Variable Flavor Number Scheme (VFNS) for Final State Jets

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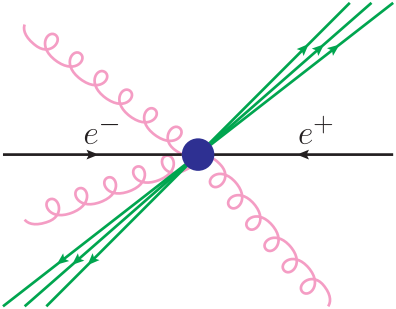
Motivation

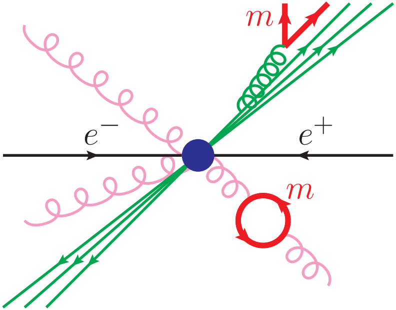
- interesting times for high-energy collider phenomenology
- typical event with QCD radiation (e^+e^--collision):



- nowadays: QCD is precision physics
  $\rightarrow$ extraction of fundamental parameters with small uncertainties
  $\rightarrow$ prediction of background for new physics searches
- at high accuracy: quark mass effects important

Motivation

- aim: incorporation of heavy quarks ($m \gg \Lambda_{\rm QCD}$) in collider processes with jets
  $\leftrightarrow$ missing: systematic treatment of virtual and real **secondary** massive quarks



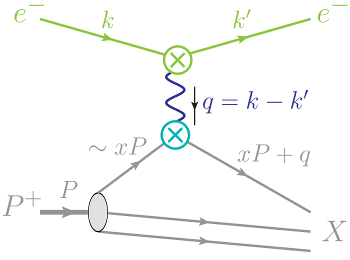
- main emphasis: development of factorization setup

Outline

1. VFNS for DIS in the classical (OPE) region $1 - x \sim \mathcal{O}(1)$
2. VFNS for DIS in the endpoint region $1 - x \ll 1$
3. VFNS for event shapes in the dijet region
4. Summary & Outlook

Outline

Deep inelastic scattering: $e^- P^+ \to e^- X$



- two relevant scales: $q^2 = -Q^2$, $P^2 \sim \Lambda^2_{\mathrm{QCD}}$
- $x = \frac{Q^2}{2P\cdot q}$: $0 \leq x \leq 1$, classical region: $1 - x \sim \mathcal{O}(1)$
- diff. cross section $\frac{d\sigma}{dQ^2 dx} \rightsquigarrow$ structure functions $F_{1,2}$

Factorization in DIS

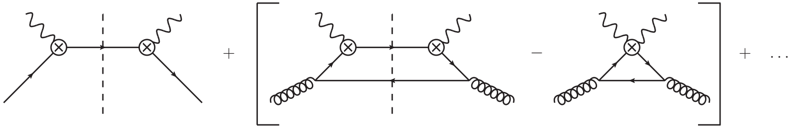
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q} \sum_{j=q,g} H_{ij}(\mu_H) \otimes U^{\Phi}_{jk}(\mu_H, \mu_\Phi) \otimes \Phi_{k/P}(\mu_\Phi) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

Collins, Soper, Sterman (1988), Bauer et al.(2002)

Ingredients:

- hard function $H_{ij}(\mu_H \sim Q)$: difference QCD − SCET (low-energy EFT)



- parton distribution function (PDF) $\Phi_{k/P}(\mu_\Phi \sim \Lambda_{\rm QCD})$: nonperturbative!

$$\Phi_{k/P}(x, \mu_\Phi) = \langle P^+ | \mathcal{O}_k(x, \mu_\Phi) | P^+ \rangle$$

- RG factor $U^{\Phi}_{jk}(\mu_H, \mu_\Phi)$ resums logs $\sim \ln(\frac{\mu_H}{\mu_\Phi})$ (implicit in the following)

Mass effects in DIS

Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q} \sum_{j=q,g} H_{ij}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

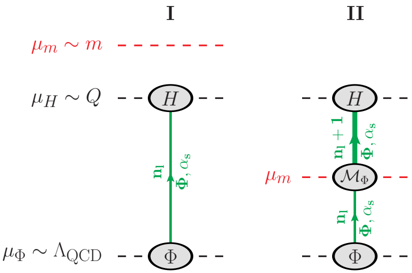
How to incorporate heavy quark mass effects (here: n_l massless + 1 massive flavor)?

$\rightarrow$ resummation of all logarithms $\ln\left(\frac{m}{\Lambda_{\rm QCD}}\right)$, $\ln\left(\frac{Q}{m}\right)$

$\rightarrow$ correct limits for H_{ij} (decoupling for $m \rightarrow \infty$ + massless limit for $m \rightarrow 0$)

$\rightarrow$ continuous description for arbitrary masses

$\Rightarrow$ ACOT scheme `Aivazis, Collins, Olness, Tung (1994)`



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1. VFNS for DIS in the classical (OPE) region $1 - x \sim \mathcal{O}(1)$
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Massless factorization theorem for $x \to 1$

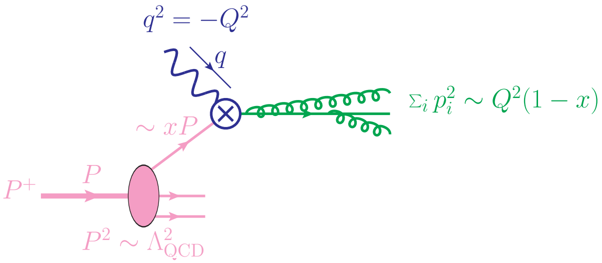
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\mathrm{DIS}}(\mu_H) J_{\mathrm{DIS}}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi) \left[1 + \mathcal{O}(1-x)\right]$$

`Sterman (1987), Manohar (2003), Becher, Neubert, Pecjak (2006), ...`

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\mathrm{DIS}}(\mu_H)$
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\mathrm{DIS}}(\mu_J)$
- at $\mu_\Phi \sim \Lambda_{\mathrm{QCD}}$: PDF $\Phi_{q/P}(\mu_\Phi)$

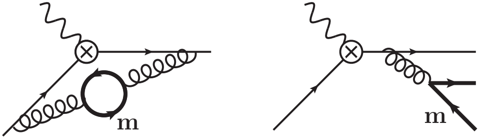


Massive quark effects

Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi) \, [1 + \mathcal{O}(1-x)]$$

- only flavor-diagonal contributions in EFT components
- same for massive quarks
  $\Rightarrow$ only "secondary" massive corrections to light quark inititated processes



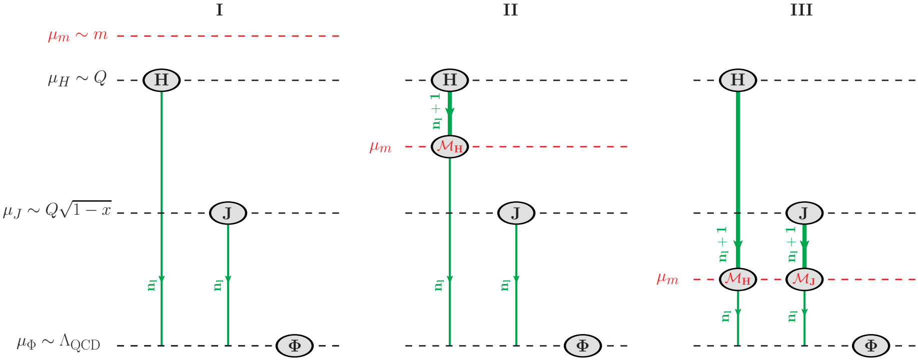
- aim: factorization setup with secondary massive quarks incorporating
  $\rightarrow$ summation of large logarithms
  $\rightarrow$ correct limits for $H_{\rm DIS}$ and $J_{\rm DIS}$
  $\rightarrow$ continuous behavior in between with full singular mass dependence

Mass factorization: Overview

scaling hierarchies for a heavy quark ($m \gg \Lambda_{\rm QCD}$) in the endpoint region ($1 - x \ll 1$):

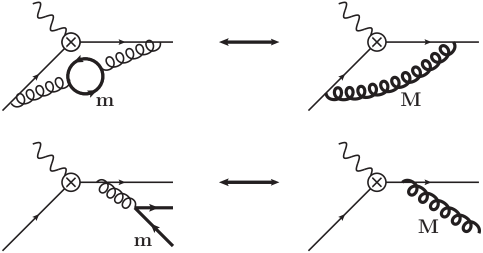
$$\text{I.}\ m > Q\,, \quad \text{II.}\ Q > m > Q\sqrt{1-x}\,, \quad \text{III.}\ Q\sqrt{1-x} > m > \Lambda_{\rm QCD}$$

here: top-down evolution → final renormalization scale $\mu = \mu_\Phi$

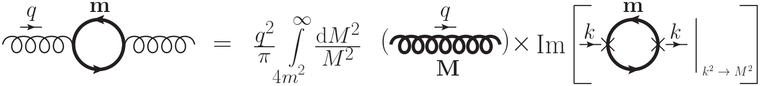


Computation of secondary massive quark effects

- massive quark corrections at $\mathcal{O}(\alpha_s^2 C_F T_F) \leftrightarrow$ "massive gluon" corrections at $\mathcal{O}(\alpha_s)$



- connection: dispersion relations



$\rightarrow$ generic scaling $M \sim m$ is inherited, same field theoretic setup

$\Rightarrow$ hard and jet functions at $\mathcal{O}(\alpha_s^2 C_F T_F)$ $\checkmark$

Massive threshold corrections

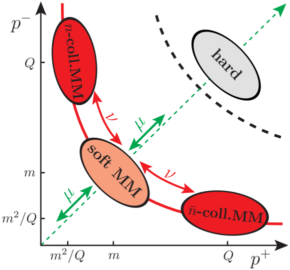
- e.g. threshold correction in jet sector

$$\mathcal{M}_J(s, m, \mu) = J^{\mathrm{OS}}(s, m, \mu) \otimes \left(J^{\overline{\mathrm{MS}}}(s, m, \mu)\right)^{-1}$$

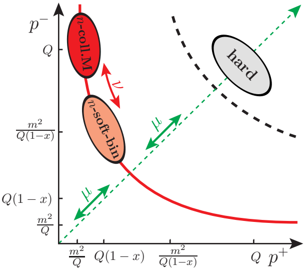
$\rightarrow$ related to virtual massive quark corrections
$\rightarrow$ continuity by construction

- rapidity logarithms

in $\mathcal{M}_H$:
large logarithm $\sim \ln\left(\frac{Q^2}{m^2}\right)$

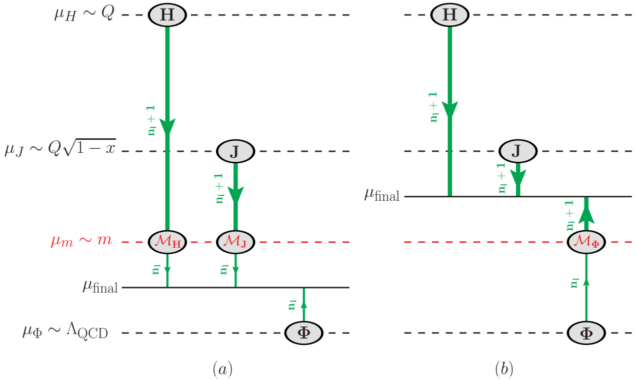


in $\mathcal{M}_J$:
large logarithm $\sim \ln\left(\frac{Q^2(1-x)}{m^2}\right)$



$\rightarrow$ exponentiation (used rapidity RGE: Chiu et al. (2012))

Consistency conditions (for $Q\sqrt{1-x} > m > \Lambda_{QCD}$)



physical cross section independent of μ_{final} $\rightarrow$ (a) and (b) equivalent
$\rightarrow$ relation between evolution factors

$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l + 1$$

$\rightarrow$ relation between massive threshold factors

$$\boxed{\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi}$$

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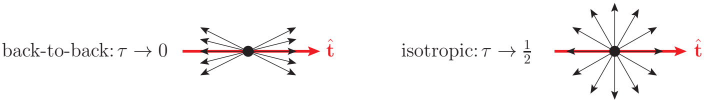
Event shapes

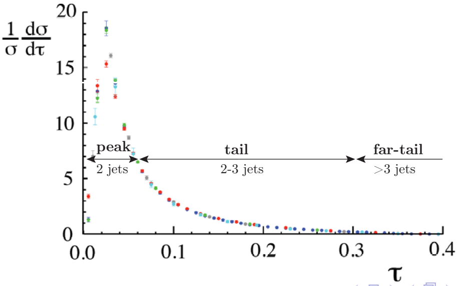
- event shape variables: geometric description of final state kinematics
- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i E_i} \in [0, \frac{1}{2}]$



peak + tail

far-tail

- thrust distribution from LEP data ($e^+e^- \to jets$)



Factorization theorem for massless quarks

Massless factorization theorem for $\tau \ll 1$:

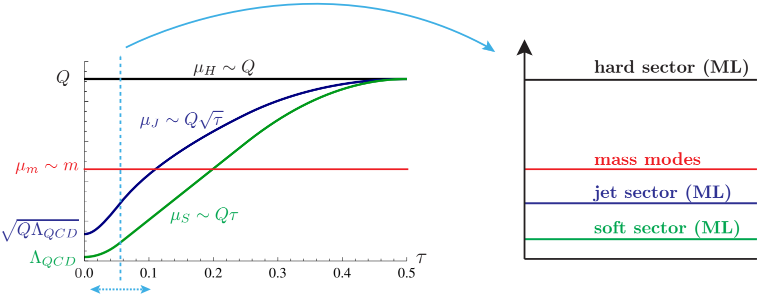
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H_\tau(\mu_H) J_\tau(\mu_J) \otimes S_\tau(\mu_S)\,[1 + \mathcal{O}(\tau)]$$

Berger, Kucs, Sterman (2003), Fleming, Hoang, Mantry, Stewart (2007), Bauer, Fleming, Lee, Sterman (2008),...

- compared to DIS: $H_\tau = H_{\mathrm{DIS}}(Q^2 \to -Q^2)$, $J_\tau \to J_{\mathrm{DIS}} \otimes J_{\mathrm{DIS}}$
- main difference concerns soft physics: $S_\tau \leftrightarrow \Phi_{i/P}$
  $\to$ in tail region ($\tau \gg \Lambda_{\mathrm{QCD}}/Q$): $\mu_S \sim Q\tau \gg \Lambda_{\mathrm{QCD}}$

Scale hierarchies with massive quarks

- profile functions: Parametrization of renormalization scales in terms of thrust $\rightarrow$ continuous transition between peak, tail and far-tail region
- include massive quark effects $\rightarrow$ scales and hierarchies:



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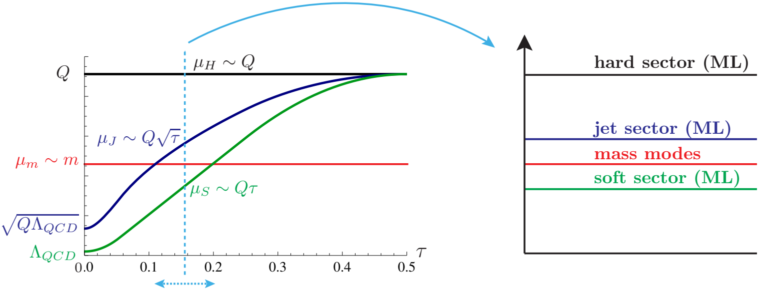

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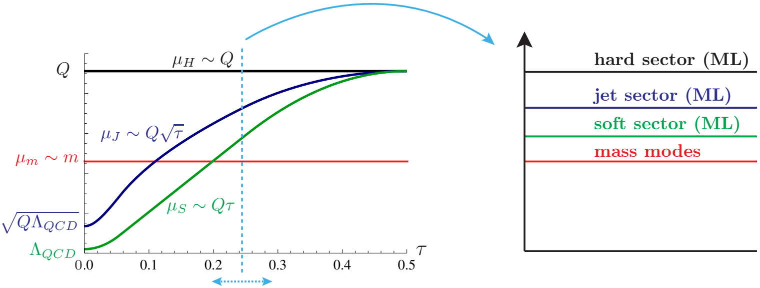
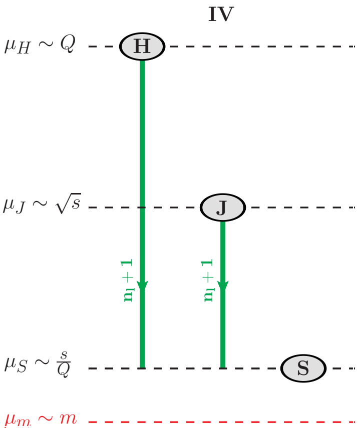


Setup for secondary massive quarks

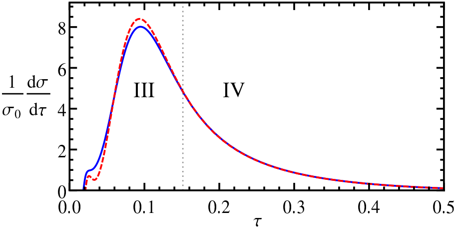
- Setup for event shape distribution ≈ Setup for endpoint DIS
- now: additional hierarchy possible $m < Q\tau \sim \mu_S$
  $\rightarrow$ $\overline{\text{MS}}$ renormalization for all structures ($n_l + 1$ active flavors)
  $\rightarrow$ massive contributions to soft function √



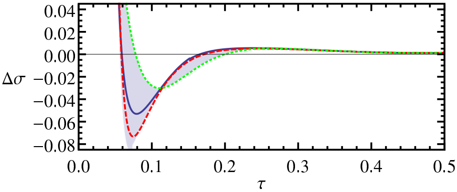
Secondary massive bottom effects for $Q = 14$ GeV

comparison between massless and massive thrust distribution
ML: $n_l = 5$, M: $n_l = 4$ & massive b ($m_b = 4.2$ GeV)

massive vs. massless



relative deviation massive vs. massless
$\mu_m = m$, $\mu_m = m/2$, $\mu_m = 2m$



Outline

Summary & Outlook

- achievements:
  1. general framework for secondary massive quark effects in multiscale processes
  2. DIS for $1 - x \sim \mathcal{O}(1)$: re-interpretation + results at $\mathcal{O}(\alpha_s^2 C_F T_F)$
  3. DIS for $1 - x \ll 1$: setup + results at $\mathcal{O}(\alpha_s^2 C_F T_F)$
  4. thrust distribution for $\tau \to 0$: setup + results at $\mathcal{O}(\alpha_s^2 C_F T_F)$ + numerical analysis (partially also for primary massive quarks)

Gritschacher, Hoang, Jemos, P.P., Phys.Rev.D88 (2013) 034021
Gritschacher, Hoang, Jemos, P.P., Phys.Rev.D89 (2014) 014035
Gritschacher, Hoang, Jemos, Mateu, P.P., to appear in Phys.Rev.D90
Hoang, P.P., Samitz (in preparation)
Hoang, Pathak, P.P., Stewart (in preparation)

- possible applications:
  1. analysis of low energy collider data (e.g. event shapes)
  2. top quark production at the LHC
  3. beam functions with massive quarks
  4. . . .

Acknowledgements

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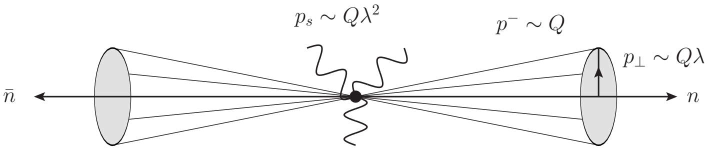
Moritz Preißer
Daniel Samitz
Maximilian Stahlhofen

Outline

Scales in jets (SCET I)

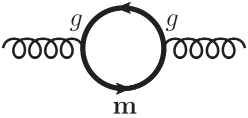


- description of jets: lightcone coordinates $n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$
  $\rightarrow p^\mu = \bar{n}\cdot p\,\frac{n^\mu}{2} + n\cdot p\,\frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv (p^-, p^+, p_\perp)$
  $\rightarrow p^2 = p^- p^+ + p_\perp^2$
- for collimated jets: $Q^2 \gg m_{jet}^2 \gg \Lambda_{QCD}^2$
  $\rightarrow$ language of effective field theory appropriate
  $\rightarrow p^- \gg p_\perp \gg p^+$ (n-direction) $\longrightarrow p^- \sim Q$, $p_\perp \sim Q\lambda$, $p^+ \sim Q\lambda^2$
  with power counting parameter $\lambda = m_{jet}/Q$
- ultrasoft particles: $p^- \sim p_\perp \sim p^+ \sim Q\lambda^2$

$\Rightarrow$ Soft-collinear effective theory (SCET)

Bauer, Fleming, Luke (2000); Bauer, Fleming, Pirjol, Stewart (2001)

Renormalization of the strong coupling: Massive quark contributions



$$\Pi(0) = \frac{\alpha_s T_F}{3\pi}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{m^2}\right) - \gamma_E + \ln(4\pi) + \mathcal{O}(\epsilon)\right]$$

Renormalization for $\alpha_s \equiv g^2/4\pi$:

$$\alpha_s = \mu^{2\epsilon} Z_\alpha^{\overline{\rm MS}} \alpha_s^{\overline{\rm MS}}(\mu) = \mu^{2\epsilon} Z_\alpha^{\rm OS} \alpha_s^{\rm OS}(\mu) = \ldots$$

$\rightarrow$ $\overline{\rm MS}$ renormalization: $Z_\alpha^{\overline{\rm MS}} = 1 + \frac{\alpha_s^{\overline{\rm MS}} T_F}{3\pi}\frac{1}{\epsilon} + \text{const} + \ldots$ (default for massless partons)
$\rightarrow$ OS (on-shell) renormalization: $Z_\alpha^{\rm OS} = 1 + \Pi(0) + \ldots$

Anomalous dimension for resummation of logarithms (RGE)

$$\beta^{\overline{\rm MS}} = \frac{d\alpha_s^{\overline{\rm MS}}}{d\ln\mu^2} + \epsilon\alpha_s^{\overline{\rm MS}} = -\mu^{2\epsilon}\alpha_s^{\overline{\rm MS}}\frac{d\ln Z_\alpha^{\overline{\rm MS}}}{d\ln\mu^2} = \beta^{(n_l+1)} \quad \rightarrow \alpha_s^{\overline{\rm MS}} \equiv \alpha_s^{(n_l+1)}(\mu)$$

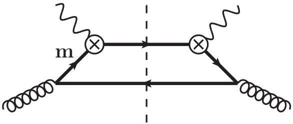
$$\beta^{\rm OS} = \frac{d\alpha_s^{\rm OS}}{d\ln\mu^2} + \epsilon\alpha_s^{\rm OS} = -\mu^{2\epsilon}\alpha_s^{\rm OS}\frac{d\ln Z_\alpha^{\rm OS}}{d\ln\mu^2} = \beta^{(n_l)} \quad \rightarrow \alpha_s^{\rm OS} \equiv \alpha_s^{(n_l)}(\mu)$$

Massive quark corrections for $m \gtrsim Q$

$$m \gtrsim Q: \boxed{F_{1,2} = \sum_{i=q,Q} \sum_{j=q,g} H^l_{ij}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)}$$

use OS renormalization = low-momentum subtraction for PDFs and α_s

- evolution always with n_l flavors
- only full QCD contributions in H^l_{ij}, e.g. at one-loop to H^l_{Qg}:



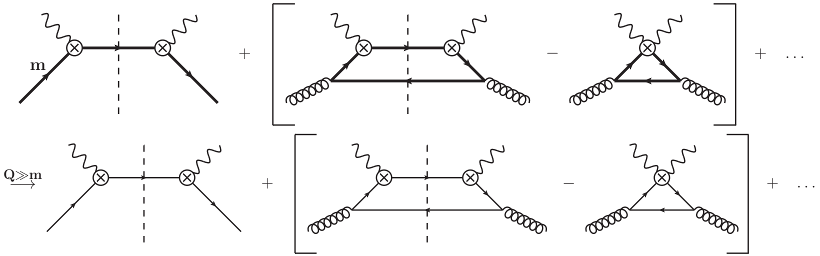
$\rightarrow$ for $m \gg Q$: automatic decoupling $\surd$
$\rightarrow$ for $m \ll Q$: unresummed logarithms $\sim \ln(m^2/Q^2)$ ↯

Massive quark corrections for $m \lesssim Q$

$$m \lesssim Q: \quad F_{1,2} \sim \sum_{i=q,Q}\sum_{j=q,Q,g}\sum_{k=q,g} H_{ij}^{ll}(\mu_H) \otimes \mathcal{M}_{jk}^{\phi}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)$$

use $\overline{\mathrm{MS}}$ renormalization above the mass scale for PDFs and α_s

- evolution with $n_l + 1$ flavors above μ_m
- massive quark contributions from full QCD and SCET to H_{ij}^{ll}, e.g. at one loop



→ for $m \ll Q$: correct massless limit for H_{ij}^{ll} √

Massive quark corrections for $m \lesssim Q$

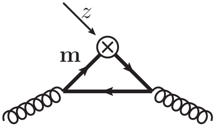
$$m \lesssim Q: \quad \boxed{F_{1,2} \sim \sum_{i=q,Q} \sum_{j=q,Q,g} \sum_{k=q,g} H_{ij}^{ll}(\mu_H) \otimes \mathcal{M}_{jk}^{\phi}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)}$$

use $\overline{\mathrm{MS}}$ renormalization above the mass scale for PDFs and α_s

- evolution with $n_l + 1$ flavors above μ_m
- now massive quark contributions from full QCD and SCET to H_{ij}^{ll}

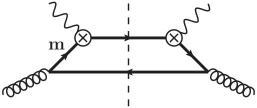
use OS renormalization below the mass scale for PDFs and α_s

- evolution with n_l flavors below μ_m
- scheme change $\leftrightarrow$ PDF matching $\mathcal{M}_{ij}^{\phi}$, e.g. at one-loop for $\mathcal{M}_{Qg}^{\phi} = \langle g|\mathcal{O}_Q|g\rangle$



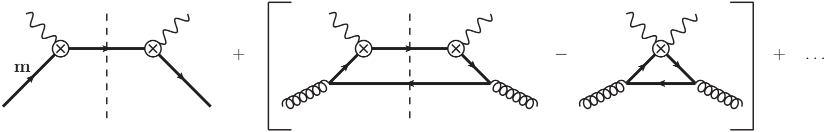
- I. $m \gtrsim Q$: $F_{1,2} = \sum_{i=q,Q} \sum_{j=q,g} H^{I}_{ij}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to H^{I}_{ij} at one-loop

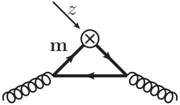


- II. $m \lesssim Q$: $F_{1,2} \sim \sum_{i=q,Q} \sum_{j=q,Q,g} \sum_{k=q,g} H^{II}_{ij}(\mu_H) \otimes \mathcal{M}^{\phi}_{jk}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to H^{II}_{ij} at one-loop

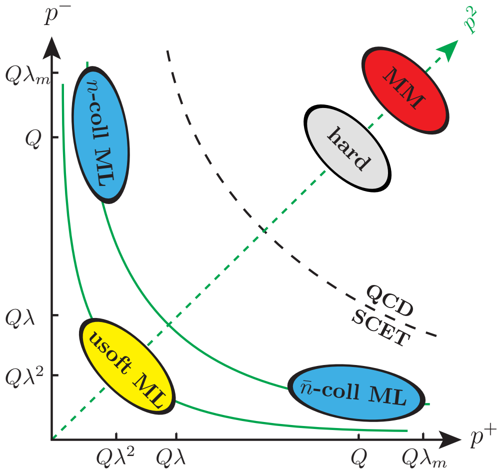


  $\rightarrow$ massive contributions to PDF matching $\mathcal{M}^{\phi}_{ij}$ at one-loop



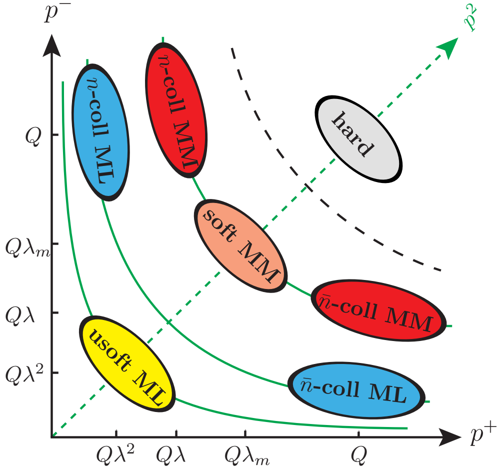
$\Rightarrow$ for $\mu_m \sim m \sim Q \sim \mu_H$: continuous transition at $\mathcal{O}(\alpha_s)$ $\surd$

Scenario I. $\lambda_m > 1 > \lambda > \lambda^2$



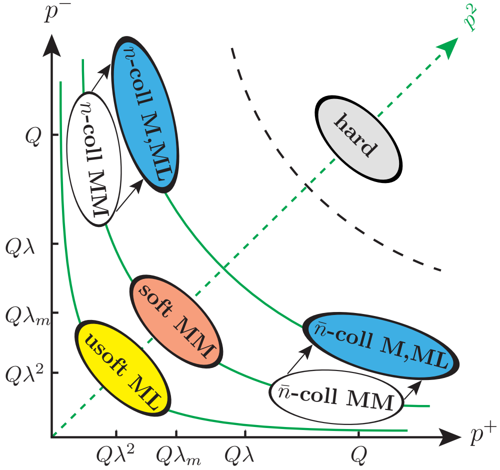
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario II. $1 > \lambda_m > \lambda > \lambda^2$



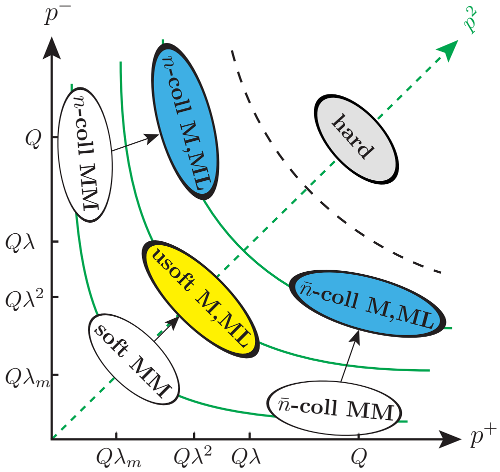
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario III. $1 > \lambda > \lambda_m > \lambda^2$



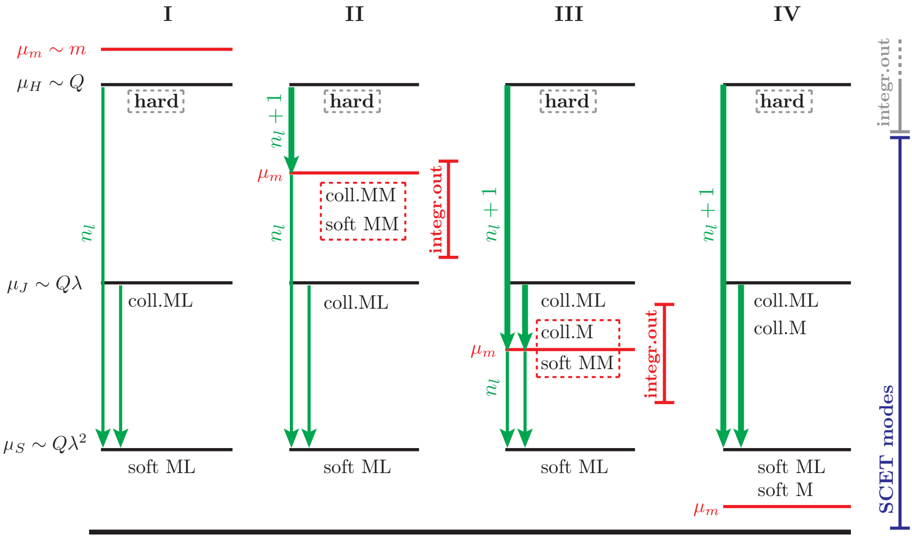
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Scenario IV. $1 > \lambda > \lambda^2 > \lambda_m$



mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft M	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

Mass mode setup: Summary



MM = mass-mode, ML = massless, M = massive

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\frac{d\sigma}{d\tau} \sim H_l^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

modification of hard matching coefficient due to massive quark
$\rightarrow$ use OS renormalization for current
$\Rightarrow$ decoupling for $m \gg Q$, but mass-singularities for $m \rightarrow 0$

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\frac{d\sigma}{d\tau} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- II. $Q > m > Q\sqrt{1-x}$: virtual mass mode contributions in SCET

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

modification of hard matching coefficient, subtractions due to SCET diagrams
$\rightarrow$ use $\overline{\mathrm{MS}}$ renormalization for hard current matching
$\Rightarrow$ correct massless limit for $m \ll Q$
additional current mass mode matching contribution $\mathcal{M}_H$ at μ_m

Factorization theorems

- I. $m > Q$: massive quark integrated out when matching to SCET

$$\frac{d\sigma}{d\tau} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- II. $Q > m > Q\sqrt{1-x}$: virtual mass mode contributions in SCET

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- III. $Q\sqrt{1-x} > m > \Lambda_{\mathrm{QCD}}$: virtual and real mass mode contributions in SCET

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l+1)}(\mu_J) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

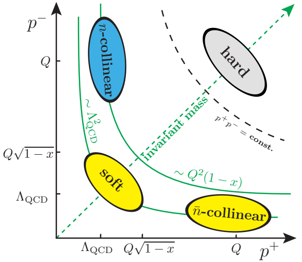
modification of the jet function at μ_J due to massive quark
$\rightarrow$ use $\overline{\mathrm{MS}}$ renormalization $\Rightarrow$ correct massless limit for $m \ll Q\sqrt{1-x}$

$$J^{(n_l+1)}(s, m, \mu_J) = J_0^{(n_l+1)}(s, \mu_J) + \delta J_m^{\mathrm{dist}}(s, m, \mu_J) + \theta(s - 4m^2)\delta J_m^{\mathrm{real}}(s, m)$$

additional jet mass mode matching contribution $\mathcal{M}_J$ at μ_m

Massless mode setup and factorization

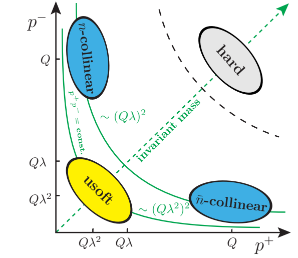
- DIS



Factorization theorem for $1 - x \ll 1$:

$$F_1 \sim \sum_{i=q} H_{\mathrm{DIS}}(\mu_H) J_{\mathrm{DIS}}(\mu_J) \otimes \underbrace{S_{\mathrm{DIS}}(\mu_\Phi) \otimes f_{i/P}(\mu_\Phi)}_{=\Phi_{i/P}(\mu_\Phi)}$$

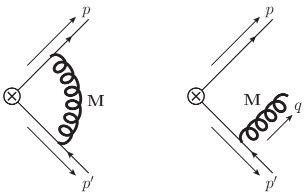
- Thrust



Factorization theorem for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} \sim H_\tau(\mu_H) J_\tau(\mu_J) \otimes S_\tau(\mu_S)$$

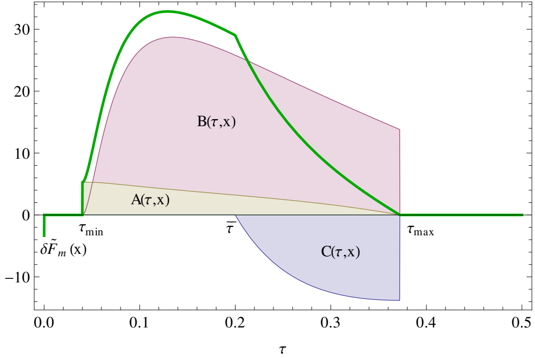
Full theory result



$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \frac{\alpha_s C_F}{4\pi}\Big\{\delta(\tau)\,\delta\tilde{F}_m^{\rm QCD}\left(\frac{M^2}{Q^2}\right) + \theta(\tau-\tau_{\rm min})\left[A\left(\tau,\frac{M^2}{Q^2}\right)+B\left(\tau,\frac{M^2}{Q^2}\right)\right] + \theta(\tau-\bar{\tau})\,C\left(\tau,\frac{M^2}{Q^2}\right)\Big\}$$

$\tau_{\rm min} = \frac{M^2}{Q^2} \rightarrow$ threshold for real jet radiation

$\bar{\tau} = \frac{M}{Q} \rightarrow$ threshold for real soft radiation



Massive threshold corrections

Example: threshold correction in jet sector

- bare jet function:

$$J^{\text{bare}} = Z_J^{OS} \otimes J^{\text{OS}} = Z_J^{\overline{\text{MS}}} \otimes J^{\overline{\text{MS}}}$$

- in OS renormalization:

$$J^{\text{OS}}(s,m,\mu) = J^{(n_l)}(s,\mu) + \theta(s-4m^2)\delta J_m^{\text{real}}(s,m) \stackrel{m\gg s}{\longrightarrow} J^{(n_l)}(s,\mu)$$

- in $\overline{\text{MS}}$ renormalization:

$$J^{\overline{\text{MS}}}(s,m,\mu) = J^{(n_l+1)}(s,\mu) + \delta J_m^{\text{dist}}(s,m,\mu) + \theta(s-4m^2)\delta J_m^{\text{real}}(s,m) \stackrel{m\ll s}{\longrightarrow} J^{(n_l+1)}(s,\mu)$$

- threshold correction:

$$\boxed{\Rightarrow \mathcal{M}_J(s,m,\mu) = J^{\text{OS}}(s,m,\mu) \otimes \left(J^{\overline{\text{MS}}}(s,m,\mu)\right)^{-1}}$$

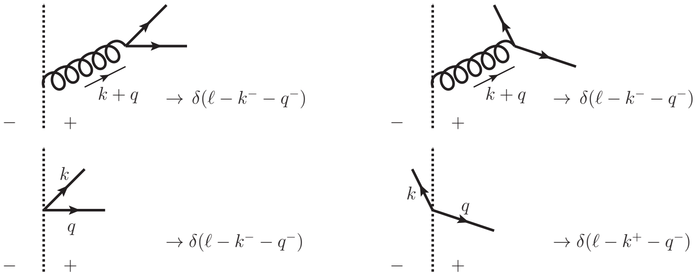
$\Rightarrow$ continuity by construction

Two-loop computations

dispersive technique appropriate for $\rightarrow$ Wilson coefficient $\surd$
$\rightarrow$ jet function $\surd$
$\rightarrow$ soft function ...

Two-loop computations

dispersive technique appropriate for $\rightarrow$ Wilson coefficient $\surd$
$\rightarrow$ jet function $\surd$
$\rightarrow$ soft function ... ↯ hemisphere definition!



same hemisphere: $k^+ > k^-\,,\; q^+ > q^-$

opposite hemisphere: $k^- > k^+\,,\; q^+ > q^-$
$k^+ + q^+ > k^- + q^-$

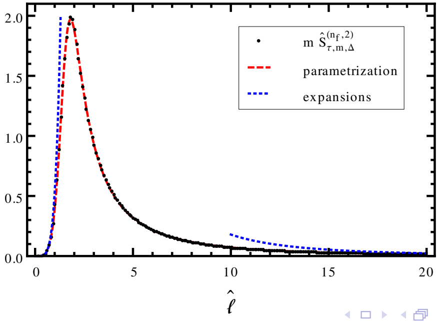
$\Rightarrow$ Opposite hemisphere contribution accounted for in different ways

1. "gluon hemisphere" prescription $\rightarrow S^{(g)}(\ell, m, \mu)$
2. "quark hemisphere" prescription $\rightarrow S^{(qq)}(\ell, m, \mu)$ (correct soft function)

Soft function computation

So how to compute the thrust soft function with the correct hemisphere prescription?

- $S^{(qq)}(\ell, m, \mu) = S^{(g)}(\ell, m, \mu) + \underbrace{\left(S^{(qq)}(\ell, m, \mu) - S^{(g)}(\ell, m, \mu)\right)}_{=\Delta S(\ell, m)}$
- $S^{(g)}(\ell, m, \mu)$: distributions and threshold $\theta(\ell - 2m)$
- hemisphere misalignment contribution $\Delta S(\ell, m)$ is finite
  $\rightarrow$ numerical integration in 4D possible
  $\rightarrow$ correct massless limit Kelley, Schabinger, Schwartz, Zhu (2011)
  $\rightarrow$ good parametrization with correct normalization and asymptotic behaviour



Soft function computation

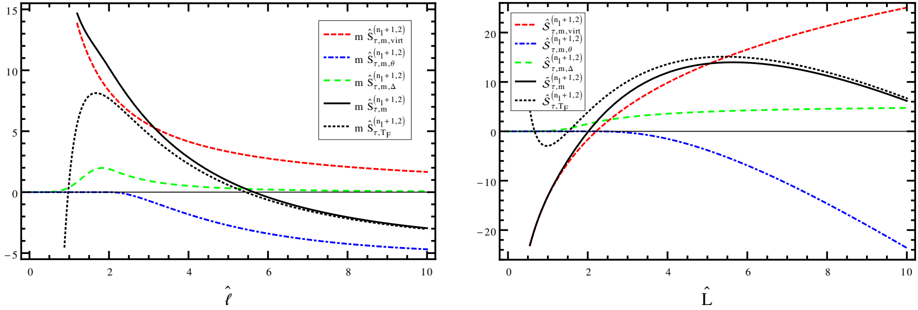


Figure : Massive quark contributions to the thrust soft function (left) + cumulant (right) for $\mu = m$, normalized by $(\alpha_s^{(n_l+1)})^2 C_F T_F / 16\pi^2$.

R-evolution with massive quarks

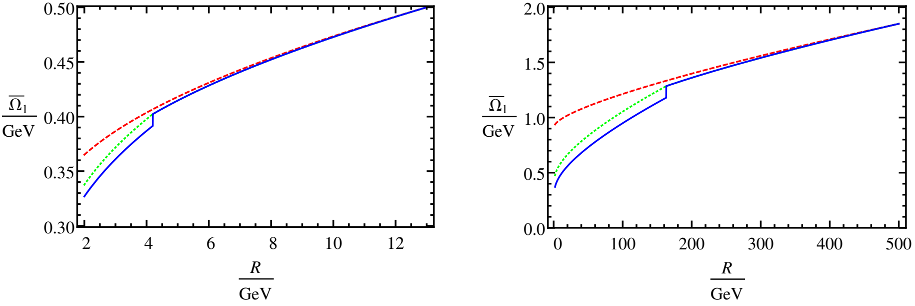
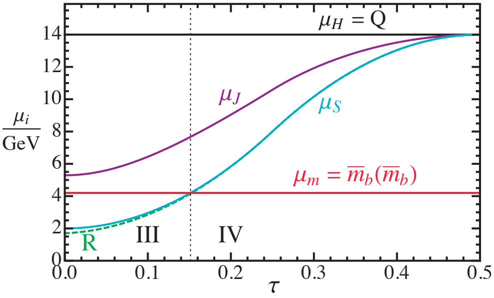


Figure : R-evolution of $\bar{\Omega}_1(R, \mu = R)$ with a massive bottom (left) and a massive top quark (right). Massless evolution (red, dashed), massive evolution incl. threshold matching (blue, solid) at $\overline{m}_b(\overline{m}_b)/\overline{m}_t(\overline{m}_t)$ and massive evolution without threshold matching (green, dotted).

Analysis of secondary massive bottom effects

- analysis for $Q = 14, 22, 35$ GeV $\leftrightarrow$ bottom mass effects relevant
- ingredients for analysis at N^{3}LL in the dijet region $\tau \ll 1$ $\surd$
- numerical code (incl. nonperturbative model function & gap subtractions) $\surd$
- profile functions for $Q = 14$ GeV:



Plots: secondary massive bottom quarks

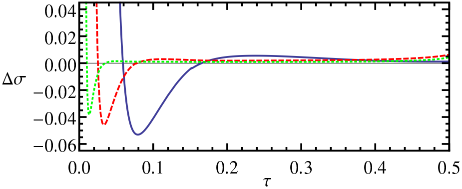


Figure : $Q = 14$ GeV (blue, solid), $Q = 35$ GeV (red, dashed) and $Q = m_Z$ (green, dotted).

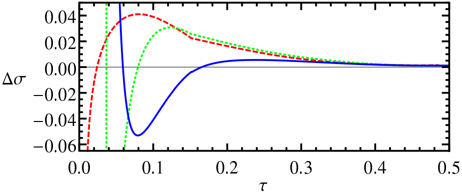


Figure : $Q = 14$ GeV: partonic (red, dashed), incl. the nonperturbative soft model function (green, dotted) and + gap formalism = default (blue, solid).

Plots: secondary massive top quarks

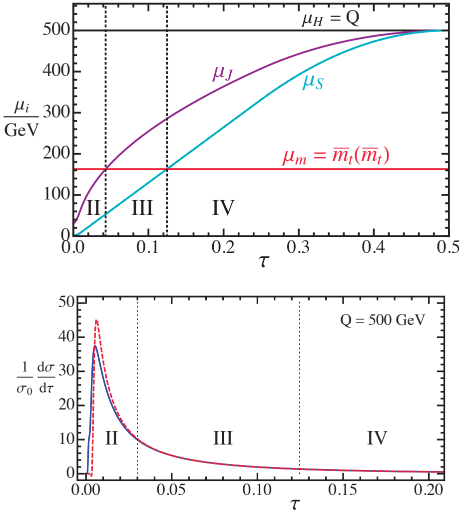


Figure : $Q = 500$ GeV: massive (blue, solid) vs. massless (red, dashed).

Plots: secondary massive top quarks

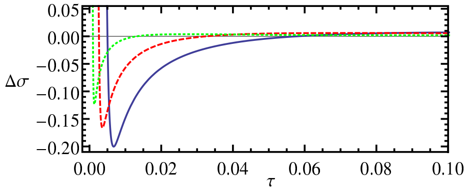


Figure : $Q = 500$ GeV (blue, solid), $Q = 1000$ GeV (red, dashed) and $Q = 3000$ GeV (green, dotted).

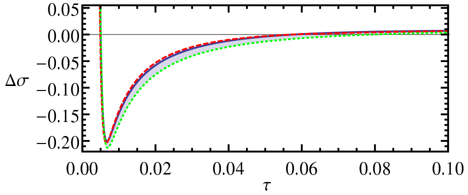


Figure : $Q = 500$ GeV: $\mu_m = m_t$ (blue, solid), $\mu_m = m_t/2$ (red, dashed) and $\mu_m = 2m_t$ (green, dotted).