Glauber Gluons and Multiple Parton Interactions.

Jonathan Gaunt, DESY

Vienna University, Seminar on particle physics, 28th April 2015

Based on **JHEP 1407 (2014) 110**
And **work in progress** together with
Markus Diehl, Daniel Ostermeier, Peter Plössl and Andreas Schäfer

Outline

- Review of the traditional Collins-Soper-Sterman method of factorisation analysis.
- 'Standard factorisation formulae' with soft, hard and collinear functions. Glauber gluons, and the need for their cancellation to obtain a standard factorisation formula.
- Analysis of Glauber gluons for the observable E_T, with $E_T \ll Q$, and demonstration of lack of cancellation at the level of two Glauber gluon exchanges between spectators (plus review of successful cancellation for p_T).
- Connection of these noncancelled Glauber exchange diagrams to MPI and (at higher orders) Regge physics.
- Other MPI sensitive variables for which 'standard' factorisation fails.
- Discussion of Glauber modes in double parton scattering (DPS) – do Glauber modes cancel here?

Factorisation Formulae

Factorisation formulae are essential to make predictions at LHC.

Separate out short distance interaction of interest from long-distance QCD-dominated interactions. Low-momentum part of long-distance piece will not be calculable perturbatively, but is (hopefully) universal.

Examples of factorisation formulae:

Collinear factorisation for pp → V + X inclusive total cross section, V colourless

PDFs (long distance physics, universal)

$$\sigma = \int dx_A dx_B \hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s) f_i(x_A) f_j(x_B) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^4}\right)$$

Parton-level cross section/coefficient function (short distance physics)

Corrections suppressed by Λ^2/Q^2

Factorisation Formulae

TMD factorisation for pp → V + X cross section differential in p_T, $p_T \ll Q$, V colourless

Hadronic tensor

Approximated momenta (transverse momenta → 0)

Rapidity regulator

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}_f\left(x_A, \mathbf{b}_T; \zeta_A\right) \tilde{f}_{\bar{f}}\left(x_B, \mathbf{b}_T; \zeta_B\right) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

Transverse momentum dependent PDFs (long distance physics)

Coefficient function (short distance physics)

Corrections suppressed by p_T^2/Q^2 (can be augmented to Λ^2/Q^2 by adding matching to fixed order)

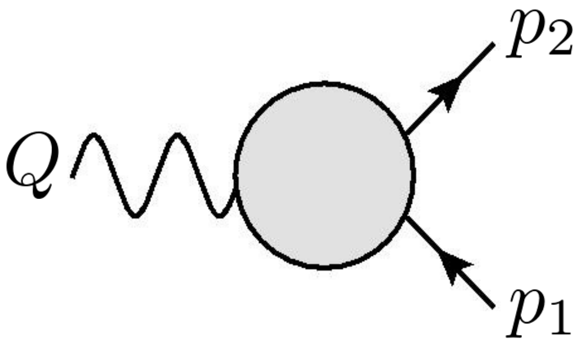
Both formula rigorously proved to leading power by Collins, Soper and Sterman

Bodwin Phys. Rev. 31 (1985) 2616
Collins, Soper, Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833
Collins, pQCD book

Establishing factorisation – the CSS approach

How do we establish a leading power factorisation for a given observable?

Many of you will be familiar with the SCET approach – but here I want to review the original Collins-Soper-Sterman (CSS) method

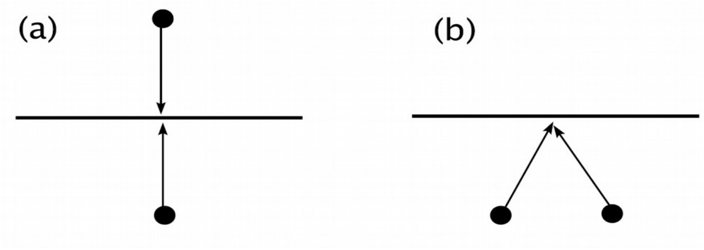


Consider a simple process – Sudakov form factor. This is electromagnetic form factor of an elementary particle (here a quark) at large Q.

To obtain a factorisation formula, need to identify non-UV leading power regions of Feynman graphs – i.e. small regions around the points at which certain particles go on shell, which despite being small are leading due to propagator denominators blowing up.

Pinch singularities

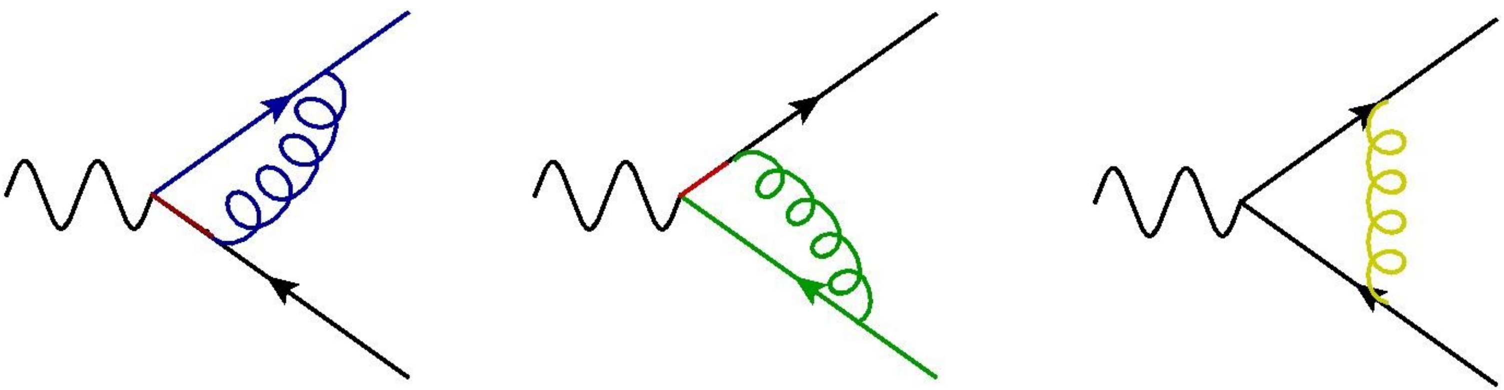
More precisely, need to find regions around pinch singularities – these are points where propagator denominators pinch the contour of the Feynman integral.



Pinched   Non-pinched

Pinch singularities in Feynman graphs correspond to physically (classically) allowed processes. This is the Coleman-Norton theorem.

e.g. for one-loop Sudakov form factor, pinch surfaces are:



'Gluon collinear to antiquark'   'Gluon collinear to quark'   'Soft gluon'

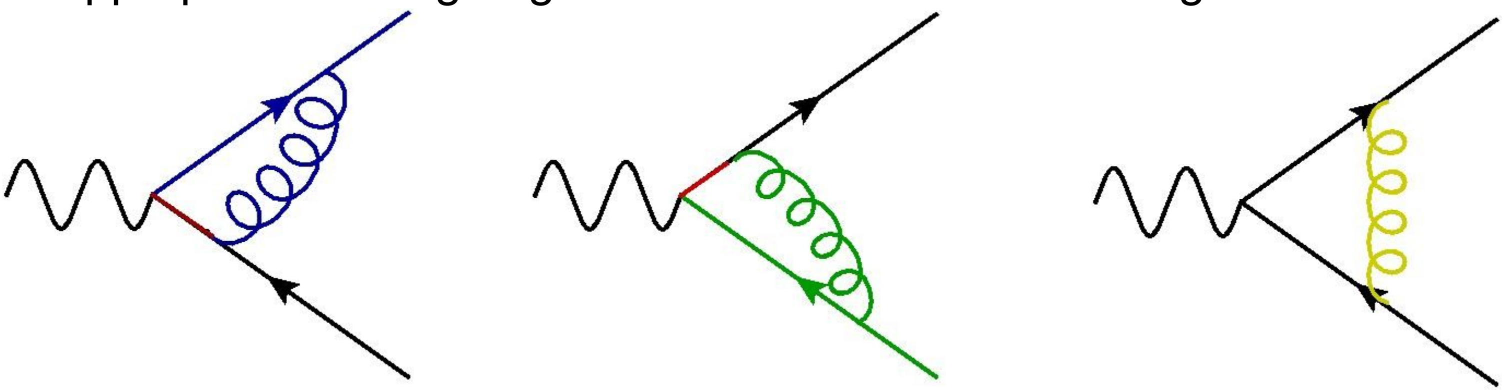
Scaling Variables

This analysis tells us where the singularities are, but not their strength. Need to supplement this with a power counting analysis to determine if region around singularity gives a leading contribution, and what the shape of this region is.

Let's introduce lightcone coordinates (A^+, A^-, $A_\perp$) according to:

$$A = A^+ p + A^- n + A_\perp \qquad p \propto p_1, n \propto p_2, n.p = 1, n^2 = p^2 = 0$$

Can determine that regions around all Sudakov one-loop pinches are leading, and that the appropriate scaling of gluon momentum *k* for each region is as follows.



$$k \sim Q(\lambda^2, 1, \lambda) \qquad k \sim Q(1, \lambda^2, \lambda) \qquad k \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

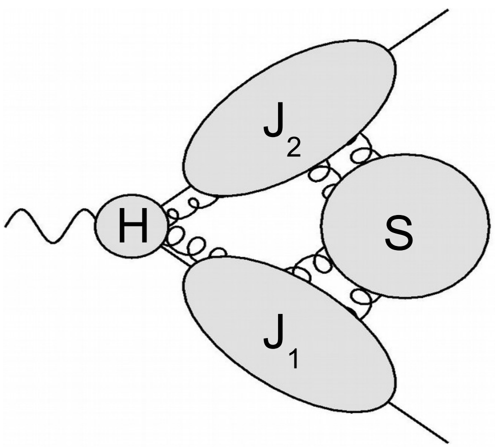
COLLINEAR SOFT

(familiar collinear and soft modes from SCET!)

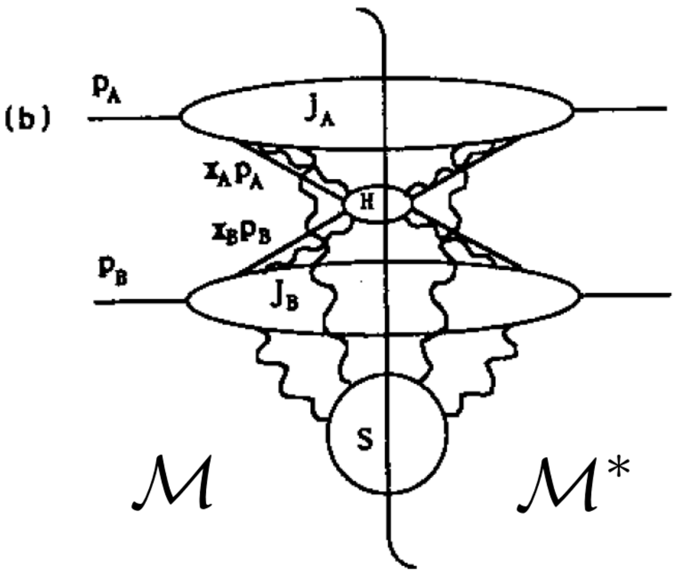
Beyond One Loop

Going beyond one loop – things become more complicated. Various leading contributions, coming from the different loop momenta scaling in different ways.

However, can repeat the pinch + power counting analysis and obtain most general leading region at all loops:



Sudakov factor



pp → V + X with associated scale Q

Momentum Regions

Scalings of loop momenta that can give leading power contributions:

1) Hard region – momentum with large virtuality (order Q)

n/- component

p/+ component

transverse component

$$k \sim Q\,(1, 1, 1)$$

2) Collinear region – momentum close to some beam/jet direction

$$k \sim Q\,(1, \lambda^2, \lambda)$$ (for example)

3) (Central) soft region – all momentum components small and of same order

$$k \sim Q\,(\lambda^n, \lambda^n, \lambda^n)$$

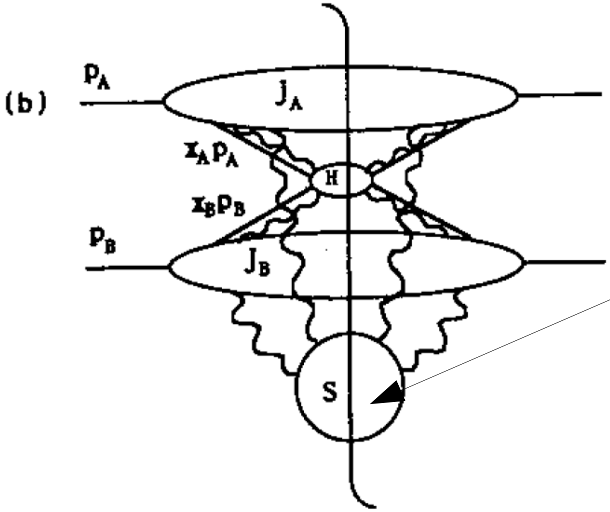
Momentum regions

AND...

4) Glauber region – all momentum components small, but transverse components much larger than longitudinal ones

$$|k^+k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

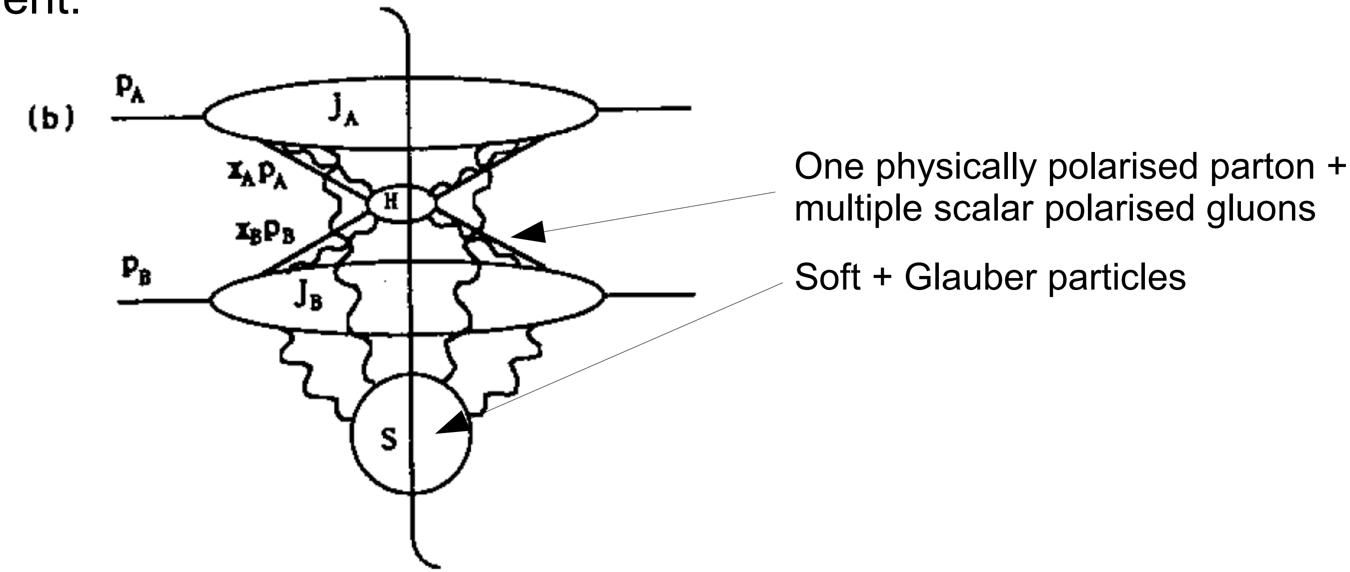
Canonical example: $k \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$



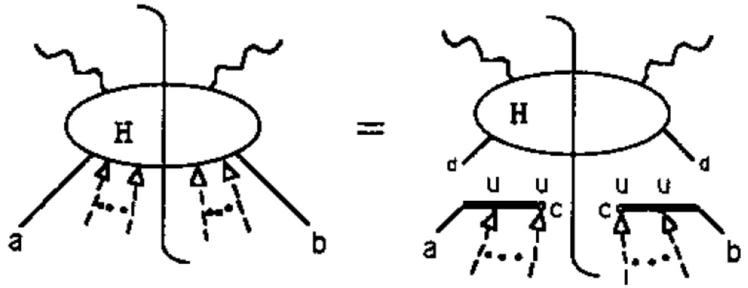
Soft + Glauber particles

Obtaining a factorisation formula

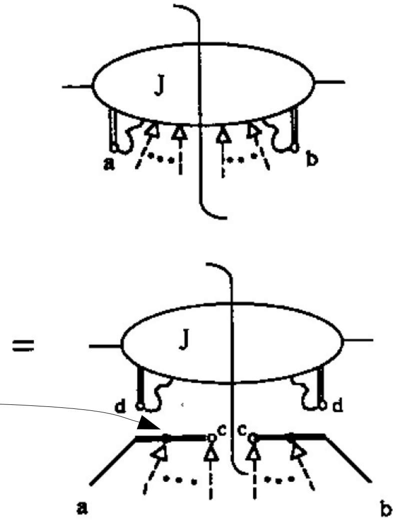
Already with this leading diagram we are close to some kind of a factorisation formula, but must separate different pieces – too many connections between H, J , S at the moment:



Collinear scalar polarised gluons can be stripped from hard by using Ward identities, physically polarised parton detached using some projector.



Obtaining a factorisation formula



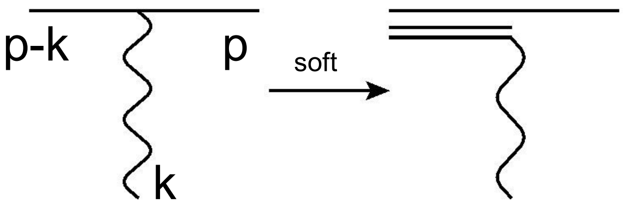
If blob S only contained central soft, then we could strip soft attachments to collinear J blobs using Ward identities too.

Wilson line in direction of J

(In SCET this is achieved by the BPS field redefinition).

Bauer, Pirjol, Stewart Phys. Rev. D 65 (2002) 054022

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p \cdot k + k^2 \xrightarrow{\text{soft}} -2p \cdot k$$

Eikonal piece

Glauber Gluons and Factorisation

The manipulation used for soft gluons is NOT POSSIBLE for Glauber gluons

Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \not\to -2p\cdot k$$

Two terms in denominator are of same order in Glauber region

How do we get around this problem?

One approach: try and show that that contribution from the Glauber region cancels (used by CSS for total cross section and p_T of V)

Will review later

Possibility of factorisation formulae including Glaubers? (Glaubers and soft gluons treated differently).

But work ongoing by Stewart, Rothstein

Note that SCET up till now does not have Glauber modes built into it! Any factorisation formulae derived in SCET hold 'in absence of Glaubers'.

Resummation

A further approach to obtain some kind of 'factorisation' formula (e.g. for p_T of V) is the 'resummation' approach:

Start with factorisation formula for total cross section, and assume this can also be applied in a straightforward way to the observable of interest (p_T):

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} = \int dx_A dx_B \frac{d\hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s)}{dp_T^2} f_i(x_A) f_j(x_B)$$

Then one notices that for $p_T << Q$, we get a tower of large logarithms that spoils the convergence of the perturbative series:

$$\frac{1}{\sigma}\frac{d\hat{\sigma}}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1 \alpha_S \log\frac{Q^2}{p_T^2} + A_2 \alpha_S^2 \log^3\frac{Q^2}{p_T^2} + \ldots + A_n \alpha_S^{2n-1}\frac{Q^2}{p_T^2} + \ldots\right]$$

Resummation

If possible, rearrange these terms into an exponential (using factorisation properties of QCD matrix elements in soft/collinear limits, plus some transform to factorise measurement):

$$\frac{d\sigma^{(\text{sing})}}{dy\, dq_T^2} = \frac{M^2}{s}\, \sigma_H^{(0)}(\alpha_s(M^2)) \int_0^{+\infty} db\, \frac{b}{2}\, J_0(bq_T)\, S_g(M,b)$$

Exponential

$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2}\, \left[H^F C_1 C_2\right]_{gg;\, a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \ ,$$

Collection of coefficient functions

Can rearrange this expression to resemble CSS factorisation formula for p_T

Unlike for CSS method, you do not make any claims to have all contributions at leading power – in particular Glauber contributions (you just summed up soft and collinear emissions).

Glauber Gluons and Factorisation

So in both SCET and the resummation approach we effectively ignore Glaubers and obtain the following type of factorisation/resummation formula:

$$\frac{d\sigma}{dO} = \underset{\text{Hard}}{H} \times \left[\underbrace{B_a \otimes B_b \otimes J_1 \otimes \ldots \otimes J_n}_{\text{Collinear}} \otimes \underset{\text{Soft}}{S}\right](O)$$

Here we will take this sort of formula as the 'standard' factorisation, and say that factorisation breaks/fails for an observable if such a formula does not completely capture the leading contribution.

Other definitions of factorisation breaking are possible: breakdown of collinear factorization involving PDFs, failure of universality for collinear functions,...

Hadronic Transverse Energy

Definition:

In p + p → V + X: $$E_T = \sum_{i \in X} \sqrt{\mathbf{p}_{Ti}^2 + m_i^2}$$

Standard factorisation/resummation formula for E_T obtained by Papaefstathiou, Smillie, Webber:

$$\left[\frac{d\sigma_H}{dQ^2\, dE_T}\right]_{\text{res.}} = \frac{1}{2\pi} \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau\; \mathrm{e}^{-i\tau E_T}\; f_{a/h_1}(x_1,\mu)\; f_{b/h_2}(x_2,\mu) \cdot W_{ab}^H(x_1 x_2 s; Q, \tau, \mu)$$

$$W_{ab}^H(s; Q, \tau, \mu) = \int_0^1 dz_1 \int_0^1 dz_2\; C_{ga}(\alpha_S(\mu), z_1; \tau, \mu)\; C_{gb}(\alpha_S(\mu), z_2; \tau, \mu)\; \delta(Q^2 - z_1 z_2 s) \cdot \sigma_{gg}^H(Q, \alpha_S(Q))\; S_g(Q, \tau)\ .$$

Papaefstathiou, Smillie, Webber JHEP 1004 (2010) 084
Grazzini, Papaefstathiou, Smillie, Webber JHEP 1409 (2014) 056

Calculated up to (approximate) NNLL + NLO for Higgs production

Grazzini, Papaefstathiou, Smillie, Webber JHEP 1409 (2014) 056

Hadronic Transverse Energy

$$\left[\frac{d\sigma_H}{dQ^2\, dE_T}\right]_{\text{res.}} = \frac{1}{2\pi}\sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau\; e^{-i\tau E_T}\; f_{a/h_1}(x_1,\mu)\; f_{b/h_2}(x_2,\mu)$$
$$\cdot\; W_{ab}^H(x_1 x_2 s; Q, \tau, \mu)$$

$$W_{ab}^H(s;Q,\tau,\mu) = \int_0^1 dz_1 \int_0^1 dz_2\; C_{ga}(\alpha_S(\mu), z_1; \tau, \mu)\; C_{gb}(\alpha_S(\mu), z_2; \tau, \mu)\; \delta(Q^2 - z_1 z_2 s)$$
$$\cdot\; \sigma_{gg}^H(Q, \alpha_S(Q))\; S_g(Q,\tau)\ .$$

Contains effects of collinear, soft, hard radiation, could be rearranged into HxJxS form:

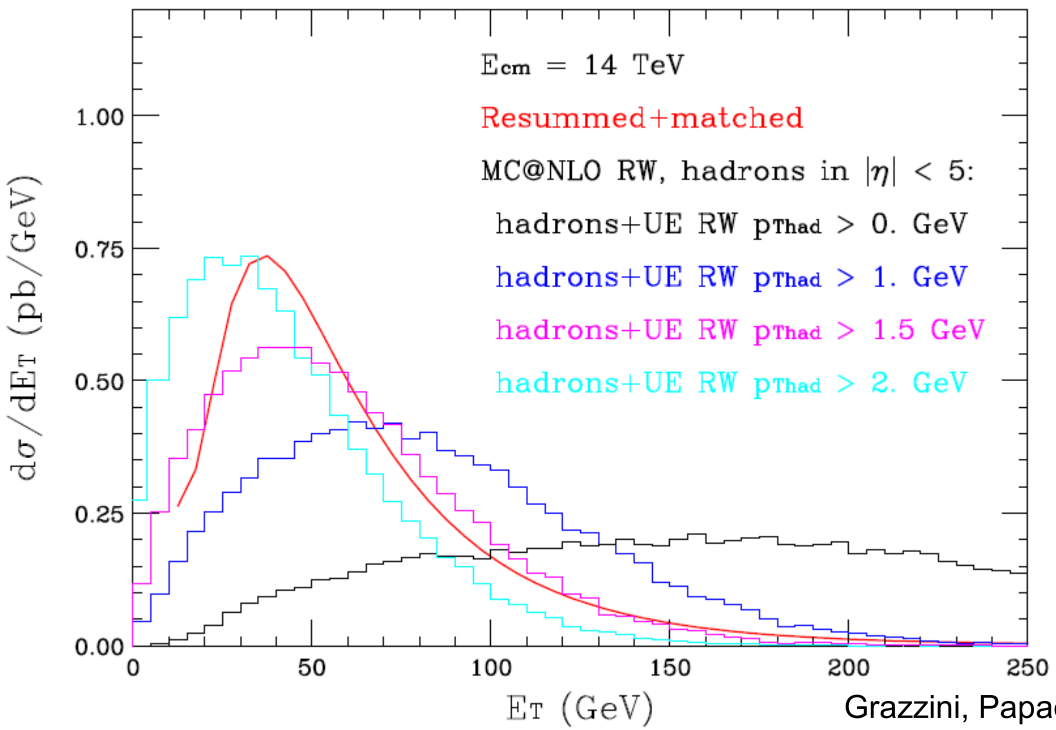
$$\frac{d\sigma}{dE_T} = \sigma_0\, H_{gg}(m_H,\mu) \int dY \int dk_a\, dk_b$$
$$\times\, B_g(m_H, k_a, x_a, \mu, \nu)\, B_g(m_H, k_b, x_b, \mu, \nu)$$
$$\times\, S_T^{gg}\Big(E_T - k_a - k_b, \mu, \nu\Big)\,.$$

Tackmann, Walsh, Zuberi
Phys. Rev. D 86 (2012) 053011

Does this give the leading contribution?

Hadronic Transverse Energy

Monte Carlo study with Herwig++:



Grazzini, Papaefstathiou, Smillie, Webber
arXiv:1403.3394

Apparently not! Once we turn on UE, event shape completely changes

Can we see this from a factorisation point of view? Must be related to Glauber gluons.

Model Setup

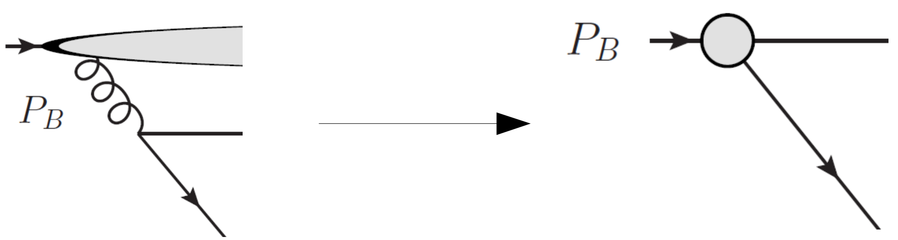
We will see if the effects of Glauber gluons can be cancelled for E_T, as was demonstrated to occur for p_T by CSS, when $E_T \ll Q$

Model setup: Each 'proton' is composed from a quark-antiquark pair

Central 'hard process' is $q\bar{q} \to V$ with V colourless and associated scale Q

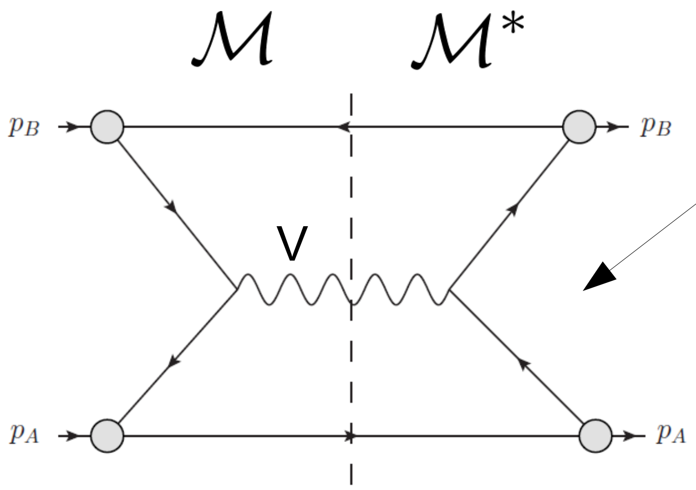
Assume momentum of proton A mainly along p/+ direction and that of B mainly along n/- direction, but with small masses. All partons taken to be massless.

We assume little about the coupling of the quark-antiquark pair to the 'proton' – could either represent some soft nonperturbative coupling (appropriate when $E_T \sim \Lambda$) or the perturbative quark-antiquark-gluon coupling (appropriate when E_T is perturbative)



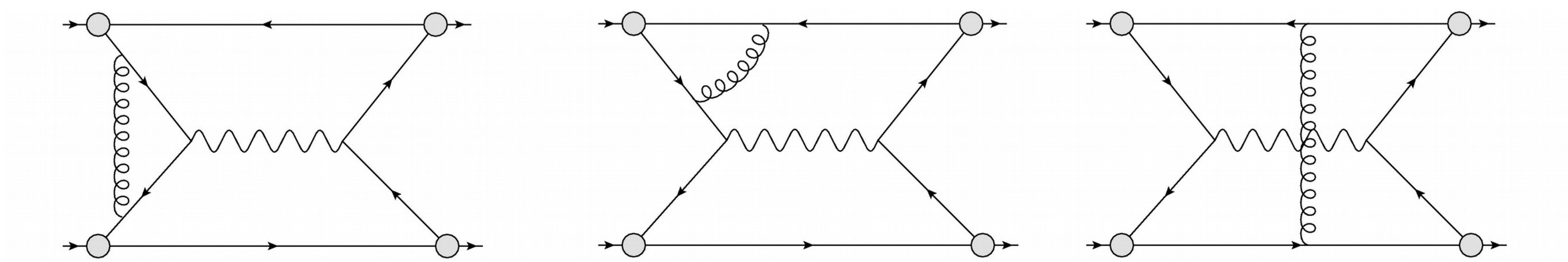
Calculation follows closely discussion in Collins pQCD book

One-Glauber Exchange



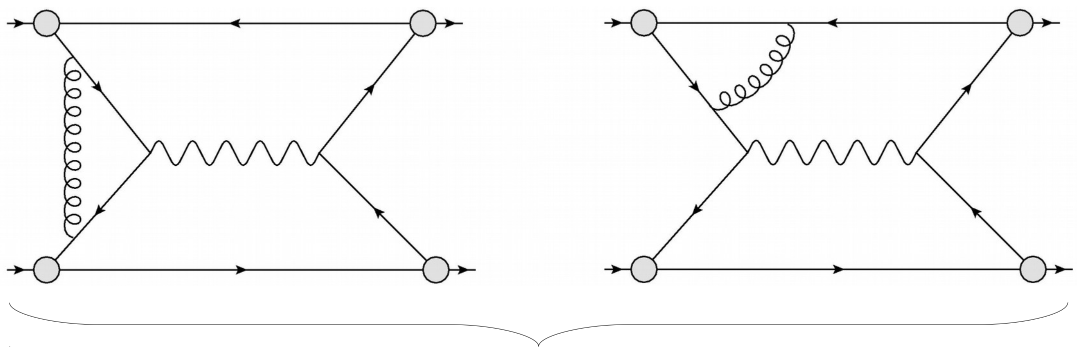
Lowest order 'parton model' process for p + p → V + X

Consider one gluon corrections to this picture. Various possibilities:

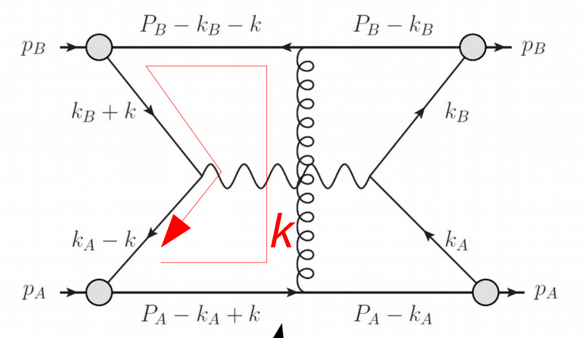


Active-active interaction Active-spectator interaction Spectator-spectator interaction

One-Glauber Exchange



For these graphs, gluon momentum is not trapped in the Glauber region however → can deform loop integration contour and handle these using usual soft/collinear/hard regions. Collins, Phys.Rev. D57 (1998) 3051–3056
Collins, Metz, Phys.Rev.Lett. 93 (2004) 252001



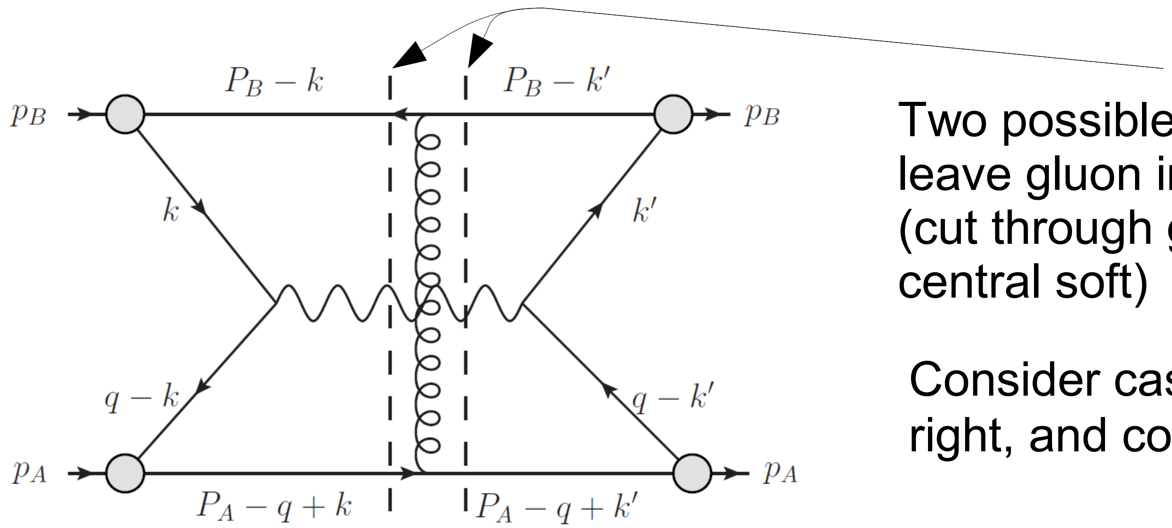
$$\int \frac{dk^+ dk^-}{(2\pi)^2} \frac{\text{numerator}}{2k^+k^- - \mathbf{k}_T^2 + i0}$$
$$\times \frac{1}{[-2k^+(P_B^- - k_B^-) + \ldots + i0][2k^+k_B^- + \ldots + i0]}$$
$$\times \frac{1}{[-2k^-k_A^+ + \ldots + i0][2k^-(P_A^+ - k_A^+) + \ldots + i0]}$$

In this graph gluon is trapped in Glauber region

Traps k⁺ small

Traps k⁻ small

One-Glauber Exchange



Two possible cuts of graph that leave gluon in Glauber region (cut through gluon forces it into central soft)

Consider case where cut is to the right, and consider k^+ integral.

In top half of graph can ignore $q^- - k^-$ compared to large components (q^-, $P_B^- - q^-$)

In bottom half of graph can ignore k^+ compared to large components (q^+, $P_A^+ - q^+$)

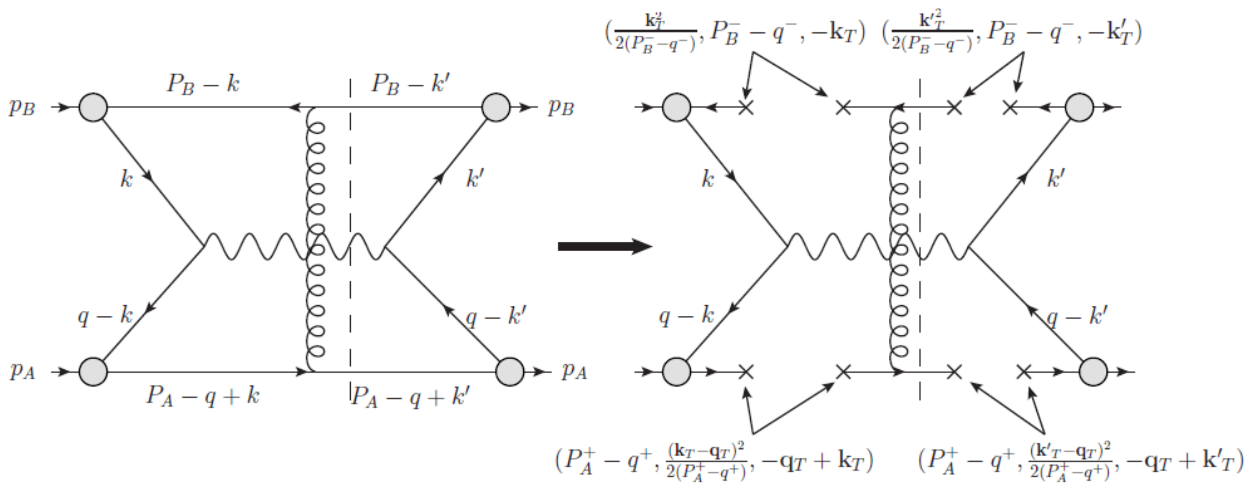
In gluon propagator can ignore lightcone components of k compared to transverse components

$$\int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} \frac{i}{(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)}$$
$$= \frac{i}{2(P_B^- - q^-)} \frac{i}{2q^- k^+_{\text{on-shell}} - \mathbf{k}_T^2 + i0}$$
$$= \int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} 2\pi\delta(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)$$

Net effect – set P_B-k line on shell!

One-Glauber Exchange

Repeat with k^-, k'^+, k'^- integrations:



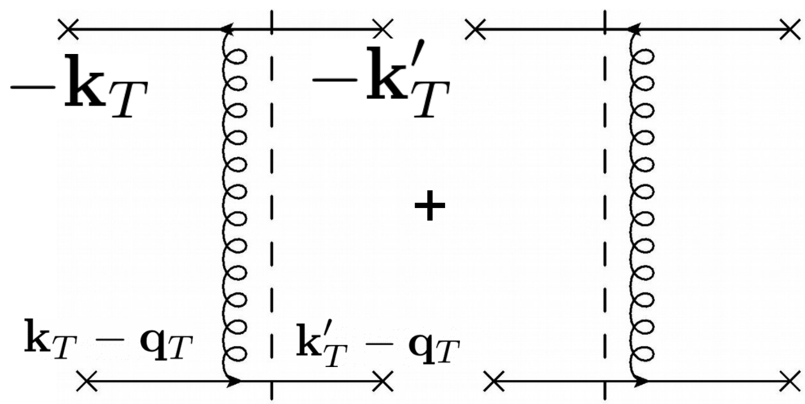
Factor out on-shell Glauber exchange graph!

Can do a similar procedure when cut is on left.

Measured Transverse Momentum

Consider case where we measure p_T of V. For given momenta in the two decomposed graphs, the value of the measurement is the same.

Therefore, we can factor out the parton model graph and measurement and add together the two Glauber subgraphs:



$\mathcal{M} = -iL$

Graph level: $L(\mathbf{k}_T \to \mathbf{k}'_T) + L^*(\mathbf{k}'_T \to \mathbf{k}_T)$

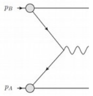
Amplitude level: $i\mathcal{M}(\mathbf{k}_T \to \mathbf{k}'_T) - i\mathcal{M}^*(\mathbf{k}'_T \to \mathbf{k}_T)$

= 0

CUTKOSKY RULE

Measured Hadronic Transverse Energy

For E_T, can't do the same thing – it equals $|\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T|$ for cut to left and $|\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T|$ for cut to right. Can still arrange cancellation by change of transverse variables:

Parton model graph 

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L(\mathbf{k}_T \to \mathbf{k'}_T)\delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|)+$$

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L^*(\mathbf{k'}_T \to \mathbf{k}_T)\delta(E_T = |\mathbf{k}_T| + |\mathbf{k}_T - \mathbf{q}_T|)$$

Relabel $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$ in second term

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, \delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|)\times$$

$$[f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) L(\mathbf{k}_T \to \mathbf{k'}_T) + f_P(\mathbf{k'}_T) f_P^*(\mathbf{k}_T) L^*(\mathbf{k}_T \to \mathbf{k'}_T)]$$

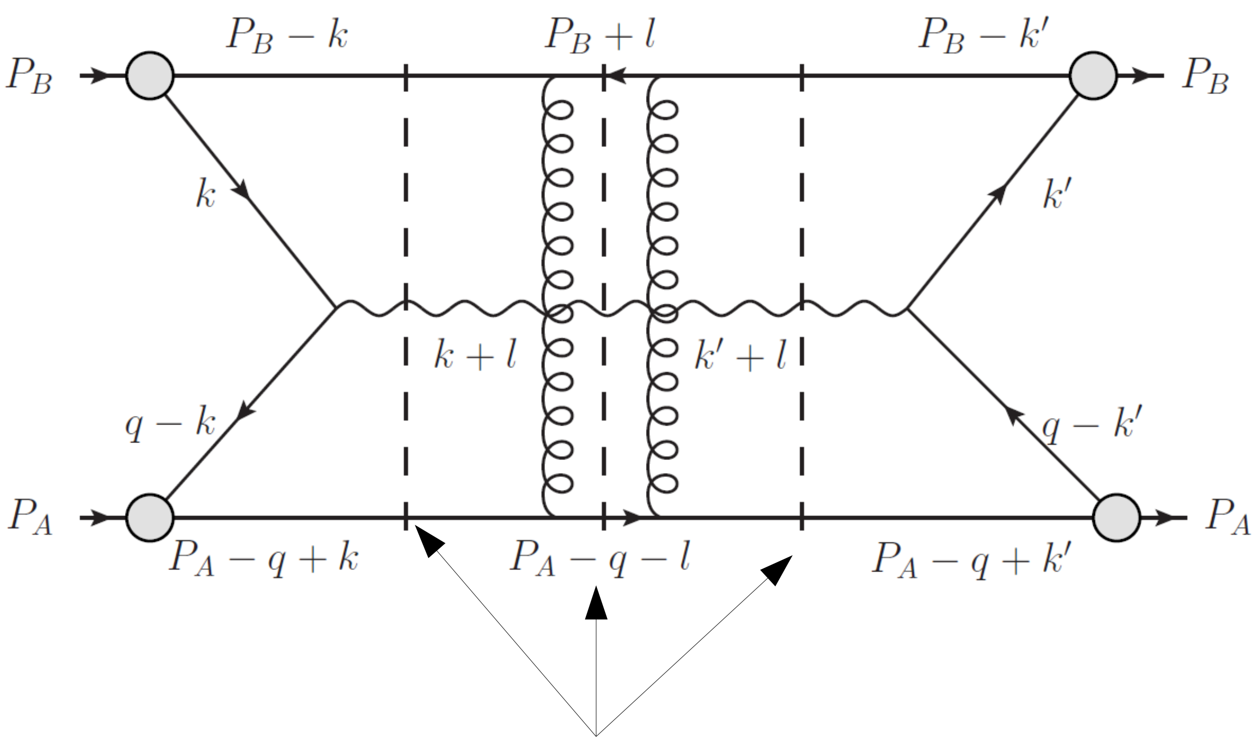
$L(\mathbf{k}_T \to \mathbf{k}'_T) \propto 1/(\mathbf{k}_T - \mathbf{k}'_T)^2 = L(\mathbf{k}'_T \to \mathbf{k}_T)$ & $f_P = -f_P^*$

$$\int \mathrm{d}^{d-2}\mathbf{k}_T\, \mathrm{d}^{d-2}\mathbf{k'}_T\, \delta(E_T = |\mathbf{k'}_T| + |\mathbf{k'}_T - \mathbf{q}_T|)\times$$

$$f_P(\mathbf{k}_T) f_P^*(\mathbf{k'}_T) \left[L(\mathbf{k}_T \to \mathbf{k'}_T) + L^*(\mathbf{k'}_T \to \mathbf{k}_T)\right] \longrightarrow 0$$

Two-Glauber Exchange

Now add in one more (Glauber) gluon between spectators:

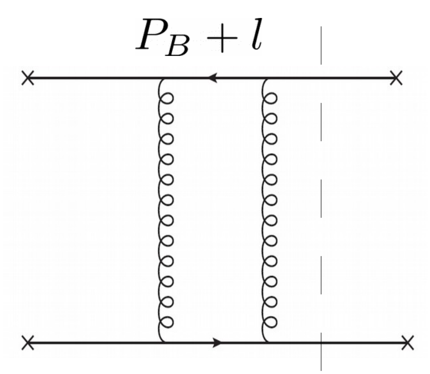


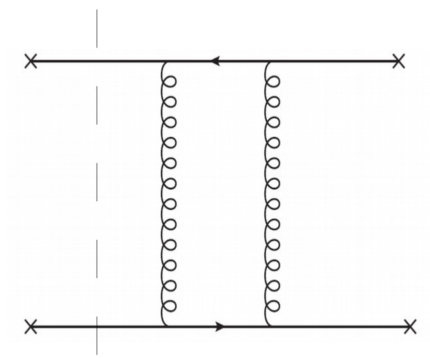
Now 3 cuts that leave both gluons in Glauber region

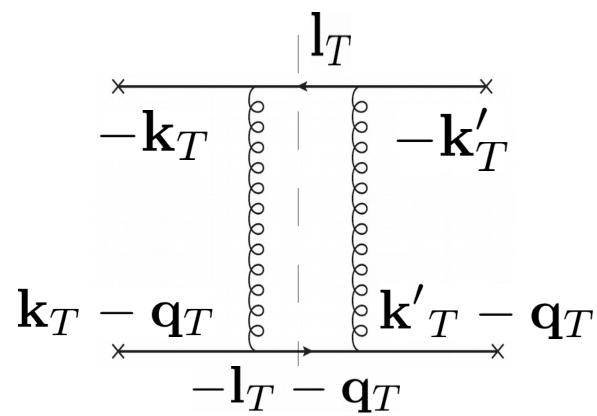
Factor out Glauber subgraphs as before.

Measured Transverse Momentum

p_T measured: Again, for given momenta in (decomposed) graphs, measurement is the same – factor measurement and parton model graphs, and combine Glauber subgraphs:







$$L(\mathbf{k}_T \to \mathbf{k'}_T; l) + L^*(\mathbf{k'}_T \to \mathbf{k}_T; l) \quad + \quad \int \Phi_2 L(\mathbf{k}_T \to -\mathbf{l}_T) L^*(\mathbf{k'}_T \to -\mathbf{l}_T)$$

$$i\mathcal{M}(\mathbf{k}_T \to \mathbf{k'}_T; l) - i\mathcal{M}^*(\mathbf{k'}_T \to \mathbf{k}_T; l) \quad + \quad \int \Phi_2 \mathcal{M}(\mathbf{k}_T \to -\mathbf{l}_T) \mathcal{M}^*(\mathbf{k'}_T \to -\mathbf{l}_T)$$

-Imaginary Part

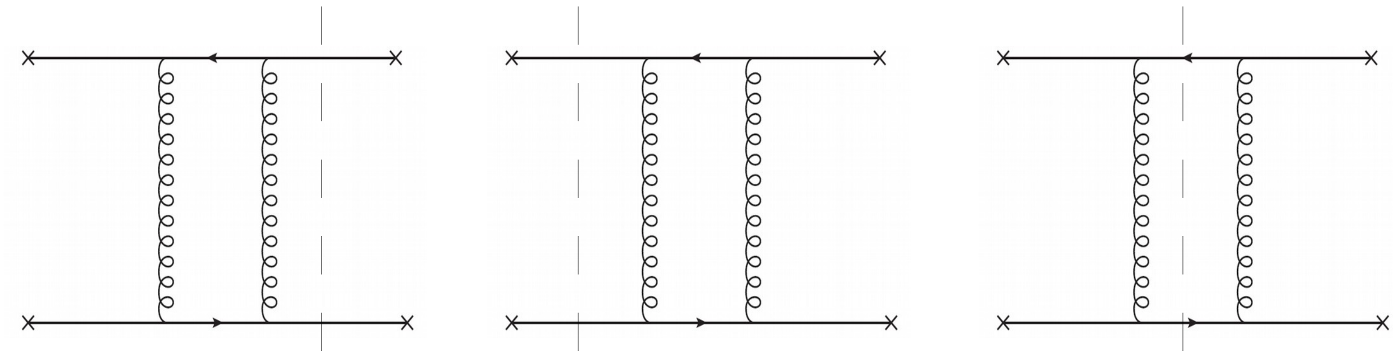
Sum over (1) cuts

=0 using Cutkosky rules

Measured Hadronic Transverse Energy

For E_T cancellation of Glauber subgraphs fails, because E_T for central/real cut depends on loop momentum, whilst same is not true for external/absorptive cuts:

$$E_T: \quad |\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T| \qquad |\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T| \qquad |\mathbf{l}_T| + |\mathbf{q}_T + \mathbf{l}_T|$$

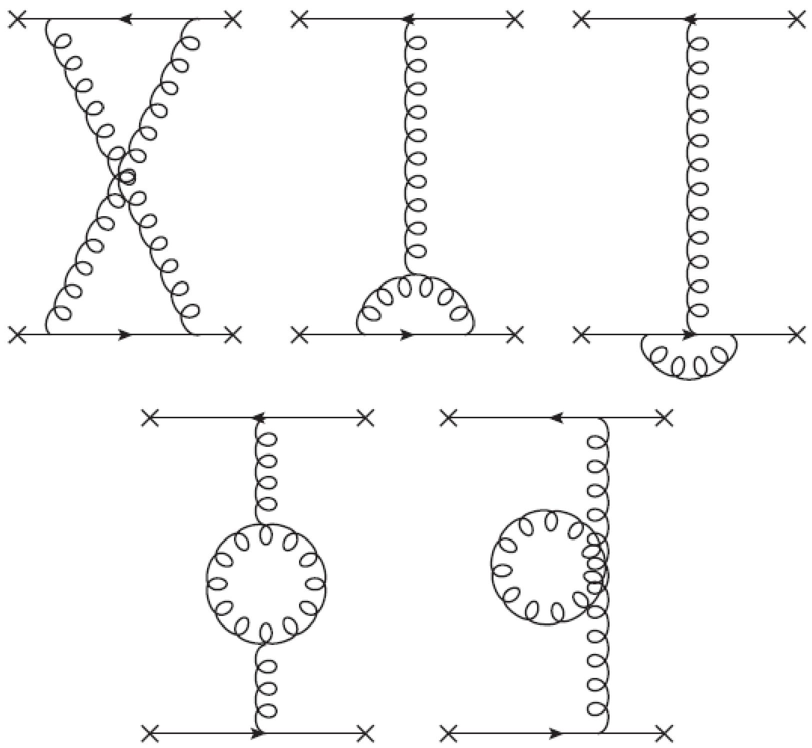


Maybe some change of loop and external variables is possible to arrange cancellation?

I argue no such change of variables is possible. Cutkosky cancellation is one that occurs point by point in spatial momentum – should match parameterisation of loop momentum between graphs.

Sterman, hep-ph/9606312

Other one-loop spectator interactions

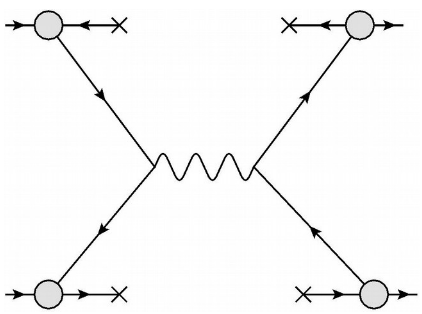


Might wonder if you can cancel contribution from spectator-spectator box graph against some of the other one-loop spectator-spectator interactions.

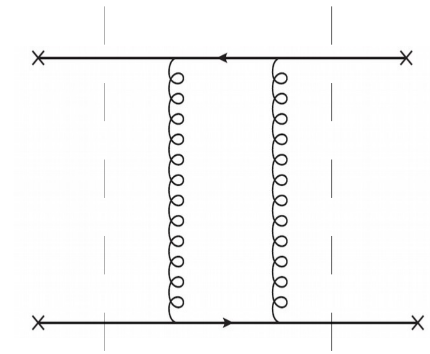
These graphs have no internal cuts → can cancel the contribution of these to E_T spectrum using same argument as applied for one-gluon exchange.

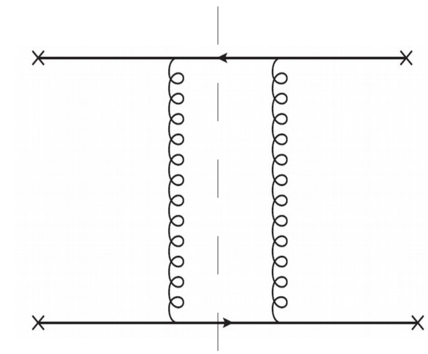
Glauber Gluons and MPI

The factorisation breaking for E_T is associated with:



$\otimes$





But this can be interpreted as:

Primary hard interaction

Secondary low scale absorptive process ('MPI did not occur')

Secondary low scale scattering ('MPI occurred')

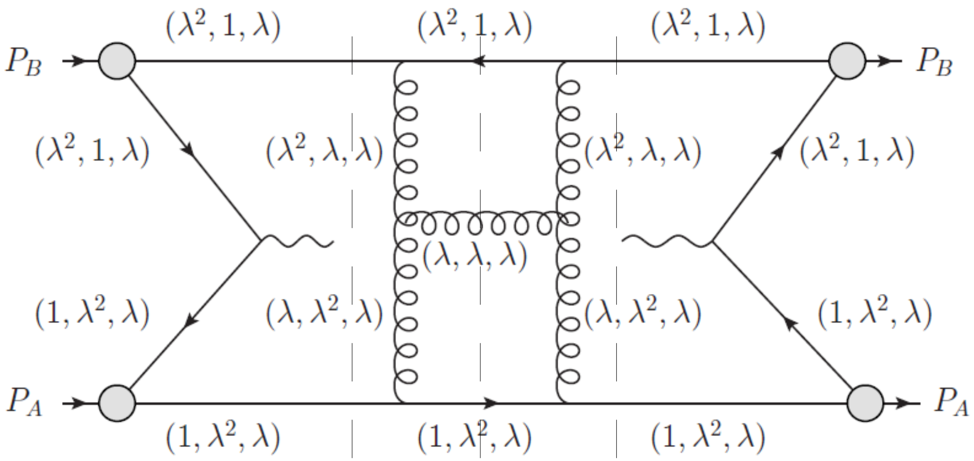
Factorisation breaking effects are due to MPI, as was also found in MC studies. Close connection between Glauber gluons and MPI!

Absorptive process included in MCs via unitarity constraints.

NOTE: Contribution from Glauber region cancels for p_T because p_T of V is insensitive to whether extra interactions occurred or not, NOT because MPI is cancelled.

Glauber Gluons and Regge Physics

Also achieve a leading contribution by inserting central soft rung between Glauber verticals:



3 cuts again don't cancel for E_T

This graph suppressed by additional power of α_s compared to zero-rung graph, but enhanced by rapidity (BFKL) logarithm. Can insert arbitrary number of rungs (forming Pomeron type object) and still be at leading log order in BFKL sense.

→ should need good control of BFKL effects in MPI to describe E_T well.

Other observables

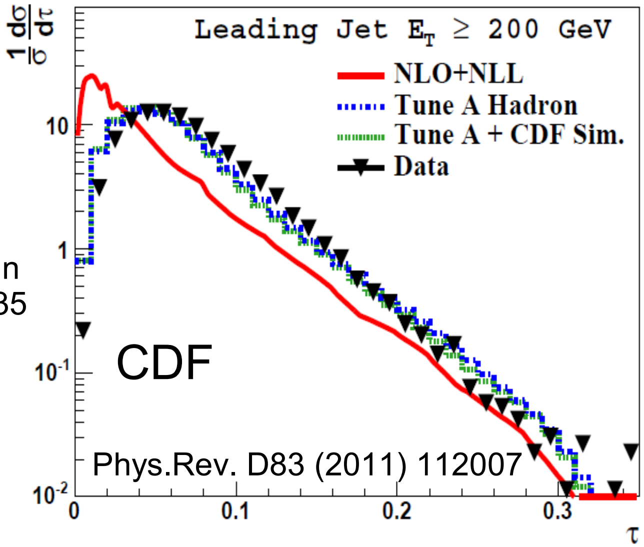
Same effect should occur for any other observable which is sensitive to whether an additional interaction occurred or not (MPI sensitive observables)

For example:

Beam thrust $B_a = \sqrt{2} \sum_{i \in X,a} p_i \cdot p \quad B_b = \sqrt{2} \sum_{i \in X,b} p_i \cdot n$

Stewart, Tackmann, Waalewijn
Phys. Rev. D 81 (2010) 094035

Transverse thrust $\max_{\mathbf{n}_{iT}} \sum_i |\mathbf{q}_{iT} \cdot \mathbf{n}_{iT}| / \sum_i |\mathbf{q}_{iT}|$



Usually global event shapes. Jet observables much less MPI sensitive as particles from MPI only collected up over jet radius → MPI suppressed by radius *R*.

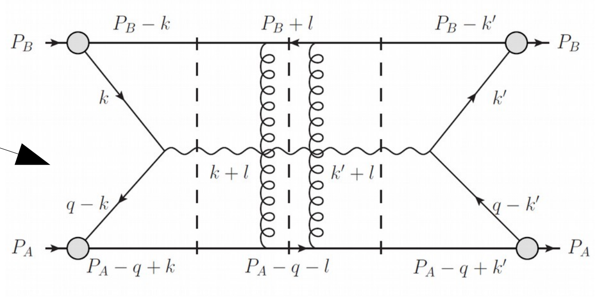
It was noted earlier that underlying event, or MPI effects are suppressed by jet radius R

M. Dasgupta, L. Magnea, and G. P. Salam, JHEP 0802 (2008) 055
Tackmann, Waalewijn, Stewart, Phys. Rev. Lett. 114, 092001 (2015)

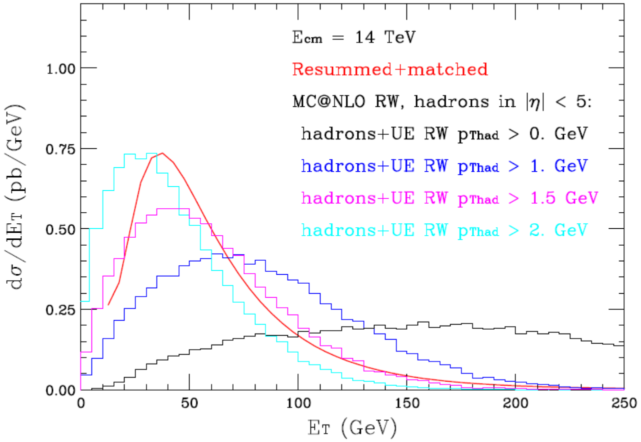
MPI sensitive variable of order of hard scale

What about when MPI sensitive observable O_s is of order of the hard scale Q? Then for the cumulant of O_s, are we inclusive enough that standard factorisation formula can be used?

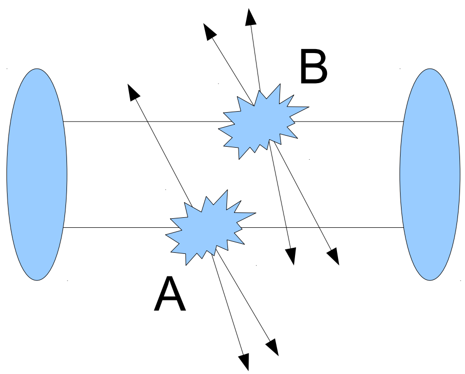
Miscancellation of cuts in this graph now only smears observable by some power suppressed amount



The trouble is that we can have Glauber miscancellations on multiple spectator legs adding up to produce a big smearing of O_s, even when $O_s \sim Q$



Double Parton Scattering



Double parton scattering (DPS) is where we have two hard interactions in one proton-proton collision.

DPS can be a background to rare (e.g. new physics) signals, and reveals new information about proton – i.e. correlations between partons.

Measure inclusive cross section or cross section differential in p_Ts

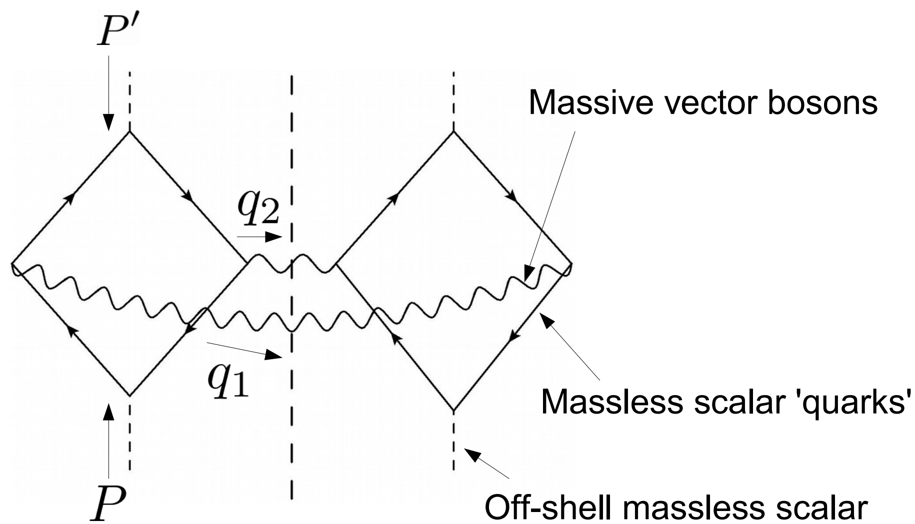
Of course there is also a (large) contribution to such observables from single scattering (SPS), plus some interplay between the two mechanisms.

This process is (funnily enough) MPI insensitive – doesn't care if further (soft) interactions occurred or not. Do Glauber effects cancel for this observable?

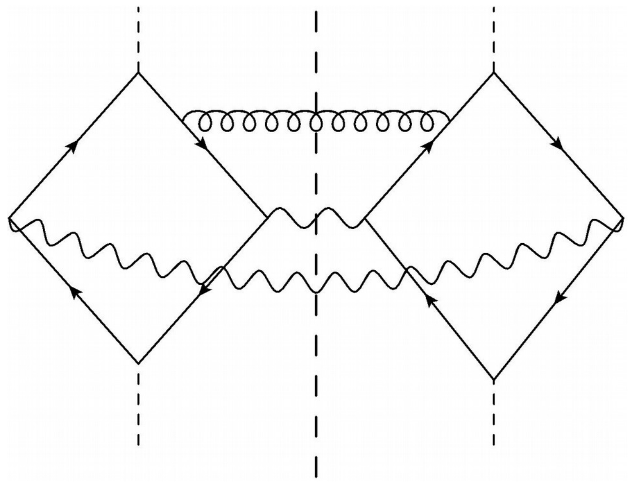
Glauber in DPS – one loop model calculation

One loop model calculation

'Parton-model' process:



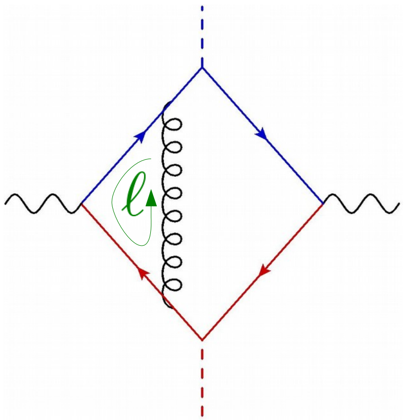
Real corrections:



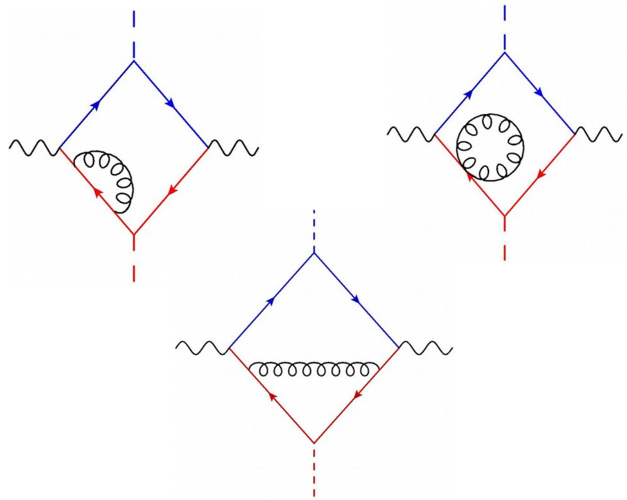
$\propto [Tr(t^A)]^2 = 0$

Glauber in DPS – one loop model calculation

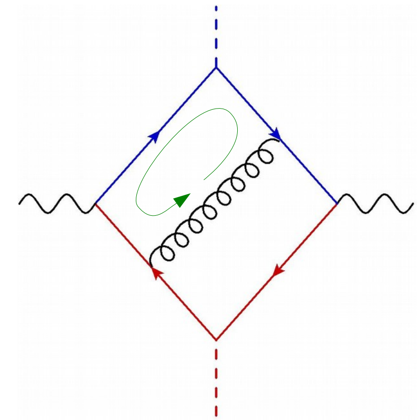
Virtual corrections:



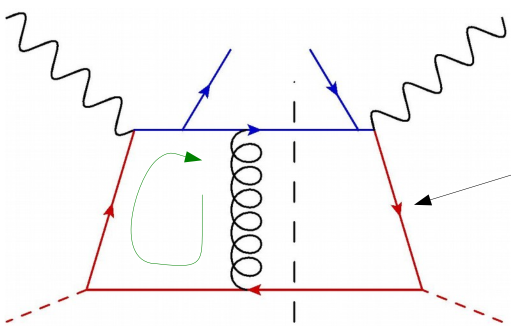
Neither l^+ nor l^- is trapped small



'Topologically factored graphs'



l^+ only is trapped small – l^- can be freely deformed away from origin (into region where l is collinear to P').



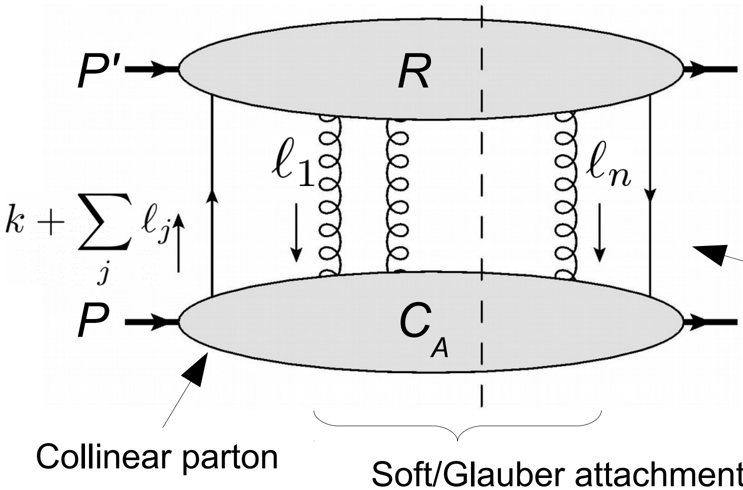
Very similar to situation in SIDIS – no Glauber contribution there too.

More detailed checks that Glauber contributions are absent in the one-loop calculation will be in the upcoming paper.

Glauber in SPS – all-order analysis

How did CSS prove Glaubers cancel to all orders for SPS Drell-Yan (inclusive cross section or $d\sigma/dp_T$)?

Collins, Soper, Sterman Nucl. Phys. B308 (1988) 833
Collins, pQCD book



1) Factor all longitudinally polarised collinear gluon connections to hard into Wilson lines in collinear factors

2) Partition leading order region into one collinear factor C_A and the remainder R

Partioning of soft vertex attachments in C_A between amplitude and conjugate

All compatible cuts of C_A

In C_A can approximate

$$\ell_j \to \tilde{\ell}_j = (0, \ell_j^-, \boldsymbol{\ell}_j)$$

but can't yet set

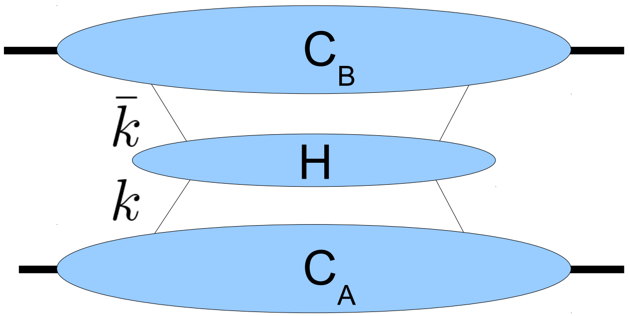
$$\hat{\ell}_j \to (0, \ell_j^-, \mathbf{0})$$

$$G_L = \int \frac{dk_1^+ d^2\boldsymbol{k}}{(2\pi)^3} \int \frac{d^2\boldsymbol{\ell}_j d\ell_j^-}{(2\pi)^3} \prod_{j=1}^{n} \sum_V \sum_{F_A \in \mathcal{A}(V)} C_A^{F_A}(k, r, \{\tilde{\ell}_j\})$$

$$\times \int \frac{d\ell_j^+}{(2\pi)} \sum_{F_R \in \mathcal{R}(V)} R^{F_R}(k^+, \boldsymbol{k}, \{\ell_j\})$$

All compatible cuts of R

Glauber in SPS – all-order analysis



3) Consider $\displaystyle \int \frac{d\ell_j^+}{(2\pi)} \sum_{F_R \in \mathcal{R}(V)} R^{F_R}(k^+, \boldsymbol{k}, \{\ell_j\})$

For a given V, this is:

$$\int \frac{d\bar{k}^- d^2\bar{\boldsymbol{k}}}{(2\pi)^3} \sum_{C_H} H^{C_H}(k^+, \bar{k}^-)\delta^{(2)}(\boldsymbol{Q} = \boldsymbol{k} + \bar{\boldsymbol{k}})$$

$$\times \int d^4 z_1 \boxed{\delta(z_1^-)} e^{i\bar{k}\cdot z_1} \left\{ \prod_j \int d^4 x_j \boxed{\delta(x_j^-)} e^{ix_j \cdot \ell_j} \right\}$$

$$\times \langle P|\bar{T}\left\{ \bar{\psi}\left(-\tfrac{1}{2}z_1\right) \prod_{j>F} A(x_j) \right\} \times T\left\{ \psi\left(\tfrac{1}{2}z_1\right) \prod_{j<F} A(x_j) \right\}|P\rangle \qquad (♣)$$

This delta function is due to usual short-circuiting of small mtm component in PDF

This delta function is here because we integrated over ℓ_j^+

Due to the delta functions, the fields in (♣) all have the same value of x^-

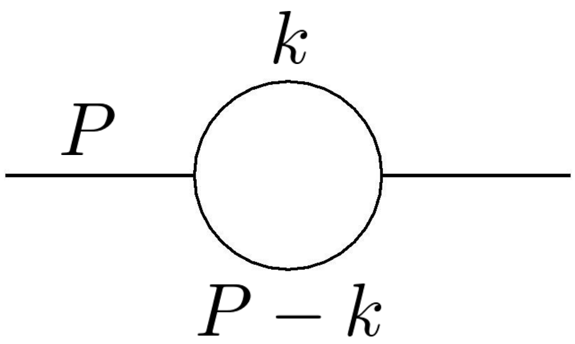
→ with generally nonzero spacelike separations, they all commute

→ R does not depend on the partitioning V

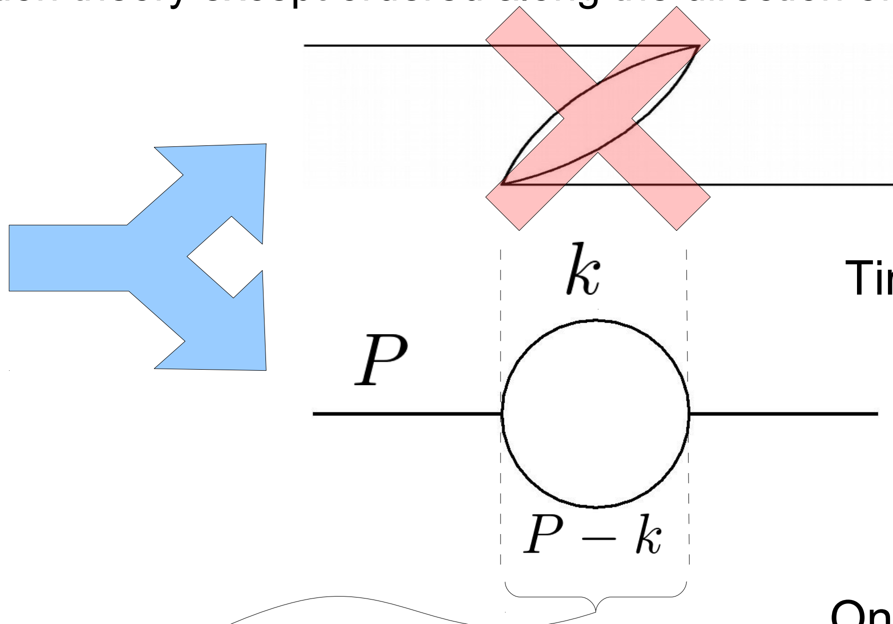
Glauber in SPS – all-order analysis

4) Now we can consider $\sum_{V} \sum_{F_A \in \mathcal{A}(V)} C_A^{F_A}(k, r, \{\tilde{\ell}_j\}) = \sum_{\text{all cuts}} C_A^{F_A}(k, r, \{\tilde{\ell}_j\})$

Consider this factor in lightcone ordered perturbation theory (LCPT) – this is like old-fashioned time ordered perturbation theory except ordered along the direction of the P-jet.



Feynman graph



Time orderings

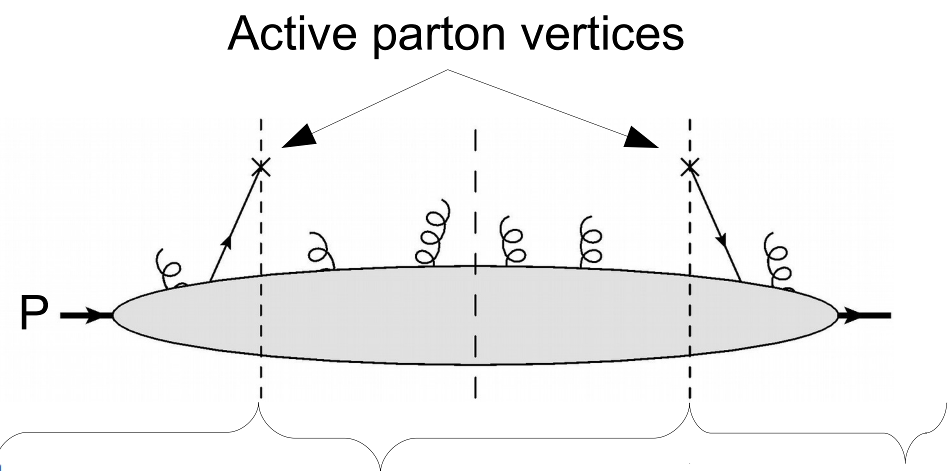
Denominator associated with state ξ:

$$P^- - \frac{\mathbf{k}^2 + m^2}{2k^+} - \frac{\mathbf{k}^2 + m^2}{2(P^+ - k^+)} + i\epsilon$$

Total minus momentum entering state from left

On-shell minus momenta of lines in state

Glauber in SPS – all-order analysis



$$\prod_{\substack{\text{states } \xi \\ \xi<H}} \frac{1}{P^- + \sum\limits_{\substack{\text{vertices } j \\ j<\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L \\ L\in\xi}} \kappa_L + i\epsilon}$$

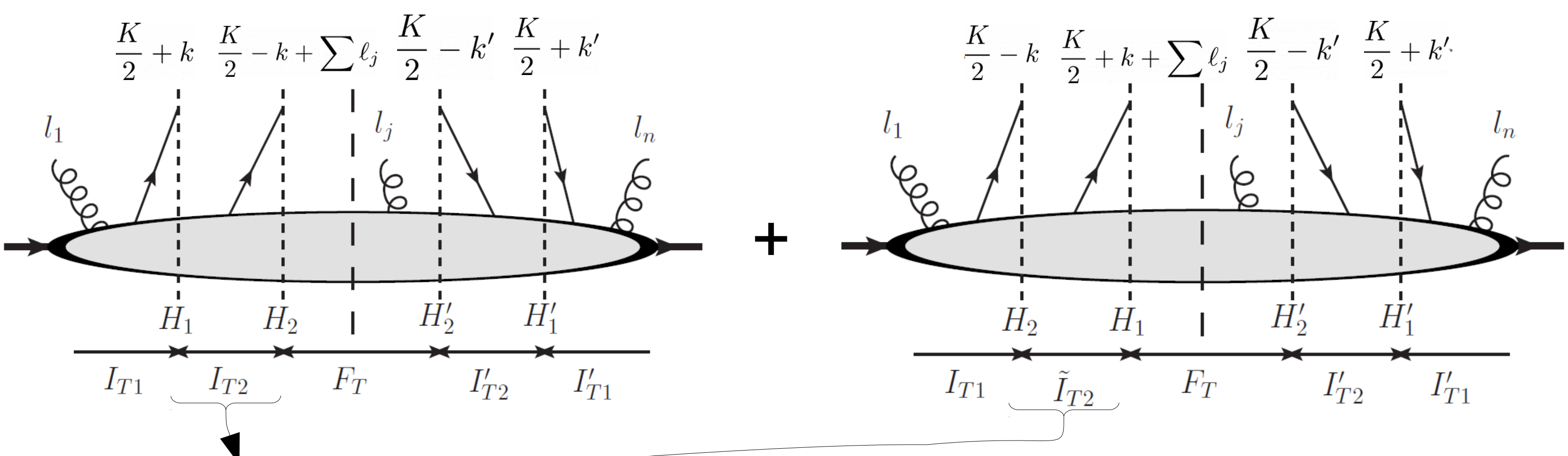
$$\prod_{\substack{\text{states } \xi \\ H'<\xi}} \frac{1}{P^- - \sum\limits_{\substack{\text{vertices } j \\ j>\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L \\ L\in\xi}} \kappa_L - i\epsilon}$$

Sum over cuts of this either gives 0, if there is more than one state in between active parton vertices, or 1 if there is one. Due to unitarity (LCPT version of Cutkosky rules)

Glauber in DPS – all-order analysis

All-order analysis for double Drell-Yan can be done using the same method as for single DY:

LCPT graphs for C_A in DPS:



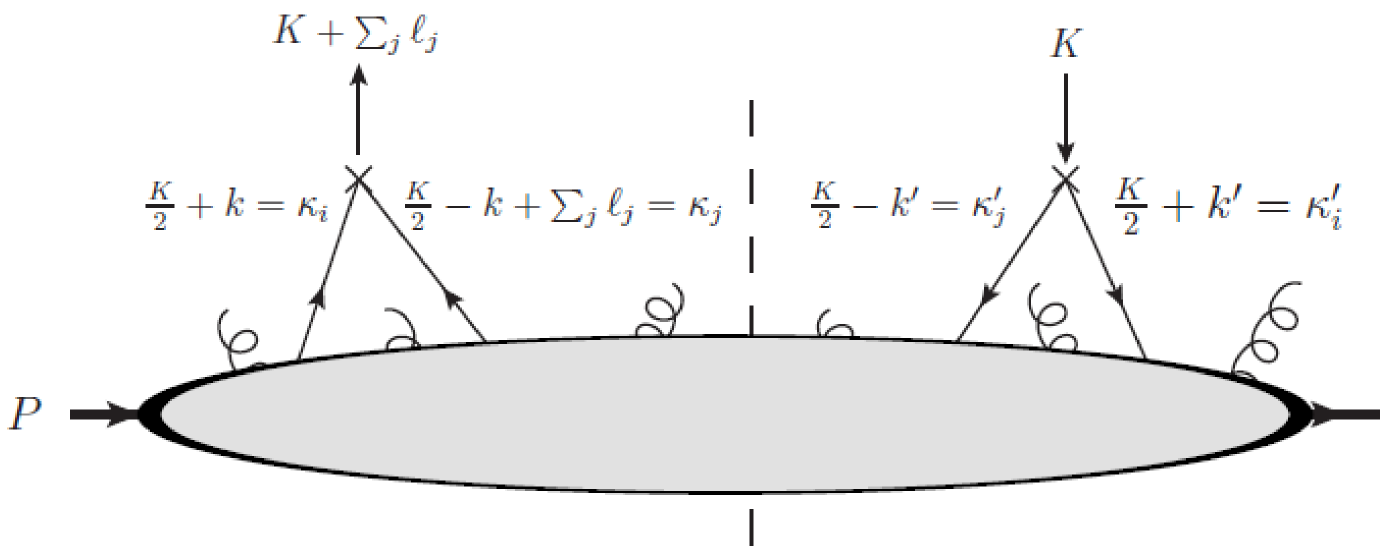
$$I_{T2}+\tilde{I}_{T2}$$

$$= \int \frac{dk^-}{2\pi} \left(\frac{1}{P^- - K^-/2 - k^- - \sum\limits_{\substack{\text{lines } L \\ L\in\xi}} \kappa_L + i\epsilon} + \frac{1}{P^- - K^-/2 + k^- - \sum\limits_{\substack{\text{lines } L \\ L\in\xi}} \kappa_L + i\epsilon} \right)$$

$$= -i$$

Glauber in DPS – all-order analysis

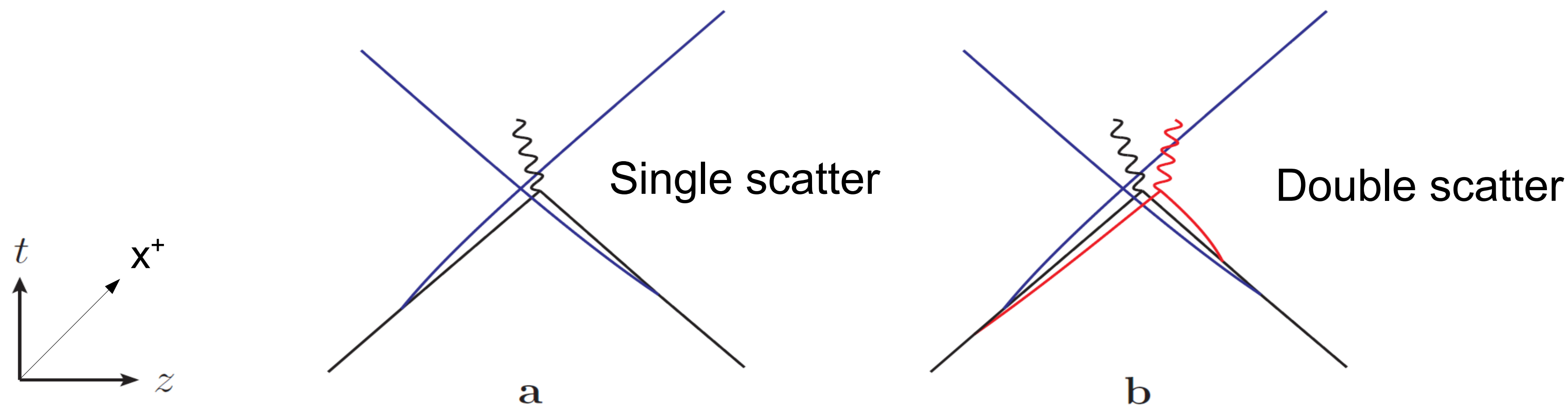
Repeat for k' in conjugate – end up with the following picture:



Just one external vertex in amplitude and conjugate – diagram looks essentially identical to SPS C_A and cancellation of Glaubers proceeds as for SPS.

Glauber in DPS – space-time structure

Basic reason why Glauber modes cancels for double Drell-Yan, just as it does for single Drell-Yan – spacetime structure of pinch surfaces for single and double scattering are rather similar:



Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

More details will be in upcoming paper

Conclusions

- CSS-style cancellation of Glauber region fails for E_T (<< Q), at the level of two Glauber gluons exchanged between spectators.
- Can connect these diagrams to events with additional soft scatterings – connection between Glauber gluons and MPI.
- Also lack of cancellation for more general Pomeron-type exchanges – connection between Glauber gluons and Regge physics.
- Standard factorisation with only collinear, soft and hard functions also fails for a wider class of MPI sensitive observables – e.g. beam thrust, transverse thrust.
- Double parton scattering observables are not MPI sensitive. Glauber cancellation appears to go through for double Drell-Yan similarly as for single Drell-Yan.