Dispersive Approach to Hadronic Light-by-Light Scattering and the Muon $g-2$

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1. Introduction
2. Standard Model vs. Experiment
3. Dispersive Approach to HLbL Scattering
4. Conclusion and Outlook

1 Introduction
- The Anomalous Magnetic Moment of the Muon
- Status of Theory and Experiment

2 Standard Model vs. Experiment

3 Dispersive Approach to HLbL Scattering

4 Conclusion and Outlook

Magnetic moment

- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio
- Dirac theory predicted $g_e = 2$, twice the value of the gyromagnetic ratio for orbital angular momentum
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$

Anomalous magnetic moments: a bit of history

- 1928, Dirac: $g_e = 2$
- 1934, Kinster & Houston: experimental confirmation (large errors)
- 1948, Kusch & Foley, hyperfine structure of atomic spectra: $g_e = 2.00238(10) \Rightarrow a_e = 0.00119(5)$
- 1948, Schwinger: $a_\ell = \alpha_{\mathrm{QED}}/(2\pi) \approx 0.00116$
- helped to establish QED and QFT as the framework for elementary particle physics

Anomalous magnetic moments: a bit of history

- 1957, Lee & Yang: parity violation
  $\Rightarrow \pi^+ \to \mu^+ \nu_\mu$ produces polarised muons
- 1957: g_μ through spin precession experiments
- 1960, Columbia: $a_\mu = 0.00122(8)$
- 1961, CERN: establishing muon as a ‘heavy electron’
- 1969-1976, CERN muon storage ring: 7 ppm
- 2000-2004, BNL E821: 0.54 ppm
- probing not only QED but entire SM

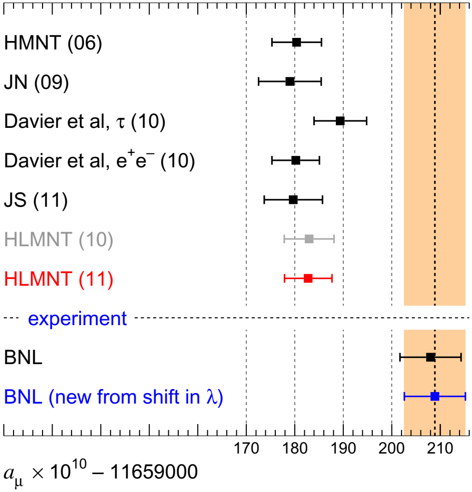
Electron vs. muon magnetic moments

- influence of heavier virtual particles of mass M scales as

$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

- a_e used to determine α_{QED}
- $(m_\mu/m_e)^2 \approx 4 \cdot 10^4 \Rightarrow$ muon is much more sensitive to new physics, but also to EW and hadronic contributions
- a_τ experimentally not yet known precisely enough

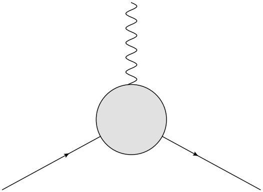
a_μ: comparison of theory and experiment



→ Hagiwara et al. 2012

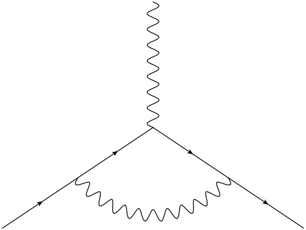
1. Introduction
2. Standard Model vs. Experiment
   - QED Contribution
   - Electroweak Contribution
   - Hadronic Vacuum Polarisation
   - Hadronic Light-by-Light Scattering
   - Summary and Prospects
3. Dispersive Approach to HLbL Scattering
4. Conclusion and Outlook

Interaction of a muon with an external electromagnetic field



Anomalous magnetic moment given by one particular form factor

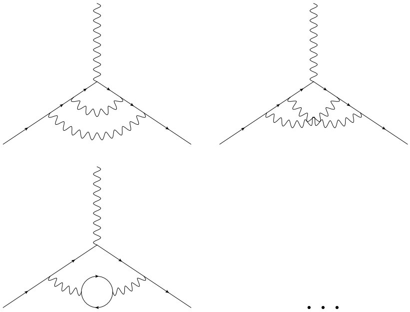
QED at $\mathcal{O}(\alpha)$



Schwinger term:

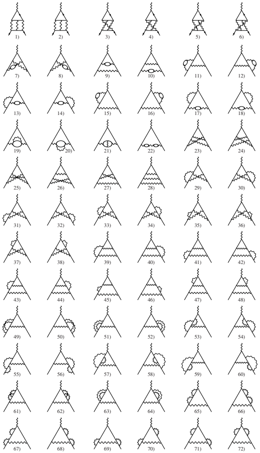
$$a_\mu = \frac{\alpha_{\text{QED}}}{2\pi}$$

QED at $\mathcal{O}(\alpha^2)$



- 7 diagrams
- Petermann and Sommerfeld 1957: universal part
- full calculation 1966

QED at $\mathcal{O}(\alpha^3)$



- 72 diagrams
- Remiddi et al. 1969: universal part
- Laporta, Remiddi 1996: full calculation

→ figure taken from Jegerlehner 2007

QED at $\mathcal{O}(\alpha^4)$ and $\mathcal{O}(\alpha^5)$

- complete numerical calculation of $\mathcal{O}(\alpha^4)$ contribution by Kinoshita et al. (took about 30 years)
- full $\mathcal{O}(\alpha^5)$ calculation by Kinoshita et al. 2012 (involves 12672 diagrams!)

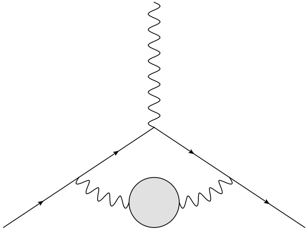
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.95$	0.08
Theory total	$116\,591\,855$	59

Electroweak

- contributions with EW gauge bosons and Higgs
- calculated to two loops
- three-loop terms estimated to be negligible

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
EW	153.6	1.0
Theory total	$116\,591\,855$	59

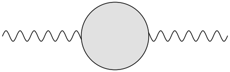
Leading hadronic contribution: $\mathcal{O}(\alpha^2)$



- problem: QCD is non-perturbative at low energies
- first principle calculations (lattice QCD) may become competitive in the future
- current evaluations based on dispersion relations and data

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$

Photon hadronic vacuum polarisation function:



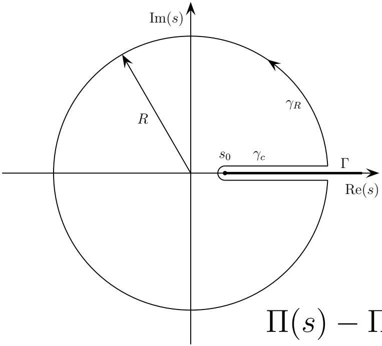
$$= -i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma_{\mathrm{tot}}(e^+e^- \to \gamma^* \to \mathrm{hadrons})$$

Dispersion relation

Causality implies analyticity:



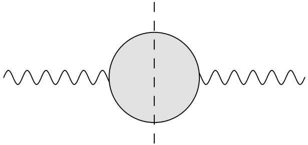
Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i}\oint_\gamma \frac{\Pi(s')}{s'-s}ds'$$

Deform integration path:

$$\Pi(s) - \Pi(0) = \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s'-s-i\epsilon)s'}ds'$$

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$



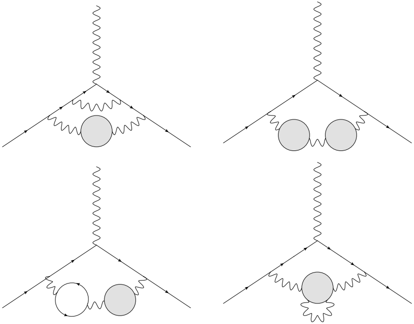
- basic principles: unitarity and analyticity
- direct relation to experiment: total hadronic cross section $\sigma_{\mathrm{tot}}(e^+e^- \to \gamma^* \to \mathrm{hadrons})$
- one Lorentz structure, one kinematic variable

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$

- at present: dominant theoretical uncertainty
- theory error due to experimental input
- can be systematically improved: dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE2, SND)

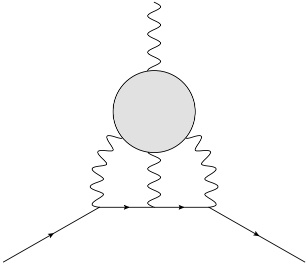
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
LO HVP	$6\,949$	43
Theory total	$116\,591\,855$	59

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$



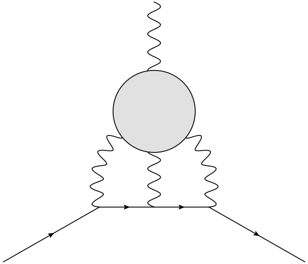
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
NLO HVP	-98	1
Theory total	$116\,591\,855$	59

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$ Hadronic light-by-light (HLbL) scattering



- hadronic matrix element of four EM currents
- up to now, only model calculations
- lattice QCD not yet competitive

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$
Hadronic light-by-light (HLbL) scattering



- uncertainty estimate based rather on consensus than on a systematic method
- will dominate theory error in a few years
- "dispersive treatment impossible"

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$	
BNL E821	$116\,592\,091$	63	→ PDG 2013
QED $\mathcal{O}(\alpha)$	$116\,140\,973.32$	0.08	
QED $\mathcal{O}(\alpha^2)$	$413\,217.63$	0.01	
QED $\mathcal{O}(\alpha^3)$	$30\,141.90$	0.00	
QED $\mathcal{O}(\alpha^4)$	381.01	0.02	
QED $\mathcal{O}(\alpha^5)$	5.09	0.01	
QED total	$116\,584\,718.95$	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
Hadronic total	6982	59	
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QED total	$116\,584\,718.95$	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
LO HVP	$6\,949$	43	→ Hagiwara et al. 2011
NLO HVP	-98	1	→ Hagiwara et al. 2011
NNLO HVP	12.4	0.1	→ Kurz et al. 2014
LO HLbL	116	40	→ Jegerlehner, Nyffeler 2009
NLO HLbL	3	2	→ Colangelo et al. 2014
Hadronic total	6982	59	
Theory total	$116\,591\,855$	59	

a_μ: theory vs. experiment

- theory error completely dominated by hadronic effects
- discrepancy between Standard Model and experiment $\sim 3\sigma$
- hint to new physics?

Future experiments

- current experimental determination is limited by statistical error
- new experiments aim at reducing the experimental error by a factor of 4
- FNAL: reuses BNL storage ring
- J-PARC: completely different systematics

BNL storage ring moves to FNAL

June 22, 2013

→ credit: Brookhaven National Laboratory

BNL storage ring moves to FNAL

→ photo: Darin Clifton/Ceres Barge

BNL storage ring moves to FNAL

July 26, 2013

→ credit: Fermilab

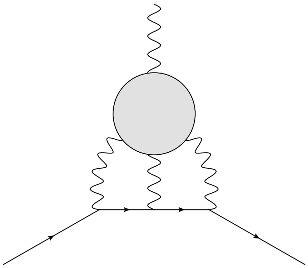
1 Introduction

2 Standard Model vs. Experiment

3 Dispersive Approach to HLbL Scattering
- Lorentz Structure of the HLbL Tensor
- Mandelstam Representation

4 Conclusion and Outlook

How to improve HLbL calculation?



- "dispersive treatment impossible": no!
- relate HLbL to experimentally accessible quantities
- make use of unitarity, analyticity, gauge invariance and crossing symmetry

The HLbL tensor

- object in question: $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$
- a priori 138 Lorentz structures
- gauge invariance: 95 linear relations
  $\Rightarrow$ (off-shell) basis: 43 independent structures
- six dynamical variables, e.g. two Mandelstam variables

  $$s = (q_1 + q_2)^2, \quad t = (q_1 + q_3)^2$$

  and the photon virtualities q_1^2, q_2^2, q_3^2, q_4^2
- complicated analytic structure

HLbL tensor: Lorentz decomposition

Problem: find a decomposition

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_i T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

with the following properties:

- Lorentz structures $T_i^{\mu\nu\lambda\sigma}$ manifestly gauge invariant
- scalar functions Π_i free of kinematic singularities and zeros

HLbL tensor: Lorentz decomposition

Recipe by Bardeen, Tung (1968) and Tarrach (1975):

- apply gauge projectors to the 138 initial structures:

$$I_{12}^{\mu\nu} = g^{\mu\nu} - \frac{q_2^\mu q_1^\nu}{q_1 \cdot q_2}, \quad I_{34}^{\lambda\sigma} = g^{\lambda\sigma} - \frac{q_4^\lambda q_3^\sigma}{q_3 \cdot q_4}$$

- remove poles taking appropriate linear combinations
- Tarrach: no kinematic-free basis of 43 elements exists
- extend basis by additional structures taking care of remaining kinematic singularities

HLbL tensor: Lorentz decomposition

Solution for the Lorentz decomposition:

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

- Lorentz structures manifestly gauge invariant
- crossing symmetry manifest
- scalar functions Π_i free of kinematics
  $\Rightarrow$ ideal quantities for a dispersive treatment

Master formula: contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HLbL}} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p + q_1)^2 - m_\mu^2][(p - q_2)^2 - m_\mu^2]}$$

- $\hat{T}_i$: known integration kernel functions
- $\hat{\Pi}_i$: linear combinations of the scalar functions Π_i
- five loop integrals can be performed with Gegenbauer polynomial techniques

Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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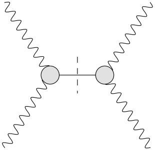
one-pion intermediate state: 

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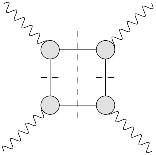
two-pion intermediate state in both channels:



Mandelstam representation

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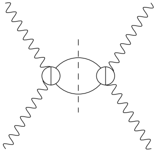
two-pion intermediate state in first channel: 

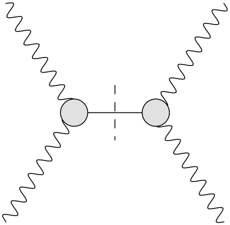
Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

neglected: higher intermediate states

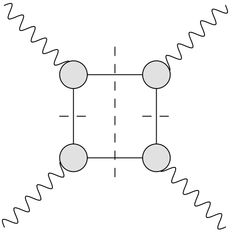
Pion pole



- input the doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- dispersive analysis of transition form factor:

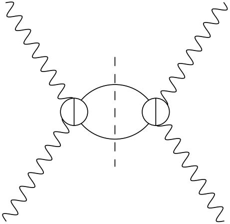
  $\rightarrow$ Hoferichter et al., arXiv:1410.4691 [hep-ph]

Box contributions



- simultaneous two-pion cuts in two channels
- analytic properties correspond to sQED loop
- q^2-dependence given by multiplication with pion vector form factor $F_\pi^V(q^2)$ for each off-shell photon ($\Rightarrow$ 'FsQED')

Rescattering contribution



- multi-particle intermediate states in crossed channel approximated by polynomial
- two-pion cut in only one channel
- expand into partial waves
- unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$

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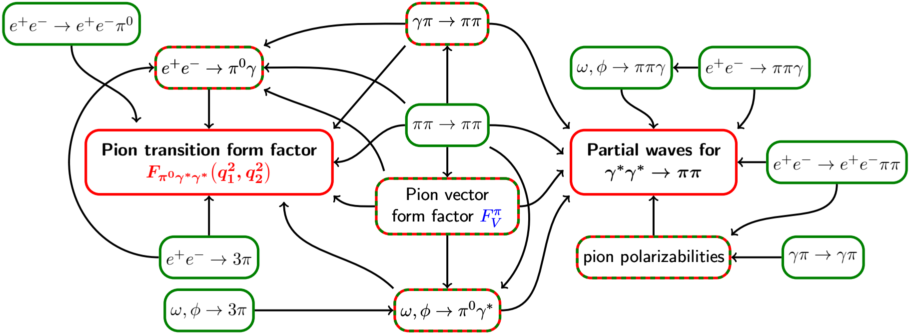
Summary

- our dispersive approach to HLbL scattering is based on fundamental principles:
  - gauge invariance, crossing symmetry
  - unitarity, analyticity
- we take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- a step towards a model-independent calculation of a_μ
- numerical evaluation is work in progress

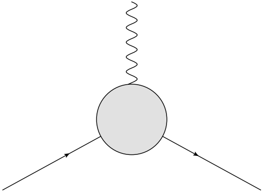
A roadmap for HLbL



$\to$ Flowchart by M. Hoferichter

Backup

Interaction of a muon with an external electromagnetic field



$$= (-ie)\langle \mu^-(p_2, s_2)|j^\mu_{\rm em}(0)|\mu^-(p_1, s_1)\rangle$$
$$= (-ie)\bar{u}(p_2)\Gamma^\mu(p_1, p_2)u(p_1)$$

$\Gamma^\mu(p_1, p_2)$: vertex function

Form factors of the vertex function

Lorentz decomposition:

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_E(k^2) - i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_M(k^2) - \frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5 F_D(k^2) + \left(\gamma^\mu + \frac{2mk^\mu}{k^2}\right)\gamma_5 F_A(k^2)$$

Form factors depend only on $k^2 = (p_1 - p_2)^2$.

Form factors of the vertex function

Lorentz decomposition:

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Form factors depend only on $k^2 = (p_1 - p_2)^2$.

electric charge or Dirac form factor, $F_E(0) = 1$

Form factors of the vertex function

Lorentz decomposition:

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_E(k^2) - i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_M(k^2) - \frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5 F_D(k^2) + \left(\gamma^\mu + \frac{2mk^\mu}{k^2}\right)\gamma_5 F_A(k^2)$$

Form factors depend only on $k^2 = (p_1 - p_2)^2$.

magnetic or Pauli form factor, $F_M(0) = a_\mu$

Form factors of the vertex function

Lorentz decomposition:

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_E(k^2) - i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_M(k^2) - \frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5 F_D(k^2) + \left(\gamma^\mu + \frac{2mk^\mu}{k^2}\right)\gamma_5 F_A(k^2)$$

Form factors depend only on $k^2 = (p_1 - p_2)^2$.

electric dipole form factor, $F_D(0)$ gives the CP-violating EDM

Form factors of the vertex function

Lorentz decomposition:

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_E(k^2) - i\frac{\sigma^{\mu\nu}k_\nu}{2m}F_M(k^2) - \frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5 F_D(k^2) + \left(\gamma^\mu + \frac{2mk^\mu}{k^2}\right)\gamma_5 F_A(k^2)$$

Form factors depend only on $k^2 = (p_1 - p_2)^2$.

anapole form factor, P-violating

Model calculations of HLbL

Table 13
Summary of the most recent results for the various contributions to $a_\mu^{\mathrm{LbL;had}} \times 10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

→ Jegerlehner, Nyffeler 2009

- pseudoscalar pole contribution most important
- pion-loop second most important
- differences between models, large uncertainties