Beyond Infrared-safe Jet Observables

Wouter Waalewijn

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Outline

• Introduction to Jets
• Infrared Safety
• Jet Charge
• Ratio of Two Angularities
• Generalized Angularities
• Out-of-jet Hadron Multiplicity
• Conclusions
Introduction
What is a Jet?

Energetic quarks and gluons radiate and hadronize → Produce jets of hadrons
Jet Algorithms

- Repeatedly cluster nearest “particles” \( p_i, p_j \rightarrow p_i + p_j \)
- Cut off by jet “radius” \( R \)

\[
\text{distance} = (\Delta y)^2 + (\Delta \phi)^2
\]
Jet Algorithms

- Repeatedly cluster nearest “particles” \( p_i, p_j \rightarrow p_i + p_j \)
- Cut off by jet “radius” \( R \)
- Default at LHC: anti-\( k_T \) (Cacciari, Salam, Soyez)

\[
\begin{align*}
p_T \text{[GeV]} & \quad \text{Azimuthal angle} \\
\text{Cam/Aachen, } R=1 & \quad \text{anti-}k_T, \ R=1
\end{align*}
\]

\( \text{(arXiv:0802.1189) } \)
Jets at the LHC

- Most measurements involve jets as signal or background.
Jet Cross Sections

- Bin by jet multiplicity to improve background rejection

\[ \sigma(H + 0 \text{ jets}) \propto 1 - \frac{6\alpha_s}{\pi} \ln^2 \frac{p_T^{\text{cut}}}{m_H} + \ldots \]

- Large logarithms lead to large theory uncertainties

\( \sqrt{s} = 8 \text{ TeV}, \int L dt = 20.3 \text{ fb}^{-1} \)

- Obs ± stat
- Exp ± syst

\( \begin{align*}
\text{DY} & \quad \text{Higgs} \\
\text{Top} & \quad \text{VV} \\
\text{Misid} & \quad \text{WW}
\end{align*} \)
Jet Substructure for Boosted Objects

- New heavy particles could produce boosted top, W, Higgs decay products lie within one “fat” jet
- Distinguish from QCD jets using jet substructure
- Avoids combinatorial background

Hadronic decay of top quark

![Hadronic decay of top quark diagram](image)
• One leptonic and one hadronic top
• Boosted analysis crucial for large $m_{Z'}$
Jet Substructure for Quark/Gluon Discrimination

- New physics often more quarks than QCD backgrounds
- Extensive Pythia study (Gallicchio, Schwartz)

- Charged hadron multiplicity and jet “girth” are good

\[
girth = \sum_{i \in \text{jet}} \frac{p_T^i}{p_T^J} \sqrt{(y_i - y_J)^2 + (\phi_i - \phi_J)^2}
\]

- More variables only give marginal improvement
Infrared Safety
Soft and Collinear Divergences

- **Tree:**

- **Real:** divergences from phase-space integration

- **Virtual:** divergences from loop integration

\[\sigma_R \sim \mu^{2\epsilon} \int \frac{dE}{E^{1+2\epsilon}} \frac{d\theta}{\theta^{1+2\epsilon}}\]

\[\sigma_V \sim \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[ -\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} + \ldots \right]\]
Infrared Safety

• IR divergences cancel between real and virtual corrections for IR safe measurements (Kinoshita, Lee, Nauenberg)

• IR safe: measure same value as for virtual when $E$ or $\theta \to 0$

• E.g. quark energy $E_q$ is not IR safe in $\theta \to 0$ limit

\[ +\infty \delta(\theta) \delta(E_q - Q/2 - E) \quad -\infty \delta(\theta) \delta(E_q - Q/2) \]

• IR safety has historically been an issue for jet algorithms
Hadronization Effects

- IR divergences are cut off by $\Lambda_{QCD}$ in QCD
- IR unsafe $\rightarrow$ not (fully) calculable in perturbation theory
- One example: the energy fraction $z$ of a hadron in $e^+e^-$

$$\frac{d\sigma_h}{dz} = \sum_{i=q,\bar{q},g} \int \frac{dx}{x} \frac{d\hat{\sigma}_i}{dx} D^h_i \left( \frac{z}{x} \right)$$

$e^+e^- \rightarrow i(x) + \ldots$

$i(x) \rightarrow h(z) + \ldots$

- Fragmentation function $D$ is nonperturbative but process independent, “absorbing” divergences (Collins, Soper; …)

\[\text{(hep-ph/0702250)}\]
Motivation for IR Unsafe Jet Observables

• Experimentalists use them, e.g. in quark/gluon discrimination

• Questions for theorists:
  • Does this allow for powerful new observables?
  • How can we calculate them?
Jet Charge

Krohn, Lin, Schwartz, WW (arXiv:1209.2421)
WW (arXiv:1209.3091)
Defining Jet Charge

\[ Q_\kappa = \sum_{i \in \text{jet}} Q_i \left( \frac{p_T^i}{p_T^f} \right)^\kappa \]

(Feynman, Field)

- If \( \kappa \) too small: sensitive to soft hadrons contamination from other jets etc.
- If \( \kappa \) too large: need more statistics

Pythia
\[ \kappa = 0.5 \]

\[ (1/\sigma) d\sigma / dQ_\kappa \] [e\(^{-1}\)]

- If \( \kappa \) too small: sensitive to soft hadrons \( \rightarrow \) contamination
- If \( \kappa \) too large: only sensitive to most energetic hadron \( \rightarrow \) need more statistics
Historical Applications

- Test parton model

\[ Q_{0.5} \text{ [e]} \]

\[ \nu_\mu p \rightarrow \mu^- u X \]

\[ \bar{\nu}_\mu p \rightarrow \mu^+ d X \]

- Jet charge at LEP:
  - Forward-backward charge asymmetry (AMY (1990), ...)
  - \( B^0 \leftrightarrow \bar{B}^0 \) mixing (ALEPH (1992), ...)

\[ \kappa = 0.5 \]

(Nucl. Phys. B184, 13 (1981))

\[ (1/\sigma) d\sigma / dQ_{0.5} \text{ [e]} \]
Possible LHC application: $W'$ vs. $Z'$

- Leptophobic $W'$ or $Z'$ with 1 TeV mass
- 2-dim. likelihood discriminant based on both jet charges

$Z' \rightarrow u\bar{u}$

$Z' \rightarrow d\bar{d}$

vs.

$W' \rightarrow u\bar{d}$

$W' \rightarrow d\bar{u}$
LHC Challenges

- Trade off between soft contamination and statistics
- We did not include: backgrounds, detector effects, …
LHC Challenges

• Trade off between soft contamination and statistics
• We did not include: backgrounds, detector effects, …
• Various sources of contamination:
  • Initial-State Radiation
  • Multiparton Interactions
  • Pile-up
• All soft $\rightarrow$ increase $\kappa$

$W'$ vs. $Z'$

![Graph showing significance vs. $\kappa$ for different scenarios: FSR only, FSR+MI+ISR, FSR+MI+ISR+trim, Npileup=10, Npileup=10 + trim. The graph compares 50 events of $W'$ vs. $Z'$ using Pythia.](image)
Jet Charge Not Infrared Safe

- Consider $q \rightarrow qg$ in collinear limit
- $Q_q z^\kappa_q \neq Q_q \rightarrow$ divergences don’t cancel between real/virtual
Jet Charge Not Infrared Safe

- Consider $q \rightarrow qg$ in collinear limit
- $Q_q z^\kappa \neq Q_q \rightarrow$ divergences don’t cancel between real/virtual
- Jet charge only defined for hadrons

\[
Q_q = z^\kappa - 1.0 - 0.5 + 0.0 + 0.5 + 1.0
\]

\[
\frac{d}{dQ} q(1 - z)
\]

\[
\frac{1}{\sigma} d\sigma/dQ_{0.5} \left[ e^{-1} \right]
\]

Pythia

- d-quark
- hadronic
- partonic
Average Jet Charge Calculation

\[ \langle Q_\kappa \rangle = \sum_h \int dz \; Q_h z^\kappa \left( \frac{1}{\sigma_{\text{jet}}} \frac{d\sigma_{h\in\text{jet}}}{dz} \right) \]

- At LO, weight = fragmentation function \( D^h_q(z, \mu \sim p_T^J R) \)

Jet scale
Average Jet Charge Calculation

\[ \langle Q_\kappa \rangle = \sum_h \int dz \quad Q_h z^\kappa \quad \frac{1}{\sigma_{\text{jet}}} \quad \frac{d\sigma_{h \in \text{jet}}}{dz} \]

- At LO, weight = fragmentation function \( D^h_q(z, \mu \sim p^J_T R) \)
- Calculate \( p^J_T, R \) dependence from evolution to \( \mu \sim \Lambda_{\text{QCD}} \)
- \( D^h_q(z, \mu \sim \Lambda_{\text{QCD}}) \) describes hadronization
RG Evolution vs. Pythia’s Parton Shower

\[
\langle Q_\kappa(p_T^J R, \text{flavor}) \rangle = \text{perturbative}(\kappa, p_T^J R) \times \text{hadronization}(\kappa, \text{flavor})
\]

perturbative splitting + evolution

- Normalize average jet charge:
  \[
  \frac{\langle Q_\kappa(p_T^J R) \rangle}{\langle Q_\kappa(50 \ GeV) \rangle}
  \]

→ Hadronization (and flavor dependence) drops out

✓ Good agreement
Fragmentation Functions vs. Pythia’s Hadronization

- Average jet charge at $p_T^J = 100$ GeV, $R = 0.5$

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$u$-quark</th>
<th>$d$-quark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PYTHIA</td>
<td>DSS</td>
</tr>
<tr>
<td>0.5</td>
<td>0.271</td>
<td>0.237</td>
</tr>
<tr>
<td>1</td>
<td>0.144</td>
<td>0.122</td>
</tr>
<tr>
<td>2</td>
<td>0.055</td>
<td>0.046</td>
</tr>
</tbody>
</table>

(DSS = De Florian, Sassot, Stratmann, AKK08 = Albino, Kniehl, Kramer)

✓ Pythia consistent with fragmentation functions

- Large uncertainties as we need $D_{q}^{h^+} - D_{q}^{h^-} = D_{\bar{q}}^{h^+} - D_{\bar{q}}^{h^+}$

Most fragmentation data is $e^+e^-$ giving $D_{q}^{h^+} + D_{\bar{q}}^{h^+}$
Average Dijet Charge at the LHC

- Depends on proton structure and scattering process
- Pure QCD measurement of valence structure of proton
- Study of scale violation effect is ongoing

\[ \int \mathcal{L} dt = 5.8 \text{ fb}^{-1} \]

\[ \sqrt{s} = 8 \text{ TeV}, s = 1.0, \kappa < 70 \text{ GeV} \]

- Dijet Mass [GeV]
- Dijet Charge [e]

ATLAS Preliminary

\[ \kappa = 1.0 \]
\[ \kappa = 0.3 \]

Data
Pythia Dijets

Data/MC

(ATLAS-CONF-2013-086)
Full Jet Charge Distribution

- **Perturbative splitting** reduces $\mu$-dependence (Jain, Procura, WW)
- **Hadronization** depends on full charge distribution $D_i(Q_\kappa, \mu)$
- Moments related to multi-hadron fragmentation functions
Full Jet Charge Distribution

Perturbative splitting

\[ \mu \sim p_T^J R \]

Shower-like evolution

\[ j(z) \]

\[ i \]

Hadronization

\[ \mu \sim \Lambda_{QCD} \]

\[ \mu \frac{d}{d\mu} D_i(Q_\kappa, \mu) = \sum_j \int dz \frac{\alpha_s}{2\pi} P_{ji}(z) \int dQ^a_\kappa D_j(Q^a_\kappa, \mu) \int dQ^b_\kappa D_k(Q^b_\kappa, \mu) \times \delta[Q_\kappa - z^\kappa Q^a_\kappa - (1 - z)^\kappa Q^b_\kappa] \]

Sample over distributions of branches

Splitting probability

Charge is (weighted) sum of branches

• RGE:

\[ \mu \frac{d}{d\mu} D_i(Q_\kappa, \mu) = \sum_j \int dz \frac{\alpha_s}{2\pi} P_{ji}(z) \int dQ^a_\kappa D_j(Q^a_\kappa, \mu) \int dQ^b_\kappa D_k(Q^b_\kappa, \mu) \times \delta[Q_\kappa - z^\kappa Q^a_\kappa - (1 - z)^\kappa Q^b_\kappa] \]
RG Evolution vs. Pythia’s Parton Shower

✓ Use Pythia as input and evolve → good agreement

• Can go to higher orders, which involves $1 \rightarrow n$ splittings

• Distribution changes more slowly than fragmentation functions or parton distribution functions
Ratio of Two Angularities

Thaler, Larkoski (arXiv:1307.1699)
Larkoski, Moult, Neill (arXiv:1401.4458)
Procura, WW, Zeune (arXiv:1410.6483)
Ratios of Observables are Not IR Safe

- Angularities $e_\alpha$ probe the radial energy distribution in jet

\[
e_\alpha = \sum_{i \in \text{jet}} \frac{p_T^i}{p_T^j} \left( \frac{\theta_i}{R} \right)^\alpha
\]

- $e_\alpha$ is IR safe: $p_T^i \theta_i^\alpha + p_T^j \theta_j^\alpha \xrightarrow{\text{coll.}} (p_T^i + p_T^j) \theta_i^\alpha$

(Berger, Kucs, Sterman; Almeida et al.)
Ratios of Observables are Not IR Safe

- Angularities $e_\alpha$ probe the radial energy distribution in jets

\[ e_\alpha = \sum_{i \in \text{jet}} \frac{p^i_T}{P^j_T} \left( \frac{\theta_i}{R} \right)^\alpha \]

- $e_\alpha$ is IR safe: $p^i_T \theta_i^\alpha + p^j_T \theta_j^\alpha \xrightarrow{\text{coll.}} (p^i_T + p^j_T) \theta_i^\alpha$

- Ratio $r = e_\alpha / e_\beta$ is not IR safe

(Soyez, Salam, Kim, Dutta, Cacciari)

\[ \frac{d\sigma}{dr} = \int d\epsilon_\alpha d\epsilon_\beta \frac{d^2\sigma}{d\epsilon_\alpha d\epsilon_\beta} \delta \left( r - \frac{e_\alpha}{e_\beta} \right) \]

IR divergence

(Berger, Kucs, Sterman; Almeida et al.)
Ratios of Observables are Sudakov Safe

- Resummation of $\ln e_\alpha, \ln e_\beta$ is required when $e_\alpha, e_\beta \ll 1$
- IR divergence is Sudakov suppressed (Larkoski, Thaler)

$$
\frac{d^2 \sigma^{LL}_i}{de_\alpha de_\beta} = \frac{\partial}{\partial e_\alpha} \frac{\partial}{\partial e_\beta} \exp \left[ - \frac{\alpha_s C_i}{\pi} \left( \frac{1}{\beta} \ln^2 e_\beta + \frac{1}{\alpha - \beta} \ln^2 \frac{e_\alpha}{e_\beta} \right) \right]
$$

$$
C_i = C_F \text{ (quark), } C_A \text{ (gluon)}
$$

- No valid expansion in $\alpha_s$

$$
\frac{d\sigma}{dr} = \int de_\alpha de_\beta \frac{d^2 \sigma}{de_\alpha de_\beta} \delta \left( r - \frac{e_\alpha}{e_\beta} \right)
= \sqrt{\alpha_s} \frac{\sqrt{C_F} \beta}{\alpha - \beta} \frac{1}{r} + \mathcal{O}(\alpha_s)
$$

(LL Resummation
Ratio $r_{\alpha, \beta}, \alpha = 2$
- $\beta = 1.5$
- $\beta = 1.0$
- $\beta = 0.5$
- $\beta = 0.0$

(arXiv:1307.1699)
Two Angularities Beyond Leading Log

- Degrees of freedom in Soft-Collinear Effective Theory

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</tr>
</thead>
<tbody>
<tr>
<td>collinear</td>
<td>$p_T^J(1, e_\beta^{2/\beta}, e_\beta^{1/\beta})$</td>
<td>$p_T^J e_\beta^{1/\beta}$</td>
</tr>
<tr>
<td>soft</td>
<td>$p_T^J(e_\alpha, e_\alpha, e_\alpha)$</td>
<td>$p_T^J e_\alpha$</td>
</tr>
</tbody>
</table>

- Leads to two factorization theorems (Larkoski, Moult, Neill)

\[
\frac{d^2\sigma_i}{de_\alpha de_\beta} e_\alpha \sim e_\beta = H_i(p_T^J, \mu) \int de'_\beta J_i(e_\beta - e'_\beta, \mu) S_i(e_\alpha, e'_\beta, \mu)
\]

\[
\frac{d^2\sigma_i}{de_\alpha de_\beta} e_\alpha \sim e_\beta = H_i(p_T^J, \mu) \int de'_\alpha J_i(e'_\alpha, e_\beta, \mu) S_i(e_\alpha - e'_\alpha, \mu)
\]

- Convolution structure follows from measurement:

\[
e_\alpha p_T^J R = \sum_{c \in \text{jet}} p_T^c \theta_c^\alpha + \sum_{s \in \text{jet}} p_T^s \theta_s^\alpha \sim p_T^J \times e_\beta^{\alpha/\beta} + e_\alpha p_T^J \times 1
\]

\[
\text{power suppressed unless } e_\alpha^\alpha \sim e_\beta^\beta
\]
Resummation for the Phase-Space Boundaries

- \( H, J \) and \( S \) calculated from their field-theoretic definitions
  (Lee, Hornig, Ovanesyan; Ellis et al.; Larkoski, Moult, Neill)

- Each depends on one scale

\[ \mu_H \sim p_T^J, \quad \mu_J \sim p_T^J e^{1/\beta}, \quad \mu_S \sim p_T^J e^\alpha \]

- sum logs with RG evolution

- Factorization theorems correspond to phase-space boundaries

- Conjecture for NLL interpolation
  (Larkoski, Moult, Neill)
Factorization in Intermediate Regime

- Power expansion for $e_{\beta}^{\alpha/\beta} \ll e_{\alpha} \ll e_{\beta}$ gives

$$\frac{d^2\sigma_i}{de_{\alpha}de_{\beta}} = H_i(p_T^J, \mu) J_i(e_{\beta}, \mu) S_i(e_{\alpha}, \mu) \quad \rightarrow \quad \text{inconsistent!}$$
Factorization in Intermediate Regime

- Power expansion for \( e^\alpha/\beta \ll e_\alpha \ll e_\beta \) gives

\[
\frac{d^2 \sigma_i}{de_\alpha \, de_\beta} = H_i(p_T^J, \mu) J_i(e_\beta, \mu) S_i(e_\alpha, \mu) \quad \rightarrow \quad \text{inconsistent!}
\]

- **Collinear-soft** mode missing (Procura, WW, Zeune)

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<td>( p_T^J(1, e^{2/\beta}<em>\beta, e^{1/\beta}</em>\beta) )</td>
<td>( p_T^J e^{1/\beta}_\beta )</td>
</tr>
<tr>
<td>collinear-soft</td>
<td>( p_T^J(e^{-(\beta-\alpha)}<em>\beta e^{(\alpha-\beta)}</em>\beta, e^{(\alpha-\beta)}<em>\beta, e^{(\alpha-\beta)}</em>\beta, e^{(\alpha-\beta)}_\beta) )</td>
<td>( p_T^J e^{(1-\beta)}<em>\alpha e^{(\alpha-1)}</em>\beta )</td>
</tr>
<tr>
<td>soft</td>
<td>( p_T^J(e_\alpha, e_\alpha, e_\alpha) )</td>
<td>( p_T^J e_\alpha )</td>
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\[
\frac{d^2 \sigma_i}{de_\alpha \, de_\beta} = H_i(p_T^J, \mu) \int de'_\alpha \, de'_\beta \ J_i(e_\beta - e'_\beta, \mu) S_i(e'_\alpha, e'_\beta, \mu) S_i(e_\alpha - e'_\alpha, \mu)
\]

- **Collinear-soft** merges with **collinear** or **soft** on boundaries
Two Angularities at Next-to-Leading Log Order

• Collinear-soft function is matrix element of Wilson lines

\[ S_i(e_\alpha, e_\beta) = \frac{1}{N_c} \left\langle 0 \left| \text{Tr} \left[ \overline{\mathbf{T}}(X_n^+(0)V_n(0)) \delta \left( e_\alpha - \sum_i \frac{p_i^j}{P_T} \left( \frac{\theta_i}{R} \right)^\alpha \right) \delta \left( e_\beta - \sum_i \frac{p_i^j}{P_T} \left( \frac{\theta_i}{R} \right)^\beta \right) \mathbf{T}(V_n^+(0)X_n(0)) \right] \right| 0 \right\rangle \]

“collinear” emissions “soft” emissions

• NLL result in terms of evolution kernels \( K_X^i, \eta_X^i \)

\[ \Sigma_i(e_\alpha, e_\beta) = \int_0^{e_\alpha} de'_\alpha \int_0^{e_\beta} de'_\beta \frac{d^2 \sigma}{de'_\alpha de'_\beta} = \frac{e^{K_H^i + K_J^i + K_S^i - \gamma_E^i \eta_J^i - \gamma_E^i \eta_S^i}}{\Gamma(1 + \eta_J^i) \Gamma(1 + \eta_S^i)} \]

• Differs from NLL conjecture away from boundaries at \( \mathcal{O}(\alpha_s^2) \)

\[ \Gamma(1 + \eta_J^i) \Gamma(1 + \eta_S^i) \neq \Gamma(1 + \eta_J^i + \eta_S^i) \]

• Non-logarithmic corrections at boundaries beyond NLL:

\[ \int de'_\alpha S_i(e'_\alpha, e_\beta, \mu) S_i(e_\alpha - e'_\alpha, \mu) = S_i(e_\alpha, e_\beta, \mu) + \mathcal{O}\left(\frac{e_\alpha}{e_\beta}\right) \]
Many Possible Applications

- Ratio observables:
  - $N$-subjettiness (Thaler, Van Tilburg)
  - Planar flow (Thaler, Wang; Almeide et al.)
  - Energy correlation functions (Banfi, Salam, Zanderighi; Larkoski, Salam, Thaler; Larkoski, Moul, Neill)
  - Multivariate analyses (an example in next section)

- Jet production with hierarchies in jet energies or angles (Bauer, Tackmann, Walsh, Zuberi; Pietrulewicz, Tackmann, WW; …)

- …
Generalized Angularities

Larkoski, Thaler, WW (arXiv:1408.3122)
Generalized Angularities

\[ \lambda_\beta^\kappa = \sum_{i \in \text{jet}} \left( \frac{p_T^i}{P_T^j} \right)^\kappa \left( \frac{\theta_i}{R} \right)^\beta \]

- \( \kappa = 1 \): IR safe, angularities
- \( \beta = 0 \): very IR unsafe, similar to jet charge
- blue: a bit IR unsafe, one nonpert. parameter at NLL

Can we use \( \lambda_\beta^\kappa \) to distinguish quark jets from gluon jets?
Mutual Information

\[ I(A; B) = \int da \, db \, p(a, b) \log_2 \frac{p(a, b)}{p(a)p(b)} \]

- Can directly be calculated from double diff. cross section

\[ p(a, b) = \frac{1}{\sigma} \frac{d^2\sigma}{da \, db} \]

- Quark/gluon discrimination is one bit of (truth) information

Number of bits of shared information
Quark/Gluon Discrimination with $\lambda_{n}^{\kappa_{\beta}}$

- (N)LL valid in grey bounds
- LL is constant (Casimir scaling)
- Significant differences
- Interesting region outside our bounds

Calculation uses arXiv:1306.6630 (Chang, Procura, Thaler, WW)
Quark/Gluon Discrimination with $e_\alpha, e_\beta$

- LL not constant
- Similar behavior, different size:
  - LL ~ Herwig
  - NLL ~ Pythia

Calculation uses arXiv:1401.4458 (Larkoski, Moult, Neill)
Quark/Gluon Discrimination with $\lambda^\rho_\alpha$, $\lambda^\kappa_\beta$

- (N)LL valid in grey bounds
- LL not constant
- Significant differences
- But similar $\lambda^\rho_\alpha$, $\lambda^\kappa_\beta$ correlations

Calculation uses arXiv:1401.4458 (Larkoski, Moult, Neill)
Out-of-jet Hadron Multiplicity

Ritzmann, WW (in progress)
Hadron Multiplicity

- Extensively studied in $e^+e^-$: (Malaza, Webber; Lupia, Ochs; Eden, Gustafson; Capella et al.; Bolzoni, Kniehl, Kotikov)
  - Frag. function evolution
  - Small $x$ resummation

![Graph showing hadron multiplicity versus $Q$ (GeV) for gluon and quark jets.](arXiv:1305.6017)
Hadron Multiplicity

- Extensively studied in $e^+e^-$: (Malaza, Webber; Lupia, Ochs; Eden, Gustafson; Capella et al.; Bolzoni, Kniehl, Kotikov)
- Frag. function evolution
- Small $x$ resummation
- Underlying event studies measure hadrons away from jets
  - Not well modelled
  - Ambiguity between primary vs. secondary collisions
Out-of-jet Hadron Multiplicity in $e^+ e^-$

- Hadrons produced by soft radiation when $E_{\text{out}} \ll Q$, $R \ll 1$

$$\langle n_{\text{out}} \rangle = \frac{S(\langle n_{\text{out}} \rangle, E_{\text{out}}, R, \mu)}{S(E_{\text{out}}, R, \mu)} + \mathcal{O}\left(\frac{E_{\text{out}}}{Q}, R^2\right)$$

$$= \int_{-\eta_{\text{cut}}}^{\eta_{\text{cut}}} d\eta \int_0^{2\pi} d\phi \int_0^1 dr \frac{1}{N_c} \text{Tr} \langle 0| T [Y_n^\dagger Y_{\bar{n}}] \hat{n}(y, \phi, r) T [Y_{\bar{n}}^\dagger Y_n] |0 \rangle + \mathcal{O}(\alpha_s)$$

$$= 4\pi \eta_{\text{cut}} = M(r, \mu)$$

- By boost and rotational invariance $M$ only depends on $r = p_T^2 / (p_T^2 + m^2)^{1/2}$, encoding hadron mass effects
Some Preliminary Results

\[ \langle n_{\text{out}}(\mu) \rangle = 4\pi \eta_{\text{cut}} \int_{0}^{1} dr \, M(r, \mu) \]

- Calculating \( S(E_{\text{out}}, R, \mu) \) at NLO suggests \( \mu \sim E_{\text{out}} R^{0.5} \)
- RG evolution \( \frac{d}{d \ln \mu} \ln M(r, \mu) = -\frac{\alpha_s C_A}{\pi} \ln(1 - r^2) \)

(From similar calculation for transverse velocity operator by Mateu, Stewart, Thaler)
Some Preliminary Results

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(From similar calculation for transverse velocity operator by Mateu, Stewart, Thaler)

- Find \( R^{0.5} \to R^{0.75} \) depending on Monte Carlo
- Fits \( \langle n_{\text{out}}(\mu) \rangle = a + b \ln \mu \)
- Strong running since:
  \[ 1 - r^2 \approx \left( \frac{m_\pi \langle n_{\text{out}} \rangle}{E_{\text{out}}} \right)^2 \]
Summary

- Jet substructure provides a new set of tools at the LHC
- IR unsafe observables have interesting applications

**Sudakov safe:**
Calculable in resummed perturbation theory

**Collinear unsafe:**
Unknown distribution
Known evolution

**Soft unsafe:**
Unknown distribution
“Known” evolution
Depends on all jets

N-subjettiness ratio
Jet charge
Out-of-jet hadron multiplicity

Track-based IR-safe observables
Hadron multiplicity

Planar flow

**Generalized angularities**

*thank you!*