On the loop-tree Duality

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LHCphenonet

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The Duality Hall of Fame

- Bierenbaum,I.
- Buchta, S
- Catani, S
- Chachamis, G
- Draggiotis, P
- Gleisberg, T
- Krauss, F
- M.I.
- Rodrigo, G
- Winter, J-C

- [Catani,Gleisberg,Krauss,Rodrigo,Winter, JHEP0809(2008)065]
- [Bierenbaum,Catani,Draggiotis,Rodrigo, JHEP1010(2010)073]
- [Bierenbaum,Buchta,Draggiotis,M.I.,Rodrigo, JHEP 1303(2013)025]
- [Buchta, Chachamis,Draggiotis, M.I.,Rodrigo, in preparation]

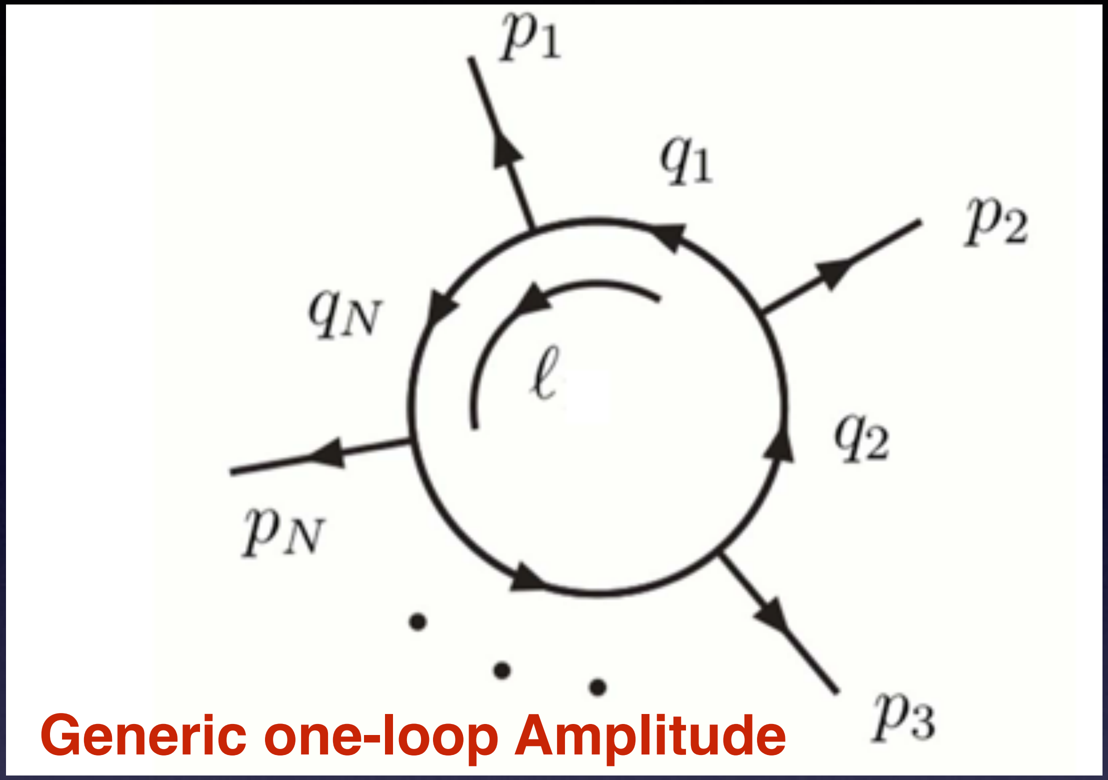
Outline of the talk

- Feynman Tree theorem and Duality theorem
- Duality theorem for higher loops
- Singularities of the loop integrands
- Example of cancellation of singularities

Outline

- Numerical Implementation
- Extensions ?
- Conclusions

Notation

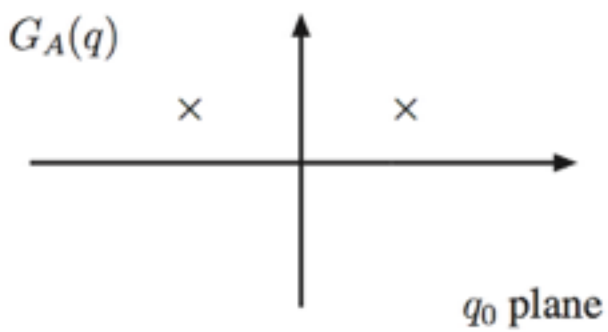


$$q_i = \ell + k_i \quad \text{with} \quad k_i = p_1 + \cdots + p_i$$

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} \quad \text{and} \quad \int_\ell = -i \int \frac{d^d\ell}{(2\pi)^d}$$

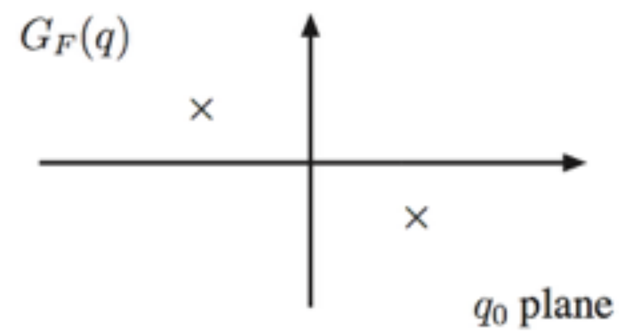
- All momenta outgoing

Feynman´s tree theorem and a Duality theorem



$$G_A(q) \equiv \frac{1}{q^2 - i0\, q_0}$$

Both poles are placed above the real axis, independently of the sign of the energy



$$G_F(q) \equiv \frac{1}{q^2 + i0}$$

$+i0$: positive frequencies are propagated forward in time, negatives backward

$$G_A(q) \;\; \equiv \;\; G_F(q) + \widetilde{\delta}\,(q)\ , \qquad \widetilde{\delta}\,(q) \equiv 2\pi\, i\, \theta(q_0)\, \delta(q^2) = 2\pi\, i\, \delta_+(q^2)$$

- Advanced one-loop integral vanishes
- Amplitude is given as an integral of Feynman propagators

$$0 \;\; = \;\; L_A^{(1)}(p_1, p_2, \dots, p_N) = \int_q \prod_{i=1}^{N} G_A(q_i) \;\; = \int_q \prod_{i=1}^{N} \left[G_F(q_i) + \widetilde{\delta}\,(q_i) \right]$$

↘

$$L^{(1)}(p_1, p_2, \ldots, p_N) = -\left[L^{(1)}_{1-\text{cut}}(p_1, p_2, \ldots, p_N) + \cdots + L^{(1)}_{\text{N}-\text{cut}}(p_1, p_2, \ldots, p_N) \right]$$

Feynman´s tree theorem

- ¨N-cut¨ is the term with N delta functions (For N>d terms vanish)
- The Duality produces the one-loop Amplitude with only one cut
- Apply the Cauchy residue theorem and select residues with positive energy and negative imaginary part

- Apply the Cauchy theorem for Residues with positive energy:

$$L^{(1)}(p_1, p_2, \ldots, p_N) \quad = \quad -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\mathrm{Im}\, q_0<0} \left[\prod_{j=1}^{N} G_F(q_j) \right]$$

- Notice that:

$$\left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0 \, \delta_+(q_i^2)$$

$$\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \left[\prod_{j=1}^{N} G_F(q_j) \right] \quad = \quad \left[\mathrm{Res}_{\{i-\mathrm{th\ pole}\}} \, G_F(q_i) \right] \left[\prod_{j \neq i} G_F(q_j) \right]_{\{i-\mathrm{th\ pole}\}}$$

which leads to:

$$\left[\prod_{j\neq i} G_F(q_j)\right]_{\{i-\text{th pole}\}} = \left[\prod_{j\neq i} \frac{1}{q_j^2 + i0}\right]_{\{q_i^2=-i0\}} = \prod_{j\neq i} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

We define the Dual propagator (notice the different i0 prescription)

$$G_D(q_i;q_j) := \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

- The first argument in the parenthesis stands for the cut propagator

- The new i0 prescription does not depend on the loop momentum!

- *n is a future-like momentum,* its dependence should (and does) cancel when summing all contributions

$$\eta_0 \geq 0, \ \eta^2 = \eta_\mu \eta^\mu \geq 0$$

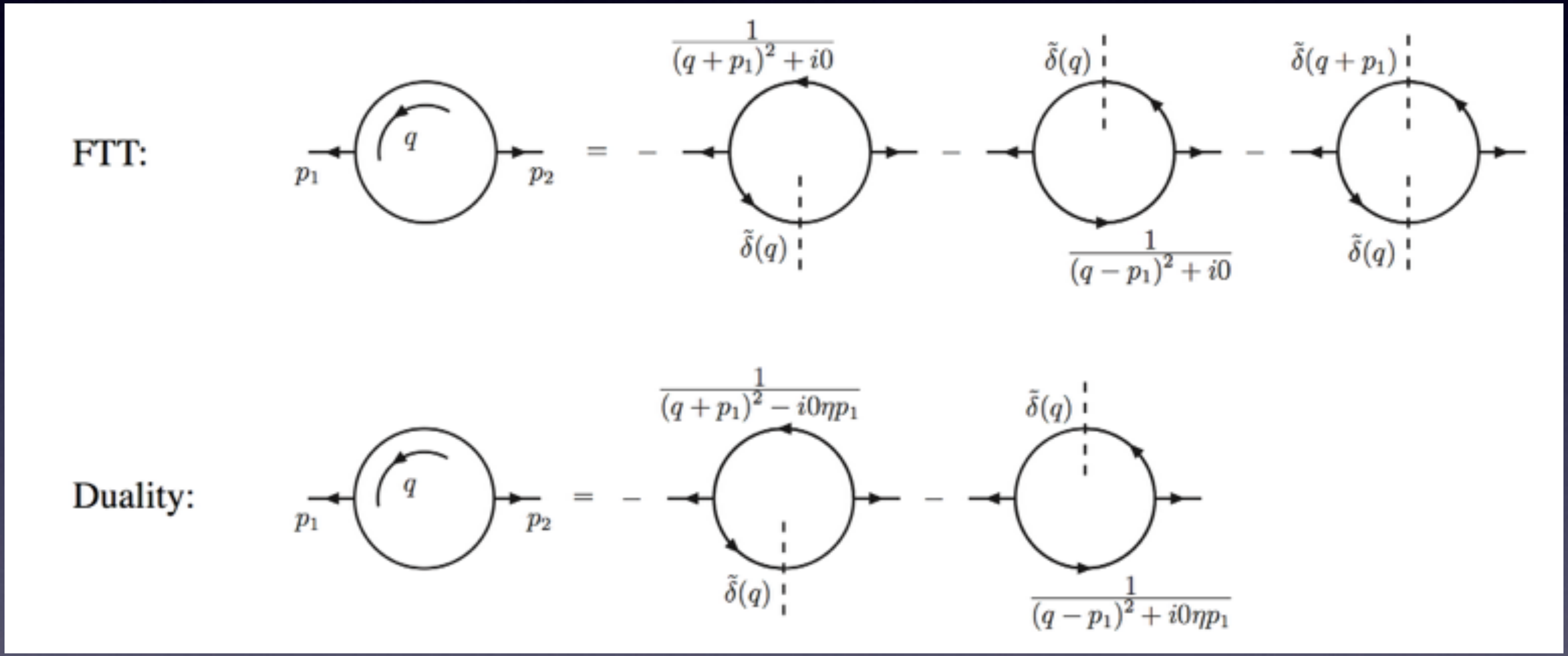
↘

$$L^{(1)}(p_1, ..., p_N) \quad = \quad -\sum_{i=1}^{N} \int_q \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

Loop-tree Duality theorem

- Virtual contributions take similar form to the real corrections (return to that later)

- <u>Example-The two point function</u>



The new i0 compensates for the absence of multi-cuts

Feynman and Dual propagators are related through

$$\tilde{\delta}(q_i)\ G_D(q_i;q_j) = \tilde{\delta}(q_i)\ \left[G_F(q_j) + \tilde{\theta}(q_j - q_i)\ \tilde{\delta}(q_j)\right], \qquad \tilde{\theta}(q) = \theta(\eta q)$$

which also connects the FTT and the Duality theorem

- Method can be extended to Amplitudes-(Unitary and local)

Duality theorem at higher orders

-Bierenbaum, Catani,Draggiotis, Rodrigo, JHEP 10(2010)073

-Bierenbaum, Buchta ,Draggiotis,M.I. Rodrigo, JHEP 03(2013)025

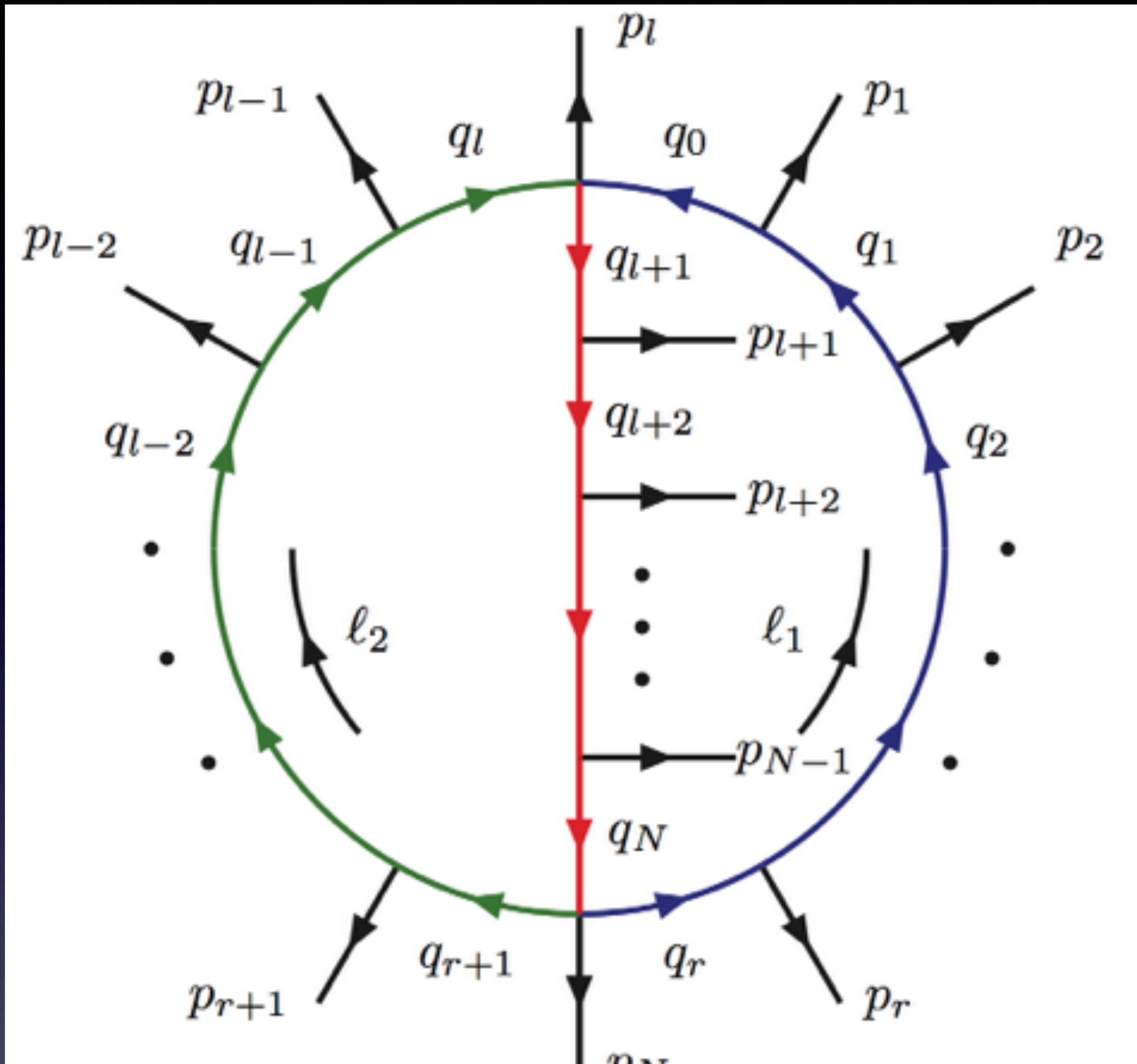
- Duality can be extended to higher loops

- Two options
  - Number of cuts= Number of loops- i0 prescription depends on loop momenta
  - Cut more to disconnect graphs to keep the i0 prescription as in the one loop case

- Define sets of propagators with the same loop momentum

The “Loop Lines”

$$\alpha_1 \equiv \alpha_1(\ell_1) \equiv \{0, 1, ..., r\} ,$$
$$\alpha_2 \equiv \alpha_2(\ell_2) \equiv \{r+1, r+2, ..., l\} ,$$
$$\alpha_3 \equiv \alpha_3(\ell_1 + \ell_2) \equiv \{l+1, l+2, ..., N\}$$



Generic two loop Amplitude

- For these sets of momenta notice:

$$G_{F(A,R)}(\alpha_k) = \prod_{i\in\alpha_k} G_{F(A,R)}(q_i)$$

$$G_D(\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}(q_i) \prod_{\substack{j\in\alpha_k \\ j\neq i}} G_D(q_i;q_j)$$

- From the following equation

$$G_A(\alpha_k) \quad = \quad G_F(\alpha_k) + G_D(\alpha_k)$$

we can find the expression for the dual propagators over a union of sets

$$G_D(\alpha_1 \cup \alpha_2 \cup \ldots \cup \alpha_N) = \sum_{\beta_N^{(1)} \cup \beta_N^{(2)} = \beta_N} \prod_{i_1 \in \beta_N^{(1)}} G_D(\alpha_{i_1}) \prod_{i_2 \in \beta_N^{(2)}} G_F(\alpha_{i_2}) \, .$$

The sum runs over all partitions of β_N into exactly two blocks $\beta_N^{(1)}$ and $\beta_N^{(2)}$ with elements $\alpha_i, i \in \{1,\ldots,N\}$, where we include the case: $\beta_N^{(1)} \equiv \beta_N, \beta_N^{(2)} \equiv \emptyset$.

We can derive the formula for the two-loop duality theorem

$$L^{(2)}(p_1, \dots, p_N) = \int_{\ell_1} \int_{\ell_2} [-G_D(\alpha_1) G_F(\alpha_2) G_D(\alpha_3) + G_D(\alpha_1)\, G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2) G_D(\alpha_3)]$$

- Each term includes two Dual propagators (= two cuts)

- However, using

$$G_D(\alpha_1 \cup \alpha_2) = \underbrace{G_D(\alpha_1)\, G_F(\alpha_2) + G_F(\alpha_1)\, G_D(\alpha_2)}_{\text{single cut}} + \underbrace{G_D(\alpha_1)\, G_D(\alpha_2)}_{\text{double cut}} .$$

we can cut more up to disconnected diagrams,

keeping the i0 prescription independent of any loop momentum

- The extension of the duality theorem to even higher loops is also known

- In the case of double poles, either use Cauchy theorem, either IBP´s

-Bierenbaum, Buchta ,Draggiotis,M.I. Rodrigo, JHEP 03(2013)025

Singularities of the loop integrands

- Motivation: Calculate (numerically) amplitudes

↘

  - Need to identify singular contributions

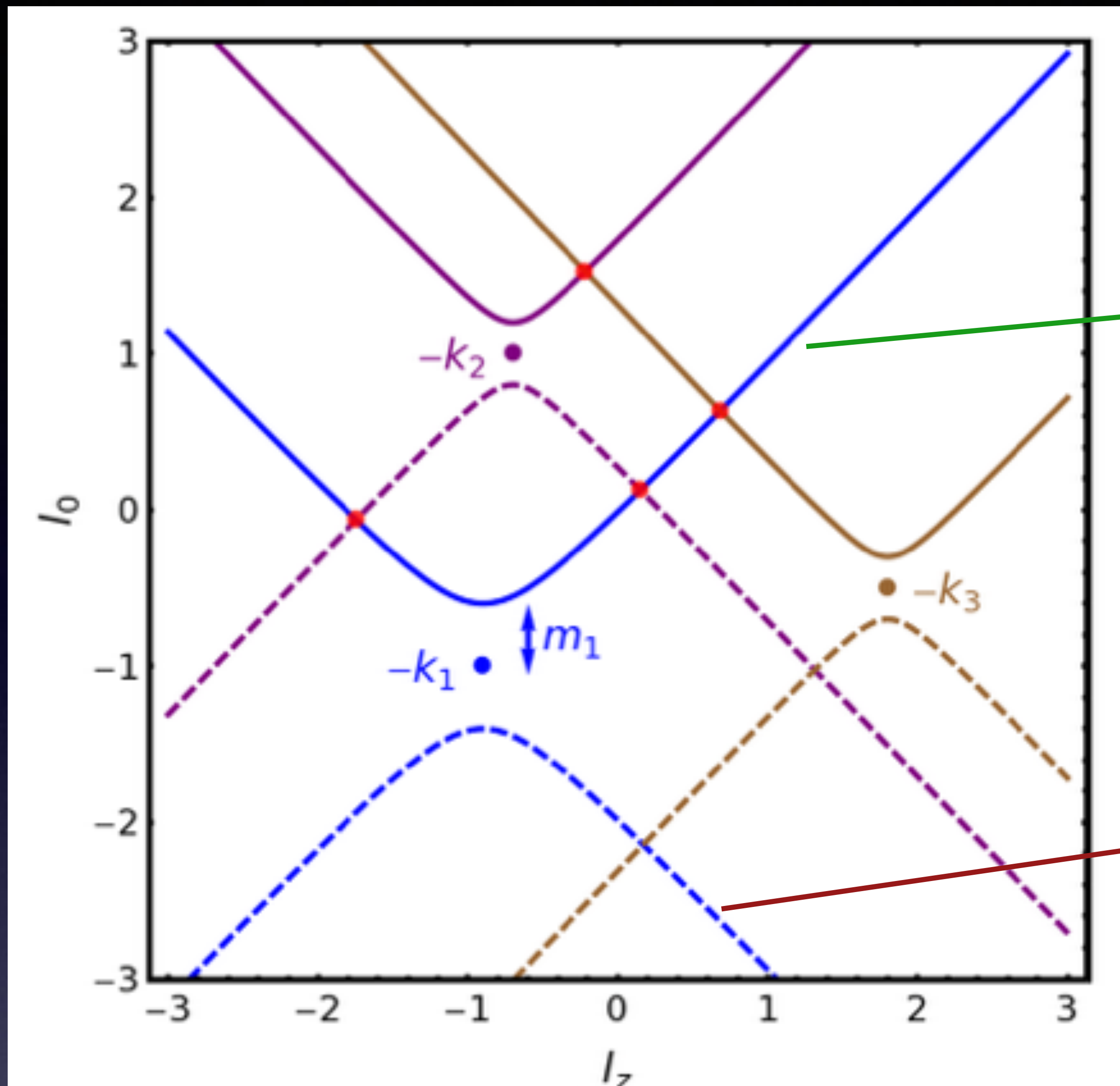
- Assume for the moment that UV divergencies have been subtracted

- Duality helps us identify IR contributions that cancel each other (Virtual-Real)

↘

Loop integrals can be viewed as Phase-Space integrals (slightly modified P-S)

- Threshold singularities are integrable but can lead to numerical instabilities



Positive energy solution
for a vanishing propagator
(Forward light-cone)

Negative energy solution
(Backward light-cone)

- The hyperboloids above are the lines where: $G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$
- Duality means integrate along the positive lines (for every contribution one positive line)
- At the intersection points more than one propagators become zero → singularities
- Dual integrals are positive inside the light cone and negative outside

Study of the different types of intersections

- To make the study of intersections of propagators easier, notice that dual propagators can be written in the following form :

$$\tilde{\delta}(q_i)\, G_D(q_i;q_j) = i\,2\pi \frac{\delta(q_{i,0} - q_{i,0}^{(+)})}{2q_{i,0}^{(+)}} \frac{1}{(q_{i,0}^{(+)} + k_{ji,0})^2 - (q_{j,0}^{(+)})^2}$$

with $q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$ (after some simple algebra)

- The intersection is now explicit and happens when one of the following condition is fulfilled

$$q_{i,0}^{(+)} + q_{j,0}^{(+)} + k_{ji,0} = 0\,,$$
$$q_{i,0}^{(+)} - q_{j,0}^{(+)} + k_{ji,0} = 0\,.$$

Forward-Forward intersection (second condition)

Forward of -k_i with Backward of -k_j (first condition)

Notation here: $k_{ji,\mu} = (q_j - q_i)_\mu$.

Cancellation of threshold singularities

- Imagine for example the intersection of two propagators when $q_{i,0}^{(+)} - q_{j,0}^{(+)} + k_{ji,0} = 0$

(Forward-Forward intersection)

- Two relevant contributions from the two dual integrals

- One intersection point- the two contributions have a different sign coming from crossing in an opposite way the intersection point where dual propagators change sign

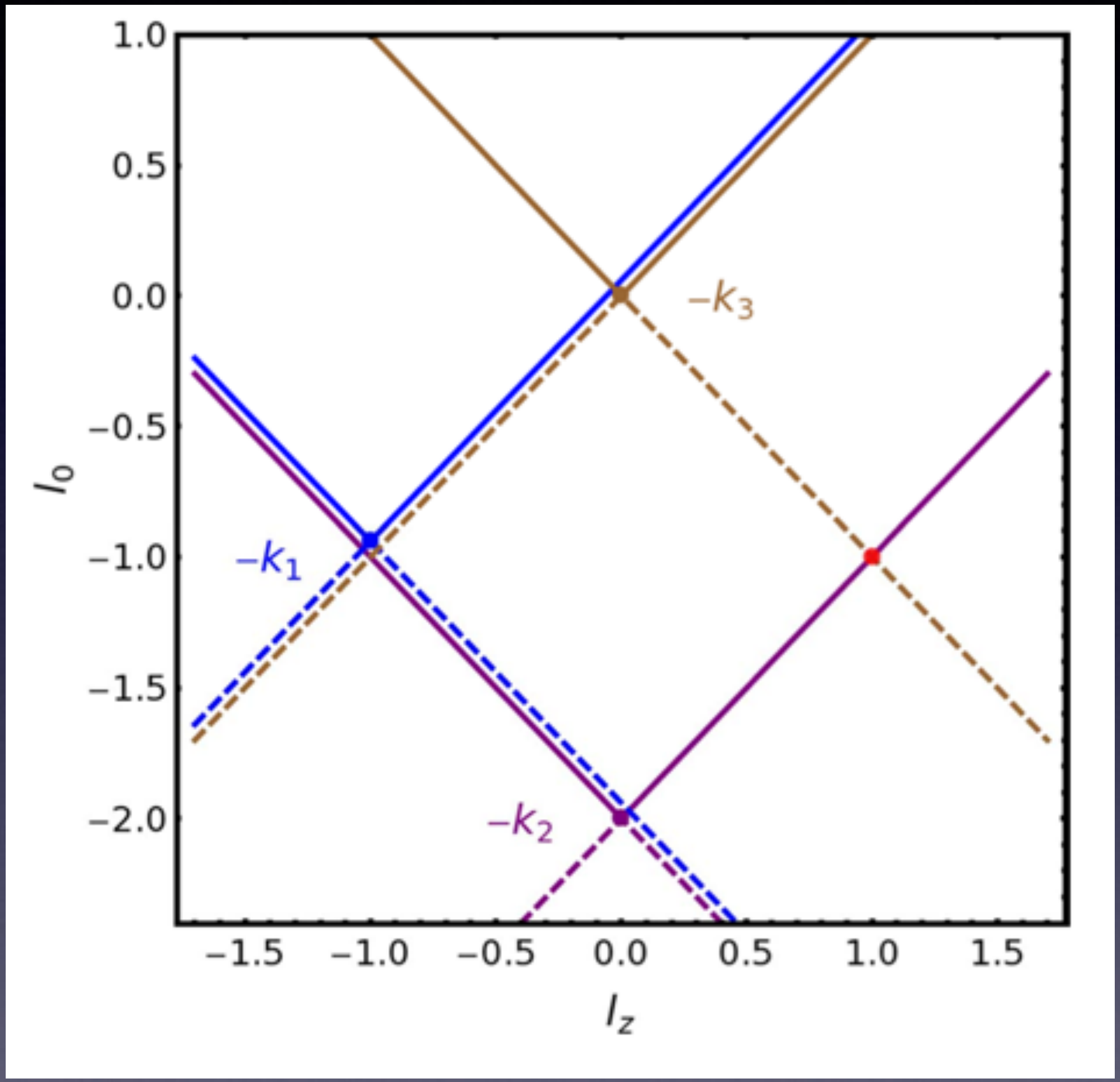
- Setting : $x = q_{i,0}^{(+)} - q_{j,0}^{(+)} + k_{ji,0}.$ and taking the limit $x \to 0$

one can prove that the singularity cancels

$$\lim_{x\to 0} \left(\tilde{\delta}\left(q_i\right) G_D(q_i; q_j) + (i \leftrightarrow j) \right) = i\, 2\pi \left(\frac{1}{x} - \frac{1}{x} \right) \frac{1}{2q_{i,0}^{(+)}} \frac{1}{2q_{j,0}^{(+)}} \delta(q_{i,0} - q_{i,0}^{(+)}) + \mathcal{O}(x^0) \,,$$

- The same is true for intersection of 3 or more propagators

Massless cases-IR divergencies

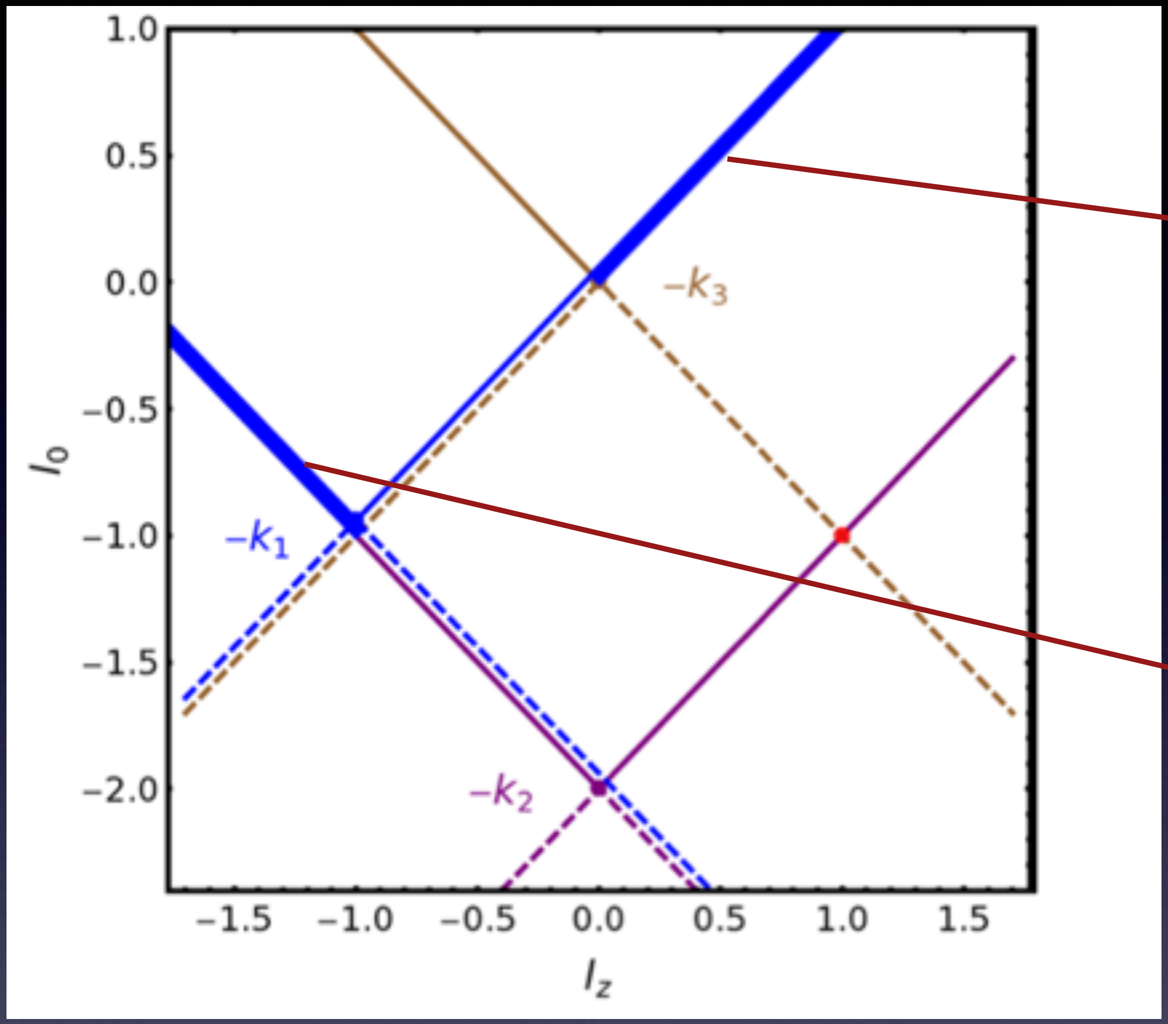


-Interested in cases with massless internal lines and external momenta on-shell

- The intersections are tangental

-Collinear divergencies, intersections lines

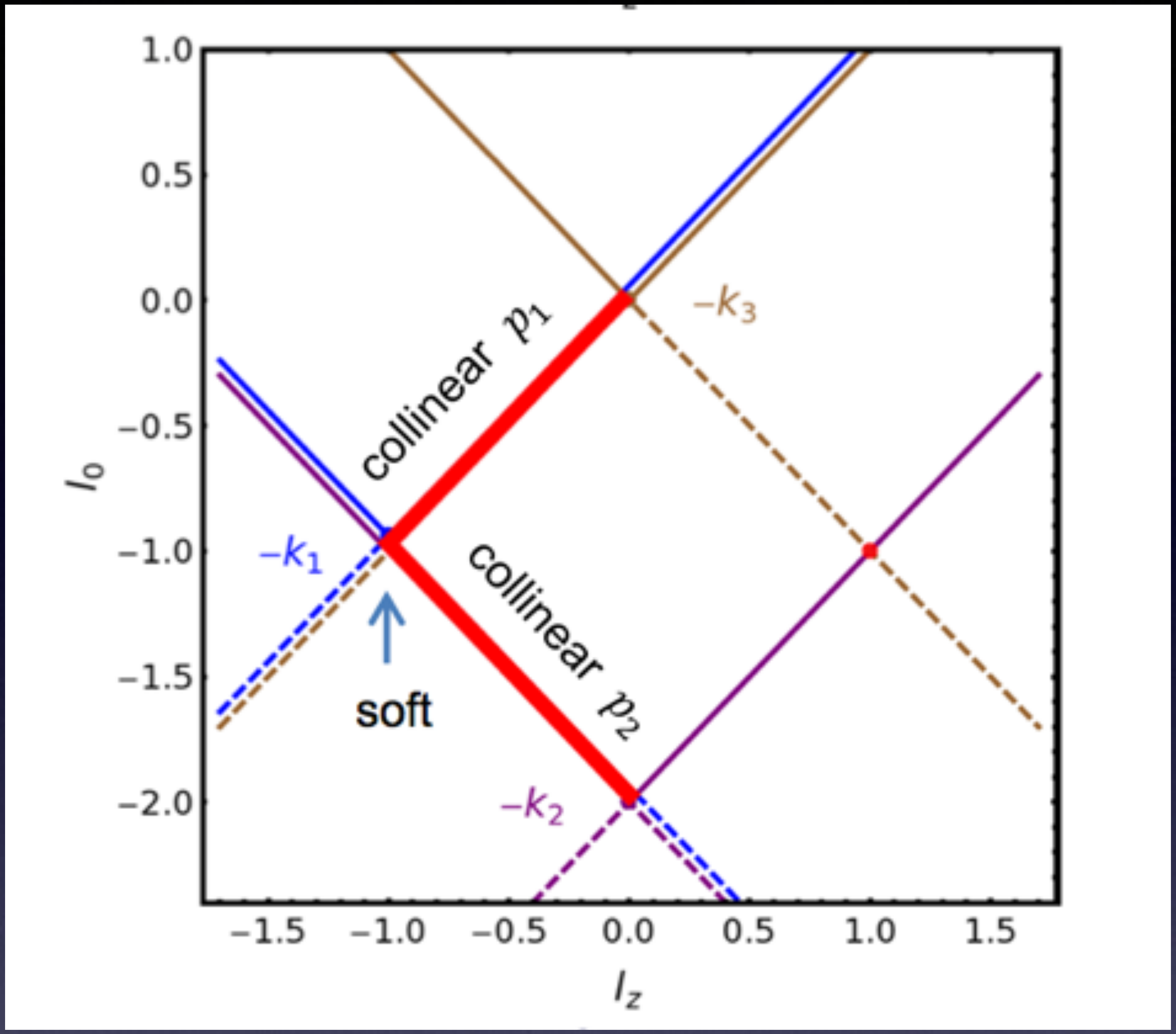
-Soft divergencies, intersection points



Forward-Forward intersection of Dual propagators 1 and 3-Cancels

F-F of 1 and 2, Cancels

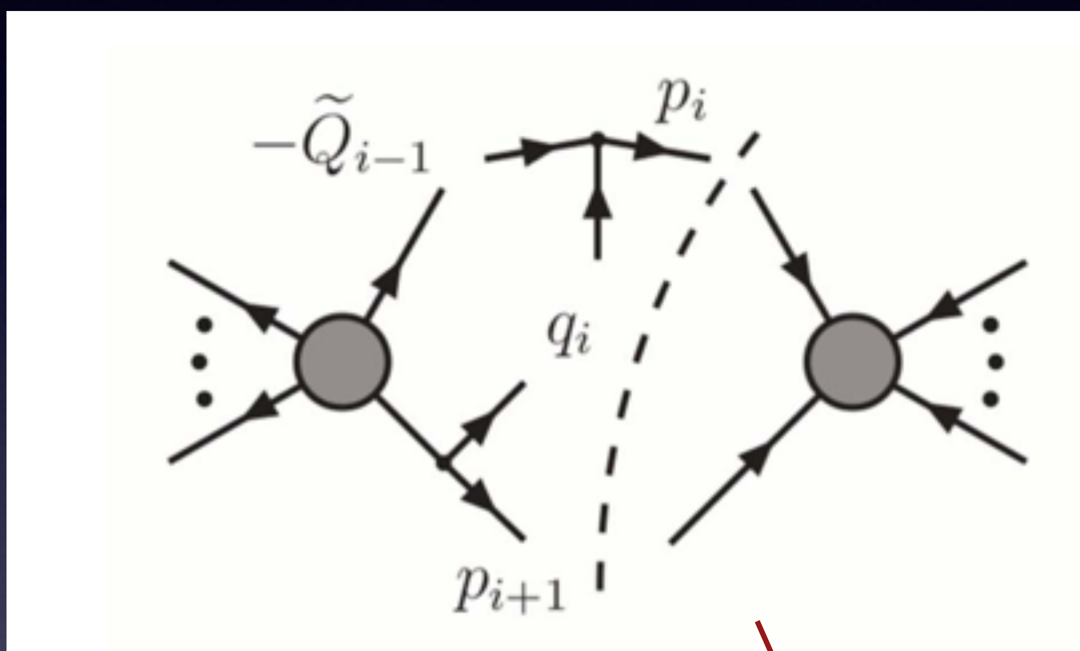
- Forward-Backward singularities survive but…



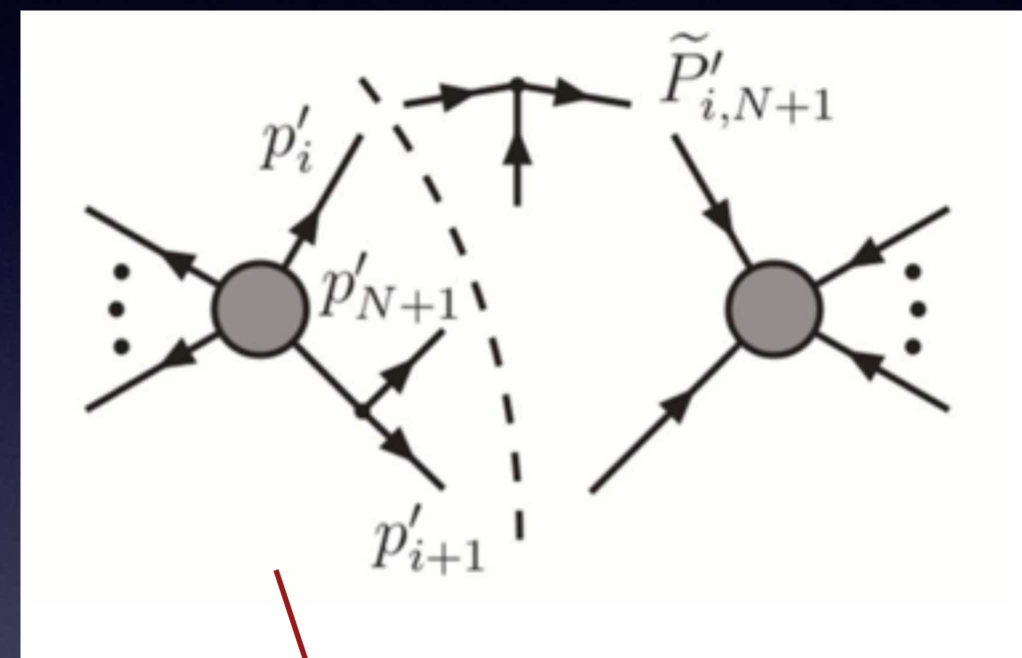
Now they are restricted to a finite region and they can be mapped to some phase-space contribution

Cancellation of collinear divergencies-Example

For splitting function also look at -Catani, de Florian, Rodrigo, PLB586(2004), JHEP 07(2012)026



Virtual



Real

$$I_i^{(1)} = -2\mathrm{Re}\left(\prod_{i=2}^{N}\int_{p_i}\tilde{\delta}\,(p_i)\right)\,\delta^{(d)}(\sum_{i=1}^{N} p_i)$$
$$\times \int_\ell \tilde{\delta}\,(q_i)\;\theta(p_{i,0}-q_{i,0}^{(+)})\;\langle \mathcal{M}_N^{(0)}(\ldots,p_i,p_{i+1},\ldots)|\,\mathcal{M}_{N+2}^{(0)}(\ldots,p_i,-q_i,q_i,p_{i+1},\ldots)\rangle$$

$$I_{ab}^{(0)} = 2\mathrm{Re}\left(\prod_{i=1}^{N+1}\int_{p'_i}\tilde{\delta}\,(p'_i)\right)\,\delta^{(d)}(\sum_{i=2}^{N+1} p'_i)\;\langle \mathcal{M}_{N+1}^{(0),a}(p'_1,\ldots,p'_{N+1})|\,\mathcal{M}_{N+1}^{(0),b}(p'_1,\ldots,p'_{N+1})\rangle$$

- In the virtual part, Duality has been used to open the loop to tree

$$\left|M_N^{(1)}(p_1, \dots, p_N)\right\rangle \rightarrow \left|M_{N+2}^{(0)}(\dots, p_i, -q_i, q_i, \dots)\right\rangle$$

- Phase-Space different-need some mapping to show the cancellation

- When p_i and q_i become collinear

$$\begin{aligned} |\mathcal{M}_{N+2}^{(0)}(\dots, p_i, -q_i, q_i, p_{i+1}, \dots)\rangle &= \boldsymbol{Sp}^{(0)}(p_i, -q_i; -\widetilde{Q}_{i-1}) \\ &\times |\overline{\mathcal{M}}_{N+1}^{(0)}(\dots, p_{i-1}, -\widetilde{Q}_{i-1}, q_i, p_{i+1}, \dots)\rangle + \mathcal{O}(q_{i-1}^2) \end{aligned}$$

where

$$\widetilde{Q}_{i-1}^{\mu} = q_{i-1}^{\mu} - \frac{q_{i-1}^2 \, n^{\mu}}{2nq_{i-1}}$$

- Similarly

$$\langle \mathcal{M}_{N+1}^{(0),a}(p_1', \ldots, p_{N+1}')| = \langle \overline{\mathcal{M}}_N^{(0)}(\ldots, p_{i-1}', \widetilde{P}_{i,N+1}', p_{i+1}', \ldots)| \, \boldsymbol{Sp}^{(0)\dagger}(p_i', p_{N+1}'; \widetilde{P}_{i,N+1}') + \mathcal{O}(s_{i,N+1}')$$

where $s_{i,N+1}' = (p_i' + p_{N+1}')^2$, and

$$\widetilde{P}'^{\mu}_{i,N+1} = (p_i' + p_{N+1}')^\mu - \frac{s_{i,N+1}' \, n^\mu}{2n(p_i' + p_{N+1}')}$$

in the collinear limit of p´_i and p´_N+1

- From the two graphs we can see that the mapping should be the following

$$\begin{aligned} p_i &= \widetilde{P}_{i,N+1}' \\ p_j &= p_j' \qquad j \neq i \\ -\widetilde{Q}_{i-1} &= p_i' \\ q_i &= p_{N+1}' \end{aligned}$$

- Under this mapping some of the momenta and the matrix elements match completely

- Difference in the propagators of splitting functions and some integration measurements

From

$$\frac{1}{(q_i - p_i)^2} \to \frac{1}{(p'_{N+1} - \widetilde{P}'_{i,N+1})^2} = -\frac{n\widetilde{P}'_{i,N+1}}{np'_i} \frac{1}{(p'_i + p'_{N+1})^2}$$

we get

$$\boldsymbol{Sp}^{(0)\dagger}(p'_i, p'_{N+1}; \widetilde{P}'_{i,N+1}) = -\frac{np'_i}{n\widetilde{P}'_{i,N+1}} \boldsymbol{Sp}^{(0)}(\widetilde{P}'_{i,N+1}, -p'_{N+1}; p'_i) .$$

- We get the cancellation if

$$\int d\Phi_N(p'_{j\neq i}, \cdots, \widetilde{P}'_{i,N+1})\tilde{\delta}\,(p'_{N+1}) \frac{1}{(p'_{N+1} - \widetilde{P}'_{i,N+1})^2} = -\int d\Phi_{N+1}(p'_i, \cdots, p'_{N+1}) \frac{1}{(p'_i + p'_{N+1})^2}$$

in the collinear limit

$$p'_i + p'_{N+1} = \widetilde{P}'_{i,N+1} + \mathcal{O}(s_{i,N+1})$$

We start from the left hand side and perform some trivial delta integrations

$$\int \tilde{\delta}\left(\widetilde{P}'_{i,N+1}\right)\tilde{\delta}\left(p'_{N+1}\right)\frac{1}{(p'_{N+1}-\widetilde{P}'_{i,N+1})^2} = \frac{1}{4|\vec{p}'_{N+1}||\vec{\widetilde{P}'}_{i,N+1}|}\,\frac{-1}{2(|\vec{p}'_{N+1}||\vec{\widetilde{P}'}_{i,N+1}| - \vec{p}'_{N+1}\vec{\widetilde{P}'}_{i,N+1})}$$

focusing only on the terms that don´t match

- We take now the collinear limit : $\vec{p}'_{N+1} = x\vec{\widetilde{P}'}_{i,N+1} - \vec{l}_T$

with $0 < x < 1,\ \widetilde{P'}_{i,N+1}\cdot\vec{l}_T = 0$ and $\vec{l}_T \to 0$

- We get (after expansion) :

$$\int \tilde{\delta}\left(\widetilde{P}'_{i,N+1}\right)\tilde{\delta}\left(p'_{N+1}\right)\frac{1}{(p'_{N+1}-\widetilde{P}'_{i,N+1})^2} = \frac{-1}{4\vec{l}_T^2\,\vec{\widetilde{P'}}^2_{i,N+1}}(1+\mathcal{O}(\vec{l}_T^{\,2}))$$

- For the right hand side we need to perform a Phase-Space decomposition first

$$d\Phi_{N+1}(p'_i,\cdots,p'_{N+1}) = d\Phi_N(p'_{i,N+1},p'_{j\neq i},\cdots,p'_N)d\Phi_2(p'_{i,N+1};p'_i,p'_{N+1})ds'_{i,N+1}$$

- Performing again some delta integrations we are able to show

$$\int d\Phi_{N+1}(p'_i,\cdots,p'_{N+1})\frac{1}{(p'_i+p'_{N+1})^2} = \int \delta^4(P-p'_{i,N+1}-\sum_{j\neq i}^{N}p'_j)\left(\prod_{i\neq j}^{N}\frac{d^3\vec{p}'_j}{2|\vec{p'}_j|}\right)$$
$$\frac{d^3\vec{p'}_{N+1}}{2|\vec{p'}_{N+1}|}\frac{d^3\vec{p'}_{i,N+1}}{2|\vec{p'}_{i,N+1}-\vec{p'}_{N+1}|}\frac{1}{(|\vec{p'}_{i,N+1}-\vec{p'}_{N+1}|+|\vec{p'}_{N+1}|)^2-(\vec{p'}_{i,N+1})^2}$$

- We take the collinear limit as before $\vec{p'}_{N+1} = x\vec{P'}_{i,N+1}-\vec{l}_T$

and we get

$$\frac{1}{4|\vec{p'}_{N+1}||\vec{p'}_{i,N+1}-\vec{p'}_{N+1}|}\frac{1}{(|\vec{p'}_{i,N+1}-\vec{p'}_{N+1}|+|\vec{p'}_{N+1}|)^2-(\vec{p'}_{i,N+1})^2} = \frac{1}{4\vec{l}_T^2\vec{P'}^2_{i,N+1}}(1+\mathcal{O}(\vec{l_T}^2))$$

- The two terms cancel exactly
- In a similar way, all possible collinear divergences cancel
- Cancellation of the soft divergences (omitted in the presentation)
- Loop-tree duality $\Longleftrightarrow$ Virtual-Real duality

Numerical Implementation

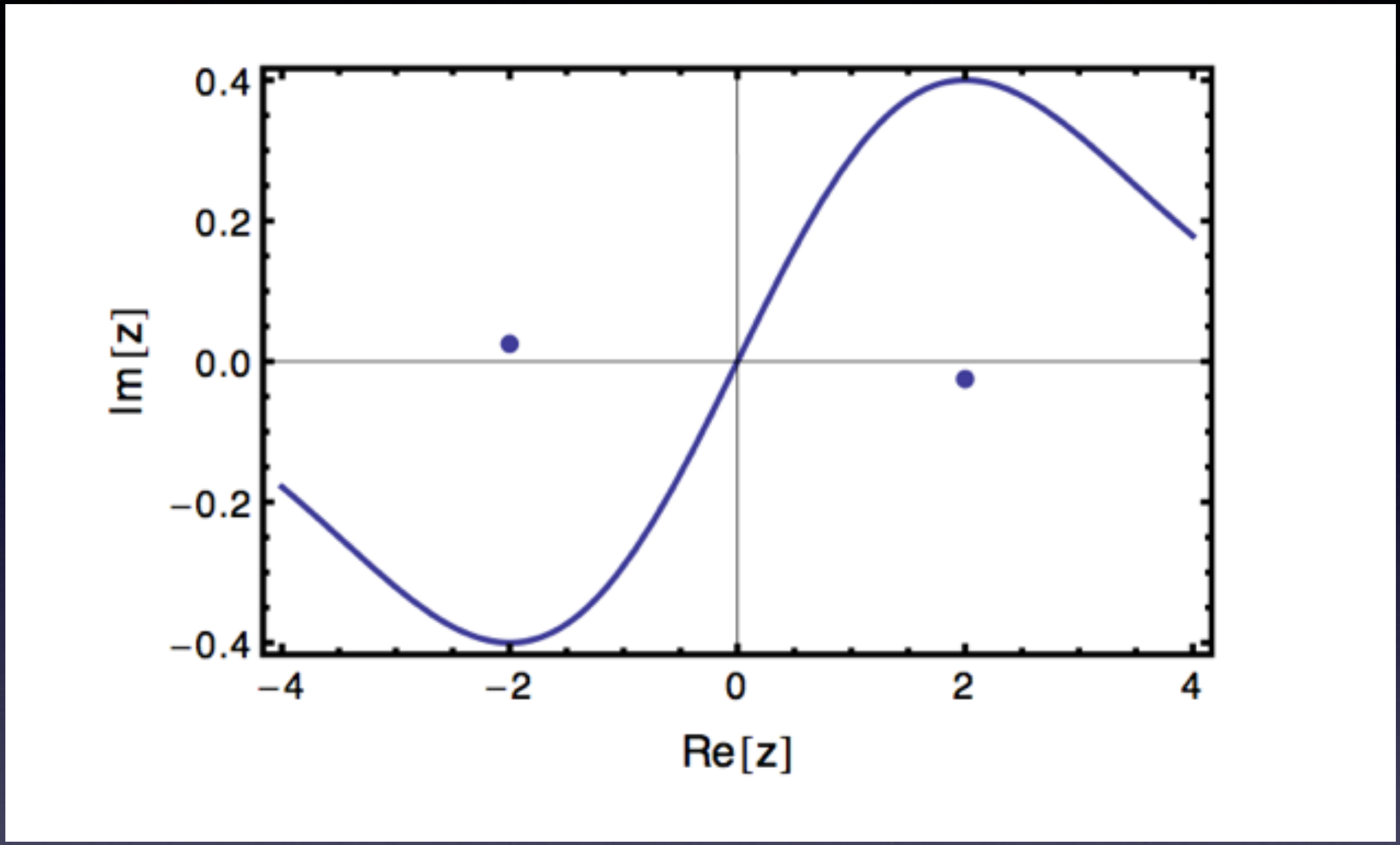
- Contour Deformation

$$f(x) = \frac{1}{x^2 - a^2 + i0} \quad , \quad I = \int_{-\infty}^{\infty} dx\, f(x)$$

Poles are located at $x = \pm(a - i0)$

- Deform the contour $z = x + i\lambda x \exp^{-\frac{x^2 - y^2}{2a^2}}$

$$I = \int_C dz\, f(z) = \int_{-\infty}^{\infty} dx \frac{\partial z}{\partial x} f(z)$$



Contour Deformation

- Deformation maximum around the poles
- Vanishes away from the poles

Contour deformation for dual integrals

(work in progress…)

$$\vec{l} \to z = \vec{l} + i\bar{z},\ \mathbf{q_i} \to \mathbf{z_i} = \mathbf{q_i} + i\mathbf{\bar{z}}$$

Now we analyse the factor

$$q_{i,0}^{+} = \sqrt{\mathbf{q_i^2} + m_i^2 - \mathbf{\bar{z}^2} + 2i\mathbf{q_i} \cdot \mathbf{\bar{z}} - i0}$$

To match the i0 prescription ,for every i : $\mathbf{q_i} \cdot \mathbf{\bar{z}} < 0$

but since this is not possible we define:

$\mathbf{q_i} \to \mathbf{z_i} = \mathbf{q_i}(1 - ic_i)$,with $c_i > 0$. For c_i we use

$$c_i = \lambda_i \left(\prod_{j \neq i} h^S_{ji} \right) \left(\prod_{j \neq i} h^T_{ji} \right)$$

- $c_i = 0$ when there is no time-like contribution
- λ_i is a scailing parameter
  - For time-like distances deform only momenta inside the forward light-cone

$$h^T_{ji} = \theta(-k_{ji,0})\theta(k^2_{ji} - (m_i + m_j)^2)\exp(-\frac{1}{2\sigma^2_{ji}} G_D^{-2}(q_i; q_j))$$

- For space-like distances , coefficient c_i vanishes where forward light cones intersect

$$h_{ji}^{S} = \theta(-k_{ji}^2 + (m_j - m_j)^2)\left(1 - \exp(-\frac{1}{2\sigma_{ji}^2} G_D^{-2}(q_i; q_j))\right)$$

- σ_{ji} determines the width of the deformation

- Jacobian and scailing parameter

Duality from Integrand Reduction-Extensions?

- Inverse Feynman propagator

$$D_i = G_F(q_i)^{-1} = q_i^2 - m_i^2 + i0$$

- Solutions $\quad q_{i,0}^{\pm} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$

- Partial fractioning in the q_0 component

$$I^{(q)}(p_1, p_2, \cdots, p_N) = \sum_{i=1}^{N} F_i(\eta q_i) G_F(q_i)$$

with

$$1 = F_1(\eta q_1) D_2 \cdots D_N + \cdots + F_N(\eta q_N) D_1 \cdots D(q_{N-1})$$

- Conjecture for F´s: $F_i(\eta q_i) = a_i + b_i(\eta q_i)$

- Find the coefficients using the solutions of the cuts

$$a_i = \frac{1}{2}\left(\prod_{i \neq j} G_F(q_j)|_{q_{i+}} + \prod_{i \neq j} G_F(q_j)|_{q_{i-}} \right)$$

$$b_i = \frac{1}{2\eta q_{i+}}\left(\prod_{i \neq j} G_F(q_j)|_{q_{i+}} - \prod_{i \neq j} G_F(q_j)|_{q_{i-}} \right)$$

- Integrate over q_0 and get Duality

$$L^{(1)}(p_1, \cdots, p_N) = -\int \sum_{i=1}^{N} \frac{a_i + b_i \eta q_i}{D_i} = -2\pi i \int \sum_{i=1}^{N} \frac{1}{2\eta q_{i+}} \prod_{i \neq j} \frac{1}{D_j(q_{i+})}$$

- The new i0 prescription comes from the i0 in the solutions

- b_i contributions add up to zero (Spurious terms)

- Partial fractioning in more components- more cuts?

Reduction in two components

$$L^{(1)}(p_1, \cdots, p_N) = \int \sum_{i<j} F_{ij}(\tilde{\eta}_{ji} q_i) G_F(q_i) G_F(q_j)$$

with

$$1 = F_{12}(\tilde{\eta}_{21} q_1) D_3 \cdots D_N + \cdots + F_{N-1,N}(\eta_{N-1,N} q_N) D_1 \cdots D_{N-2}$$

- With the new conjecture

$$F_{ij}(\tilde{\eta}_{ij} q_i) = a_{ij} + b_{ij} \tilde{\eta}_{ji} (q + \frac{k_i + k_j}{2})$$

- Solutions of the cut are more complicated :

$$q^\mu_\pm = c_1 \eta^\mu_{ji} + d_\pm \tilde{\eta}^\mu_{ji} + q^\mu_\perp$$

with

$$c_1 = \frac{1}{2k_{ji} \cdot \eta_{ji}} \left(m_j^2 - m_i^2 + k_i^2 - k_j^2 - 2k_{ji} \cdot q_\perp \right)$$

$$d_\pm = \frac{-k_i \cdot \tilde{\eta}_{ji} \pm \sqrt{(k_i \cdot \tilde{\eta}_{ji})^2 - \tilde{\eta}_{ji}^2 C}}{\tilde{\eta}_{ji}^2}$$

$$C = c_1^2 \eta_{ji}^2 + q_\perp^2 + 2c_1 k_i \cdot \eta_{ji} + 2k_i \cdot q_\perp + k_i^2 - m_i^2$$

- Solving for the coefficients :

$$a_{ij} = \frac{1}{2}\left(\prod_{k\neq i,j} G_F(q_k)|_{q_+} + \prod_{k\neq i,j} G_F(q_k)|_{q_-}\right)$$

$$b_{ij} = \frac{1}{2\sqrt{(k_i \cdot \tilde{\eta}_{ji})^2 - \tilde{\eta}_{ji}^2 C}}\left(\prod_{k\neq i,j} G_F(q_k)|_{q_+} - \prod_{k\neq i,j} G_F(q_k)|_{q_-}\right)$$

- Again contributions from b_{ij} cancel
- Integrate in the two components, define double Dual propagators,…
- OPP method

Conclusions and Future

- Duality is a method for calculating loop Amplitudes and has interesting properties
- The method is extended to higher loops
- Threshold and IR singularities among dual integrals cancel when the intersections happen at the Forward-Forward light cone
- The remaining singularities are restricted in a finite region of the loop momentum

- Numerical implementation (to finish soon)

- UV divergences (future)

- Can you write a Feynman integral with double cuts ?-

Extensions

THANK YOU