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Towards improved predictions for electroweak vector boson pair production at the LHC

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Outline

1) Motivation

2) How to compute two-loop amplitudes for gg -> V_1V_2 and qq -> V_1V_2 ;

3) QCD corrections to gg-> ZZ through the massive top quark loop;

4) Conclusions

Importance of vector boson production

Production of pairs of electroweak vector bosons in hadron collisions is an interesting process for several reasons. For example,

1) it is an important SM benchmark with small, but not tiny cross-section; this process may contain missing energy, semi-leptonic final states, additional jets -- lots of physics and many features used to define a generic "signal" !

2) it is sensitive to triple gauge boson couplings;

3) it is a background to Higgs boson signal in ZZ^* and WW^* final states;

4) it is a background for searches with multi-lepton final states and jets (SUSY);

5) it can be used to put constraints on the Higgs boson width;

6) it can be used to probe for anomalous couplings in the Higgs sector

These are clearly good enough reasons to push for the best possible description of this process in perturbative QCD. This topic is also quite challenging theoretically as problems that need to be addressed to achieve high precision are quite non-trivial.

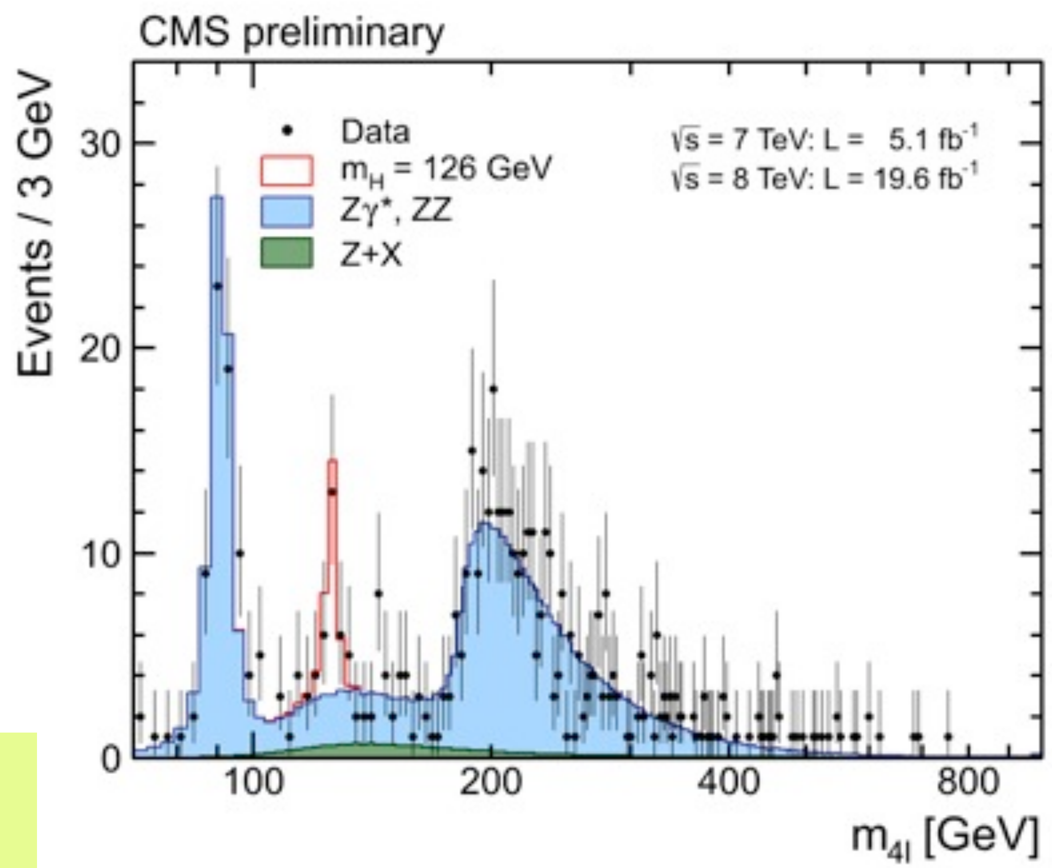
Issues we need to address to make NNLO accuracy useful

Our goal is to extend the description of vector boson pair production to NNLO in perturbative QCD. This is a non-trivial task per se but, in order to address realistic physics, we should be careful to take a few points into account.

For example, we should have access to vector bosons on- (VV production proper) and off- (background to Higgs signal) the mass-shell.

We should be able to monitor hadronic activity (jet selection criteria) since this is how the Higgs signal is separated from many backgrounds in the WW channel.

Therefore, we need to construct a fully-exclusive partonic level generator that can describe production of four lepton final states in perturbative QCD including both double-resonant and single-resonant components. In a way, we would like to be able to provide NNLO QCD corrections to the plot on the right for all values of the four-lepton invariant mass.



The NNLO QCD calculation exists for total cross sections for ZZ and WW production at the LHC.

Grazzini, Gehrmann, Pozzorini,Rathlev, Tancredi

Anatomy of a NNLO computation

A NNLO QCD computation for a process pp -> X requires several well-defined things:

1) Two-loop amplitudes for i+j -> X where i and j are parton labels;

2) One-loop amplitudes for i+j -> X+k;

3) Tree amplitudes for i+j -> X+k+m

Putting those things together is a non-trivial procedure that is currently being explored in several ways. We have seen significant process in our understanding of how to organize the NNLO computations in recent years. This could have been a topic of a separate seminar...

However, my goal today is different. I would like to talk about point 1) -- calculation of two-loop amplitudes for vector boson pair production -- in some detail. This is a small well-defined part of the NNLO program that can be discussed separately.

Two-loop calculations in QCD

Calculation of two-loop virtual corrections is an integral part of any NNLO QCD computation for the LHC. Technology for such computations is being continuously developed since early 2000's. It can be summarized as a sequence of four steps

1) Parametrization of scattering amplitudes in terms of Lorentz-invariant form factors;

2) Determination of operators that project an amplitude on individual form factors ;

3) Application of integration-by-parts identities and reduction of scalar Feynman integrals to master integrals;

4) Calculation of master integrals;

Each of these steps is relatively well-established, but for each step there are complications;

1) parametrization of scattering amplitudes becomes non-trivial in cases when we have to deal with the Dirac matrix γ_5 in closed fermion loops (in fact, so far we were always able to argue such contributions away);

2) integration-by-parts can be dealt with using existing programs (AIR,FIRE, REDUZE) but it becomes very difficult for processes with large number of kinematic invariants;

3) calculation of master integrals was always an ``art'' rather than `science" and it a continues to remain this way.

gg -> VV scattering amplitude

As an illustration, consider computation of a two-loop gg -> VV amplitude. For generic vector bosons, the amplitude can be written in a form with all electroweak couplings factored out (massless quarks only)

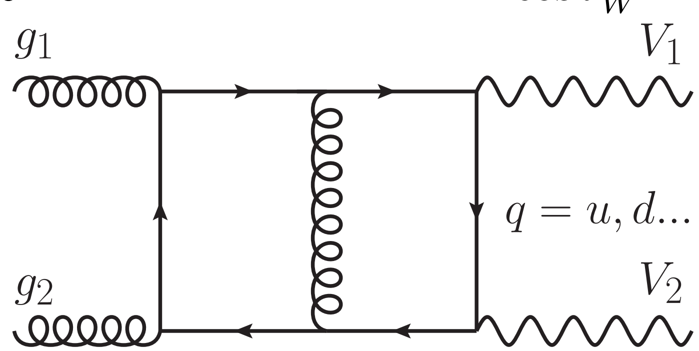
$$\mathcal{M}(\lambda_{g_1},\lambda_{g_2},\lambda_5,\lambda_7) = i\left(\frac{g_W}{\sqrt{2}}\right)^4 \delta^{a_1 a_2}\mathcal{D}_3\mathcal{D}_4 C^{\lambda_7}_{l,V_2} C^{\lambda_5}_{l,V_1}\epsilon_3^{\mu}(\lambda_5)\epsilon_4^{\nu}(\lambda_7)\; C_{V_1V_2}\; \mathcal{A}_{\mu\nu}(p_1^{\lambda_{g_1}},p_2^{\lambda_{g_2}};p_3,p_4),$$

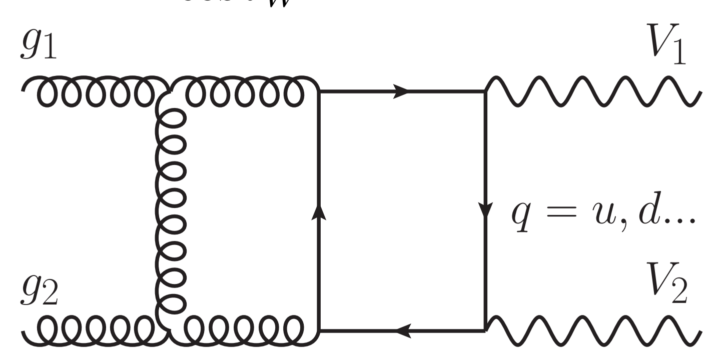
Couplings to leptons and quarks are shown below; note the absence of vector-axial current correlators (C-parity) and the equality of the vector-vector and axial-axial current correlators.

$$C_{\gamma}^{L,R} = -\sqrt{2}Q_l\sin\theta_W, \qquad C_{l,Z}^{L,R} = \frac{1}{\sqrt{2}\cos\theta_W}\left(V_l \pm A_l\right), \qquad C^{\lambda}_{lW^+} = C^{\lambda}_{lW^-} = \delta_{\lambda L}.$$

$$V_e = -1/2 + 2\sin^2\theta_W, \quad V_\nu = 1/2, \quad A_e = -1/2, \quad A_\nu = 1/2$$

$$C_{\gamma\gamma} = \frac{20\sin\theta_W^2}{9}, \qquad C_{ZZ} = \frac{(V_u^2+V_d^2+A_u^2+A_d^2)}{\cos\theta_W^2}, \quad C_{Z\gamma} = -\frac{2\sin\theta_W}{\cos\theta_W}\left(V_uQ_u + V_dQ_d\right), \qquad C_{W^+W^-} = 1,$$





The primary object to compute is the amplitude A(p1,p2,p3,p4) contracted with the polarization vectors of vector bosons. Only vector couplings of electroweak bosons are needed.

gg -> VV scattering amplitude

The problem with computing the amplitude $A(p_1,p_2,p_3,p_4)$ "as is" is that it is too complicated at two-loops. Indeed, integration-by-parts technology can not be used efficiently if there are many external vectors (vector boson polarizations and/or lepton momenta) in the calculation.

To remove all external vectors, we need to express the amplitude through invariant form factors. If this is done without imposing reasonable physics conditions, the number of form factors becomes very large, O(150) !

Using transversality and gauge-fixing conditions $\epsilon_1 \cdot p_{1,2} = 0, \quad \epsilon_2 \cdot p_{1,2} = 0, \quad \epsilon_3 \cdot p_3 = 0, \quad \epsilon_4 \cdot p_4 = 0.$
we express the amplitude through just 20 form factors.

$$
\begin{aligned}
\mathcal{A} &= T_1 \left(\epsilon_1 \cdot \epsilon_2\right)\left(\epsilon_3 \cdot \epsilon_4\right) + T_2 \left(\epsilon_1 \cdot \epsilon_3\right)\left(\epsilon_2 \cdot \epsilon_4\right) + T_3 \left(\epsilon_1 \cdot \epsilon_4\right)\left(\epsilon_2 \cdot \epsilon_3\right) \\
&+ \left(\epsilon_1 \cdot \epsilon_2\right)\left(T_4 (p_1 \cdot \epsilon_3)\left(p_1 \cdot \epsilon_4\right) + T_5 (p_1 \cdot \epsilon_3)\left(p_2 \cdot \epsilon_4\right) + T_6 (p_2 \cdot \epsilon_3)\left(p_1 \cdot \epsilon_4\right) + T_7 (p_2 \cdot \epsilon_3)\left(p_2 \cdot \epsilon_4\right)\right) \\
&+ \left(\epsilon_1 \cdot \epsilon_3\right)\left(p_\perp \cdot \epsilon_2\right)\left(T_8 (p_1 \cdot \epsilon_4) + T_9 \left(p_2 \cdot \epsilon_4\right)\right) + \left(\epsilon_1 \cdot \epsilon_4\right)\left(p_\perp \cdot \epsilon_2\right)\left(T_{10} (p_1 \cdot \epsilon_3) + T_{11} \left(p_2 \cdot \epsilon_3\right)\right) \\
&+ \left(\epsilon_2 \cdot \epsilon_3\right)\left(p_\perp \cdot \epsilon_1\right)\left(T_{12} (p_1 \cdot \epsilon_4) + T_{13} \left(p_2 \cdot \epsilon_4\right)\right) + \left(\epsilon_2 \cdot \epsilon_4\right)\left(p_\perp \cdot \epsilon_1\right)\left(T_{14} (p_1 \cdot \epsilon_3) + T_{15} \left(p_2 \cdot \epsilon_3\right)\right) \\
&+ \left(\epsilon_1 \cdot p_\perp\right)\left(\epsilon_2 \cdot p_\perp\right)\left(T_{17} (p_1 \cdot \epsilon_3)\left(p_1 \cdot \epsilon_4\right) + T_{18} (p_1 \cdot \epsilon_3)\left(p_2 \cdot \epsilon_4\right) + T_{19} (p_2 \cdot \epsilon_3)\left(p_1 \cdot \epsilon_4\right)\right. \\
&\left. + T_{20} (p_2 \cdot \epsilon_3)\left(p_2 \cdot \epsilon_4\right)\right) + \left(\epsilon_3 \cdot \epsilon_4\right)\left(p_\perp \cdot \epsilon_1\right)\left(p_\perp \cdot \epsilon_2\right) T_{16}
\end{aligned}
$$

The transverse momentum is defined through the Sudakov decomposition w.r.t p_1 and p_2.

The invariant form-factors T_1-T_{20} are functions of Mandelstam variables only; if we construct operators that project A -> $T_{i=1..16}$, we are able to apply integration-by-parts technology without much problem.

gg -> VV scattering amplitude

It is relatively straightforward to construct such projection operators (their choices are by far not unique). We start by defining an auxiliary object (note projections on physical polarizations)

$$\mathcal{O}^{\mu_1\mu_2\mu_3\mu_4} = \bar{A}_{\nu_1\nu_2\nu_3\nu_4}\mathcal{P}_{12}^{\nu_1\mu_1}\mathcal{P}_{12}^{\nu_2\mu_2}\mathcal{P}_{3}^{\nu_3\mu_3}\mathcal{P}_{4}^{\nu_4\mu_4}.$$

$$\mathcal{P}_{12}^{\mu\nu} = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1\cdot p_2}, \quad \mathcal{P}_{3}^{\mu\nu} = -g^{\mu\nu} + \frac{p_3^\mu p_3^\nu}{p_3^2}, \quad \mathcal{P}_{3}^{\mu\nu} = -g^{\mu\nu} + \frac{p_4^\mu p_4^\nu}{p_4^2}.$$

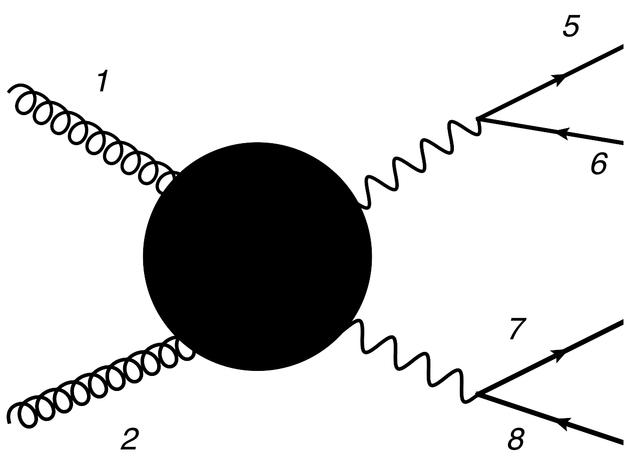
We then contract it with various vectors and tensors and find a mapping $G_{1..20}$ -> $T_{1..20}$

$$\begin{aligned}
G_1 &= \mathcal{O}^{\mu_1\mu_2\mu_3\mu_4} g_{\mu_1\mu_2} g_{\mu_3\mu_4}, & G_2 &= \mathcal{O}^{\mu_1\mu_2\mu_3\mu_4} g_{\mu_1\mu_3} g_{\mu_2\mu_4},\\
G_3 &= \mathcal{O}^{\mu_1\mu_2\mu_3\mu_4} g_{\mu_1\mu_4} g_{\mu_2\mu_3}, & G_4 &= p_\perp^{-4} s^{-2} \mathcal{O}^{p_\perp p_\perp p_1 p_1},\\
G_5 &= p_\perp^{-4} s^{-2} \mathcal{O}^{p_\perp p_\perp p_1 p_2}, & G_6 &= p_\perp^{-4} s^{-2} \mathcal{O}^{p_\perp p_\perp p_2 p_1},\\
G_7 &= p_\perp^{-4} s^{-2} \mathcal{O}^{p_\perp p_\perp p_2 p_2}, & G_8 &= 4p_\perp^{-6} s^{-2} \mathcal{O}^{p_\perp p_\perp \mu_3\mu_4} t_{\mu_3\mu_4},\\
G_9 &= 4p_\perp^{-6} s^{-6} \mathcal{O}^{\mu_1\mu_2 p_1 p_1} t_{\mu_1\mu_2}, & G_{10} &= 8p_\perp^{-4} s^{-3} \mathcal{O}^{p_\perp \mu_2\mu_3 p_1} t_{\mu_2\mu_3},\\
G_{11} &= 4p_\perp^{-6} s^{-3} \mathcal{O}^{p_\perp \mu_2\mu_3 p_\perp} t_{\mu_2\mu_3}, & G_{12} &= 8p_\perp^{-4} s^{-3} \mathcal{O}^{\mu_1 p_\perp p_1 \mu_4} t_{\mu_1,\mu_4},\\
G_{13} &= 4p_\perp^{-6} s^{-3} \mathcal{O}^{\mu_1 p_\perp p_\perp \mu_4} t_{\mu_1,\mu_4}, & G_{14} &= 8p_\perp^{-4} s^{-3} \mathcal{O}^{\mu_1 p_\perp \mu_3 p_2} t_{\mu_1,\mu_3},\\
G_{15} &= 4p_\perp^{-6} s^{-3} \mathcal{O}^{\mu_1 p_\perp \mu_3 p_\perp} t_{\mu_1,\mu_3}, & G_{16} &= 8p_\perp^{-4} s^{-3} \mathcal{O}^{p_\perp \mu_2 p_1 \mu_4} t_{\mu_2,\mu_4},\\
G_{17} &= 4p_\perp^{-6} s^{-3} \mathcal{O}^{p_\perp \mu_2 p_\perp \mu_4} t_{\mu_2,\mu_4}, & G_{18} &= 4p_\perp^{-6} s^{-6} \mathcal{O}^{\mu_1\mu_2 p_1 p_2} t_{\mu_1\mu_2},\\
G_{19} &= 4p_\perp^{-6} s^{-6} \mathcal{O}^{\mu_1\mu_2 p_2 p_1} t_{\mu_1\mu_2}, & G_{20} &= 4p_\perp^{-6} s^{-6} \mathcal{O}^{\mu_1\mu_2 p_2 p_2} t_{\mu_1\mu_2}.
\end{aligned}$$

$$t^{\mu_1}_{\nu_1} = \delta^{\mu_1 p_1 p_2 p_\perp}_{\nu_1 p_1 p_2 p_\perp} = \delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\; p_1^{\nu_2} p_2^{\nu_3} p_\perp^{\nu_4} p_{1,\mu_2} p_{2,\mu_3} p_{\perp,\mu_4} \qquad \delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4} = \det|g^{\mu_{i\in\{1...4\}}}_{\nu_{j\in\{1..4\}}}|.$$

gg -> VV scattering amplitude

It turns out that it is easier to construct helicity amplitudes from projections that we just described; one requires just two independent helicity amplitudes (polarizations of gluons are either the same or different); each amplitude is expressed in terms of nine form factors. Vector boson polarization vectors are $\epsilon_3^\mu = \langle 5|\gamma^\mu|6], \quad \epsilon_4^\mu = \langle 7|\gamma^\mu|8].$



$$
\begin{aligned}
\mathcal{A}_{3L4L}^{\lambda_1\lambda_2} =& \mathcal{N}_{\lambda_1\lambda_2}\Big\{\Big(F_1^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_2^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 17\rangle[81] \\
&+ \Big(F_3^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_4^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 27\rangle[82] + 2F_5^{\lambda_1\lambda_2}\langle 57\rangle[86] \\
&+ \frac{1}{2}\Big(F_6^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_7^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\Big(\langle 12\rangle\langle 78\rangle[81][82] + \langle 17\rangle\langle 27\rangle[21][87]\Big) \\
&- \frac{1}{2}\Big(F_8^{\lambda_1\lambda_2}\langle 17\rangle[81] + F_9^{\lambda_1\lambda_2}\langle 27\rangle[82]\Big)\Big(\langle 12\rangle\langle 56\rangle[61][62] + \langle 15\rangle\langle 25\rangle[21][65]\Big)\Big\}.
\end{aligned}
$$

Examples of relations between F and G form factors are shown below. Note that no spurious d-4 singularities are present in these relations !

$$
\begin{aligned}
F_6^{LL} &= \frac{(m_3^2-u)(G_{12}+G_{16})}{(d-3)(m_3^2-t)} + \frac{s(m_3^2+p_\perp^2)(G_{13}+G_{17})}{(d-3)(m_3^2-t)}, \\
F_7^{LL} &= \frac{G_{12}+G_{16}+(t-m_3^2)(G_{13}+G_{17})}{d-3}, \\
F_8^{LL} &= \frac{(m_4^2-t)G_{10}-s(m_4^2+p_\perp^2)G_{11}}{(d-3)(m_4^2-u)} + \frac{G_{14}+(m_4^2-t)G_{15}}{d-3}, \\
F_9^{LL} &= \frac{(u-m_4^2)G_{11}-G_{10}}{d-3} + \frac{(u-m_4^2)G_{14}+s(m_4^2+p_\perp^2)G_{15}}{(d-3)(m_4^2-t)}.
\end{aligned}
$$

gg -> VV scattering amplitude

As the result, it appears that form factors can be computed in a straightforward way. We need to:

1) generate diagrams (QGRAPH);
2) project on relevant operators (Form);
3) express the result in terms of various two-loop four-point integrals (Form) ;
4) apply integration-by-parts identities to reduce these integrals to master integrals (FIRE);
5) combine the results into physical form factors;
6) write a numerical program (Fortran,C++) that can turn the analytic formulas into numbers;

Even if every step sounds straightforward, most of them are non-trivial and demanding. A particular problem is the size of expressions that affects both reduction to master integrals and procession of the final expressions. For example, the size of Fortran files (not executables) for the final two-loop amplitude gg -> VV is O(100 MB) !)

The remaining problem to address is the computation of master integrals and this is what we will discuss now.

Master integrals

Calculation of two-loop integrals relies on a number of methods (Feynman parameter integration, Mellin-Barns, differential equations). The method of differential equations has been used to find master integrals for a long time, starting from papers by Kotikov and Remiddi in the early 1990s. However it was never considered to be ``the" method.

$$\partial_{p_i \cdot p_j} I_\alpha = \sum c_{\alpha,ij,\beta}(\{p\},d) I_\beta$$

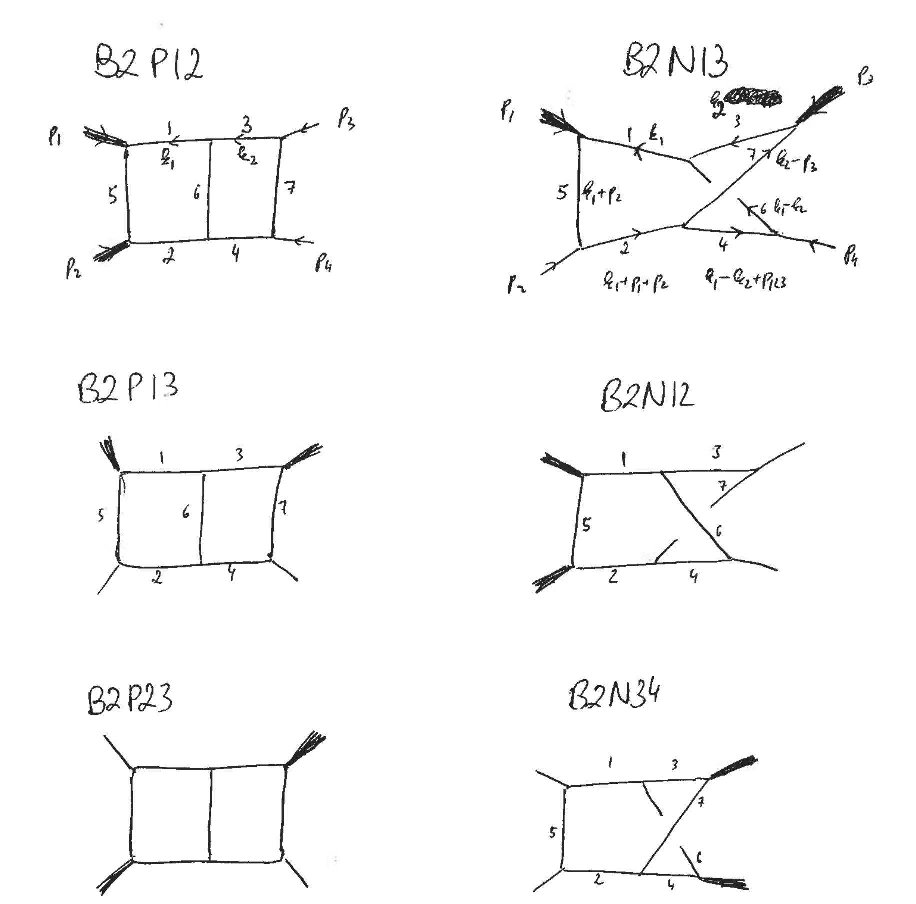
An interesting recent development in this field is the suggestion by J. Henn to streamline application of differential equations in external kinematic variables to compute master integrals. Henn suggests to write the differential equations in the following form:

$$\partial_x \vec{f} = \epsilon \hat{A}_x(x,y,z\dots)\vec{f} \qquad \vec{f} = \sum_{n=0}^{\infty} \epsilon^n \vec{f}^{(n)}$$

The important point is that on the right-hand side, the dimensional regularization parameter appears explicitly, and only as a multiplicative pre-factor. It is then possible to solve these equations iteratively order-by-order in (d-4) since in each order of this expansion the above equation contains no homogeneous terms (so that in each order in epsilon, the right-hand side is a source for the left-hand side).

The idea by Henn streamlines and simplifies such computations significantly, making bookkeeping particularly straightforward.

Two-loop virtual corrections: gg->V_1V_2



For the case of double vector boson production, we can identify six different two-loop topologies; the differential equations can be "rationalized" with the following change of variables (topology B2P12)

$$\frac{s}{m_3^2} = (1+x)(1+xy), \quad \frac{t}{m_3^2} = -xz, \quad \frac{m_4^2}{m_3^2} = x^2 y$$

$$\sqrt{(s-m_3^2-m_4^2)^2 - 4m_3^2 m_4^2} = m_3^2 x(1-y)$$

The differential equation is written in the following form where elements of the alphabet are rational functions of x,y and z (topology B2P12)

$$d\vec{f} = \epsilon(dA) \times f, \quad A = \sum A_i \log \alpha_i$$

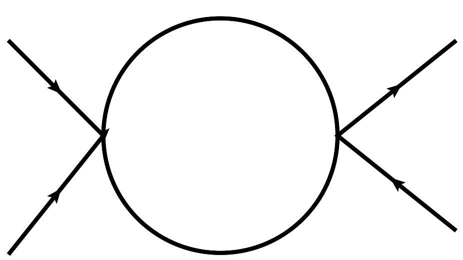
$$\begin{aligned}\alpha = \{&x, y, z, 1+x, 1-y, 1+xy, z-y, 1+y(1+x)-z,\\ &xy+z, 1+x(1+y-z), 1+xz,\\ &1+y-z, z+x(z-y)+xyz, z-y+yz+xyz\}\end{aligned}$$

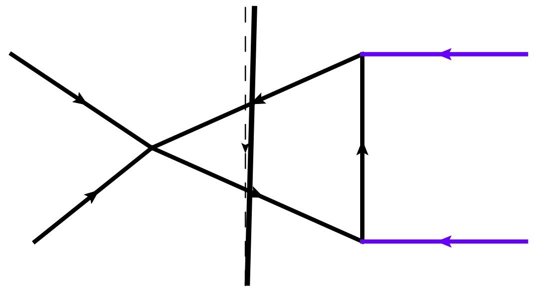
Canonical basis at one-loop

Finding canonical basis is, in general, non-trivial. If it exists, integrals have to be dimensionless quantities of uniform transcendentality. This means that only very particular terms are allowed to appear in each order of the ep-expansion of master integrals. One thing that is prohibited are rational polynomials in epsilon.

Some ideas on how to find the basis can be illustrated by looking at the one-loop example. There are three types of master integrals -- box, triangle and bubble. Bubbles is easy to fix by hand. For triangles and boxes, considering multiple cuts is useful to realize that one must multiply original integrals by certain kinematic functions to obtain candidates for integrals in the canonical basis.

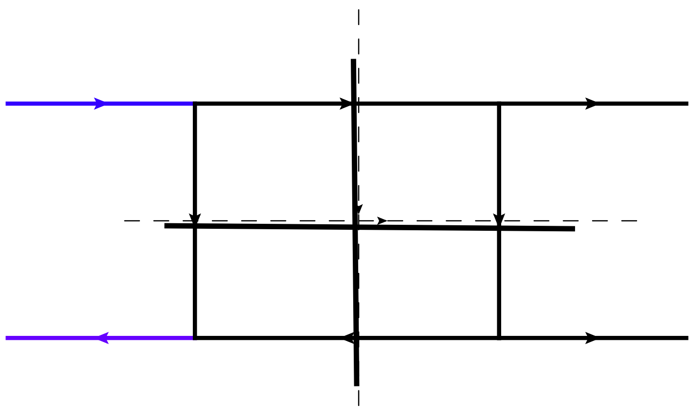
$$\mathrm{Bub}(1,1) \sim \frac{(p^2)^{-\epsilon}}{\epsilon(1-2\epsilon)} \to (1-2\epsilon)\mathrm{Bub}(1,1) \sim p^2\mathrm{Bub}(2,1)$$





$$\mathrm{Tri} \to \sqrt{\Delta}\mathrm{Tri}$$

$$\mathrm{Box} \to st\ \mathrm{Box}$$

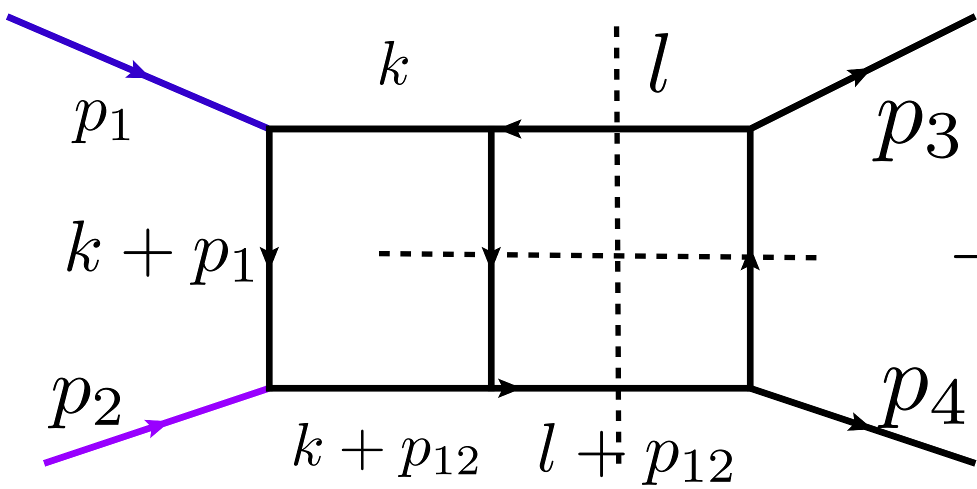


If an integral belongs to canonical basis, then cut integrals should also exhibit all the properties of the canonical basis integrals. But cut integrals are easier to compute....

Canonical basis at two-loops: an example

The idea that cut integrals should be also of fixed transcendentality becomes especially fruitful at higher loops. For example, one can understand the appearance of particular tensor integrals as two-loop master integrals in this way.

$$I = \int \mathrm{d}^d k \mathrm{d}^d l \frac{N(k)}{k^2 (k+p_1)^2 (k+p_{12})^2 l^2 (l+p_{12})^2 (l+p_3)^2 (k-l)^2}$$

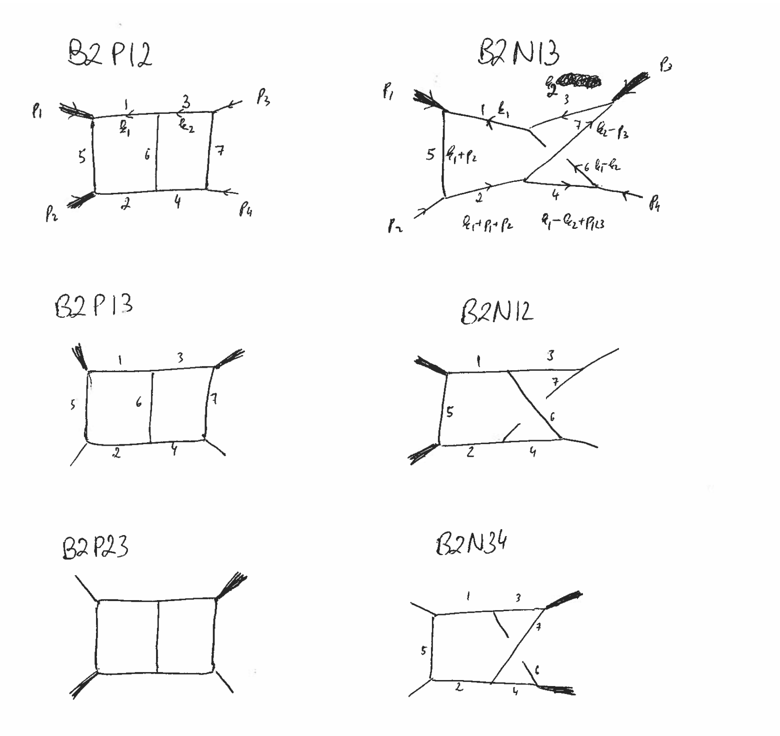


$$\to \int \mathrm{d}^d k \left[\frac{1}{s(k+p_3)^2} \right] \frac{N(k)}{k^2 (k+p_1)^2 (k+p_{12})^2}$$

Candidate integrals obtained from various cuts, consistent with canonical one-loop integrals, are require numerator functions shown below; this construction, eventually, leads to correct canonical basis at two loops.

$$N(k) = s^2 t, \quad N(k) = s(k + p_3)^2 \sqrt{\Delta}$$

Integrating system of equations



Integrating systems of differential equations in Henn's basis becomes straightforward: no homogeneous terms appear; recursive integration is possible, results expressed through Goncharov polylogarithms.

$$G(a_n, a_{n-1}, \ldots, a_1, t) = \int\limits_0^t \frac{\mathrm{d}t_n}{t_n - a_n} G(a_{n-1}, \ldots a_1, t_n)$$

Important issue -- boundary conditions. They (obviously) need to be computed. One option is calculation of boundary conditions and integrals in an Euclidean point followed by the analytic continuation. We found the analytic continuation difficult, especially because of non-linear changes of variables required to rationalize the system.

On the other hand, it is possible to compute boundary conditions in Minkowski region. We do it in a singular threshold point $s \to (m_3 + m_4)^2, \quad m_3 \to \infty, \quad m_4 \to 0.$

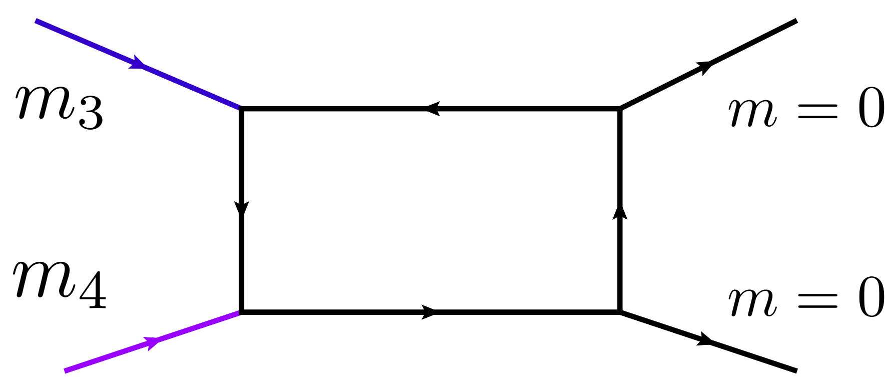
This point is not ideal. For example, there are non-trivial limits, e.g. double-parton scattering singularities in certain diagrams.

Interestingly, since we write differential equations in many variables, it happens often that differential equations in a variable y, constrain boundary conditions in a variable x....

Boundary conditions

In principle, boundary conditions need to be "just" calculated. This is possible but difficult. However, often such calculations are even not necessary since systems of differential equations contain a lot of information about relations between boundary conditions for various integrals provided we understand their singularities.

Consider an example of an "easy" box. Writing a differential equation in the transverse momentum, we find a branch cut at $p_t = 0$. But an easy box does not have a branch cut like this. This gives a relation between various integrals in brackets on the right-hand side in $p_t = 0$ limit.



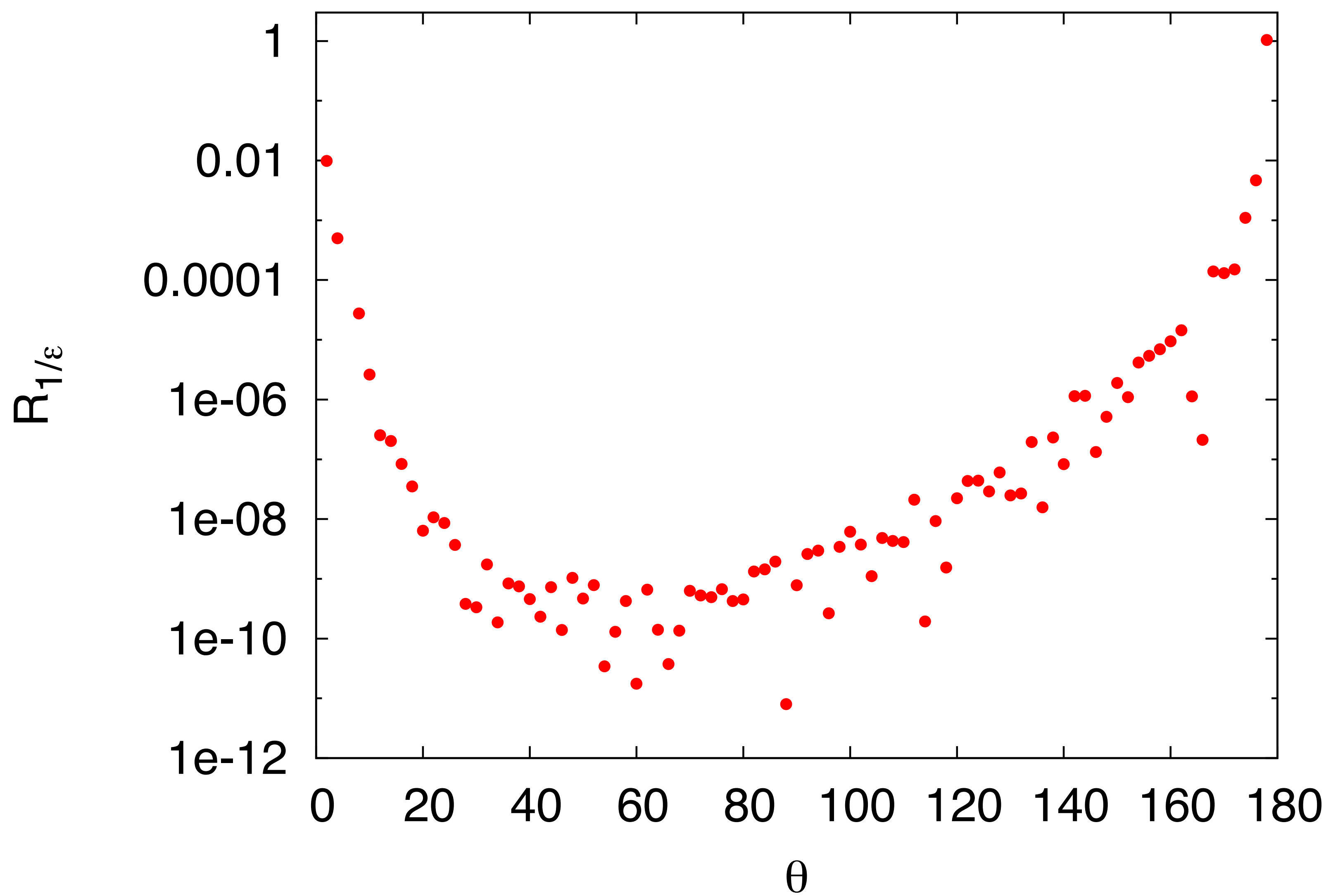
$$\partial_{p_\perp} D = \frac{\epsilon}{p_\perp}(B_1 + B_2 + B_s - 2B_t - C - D)$$

$$\lim_{p_\perp \to 0}(B_1 + B_2 + B_s - 2B_t - C) = \lim_{p_\perp \to 0} D$$

Similar ideas can be applied to the two-loop case; this is obviously more complex but the principle is the same -- many boundary conditions can be deduced from the DEqs.

Two-loop amplitudes qq->V_1V_2 and gg -> V_1V_2

Analytic results for two-loop amplitudes are implemented in a Fortran code. Numerical results for the poles can be checked against the Catani's formula, which connects the infra-red poles of NNLO amplitudes with LO and NLO amplitudes.



Kinematic point

$$\sqrt{s} = 125 \text{ GeV}, \quad m_1 = 80 \text{ GeV}, \quad m_2 = 25 \text{ GeV}.$$

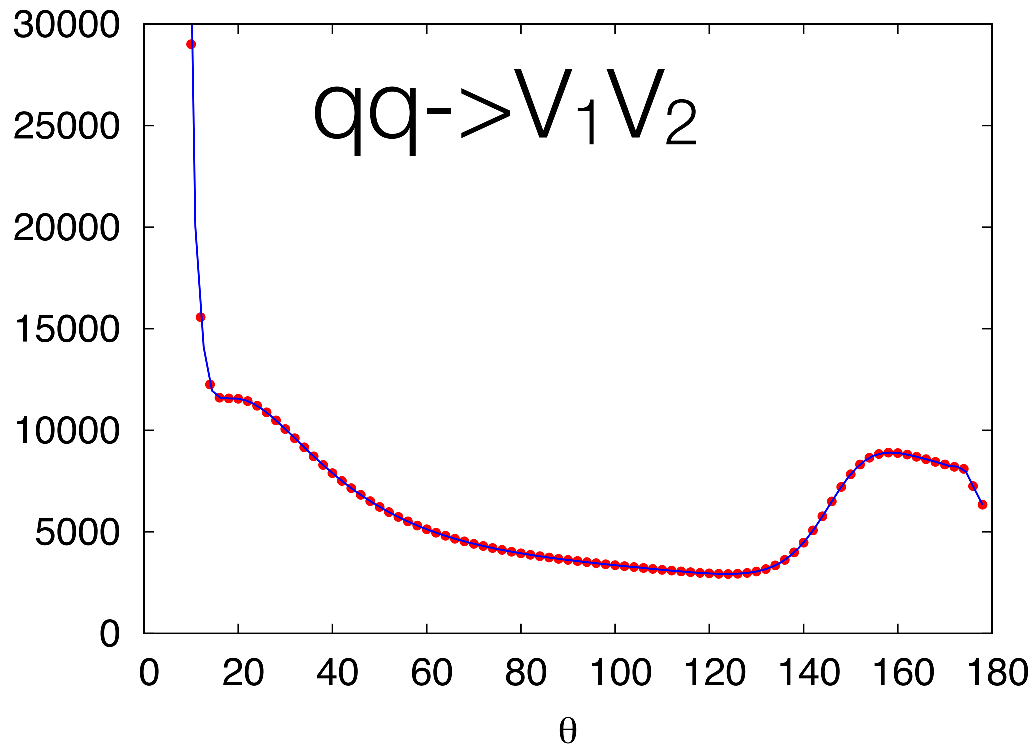
Comparison of 1/ep poles of qq ->V_1V_2 amplitude with predictions bases on Catani's formula in dependence of the scattering angle of electroweak bosons

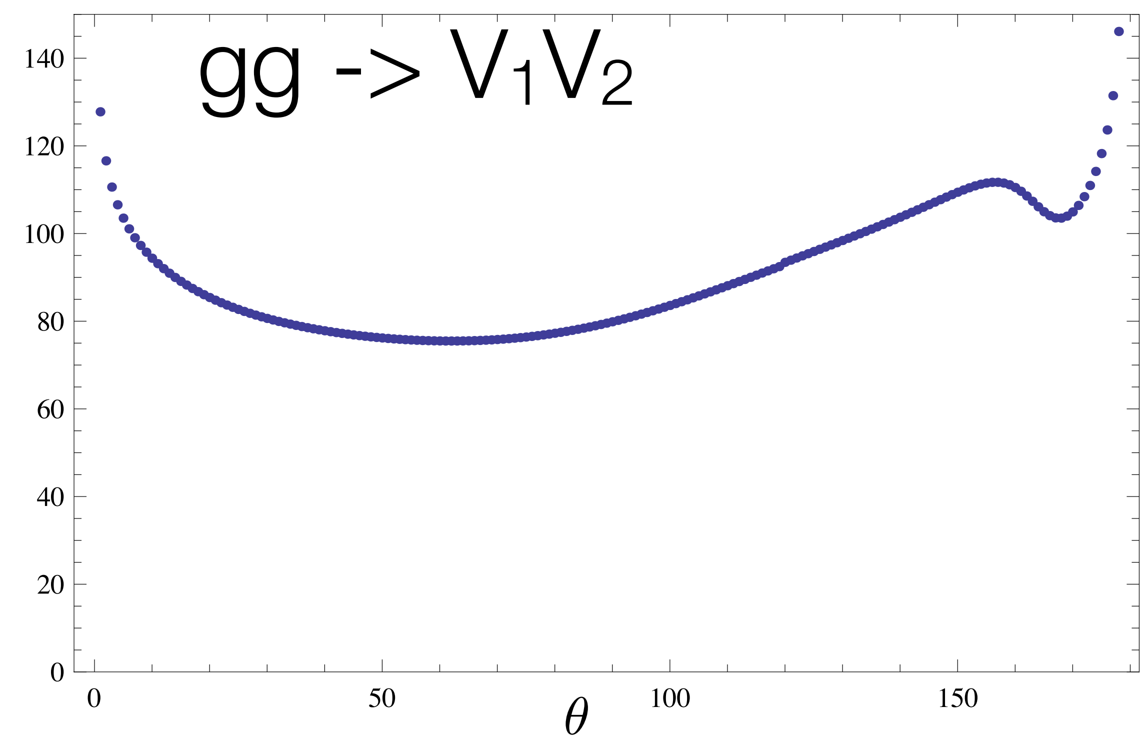
Two-loop amplitudes qq->V_1V_2 and gg -> V_1V_2

Numerical results for finite parts of two-loop amplitudes can be obtained. Important issues are numerical stability and evaluation speed.

Evaluation speed (all amplitudes / per phase-space point) close to one minute. The time is spent on the computation of master integrals (GINAC). Not ideal, but probably not a disaster.

Numerical stability needs to be explored further; results do have spurious singularity associated with zero transverse momentum; recall that even one-loop calculations were performed with the transverse momentum cut until very recently.





Ratios of various two- and tree- (one-loop) amplitudes for quark and gluon annihilation to vector bosons

Two-loop calculations: summary

Calculation of complicated two-loop scattering amplitudes are now possible thanks to recent developments in techniques related to integration-by-parts reductions and computation of master integrals.

Calculation of amplitudes requires construction of projection operators that allow a decomposition of an amplitude into invariant form factors.

Form factors are expressed through master integrals that, in turn, are computed using differential equations that follow from the integration-by-parts identities.

Calculation of master integrals in the canonical (universal transcendentality) basis is ``mechanical”; important issues are how to find this basis and how to find boundary conditions for differential equations. There are systematic considerations of how to do the former (only for single-variable problems) and not so many systematic considerations of how to do the latter. It turns out to be useful to compute boundary conditions in Minkowski space and in this way avoid analytic continuation.

From two-loop amplitudes to cross-section

Suppose that a two-loop amplitude is, actually, available. How do we get to the cross-section?
This question has many facets which are actually different for $qq \to V_1V_2$ and $gg \to V_1V_2$.

For quark annihilation, the leading order process is tree-level; implementation of two-loop amplitude requires full-fledged NNLO computation. This can be done but it is non-trivial.

For gluon annihilation, the story is different since leading order process in this case is one-loop. Therefore the two-loop amplitude has to be combined with a single real-emission process. The amplitude for single real-emission process gg -> VV+g is complicated but it is well-understood how to combine virtual and real corrections in this case.

An essential limitation of the calculation of the virtual amplitude that I just described is that it is applicable to massless quarks only. For certain type of physics this is sufficient and for some other physics it is not.

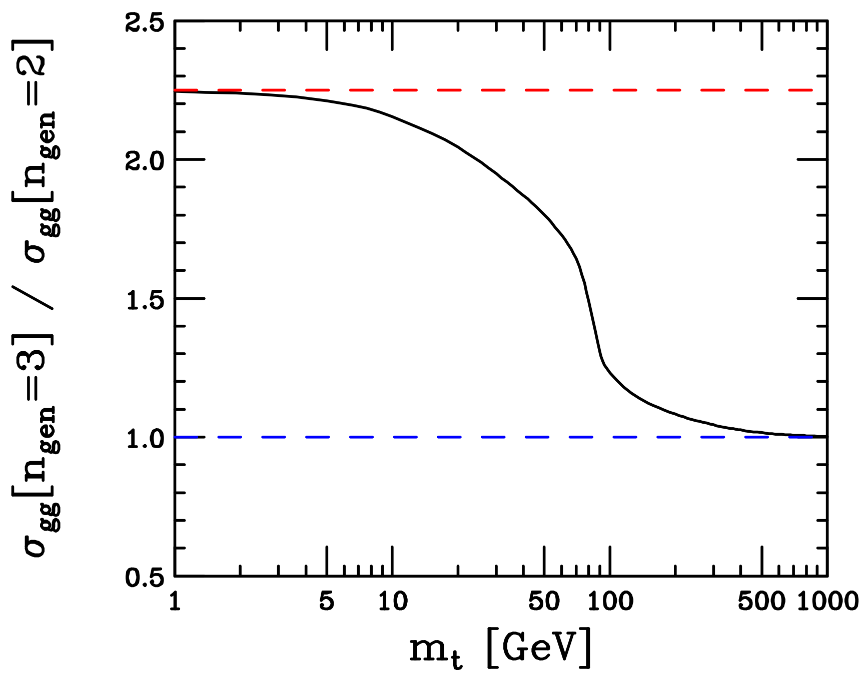
In what follows I want to focus on a gluon annihilation to ZZ and discuss a massive quark contribution to this process. I will explain the precise reasons for that on the next slide.

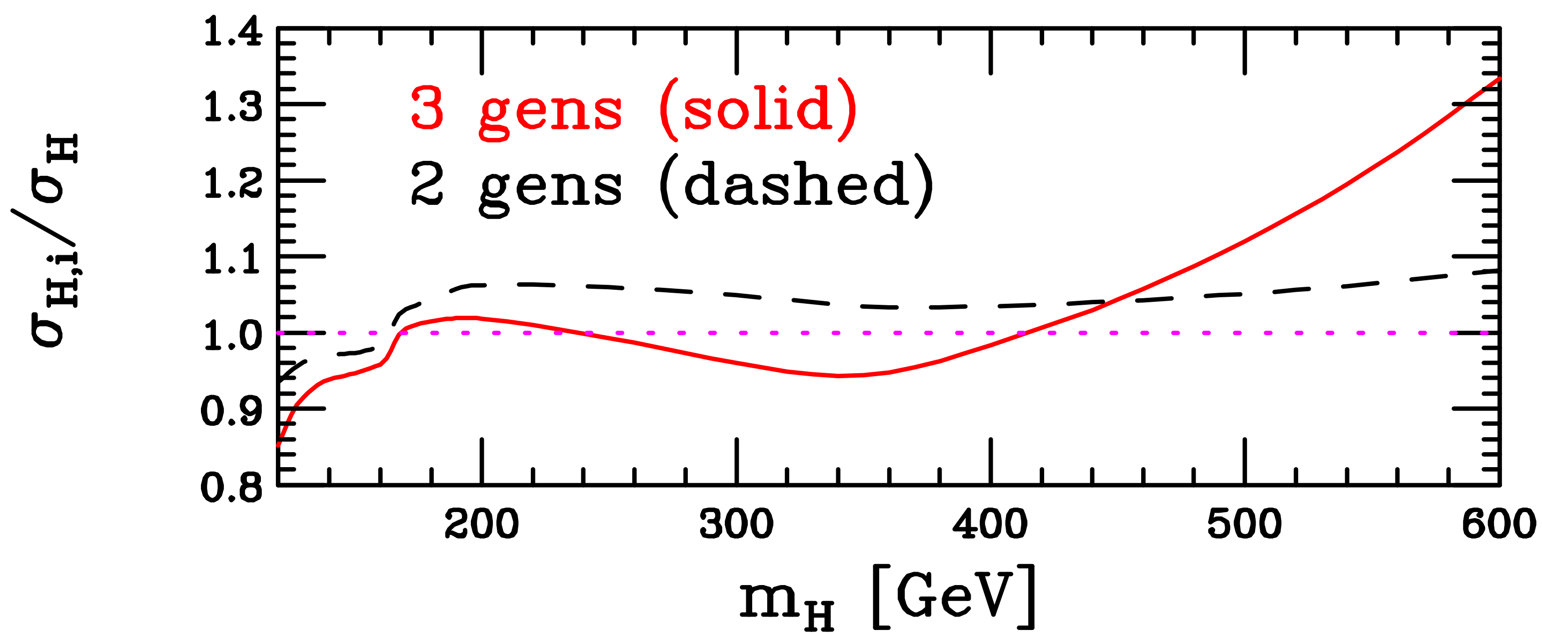
Massive quarks and gg -> VV amplitudes

Consider gg->ZZ production. It is facilitated by massless and massive (top) loops; the formalism that we discussed works for massless loops; for massive loops absolutely nothing is known (no reductions, no master integrals etc.).

Are massive loops needed? For cross-sections, perhaps not (O(1%) contributions to cross sections) but for aggressive cuts or for Higgs boson off-shell measurements where interference of gg ->H -> ZZ and gg -> ZZ is important, contributions of top quark loops become more relevant...

Also, as we will see, for massive top quarks we can get full results for cross-sections and obtain a reasonable estimate of the K-factor (whose value was the subject of much discussion recently).





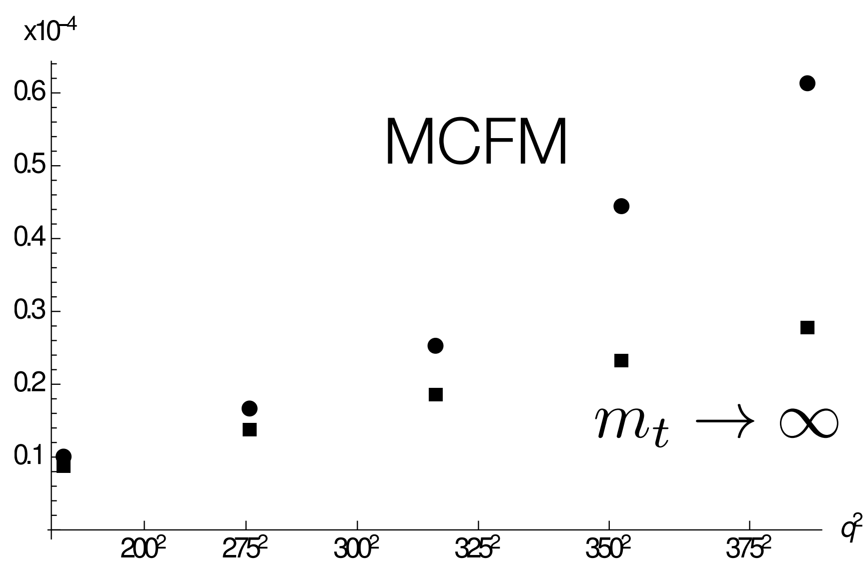
Contribution of the 3rd generation to gg -> W^+W^- processes.

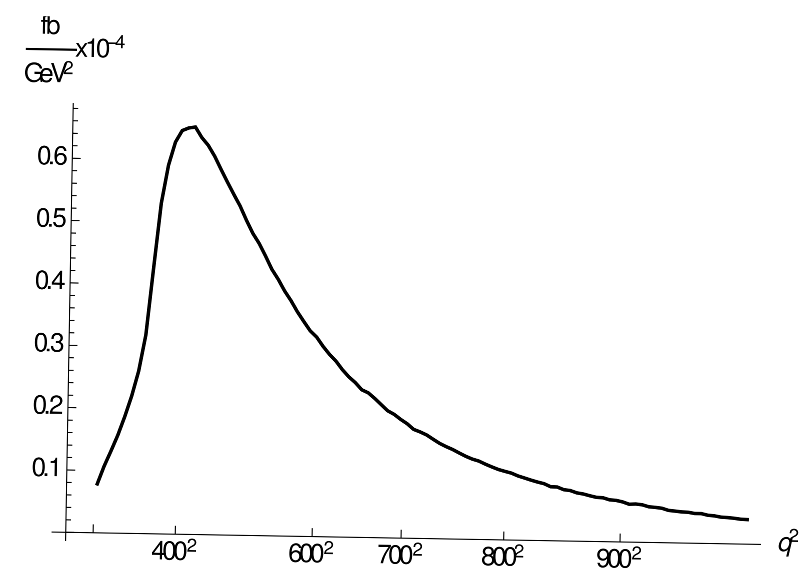
Top quark contribution to gg -> ZZ

As we just said doing exact-in-m_t calculation is not possible. Therefore, we should invent an approximation that will allow us to compute gg -> ZZ with the NLO (two-loop) accuracy. The idea is to perform an expansion in the inverse mass of the top quark.

This is not an ideal expansion, but for ZZ invariant mass pairs between 180 GeV and 350 GeV, it may actually converge. For those values of the invariant masses, it will give us an idea about the magnitude of the radiative corrections.

Moreover, we know from gg->H and from gg->HH that QCD corrections to cross-sections are not strongly affected by mass effects, in contrast to cross-sections themselves. For this reason, we expect that K-factor for gg -> ZZ, computed in the approximation of an infinitely large top quark mass is a good approximation to the actual m_t-dependent K-factor.





Comparison of the exact and expanded (leading term) one-loop results for pp->ZZ cross-section (gluon fusion, top loops). Right pane: MCFM cross-section for gg ->ZZ (top loop)

Why the expansion in the inverse top mass helps

The expansion in the inverse top quark mass helps because non-trivial two-loop four-point integrals are reduced to either two-loop vacuum bubbles or to products of one-loop integrals.

Key technique -- asymptotic expansion in the inverse large mass of the top quark. Consider a one-loop amplitude. A typical integral reads ($p_{1..4}$ are external momenta).

$$I = \int \frac{\mathrm{d}l}{(2\pi)^d} \frac{1}{((l+p_1)^2 - m_t^2)((l+p_2)^2 - m_t^2)\ldots((l+p_4)^2 - m_t^2)}$$

The essence of the large mass expansion procedure is the statement that the correct result for this integral is obtained if one expands the integrand in Taylor series in p/l, p/m (loop momentum "hard"). In principle, one can consider another expansion of the integrand in l/m, p/m (loop momentum "soft") but this expansion leads to massless tadpoles and therefore vanishes.

$$I \to \int \frac{\mathrm{d}l}{(2\pi)^d} \frac{\mathrm{Num}(l,p)}{(l^2 - m_t^2)^n}, \quad n \in 1..4 \qquad \int \mathrm{d}\Omega_d \, l_\alpha l_\beta = \frac{g_{\alpha\beta} l^2}{d}$$

Vacuum integrals are well-known. They can be computed directly, with or without tensor structures in the numerator. The increase in complexity from the one-loop to the two-loop case is minor; the only new element is that one of the loop momenta can become soft, leading to factorized one-loop integrals.

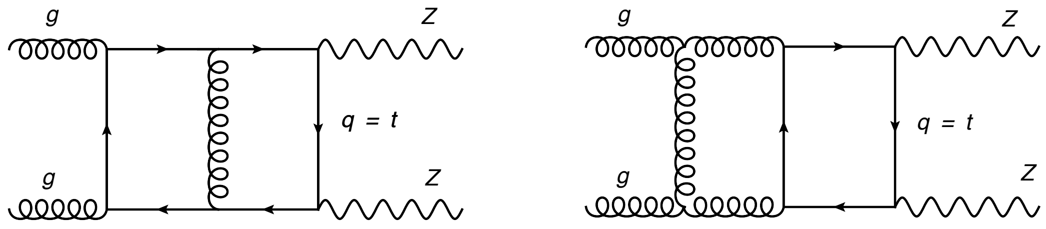
Virtual amplitudes

We work to leading order in the inverse top quark mass in a theory where Z bosons only couple to top quark loops (anomalous theory, but for us this will be of no concern).

$$Z\bar{t}t \in -i\gamma^\mu(g_V + g_A\gamma_5), \quad g_V = e/(2\sin 2\theta_W)(1 - 8/3\sin^2\theta_W), \quad g_A = e/(2\sin 2\theta_W)$$

$$\mathcal{A}_{gg\to ZZ} = i a_s \delta^{a_1 a_2}\left(g_A^2 \mathcal{A}^{aa} + g_V^2 \mathcal{A}^{vv} + g_A g_V \mathcal{A}^{av}\right),$$

$$a_s = \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \frac{\alpha_s(\mu)}{\pi},$$



The vector-axial contribution to the amplitude vanishes thanks to C-parity. The vector-vector and axial-axial contributions are both present but behave differently under $1/m_t$ expansion. The difference is that vector current is conserved and the axial is not. As the result

$$\mathcal{A}_{vv} \sim \frac{s^2}{m_t^4}, \qquad \mathcal{A}_{aa} \sim \frac{s}{m_t^2} \qquad \leftrightarrow \qquad \partial_\mu\left[\bar{\psi}\gamma^\mu\psi\right] = 0, \qquad \partial^\mu\left[\bar{\psi}\gamma^\mu\gamma_5\psi\right] = 2m_t\bar{\psi}\psi$$

Given that ttZ vector coupling is 3 times smaller than the axial coupling, the A_{vv} contribution is largely irrelevant in practice.

As the first step, we will compute the gg -> ZZ cross-section to leading order in $1/m_t$ (a path taken earlier for gg -> H, gg -> HH, gg -> HZ etc.).

Results for gg-> ZZ amplitude

The gg -> ZZ amplitudes at leading order in 1/m$_t$ are simple to write down. To emphasize constraints that follow from gauge-invariance, I will write the amplitudes using the field-strength tensor for gluons

$$\mathcal{A}^{aa} = \frac{1}{3m_t^2}\left(\frac{\mu}{m_t}\right)^{2\epsilon}\left\{\mathcal{A}_1^{aa} + a_s\left(\frac{\mu}{m_t}\right)^{2\epsilon}\mathcal{A}_2^{aa}\right\}.$$

$$f^{i,\mu\nu} = p_i^\mu \epsilon_i^\nu - p_i^\nu \epsilon_i^\mu, \quad i = 1,2. \qquad t_{34}^{\rho\beta} = \epsilon_3^\rho\epsilon_4^\beta + \epsilon_4^\rho\epsilon_3^\beta$$

$$\mathcal{A}_1^{aa} = (1+\epsilon)\left(f_{\mu\rho}^1 f_\beta^{2,\mu} - \frac{g_{\rho\beta}}{2} f_{\mu\rho}^1 f^{2,\mu\rho}\right)t_{34}^{\rho\beta},$$

$$\mathcal{A}^{aa,2} = \left(-\left(\frac{3}{2\epsilon^2}+\frac{\beta_0}{2\epsilon}\right)\left(\frac{-s-i0}{m_t^2}\right)^{-\epsilon} - \frac{\beta_0}{2}L_{s\mu} + \frac{11}{4}L_{sm} + \frac{\pi^2}{4} - \frac{175}{36}\right)\mathcal{A}^{aa,1}$$
$$+\frac{1}{2}f_{\mu\rho}^1 f^{2,\mu\rho} t_{34\beta}^\beta\left(-\frac{385}{72}+\frac{11}{8}L_{sm}\right) - \frac{1}{2s}f_{\mu\nu}^1 f^{2,\mu\nu} t_{34}^{\rho,\beta}(p_{1,\rho}p_{1,\beta} + p_{2,\rho}p_{2,\beta})$$
$$+\frac{3}{2s}f_{\mu\rho}^1 p_1^\mu f_{\nu\beta}^2 p_2^\nu t_{34}^{\rho,\beta} + \mathcal{O}(\epsilon)$$

The result shows a proper structure of divergences consistent with Catani's formula; it can be interpreted as the leading-term of the effective Lagrangian (generalization of the Euler-Heisenberg Lagrangian to the case where axial currents are present).

The partonic cross-section

To obtain the gg -> ZZ cross-section, we need to combine the elastic gg -> ZZ process with the inelastic process gg -> ZZ+g. The amplitude for the latter is complicated even in the limit of the large top mass. Nevertheless, the resulting formula for partonic cross section (differential in the invariant mass of the pair) is simple enough to be shown (note strong dependence on q/m_Z).

$$q^2 \frac{\mathrm{d}\sigma_{gg\to ZZ}}{\mathrm{d}q^2}(s, q^2)|_{s=q^2/z} = \sigma_0 z G(z, q^2),$$

$$G(z, q^2) = \left[\Delta_0 \delta(1-z) + a_s \Big(\Delta_V \delta(1-z) + 6\Delta_0 \left(2D_1(z) + \ln\frac{q^2}{\mu^2} D_0(z)\right) + \Delta_H\Big)\right],$$

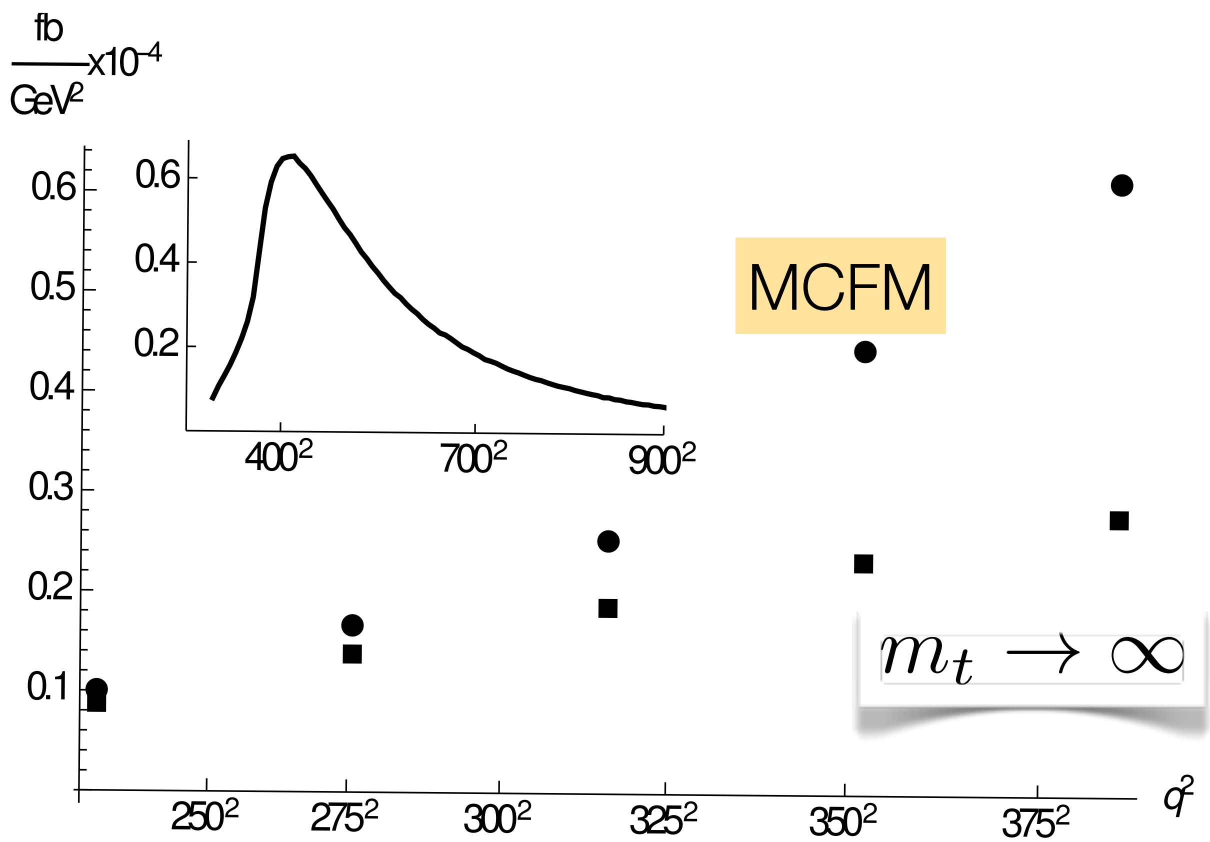
$$\Delta_0 = \frac{73}{270} - \frac{2r}{15} + \frac{34r^2}{135}, \quad r = q^2/(4m_Z^2)$$

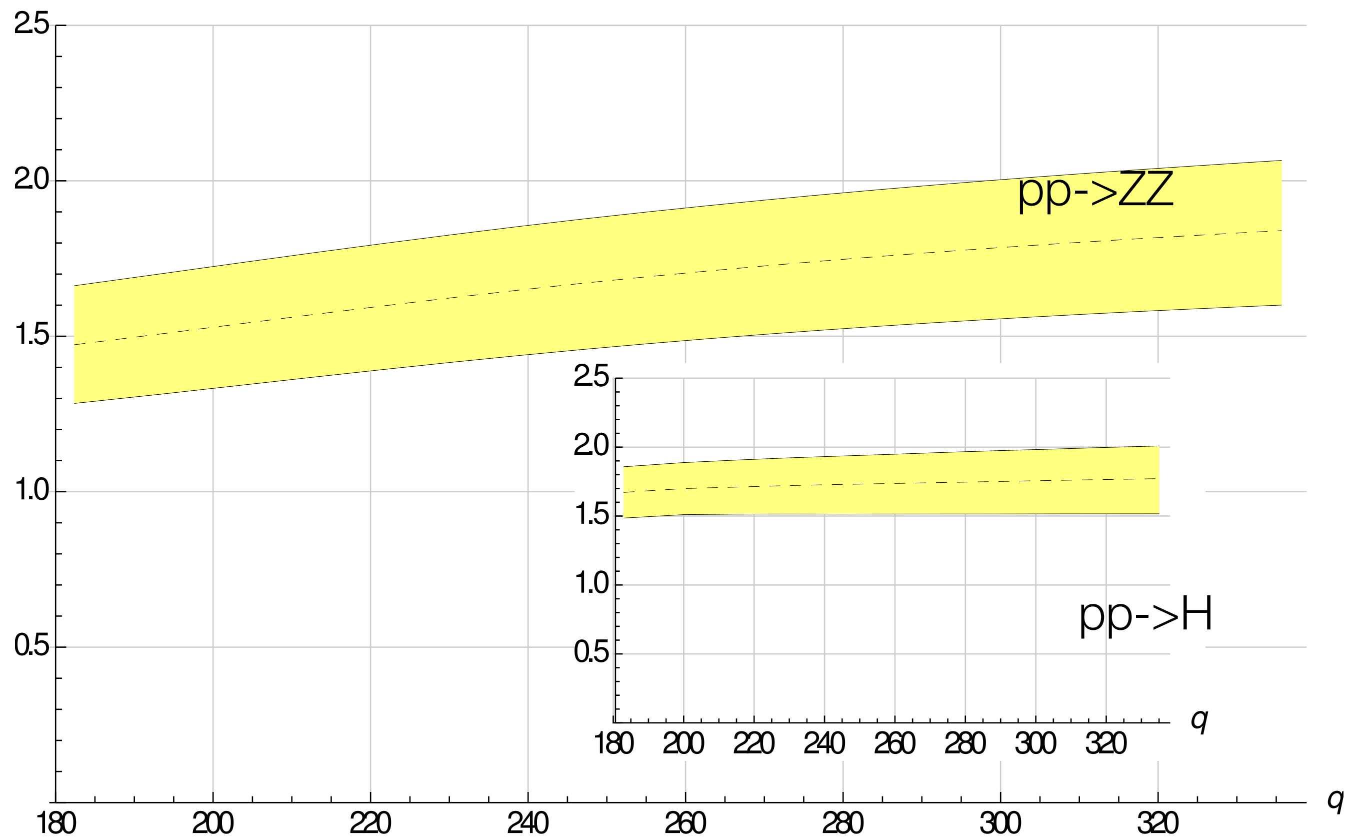
$$\Delta_V = \frac{2473 - 8661r + 5798r^2}{2430} + \frac{(73 - 36r + 68r^2)\pi^2}{270} + \frac{11(7 + 6r + 2r^2)}{135} \ln\frac{q^2}{m_t^2}.$$

$$\Delta_H = \frac{6\Delta_0}{z}\left((\omega(z) - z\kappa(z))\ln\left(\frac{q^2(1-z)^2}{\mu^2}\right) - \omega(z)^2 \frac{\ln(z)}{(1-z)}\right) + (1-z)\left[\frac{r(11\kappa(z) - 46z)}{15z} - \frac{r^2(187\kappa(z) - 302z)}{135z} - \frac{(803\kappa(z) - 598z)}{540z}\right],$$

Numerical results for cross-sections and K-factors

Integrating partonic cross-sections with PDFs (NNPDF3.0) we obtain predictions for cross-sections and for K-factors. We again show the comparison of the MCFM (full top quark mass dependence) and our (leading top mass dependence) cross-sections. We also show K-factors for pp->ZZ (gluon fusion through top loops only) and for pp -> H .





The close proximity of the Higgs and gg->ZZ K-factors is striking. Note also that the K-factor is large, O(1.5 - 2). If a similar result holds for the massless quark contribution to gluon fusion (most likely in these cases the K-factor is smaller), the gluon fusion contribution to pp -> ZZ will be be increased by a factor 2. This is not negligible compared to current error estimates on NNLO QCD theory predications for pp->ZZ .

Conclusions

Four-lepton production processes at the LHC are important for a variety of phenomenological reasons (anomalous couplings, Higgs backgrounds, SUSY backgrounds, the Higgs width).

Calculation of two-loop QCD corrections to this process was hindered for a long time by uncalculated two-loop amplitudes for qq -> V_1V_2.

Similarly, large gluon flux makes it desirable to compute gg -> V_1V_2 to NLO (i.e. two-loop virtual again). This requires gg -> V_1V_2 amplitude.

These amplitudes have now been computed. This was made possible by recent improvements in technology of reductions of loop integrals to master integrals and of computing master-integrals. I described how computation of these two-loop amplitudes (including master integrals) can be performed. Phenomenology of four-lepton final states is the next step.

It is also interesting to discuss how massive quarks contribute to gg -> ZZ . In this case, one can use the asymptotic expansions in the inverse top quark mass. Our computation shows large K-factors that are very similar to the ones for gg -> H process. Physically, this means that gluon fusion contribution to ZZ production is (probably) underestimated by a factor between 1.5 and 2. Such large radiative corrections to gluon fusion will have important consequences for estimating uncertainty in theory predictions for pp->ZZ.