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*Wien, 29. April 2014*

*Effective-theory approach to  
top quark production at hadron colliders*

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## introduction

- background

### Part I: off-shell effects ↔ non-factorizable corrections

## effective theory setup

- effective theory approach
- hard/soft modes, method of regions
- total cross section

## single top production

- adapting ET approach to real corrections
- structure of fully differential NLO calculation
- results and comparison with complex-mass-scheme calculation

## top pair production

- structure of fully differential NLO calculation
- results and impact on  $m_t$  determination

### Part II: approximate NNLO fully differential cross sections

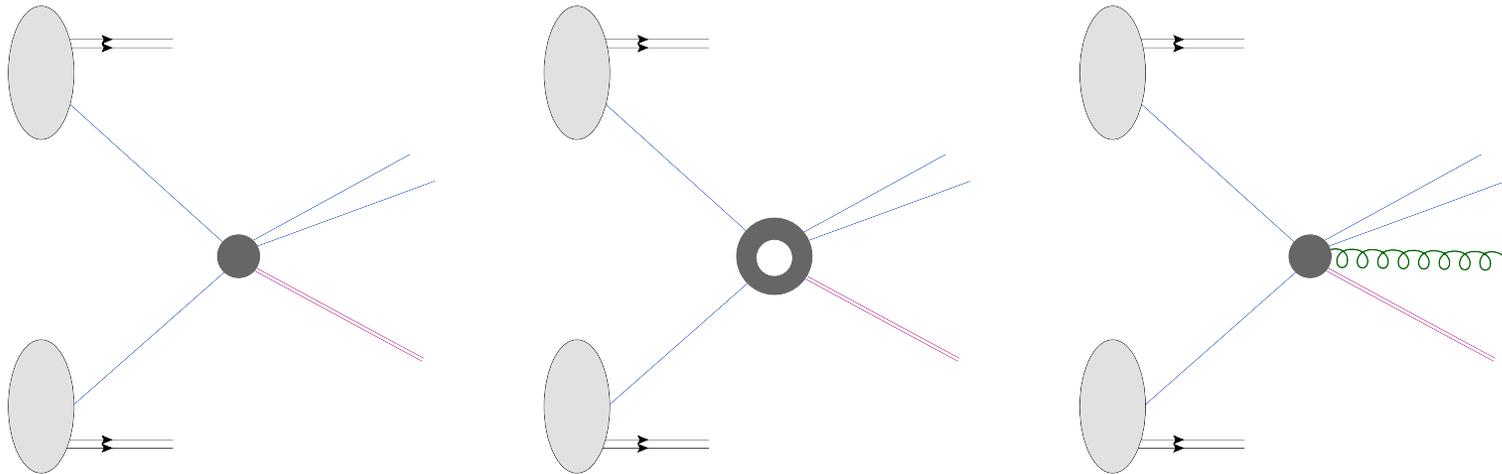
## top pair production

- PIM and 1PI kinematics
- implementation and sample results

## conclusions

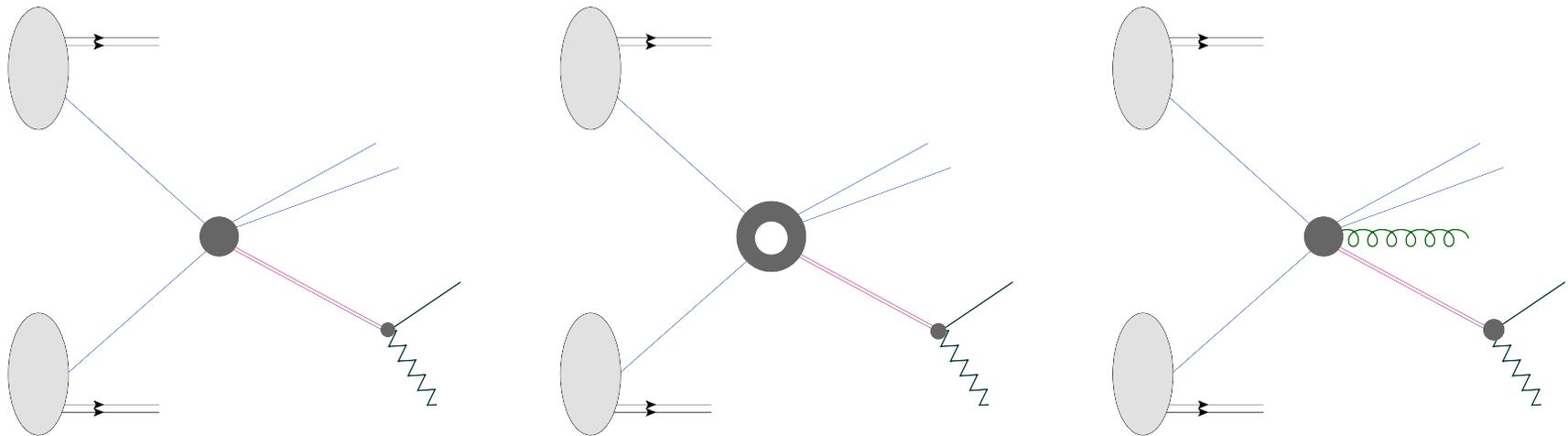
- most (all?) interesting phenomena at the LHC involve heavy unstable particles (SM:  $W^\pm$ ,  $Z$ ,  $t$ ,  $H$  and BSM  $Z'$ ,  $X_{\text{susy}} \dots$ )
  - these particles are detected/studied through their decay products (neither the Tevatron nor the LHC has seen a single top quark!!)
  - for realistic applications need to include the decay of these particles in theoretical description
  - such particles are not proper external states in a QFT, but only intermediate states
  - for most applications it is sufficient to ignore this and treat heavy particles as external
- but:
- very precise determinations of observables (e.g.  $m_t$ ) require to have full control of small effects
  - it is also an interesting problem in QFT

Production of an **on-shell heavy** (unstable) particle  $X$ :  $p_X^2 = m_X^2$



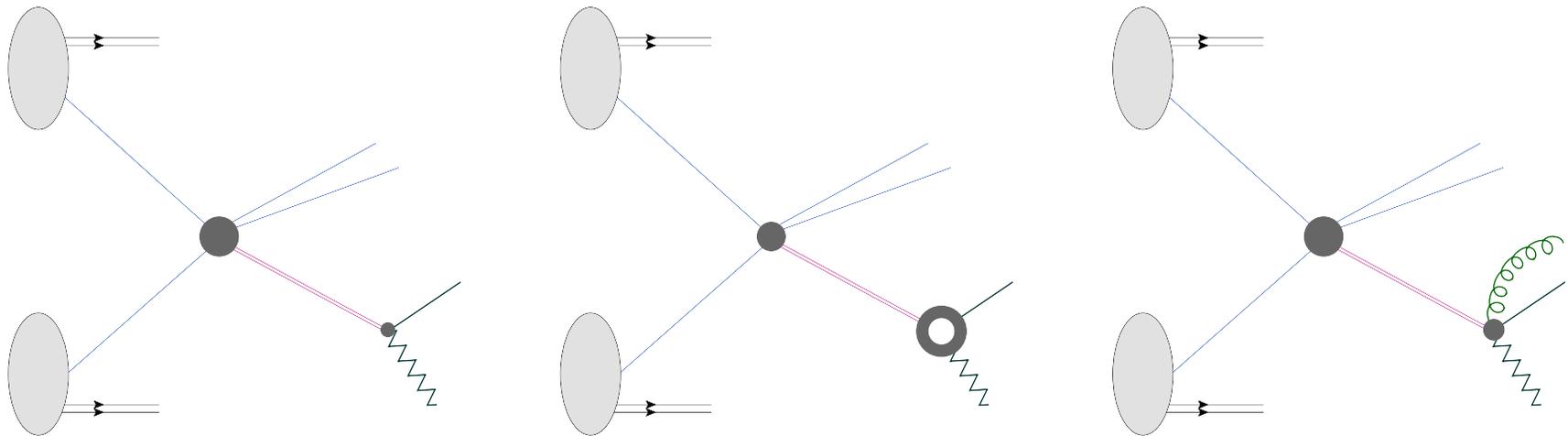
- often this is a reasonable approximation **but**
- cuts on decay products not possible
- off-shell effects of  $X$  not taken into account

Production of an **on-shell heavy** (unstable) particle  $X$ , including decay:  $p_X^2 = m_X^2$



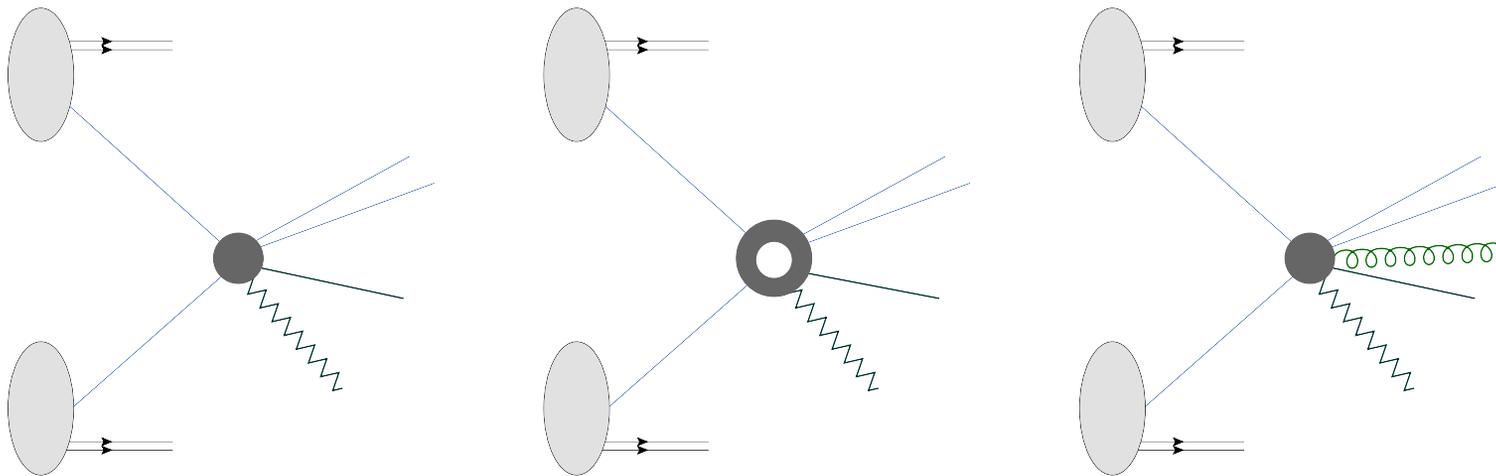
- (improved) narrow width approximation,  $M_{\text{decay}}^2 = m_X^2$
- NLO correction of production and decay included
- cuts on decay products possible but off-shell effects of  $X$  not taken into account

Production of an **on-shell heavy** (unstable) particle  $X$ , including decay:  $p_X^2 = m_X^2$



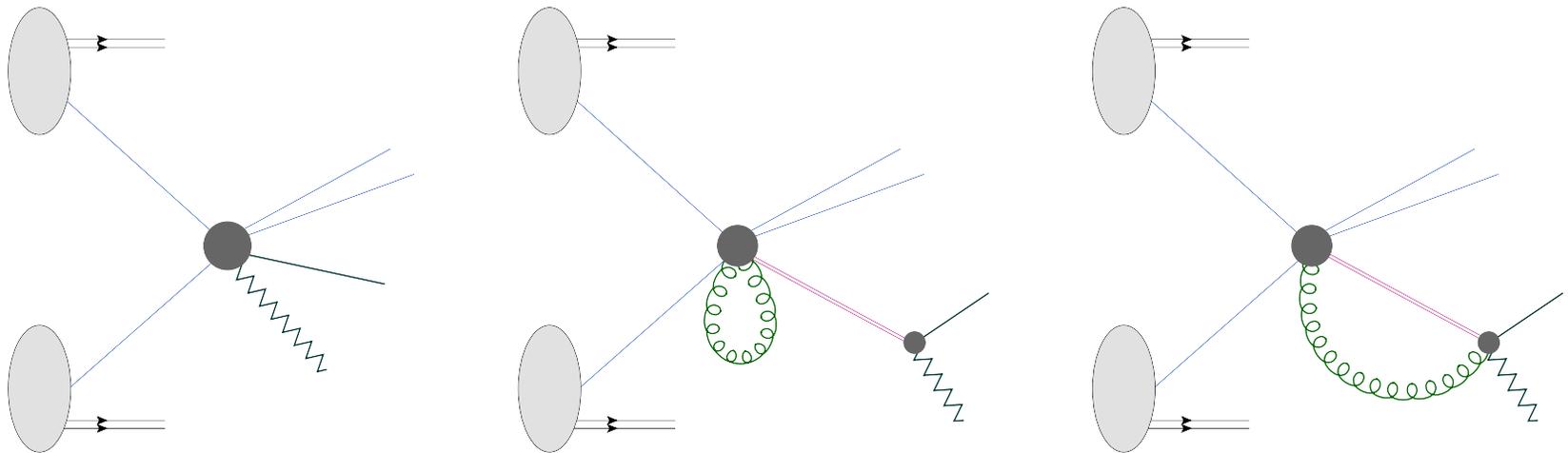
- (improved) narrow width approximation,  $M_{\text{decay}}^2 = m_X^2$
- NLO correction of production and decay included
- cuts on decay products possible but off-shell effects of  $X$  not taken into account

Production of an **off-shell heavy** (unstable) particle  $X$ , including decay:  $p_X^2 \neq m_X^2$



- tree-level background diagrams (no particle  $X$ , but same final state)
- virtual and real background diagrams
- valid for any  $p_X^2$ , (off-shell effects taken into account) but calculation complicated (e.g. complex-mass scheme)

Production of a **resonant heavy** (unstable) particle  $X$ , including decay:  $p_X^2 \sim m_X^2$



- tree-level background diagrams (no particle  $X$ , but same final state)
- use pole approximation (at one loop)
  - factorizable corrections
  - non-factorizable corrections
 } gauge invariant separation
- real background diagrams
- valid for  $p_X^2 \sim m_X^2$ , off-shell effects of  $X$  are taken into account, calculation simplified

non-factorizable corrections have been extensively studied

[Fadin et al; Melnikov et al; Beenakker et al; Denner et al; Bevilacqua et al; Jadach et al; . . .]

but are usually neglected at hadron colliders, because:

- they seem to be more difficult to compute (not really)
- they are generally small [Beenakker et al; Pittau]
  - resonant  $\rightarrow$  non-resonant propagator unless  $E \lesssim \Gamma$  is small (soft)
  - cancellations for “inclusive” observables [Fadin, Khoze, Martin]

purpose of this work (part I):

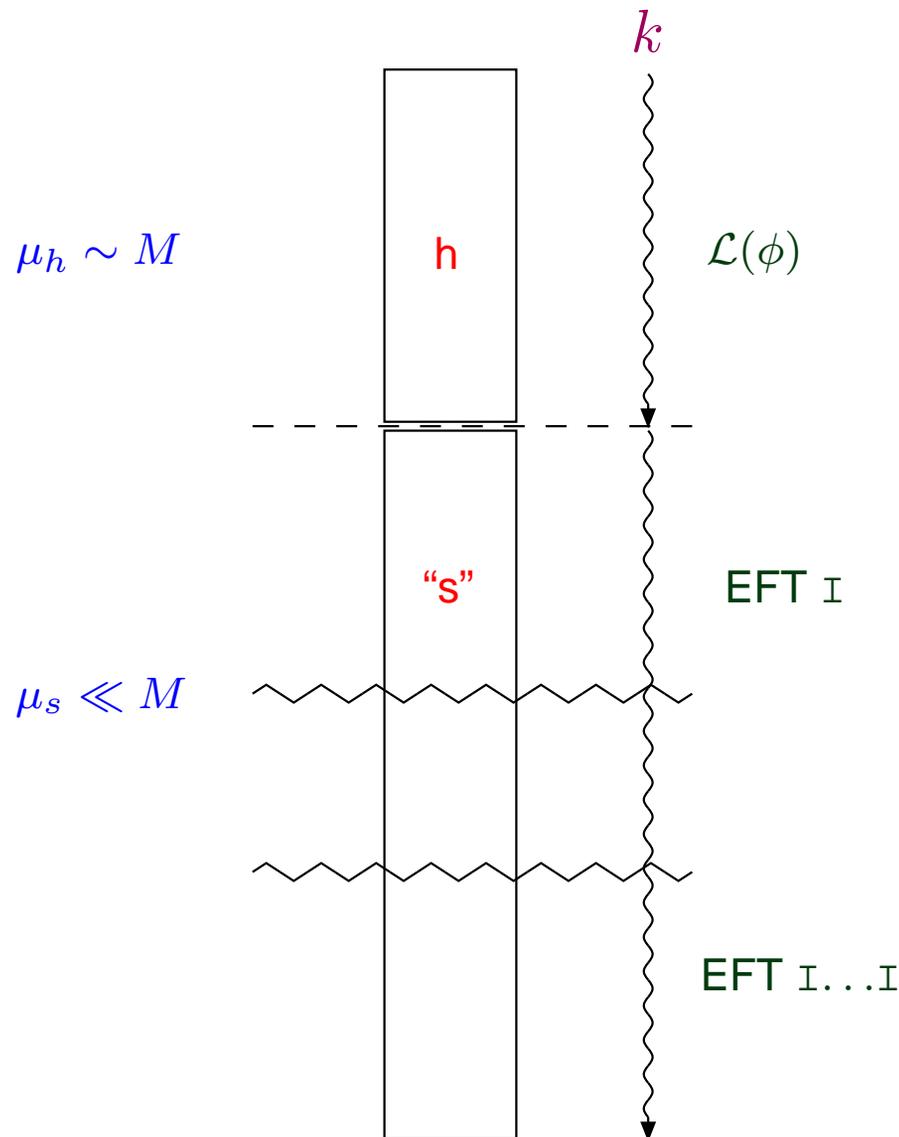
- do not neglect non-factorizable corrections
- consistently combine with propagator corrections
- try to obtain an efficient way to identify and compute minimal amount required
- do this for fully differential cross section

- **the feature:** hierarchy of scales, exploit via effective theory (ET) approach
  - hard  $k \sim m_X$
  - soft:  $k \sim (p_X^2 - m_X^2)/m_X \sim \delta m_X \ll m_X$
- expand in all small parameters  $\alpha$  and  $\delta = (p_X^2 - m_X^2)/m_X^2 \rightarrow$  power counting:
 
$$\alpha \sim \frac{p_X^2 - m_X^2}{m_X^2} \sim \frac{\Gamma_X}{m_X} \sim \delta \ll 1 \quad \text{for top: } \alpha \equiv \alpha_{ew} \sim \alpha_s^2$$
- **integrate out hard modes:**  $\mathcal{L}(\phi, A, \psi) \rightarrow \mathcal{L}_{\text{eff}}(c_i, \phi_s, A_s, A_c, \psi_s, \psi_c)$   
UPET is nothing but a mix between SCET and H'Q'ET
- virtual corrections and total cross section
  - expand integrand, **method of regions** [Beneke, Smirnov]
  - new identification [Chapovsky, Khoze, AS, Stirling]  
factorizable corrections = **hard corrections** (ET)  
non-factorizable corrections = **soft corrections** (ET)
  - applications for total cross section:
    - $e^+e^- \rightarrow t\bar{t}$  near threshold [Hoang et al; Beneke et al; Melnikov et al; ...]
    - “Higgs” production in toy model [Beneke et al.]
    - $e^+e^- \rightarrow W^+W^-$  near threshold [Beneke et al.]
- arbitrary real corrections problematic (new scales from definition of observable)

Let  $p^2 \ll M^2$  and assume we want to compute (the first few terms in an expansion in  $p^2/M^2 \ll 1$  of) the integral

$$\begin{aligned}
 & \int \frac{d^d k}{(k^2 - p^2)(k^2 - M^2)} \\
 &= \underbrace{\int \frac{d^d k}{(k^2 - p^2)} \sum_n \frac{(k^2)^n}{(M^2)^{n+1}}}_{\text{soft}} + \underbrace{\int \frac{d^d k}{(k^2 - M^2)} \sum_n \frac{(p^2)^n}{(k^2)^{n+1}}}_{\text{hard}}
 \end{aligned}$$

- identify modes: soft ( $k \sim p$ ) and hard ( $k \sim M$ )  
(in general there are other modes, e.g. potential, collinear . . .)
- expand integrand in each region to whatever order required
- each term has a well-defined scaling in  $p^2/M^2 \rightarrow$  power counting
- no explicit cutoff needed (dimensional regularization is important)
- the soft part generates UV singularities, the hard part generates IR singularities



underlying theory

$$\mathcal{L}(\partial, \phi) = \mathcal{L}(\phi_h, \phi_s, \dots)$$

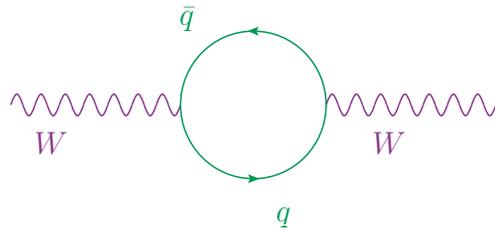
effective theory I

$$\mathcal{L} = \sum_i c_i(h, \mu_h) O_i(\phi_s, \dots, \{\mu_s\})$$

- UV divergences due to operators  $O_i$  are compensated by IR singularities in matching coefficients  $c_i$
- anomalous dimensions of  $O_i \rightarrow$  resummation of  $\ln \mu_h / \mu_s$
- outside region of validity of EFT match to underlying theory

use method of regions [Beneke, Smirnov] and expand integrand (in principle to any order):

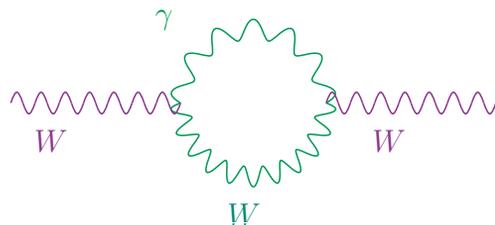
- hard corrections  $l \sim m_X$  (= factorizable corrections)
- soft corrections  $l \sim m_X \delta$  (= non-factorizable corrections)



$$\int \frac{d^d l}{(p+l)^2 l^2}$$

hard: full

soft:  $\int \frac{d^d l (2p \cdot l)}{p^2 l^2} = 0$



$$\int \frac{d^d l}{(l^2 + 2p \cdot l + \Delta) l^2}$$

hard:  $\int \frac{d^d l}{l^2 (l^2 + 2p \cdot l)} \neq 0$

soft:  $\int \frac{d^d l}{l^2 (2p \cdot l + \Delta)} \neq 0$

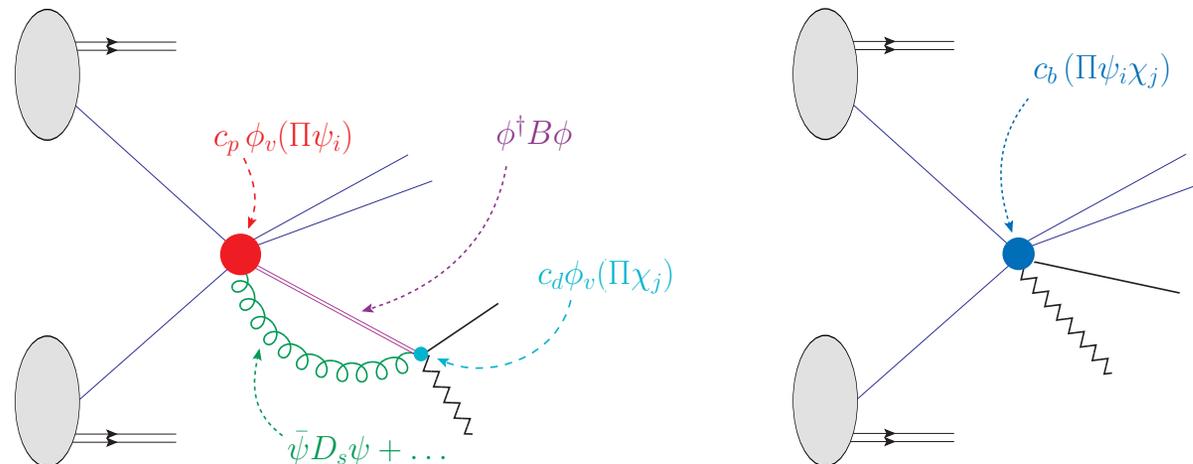
- leads to resummation of **hard part** (= leading part in  $\Delta$ ) of self-energy insertions

$$\mathcal{L}_{\text{eff}} = 2m_X \phi_s^\dagger \left( i v \cdot \partial - \frac{c_{\phi\phi}}{2} \right) \phi_s + \dots$$

- matching coefficients are gauge invariant ( $c_{\phi\phi} = -i\Gamma$  in pole scheme)  
full result is gauge invariant at each order in  $\delta$ , but gauge invariance is **not** an input

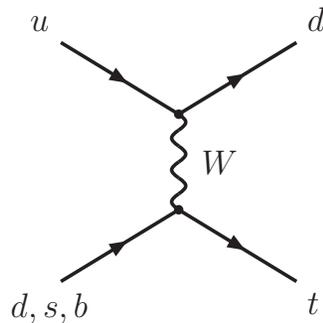
integrate out hard modes  $\rightarrow$  effective Lagrangian

$$\mathcal{L} = \phi^\dagger B \phi + c_p \phi(\Pi\psi_i) + c_d \phi(\Pi\chi_j) + c_b (\Pi\psi_i\chi_j) + \bar{\psi} D_s \psi + \dots$$

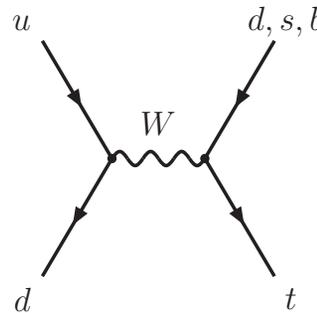


- matching coefficients  $c_i$  contain effects of hard modes
- matching done on shell,  $p_X^2 = \bar{s} = m_X^2 + \mathcal{O}(\delta)$ , with  $\bar{s}$  the complex position of pole
- soft (and collinear ...) d.o.f. still dynamical
- can be combined with further resummations (e.g. non-relativistic  $\rightarrow$  ET has more complicated structure)

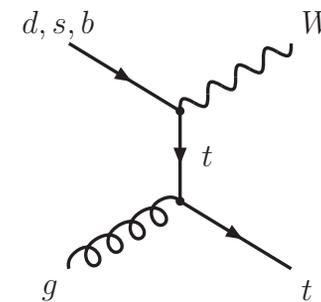
single top: t-channel



s-channel



(W t not considered here)



- total rate and distributions for on-shell top quarks at NLO known [Bordes et al; Stelzer et al; Harris et al; Campbell et al; Cao et al; ...]
- implemented in MC@NLO [Frixione et al.] and POWHEG [Alioli et al.]
- comparison 5-flavour scheme vs. 4-flavour scheme [Campbell et al.]
- EW corrections [Beccaria et al.]
- effects of BSM operators [Willenbrock et al.]
- resummation of threshold logs [Kidonakis, Wang et al.]
- full NLO calculation of  $p\bar{p} \rightarrow W J_b J_q$  in complex mass scheme [Papanastasiou et al.]

- consider  $t$  channel (i.e.  $p\bar{p} \rightarrow W J_b J_q$ )
- final state  $W(= \ell\nu)$  (treated in improved narrow-width approximation)

require  $J_b$  and  $J_q$ , apply whatever cuts

top window:  $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$

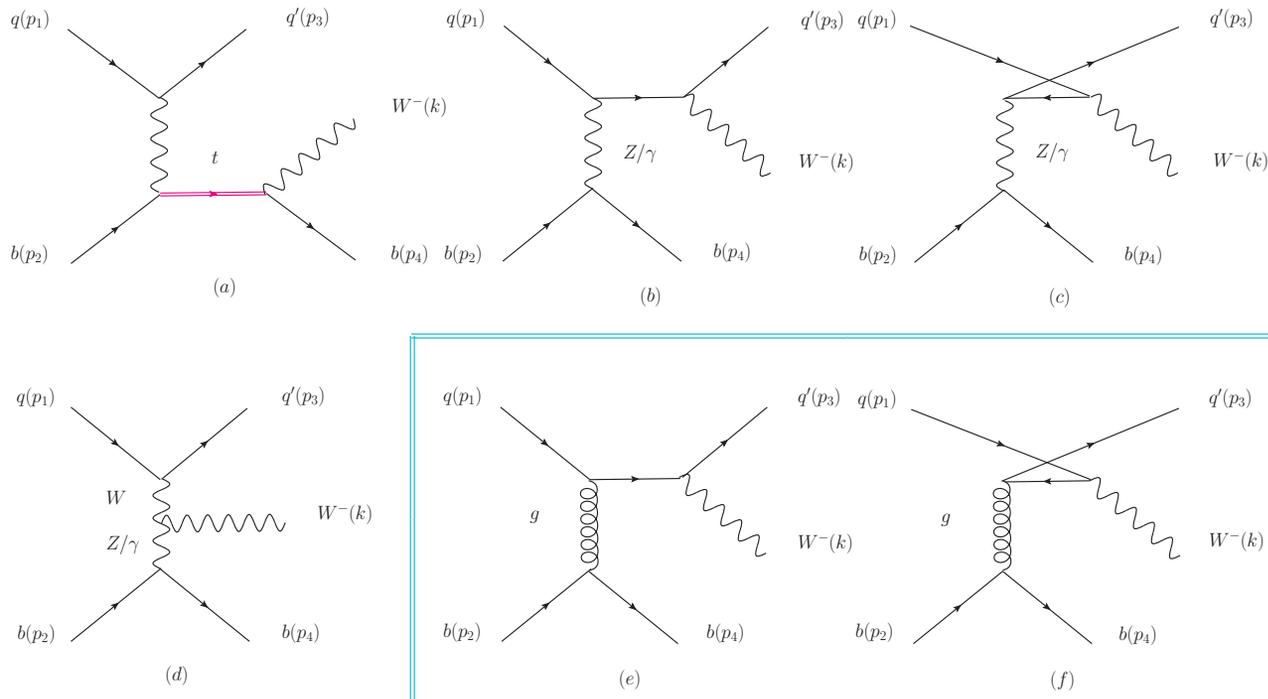
- small parameter:  $\delta \sim \frac{s_{Wb} - m_t^2}{m_t^2} \equiv \frac{D}{m_t^2}$ ;     **counting:**  $\alpha_s^2 \sim \alpha_{ew} \sim \frac{\Gamma_t}{m_t} \sim \delta \ll 1$
- use 5 flavour scheme,  $m_b = 0$ , and “fixed” order, i.e. no parton shower etc.
- resummation of “self-energies” is resummation of hard part of two-point function in scheme  $X$  (operator  $\phi^\dagger B\phi$ )

$$\mathcal{L}_{\text{EFT}}^{\text{kin}} = 2\hat{m}_X \phi_X^\dagger \left( i v \cdot \partial - \frac{\Omega_X}{2} \right) \phi_X + \dots$$

- matching coefficient  $\Omega_X = (\bar{s} - \hat{m}_X^2)/(2\hat{m}_X)$ , in pole scheme  $\Omega_{\text{pole}} = -i\Gamma_t$  but can take any scheme  $X$  as long as  $\Omega_X \sim \delta$

- propagator in ET  $\frac{1}{s_{Wb} - m_t^2} = \frac{1}{D} \Rightarrow \frac{1}{s_{Wb} - m_t^2 + im_t\Gamma_t} = \frac{1}{\Delta_t}$

$$\text{amplitude: } A^{\text{tree}} = \delta_{31}\delta_{42} \left( \underbrace{g_{ew}^3 A_{(-1)}^{(3,0)}}_{\delta^{1/2}} + \underbrace{g_{ew}^3 A_{(0)}^{(3,0)}}_{\delta^{3/2}} + \dots \right) + \underbrace{T_{31}^a T_{42}^a g_{ew} g_s^2}_{\delta \text{ signal!}} A^{(1,2)}$$

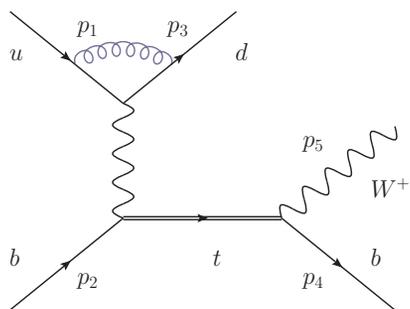


amplitude squared: (no interference due to colour  $\rightarrow$  no  $\delta^{3/2}$  term)

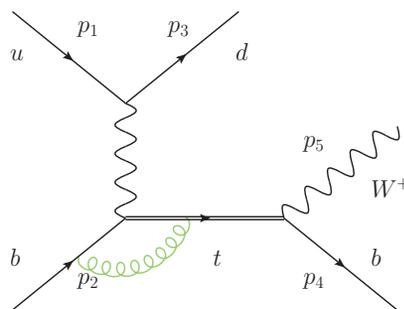
$$|M|^2 = \underbrace{g_{ew}^6 N_c^2 \left| A_{(-1)}^{(3,0)} \right|^2}_{\delta} + \underbrace{g_{ew}^6 N_c^2 2 \operatorname{Re} \left( A_{(-1)}^{(3,0)} \left[ A_{(0)}^{(3,0)} \right]^* \right)}_{\delta^2} + \underbrace{g_{ew}^2 g_s^4 N_c C_F / 2 \left| A^{(1,2)} \right|^2}_{\delta^2} + \dots$$

tree-level (squared)  $\sim \delta$ , compute all  $\sim \delta^{3/2}$  contributions to  $|M|^2$  ( $\sim \mathcal{O}(\alpha_s)$  corrections)

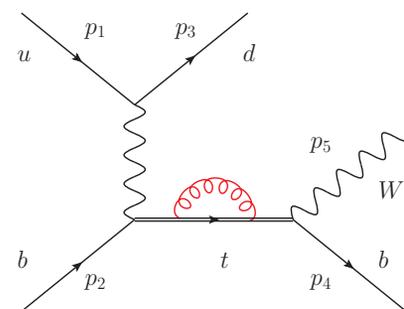
consider subset of resonant virtual diagrams (before expansion in  $\delta$  this is gauge dependent)



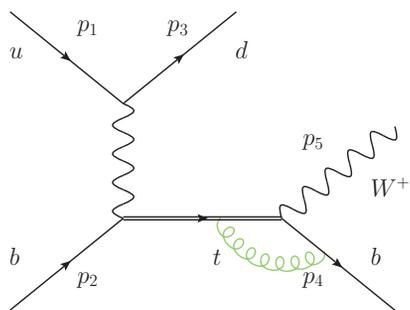
hard  $\rightarrow c_{\text{prod}}$  soft = 0



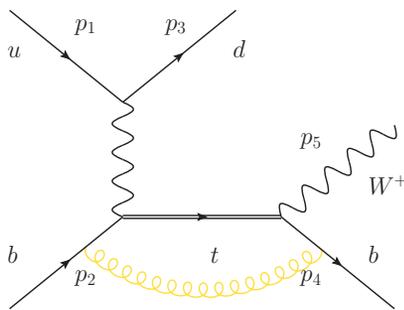
hard  $\rightarrow c_{\text{prod}}$  soft  $\rightarrow$  non-fact



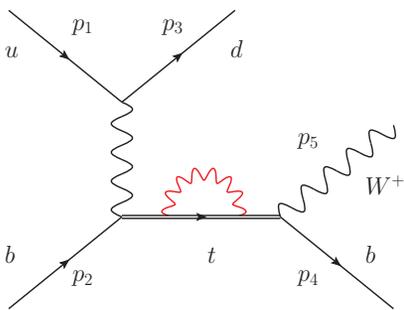
hard  $\leftrightarrow \sim 0$  soft  $\rightarrow$  non-fact



hard  $\rightarrow c_{\text{dec}}$  soft  $\rightarrow$  non-fact

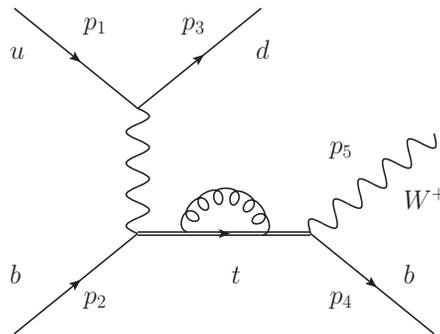


hard  $\sim \delta^2$  soft  $\rightarrow$  non-fact



hard  $\rightarrow c_{\phi\phi}$  resum

## QCD self energy



denominator  $D^2 \ell^2 [(p_t - \ell)^2 - m_t^2]$

hard  $D^2 \ell^2 [\ell^2 - 2\ell \cdot p_t]$

soft  $D^2 \ell^2 [-2\ell \cdot p_t + D]$

$$\sim \frac{g_{ew}^3 \cdot \alpha_s \cdot 1}{\delta^2 \cdot 1 \cdot 1} \sim 1$$

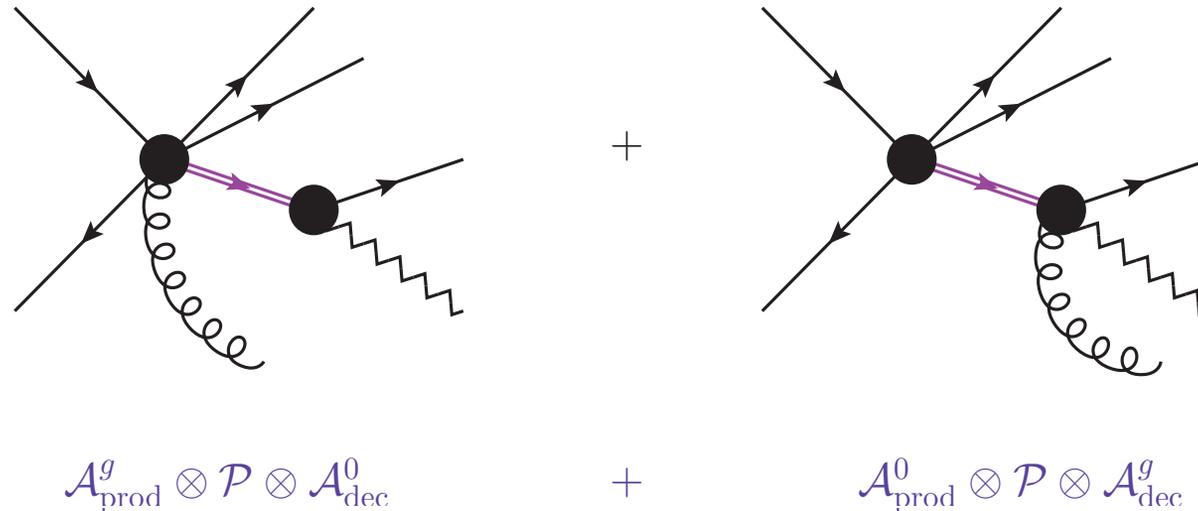
$$\sim \frac{g_{ew}^3 \cdot \alpha_s \cdot \delta^4}{\delta^2 \cdot \delta^2 \cdot \delta} \sim \delta$$

- hard part of QCD self-energy is superleading, i.e.  $\mathcal{O}(1)$  with LO amplitude  $\sim \delta^{1/2}$
- but in pole scheme the leading hard part is precisely cancelled by counter term
- can use another scheme, as long as this cancellation holds up to  $\mathcal{O}(\delta)$
- soft and subleading hard part of QCD self-energy is NLO, i.e.  $\mathcal{O}(\delta^{3/2})$  for  $|M|^2$
- hard part of EW self-energy is leading, i.e.  $\mathcal{O}(\delta^{1/2}) \rightarrow$  resum

- real corrections for “arbitrary” differential cross section cannot be done in a strict ET approach
- it is not even clear what the proper expansion parameter is (where is the gluon attached?)
- ET relies on the fact that all scales are explicit, but observable can introduce new small scale → change of structure of ET
- aim: compute real corrections for “arbitrary” observable with the implicit assumption no new small scale is introduced (e.g. for a  $p_t$  distribution result is unreliable for small  $p_t$ )
- if there is a new small scale → large logs → resummation → requires dedicated ET (or other) calculation
- expand real amplitude in  $\delta$  and  $\alpha$  under the assumption that there is no further small scale (compare to parton showers)

$$\mathcal{A}_{\text{real}} = \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^0 + \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^g$$

- the restriction to no new small scales is generic for fixed-order calculations



corrections to production (soft and coll singularities):

$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^0 \right|^2 \text{ combined with (hard) Wilson coeff. for production is IR finite}$$

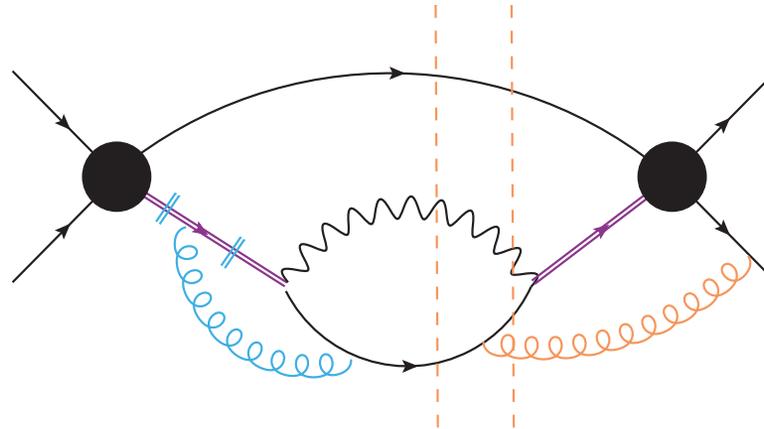
corrections to decay (soft and coll singularities):

$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^g \right|^2 \text{ combined with (hard) Wilson coefficient for decay is IR finite}$$

non-factorizable corrections (soft singularities only):

$$\int d\Phi_{n+1} 2 \text{Re} \left( \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^g \right) \left( \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^0 \right)^* \text{ plus soft virtual is IR finite}$$

## Comparison between (ET) and earlier (non-ET) NLO calculations [Campbell et.al, Yuan et.al.]



- ET virtual: hard part vanishes (at this order), soft part contributes and is included
- ET real: interference between production and decay radiation included  
after expansion this cancels corresponding virtual IR singularities
- non-ET real and virtual: not included
- ET: both top quarks can be off-shell, hard and soft part contribute
- non-ET: one top is always on-shell

## 7 TeV LHC 't'-channel:

 $m_t = 171.3 \text{ GeV}$ , MSTW 2008 NLO pdf,  $m_t/4 \leq \mu \leq m_t$ 

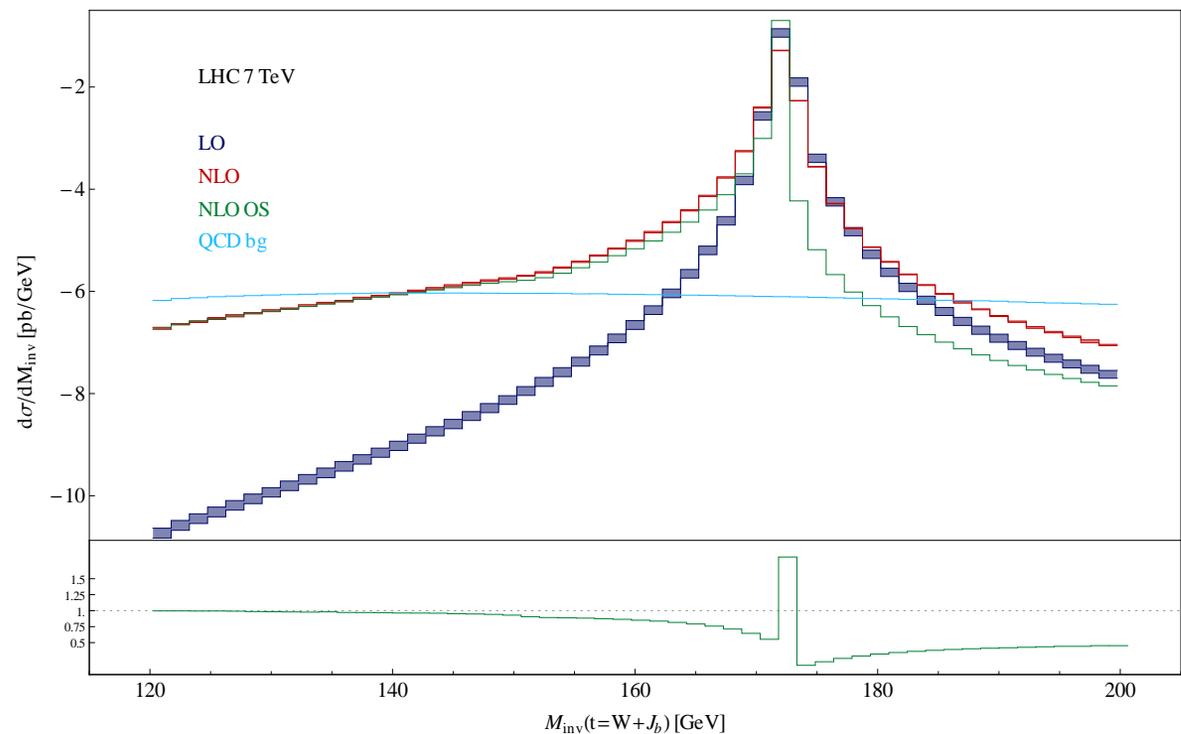
 define jets:  $k_{\perp}$  cluster algorithm  $\Rightarrow$  cuts on  $p_{\perp}(J_b)$ ,  $p_{\perp}(J_q)$ ,  $E_{\perp}$ ,  $p_{\perp}(\ell)$ 

 top window:  $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$ 

invariant mass of 'top'

$$M_{\text{inv}}^2 \equiv (p(J_b) + p(W))^2$$

effects large around  
the peak, but small for  
observables inclusive  
enough in  $M_{\text{inv}}$



## 7 TeV LHC 't'-channel:

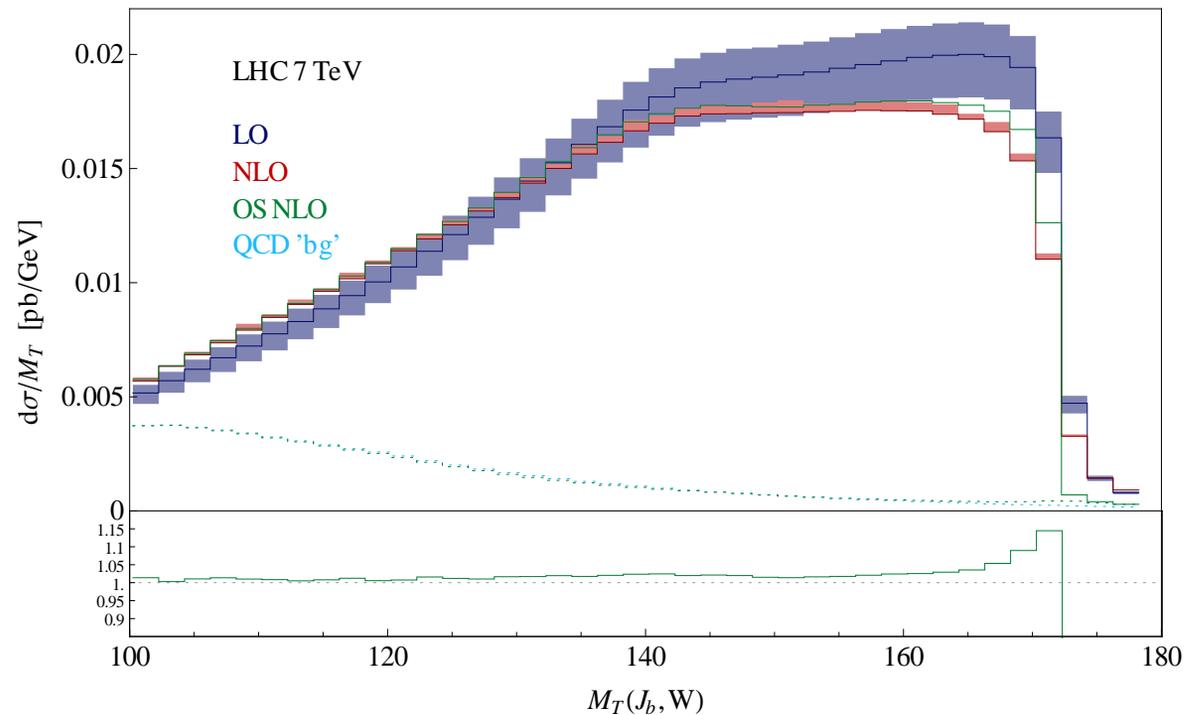
 $m_t = 171.3 \text{ GeV}$ , MSTW 2008 NLO pdf,  $m_t/4 \leq \mu \leq m_t$ 

 define jets:  $k_{\perp}$  cluster algorithm  $\Rightarrow$  cuts on  $p_{\perp}(J_b)$ ,  $p_{\perp}(J_q)$ ,  $E_{\perp}$ ,  $p_{\perp}(\ell)$ 

 top window:  $150 \text{ GeV} < \sqrt{(p(J_b) + p(\ell) + p(\nu))^2} < 200 \text{ GeV}$ 

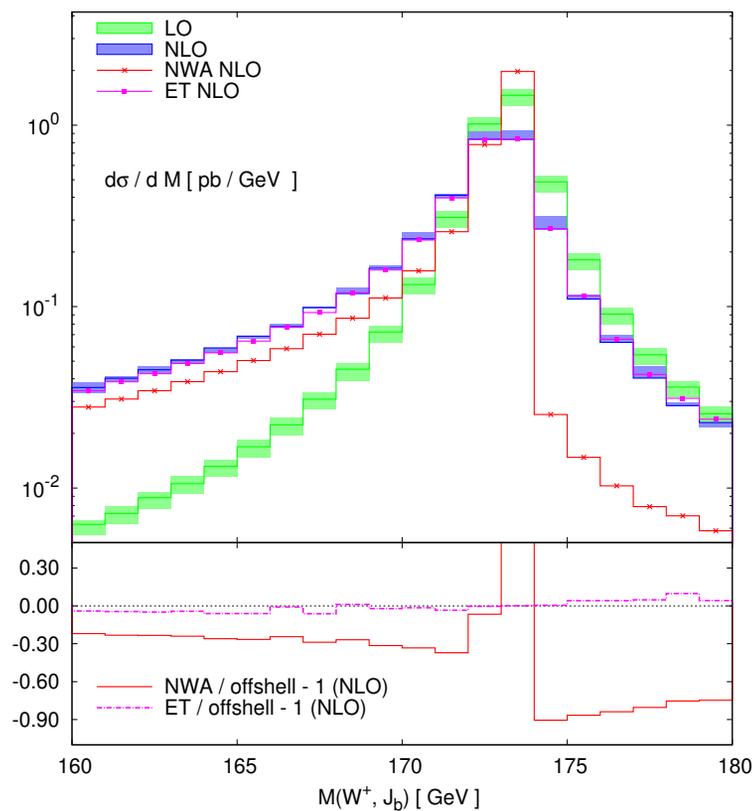
transverse mass of 'top'

$$M_T^2 = \left( \sum E_T \right)^2 - \left( \sum \vec{p}_T \right)^2$$

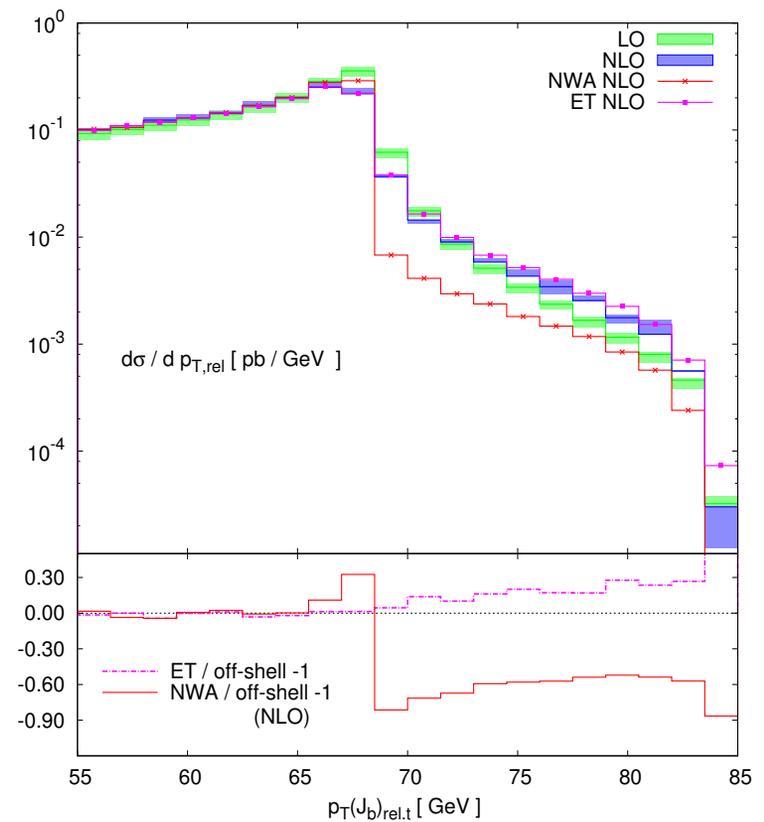
 effects tiny except at  
edges of distributions


comparison EFT approach vs complex mass scheme calculation  $\Rightarrow$  good agreement  
 [Papanastasiou et al. 1305.7088]

invariant mass

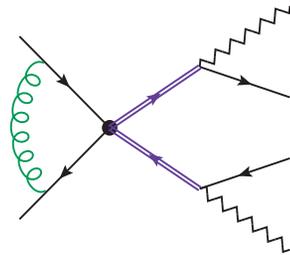


relative transverse b-jet momentum

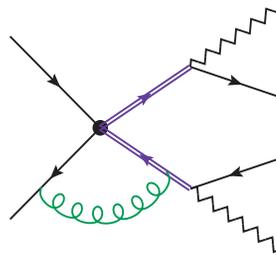


## SM $t\bar{t}$ theory status

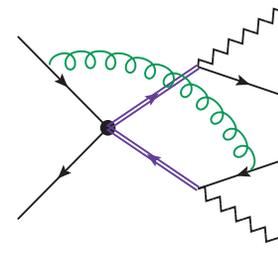
- fully exclusive known at  $\sim$  one-loop
  - electroweak corrections known [Bernreuther et al., Kuhn et al.]
  - spin correlations included [Bernreuther et al., Melnikov et al.]
  - full one-loop  $2 \rightarrow 4$  computed [Denner et al., Bevilacqua et al.]
  - included in MC@NLO and POWHEG [Frixione, Nason, Webber . . . . .]
  - two-loop corrections on their way . . .
- inclusive cross section(s) known at  $\sim$  two-loop
  - two-loop known [Czakon et al.]
  - bound-state effects computed [Hagiwara et al., Kiyo et al.]
  - non-factorizable corrections computed [Beenakker et al.]
  - resummation of logs under control [Ahrens et al, Beneke et al . . .]

virtual correction  $q\bar{q} \rightarrow W^+\bar{b}W^-b$ 

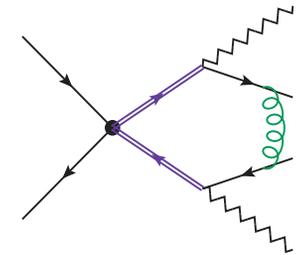
hard only



hard and soft



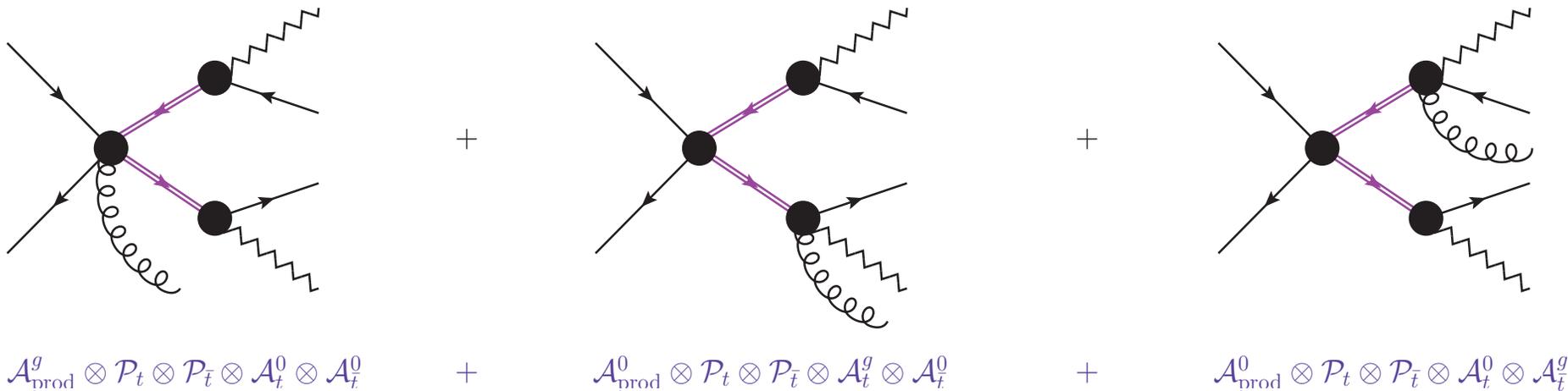
soft only



soft only

- hard parts  $\Rightarrow$  matching coefficients
- soft parts  $\Rightarrow$  explicit diagrams in ET
- integrals with more than 4 legs only needed in soft approximation
- hard integrals:  $\left(\frac{\mu^2}{s}\right)^\epsilon \Rightarrow$  hard scale
- soft integrals:  $\left(\frac{\mu^2}{\Delta_t m_t}\right)^\epsilon \Rightarrow$  soft scale

## top pair production: structure of real amplitude



## corrections to production

$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P}_t \otimes \mathcal{A}_t^0 \otimes \mathcal{P}_{\bar{t}} \otimes \mathcal{A}_{\bar{t}}^0 \right|^2 \quad (\text{finite if combined with virtual corr. to production})$$

## and corrections to decay

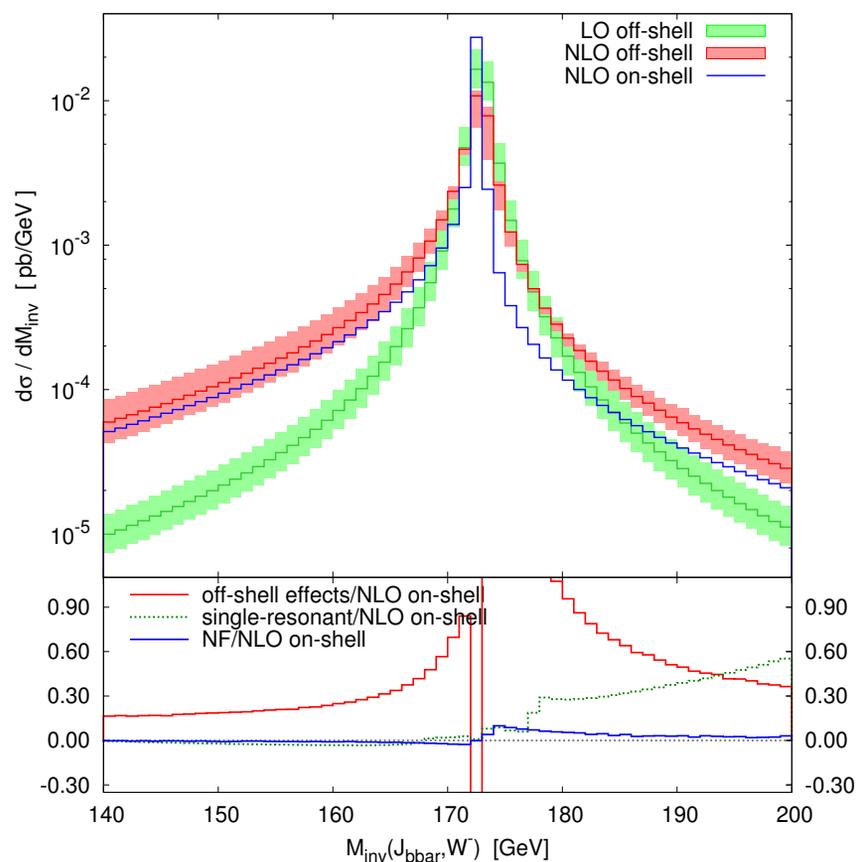
$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P}_t \otimes \mathcal{A}_t^g \otimes \mathcal{P}_{\bar{t}} \otimes \mathcal{A}_{\bar{t}}^0 \right|^2 \quad (\text{finite if combined with virtual corr. to top decay})$$

routinely taken into account [Bernreuther et al; Melnikov et al; Campbell et al;]

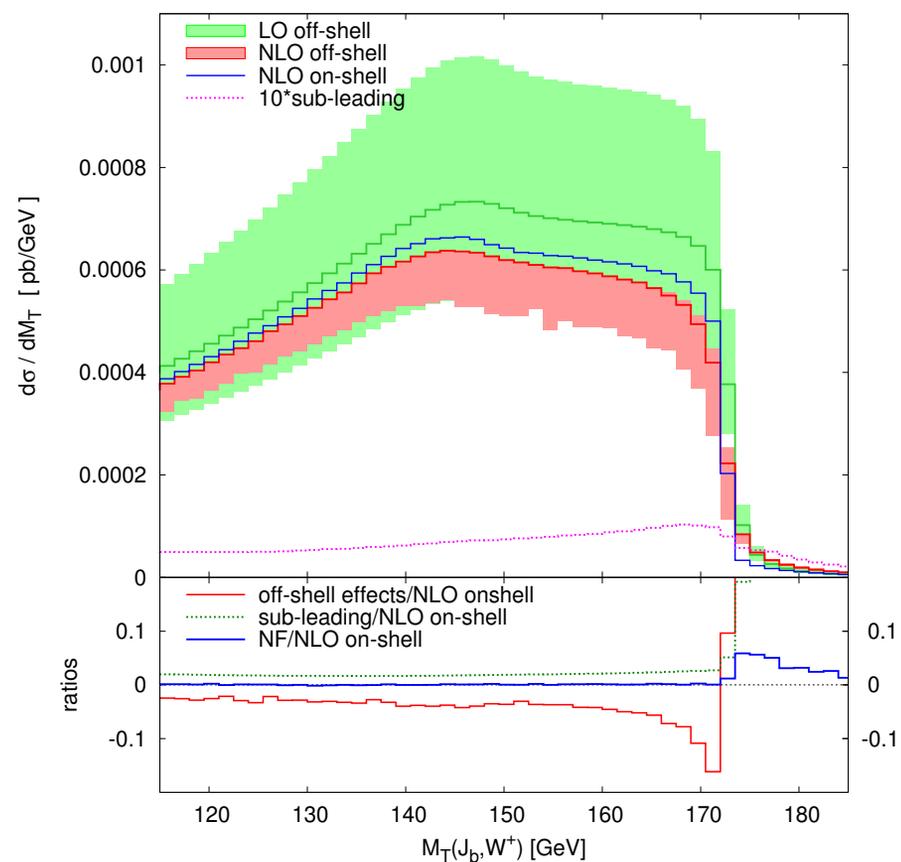
real interference contributions combined with soft virtual corrections are separately IR finite  
generally small, but study e.g. impact on  $m_t$  measurement [Falgari, Papanastasiou, AS]

sample results for Tevatron,  $q\bar{q} \rightarrow t\bar{t}$  only

invariant mass of 'top'



transverse mass of 'top'



again, effects small except at kinematic boundaries

## extraction of top mass from invariant mass

- consider mass scheme different from pole mass  $m_t$ 
  - check scheme dependence
  - avoid infrared sensitivity of pole mass (renormalons)
- many possible choices
- example used here: potential subtracted mass  $m_{\text{PS}}$  [Beneke]

$$m_{\text{PS}}(\mu_{\text{PS}}) = m_t + \frac{1}{2} \int_{q < \mu_{\text{PS}}} \frac{d^3 \vec{q}}{(2\pi)^3} V_{\text{coul}}(q) \quad \text{with} \quad \mu_{\text{PS}} \sim m \alpha_s \sim \delta^{1/2}$$

- express everything in terms of  $m_{\text{PS}}$

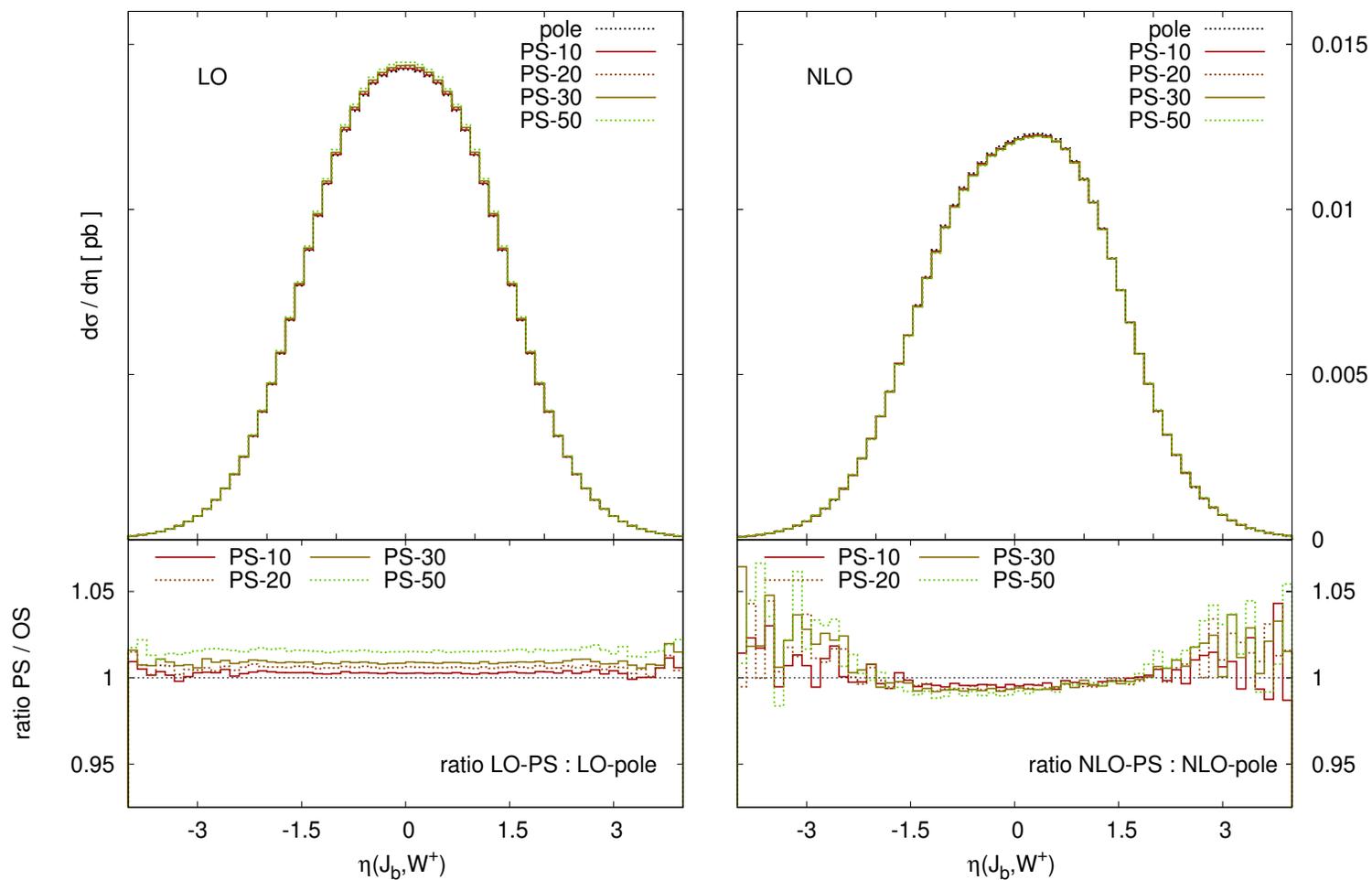
$$m_t = m_{\text{PS}}(\mu_{\text{PS}}) + \mu_{\text{PS}} \left[ \frac{\alpha_s}{2\pi} \delta_1 + \frac{\alpha_s^2}{(2\pi)^2} \delta_2 + \dots \right]$$

- (inverse of) propagator:

$$\underbrace{p^2 - m_{\text{PS}}^2 + im_{\text{PS}}\Gamma}_{\sim \delta} - \underbrace{\frac{\alpha_s}{\pi} \delta_1 \mu_{\text{PS}} m_{\text{PS}}}_{\sim \delta} - \underbrace{\frac{\alpha_s^2}{2\pi^2} \delta_2 \mu_{\text{PS}} m_{\text{PS}} + \dots}_{\sim \delta^{3/2}}$$

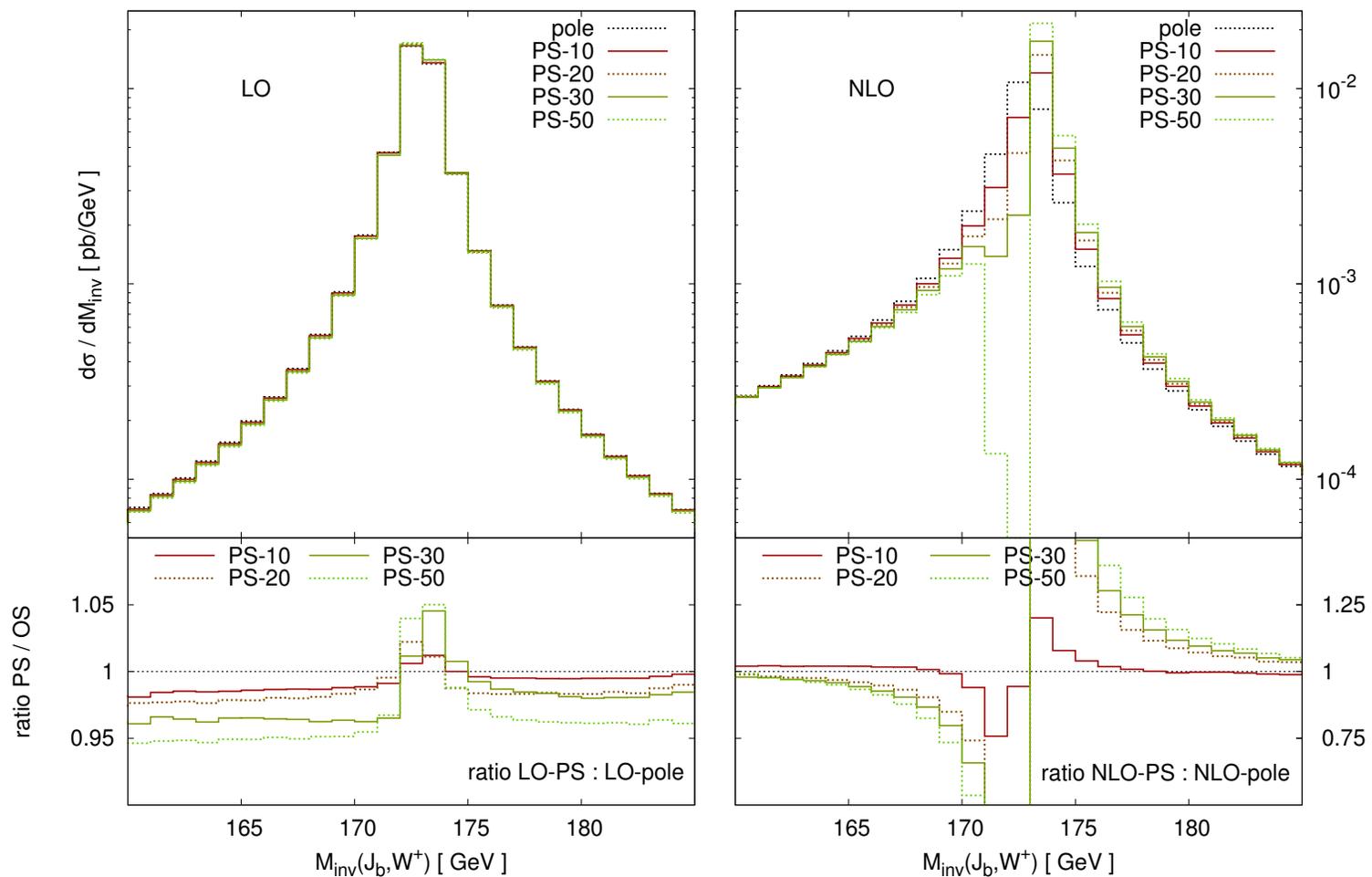
results in PS scheme  $\mu_{\text{PS}} \in \{0, 10, 20, 30, 50\}$  GeV

example of non-sensitive observable (pseudo-rapidity of 'top')



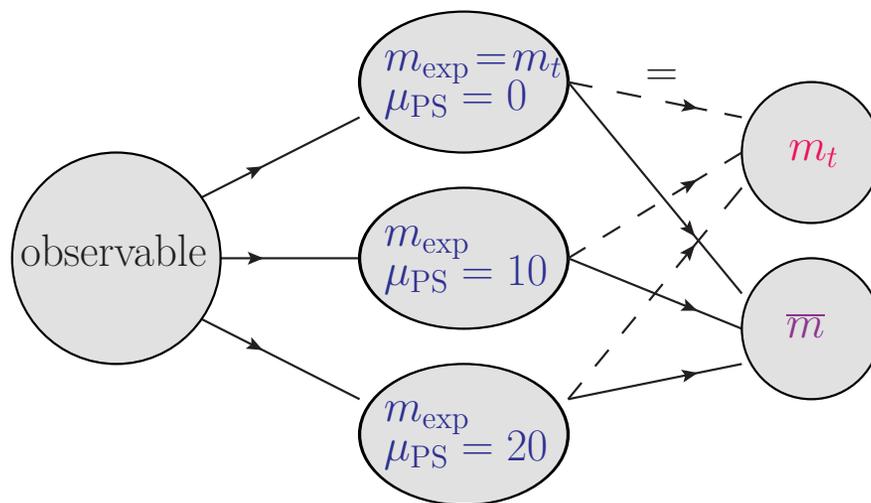
results in PS scheme  $\mu_{\text{PS}} \in \{0, 10, 20, 30, 50\}$  GeV

example of sensitive observable (invariant mass of 'top')  $\Rightarrow \mu_{\text{PS}} \lesssim 20$  GeV



consider scheme dependence of mass extraction

$\mu_{\text{PS}}$	LO			NLO		
	$m_{\text{exp}}$	$\overline{m}$	$m_t$	$m_{\text{exp}}$	$\overline{m}$	$m_t$
0	172.9	162.2	172.9	172.9	162.2	172.9
10	172.4	162.7	173.5	172.2	162.4	173.3
20	172.0	163.0	173.8	171.5	162.5	173.4



- conversion at NNNLO  
[Melnikov, Ritbergen]  
(+ Pade approximation)
- scheme ambiguity  
 $\sim 500 - 900$  MeV at LO
- scheme ambiguity  
 $\sim 400 - 500$  MeV at NLO
- $\overline{\text{MS}}$  scheme somewhat more stable

## Part II: approximate NNLO for fully differential $t\bar{t}$ production, including decay

- NNLL renormalization-group improved calculations for total cross section,  $d\sigma/(dM_{t\bar{t}} d\cos\theta)$  and  $d\sigma/(dp_T dy)$  available [Kidonakis et al, Ahrens et al. . . .]
- resummation can reproduce dominant (??) terms of fixed-order approach
- generalize resummed cross section to include decay of top quarks
- obtain approximate NNLO corrections to the production part of  $q\bar{q} \rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b}$  and  $gg \rightarrow t\bar{t} \rightarrow W^+ b W^- \bar{b}$  through expansion of resummed results
- implement these and match to fixed-order NLO to obtain 'improved' weight for parton-level Monte Carlo

$$d\sigma^{\text{approx NNLO}} = d\sigma^{\text{NLO}} + \alpha_s^2 d\sigma_2^{\text{resum}}$$

- **not** a strict approach, **not** a unique approach  
attempt to include most important features of fully differential NNLO corrections

## pair-invariant mass (PIM) kinematics

- $h_1(P_1) h_2(P_2) \rightarrow (t + \bar{t})(p_3 + p_4) + X(p_X)$
- soft limit  $z = (p_3 + p_4)^2 / \hat{s} \rightarrow 1$
- factorization of cross section [Ahrens et al.]

$$\frac{d\sigma}{dM_{t\bar{t}} d\cos\theta} \simeq \sum_{ij} \int \frac{dz}{z} \int \frac{dx}{x} f_{i/h_1}(x) f_{j/h_2}(\tau/(zx)) \left( \text{Tr} [\mathbf{H}_{ij} \cdot \mathbf{S}_{ij}] + \mathcal{O}(1-z) \right)$$

- plus distribution  $P_n(z) = \left[ \frac{\ln^n(1-z)}{1-z} \right]_+$

## one-particle inclusive (1PI) kinematics

- $h_1(P_1) h_2(P_2) \rightarrow t(p_3) + (\bar{t} + X)(p_4 + p_X)$
- soft limit  $s_4 = (p_4 + p_X)^2 - m_t^2 \rightarrow 0$
- factorization of cross section [Ahrens et al.]

$$\frac{d\sigma}{dp_T dy} \simeq \sum_{ij} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} f_{i/h_1}(x_1) f_{j/h_2}(x_2) \left( \text{Tr} [\mathbf{H}_{ij} \cdot \mathbf{S}_{ij}] + \mathcal{O}(s_4) \right)$$

- plus distribution  $P_n(s_4) = \left[ \frac{1}{s_4} \ln^n \left( \frac{s_4}{m_t^2} \right) \right]_+$

## parton-level Monte Carlo, including top decay

- compute modified hard function including top decay (in narrow-width approximation)
- soft functions and structure of renormalization group equations not affected
- obtain approximate NLO (for consistency checks only) and NNLO corrections by expansion in  $\alpha_s \rightarrow$  coefficients of plus distributions
- e.g. for PIM @ NNLL  $\rightarrow$  NNLO:

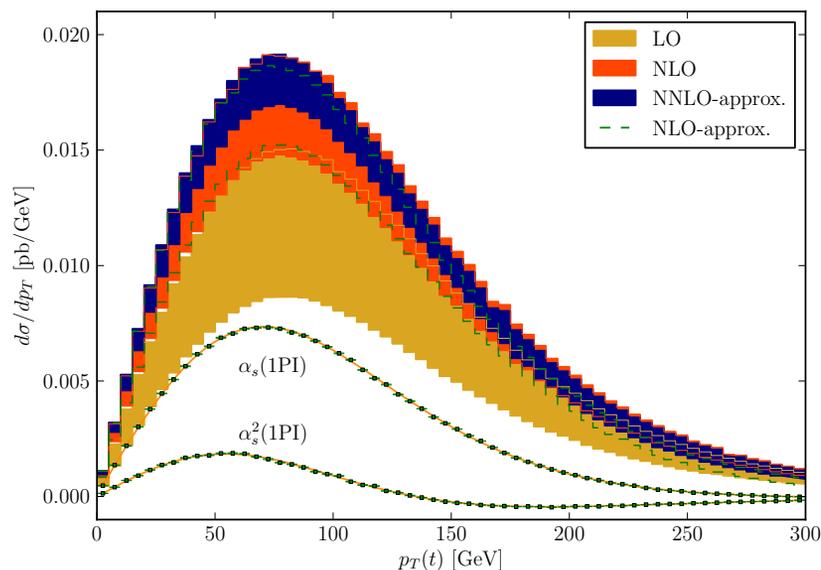
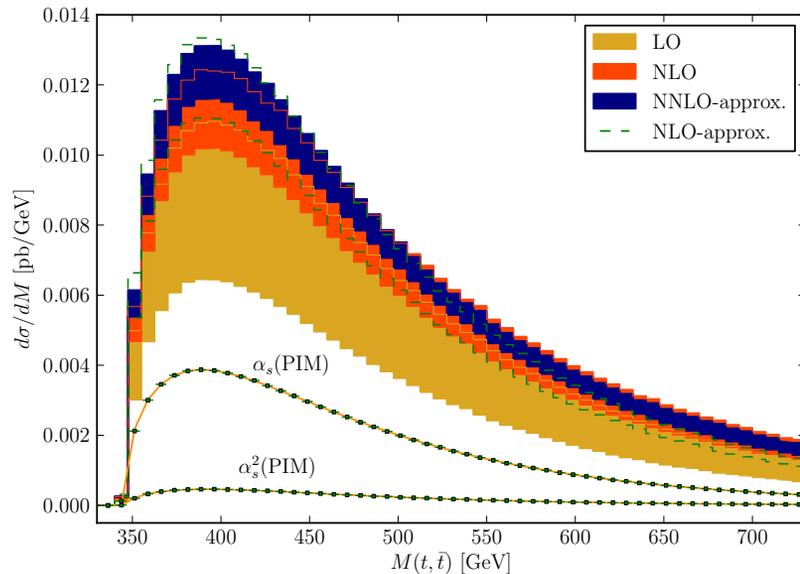
$$[\mathbf{H}_{ij} \cdot \mathbf{S}_{ij}] \sim D_3 P_3(z) + D_2 P_2(z) + D_1 P_1(z) + D_0 P_0(z) + C_0 \delta(1-z) + R(z)$$

- restore dependence on final-state particles  $\rightarrow$  weight of events in Monte Carlo

$$D_i(M_{t\bar{t}}, \cos \theta) \rightarrow D_i(\{p_i\})$$

- different resummation (PIM and 1PI) and different implementations due to treatment of subleading terms (e.g. in phase-space integration)
- take scale variation **and** variation over various implementations for estimate of theory error (take known NNLO total cross section as cross check of procedure)

## approx NNLO, cross checks, LHC 8TeV



compare  $M_{t\bar{t}}$  with [Ahrens et al.]

cluster final state partons into jets

reconstruct top  $t \doteq J_b + \ell + \nu$

here: no cuts whatsoever

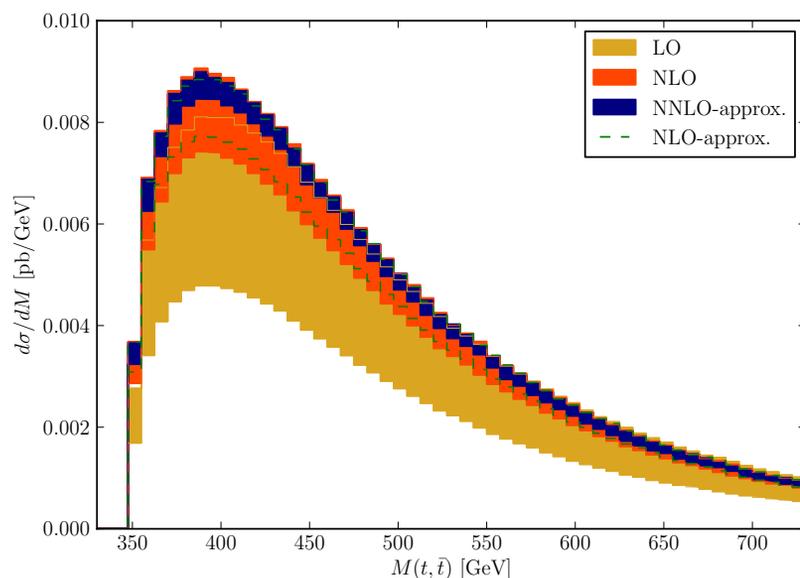
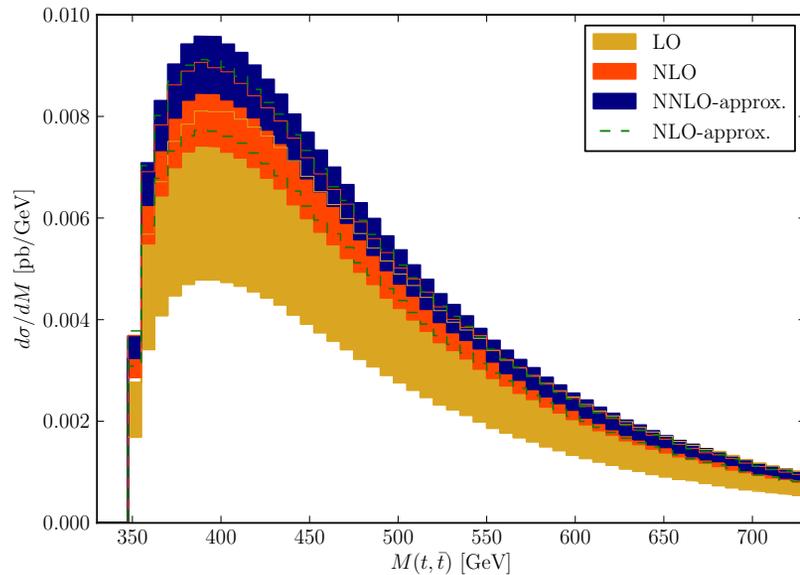
recover total cross section

compare  $p_T(t)$  with [Ahrens et al.]

stable perturbative behaviour

pdf: mstw08nlo (!)

theory error band: envelope of scale variation and phase-space implementations

realistic  $M_{t\bar{t}}$  distribution with (standard) cuts

cluster final state partons into jets

reconstruct top  $t \doteq J_b + \ell + \nu$

apply (whatever) cuts

compute  $M_{t\bar{t}}$  distribution

note: **no improvement** in treatment of decay (strict NLO)

theory error band:  $\{1\text{PI}, \text{PIM}\}_{\text{impl}}$

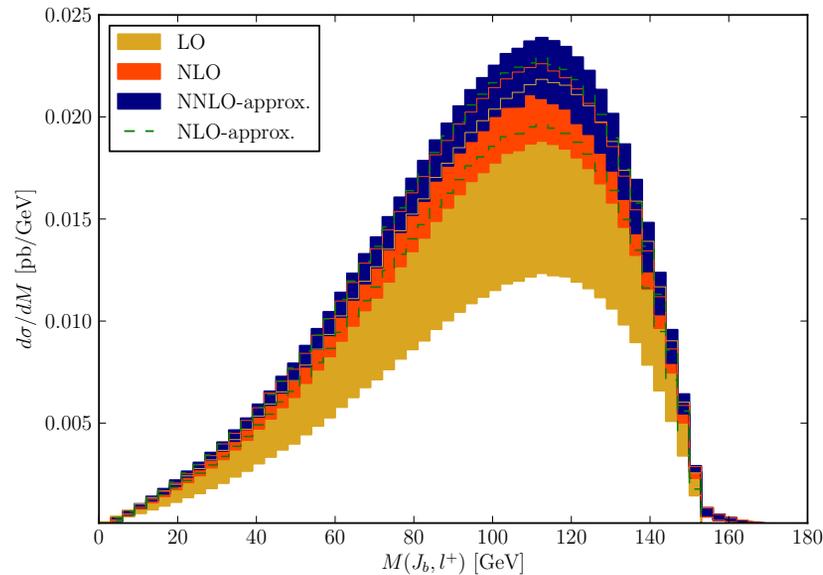
less conservative error estimate:

theory error band:  $\{1\text{PI}, \text{PIM}\}$

“wrong” resummation (1PI) gives remarkably consistent results with “correct” resummation (PIM)

pdf: mstw08nlo (!)

'go crazy' and compute other distributions with realistic cuts, e.g.  $M(J_b, \ell^+)$



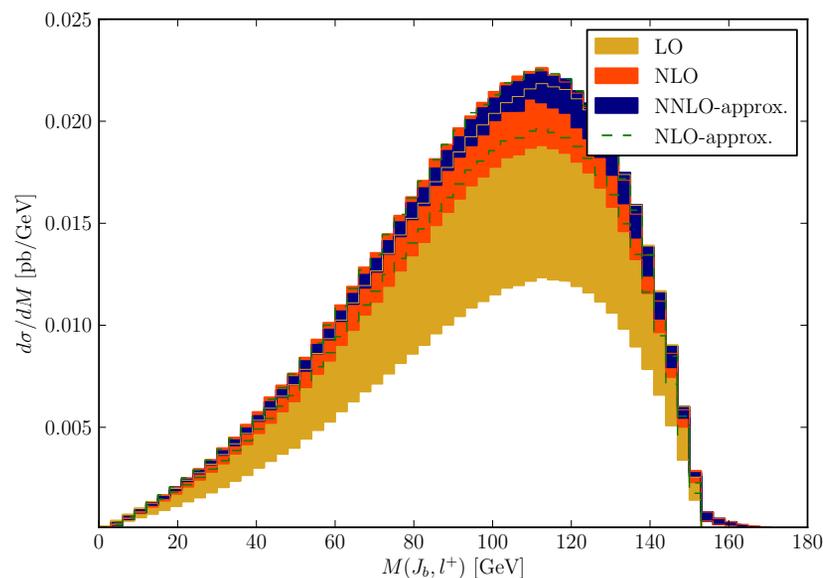
apply (whatever) cuts

compute  $M(J_b, \ell^+)$  @ approx NNLO

note: **no improvement** in treatment of decay (strict NLO)

pdf: mstw08nlo (!), approx NNLO band is lower for mstw08nnlo

theory error band:  $\{1PI, PIM\}_{impl}$



less conservative error estimate:

theory error band:  $\{1PI, PIM\}$

both resummations (1PI) and (PIM) give remarkably consistent results

- include off-shell effects at NLO for unstable particles using ET inspired approach
  - amounts to inclusion of non-factorizable (= soft) corrections (and all spin correlation effects)
  - combined with “standard” (= hard) corrections for production and decay
- applicable to unstable fermions, gauge bosons and scalars
- example single top:
  - off-shell effects  $\mathcal{O}(\alpha_s \delta)$  are small 1 – 2% for most observables
  - can be larger at kinematic end points
  - excellent agreement between ET and complex mass scheme calculations
- example top pair production:
  - off-shell effects small except for kinematic edges
  - impact on  $m_t$  determination needs to be under control for  $\delta m_t / m_t < 1\%$
- approximate NNLO for fully differential top pair cross section, including the decay
  - PIM and 1PI implemented in fully differential parton-level Monte Carlo
  - decay only at NLO