\alpha^{\frac{1-\beta}{\alpha-\beta}}\, e_\beta^{\frac{\alpha-1}{\alpha-\beta}}$
soft	$p_T^J(e_\alpha, e_\alpha, e_\alpha)$	$p_T^J e_\alpha$

$$\frac{d^2\sigma_i}{de_\alpha\, de_\beta} = H_i(p_T^J,\mu) \int de'_\alpha\, de'_\beta\, J_i(e_\beta - e'_\beta,\mu)\, \mathcal{S}_i(e'_\alpha, e'_\beta,\mu) S_i(e_\alpha - e'_\alpha,\mu)$$

- Collinear-soft merges with collinear or soft on boundaries

Two Angularities at Next-to-Leading Log Order

- Collinear-soft function is matrix element of Wilson lines

$$\mathcal{S}_i(e_\alpha, e_\beta) = \frac{1}{N_c}\left\langle 0 \middle| \mathrm{Tr}\left[\overline{\mathbf{T}}(X_n^\dagger(0)V_n(0))\,\delta\left(e_\alpha - \sum_i \frac{p_T^i}{P_T^J}\left(\frac{\theta_i}{R}\right)^\alpha\right)\delta\left(e_\beta - \sum_i \frac{p_T^i}{P_T^J}\left(\frac{\theta_i}{R}\right)^\beta\right)\mathbf{T}(V_n^\dagger(0)X_n(0))\right]\middle| 0\right\rangle$$

"collinear" emissions

"soft" emissions

- NLL result in terms of evolution kernels K_X^i, η_X^i

$$\Sigma_i(e_\alpha, e_\beta) = \int_0^{e_\alpha} de'_\alpha \int_0^{e_\beta} de'_\beta \,\frac{d^2\sigma}{de'_\alpha de'_\beta} = \frac{e^{K_H^i + K_J^i + K_S^i - \gamma_E\,\eta_J^i - \gamma_E\,\eta_S^i}}{\Gamma(1+\eta_J^i)\Gamma(1+\eta_S^i)}$$

- Differs from NLL conjecture away from boundaries at $\mathcal{O}(\alpha_s^2)$

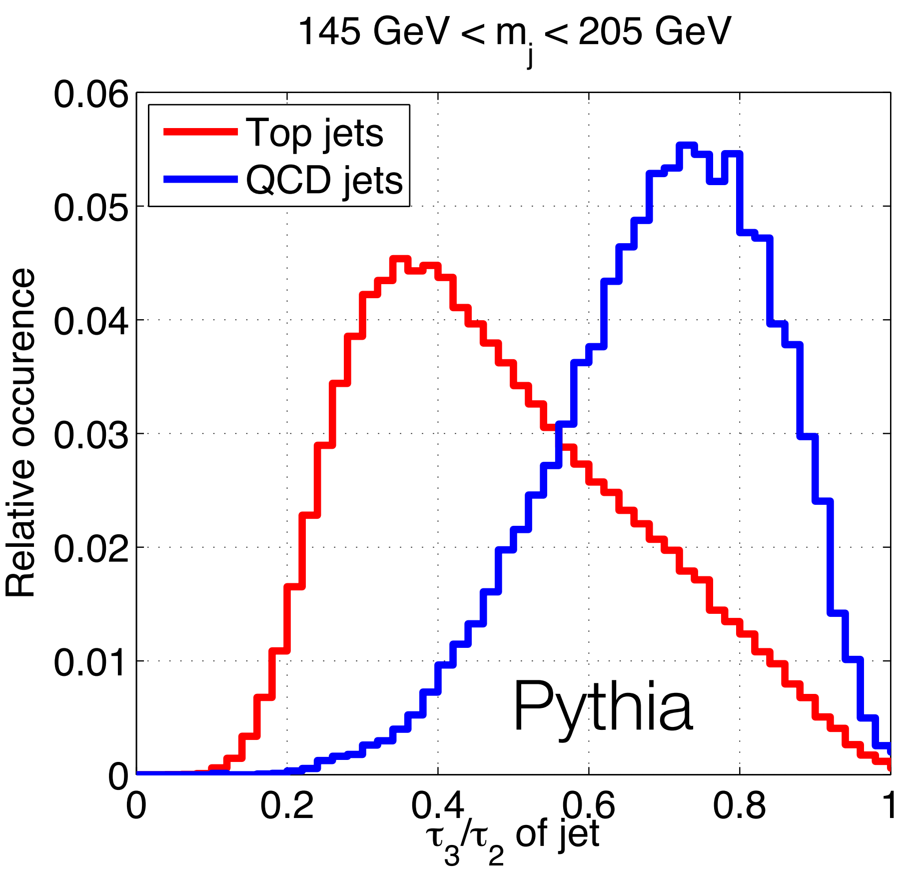
$$\Gamma(1+\eta_J^i)\Gamma(1+\eta_S^i) \neq \Gamma(1+\eta_J^i+\eta_S^i)$$

- Non-logarithmic corrections at boundaries beyond NLL:

$$\int de'_\alpha\, \mathcal{S}_i(e'_\alpha, e_\beta, \mu) S_i(e_\alpha - e'_\alpha, \mu) = S_i(e_\alpha, e_\beta, \mu) + \mathcal{O}\left(\frac{e_\alpha}{e_\beta}\right)$$

Many Possible Applications

- Ratio observables:
  - *N*-subjettiness (Thaler, Van Tilburg)
  - Planar flow (Thaler, Wang; Almeide et al.)
  - Energy correlation functions (Banfi, Salam, Zanderighi; Larkoski, Salam, Thaler; Larkoski, Moult, Neill)



(arXiv:1011.2268)

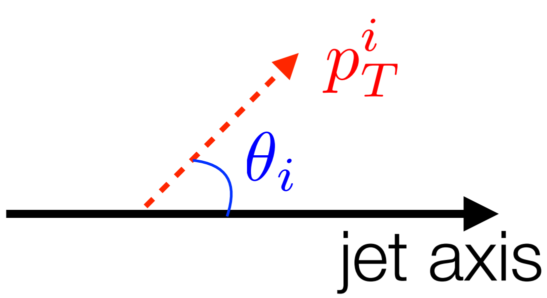
- Multivariate analyses (an example in next section)
- Jet production with hierarchies in jet energies or angles (Bauer, Tackmann, Walsh, Zuberi; Pietrulewicz, Tackmann, WW; …)
- …

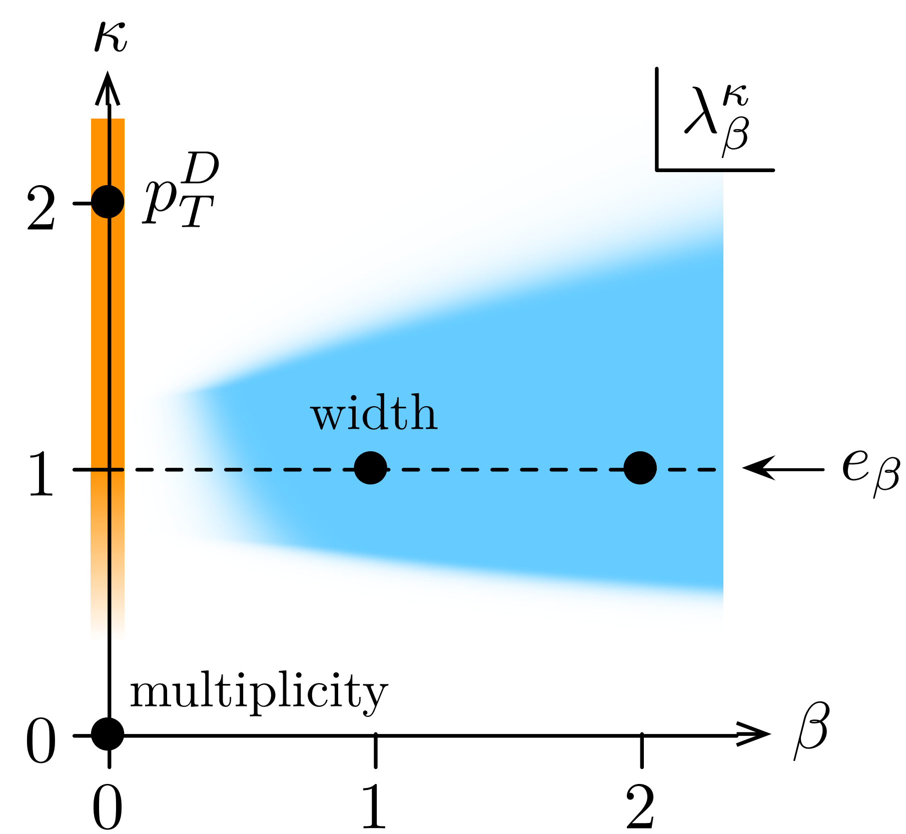
Generalized Angularities

Larkoski, Thaler, WW (arXiv:1408.3122)

Generalized Angularities

$$\lambda_\beta^\kappa = \sum_{i \in \text{jet}} \left(\frac{p_T^i}{P_T^J}\right)^\kappa \left(\frac{\theta_i}{R}\right)^\beta$$



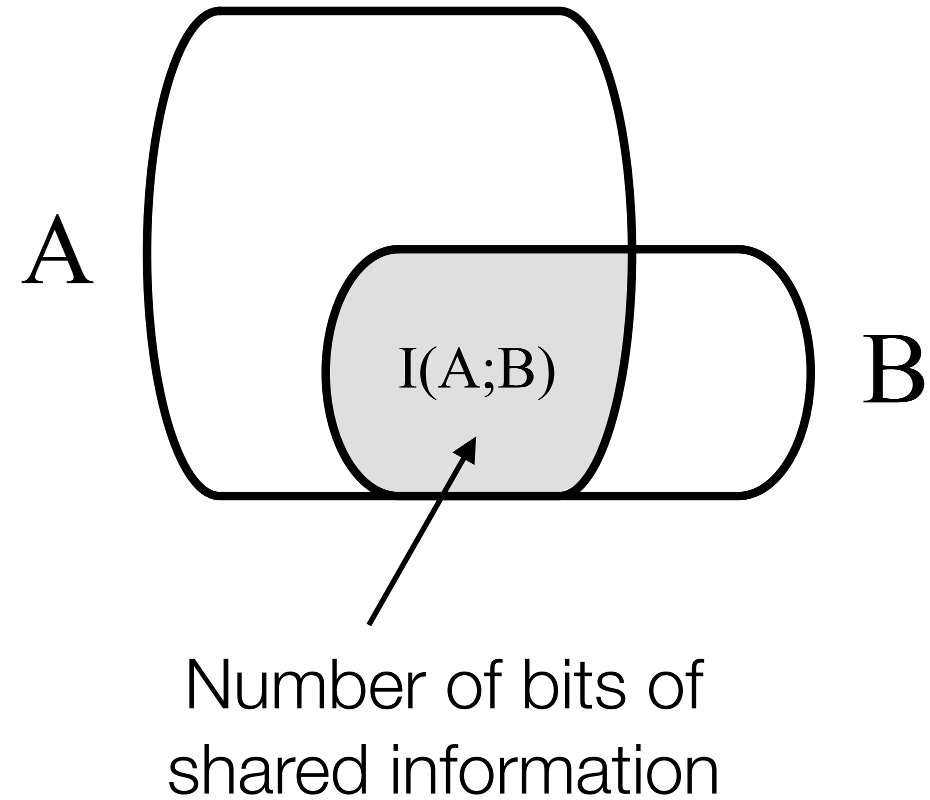


- $\kappa = 1$: IR safe, angularities
- $\beta = 0$: very IR unsafe, similar to jet charge
- blue: a bit IR unsafe, one nonpert. parameter at NLL

Can we use λ_κ^β to distinguish quark jets from gluon jets?

Mutual Information

$$I(A;B) = \int da\, db\, p(a,b) \log_2 \frac{p(a,b)}{p(a)p(b)}$$

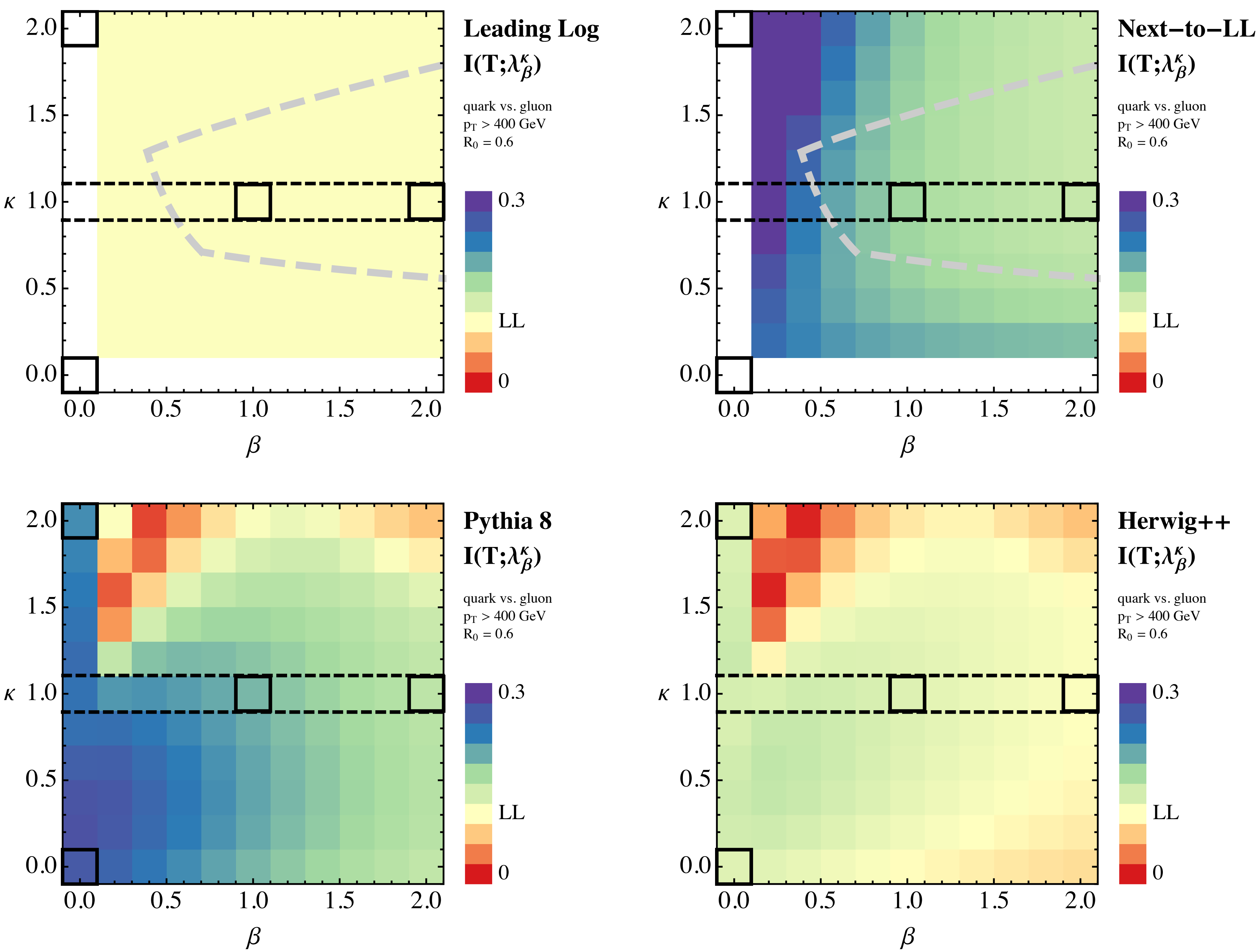


- Can directly be calculated from double diff. cross section

$$p(a,b) = \frac{1}{\sigma} \frac{d^2\sigma}{da\, db}$$

- Quark/gluon discrimination is one bit of (truth) information

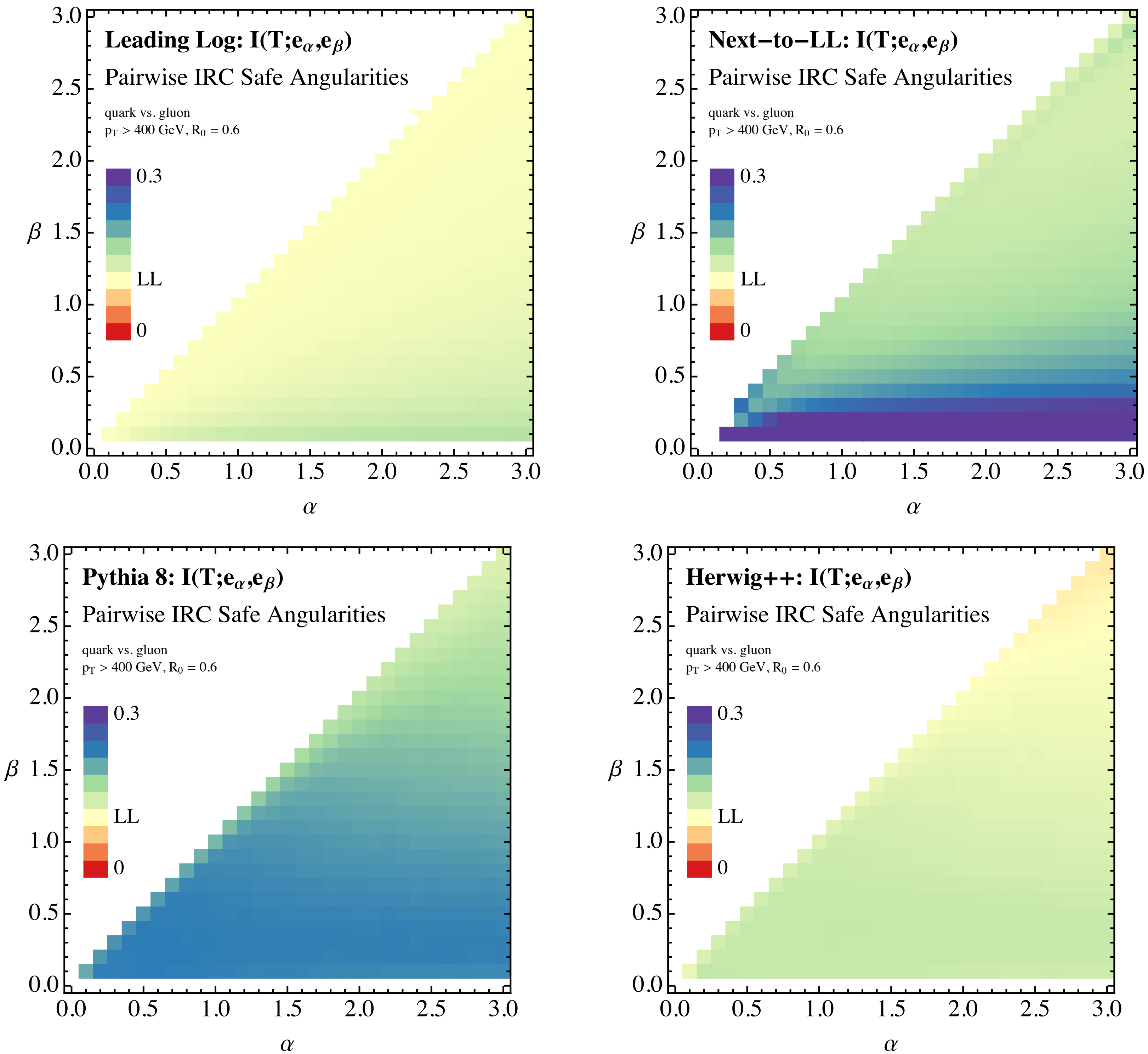
Quark/Gluon Discrimination with λ_β^κ



- (N)LL valid in grey bounds
- LL is constant (Casimir scaling)
- Significant differences
- Interesting region outside our bounds

Calculation uses arXiv:1306.6630 (Chang, Procura, Thaler, WW)

Quark/Gluon Discrimination with e_α, e_β



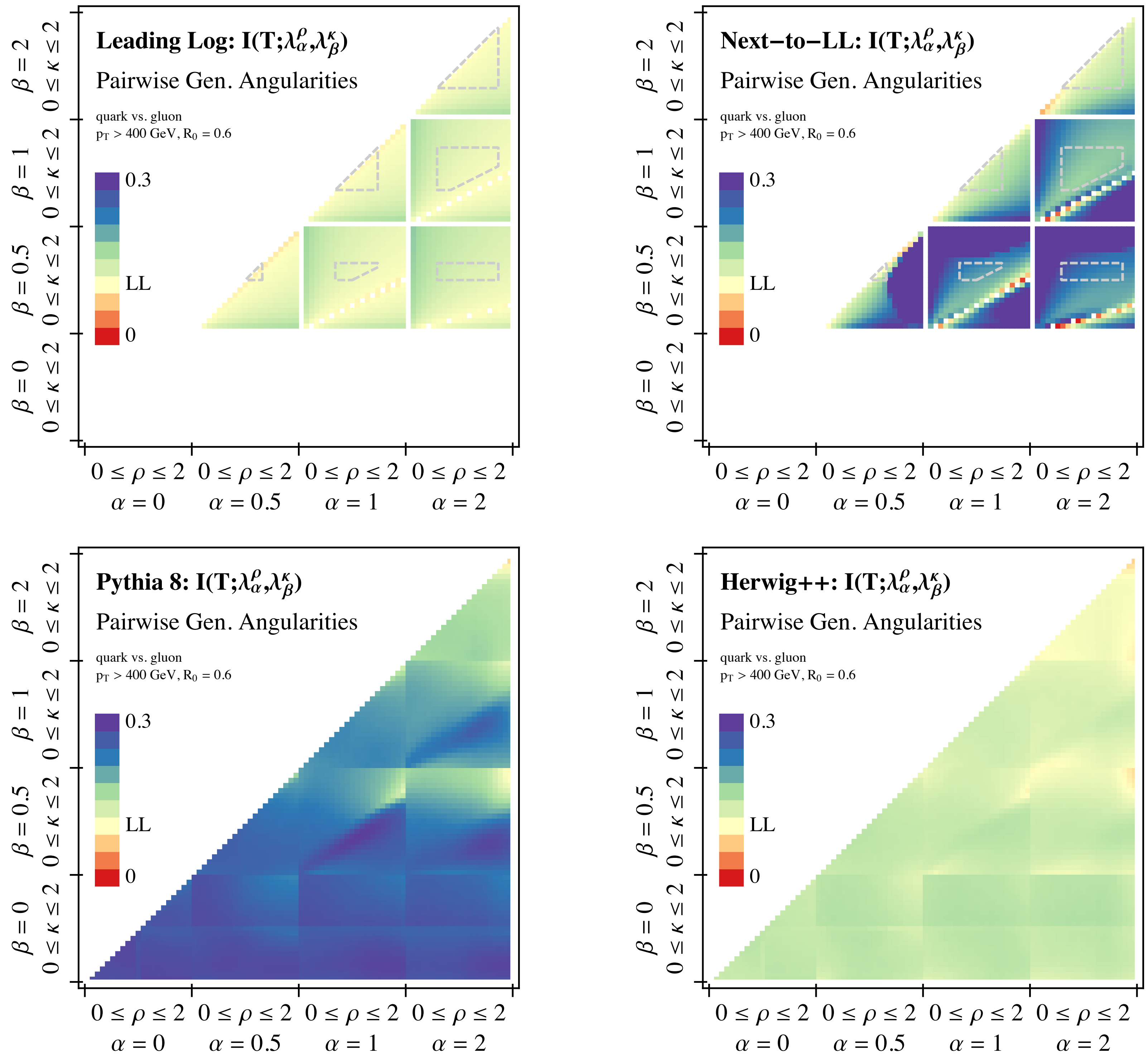
- LL not constant
- Similar behavior, different size:

  LL ~ Herwig

  NLL ~ Pythia

Calculation uses arXiv:1401.4458 (Larkoski, Moult, Neill)

Quark/Gluon Discrimination with $\lambda_\alpha^\rho, \lambda_\beta^\kappa$



- (N)LL valid in grey bounds
- LL not constant
- Significant differences
- But similar $\lambda_\alpha^\rho, \lambda_\beta^\kappa$ correlations

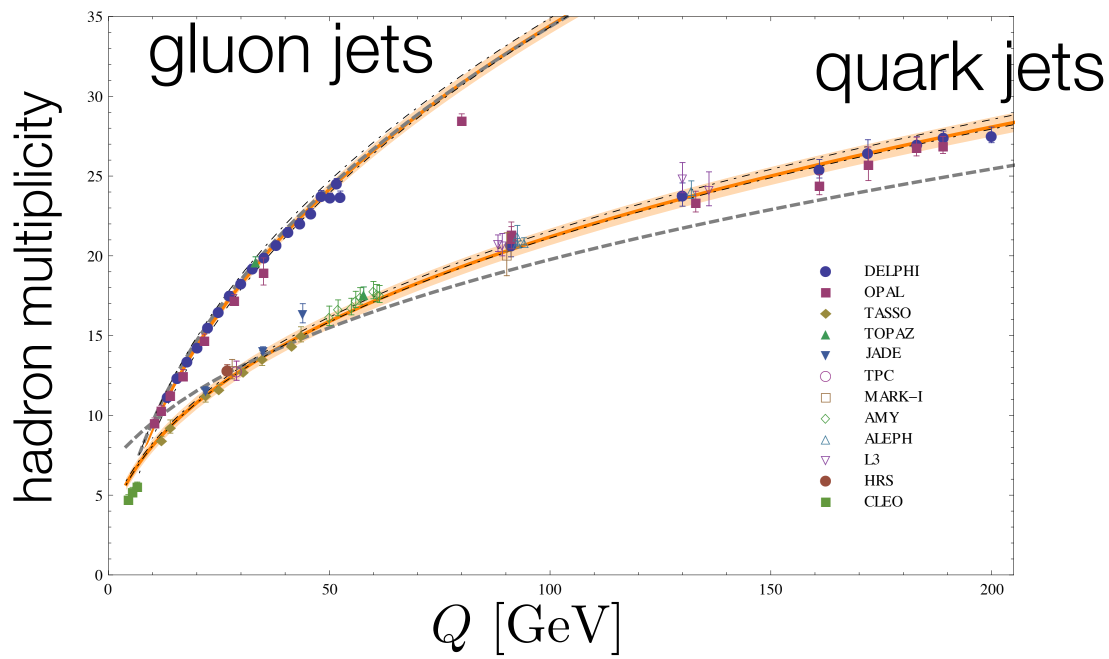
Calculation uses arXiv:1401.4458 (Larkoski, Moult, Neill)

Out-of-jet Hadron Multiplicity

Ritzmann, WW (in progress)

Hadron Multiplicity

- Extensively studied in e^+e^-:
  (Malaza, Webber; Lupia, Ochs; Eden, Gustafson; Capella et al.; Bolzoni, Kniehl, Kotikov)
  - Frag. function evolution
  - Small x resummation



(arXiv:1305.6017)

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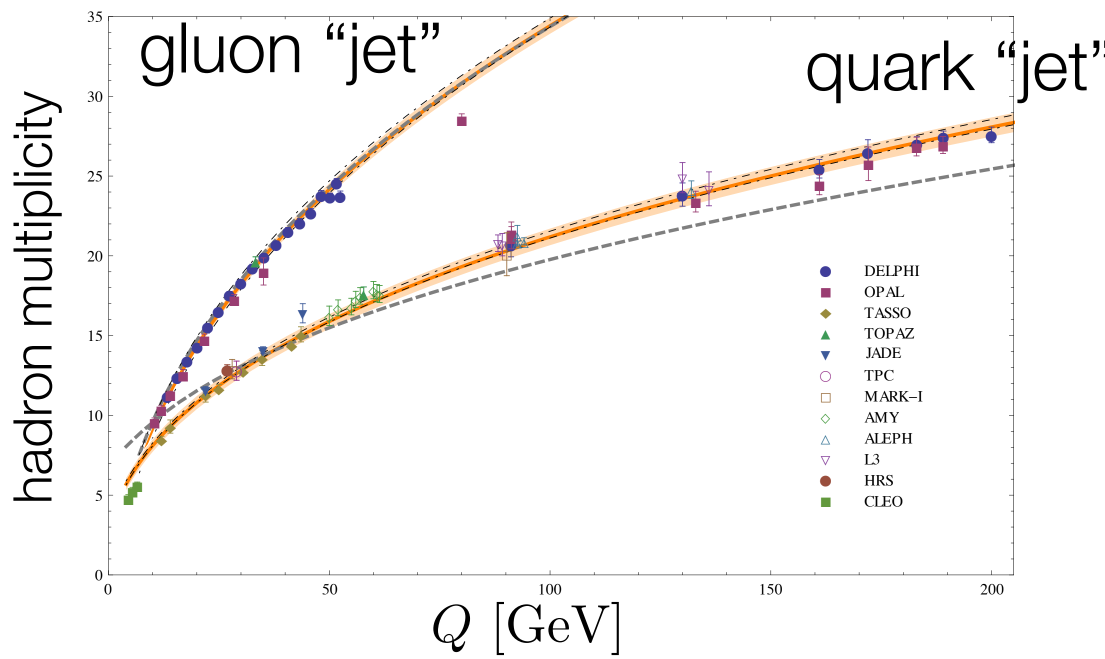
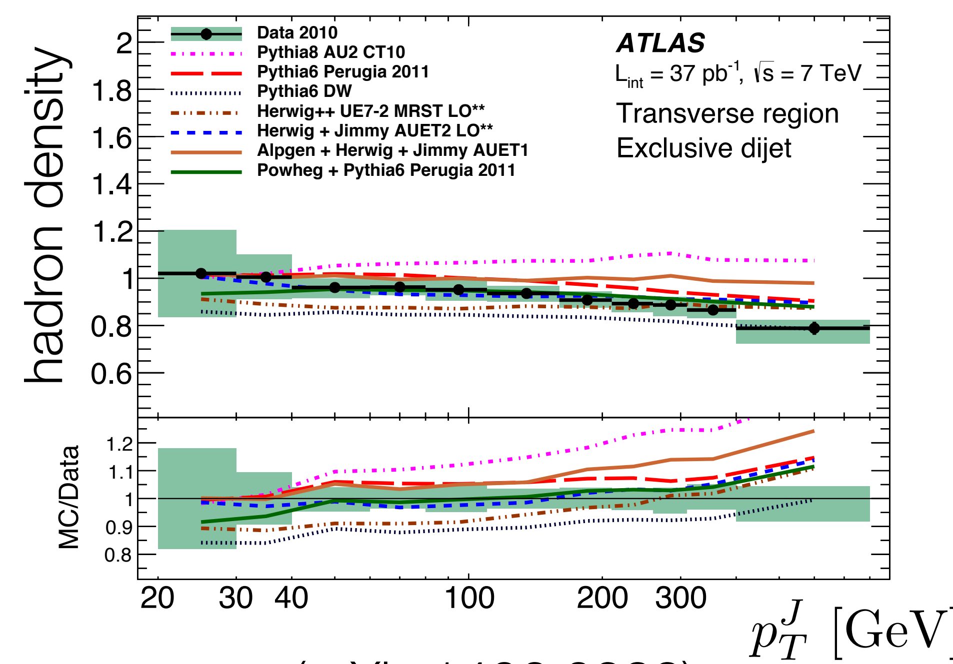


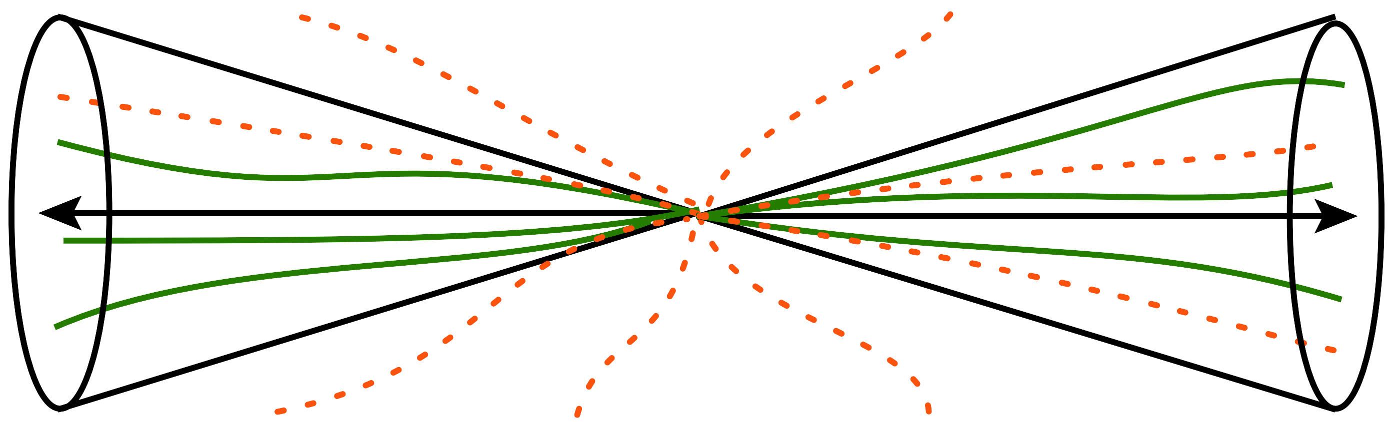
(arXiv:1305.6017)

- Underlying event studies measure hadrons away from jets
  - Not well modelled
  - Ambiguity between primary vs. secondary collisions



(arXiv:1406.0392)

Out-of-jet Hadron Multiplicity in e^+e^-



- Hadrons produced by soft radiation when $E_{\text{out}} \ll Q,\ R \ll 1$

$$\langle n_{\text{out}}\rangle = \frac{S(\langle n_{\text{out}}\rangle, E_{\text{out}}, R, \mu)}{S(E_{\text{out}}, R, \mu)} + \mathcal{O}\Big(\frac{E_{\text{out}}}{Q}, R^2\Big)$$

$$= \underbrace{\int_{-\eta_{\text{cut}}}^{\eta_{\text{cut}}} d\eta \int_0^{2\pi} d\phi}_{=4\pi\eta_{\text{cut}}} \int_0^1 dr\, \underbrace{\frac{1}{N_c}\,\mathrm{Tr}\big\langle 0\big|\bar{T}\big[Y_n^\dagger Y_{\bar{n}}\big]\,\hat{n}(y,\phi,r)T\big[Y_{\bar{n}}^\dagger Y_n\big]\big|0\big\rangle}_{=M(r,\mu)} + \mathcal{O}(\alpha_s)$$

- By boost and rotational invariance M only depends on $r = p_T^2/(p_T^2+m^2)^{1/2}$, encoding hadron mass effects

Some Preliminary Results

$$\langle n_{\rm out}(\mu)\rangle = 4\pi\eta_{\rm cut}\int_0^1 dr\, M(r,\mu)$$

- Calculating $S(E_{\rm out}, R, \mu)$ at NLO suggests $\mu \sim E_{\rm out} R^{0.5}$
- RG evolution $\frac{d}{d\ln\mu}\ln M(r,\mu) = -\frac{\alpha_s C_A}{\pi}\ln(1-r^2)$

  (From similar calculation for transverse velocity operator by Mateu, Stewart, Thaler)

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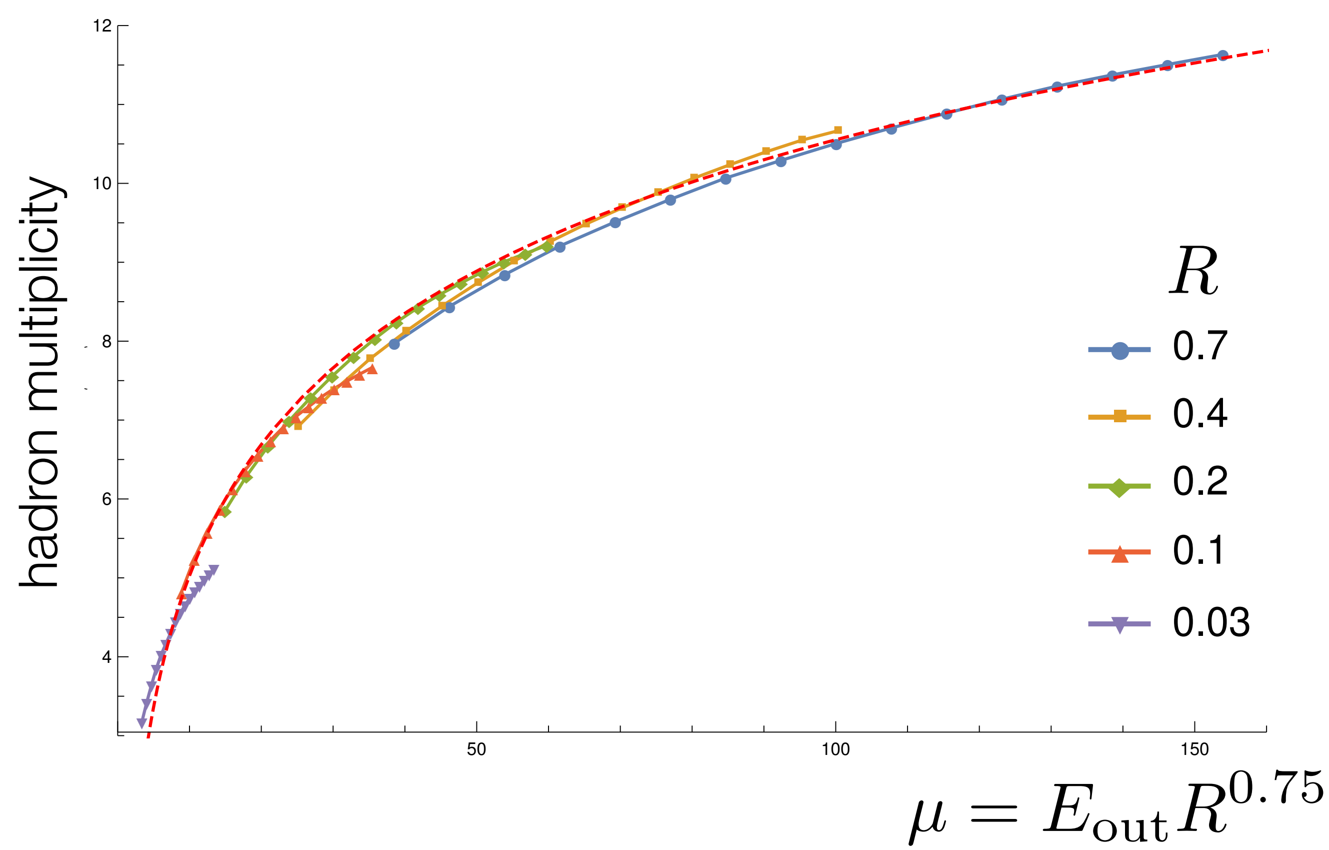
  (From similar calculation for transverse velocity operator by Mateu, Stewart, Thaler)
- Find $R^{0.5} \to R^{0.75}$ depending on Monte Carlo
- Fits $\langle n_{\rm out}(\mu)\rangle = a + b\ln\mu$
- Strong running since:

$$1 - r^2 \approx \left(\frac{m_\pi \langle n_{\rm out}\rangle}{E_{\rm out}}\right)^2$$



Summary

- Jet substructure provides a new set of tools at the LHC
- IR unsafe observables have interesting applications

Sudakov safe:
Calculable in resummed perturbation theory

Collinear unsafe:
Unknown distribution
Known evolution

Soft unsafe:
Unknown distribution
"Known" evolution
Depends on all jets

N-subjettiness ratio

Jet charge

Track-based IR-safe observables

Out-of-jet hadron multiplicity

Planar flow

Hadron multiplicity

Generalized angularities

thank you!