# Topology and Size of the Universe from CMB Temperature and Polarization

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# Talk based on:

- G. Aslanyan, A.V. Manohar and A.P.S. Yadav
  *The Topology and Size of the Universe from CMB Temperature and Polarization Data*,
  JCAP 1308:009 (2013).

- G. Aslanyan, A.V. Manohar and A.P.S. Yadav,
  *Limits on Semiclassical Fluctuations in the Primordial Universe*,
  JCAP 1302:040 (2013).

- G. Aslanyan and A.V. Manohar,
  *The Topology and Size of the Universe from the Cosmic Microwave Background*,
  JCAP 1206:003 (2012).

An overview of properties of the CMB.

# Current status

- General Relativity is a local theory, and constrains the local properties of spacetime
- The Standard Model is also a local theory
- Nothing in the physical laws says anything about global properties
- We can only see a finite portion of space (horizon)
- Nothing known from the basic laws. Global structure has to be tested experimentally by observations on cosmological scales.

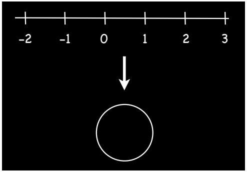
Many possibilities:

Take the usual flat, closed or open universes ($\mathbb{R}^3$, $\mathbb{S}^3$, $\mathbb{H}^3$) and take the quotient by the action of a discrete group.

Result is a finite manifold of constant curvature which is locally indistinguishable from the original covering space.

Simple $1D$ example:

$$\mathbb{R}/\mathbb{Z} = S^1, \qquad \text{under action} \qquad x \to x + n, \quad n \in \mathbb{Z}$$



# Universe Flat

Observations (Planck + BAO + highL + WP):

$$\Omega_k = -0.0005 \begin{array}{l} +0.0065 \\ -0.0066 \end{array}$$

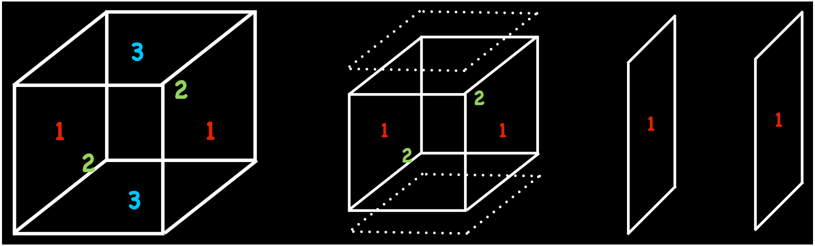
Theoretically:

The probability of quantum creation of positive curvature universes is exponentially suppressed.

Much easier to think of flat space.

# 3 Possible Flat manifolds

$$\mathcal{M}_0 = \mathbb{T}^3 \qquad \mathcal{M}_1 = \mathbb{T}^2 \times \mathbb{R} \qquad \mathcal{M}_2 = \mathbb{S}^1 \times \mathbb{R}^2$$




- Opposite edges are identified (periodic boundary conditions).
- All sides have equal length $L$ (to reduce number of parameters)
- Break isotropy of space

# Particle Horizon and LSS

The particle horizon (PH) is the portion of space from where light could have reached us.

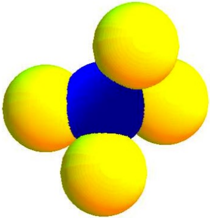
The *last scattering surface* (LSS) is where the CMB comes from. Photons free stream after that.

$$R_{LSS} \equiv L_0 = 14.4\,\text{Gpc}$$

If the global structure of space is smaller than the particle horizon, then we can (in principle) see it, and hence constrain it by observations.

# Circles in the Sky

Main method — circles in the sky



Cornish, Spergel, Starkman astro-ph/9801212

LSS crosses with itself on circles if $2L_0 > L$. Looks for these patterns in the sky.

# Limits

WMAP1 (Cornish, Spergel, Starkman, Komatsu, astro-ph/0310233)

$$L > 24\,\text{Gpc}$$

WMAP7 (Bielewicz, Banday, arXiv:1012.3549)

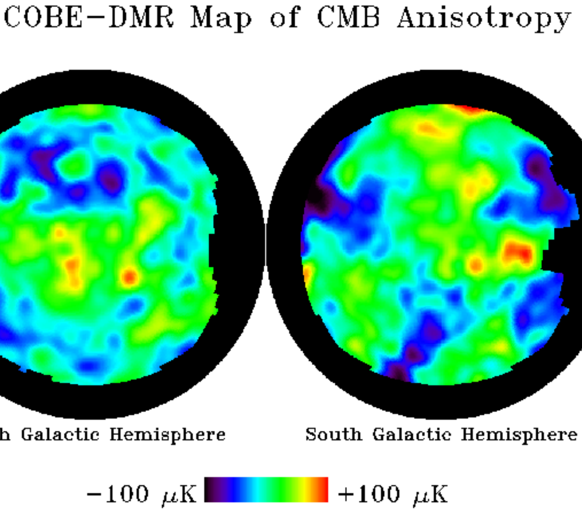
$$L > 27.9\,\text{Gpc}$$

Method does not constrain $L > 2L_0 = 28.8\,\text{Gpc}$.

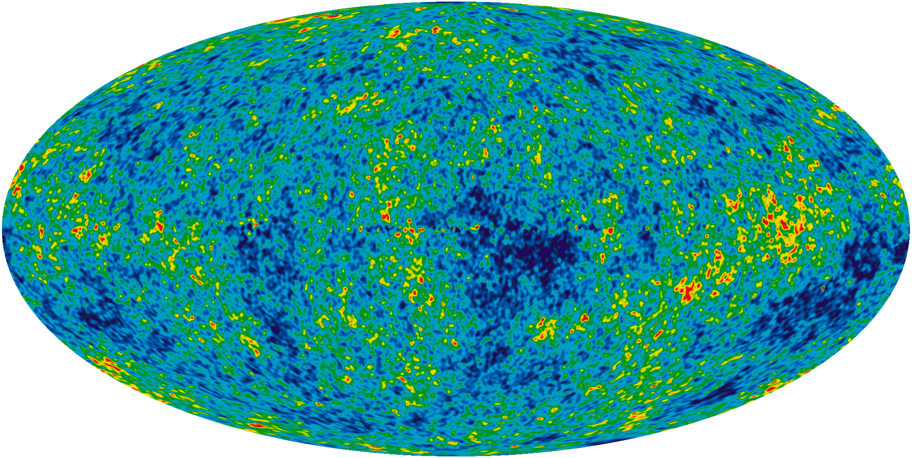
Can one can see beyond $2L_0$?

Turns out you can do better using the CMB.

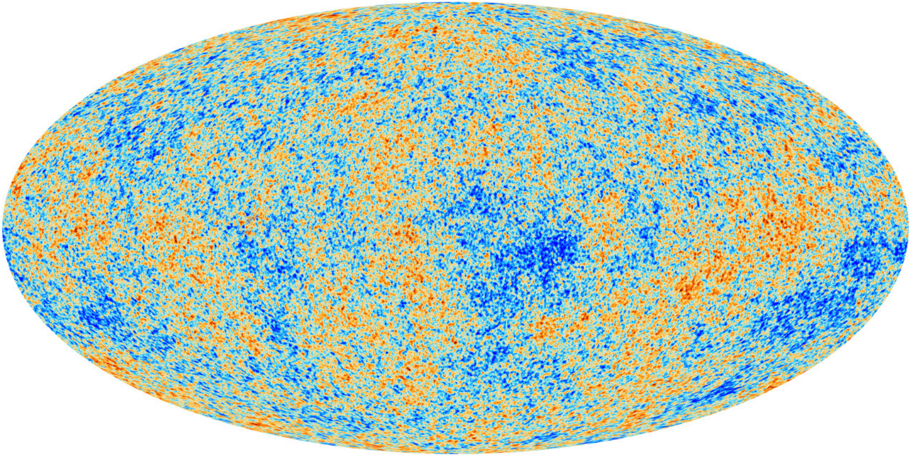
# COBE Temperature



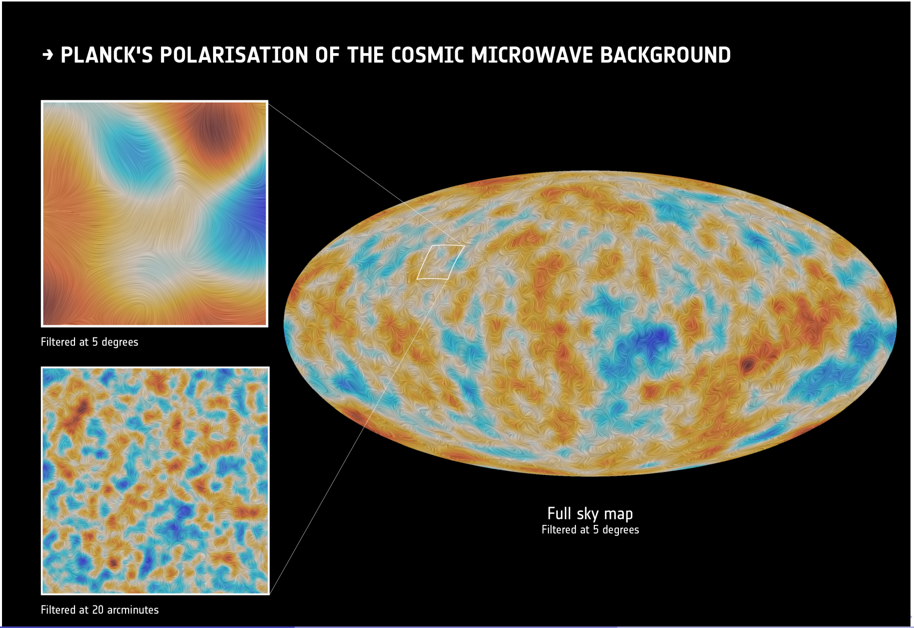
# WMAP9 Temperature



# Planck Temperature

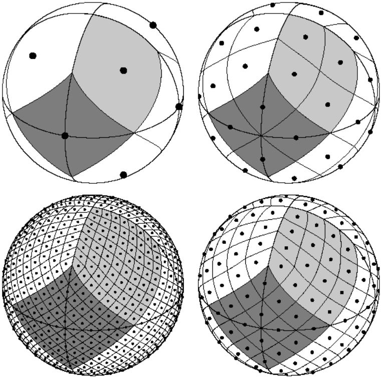


# Plank CMB Polarization



# HEALPix

Gorski et al.



Divide the sky into 12 pixels of equal solid angle, and then subdivide.

$$N_{\text{pixel}} = 12\, N_{\text{side}}^2$$

with $N_{\text{side}} = 1, 2, 4, \ldots$. Allows for a fast angular Fourier transform.

# Temperature and Polarization

For $T$, subtract out the average temperature and the dipole, which gives our motion relative to the CMB.

Polarization:

$$\mathbf{E} = \text{Re}\, \mathbf{E}_0\, e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$$

For a wave travelling in the $z$ direction, let

$$|\psi\rangle = \begin{bmatrix} e_x\, e^{i\theta_x} \\ e_y\, e^{i\theta_y} \end{bmatrix}$$

so that

$$E_x = e_x \cos(\omega t - \theta_x)$$
$$E_y = e_y \cos(\omega t - \theta_y)$$

# Polarization

The intensity matrix is

$$\rho = |\psi\rangle \langle\psi| = \begin{bmatrix} e_x^2 & e_x e_y e^{i(\theta_x-\theta_y)} \\ e_x e_y e^{-i(\theta_x-\theta_y)} & e_y^2 \end{bmatrix}$$

Decompose the density matrix as

$$\rho = \frac{1}{2}(a_0 + \mathbf{a}\cdot\boldsymbol{\sigma}) \qquad a^\mu = \mathrm{Tr}\,\rho\,\sigma^\mu \qquad \sigma^0 = 1$$

Stokes' parameters defined by

$$a^\mu = (I, U, V, Q) \qquad \rho = \frac{1}{2}\begin{bmatrix} I+Q & U-iV \\ U+iV & I-Q \end{bmatrix}$$

$$I = e_x^2 + e_y^2 \qquad U = 2e_x e_y \cos(\theta_y - \theta_x)$$

$$Q = e_x^2 - e_y^2 \qquad V = 2e_x e_y \sin(\theta_y - \theta_x)$$

Rotate the $x - y$ axes by $\theta$:

$$I' = I \qquad V' = V \qquad \begin{bmatrix} Q' \\ U' \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} Q \\ U \end{bmatrix}$$

$$Q' \pm iU' = e^{\pm 2i\theta} (Q \pm iU)$$

So can measure $Q$ and $U$ from difference in intensities for $x$ and $y$ polarization, and $x'$ and $y'$ polarization with $\theta = \pi/4$.

Acts like a helicity $\pm 2$ object because it is quadratic in the EM field.

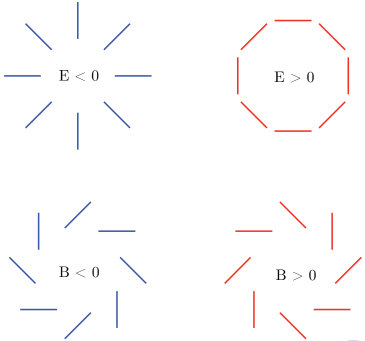
Spherical harmonic decomposition:

$$T(\hat{\mathbf{n}}) = \sum_{lm} T_{lm} Y_{lm}(\hat{\mathbf{n}})$$

$$Q(\hat{\mathbf{n}}) \pm i\, U(\hat{\mathbf{n}}) = \sum_{lm} (E_{lm} \pm i\, B_{lm})\ {}_{\pm 2}Y_{lm}(\hat{\mathbf{n}})$$

where ${}_{\pm 2}Y_{lm}(\hat{\mathbf{n}})$ are helicity spherical harmonics.

Under parity, $E \to E$ and $B \to -B$.



# Inflation

$T(\hat{\mathbf{n}}, \mathbf{x})$: CMB temperature seen by an observer at $\mathbf{x}$ in direction $\hat{\mathbf{n}}$.

$$T(\hat{\mathbf{n}}, \mathbf{x}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}(\mathbf{x}) Y_{lm}(\hat{\mathbf{n}}),$$

Fourier space temperature fluctuations $T(\hat{\mathbf{n}}, \mathbf{k})$

$$a_{lm}(\mathbf{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \int \mathrm{d}\Omega \; Y_{lm}^*(\hat{\mathbf{n}}) T(\hat{\mathbf{n}}, \mathbf{k}).$$

Observed CMB fluctuations from $a_{lm}$ correlations:

$$M_{lml'm'} \equiv \langle a_{lm}(\mathbf{x}_0) a^*_{l'm'}(\mathbf{x}_0) \rangle \; .$$

$$T(k, \mathbf{k}\cdot\hat{\mathbf{n}}) = \frac{T(k, \mathbf{k}\cdot\hat{\mathbf{n}})}{\zeta(k)} \zeta(k)$$

# Inflation

The correlations between temperature anisotropies in $k$-space are related to the initial matter power spectrum

$$\left\langle T(\mathbf{k},\hat{\mathbf{n}})\,T^*(\mathbf{k}',\hat{\mathbf{n}}')\right\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')P_\zeta(k)\,\frac{T(k,\mathbf{k}\cdot\hat{\mathbf{n}})}{\zeta(k)}\,\frac{T^*(k,\mathbf{k}\cdot\hat{\mathbf{n}}')}{\zeta^*(k)}\,,$$

The matter power spectrum is defined by

$$\left\langle \zeta(\mathbf{k})\,\zeta^*(\mathbf{k}')\right\rangle \equiv (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')P_\zeta(k)\,.$$

Transfer function computed by CAMB:

$$\frac{T(k,\mathbf{k}\cdot\hat{\mathbf{n}})}{\zeta(k)}$$

does not depend on initial conditions, since equations are linear.

# Inflation

Expanding $T(k, \mathbf{k} \cdot \hat{\mathbf{n}})$ into Legendre polynomials

$$T(k, \mathbf{k} \cdot \hat{\mathbf{n}}) = \sum_l (-i)^l (2l+1) P_l(\hat{k} \cdot \hat{\mathbf{n}}) T_l(k) \,,$$

gives

$$M_{lml'm'} = (4\pi)^2 (-i)^l i^{l'} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} P_\zeta(k) \frac{T_l(k)}{\zeta(k)} \frac{T^*_{l'}(k)}{\zeta^*(k)} Y^*_{lm}(\hat{\mathbf{k}}) Y_{l'm'}(\hat{\mathbf{k}}) \,.$$

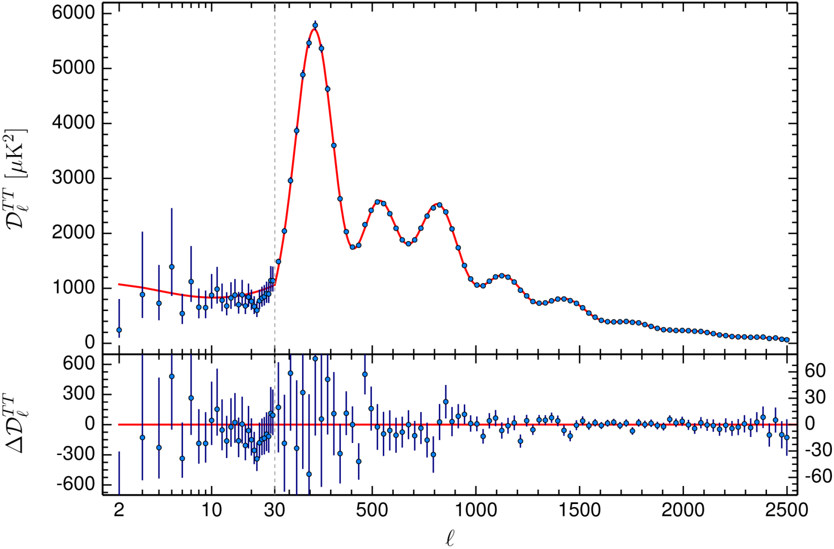
Usual infinite universe case:

$$M_{lml'm'} = \delta_{ll'} \delta_{mm'} C_l \,,$$

with

$$C_l = \frac{2}{\pi} \int \mathrm{d}k \; k^2 \, P(k) \left| \frac{T_l(k)}{\zeta(k)} \right|^2 \,.$$

# Planck 2015




$$D_l = \frac{l(l+1)C_l}{2\pi}$$

# Cosmic Variance

Measure $a_{lm}$ and get

$$C_l^{\text{obs}} \equiv \frac{1}{2l+1} \sum_m \left| a_{lm}^{\text{obs}} \right|^2$$

Each $a_{lm}$ like a measurement of $C_l$. Large cosmic variance for small $l$.

$$\left\langle C_l^{\text{obs}} \right\rangle = C_l$$

$$\left\langle \left( \Delta C_l^{\text{obs}} \right)^2 \right\rangle = \frac{2 C_l}{2l+1}$$

$\langle \cdot \rangle$ is w.r.t. universes.

# Finite Case

$$\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \to \frac{1}{L_1 L_2 L_3} \sum_{\mathbf{k}},$$

over the discrete **k** values

$$k_1 = \frac{2\pi}{L_1} n_1, \quad k_2 = \frac{2\pi}{L_2} n_2, \quad k_3 = \frac{2\pi}{L_3} n_3, \qquad n_i \in \mathbb{Z}$$

The derivation remains the same in the finite case except that the **k** integral must be replaced by the sum

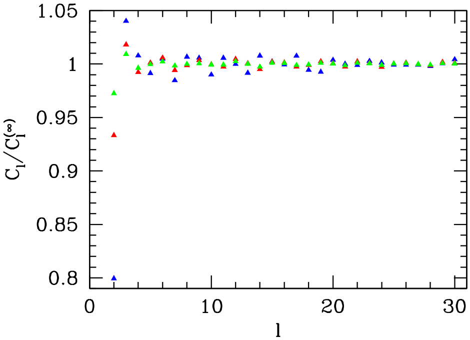
$$M_{lml'm'} = (4\pi)^2 (-i)^l i^{l'} \frac{1}{L_1 L_2 L_3} \sum_{\mathbf{k}} P(k) \frac{T_l(k)}{\zeta(k)} \frac{T^*_{l'}(k)}{\zeta^*(k)} Y^*_{lm}(\hat{\mathbf{k}}) Y_{l'm'}(\hat{\mathbf{k}}).$$

CAMB evolution is local, due to CMB scattering, etc. and is unaffected.

# Finite Case

Find

$$2 \mid l' - l \qquad\qquad 4 \mid m' - m$$




Plot of the ratio of $C_l$ for $\mathcal{M}_0 = \mathbb{T}^3$ with $L/L_0 = 1.8$ (blue), $\mathcal{M}_1 = \mathbb{T}^2 \times \mathbb{R}^1$ with $L/L_0 = 1.9$ (red), and $\mathcal{M}_3 = S^1 \times \mathbb{R}^2$ with $L/L_0 = 1.9$ (green), to that for infinite space $\mathbb{R}^3$.

# Inflation

Define the transfer functions

$$g_l^X(k) = \frac{X_l(k)}{\zeta(k)}, \qquad X = T, E, B$$

Power spectrum:

$$M^{XY}_{lml'm'} = (4\pi)^2(-i)^l i^{l'} \frac{1}{L_1 L_2 L_3} \sum_{\mathbf{k}} P_\zeta(k) g_l^X(k) g_{l'}^{Y*}(k) Y^*_{lm}(\hat{\mathbf{k}}) Y_{l'm'}(\hat{\mathbf{k}}) \,.$$

and $TT$, $TE$, $EE$ and $BB$ are non-zero. $BB$ requires gravitational waves (tensor perturbations) in the initial conditions.

$$M^{TB} = 0 \qquad M^{EB} = 0 \qquad M^{BB} = 0$$

# Likelihood

The WMAP and Planck collaborations make public the data in the form of maps in pixel space.

$m = (T, Q, U)$ are the temperature and polarization maps treated as a column vector with index $\hat{\mathbf{n}}_i$ in pixel space.

Define $S$ and $N$ as the signal and noise correlation matrices, with indices in pixel space.

$N$ is provided by the experimental collaborations

$S$ is the theoretical prediction based on the cosmological model.

The likelihood is given by

$$\mathcal{L}(m|S)\mathrm{d}m = \frac{\exp\left[-\frac{1}{2}m^t(S+N)^{-1}m\right]}{(2\pi)^{3n_p/2}\det(S+N)^{1/2}}\mathrm{d}m$$

Spherical harmonic decomposition:

$$T(\hat{\mathbf{n}}) = \sum_{lm} T_{lm} Y_{lm}(\hat{\mathbf{n}})$$

$$Q(\hat{\mathbf{n}}) \pm i\, U(\hat{\mathbf{n}}) = \sum_{lm} (E_{lm} \pm i\, B_{lm})\ {}_{\pm 2}Y_{lm}(\hat{\mathbf{n}})$$

so we can compute pixel correlations in terms of $M^{XY}_{lml'm'}$

$$\left\langle X(\hat{\mathbf{n}}) Y(\hat{\mathbf{n}}')^* \right\rangle$$

$lm \leftrightarrow \hat{\mathbf{n}}$ is like a change of basis with transformation $Y_{lm}(\hat{\mathbf{n}})$

Noise in low resolution temperature maps is negligible. Can separate out the temperature from the polarization maps.

$$\widetilde{E}_{lm} = E_{lm} - \left[ M^{ET} \left( M^{TT} \right)^{-1} T \right]_{lm}$$

so that

$$\left\langle \widetilde{E}_{lm} T^*_{lm} \right\rangle = 0$$

Define

$$\widetilde{Q}(\hat{\mathbf{n}}) = \frac{1}{2} \sum_{lm} \widetilde{E}_{lm} \left[ {}_2Y_{lm}(\hat{\mathbf{n}}) + {}_{-2}Y_{lm}(\hat{\mathbf{n}}) \right] + i\widetilde{B}_{lm} \left[ {}_2Y_{lm}(\hat{\mathbf{n}}) - {}_{-2}Y_{lm}(\hat{\mathbf{n}}) \right]$$

$$i\widetilde{U}(\hat{\mathbf{n}}) = \frac{1}{2} \sum_{lm} \widetilde{E}_{lm} \left[ {}_2Y_{lm}(\hat{\mathbf{n}}) - {}_{-2}Y_{lm}(\hat{\mathbf{n}}) \right] + i\widetilde{B}_{lm} \left[ {}_2Y_{lm}(\hat{\mathbf{n}}) + {}_{-2}Y_{lm}(\hat{\mathbf{n}}) \right]$$

$$\left\langle \widetilde{Q}(\hat{\mathbf{n}}) T(\hat{\mathbf{n}}') \right\rangle = 0 \qquad \left\langle \widetilde{U}(\hat{\mathbf{n}}) T(\hat{\mathbf{n}}') \right\rangle = 0$$

Define $P$ to be the $Q, U$ part, and $T$ to be the temperature part. Then

$$\mathcal{L}(m|S)\mathrm{d}m = \frac{\exp\left[-\frac{1}{2}\widetilde{m}_P^t(\widetilde{S}_P + N_P)^{-1}m_P\right]}{(2\pi)^{n_p}\det(\widetilde{S}_P + N_P)^{1/2}}\mathrm{d}\widetilde{m}_P \frac{\exp\left[-\frac{1}{2}\widetilde{T}^t(S_T)^{-1}T\right]}{(2\pi)^{n_p/2}\det(S_T)^{1/2}}\mathrm{d}T$$

$\widetilde{S}_P$ can be derived from

$$\left\langle \widetilde{E}_{lm}\widetilde{E}'_{lm}\right\rangle = M^{\widetilde{E}\widetilde{E}}_{lml'm'} = \left[M^{EE} - M^{ET}\left(M^{TT}\right)^{-1}M^{TE}\right]_{lml'm'}$$

Main difficulty is numerical: the maps have $N_p \sim 3072$ and so the covariance matrices are $3072 \times 3072$. Takes a lot of computing power.

# Previous Results based on Temperature

COBE: de Oliviera Costa, Smoot, astro-ph/9412003:

$$L > 4.32h^{-1}\text{ Gpc for } M_0 \quad (95\%)$$

de Oliviera Costa, Smoot, Starobinsky, astro-ph/9510109:

$$L > 3.0h^{-1}\text{ Gpc for } M_{1,2} \quad (95\%)$$

WMAP1: Phillips, Kogut, astro-ph/0404400:

$$M_0 : L > 1.2L_0(95\%), \qquad L > 2.1L_0(68\%) \qquad \text{best fit } L = 2.1L_0$$

Kunz et al. astro-ph/0510164:

$$M_0 : L > 19.3\text{Gpc} \qquad\qquad M_2 : L > 14.4\text{Gpc}$$

# Data Used

WMAP temperature maps: Low resolution ILC, V, W, Q bands at $N_{\text{side}} = 16$ smoothed to $9.183^\circ$ degrees and masked with Kp2.

In pixel space, just drop the pixels corresponding to the mask.

$1\mu$K white noise added to regularize the numerical inversion of the covariane matrix.

Polarization maps: Combination of Ka, Q, V bands at $N_{\text{side}} = 8$ masked with P06.

Analyze WMAP7 and WMAP9 data
CAMB used to calculate the transfer functions
WMAP likelihood code used

CAMB modified to use a sum on $k$, and WMAP modified to use the new covariance matrices.

# Orientation

Isotropy of space broken, so have to look at orientations of the compactification direction, in terms of Euler angles $\phi, \theta, \psi$.

Huge increase in computer time to map out these orientations. Some simplification using symmetries of a cube,

$$R(\phi, \theta, \psi) = R(g) R(\phi', \theta', \psi')$$

where $R(g)$ is a symmetry of $M$, e.g. a $\pi/2$ rotation about an axis of the cube.

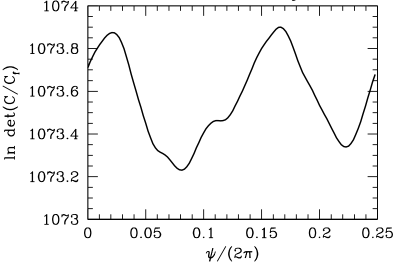
Only studied all sides equal.

# Variation with Orientation

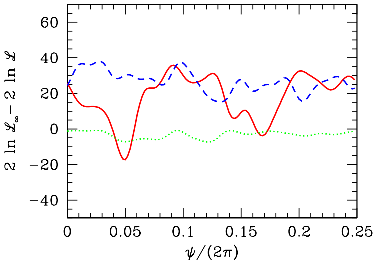
$$-2\ln\mathcal{L} = \chi^2 + \ln\det C/C_f + \ln\det(2\pi C_f)$$

Look at $M_0$:

Det varies weakly




Likelihood varies strongly




$L/L_0 = 1.8$ with $\theta, \phi$ fixed.

red is best fit direction, blue is random direction for $L/L_0 = 1.8$ with $\theta, \phi$ fixed. green is best fit direction for $L/L_0 = 2.2$.

# Analysis

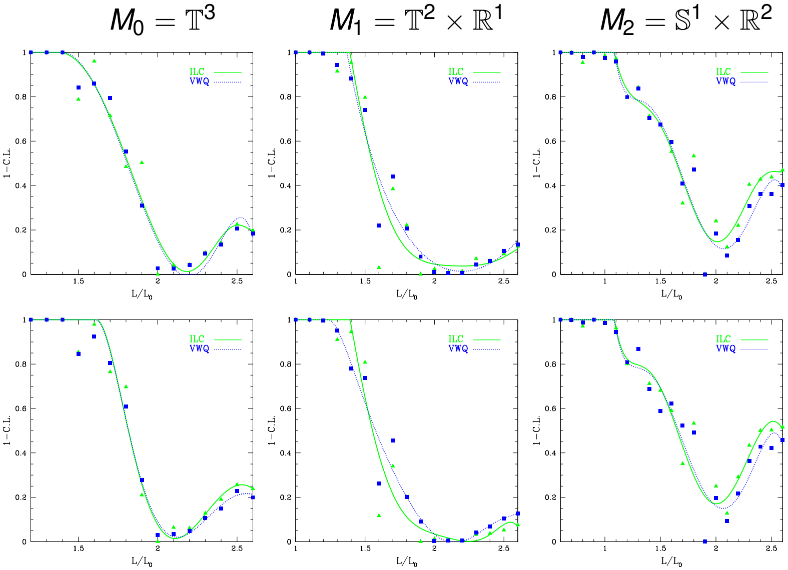
Make a scan over angle steps $0.05\pi$ and then minimize using MINUIT.

Use the Feldman-Cousins method (physics/9711021) to construct confidence intervals

1. For a point $P$ in parameter space, construct a set of simulations $S_i$ and find the best fit for each one.
2. Calculate likelihood ratios $\Delta\mathcal{L} = -2\ln\mathcal{L}(S_i|P) + 2\ln\mathcal{L}(S_i|P_{i,\text{best}})$
3. Find $\Delta\mathcal{L}_c$ such that a fraction $\alpha$ of simulations have $\Delta\mathcal{L} < \Delta\mathcal{L}_c$
4. For real data $D$ calculate $\Delta\mathcal{L}_D = -2\ln\mathcal{L}(D|P) + 2\ln\mathcal{L}(D|P_{D,\text{best}})$
5. Accept $P$ at confidence level $\alpha$ if $\Delta\mathcal{L}_D < \Delta\mathcal{L}_c$

Do 500 simulations for each size and topology, varying size in steps of $0.1L_0$.

# Results (Temp only)




upper: WMAP7, lower: WMAP9. green triangles: ILC map, the blue squares: combined data from V, W, and Q maps.

Lower bounds on $L/L_0$:

| Map | $\mathcal{M}_0$(68.3%) | $\mathcal{M}_0$(95.5%) | $\mathcal{M}_1$(68.3%) | $\mathcal{M}_1$(95.5%) | $\mathcal{M}_2$(68.3%) | $\mathcal{M}_2$(95.5%) |
|---|---|---|---|---|---|---|
| ILC (7) | 1.71 | 1.50 | 1.49 | 1.40 | 1.49 | 1.11 |
| VWQ (7) | 1.71 | 1.50 | 1.48 | 1.38 | 1.50 | 1.10 |
| ILC (9) | 1.76 | 1.66 | 1.49 | 1.41 | 1.51 | 1.10 |
| VWQ (9) | 1.76 | 1.66 | 1.47 | 1.30 | 1.51 | 1.10 |
| Planck | | 1.66 | | 1.42 | | 1.00 |

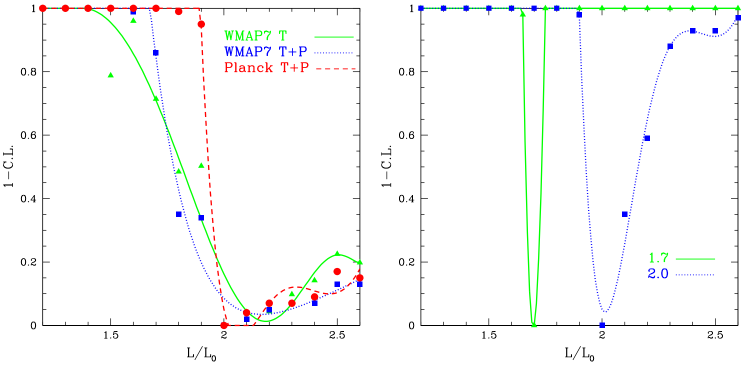
Planck 2013: XXVI 1303.5086

| Topology | Map | $\Delta\mathcal{L}$ | $L/L_0$ | $\phi$ | $\theta$ | $\psi$ |
|---|---|---|---|---|---|---|
| $\mathcal{M}_0$ | ILC (7) | 18.89 | $2.0 \pm 0.05$ | $2.328 \pm 0.036$ | $2.512 \pm 0.012$ | $0.379 \pm 0.033$ |
| | ILC (9) | 19.45 | $2.0 \pm 0.05$ | $2.330 \pm 0.035$ | $2.512 \pm 0.012$ | $0.380 \pm 0.033$ |
| $\mathcal{M}_1$ | ILC (7) | 19.30 | $1.9 \pm 0.05$ | $0.356 \pm 0.023$ | $0.932 \pm 0.024$ | $1.061 \pm 0.020$ |
| | ILC (9) | 18.46 | $1.9 \pm 0.05$ | $0.357 \pm 0.023$ | $0.928 \pm 0.022$ | $1.061 \pm 0.020$ |
| $\mathcal{M}_2$ | ILC (7) | 16.26 | $1.9 \pm 0.05$ | $1.705 \pm 0.016$ | $2.166 \pm 0.016$ | |
| | ILC (9) | 16.62 | $1.9 \pm 0.05$ | $1.704 \pm 0.016$ | $2.166 \pm 0.016$ | |

Best Fit

# Temperature + Polarization

WMAP polarization maps are very noisy, and not much improvement.
Did a forecast for Planck using 100 simulations




Bounds go up 1.73 → 1.92 (68%) and 1.68 → 1.89 (95.5%). Detect $L/L_0 = 1.7$ at $3\sigma$ and $L/L_0 = 2.0$ at $2\sigma$.
Recent paper: Planck 1502.01593 consistent with these estimates.

# Checks

Not related to the lowest multipoles. We took $M_{lml'm'}$ and replaced pieces by the infinite universe case. Seems that the signal depends on off-diagonal terms for $5 \leq \ell \leq 25$.

Not due to low $\ell$, i.e. not from the quadrupole or octupole.

# Conclusions

- Tested for global topology using CMB data
- Rule out $L/L_0 \sim 1.1 - 1.7$
- An indication of a dip at around $L/L_0 \sim 2$
- Unlikely to get much better bounds
- Also tested for deviations from inflation, can put limits in a similar way.