Variable flavor number schemes (VFNS) in QCD

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based on work with
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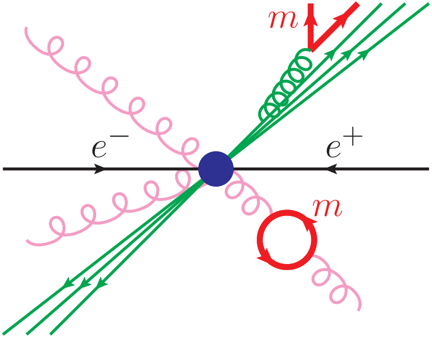
Motivation

- nowadays: precision physics in QCD processes
  $\rightarrow$ quark mass effects important (e.g. in DIS: precise extraction of pdfs!)
- aim: incorporatation of massive quarks in collider processes with jets
  $\leftrightarrow$ missing: systematic treatment of virtual and real **secondary** massive quarks

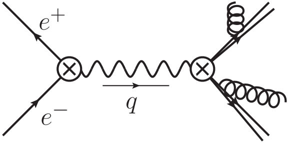


Outline

1. VFNS for the hadronic R-ratio
2. VFNS for DIS in the classical region $x \sim 1$
3. VFNS for DIS in the endpoint region $x \to 1$
4. VFNS for event shapes in the dijet region
5. Summary

Outline

Hadronic R-ratio for massless quark production



$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \mathrm{Im}\left[-\mathrm{i}\int \mathrm{dx}\, \mathrm{e}^{-\mathrm{iqx}} \langle 0|\mathrm{T}\,[\mathrm{j}^\mu(\mathrm{x})\mathrm{j}_\mu(0)]\,|\rangle\right]$$

- one relevant scale: c.o.m. energy $q^2 = Q^2$
- current conservation
  $\to$ UV divergences only related to strong coupling & field redefinitions
  $\to$ only running structure: α_s
- perturbative expansion (with $\overline{\mathrm{MS}}$-renormalized α_s with n_f light flavors)

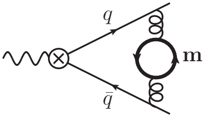
$$R_{n_f}[\alpha_s^{(n_f)}] = N_c \sum e_q^2 \left\{1 + \frac{\alpha_s^{(n_f)}(\mu)}{4\pi} r_1 + \left(\frac{\alpha_s^{(n_f)}(\mu)}{4\pi}\right)^2 \left[r_2^{(n_f)} - \beta_0 r_1 \ln\left(\frac{Q^2}{\mu^2}\right)\right]\right\}$$

$\to$ log minimized for $\mu \sim Q$

Massive quark contributions

$$R_{n_f}[\alpha_s^{(n_f)}] = N_c \sum e_q^2 \left\{ 1 + \frac{\alpha_s^{(n_f)}(\mu)}{4\pi} r_1 + \left(\frac{\alpha_s^{(n_f)}(\mu)}{4\pi} \right)^2 \left[r_2^{(n_f)} - \beta_0 r_1 \ln\left(\frac{Q^2}{\mu^2} \right) \right] \right\}$$

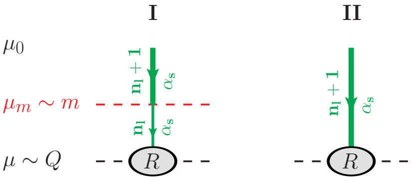
- virtual massive quark effects:



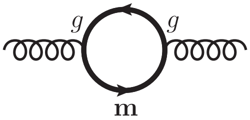
- aims for a VFNS (n_l massless + 1 massive flavor):
  - → stable perturbative prediction: resummation of all large logarithms $\ln\left(\frac{Q}{m}\right)$
  - → correct limits for R (decoupling for $m \to \infty$ + massless limit for $m \to 0$)
  - → continuous description with full mass dependence for arbitrary hierarchies

  ⇒ use of proper renormalization schemes: CWZ-scheme

  `Collins, Wilczek, Zee (1978)`



Renormalization of the strong coupling: Massive quark contributions



$$\Pi(0) = \frac{\alpha_s T_F}{3\pi}\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{m^2}\right) - \gamma_E + \ln(4\pi) + \mathcal{O}(\epsilon)\right]$$

Renormalization for $\alpha_s \equiv g^2/4\pi$:

$$\alpha_s = \mu^{2\epsilon} Z_\alpha^{\overline{\text{MS}}} \alpha_s^{\overline{\text{MS}}}(\mu) = \mu^{2\epsilon} Z_\alpha^{\text{OS}} \alpha_s^{\text{OS}}(\mu) = \ldots$$

$\rightarrow$ $\overline{\text{MS}}$ renormalization: $Z_\alpha^{\overline{\text{MS}}} = 1 + \frac{\alpha_s^{\overline{\text{MS}}} T_F}{3\pi}\frac{1}{\epsilon} + \text{const} + \ldots$ (default for massless partons)

$\rightarrow$ OS (on-shell) renormalization: $Z_\alpha^{\text{OS}} = 1 + \Pi(0) + \ldots$

Anomalous dimension for resummation of logarithms (RGE)

$$\beta^{\overline{\text{MS}}} = \frac{d\alpha_s^{\overline{\text{MS}}}}{d\ln\mu^2} + \epsilon\alpha_s^{\overline{\text{MS}}} = -\mu^{2\epsilon}\alpha_s^{\overline{\text{MS}}}\frac{d\ln Z_\alpha^{\overline{\text{MS}}}}{d\ln\mu^2} = \beta^{(n_l+1)} \quad \rightarrow \alpha_s^{\overline{\text{MS}}} \equiv \alpha_s^{(n_l+1)}(\mu)$$

$$\beta^{\text{OS}} = \frac{d\alpha_s^{\text{OS}}}{d\ln\mu^2} + \epsilon\alpha_s^{\text{OS}} = -\mu^{2\epsilon}\alpha_s^{\text{OS}}\frac{d\ln Z_\alpha^{\text{OS}}}{d\ln\mu^2} = \beta^{(n_l)} \quad \rightarrow \alpha_s^{\text{OS}} \equiv \alpha_s^{(n_l)}(\mu)$$

- I. OS renormalization for massive quark contributions to α_s: $\alpha_s^{(n_l)}(\mu \sim Q)$

$$R_{n_l,m}[\alpha_s^{(n_l)}(\mu)] \xrightarrow{m \gg Q} R_{n_l}[\alpha_s^{(n_l)}(\mu)] + \mathcal{O}\left(\frac{Q^2}{m^2}\right) \checkmark$$

$$R_{n_l,m}[\alpha_s^{(n_l)}(\mu)] \xrightarrow{m \ll Q} R_{n_l+1}[\alpha_s^{(n_l)}(\mu)] + \left(\frac{\alpha_s^{(n_l)}(\mu)}{4\pi}\right)^2 \left(\beta_0^{(n_l+1)} - \beta_0^{(n_l)}\right) r_1 \ln\left(\frac{m^2}{\mu^2}\right)$$

$\Rightarrow$ appropriate for $m \gtrsim Q$

- II. $\overline{\text{MS}}$ renormalization for massive quark contributions to α_s: $\alpha_s^{(n_l+1)}(\mu \sim Q)$

$$R_{n_l,m}[\alpha_s^{(n_l+1)}(\mu)] \xrightarrow{m \ll Q} R_{n_l+1}[\alpha_s^{(n_l+1)}(\mu)] + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \checkmark$$

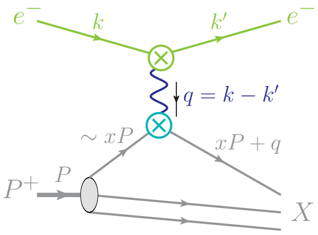
$$R_{n_l,m}[\alpha_s^{(n_l+1)}(\mu)] \xrightarrow{m \gg Q} R_{n_l}[\alpha_s^{(n_l+1)}(\mu)] - \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{4\pi}\right)^2 \left(\beta_0^{(n_l+1)} - \beta_0^{(n_l)}\right) r_1 \ln\left(\frac{m^2}{\mu^2}\right)$$

$\Rightarrow$ appropriate for $m \lesssim Q$

- for arbitrary $m \leftrightarrow Q$: use I. for $m \gtrsim Q$ and II. for $m \lesssim Q$
  $\Rightarrow$ matching at $\mu \sim Q \sim m$ = decoupling relation for α_s
  $\Rightarrow$ exact mass-dependence + correct limiting behavior

Outline

Deep inelastic scattering: $e^- P^+ \to e^- X$



- two relevant scales: $q^2 = -Q^2$, $\Lambda_{\rm QCD} \sim M_P$
- $x = \frac{Q^2}{2P\cdot q}$: $0 \leq x \leq 1$, classical region: $1 - x \sim \mathcal{O}(1)$

$$\frac{d\sigma}{dQ^2 dx} \sim W_{\mu\nu} L^{\mu\nu}$$

$\to$ leptonic tensor $L^{\mu\nu}$: purely electromagnetic (up to higher orders in $\alpha_{\rm em}$)
$\to$ hadronic tensor $W_{\mu\nu}$ $\leftrightarrow$ structure functions $F_{1,2}$

$$W_{\mu\nu} = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{1}{P\cdot q}\left(P_\mu + \frac{q_\mu}{2x}\right)\left(P_\nu + \frac{q_\nu}{2x}\right) F_2(x, Q^2)$$

Factorization in DIS

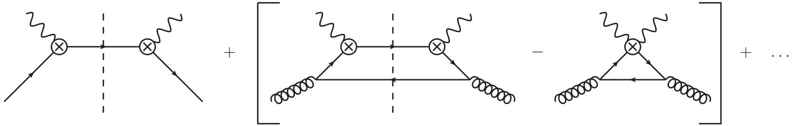
factorization = separation of quantum fluctuations at different energy scales
$\to$ conveniently achieved with effective field theories

Factorization theorem of form factors for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{i,j}(\mu_H) \otimes U^{\Phi}_{j,k}(\mu_H, \mu_\Phi) \otimes \Phi_{j/P}(\mu_\Phi) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)$$

Collins, Soper, Sterman (1988), Bauer et al.(2002)

$\to$ hard function $H_{i,j}(\mu_H \sim Q)$: difference between QCD and low-energy EFT (=SCET)



$\to$ parton distribution function (pdf) $\Phi_{j/P}(\mu_\Phi \sim \Lambda_{\text{QCD}})$: nonperturbative!

$$\Phi_{k/P}(x, \mu_\Phi) = \langle P^+ | \mathcal{O}_k(x, \mu_\Phi) | P^+ \rangle$$

$\to$ logs $\ln(\frac{\mu_H}{\mu_\Phi})$ resummed via RG factor $U^{\Phi}_{j,k}(\mu_H, \mu_\Phi)$ (suppressed in the following)

Mass effects in DIS

Factorization theorem of form factors for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{i,j}(\mu_H) \otimes \Phi_{k/P}(\mu_\Phi)$$

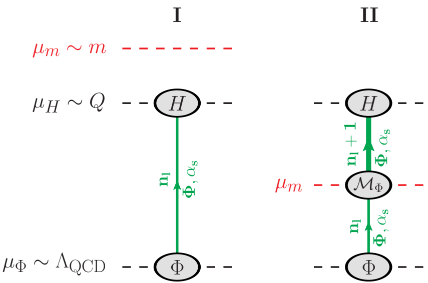
How to incorporate heavy quark mass effects ($m \gg \Lambda_{\rm QCD}$)?

$\rightarrow$ resummation of all logarithms $\ln\left(\frac{Q}{\Lambda_{\rm QCD}}\right)$, $\ln\left(\frac{m}{\Lambda_{\rm QCD}}\right)$, $\ln\left(\frac{Q}{m}\right)$

$\rightarrow$ correct limits for $H_{i,j}$ (decoupling for $m \rightarrow \infty$ + massless limit for $m \rightarrow 0$)

$\rightarrow$ continuous description for arbitrary masses

$\Rightarrow$ ACOT scheme `Aivazis, Collins, Olness, Tung (1994)`

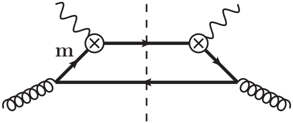


Massive quark corrections for $m \gtrsim Q$

$$m \gtrsim Q: \quad \boxed{F_{1,2} = \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},g} H^{I}_{i,j}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)}$$

use OS renormalization = low-momentum subtraction for pdfs and α_s

- evolution always with n_l flavors
- massive quark contributions vanish in SCET
- only full QCD contributions to $H_{i,j}$, e.g. at one-loop to $H^{I}_{Q,g}$:



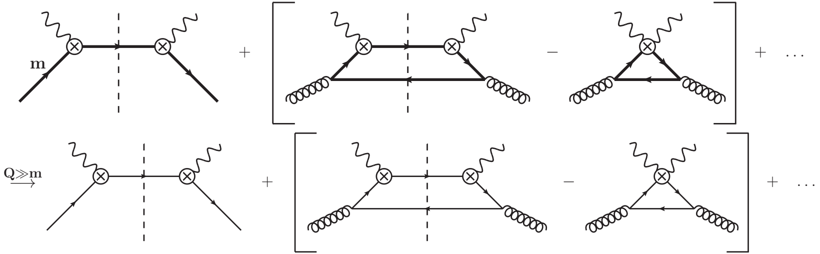
$\rightarrow$ for $m \gg Q$: automatic decoupling $\surd$
$\rightarrow$ for $m \ll Q$: unresummed logarithms $\sim \ln(m^2/Q^2)$ ↯

Massive quark corrections for $m \lesssim Q$

$$m \lesssim Q: \quad \boxed{F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},Q,\bar{Q},g} \sum_{k=q,\bar{q},g} H^{ll}_{i,j}(\mu_H) \otimes \mathcal{M}^{\phi}_{j,k}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)}$$

use $\overline{\mathrm{MS}}$ renormalization above the mass scale for pdfs and α_s

- evolution with $n_l + 1$ flavors above μ_m
- now massive quark contributions from full QCD and SCET to $H_{i,j}$, e.g. at one loop



$\rightarrow$ for $m \ll Q$: mass logarithms resummed, correct massless limit for $H_{i,j}$ $\surd$

Massive quark corrections for $m \lesssim Q$

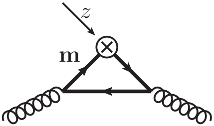
$$m \lesssim Q: \quad \boxed{F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \; \sum_{j=q,\bar{q},Q,\bar{Q},g} \; \sum_{k=q,\bar{q},g} H^{ll}_{i,j}(\mu_H) \otimes \mathcal{M}^{\phi}_{j,k}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)}$$

use $\overline{\mathrm{MS}}$ renormalization above the mass scale for pdfs and α_s

- evolution with $n_l + 1$ flavors above μ_m
- now massive quark contributions from full QCD and SCET to $H_{i,j}$
  $\rightarrow$ for $m \ll Q$: mass logarithms resummed, correct massless limit for $H_{i,j}$ $\surd$

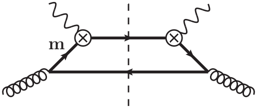
use OS renormalization below the mass scale for pdfs and α_s

- evolution with n_l flavors below μ_m
- scheme change $\leftrightarrow$ pdf matching $\mathcal{M}^{\phi}_{i,j}$, e.g. at one-loop for $\mathcal{M}^{\phi}_{Q,g} = \langle g|\mathcal{O}_Q|g\rangle$



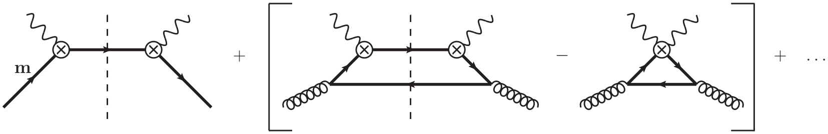
- I. $m \gtrsim Q$: $F_{1,2} = \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},g} H^{I}_{i,j}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to $H_{i,j}$ at one-loop

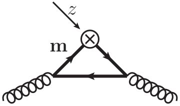


- II. $m \lesssim Q$: $F_{1,2} \sim \sum_{i=q,\bar{q},Q,\bar{Q}} \sum_{j=q,\bar{q},Q,\bar{Q},g} \sum_{k=q,\bar{q},g} H^{II}_{i,j}(\mu_H) \otimes \mathcal{M}^{\phi}_{j,k}(\mu_m) \otimes \Phi_{k/P}(\mu_\Phi)$

  $\rightarrow$ massive contributions to $H_{i,j}$ at one-loop



  $\rightarrow$ massive contributions to pdf matching $\mathcal{M}^{\phi}_{i,j}$ at one-loop

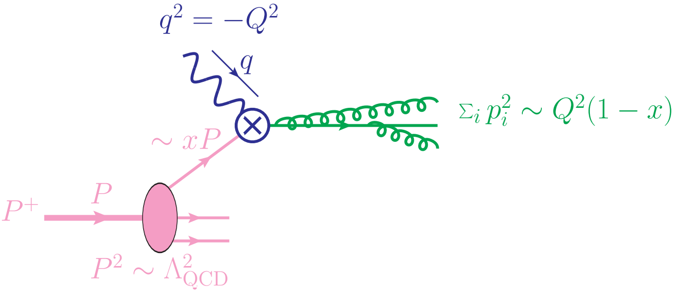


  $\Rightarrow$ continuous transition to I at $\mathcal{O}(\alpha_s)$ $\surd$

Outline

Scales for $x \to 1$

- $x \to 1$: experimentally barely accessible (small pdfs!)
  but: nontrivial factorization setup $\to$ interesting as a showcase for concepts
- use factorization theorem for $x \sim \mathcal{O}(1)$?
  unresummed logarithms in $H_{i,j}$: $\ln\left(\frac{Q^2(1-x)}{Q^2}\right) = \ln(1-x)$ ↯
  $\leftrightarrow$ additional scale: final state jet invariant mass $\sum_i p_i^2 = s \sim Q^2(1-x)$
- here: $1 - x \gg \Lambda^2_{\text{QCD}}/Q^2 \to s \gg \Lambda^2_{\text{QCD}}$



Massless factorization theorem for $x \to 1$

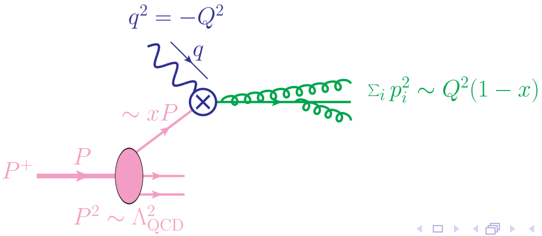
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi) \left[1 + \mathcal{O}(1-x)\right]$$

Sterman 1987, Manohar (2003), Becher, Neubert, Pecjak (2006), ...

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\rm DIS}(\mu_H) = |C(\mu_H)|^2$
  $\to$ $C(\mu_H)$: current matching between full QCD and SCET (local!)
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\rm DIS}(\mu_J)$
  $\to$ jet rate in terms of its invariant mass (nonlocal!)
- at $\mu_\Phi \sim \Lambda_{\rm QCD}$: pdf $\Phi_{q/P}(\mu_\Phi)$

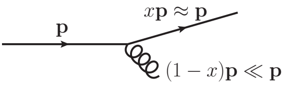


Massive quark effects

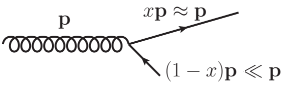
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\text{DIS}}(\mu_H) J_{\text{DIS}}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)$$

- Note: only flavor-diagonal contributions in matching and evolution



$\sim \frac{1}{1-x}$



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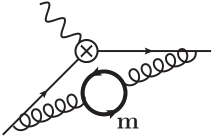
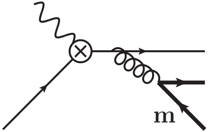
- for massive quarks: massive threshold corrections also flavor-diagonal
  $\Rightarrow$ no generation of massive quarks as initial state of the hard interaction
  $\Rightarrow$ only “secondary“ massive corrections to light quark inititated processes



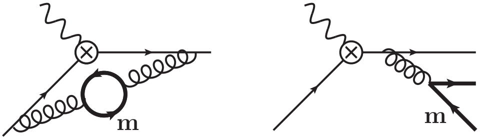


Massive quark effects

Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\mathrm{DIS}}(\mu_H) J_{\mathrm{DIS}}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)$$

- only “secondary“ massive corrections to light quark inititated processes



- aim: factorization setup with secondary massive quarks incorporating
  - $\to$ summation of large logarithms
  - $\to$ correct limits for H_{DIS}, J_{DIS} (decoupling for $m \to \infty$ + massless limit for $m \to 0$)
  - $\to$ continuous behavior in between with correct LO terms in the power counting

$\Rightarrow$ achieved by proper renormalization conditions

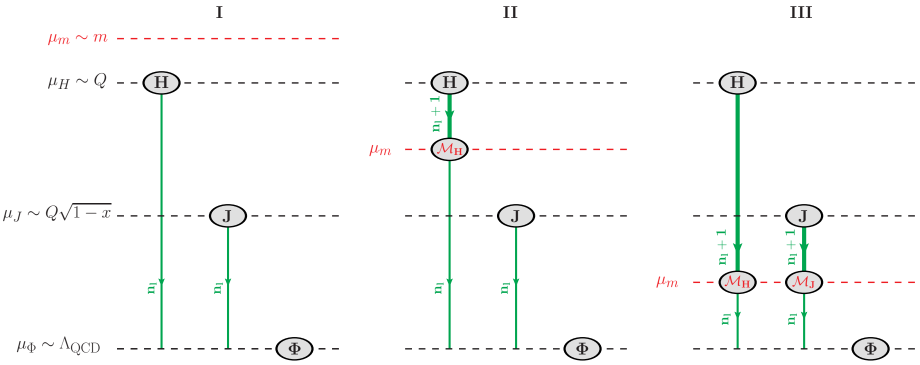
Gritschacher, Hoang, Jemos, P.P. (2013),
Gritschacher, Hoang, Jemos, Mateu, P.P. (2014)

Mass factorization: Overview

scaling hierarchies for a heavy quark ($m \gg \Lambda_{QCD}$) in the endpoint region ($1 - x \ll 1$):

$$\text{I. } m > Q\,, \quad \text{II. } Q > m > Q\sqrt{1-x}\,, \quad \text{III. } Q\sqrt{1-x} > m > \Lambda_{QCD}\,,$$

here: top-down evolution → final renormalization scale $\mu = \mu_\Phi$



Factorization theorems

- I. $m > Q$: use OS renormalization for current, jet function, pdf and α_s

$$\boxed{F_{1,2} \sim H_l^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)}$$

only full QCD contributions to hard current matching $\to H_l^{(n_l)}(\mu_H)$
$\Rightarrow$ decoupling for $m \gg Q$, but mass-singularities for $m \to 0$

Factorization theorems

- I. $m > Q$: use OS renormalization for current, jet function, pdf and α_s

$$\boxed{F_{1,2} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)}$$

- II. $Q > m > Q\sqrt{1-x}$: use $\overline{\mathrm{MS}}$ renormalization for current and α_s above μ_m

$$\boxed{\begin{array}{c} F_{1,2} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \\ \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi) \end{array}}$$

$\to$ finite subtractions to $H_I^{(n_l)}(\mu_H)$ due to different scheme (non-vanishing SCET diagrams!)

$\Rightarrow$ $H_{II}^{(n_l+1)}(\mu_H)$ has correct massless limit for $m \ll Q$

below μ_m: OS renormalization

$\to$ massive threshold contribution $\mathcal{M}_H(\mu_m) \leftrightarrow$ scheme change

Factorization theorems

- I. $m > Q$: use OS renormalization for current, jet fct, pdf and α_s

$$F_{1,2} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- II. $Q > m > Q\sqrt{1-x}$: use $\overline{\text{MS}}$ renormalization for current and α_s above μ_m

$$F_{1,2} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- III. $Q\sqrt{1-x} > m$: use $\overline{\text{MS}}$ renormalization for current, jet fct and α_s above μ_m

$$F_{1,2} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l+1)}(\mu_J) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

$\to$ modification of the jet function due to massive quark contributions
$\Rightarrow$ correct massless limit for $m \ll Q\sqrt{1-x}$

$$J^{(n_l+1)}(s, m, \mu_J) = J_0^{(n_l+1)}(s, \mu_J) + \delta J_m^{\text{dist}}(s, m, \mu_J) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m)$$

below μ_m: OS renormalization
$\to$ massive threshold contribution $\mathcal{M}_J(\mu_m) \leftrightarrow$ scheme change

Massive threshold corrections

Example: threshold correction in jet sector

bare jet function:

$$J^{\text{bare}} = Z_J^{OS} \otimes J^{\text{OS}} = Z_J^{\overline{\text{MS}}} \otimes J^{\overline{\text{MS}}}$$

in OS renormalization:

$$J^{\text{OS}}(s, m, \mu) = J^{(n_l)}(s, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \gg s} J^{(n_l)}(s, \mu)$$

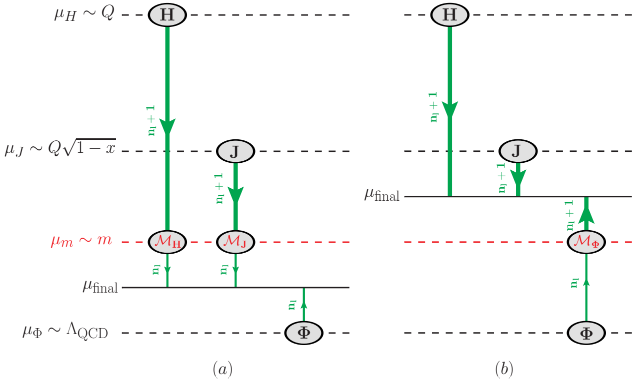
in $\overline{\text{MS}}$ renormalization:

$$J^{\overline{\text{MS}}}(s, m, \mu) = J^{(n_l+1)}(s, \mu) + \delta J_m^{\text{dist}}(s, m, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \ll s} J^{(n_l+1)}(s, \mu)$$

$$\Rightarrow \mathcal{M}_J(s, m, \mu) = J^{\text{OS}}(s, m, \mu) \otimes (J^{\overline{\text{MS}}}(s, m, \mu))^{-1}$$

$\to$ matching condition directly related to jet function
$\to$ continuity by construction

Consistency conditions for $Q\sqrt{1-x} > m > \Lambda_{\mathrm{QCD}}$



physical cross section independent of μ_{final} $\to$ (a) and (b) equivalent
$\to$ relation between evolution factors

$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l+1$$

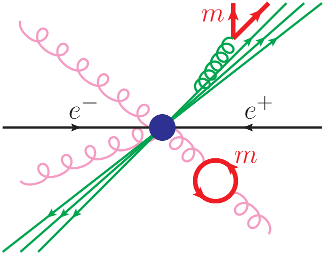
$\to$ relation between massive threshold contributions

$$\boxed{\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi}$$

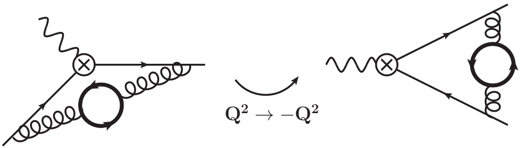
Outline

Event shapes

- goal: VFNS for differential distributions in e^+e^--collisions



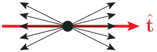
- in fact: similar to DIS (crossed process!)

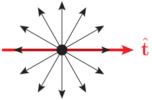


- event shape variables: geometric description of final state kinematics

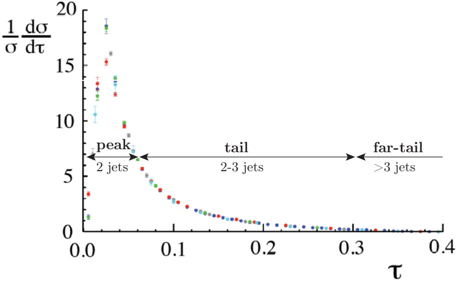
Event shapes: Thrust

- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i E_i} \in [0, \frac{1}{2}]$

back-to-back: $\tau \to 0$ 

isotropic: $\tau \to \frac{1}{2}$ 

- thrust distribution from LEP data ($e^+e^- \to jets$)



→ peak region ($\tau \sim \Lambda_{QCD}/Q$): typical jet scale $s = Q\Lambda_{QCD}$
→ tail region ($\tau \gg \Lambda_{QCD}/Q$): typical jet scale $s = Q^2\tau$

Factorization theorem for massless quarks

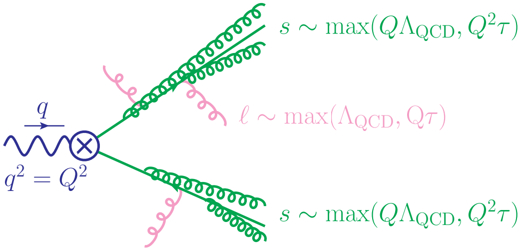
Massless factorization theorem for $\tau \ll 1$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H_\tau(\mu_H) J_\tau(\mu_J) \otimes S_\tau(\mu_S) \left[1 + \mathcal{O}(\tau)\right]$$

Berger, Kucs, Sterman (2003), Fleming, Hoang, Mantry, Stewart (2007), Bauer, Fleming, Lee, Sterman (2008),...

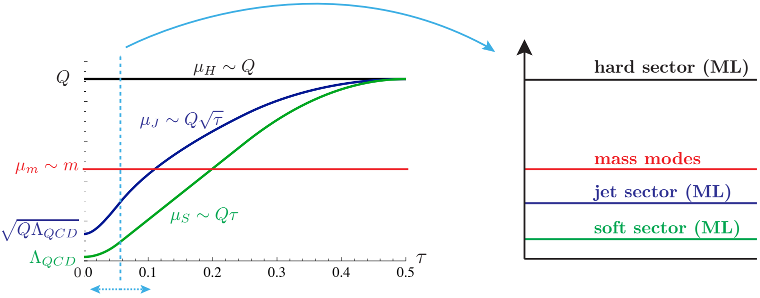
- compared to DIS: $H_\tau = H_{\mathrm{DIS}}(Q^2 \to -Q^2)$, $J_\tau \to J_{\mathrm{DIS}} \otimes J_{\mathrm{DIS}}$
- main difference concerns soft physics: $S_\tau \leftrightarrow \Phi_{i/P}$

  $\to$ in tail region ($\tau \gg \Lambda_{\mathrm{QCD}}/Q$): $\mu_S \sim Q\tau \gg \Lambda_{\mathrm{QCD}}$: $S_\tau = \hat{S} \otimes S^{\mathrm{model}}$



Scale hierarchies with massive quarks

- profile functions: Parametrization of renormalization scales in terms of thrust $\rightarrow$ continuous transition between peak, tail and far-tail region
- include massive quark effects $\rightarrow$ scales and hierarchies:



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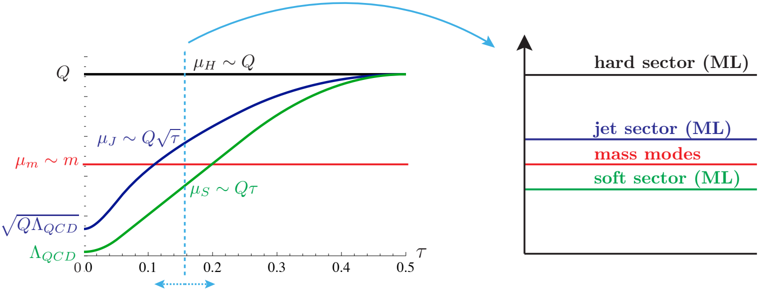

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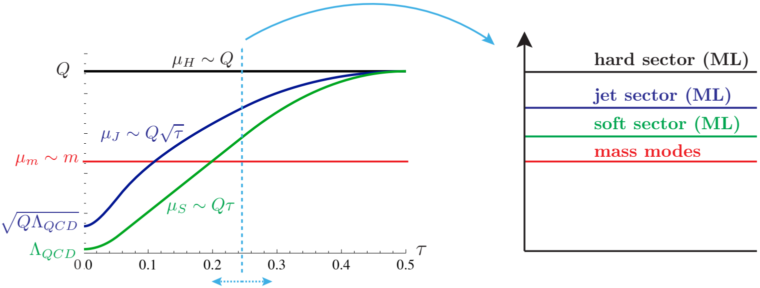
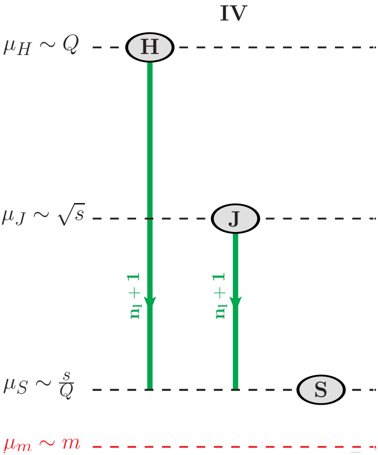


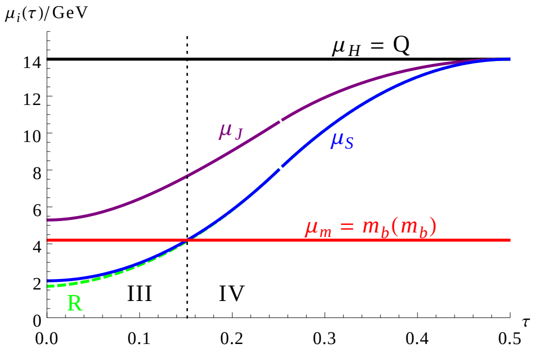
Setup for secondary massive quarks

- Setup for event shapes = Setup for DIS (same structure for factorization theorem)
- now: additional hierarchy possible $m < Q\tau \sim \mu_S$
  $\rightarrow \overline{\text{MS}}$ renormalization for all structures
  $\Rightarrow$ evolution always including massive flavor, massive contributions to soft function



Analysis of secondary massive bottom effects

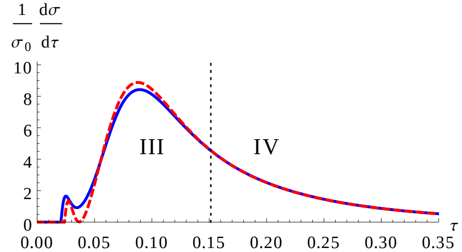
- analysis for $Q = 14, 22, 35$ GeV $\leftrightarrow$ bottom mass effects relevant
- ingredients for analysis at $\mathcal{O}(\alpha_s^2)$ in the dijet region $\tau \ll 1$ $\surd$
- numerical code (incl. a nonperturbative model function) $\surd$
- profile functions for $Q = 14$ GeV:



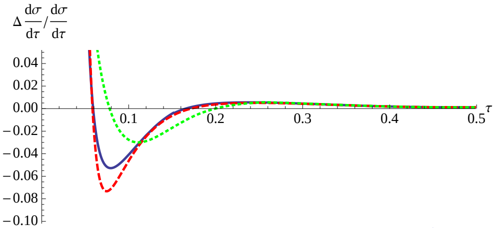
Secondary massive bottom effects for $Q = 14$ GeV

comparison between massless and massive thrust distribution
ML: $n_l = 5$, M: $n_l = 4$ & massive b ($m_b = 4.2$ GeV)

massive vs. massless



relative deviation massive vs. massless
$\mu_m = m$, $\mu_m = m/2$, $\mu_m = 2m$



Outline

1. VFNS for the hadronic R-ratio
2. VFNS for DIS in the classical region $x \sim 1$
3. VFNS for DIS in the endpoint region $x \to 1$
4. VFNS for event shapes in the dijet region
5. Summary

Summary & Outlook

- understanding of massive quark effects important in precision QCD
- VFNS cover different hierarchies between the mass scale and the kinematic scales
- use of proper renormalization schemes crucial for resummation of all logarithms and correct limiting behavior
- new: VFNS with final state jets
  $\rightarrow$ DIS for $x \rightarrow 1$: setup
  $\rightarrow$ thrust distribution for $\tau \rightarrow 0$: setup + numerical analysis

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Thank you!

Outline