Fermion mass matrices and textures

Manmohan Gupta

(Collab: Dr. Gulsheen Ahuja, Dr. Priyanka Fakay, Samandeep)

Department of Physics, Centre of Advanced Study, P.U., Chandigarh
IISER, Mohali
Chandigarh Region Innovation and Knowledge Cluster (CRIKC)

September 16th, 2014
University of Vienna, Austria.

Outline

- **General Introduction**
- **Fermion mass matrices in the SM**
- **Texture specific mass matrices**
- **Weak basis transformations**
- **Natural mass matrices**
- **Issues associated with the three approaches**
- **Relationship between mass matrices and mixing matrices**
- **Viability of mass matrices through mixing matrices**
- **Mixing matrices and unitarity**
- **Unique texture for quark mass matrices**
- **Texture specific lepton mass matrices**
- **SO(10) inspired texture specific mass matrices**
- **Summary and Conclusion**

General Introduction

- Understanding fermion masses and mixings constitutes one of the most important problems of flavor physics.

Quark masses:
m_u, m_d ~ few MeV, m_s~90 MeV, m_c~ 1.5 GeV, m_b~ 4.5GeV, m_t~ 170GeV.

Neutrino masses:
$m_{\nu 1} \leq 10^{-2}$ eV, $m_{\nu 2} \sim 10^{-2}$ eV, $m_{\nu 3} \sim 10^{-1}$ eV ($m_R \sim 10^{15}$ GeV).

$\Delta m^2_{solar} \sim 7.5 \times 10^{-5}$ eV^2, $\Delta m^2_{atm} \sim 2.5 \times 10^{-3}$ eV^2

Data from Planck satellite combined with other cosmological data limit
$\sum < 0.23$ eV at 95 % C.L.

Quark mixing angles: s_{12}~0.22, s_{23}~0.04, s_{13}~0.0035

Neutrino mixing angles: s_{12}~0.55, s_{23}~0.69, s_{13}~0.15

- **Mysteries associated:**

  1. Fermion masses span around 13 (if neutrinos are Dirac particles) or 25-27 (if neutrinos are Majorana particles) orders of magnitudes. Also, quark and neutrino masses show different hierarchy, latter not settled.

  2. Similarly, the quark mixing angles exhibit a clear cut hierarchy, this is absent in the case of neutrinos.

- Several ideas such as Grand Unified Theories, extra dimensions, horizontal symmetries, etc. have been considered to understand mass generation problem

- Unfortunately, we do not have any viable theoretical framework from the `top-down' perspective to describe the quark and lepton masses and mixings in a simplified and unified manner.

- This necessitates examining the issues from a 'bottom-up' perspective.

- To this end, texture specific mass matrices were initiated by Fritzsch (PLB 1977, 1978). Proposed ansatze based on some plausible arguments (to enhance predictability), later christened as texture specific mass matrices.

Fermion mass matrices in the SM

- The fermion masses, mixings and CP violation are encoded in the couplings of Higgs and fermions.

- Within the framework of SM, the lepton mass matrices arise from the Higgs-fermion couplings,

$$\mathcal{L}_{Yukawa} = Y_{ij}^{U}\overline{L_{Li}}HU_{Rj} \ + \ Y_{ij}^{D}\overline{L_{Li}}H^{c}D_{Rj} \ + \ h.c, \quad i,j = 1,2,3,$$

- Mass matrices are related to Yukawa couplings as

$$M_U = \frac{v}{\sqrt{2}}Y_{ij}^{U}, \quad M_D = \frac{v}{\sqrt{2}}Y_{ij}^{D},$$

- In the SM with three generations, these couplings or mass matrices are completely arbitrary and are expressed in form of complex 3×3 matrices (with 36 real free parameters).

- In the SM and its extensions where right handed quarks are singlets, mass matrices can be considered to be Hermitian using Polar Decomposition theorem (M=HX), therefore reducing the number of free parameters from 36 to 18.

Texture Specific Mass matrices

- To further enhance the predictability by reducing the number of free parameters, in texture specific mass matrices zeros are put in. The original texture 6 zero Fritzsch quark mass matrices are given by

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}$$

- A particular texture structure is said to be texture ‘n’ zero if it has ‘n’ number of non trivial zeros i.e. if the sum of number of diagonal zeros and half the number of symmetrically placed off diagonal zeros is ‘n’.

- The conditions of hermiticity, hierarchy and textures are preserved when one scales down from GUT scale to low energy scale. [Raymond,Roberts,Ross,Nucl.Phys.B(1993)]

- Reviews on textures
  1. [Fritzsch,Xing, Prog. Par. Nucl. Phys. 45, 1(2000); Xing IJMPA 19,1 (2004)]
  2. [Gupta and Ahuja , IJMPA 27, 1230033 (2012)]

Weak Basis (WB) transformations

- Within the framework of SM and its extensions, one has the freedom to make unitary transformations, referred to as Weak Basis (WB) transformations under which the quark mass matrices change but the gauge currents remain diagonal and real.

  [Branco *et. al.*,PLB.82,683 (1999); Fritzsch *et.al*.,PLB 413, 396(1997); Nucl. Phys. B 556, 49(1999)]

- In particular, one can find a unitary matrix W transforming $M_U \to W^+ M_U W$ and $M_D \to W^+ M_D W$ leading to

$$M_U = \begin{pmatrix} E_U & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} E_D & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}.$$

- Can be easily checked that such transformations preserve the hermiticity of the mass matrices.

- The CKM matrix is independent of WB transformations.

- (U_U, U_D) and (U'_U, U'_D) are the respective diagonalizing transformations of (M_U, M_D) and (M'_U, M'_D).

- For either M_U and M_D one can obtain $U'_U = W^\dagger U_U \,, \quad U'_D = W^\dagger U_D \,.$

- Using this result, the mixing matrix for the WB transformed quark mass matrices can be given as

$$V'_{\rm ckm} = U'^\dagger_u U'_d = (W^\dagger U_u)^\dagger (W^\dagger U_d) = (U_u)^\dagger W W^\dagger U_d = (U_u)^\dagger U_d = V_{\rm ckm} \,.$$

Natural Mass Matrices

- In yet another approach, advocated by Peccei and Wang [Peccei,Wang,PRD53(1996)], the concept of `natural mass matrices' has been introduced to formulate viable set of mass matrices at the Grand Unified Theories (GUTs) as well as the M_Z scale.

- Essentially, this approach, in order to avoid fine tuning of the elements of the mass matrices, involves reproducing the hierarchical nature of the mixing angles by constraining the parameter space available to the elements of the mass matrices.

- This concept of naturalness can be incorporated on mass matrices by considering the following hierarchy for the elements of the matrices [Gupta *et. al.*,IJMPA 29,144405 (2014)]

$$(1,i) \lesssim (2,j) \lesssim (3,3)\,; \quad i=1,2,3\,, \quad j=2,3\,.$$

Some Essential Details pertaining to Natural Mass Matrices

Consider the CKM matrix in the Wolfenstein parametrization

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The above matrix can approximately be written as

$$V_{\rm CKM} = \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix} .$$

To translate the constraints of the above matrix on the mass matrices, we consider a basis in which either of the two mass matrices M_U and M_D is diagonal. For example, in case we choose M_D to be diagonal and

$$M_U = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} ,$$

Keeping in mind the relation $M_U = V_{\rm ckm}^\dagger M_U^{\rm diag} V_{\rm ckm}$ and using above equations, one gets

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix}^\dagger \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \times \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix}.$$

Since $m_u << m_c << m_t$, one obtains

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \sim \begin{pmatrix} m_c\lambda^2 & -m_c\lambda & 0.6m_t\lambda^3 \\ -m_c\lambda & m_c & -0.8m_t\lambda^3 \\ 0.6m_t\lambda^3 & -0.8m_t\lambda^3 & m_t \end{pmatrix}.$$

It is interesting to note that the elements (1,3) and (3,1) in the above matrix are larger than the (1,2) and (2,1) elements. However, we still have the freedom of WB transformations. Noting that WB transformations do not affect the hierarchy of elements of mass matrices, one can eliminate the (1,3) and (3,1) elements in the above matrix. In the matrix so obtained, one can easily see that the elements of the mass matrices satisfy the natural hierarchy,

$$(1,i) \lesssim (2,j) \lesssim (3,3)\,; \quad i = 1,2,3\,, \quad j = 2,3\,.$$

Issues associated with the three approaches

Texture Specific Mass matrices

- Despite showing considerable promise, in this approach the possibility to arrive at a unique set of viable quark mass matrices emerges only by carrying out an exhaustive case by case analysis of all possible texture zero mass matrices.

Weak Basis Transformations

- Lead to a large number of viable texture zero matrices due to unconstrained parameter space of their elements, therefore, restricting the use of such mass matrices for providing vital clues for developing viable theories of flavor physics.

Natural Mass matrices

- This idea constrains the elements of the mass matrices, yet it again does not yield a finite set of viable mass matrices.

Relationship between mass matrices and mixing matrices

A general mass matrix, M_k where k = U, D can be expressed as

$$M_k = Q_k M_k{}^r P_k \quad .$$

The real matrix $M_k{}^r$ is given by

$$M_k{}^r = Q_k{}^+ M_k \; P_k{}^+$$

This matrix $M_k{}^r$ can be diagonalized by orthogonal transformation,

$$M_k{}^{diag} = (O_k{}^+ M_k{}^r O_k)$$

$$= (O_k{}^+ P_k M_k P_k^r O_k)$$

where $$M_k{}^{diag} = diag(m_1, -m_2, m_3)$$

The mixing matrix, in terms of the matrices used for diagonalizing the mass matrices is expressed as

$$V_{CKM} = O_U^T P_U P_U^+ O_D$$

Viability of mass matrices through mixing matrices

- The larger CKM picture remains intact. Shown by us (Gupta et al., IJMPA 2011) that the NP effects in the CKM paradigm if at all these are present, then they are not more than 10%.
- Viability of mass matrices is ensured by constructing CKM matrix and then examining its compatibility with the CKM matrix available in the literature.
- Usually, one compares with the CKM matrix given by PDG, UTfit, HFAG or CKMFitter, however these groups arrive at a matrix by invoking global fits which include inputs related to several parameters.
- Since the mass matrices are constructed with the assumptions of strict unitarity, therefore it is desirable to examine the viability of the corresponding CKM matrix with a unitarity based CKM matrix calculated using minimal input.
- Further, as there is a significant discrepancy between the exclusive and inclusive values of V_{ub}, therefore desirable to finds its unitarity driven value.

Mixing matrices and unitarity

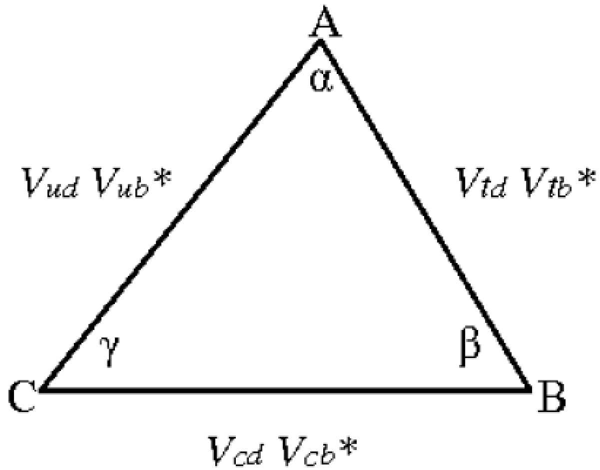
- Firstly, we would like to discuss the impact of a very precisely known Sin2β along with unitarity on the CKM element V_{ub} and the CP violating phase δ. [Gupta et al.,Phys. Lett B647 (2007) 394].

- The unitarity of the CKM matrix and the unitarity triangles have played an important role in establishing the CKM paradigm as well as the fact that a single CP violating phase δ is largely responsible for understanding the CP violation in the K and B sector.

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- The unitarity of CKM matrix implies nine relations (3 diagonal & 6 non diagonal). The non diagonal ones can be expressed as six unitarity triangles in the complex plane.

The `db' unitarity triangle



$$\alpha \equiv \arg\left[\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] = \tan^{-1}\left[\frac{s_{12}s_{23}\sin\delta}{c_{12}c_{23}s_{13} - s_{12}s_{23}\cos\delta}\right]$$

The angles of the 'db' triangle (derived in PDG parameterization)

$$\beta \equiv \arg\left[\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = \tan^{-1}\left[\frac{c_{12}s_{12}s_{13}\sin\delta}{c_{23}s_{23}(s_{12}^2 - c_{12}^2 s_{13}^2) - c_{12}s_{12}s_{13}(c_{23}^2 - s_{23}^2)\cos\delta}\right]$$

$$\gamma \equiv \arg\left[\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] = \tan^{-1}\left[\frac{s_{12}c_{23}\sin\delta}{c_{12}s_{23}s_{13} + s_{12}c_{23}\cos\delta}\right]$$

Using above equations, one can find the relationship between the CP violating phase δ and the experimentally well determined angle β

$$\tan\frac{\delta}{2} = \frac{A - \sqrt{A^2 - (B^2 - A^2C^2)\tan^2\beta}}{(B + AC)\tan\beta}$$

$$A = c_{12}s_{12}s_{13}, \quad B = c_{23}s_{23}(s_{12}{}^2 - c_{12}{}^2 s_{13}{}^2), \quad C = c_{23}{}^2 - s_{23}{}^2$$

Re-expressed as

$$\delta = -\beta + \sin^{-1}\left(\frac{s_{12}s_{23}}{c_{12}s_{13}}\sin\beta\right)$$

Also written as

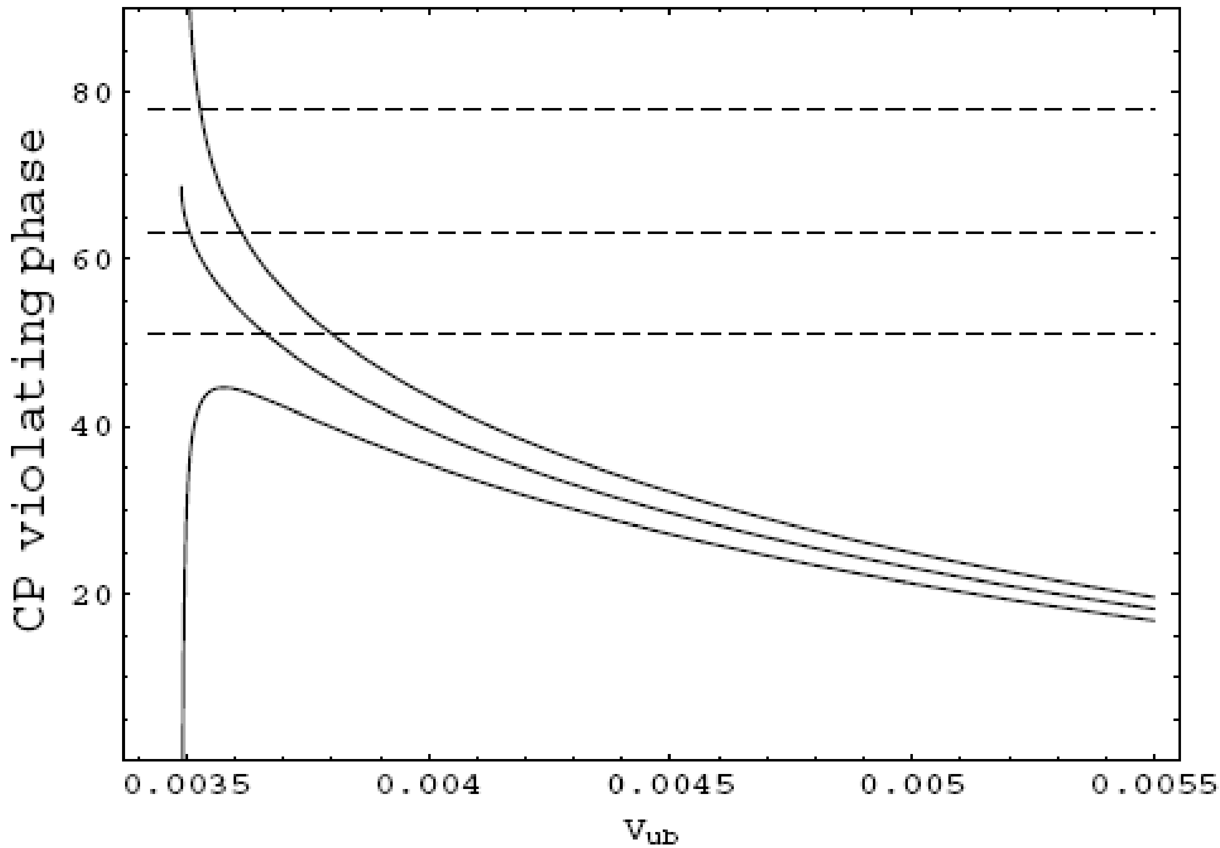
$$\frac{\sin(\delta+\beta)}{\sin\beta} = \frac{s_{12}s_{23}}{c_{12}s_{13}}$$

Using $\alpha+\beta+\gamma = \pi$, rewritten as

$$s_{13} = \frac{s_{12}s_{23}\sin\beta}{c_{12}\sin\alpha}$$

Lower bound on s_{13}

$$s_{13} \geq \frac{s_{12}s_{23}\sin\beta}{c_{12}}$$



V_{ub} versus CP violating phase δ

Figure reveals several interesting points:

For $V_{ub} > 0.00355$, δ shows a smooth decline and range gets narrower.

For $V_{ub} < 0.00355$, sharp broadening of δ.

1 σ range of inclusive Vub restricts δ to 23°-39° .

Conclusions sharpened further using unitarity based relations:

Yields lower bound $V_{ub} \geq 0.0035$.

Obtained value of $V_{ub} = 0.0035 \pm 0.0002$ ▶ a rigorous prediction of unitarity which is quite independent of the variation (20%) of α ▶ agrees with latest exclusive V_{ub} ▶ brings out so called 'tension' faced by inclusive V_{ub}.

Closure property of triangles yield $\delta = 70.8^\circ \pm 6.1^\circ$.

Using above mentioned δ and V_{ub}, along with precisely measured Vus and V_{cb}, the CKM matrix has been constructed

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.9738\text{–}0.9747 & 0.2236\text{–}0.2274 & 0.0033\text{–}0.0037 \\ 0.2234\text{–}0.2273 & 0.9729\text{–}0.9739 & 0.0401\text{–}0.0423 \\ 0.0068\text{–}0.0103 & 0.0390\text{–}0.0417 & 0.9991\text{–}0.9992 \end{pmatrix}.$$

Unique texture for quark mass matrices

Inputs pertaining to the analysis

- The quark masses and mass ratios at M_Z scale are

$$m_u = 1.38^{+0.42}_{-0.41}\,\text{MeV}, \quad m_d = 2.82 \pm 0.48\,\text{MeV}, \quad m_s = 57^{+18}_{-12}\,\text{MeV},$$

$$m_c = 0.638^{+0.043}_{-0.084}\,\text{GeV}, \; m_b = 2.86^{+0.16}_{-0.06}\,\text{GeV}, \; m_t = 172.1 \pm 1.2\,\text{GeV},$$
$$m_u/m_d = 0.553 \pm 0.043, m_s/m_d = 18.9 \pm 0.8.$$

- The latest values of quark mixing parameters are:

$$|V_{us}| = 0.22534 \pm 0.00065, \; |V_{ub}| = 0.00351^{+0.00015}_{-0.00014}, \; |V_{cb}| = 0.0412^{+0.0011}_{-0.0005},$$

$$\text{Sin}2\beta = 0.679 \pm 0.020.$$

- The parameters related to the phases of the mass matrices have been given full variation from 0 to 2π. The free parameters E_U, E_D, D_U and D_D have also been given wide variation in conformity with the condition of naturalness as well as to ensure that the elements of orthogonal diagonalizing transformations should remain real.

Essentials of the analysis

- Starting with the most general mass matrices we have made an attempt to explore the possibility of obtaining a finite set of viable texture specific mass matrices formulated by invoking weak basis transformations as well as the constraints imposed due to naturalness.

- Following Hermitian mass matrices can be considered to be the most general ones

$$M_q = \begin{pmatrix} E_q & A_q & F_q \\ A_q^* & D_q & B_U \\ F_q^* & B_q^* & C_q \end{pmatrix} \qquad (q = U, D),$$

- As a next step, to incorporate the concept of WB transformations one can introduce texture zeros in these matrices using a unitary matrix W

$$M_U = \begin{pmatrix} E_U & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} E_D & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}.$$

The above matrices can be characterized as texture 2 zero quark mass matrices.

- The matrix W can be a permutation matrix P giving rise to other possible structures for M_U and M_D wherein instead of zeros being in the (1,3) and (3,1) positions, these could be in either the (1,2) and (2,1) or (2,3) and (3,2) position.
- These different mass matrices, however, yield the same Cabibbo-Kobayashi-Maskawa (CKM) matrix, therefore while presenting the results of the analysis, it is sufficient to discuss any one of these matrices.
- Further, in order to incorporate the condition of `naturalness' on these mass matrices, one can consider the following hierarchy for the elements of the matrices

$$(1,i) < (2,j) \lesssim (3,3); \qquad i = 1,\ 2,\ 3, \quad j = 2,\ 3.$$

Numerical analysis

- Using the relation between mass matrices and mixing matrix, the resultant CKM matrix comes out to be

$$V_{CKM} = \begin{pmatrix} 0.9739-0.9745 & 0.2246-0.2259 & 0.00337-0.00365 \\ 0.2224-0.2259 & 0.9730-0.9990 & 0.0408-0.0422 \\ 0.0076-0.0101 & 0.0408-0.0422 & 0.9990-0.9999 \end{pmatrix}$$

- Fully compatible with the unitarity based CKM matrix as well as with the one given by PDG.

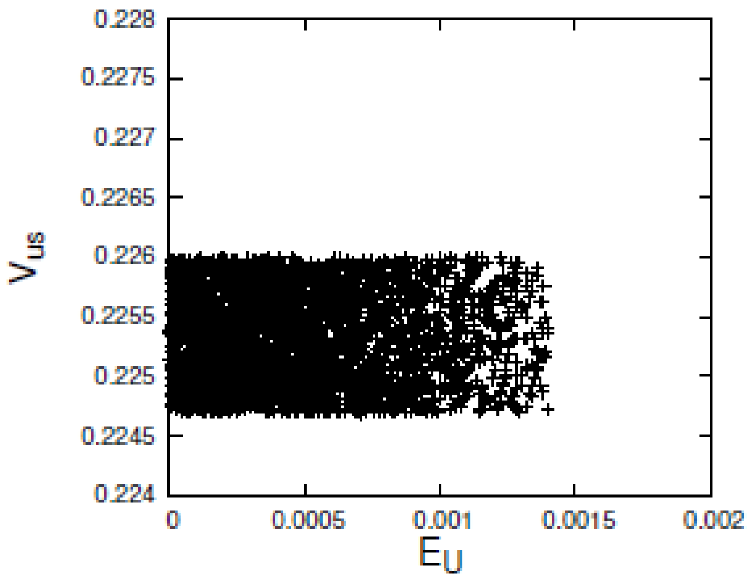
- Interesting to examine the parameter space available to various elements of matrices M_U and M_D

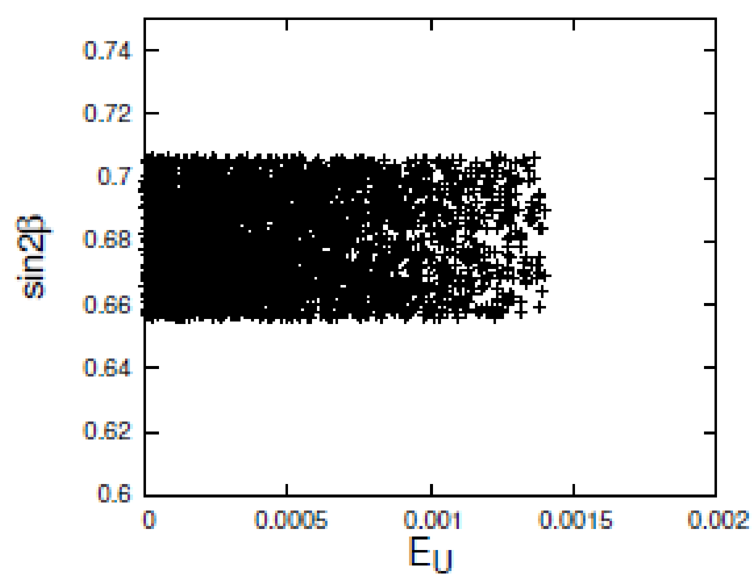
$$M_U = \begin{pmatrix} 0-0.00138 & 0.006-0.042 & 0 \\ 0.006-0.042 & 26.46-102.68 & 62.82-86.10 \\ 0 & 62.82-86.10 & 68.78-145.00 \end{pmatrix} \mathrm{GeV},$$

$$M_D = \begin{pmatrix} 0-0.00127 & 0.011-0.019 & 0 \\ 0.011-0.019 & 0.36-1.66 & 1.03-1.44 \\ 0 & 1.03-1.44 & 1.16-2.44 \end{pmatrix} \mathrm{GeV}.$$

- Curiously, the above matrices reveal that their (1,1) element (E_U, E_D) is quite small in comparison with the other non zero elements.

- Can be confirmed using the following figures





- The parameter E_U assumes quite small values, < 0.0014 GeV. Also both V_{us} and Sin2β seem independent of the range of E_U, indicating the redundancy of element E_U.

- Similar conclusions can be drawn from E_D versus the CKM matrix elements plots.

- Ignoring the elements E_U and E_D of the mass matrices, one gets M_U and M_D as

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}$$

- Carrying out a similar analysis for these matrices, the corresponding CKM matrix comes out to be

$$\mathrm{V_{CKM}} = \begin{pmatrix} 0.9741-0.9744 & 0.2246-0.2259 & 0.00337-0.00365 \\ 0.2245-0.2258 & 0.9732-0.9736 & 0.0407-0.0422 \\ 0.0071-0.0100 & 0.0396-0.0417 & 0.9990-0.9992 \end{pmatrix}.$$

- Interesting to examine how the parameter space of the elements of the mass matrices gets changed on going from texture 2 to texture 4 zero. To this end, reconstructing M_U and M_D we get

$$M_U = \begin{pmatrix} 0 & 0.031-0.041 & 0 \\ 0.031-0.041 & 13.73-98.62 & 47.70-85.80 \\ 0 & 47.70-85.80 & 72.84-157.73 \end{pmatrix} \mathrm{GeV},$$

$$M_D = \begin{pmatrix} 0 & 0.012-0.018 & 0 \\ 0.012-0.018 & 0.18-1.56 & 0.81-1.45 \\ 0 & 0.81-1.45 & 1.24-2.61 \end{pmatrix} \mathrm{GeV}.$$

- Similar to the case of texture 2 zero mass matrices, using the WB transformations, apart from the matrices given above, one gets several other possible texture 4 zero mass matrices which may or may not be related through permutations.

- Based on whether the matrices are related through permutations or not, all possible texture 4 zero mass matrices can be classified as shown in Table. The matrices which are not related to each other through permutations have been put into different categories.

	a	b	c	d	e	f
Category 1	$\begin{pmatrix} \mathbf{0} & A & \mathbf{0} \\ A^* & D & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & A \\ \mathbf{0} & C & B \\ A^* & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & A & B \\ A^* & \mathbf{0} & \mathbf{0} \\ B^* & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & A \\ \mathbf{0} & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} D & B & A \\ B^* & C & \mathbf{0} \\ A^* & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & \mathbf{0} & A \\ B^* & A^* & D \end{pmatrix}$
Category 2	$\begin{pmatrix} D & A & \mathbf{0} \\ A^* & \mathbf{0} & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & A \\ \mathbf{0} & C & B \\ A^* & B^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & A & B \\ A^* & D & \mathbf{0} \\ B & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & \mathbf{0} & A \\ \mathbf{0} & A^* & D \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & D & A \\ B^* & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & A \\ B^* & C & \mathbf{0} \\ A^* & \mathbf{0} & D \end{pmatrix}$
Category 3	$\begin{pmatrix} \mathbf{0} & A & D \\ A^* & \mathbf{0} & B \\ D^* & B^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & D & A \\ D^* & C & B \\ A^* & B & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & A & B \\ A^* & \mathbf{0} & D \\ B^* & D^* & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & C \\ B^* & \mathbf{0} & A \\ C^* & A^* & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & C & B \\ C^* & D & A \\ B^* & A^* & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & B & A \\ B^* & \mathbf{0} & C \\ A^* & C^* & D \end{pmatrix}$
Category 4	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D & B \\ \mathbf{0} & B^* & C \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & B \\ \mathbf{0} & A & \mathbf{0} \\ B^* & \mathbf{0} & D \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C & B \\ \mathbf{0} & B^* & D \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & B \\ \mathbf{0} & A & \mathbf{0} \\ B^* & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} C & B & \mathbf{0} \\ B^* & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$

- For the matrices belonging to category 1, considering both M_U and M_D as 1a type, we have already shown that these are viable and explain the quark mixing data quite well. The other matrices of this category, related through permutation matrix, also yield similar results.

- For the matrices belonging to category 4, one finds that interestingly these are not viable as in all these matrices one of the generations gets decoupled from the other two.

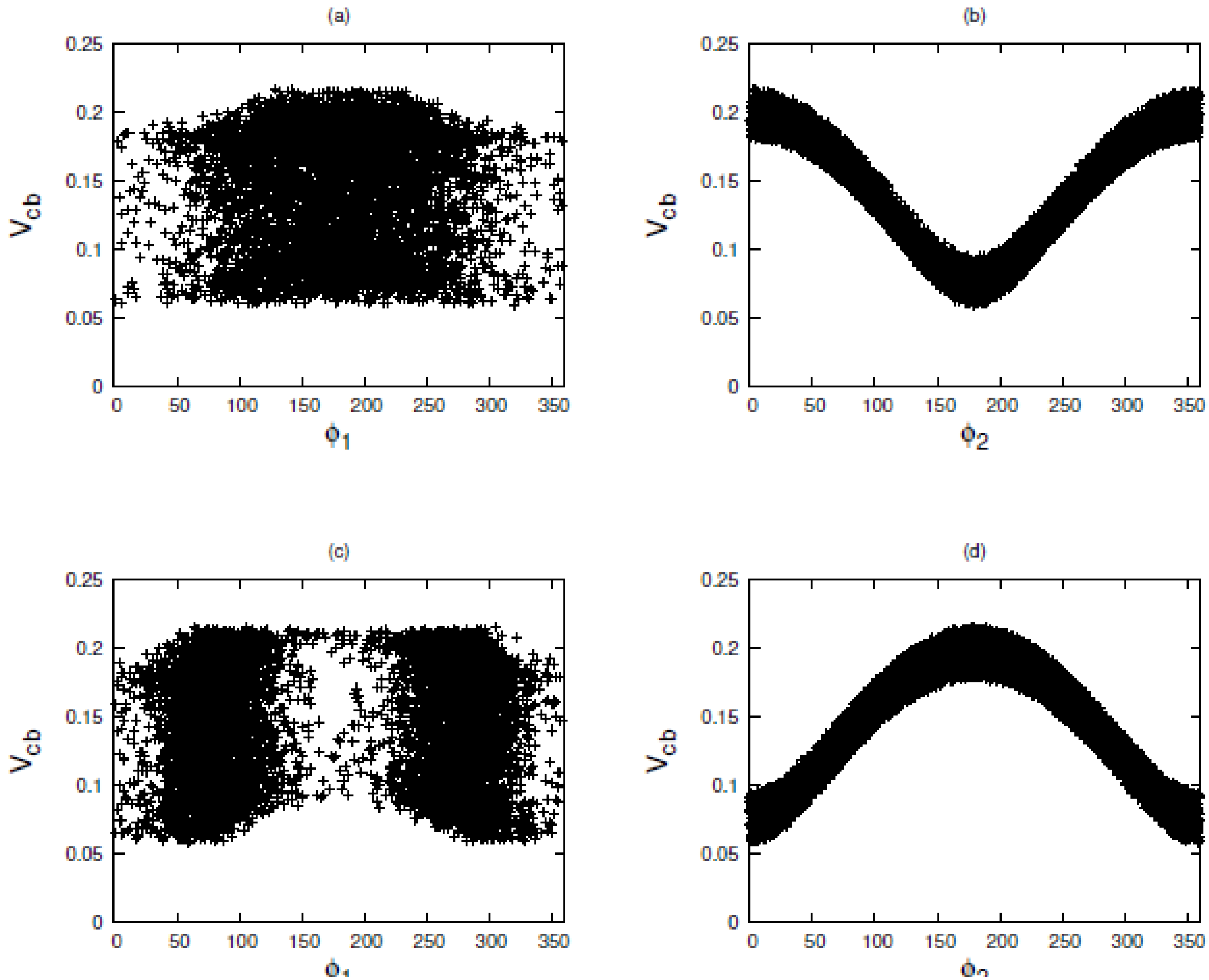
- Further, for categories 2 and 3, again a similar analysis reveals that the matrices of these classes are also not viable as can be understood from the following CKM matrices obtained for categories 2 and 3 respectively, e.g.,

$$V_{CKM} = \begin{pmatrix} 0.9740-0.9744 & 0.2247-0.2260 & 0.0024-0.0099 \\ 0.2205-0.2256 & 0.9509-0.9727 & 0.0596-0.2172 \\ 0.0140-0.0445 & 0.0584-0.2127 & 0.9905-1.0000 \end{pmatrix}$$

$$V_{CKM} = \begin{pmatrix} 0.9736-0.9744 & 0.2247-0.2260 & 0.0098-0.0331 \\ 0.2226-0.2278 & 0.9549-0.9719 & 0.0659-0.1937 \\ 0.00007-0.0340 & 0.0694-0.1928 & 0.9810-0.9976 \end{pmatrix}$$

- These matrices show no compatibility with the latest quark mixing data.

- Can be further verified from graphs plotted for categories 2 and 3 respectively.



- The plotted values of element V_{cb} have no overlap with its experimental range, therefore, these matrices can be considered to be non viable.

- The above discussion clearly brings out that only the texture 4 zero quark mass matrices belonging to category 1 of the table are found to be viable. Interestingly, the matrices considered here are quite similar to the original Fritzsch ansatze, except for their (2,2) element being non zero for both M_U and M_D.

- In case one considers more than 4 texture zero mass matrices, we find that the present data rules out texture 5 and 6 zero quark mass matrices, confirming our earlier conclusions in this regard.

- In conclusion, we would like to state that texture 4 zero quark mass matrices, similar to the original Fritzsch ansatze, and its permutations seem to be a unique finite set of quark mass matrices which are in agreement with the present mixing data.

Texture specific lepton mass matrices

- In line with our treatment of quark mass matrices, using the facility of WB transformations, one can obtain texture specific mass matrices in the lepton sector as well without any loss of generality, viz.

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \qquad (1)$$

  where M_l and $M_{\nu D}$ correspond to the charged lepton and the Dirac neutrino mass matrices respectively.

- For the case of Majorana neutrinos, the effective neutrino mass matrix is obtained by the famous Seesaw mechanism

$$M_\nu = -M_{\nu D}^T M_R^{-1} M_{\nu D}, \qquad (2)$$

  where M_R corresponds to the right handed Majorana neutrino mass matrix.

- It is well known that texture five zero as well as texture four zero mass matrices are able to accommodate the lepton mixing data quite well. However, it is interesting to note if one obtain some constraints on the unknown parameters in the neutrino sector using these most general mass matrices.
- One can carry out diagonalization of the mass matrices given in eqn. (1) by expressing it as

$$M_k = Q_k M_k^r P_k , \quad k = l, \ \nu D, \tag{3}$$

where Q_k, P_k are diagonal phase matrices given as Diag$(e^{i\alpha_k}, 1, e^{-i\beta_k})$ and Diag $(e^{-i\alpha_k}, 1, e^{i\beta_k})$ respectively and M_k^r is a real symmetric matrix. M_k^r can be diagonalized by an orthogonal transformation O_k, e.g.,

$$M_k^{diag} = O_k^T M_k^r O_k \tag{4}$$

which can be rewritten as

$$M_k^{diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k . \tag{5}$$

- The elements of the general diagonalizing transformation O_k for the mass matrices given in eqn. (1) are

$$\begin{pmatrix} \sqrt{\frac{(E_k-m_1)(D_k+E_k-m_1-m_2)(D_k+E_k-m_1-m_3)}{(D_k+2E_k-m_1-m_2-m_3)(m_1-m_2)(m_1-m_3)}} & \sqrt{\frac{(E_k-m_2)(m_3-C_k)(m_1-C_k)}{(E_k-C_k)(m_1-m_2)(m_3-m_2)}} & \sqrt{\frac{(-C_k+m_1)(-E_k+m_3)(C_k-m_2)}{(m_1-m_3)(m_3-m_2)(C_k-E_k)}} \\ \sqrt{\frac{(m_1-C_k)(m_1-E_k)}{(m_1-m_2)(m_1-m_3)}} & -\sqrt{\frac{(E_k-m_2)(C_k-m_2)}{(m_1-m_2)(m_3-m_2)}} & \sqrt{\frac{(-m_3-C_k)(E_k-m_3)}{(m_1-m_3)(m_3-m_2)}} \\ -\sqrt{\frac{(E_k-m_2)(E_k-m_3)(m_1-C_k)}{(m_1-m_2)(m_1-m_3)(E_k-C_k)}} & \sqrt{\frac{(-E_k+m_1)(C_k-m_2)(E_k-m_3)}{(m_1-m_2)(m_2-m_3)(E_k-C_k)}} & \sqrt{\frac{(E_k-m_1)(E_k-m_2)(m_3-C_k)}{(C_k-E_k)(m_1-m_3)(m_3-m_2)}} \end{pmatrix}$$

with m_1, $-m_2$, m_3 being the eigen values of M_k.

- In the case of charged leptons, because of the hierarchy $m_e \ll m_\mu \ll m_\tau$, the mass eigenstates can be approximated respectively to the flavor eigenstates. In this approximation, $m_{l1} \simeq m_e$, $m_{l2} \simeq m_\mu$ and $m_{l3} \simeq m_\tau$, one can obtain the elements of the diagonalizing matrix O_l from the equation (3), by replacing m_1, m_2, m_3 by m_e, $-m_\mu$, m_τ.
- The diagonalizing transformation for Majorana neutrinos, assuming normal hierarchy, defined as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$ as well as the corresponding degenerate case defined as $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$, can be obtained by replacing m_1, m_2, m_3 by $\sqrt{m_{\nu 1} m_R}$, $\sqrt{m_{\nu 2} m_R}$, $\sqrt{m_{\nu 3} m_R}$, where m_{ν_1}, m_{ν_2} and m_{ν_3} are neutrino masses.
- Here m_R represents the eigenvalue of the right handed Majorana neutrino mass matrix, which for the purpose of present work has been chosen to be of the form

$$M_R = m_R \,.\, Diag(1,1,1), \tag{6}$$

however our conclusion remain unaltered even on considering a more general form for M_R.

- In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case, defined as $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$, by replacing m_1, m_2, m_3 in equation (3) with $\sqrt{m_{\nu_1} m_R}$, $-\sqrt{m_{\nu_2} m_R}$, $-\sqrt{m_{\nu_3} m_R}$.
- The lepton mixing matrix, the Pontecorvo Maki Nakagawa Sakata (PMNS) matrix, in terms of the matrices used for diagonalizing the mass matrices M_l and M_ν is expressed as
$$U = (Q_l O_l \xi_l)^\dagger (P_{\nu D} O_{\nu D}), \tag{7}$$
wherein, to facilitate the construction of diagonalizing transformations for different neutrino mass hierarchies, we have introduced ξ_k defined as $\mathrm{diag}(1, e^{i\pi}, 1)$ for the case of normal hierarchy and as $\mathrm{diag}(1, e^{i\pi}, e^{i\pi})$ for the case of inverted hierarchy.
- Eliminating the phase matrix ξ_l by redefinition of the charged lepton phases, the above equation becomes
$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}\,, \tag{8}$$
where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1}, 1, e^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices as $\phi_1 = \alpha_{\nu D} - \alpha_l$, $\phi_2 = \beta_{\nu D} - \beta_l$ and can be treated as free parameters.

Inputs used for the analysis

Parameter	3σ range
Δm^2_{sol} $[10^{-5} eV^2]$	(6.99-8.18)
Δm^2_{atm} $[10^{-3} eV^2]$	(2.19-2.62)(NH); (2.17-2.61)(IH)
$sin^2\theta_{13}$ $[10^{-2}]$	(1.69-3.13)(NH); (1.71-3.15) (IH)
$sin^2\theta_{12}$ $[10^{-1}]$	(2.59-3.59)
$sin^2\theta_{23}$ $[10^{-1}]$	(3.31-6.37)(NH);(3.35-6.63)(IH)

- While carrying out our analysis, the magnitudes of atmospheric and solar neutrino mass square differences, defined as $m_2^2 - m_1^2$ and $m_3^2 - \frac{(m_1^2+m_2^2)}{2}$ respectively, are allowed full variation within their 3σ ranges. The lightest neutrino mass, m_1 for the case of NH and m_3 for the case of IH, is considered as the free parameter while the other two masses are obtained using the following relations,

$$NH: \quad m_2^2 = \Delta m^2_{sol} + m_1^2, \quad m_3^2 = \Delta m^2_{atm} + \frac{(m_1^2 + m_2^2)}{2}, \tag{9}$$

$$IH: \quad m_2^2 = \frac{2(m_3^2 + \Delta m^2_{atm}) + \Delta m^2_{sol}}{2}, \quad m_1^2 = \frac{2(m_3^2 + \Delta m^2_{atm}) - \Delta m^2_{sol}}{2}. \tag{10}$$

- Further, the phases ϕ_1, ϕ_2 and the elements $D_{l,\nu}$, $C_{l,\nu}$ are considered to be free parameters. For all the three possible mass hierarchies of neutrinos, the explored range of the lightest neutrino mass is taken to be $10^{-8}\,\mathrm{eV} - 10^{-1}\,\mathrm{eV}$, our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π.
- Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real, ensuring the mass matrices to be 'natural' as advocated by Peccei and Wang.
- The solar as well as atmospheric neutrino mass squared differences have been varied randomly within their 3σ experimental ranges. The phases ϕ_1 and ϕ_2 have also been varied randomly within the interval $[0,\ 2\pi]$. To facilitate the calculations as well as to find the parameter space available to various parameters, we have resorted to Monte Carlo simulations of various input parameters.
- Following the methodology discussed above, we find that interesting bounds can be obtained for the the parameter m_{ee}, the effective Majorana mass of the electron neutrino, which determines the rate of NDBD and is given as

$$|m_{ee}| = |m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2|. \tag{11}$$

Inverted hierarchy of neutrino masses

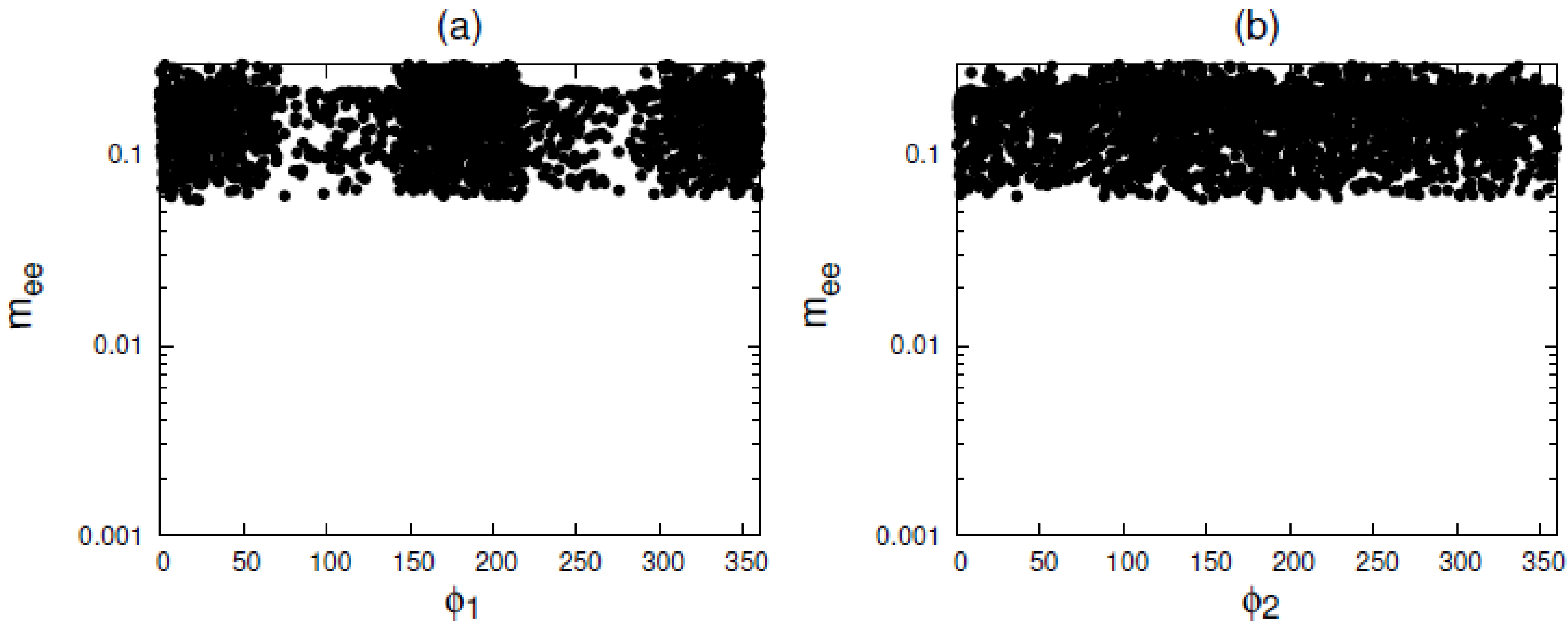


Figure: Plots showing the parameter space corresponding to m_{ee} versus (a) ϕ_1 and (b) ϕ_2 for texture two zero mass matrices (inverted hierarchy).

- It can be seen from the above figure that the parameter m_{ee} hardly shows any dependence on the phases ϕ_1 and ϕ_2, however an interesting lower bound of the order of 0.08 eV can be obtained. This bound is well within the range of m_{ee} likely to be explored by the the forthcoming experiments aiming to find a signal for NDBD.

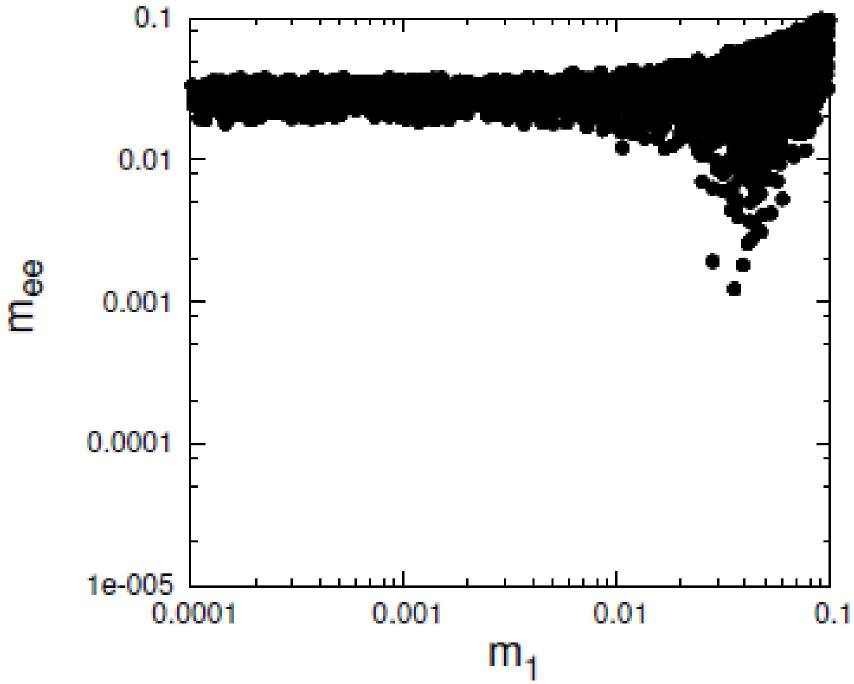


Figure: Plot showing the parameter space corresponding to m_{ee} and the lightest neutrino mass for texture two zero mass matrices (normal hierarchy).

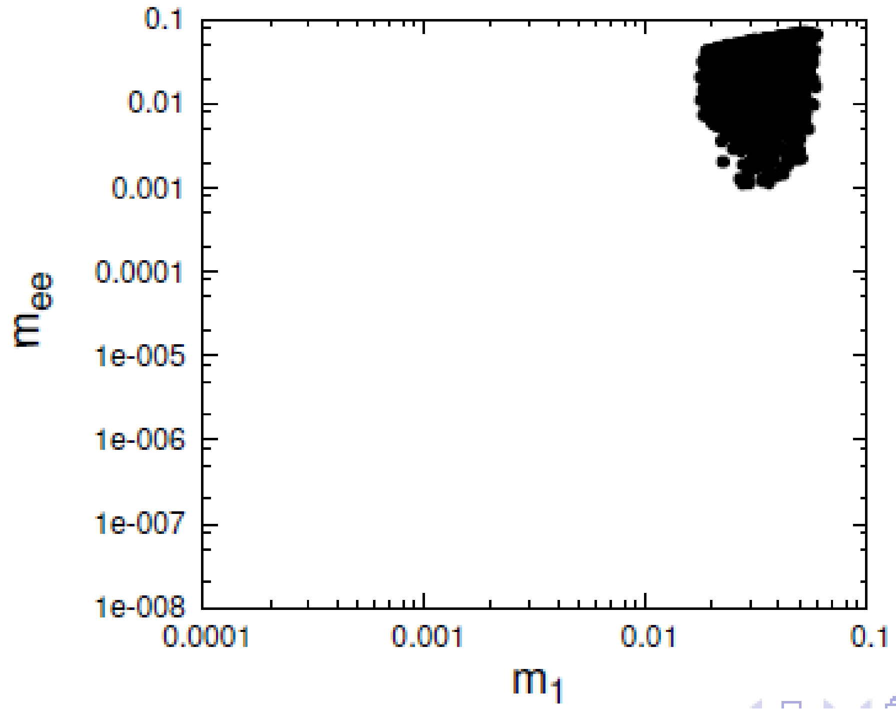
- Contrary to the observations in the IH case, here one finds that the parameter m_{ee} shows considerable variation with the lightest neutrino mass. In particular, one notices that larger values of m_{ee} are allowed for the smaller values of m_1 and vice versa.

Texture four zero lepton mass matrices

- Keeping in mind the quark lepton unification, as advocated by Smirnov as well as required by most of the grand unified theories, it becomes interesting to investigate the implications of similar type of mass matrices, e.g.

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix} \tag{12}$$

for the parameter m_{ee} and different neutrino mass hierarchies.



SO(10) inspired texture specific mass matrices

- SO(10) GUT is probably the best motivated candidate for the unification of strong and electroweak interactions.
- It unifies the family of fermions: includes $SU(4)_c$ quark-lepton symmetry and left-right (LR) symmetry.
- It includes the right handed neutrinos and through the Seesaw mechanism offers an appealing explanation for the smallness of neutrino masses.

Symmetry breaking and fermion mass matrices in SO(10)

- Since SO(10) is a rank 5 group and SM gauge group has rank 4, so there are different possible chains of symmetry breaking.
- Chain 1: SO(10) —>SU(5) x U(1) —>SU(5) —> G_{std} (ruled out by experiments)
- Chain 2 : SO(10) —> $SU(4)_c x SU(2)_L x SU(2)_R (G_{PS})$
  —> $SU(3)_c\ SU(2)_L\ SU(2)_R\ U(1)_{B-L}\ (G_{LR})$
  —> G_{std}

- Since the fermions of each generation belong to the 16 dimensional spinor representation of SO(10), the Higgs scalars which can couple to the fermions are contained in the product 16 x 16.

- Since $16 \times 16 = 10_S + 120_A + 126_S$ the fermion masses are generated when the Higgs fields of 10, 120, 126 dimensional representations of SO(10) develop non-vanishing VEVs.

- 16 = (4,2,1) + (4,1,2) and (4,2,1) (4, 1, 2) = (15, 2, 2) + (1, 2, 2) , the Dirac masses for quarks and leptons are generated when neutral components in a (1,2,2) multiplet in 10, (1,2,2) and (15,2,2) in 120 and (15,2,2) in 126 dimensional representations acquire non vanishing VEVs.

- The (10,3,1) and (10,1,3) components in 126 dimensional Higgs break the $SU(2)_L$ and $SU(2)_R$ symmetries and hence are responsible for the left and right handed Majorana neutrino masses.

- Depending upon the different choices of Higgs, numerous models of SO(10) can be constructed leading to various forms of fermion mass matrices.

- However, it can be shown that the most general mass matrices in any renormalizable SO(10) model can be given by the relations:

$$M_d = H + F + iG$$

$$M_u = r(H + sF + it_u G)$$

$$M_l = H - 3F + it_l G$$

$$M_D = r(H - 3sF + it_D G)$$

$$M_\nu = r_L F - r_R M_D F^{-1} M_D^T$$

[Grimus and Kuhbock,hep-ph/0607197]
[Joshipura et al,0903.2161]

- M_d, M_u, M_l, M_D, M_ν stand for the down, up, charged lepton, dirac and the effective neutrino mass matrices respectively.

- Here (G), H, F are complex (anti)symmetric matrices corresponding to 120, 10, 126 dimensional Higgs respectively and r, s, t_l ,t_u ,t_D , r_L ,r_R are dimensionless complex parameters out of which r, r_L , r_R can be chosen to be real without loss of generality.

- The Yukawa sector in SO(10) involves a large number of free parameters : 6 in H; 6 in F; 3 in G, along with r, s, t_u, t_e, t_D, r_L and r_R.

- This implies a total of 22 such parameters for type-II seesaw and 21 parameters for type-I seesaw.

- Only 18 experimentally observable parameters namely 6 quark masses, 3 quark mixing angles, 1 CP-violating phase in CKM matrix, 3 charged lepton masses, 3 lepton mixing angles and 2 neutrino mass square differences.

- The parameter space of the SO (10) Yukawa sector can be significantly reduced by using phenomenological structures for these fermion mass matrices.

- The "texture zero" approach involving Hermitian mass matrices has been very promising, in particular, as shown earlier, texture 4 zero mass matrices explain the recent precision experimental data for the quark and lepton mixings independently at the electro weak scale.

- Motivated by this idea, we impose the texture 2-zero structure on the fundamental Yukawa coupling matrices H, F and G in SO (10). This ensures texture 2-zero Hermitian Fritzsch-like structures for the mass matrices M_u, M_d, M_e and M_D while M_L and M_R remain texture 2-zero Fritzsch-like real symmetric mass matrices, and these may, in general, be described by the following forms:

$$M_u = \begin{bmatrix} 0 & a_u e^{i\alpha_u} & 0 \\ a_u e^{-i\alpha_u} & d_u & b_u e^{i\beta_u} \\ 0 & b_u e^{-i\beta_u} & c_u \end{bmatrix},\ M_d = \begin{bmatrix} 0 & a_d e^{i\alpha_d} & 0 \\ a_d e^{-i\alpha_d} & d_d & b_d e^{i\beta_d} \\ 0 & b_d e^{-i\beta_d} & c_d \end{bmatrix},\ M_e = \begin{bmatrix} 0 & a_e e^{i\alpha_e} & 0 \\ a_e e^{-i\alpha_e} & d_e & b_e e^{i\beta_e} \\ 0 & b_e e^{-i\beta_e} & c_e \end{bmatrix}$$

$$M_D = \begin{bmatrix} 0 & a_D e^{i\alpha_D} & 0 \\ a_D e^{-i\alpha_D} & d_D & b_D e^{i\beta_D} \\ 0 & b_D e^{-i\beta_D} & c_D \end{bmatrix},\ M_L = \begin{bmatrix} 0 & a_L & 0 \\ a_L & d_L & b_L \\ 0 & b_L & c_L \end{bmatrix},\ M_R = \begin{bmatrix} 0 & a_R & 0 \\ a_R & d_R & b_R \\ 0 & b_R & c_R \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & a_H & 0 \\ a_H & d_H & b_H \\ 0 & b_H & c_H \end{bmatrix} ; F = \begin{bmatrix} 0 & a_F & 0 \\ a_F & d_F & b_F \\ 0 & b_F & c_F \end{bmatrix} ; G = \begin{bmatrix} 0 & i\,a_G & 0 \\ -i\,a_G & 0 & i\,b_G \\ 0 & -i\,b_G & 0 \end{bmatrix}$$

$$r\,(1-s)\,a_e \mathrm{Cos}\,\alpha_e = 4\,a_u \mathrm{Cos}\,\alpha_u - (3+s)\,r\,a_d \mathrm{Cos}\,\alpha_d$$

$$r\,(1-s)\,b_e \mathrm{Cos}\,\beta_e = 4\,b_u \mathrm{Cos}\,\beta_u - (3+s)\,r\,b_d \mathrm{Cos}\,\beta_d$$

$$r\,(1-s)\,d_e = 4\,d_u - (3+s)\,r\,d_d$$

$$r\,(1-s)\,c_e = 4\,c_u - (3+s)\,r\,c_d$$

$$a_D \mathrm{Cos}\,\alpha_D = a_u \mathrm{Cos}\,\alpha_u - r\,s\,(a_d \mathrm{Cos}\,\alpha_d - a_e \mathrm{Cos}\,\alpha_e)$$

$$b_D \mathrm{Cos}\,\beta_D = b_u \mathrm{Cos}\,\beta_u - r\,s\,(b_d \mathrm{Cos}\,\beta_d - b_e \mathrm{Cos}\,\beta_e)$$

$$d_D = d_u - r\,s\,(d_d - d_e)$$

$$c_D = c_u - r\, s\, (c_d - c_e)$$

$$a_e \, \mathrm{Sin}\, \alpha_e = t_e a_d \, \mathrm{Sin}\, \alpha_d$$

$$b_e \, \mathrm{Sin}\, \beta_e = t_e b_d \, \mathrm{Sin}\, \beta_d$$

$$a_u \, \mathrm{Sin}\, \alpha_u = r\, t_u a_d \, \mathrm{Sin}\, \alpha_d$$

$$b_u \, \mathrm{Sin}\, \beta_u = r\, t_u b_d \, \mathrm{Sin}\, \beta_d$$

$$a_D \, \mathrm{Sin}\, \alpha_D = r\, t_D a_d \, \mathrm{Sin}\, \alpha_d$$

$$b_D \, \mathrm{Sin}\, \beta_D = r\, t_D b_d \, \mathrm{Sin}\, \beta_d$$

The GUT ($M_X = 2 \times 10^{16}$ GeV) scale values of the masses and mixing parameters used either as inputs or as constraints in the calculations.

$$m_d = 1.14^{+0.51}_{-0.48} \text{ MeV} \;;\; m_s = 22^{+7}_{-6} \text{ MeV} \;;\; m_b = 1.00^{+0.04}_{-0.04} \text{ GeV}$$

$$m_u = 0.48^{+0.20}_{-0.17} \text{ MeV} \;;\; m_c = 0.235^{+0.035}_{-0.034} \text{ GeV} \;;\; m_t = 74.0^{+4.0}_{-3.7} \text{ GeV}$$

$$m_e = 0.469652046 \pm 0.000000041 \text{ MeV} \;;\; m_\mu = 99.1466226 \pm 0.0000089 \text{ MeV}$$

$$m_\tau = 1.68558 \pm 0.00019 \text{ GeV} ; \Delta m_{12}^2 = (6.99 - 8.18)\times 10^{-5} \text{eV}^2 ; \Delta m_{13}^2 = (2.06 - 2.67)\times 10^{-3} \text{eV}^2$$

$$\sin^2 \theta_{12}^l = 0.265\text{-}0.364 ; \sin^2 \theta_{13}^l = 0.005\text{-}0.05 ; \sin^2 \theta_{23}^l = 0.34\text{-}0.64$$

$$\sin \theta_{12}^q = 0.2235\text{-}0.2274 ; \sin \theta_{13}^q = 0.00300\text{-}0.00455 ; \sin \theta_{23}^q = 0.0401\text{-}0.0433$$

$$|V_{us}| = 0.2234\text{-}0.2274 ; |V_{ub}| = 0.003\text{-}0.00455 ; |V_{cb}| = 0.0404\text{-}0.0433 ; \text{Sin } 2\beta = 0.656\text{-}0.706$$

Results of the analysis

$$M_u = \begin{bmatrix} 0 & (0.0094-0.0157)e^{i(2-178)} & 0 \\ (0.0094-0.0157)e^{-i(2-178)} & 13-30 & (28.4129-36.4736)e^{i(-0.2-0.03)} \\ 0 & (28.4129-36.4736)e^{-i(-0.2-0.03)} & 43.7653-60.7657 \end{bmatrix} \text{GeV}$$

$$M_e = \begin{bmatrix} 0 & (0.0076-0.0089)e^{i(47.1-97.65)} & 0 \\ (0.0076-0.0089)e^{-i(47.1-97.65)} & 0.25-0.61 & (0.7075-0.8963)e^{i(25.17-31.75)} \\ 0 & (0.7075-0.8963)e^{-i(25.17-31.75)} & 0.9769-1.3369 \end{bmatrix} \text{GeV}$$

$$M_d = \begin{bmatrix} 0 & (0.0042-0.0073)e^{i(-106-89)} & 0 \\ (0.0042-0.0073)e^{-i(-106-89)} & 0.14-0.36 & (0.3685-0.4919)e^{i(-5.12-(-1.99))} \\ 0 & (0.3685-0.4919)e^{-i(-5.12-(-1.99))} & 00.6219-0.8421 \end{bmatrix} \text{GeV}$$

$$
M_D = \begin{bmatrix} 0 & \left(-1.14-(-1.87)\right)e^{i(82.5-87.2)} & 0 \\ \left(-1.14-(-1.87)\right)e^{-i(82.5-87.2)} & -72.68-(-31.57) & \left(-89..83-(-62.67)\right)e^{i(1.24-8.46)} \\ 0 & \left(-89..83-(-62.67)\right)e^{-i(1.24-8.46)} & -149.21-(-91.44) \end{bmatrix} \mathrm{GeV}
$$

$$
M_\nu = \begin{bmatrix} 0 & \left(5.22-8.95\right)\times 10^{-12} & 0 \\ \left(5.22-8.95\right)\times 10^{-12} & \left(0.4426-7.371\right)\times 10^{-12} e^{i(-89.9-89.9)} & \left(0.2664-1.911\right)\times 10^{-11} e^{i(-89.9-89.9)} \\ 0 & \left(0.2664-1.911\right)\times 10^{-11} e^{i(-89.9-89.9)} & \left(3.921-5.137\right)\times 10^{-11} \end{bmatrix} \mathrm{GeV}
$$

$$
H = \begin{bmatrix} 0 & \left(-1.382-5.788\right)\times 10^{-3} & 0 \\ \left(-1.382-5.788\right)\times 10^{-3} & 0.1675-0.4225 & 0.4261-0.5662 \\ 0 & 0.4261-0.5662 & 0.7107-0.9658 \end{bmatrix} \mathrm{GeV}
$$

$$F = \begin{bmatrix} 0 & (-0.6396 - 1.1615)\times 10^{-4} & 0 \\ (-0.6396 - 1.1615)\times 10^{-4} & -0.0625 - (-0.0275) & -0.0746 - (-0.0585) \\ 0 & -0.0746 - (-0.0585) & -0.1249 - (-0.0880) \end{bmatrix} \text{GeV}$$

$$M_R = \begin{bmatrix} 0 & (-1.651 - 4.5289)\times 10^{11} & 0 \\ (-1.651 - 4.5289)\times 10^{11} & (-1.875 - (-0.909))\times 10^{14} & (-2.802 - (-1.548))\times 10^{14} \\ 0 & (-2.802 - (-1.548))\times 10^{14} & (-4.995 - (-1.833))\times 10^{14} \end{bmatrix} \text{GeV}$$

$$V_{CKM} = \begin{bmatrix} 0.9738 - 0.9747 & 0.2235 - 0.2274 & 0.003 - 0.00454 \\ 0.2234 - 0.2273 & 0.9729 - 0.9739 & 0.0404 - 0.0431 \\ 0.0061 - 0.0117 & 0.03917 - 0.0427 & 0.9991 - 0.9992 \end{bmatrix}$$

$$\text{Sin } 2\beta = 0.656\text{-}0.706 \; ; \; J_q = (2.089 - 4.035)\times 10^{-5}$$

$$V_{PMNS} = \begin{bmatrix} 0.7778-0.8549 & 0.5020-0.5991 & 0.0708-0.2236 \\ 0.4217-0.5688 & 0.4901-0.6877 & 0.5708-0.7457 \\ 0.1984-0.3578 & 0.4770-0.7069 & 0.6311-0.8095 \end{bmatrix}$$

$$s_{12}^2 = 0.265-0.364 \; ; \; s_{13}^2 = 0.005-0.05 \; ; \; s_{23}^2 = 0.34-0.58 \; ;$$

$$\Delta m_{12}^2 = (6.99-8.18)\times 10^{-23}\,\mathrm{GeV} \; ; \; \Delta m_{13}^2 = (2.06-2.67)\times 10^{-21}\,\mathrm{GeV} \; ;$$

$$m_{\upsilon 1} = (3.04\text{-}6.76)\times 10^{-12}\,\mathrm{GeV} \; ; \; m_{\upsilon 2} = (0.89\text{-}1.12)\times 10^{-11}\,\mathrm{GeV} \; ; \; m_{\upsilon 3} = (4.55\text{-}5.21)\times 10^{-11}\,\mathrm{GeV}$$

$$\delta_l = -87.16-71.31 \; ; \; J_l = -0.0428-0.0353$$

Summary and conclusions

Clues towards unique textures for quarks

- Our unitarity based analysis shows that the larger CKM picture remains intact and in case the NP effects are there, these are at the level of only a few percent.

- A precise value of sin2β along with other well known CKM matrix elements and unitarity allows one to find a precisely known CKM matrix and CP violating phase.

- Starting with the most general mass matrices, using the concept of weak basis transformations, one first obtains texture 2 zero quark mass matrices. Analysis of these matrices, carried out by incorporating the naturalness condition, reveals that certain elements are essentially redundant, therefore can be discarded, reducing the matrices to texture 4 zero type.

- Numerical analysis of all the texture 4 zero mass matrices, related through WB transformations, leads to a particular texture for the quark mass matrices which seems to be the unique viable option. This unique texture for quarks could be the first step towards unified textures for all fermions.

Texture specific lepton mass matrices

- To summarize, we have carried out a detailed analysis of the most general lepton mass matrices within the framework of SM, in order to examine their predictions for the effective Majorana mass in the neutrinoless double beta decay m_{ee} as well as the lightest neutrino mass pertaining to normal as well as inverted hierarchy of neutrino masses.
- We find that for the inverted hierarchy of neutrino masses a lower bound of approximately 0.08 eV for m_{ee}. In case the forthcoming experiments do not find a signal for NDBD, the inverted hierarchy scenario for the neutrino masses would be directly ruled out.
- Even for the normal hierarchy case, the non observation of NDBD would put severe constraints on the lightest neutrino mass. In case, the experiments find a signal for NDBD, then also we are likely to establish the neutrino mass hierarchy as the ranges for m_{ee} obtained for normal and inverted hierarchies overlap for a very narrow window of the lightest neutrino mass.
- In conclusion, we would like to state that currently we are in an exciting era of neutrino oscillation phenomenology aided by the series of experiments aiming to measure m_{ee} and the lightest neutrino mass. The bounds obtained in the present work would not only act as milestones for model builders, but would also provide motivation for the experimentists to push the sensitivity of the measurements further.

SO(10) inspired texture specific mass matrices

- Interestingly, the texture 4 zero fermion mass matrices are compatible with the SO(10) mass matrices with and without super symmetry.
- However, the parameter space available gets highly restricted with severe constraints on the phase structure of the mass matrices. A measurement of the leptonic CP violating phase would have deep implications for the formulation of mass matrices within GUTs.
- SO(10) constraints allow natural hierarchy for the quark sector whereas the same need not be there in the case of leptons.

Finally, we would like to state that the compatibility of texture specific mass matrices with SO(10) constraints motivates one to integrate the present approach with the horizontal / Abelian symmetries to have a comprehensive description of fermion masses and mixings.

Thank You