Light quark physics and Lattice QCD

Vittorio Lubicz

ROMA TRE UNIVERSITÀ DEGLI STUDI

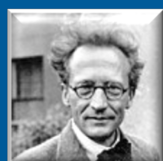

C. Doppler


J. Stefan


E. Mach


L. Boltzmann


V. F. Hess


E. Schrödinger

OUTLINE

- Lattice QCD and flavor physics: the "precision era" of LQCD
- 1. The light quark masses: m_{ud} and m_s
- 2. The CKM matrix and the first row di unitarity test: Vus, Vud ↔ f_K/f_π, $f_+(0)$
- 3. Isospin breaking effects on the lattice: mu/md, $M_{\pi+}$ - $M_{\pi0}$, M_n - M_p, $[f_K/f_\pi]^{QCD}$ …

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13 May 2014

Faculty of Physics

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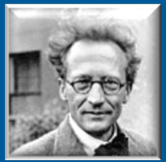
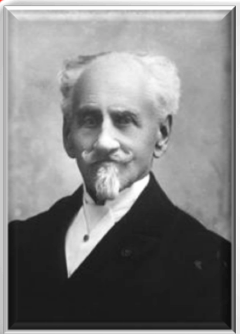


E. Schrödinger



Franz Sacher
[1816-1907]

OUTLINE

- Lattice QCD and flavor physics: the "precision era" of LQCD
- 1. The light quark masses: m_{ud} and m_s
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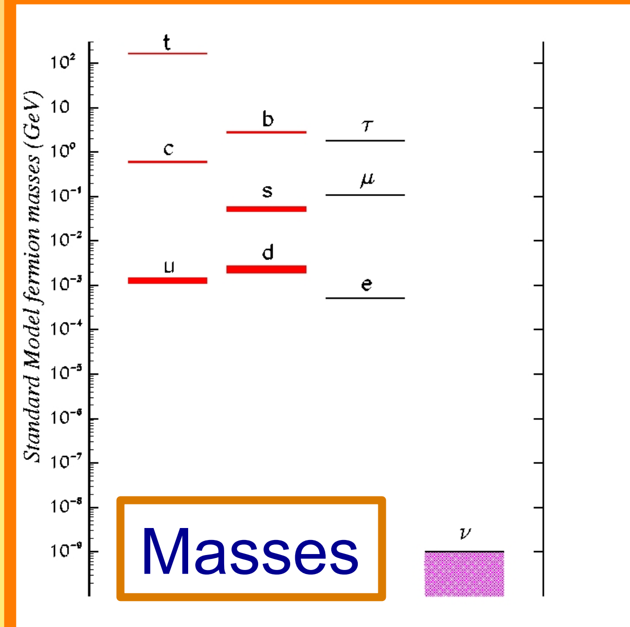
LATTICE QCD
AND FLAVOR PHYSICS:
The “precision era” of LQCD

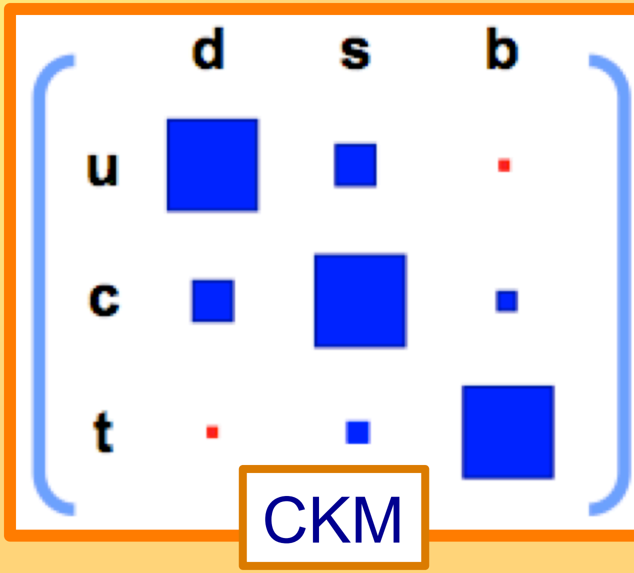
Lattice QCD and flavor physics

A fundamental task of LQCD is to provide a determination of the SM free parameters in the quark sector, particularly in the flavor sector

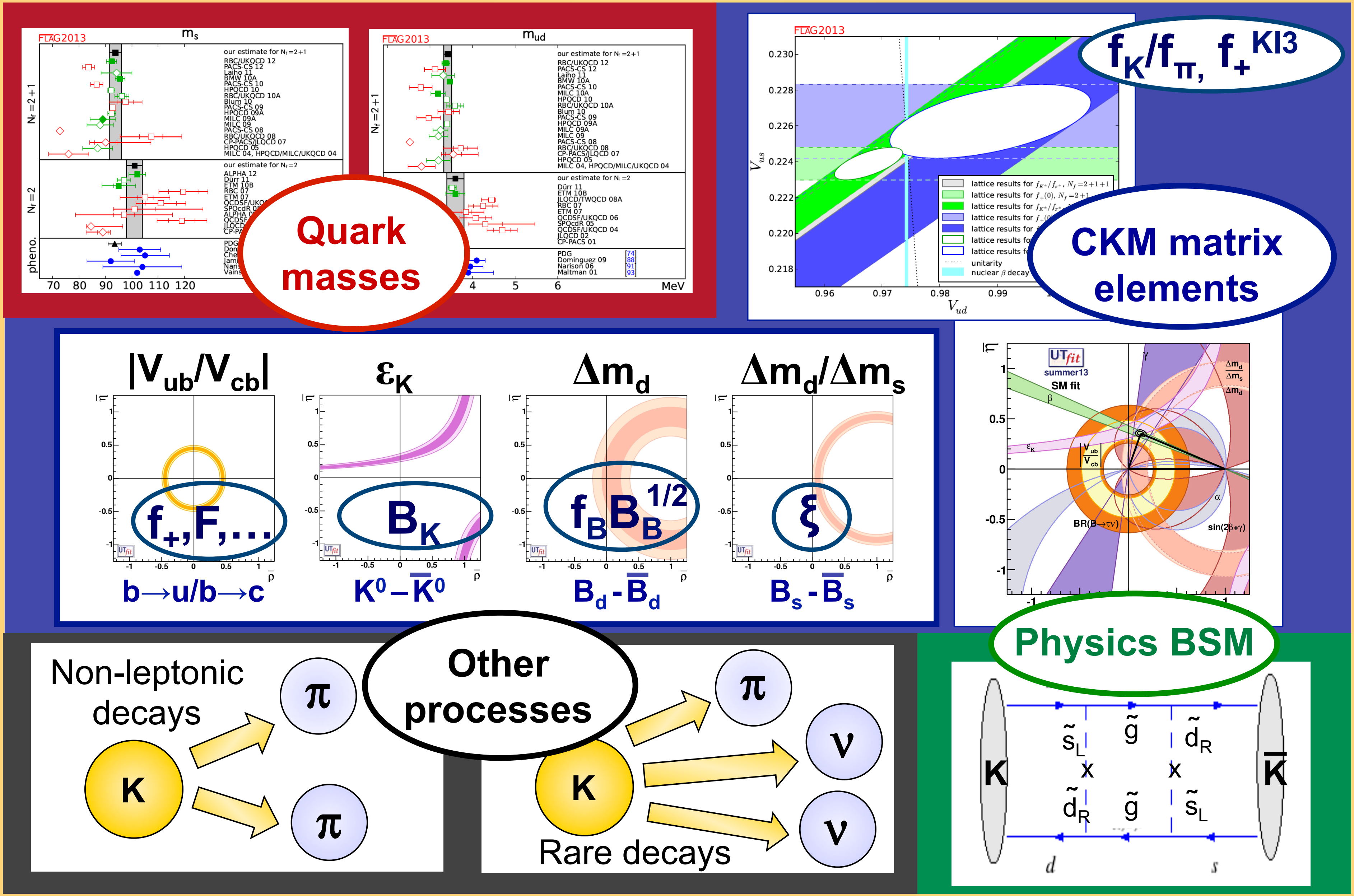
The largest number of SM free parameters is in the flavor sector and 10 parameters in the quark sector only (6 m_q + 4 CKM) with unexplained hierarchical structure

Flavor physics is (well) described but not explained in the SM

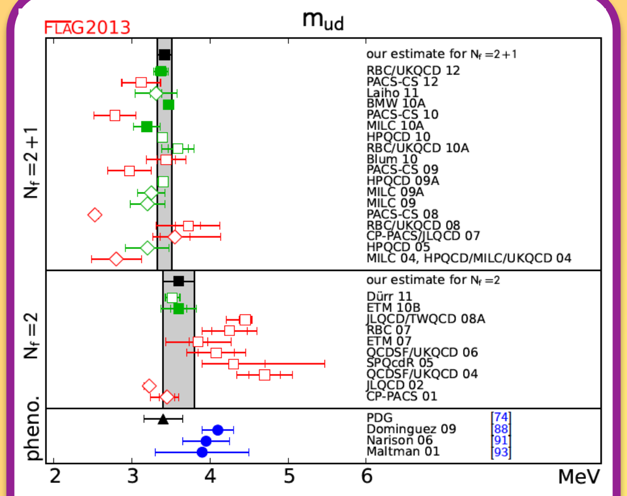




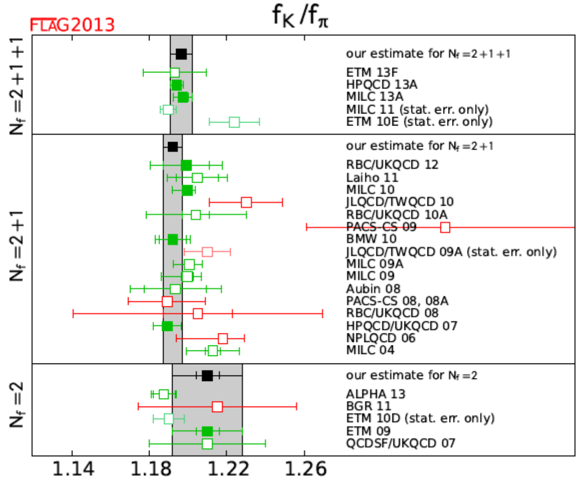
Lattice QCD and flavor physics



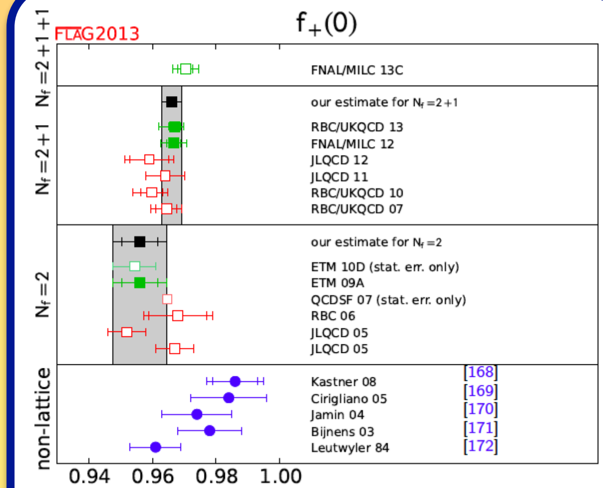
THE "PRECISION ERA" OF LQCD



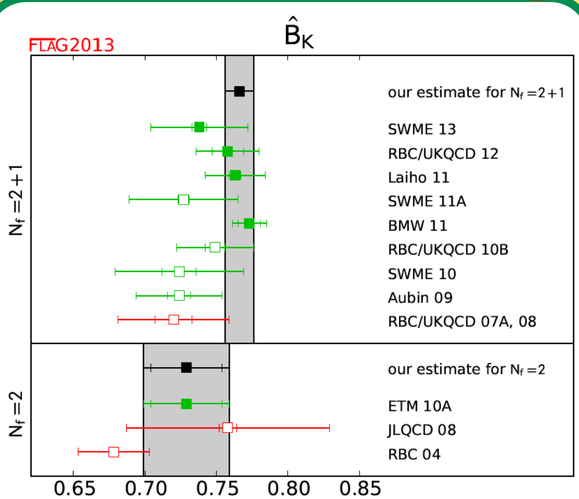
$\overline{m}_{ud}$= 3.42(9) MeV **2.6%**



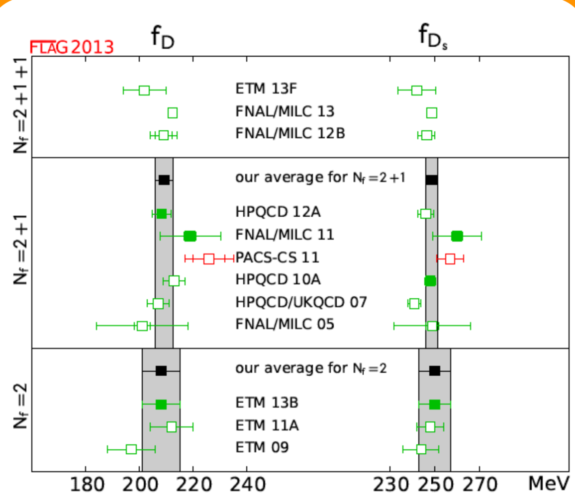
f_K / f_π = 1.192(5) **0.4%**



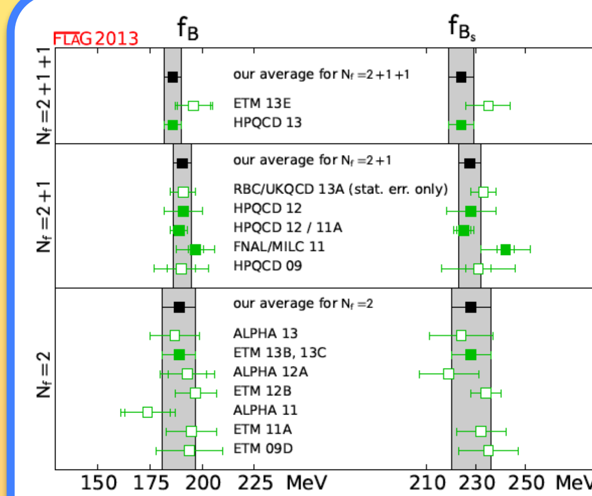
$f_+^{K\pi}(0) = 0.966(3)$ **0.3%**



$\hat{B}_K = 0.7661(99)$ **1.2%**



f_{Ds} = 250(7) MeV **2.8%**

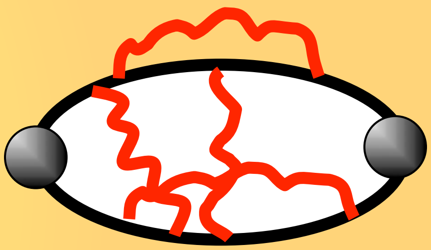


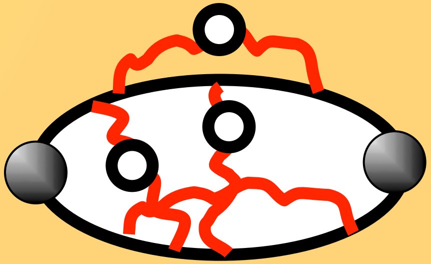
f_{Bs} = 227.7(4.5) Me **2.0%**

Uncertainties in LQCD in the "quenched era"

For many years, uncertainties in lattice calculations have been dominated by the quenched approximation (or, more precisely, by the uncertainty on the quenching error)

	f_B [MeV]	$f_{Bs}\sqrt{B_s}$ [MeV]	ξ
J.Flynn Latt'96	175(25) 14%	----	----
C.Bernard Latt'00	200(30) 15%	267(46) 17%	1.16(5) 4%
L.Lellouch Ichep'02	193(27)(10) 15%	276(38) 14%	1.24(4)(6) 6%
Hashimoto Ichep'04	189(27) 14%	262(35) 13%	1.23(6) 5%
N.Tantalo CKM'06	223(15)(19) 11%	246(16)(20) 10%	1.21(2)(5) 4%





THE "PRECISION ERA" OF LQCD

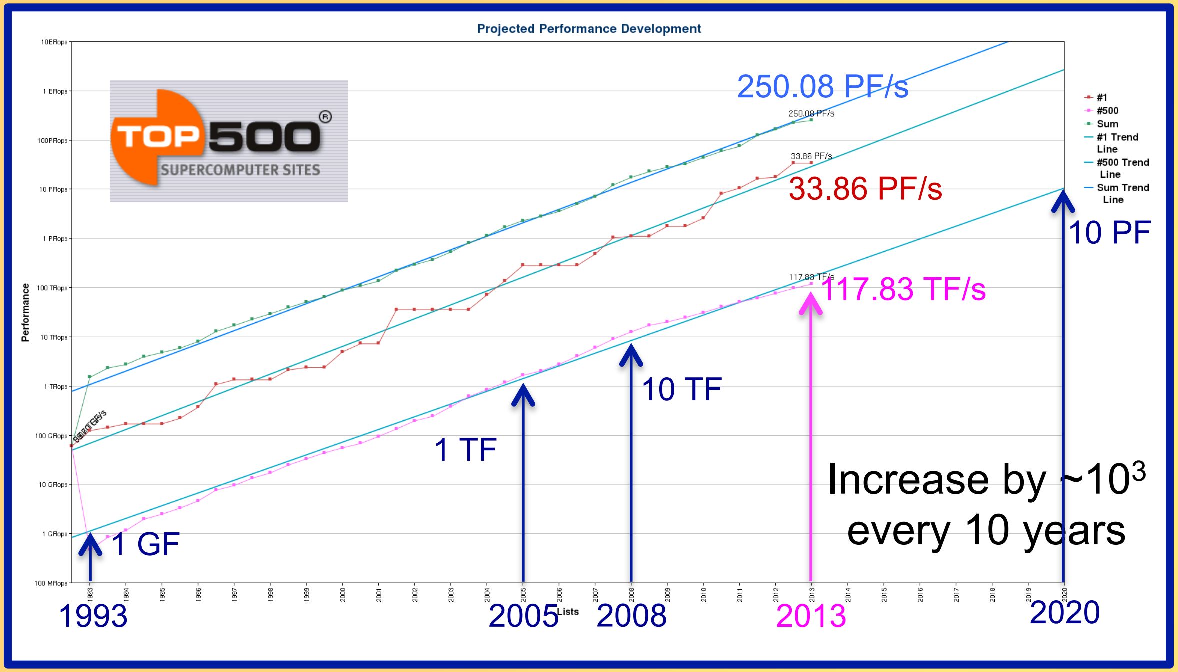
Uncertainties in lattice QCD

- Statistical errors
- Discretization errors ($a \to 0$)
- Finite volume effects ($M_\pi L \gg 1$)
- Extrapolation in quark masses, both light ($M_\pi \gg 1/L$) and heavy ($m_Q \ll 1/a$)
- Renormalization (where required)
- [Quenched approximation (N_f=0)]

All these errors can be systematically improved in time

THE “PRECISION ERA” OF LQCD

3 main reasons: 1) Increasing computational power



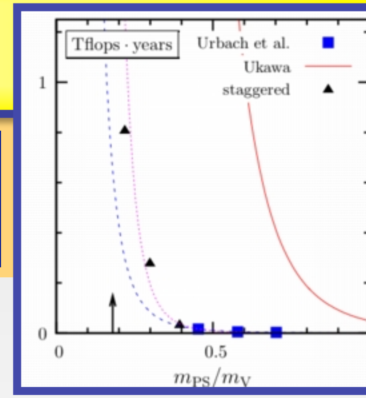
TeraFlops machines are required for unquenched LQCD simulations. They are available since few years only.

For LQCD today: ~ 100-200 TFlops

In 1989 the APE computer had a peak power of ~1 GFlops

List	Rank	System	Vendors	Cores	Rmax (GFlop/s)	Rpeak (GFlop/s)
11/2013	1	Tianhe-2 (MilkyWay-2) - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P	NUDT	3,120,000	33,862,700	54,902,400
11/2013	15	Fermi - BlueGene/Q, Power BQC 16C 1.60GHz, Custom	IBM	163,840	1,788,878	2,097,152

THE "PRECISION ERA" OF LQCD



2) Algorithmic improvements

Empirical CPU cost of a simulation (for Nf=2 Wilson fermions)

- Ukawa 2001

$$\text{TFlops-years} \simeq 3.1 \left(\frac{N_{conf}}{100}\right) \left(\frac{L_s}{3\ \text{fm}}\right)^5 \left(\frac{L_t}{2L_s}\right) \left(\frac{0.2}{\hat{m}/m_s}\right)^3 \left(\frac{0.1\ \text{fm}}{a}\right)^7$$

- Del Debbio et al. 2006

$$\text{TFlops-years} \simeq 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{L_s}{3\ \text{fm}}\right)^5 \left(\frac{L_t}{2L_s}\right) \left(\frac{0.2}{\hat{m}/m_s}\right) \left(\frac{0.1\ \text{fm}}{a}\right)^6$$

Some years ago: $M_\pi \geq 500$ MeV Today: $M_\pi \approx 140\text{-}200$ MeV

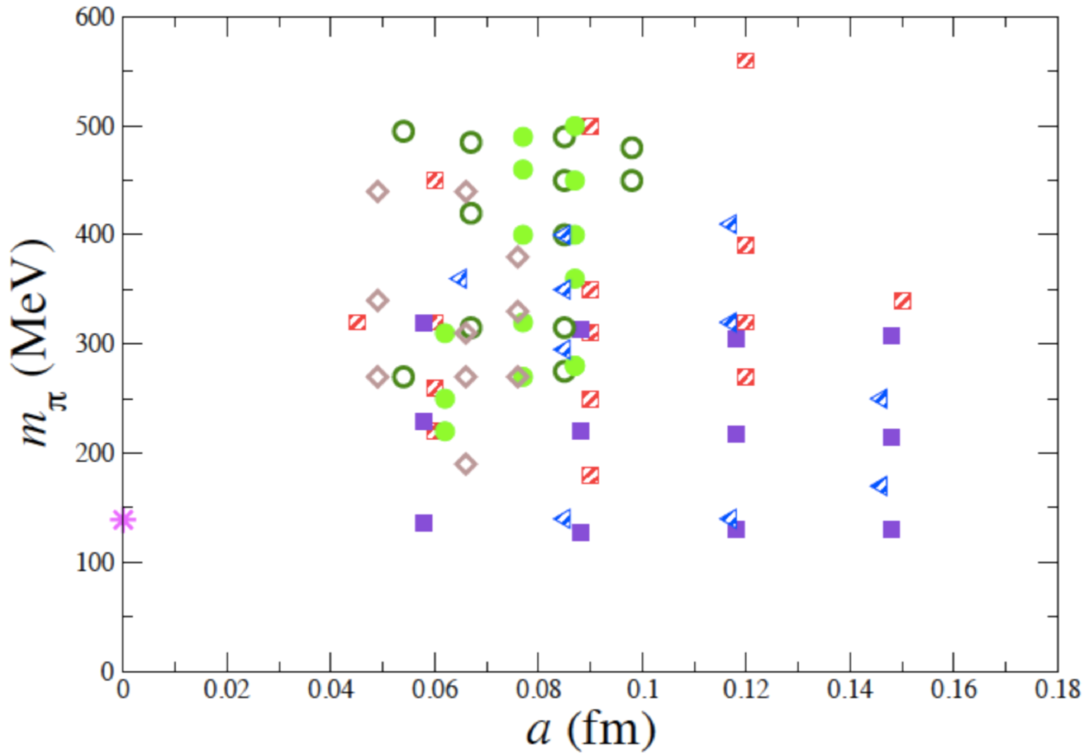
Light quark masses in the ChPT regime

3) Action improvements

- Improved chiral symmetry: GW, Domain Wall…
- Improved scaling properties: CSW, Twisted mass, …

Overview of lattice ensembles

A. El-Khadra @ Lattice 2013: "Quark Flavour Physics Review"



ETMC

MILC

RBC/UKQCD

CLS

+ BMW, HPQCD,

PACS-CS, QCDSF,

...

Nf = 2   Nf = 2 + 1   Nf = 2 + 1 + 1

FLAG 2013

Flavour Lattice Averaging Group

itpwiki.unibe.ch/flag/

FLAG-1 [2011]

Eur. Phys. J. C (2011) 71:1695
DOI 10.1140/epjc/s10052-011-1695-1

THE EUROPEAN PHYSICAL JOURNAL C

Review

Review of lattice results concerning low-energy particle physics

FLAG working group of FLAVIANET

G. Colangelo[1,a], S. Dürr[2,3], A. Jüttner[4], L. Lellouch[5], H. Leutwyler[1], V. Lubicz[6], S. Necco[4], C.T. Sachrajda[7], S. Simula[8], A. Vladikas[9], U. Wenger[1], H. Wittig[10]

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[2] Bergische Universität Wuppertal, Gaussstr. 20, 42119 Wuppertal, Germany
[3] Jülich Supercomputing Centre, Forschungszentrum Jülich, 52425 Jülich, Germany
... Geneva 23, Switzerland
... Campus de Luminy, 13288 Marseille, France[b]
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FLAG-2 [2013-14]

CP3-Origins-2013-040 DNRF90, DIAS-2013-40
FERMILAB-PUB-13-484-T, FTUAM-13-28
IFIC/13-76, IFT-UAM/CSIC-13-106
MITP/13-067, YITP-13-114

arXiv:1310.8555v2 [hep-lat] 17 Apr 2014

Review of lattice results concerning low energy particle physics

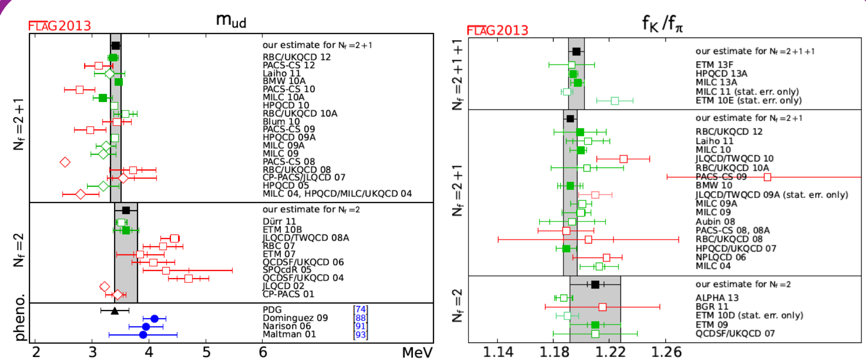
April 18, 2014

FLAG Working Group

S. Aoki[1], Y. Aoki[2,3], C. Bernard[4], T. Blum[5,3], G. Colangelo[6], M. Della Morte[7], S. Dürr[8], A. X. El-Khadra[9], H. Fukaya[10], R. Horsley[11], A. Jüttner[12], T. Kaneko[13], J. Laiho[*14], L. Lellouch[15,16], H. Leutwyler[6], V. Lubicz[17,18], E. Lunghi[19], S. Necco[6], T. Onogi[10], C. Pena[20], C. T. Sachrajda[13], S. R. Sharpe[21], S. Simula[17], R. Sommer[22], R. S. Van de Water[23], A. Vladikas[24], U. Wenger[6], H. Wittig[25]

Abstract

We review lattice results related to pion, kaon, D- and B-meson physics with the aim of making them easily accessible to the particle physics community. More specifically, we report on the determination of the light-quark masses, the form factor $f_+(0)$, arising in semileptonic $K \to \pi$ transition at zero momentum transfer, as well as the decay constant ratio f_K/f_π of decay constants and its consequences for the CKM matrix elements V_{us} and V_{ud}. Furthermore, we describe the results obtained on the lattice for some of the low-energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ Chiral Perturbation Theory and review the determination of the B_K parameter of neutral kaon mixing. The inclusion of heavy-quark quantities significantly expands the FLAG scope with respect to the previous review. Therefore, we focus here on D- and B-meson decay constants, form factors, and mixing parameters, since these are most relevant for the determination of CKM matrix elements and the global CKM unitarity-triangle fit. In addition we review the status of lattice determinations of the strong coupling constant α_s.



Our aim is to provide the answer to the frequently posed question "What is currently the best lattice value (i.e. central value and error) for a particular quantity?"

The FLAG criteria (for light quarks)

FLAG 2013
Flavour Lattice Averaging Group

arXiv:1310.8555

- Chiral extrapolation:
  - ★ $M_{\pi,\mathrm{min}} < 200$ MeV
  - ○ 200 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
  - ■ 400 MeV $< M_{\pi,\mathrm{min}}$

- Continuum extrapolation:
  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - ○ 2 or more lattice spacings, at least 1 point below 0.1 fm
  - ■ otherwise

- Finite-volume effects:
  - ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 volumes
  - ○ $M_{\pi,\mathrm{min}}L > 3$ and at least 2 volumes
  - ■ otherwise

- Renormalization (where applicable):
  - ★ non-perturbative
  - ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
  - ■ otherwise

An example of a FLAG table

arXiv:1310.8555

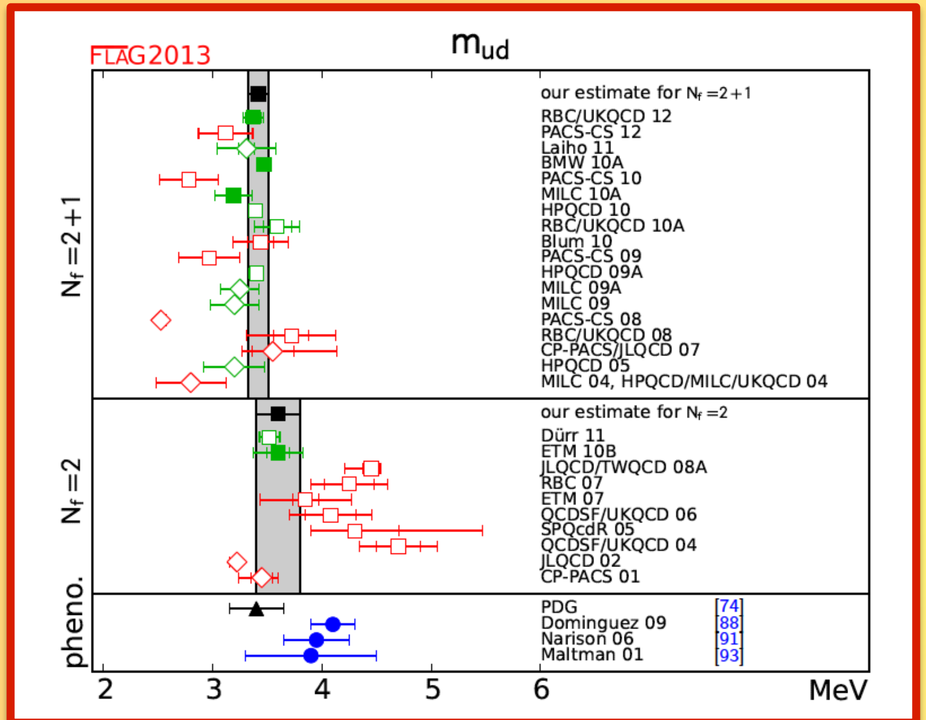
Nf=2+1

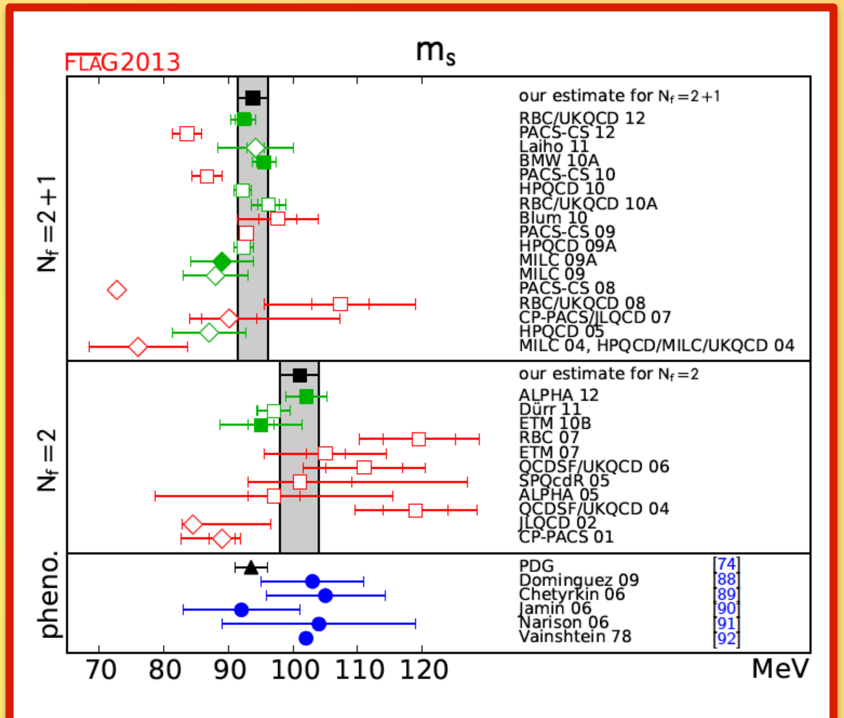
Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_{ud}	m_s
RBC/UKQCD 12$^{\ominus}$	[25]	A	★	○	★	★	a	3.37(9)(7)(1)(2)	92.3(1.9)(0.9)(0.4)(0.8)
PACS-CS 12*	[76]	A	★	■	■	★	b	3.12(24)(8)	83.60(0.58)(2.23)
Laiho 11	[77]	C	○	★	★	○	–	3.31(7)(20)(17)	94.2(1.4)(3.2)(4.7)
BMW 10A, 10B^{+}	[22, 23]	A	★	★	★	★	c	3.469(47)(48)	95.5(1.1)(1.5)
PACS-CS 10	[21]	A	★	■	■	★	b	2.78(27)	86.7(2.3)
MILC 10A	[75]	C	○	★	★	○	–	3.19(4)(5)(16)	–
HPQCD 10*	[73]	A	○	★	★	–	–	3.39(6)	92.2(1.3)
RBC/UKQCD 10A	[78]	A	○	○	★	★	a	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10†	[32]	A	○	■	○	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	[20]	A	★	■	■	★	b	2.97(28)(3)	92.75(58)(95)
HPQCD 09A$^{\oplus}$	[72]	A	○	★	★	–	–	3.40(7)	92.4(1.5)
MILC 09A	[37]	C	○	★	★	○	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	[15]	A	○	★	★	○	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	[19]	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	[79]	A	○	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[80]	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	[81]	A	○	○	○	○	–	$3.2(0)(2)(2)(0)^{\ddagger}$	$87(0)(4)(4)(0)^{\ddagger}$
MILC 04, HPQCD/ MILC/UKQCD 04	[36, 82]	A	○	○	○	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

Nf=2

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_{ud}	m_s
ALPHA 12	[59]	A	○	★	★	★	a, b		102(3)(1)
Dürr 11‡	[61]	A	○	★	○	–	–	3.52(10)(9)	97.0(2.6)(2.5)
ETM 10B	[60]	A	○	★	○	★	c	3.6(1)(2)	95(2)(6)
JLQCD/TWQCD 08A	[67]	A	○	■	■	★	–	$4.452(81)(38)(^{+0}_{-227})$	–
RBC 07†	[34]	A	■	■	★	★	–	4.25(23)(26)	119.5(5.6)(7.4)
ETM 07	[62]	A	○	■	○	★	–	3.85(12)(40)	105(3)(9)
QCDSF/ UKQCD 06	[68]	A	■	★	■	★	–	4.08(23)(19)(23)	111(6)(4)(6)
SPQcdR 05	[69]	A	■	○	○	★	–	$4.3(4)(^{+1.1}_{-0.0})$	$101(8)(^{+25}_{-0})$
ALPHA 05	[64]	A	■	○	★	★	a		$97(4)(18)^{\S}$
QCDSF/ UKQCD 04	[66]	A	■	★	■	★	–	4.7(2)(3)	119(5)(8)
JLQCD 02	[70]	A	■	■	○	■	–	$3.223(^{+46}_{-69})$	$84.5(^{+12.0}_{-1.7})$
CP-PACS 01	[63]	A	■	■	★	■	–	$3.45(10)(^{+11}_{-18})$	$89(2)(^{+2}_{-6})^{\star}$

… and examples of FLAG plots:
THE LIGHT QUARK MASSES





arXiv:1310.8555

- ■ results included in the average
- □ results that are not included in the average but pass all quality criteria
- □ all other results

1 THE LIGHT QUARK MASSES:

m_{ud} and m_s

LATTICE DETERMINATION OF QUARK MASSES

- QUARK MASSES CANNOT BE DIRECTLY MEASURED IN THE EXPERIMENTS, BECAUSE QUARKS ARE CONFINED INSIDE HADRONS

- BEING FUNDAMENTAL PARAMETERS OF THE STANDARD MODEL, QUARK MASSES CANNOT BE DETERMINED BY THEORETICAL CONSIDERATIONS ONLY.

⇒ QUARK MASSES CAN BE DETERMINED BY COMBINING TOGETHER A THEORETICAL AND AN EXPERIMENTAL INPUT. E.G.:

$$[M_{HAD}(\Lambda_{QCD}, m_q)]^{TH.} = [M_{HAD}]^{EXP.}$$

PGB

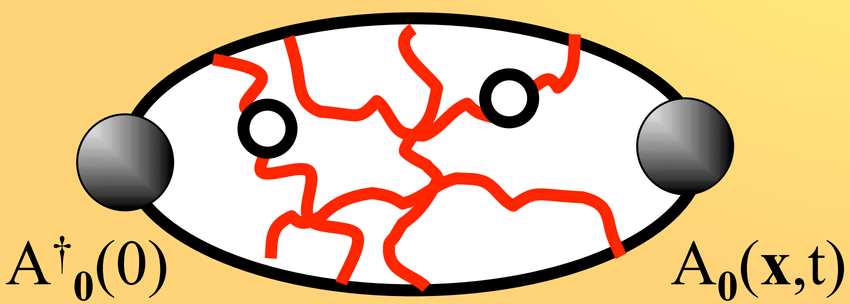
LATTICE QCD

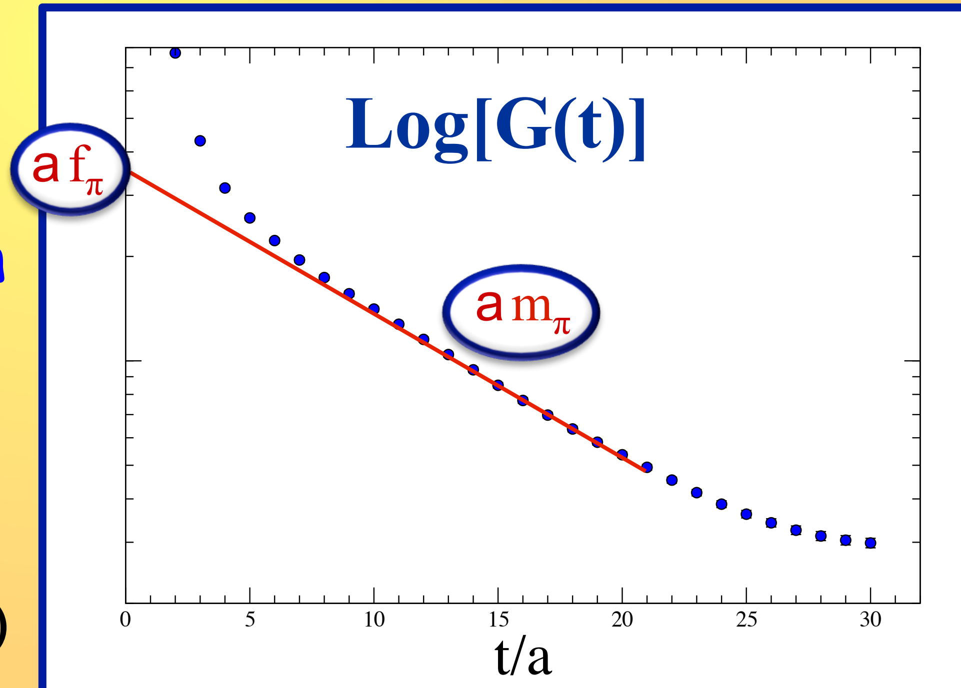
Hadron masses and matrix elements

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^\dagger{}_0(\mathbf{0},0) \rangle = \sum_n \frac{|\langle 0|\,A_0\,|\,n\rangle|^2}{2\,m_n} \exp[-m_n t]$$

$$\rightarrow \frac{|\langle 0|\,A_0\,|\,\pi\rangle|^2}{2\,m_\pi} \exp[-m_\pi t] = \frac{f_\pi^{\,2}\, m_\pi}{2} \exp[-\, m_\pi\, t]$$

Hadron mass and ⟨0|A|h⟩ matrix elements from the 2-point correlation function





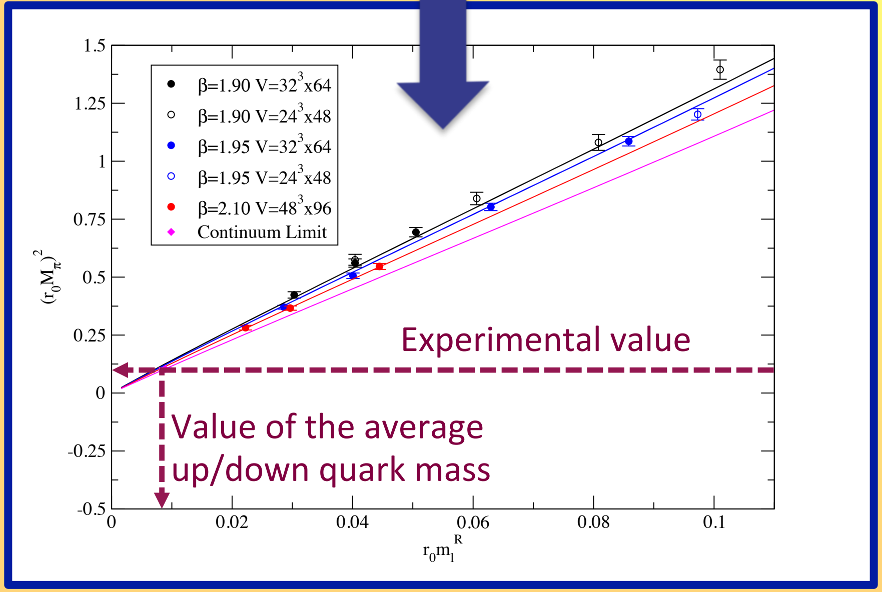
LATTICE DETERMINATION OF QUARK MASSES

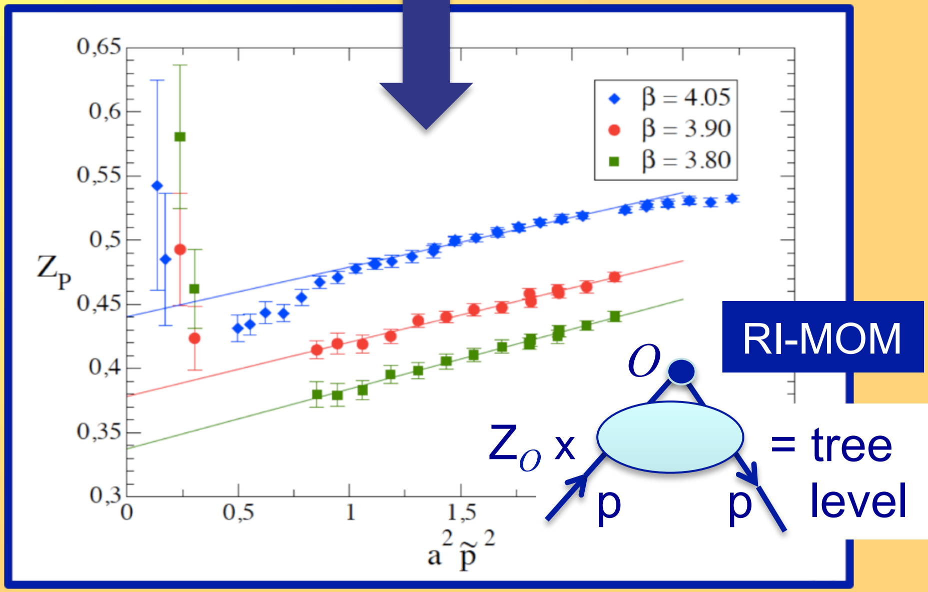
2 steps:

$$\hat{m}_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY OR NON-PERTURBATIVE METHODS





A recent lattice calculation with Nf=2+1+1

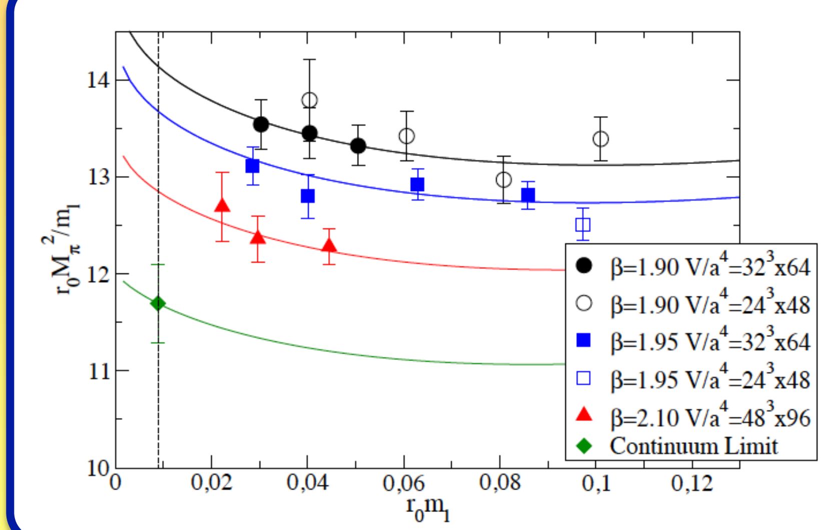
Up, down, strange and charm quark masses
with $N_f = 2+1+1$ twisted mass lattice QCD

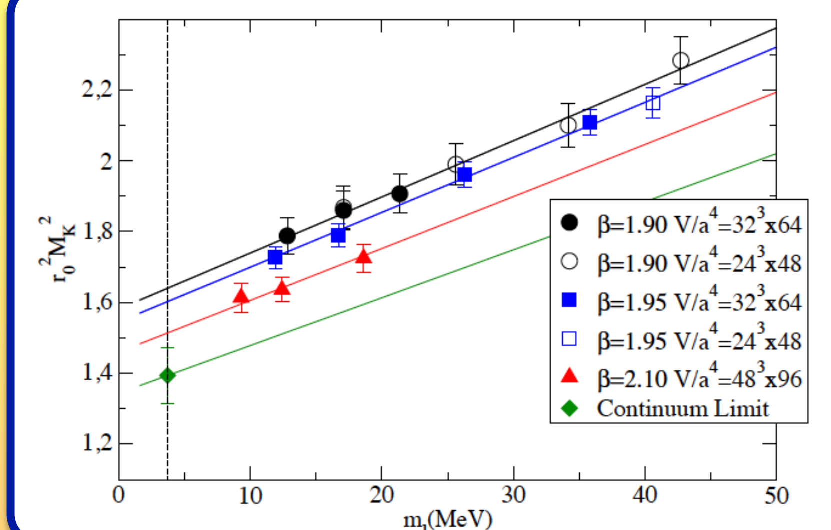
N. Carrasco[a], P. Dimopoulos[b,c], R. Frezzotti[c,d], V. Giménez[e], G. Herdoiza[f], P. Lami[g,a], V. Lubicz[g,a], A. Nube[h], D. Palao[i], L. Riggio[g,a], G.C. Rossi[c,d], F. Sanfilippo[l], L. Scorzato[m], S. Simula[a], C. Tarantino[g,a], C. Urbach[n]

ETMC 2014
(Nf=2+1+1)

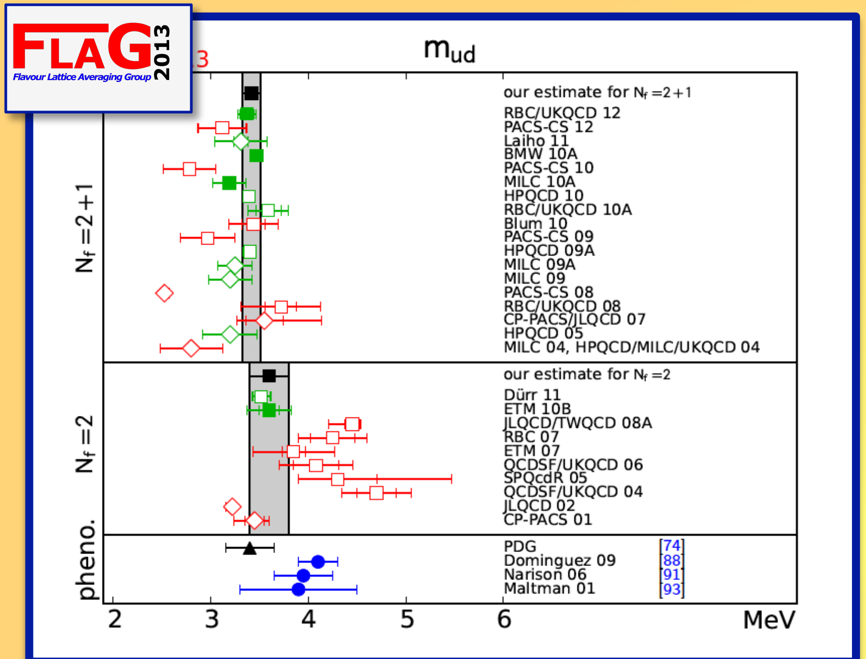
arXiv:1403.4504 [hep-lat]

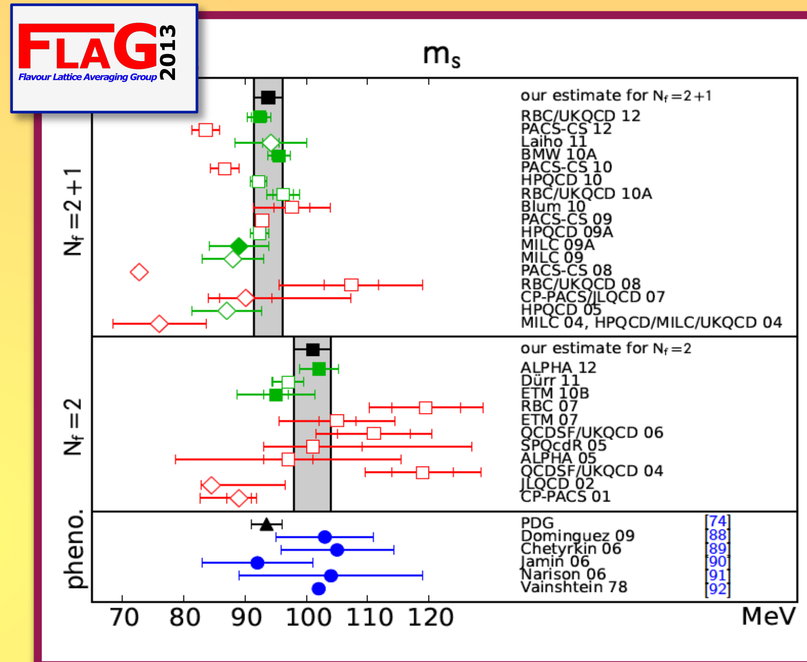
ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	N_{cfg}	$a\mu_s$	$a\mu_c$
A30.32	1.90	$32^3 \times 64$	0.0030	0.15	0.19	150	0.0145,	0.1800, 0.2200,
A40.32			0.0040			90	0.0185,	0.2600, 0.3000,
A50.32			0.0050			150	0.0225	0.3600, 0.4400
A40.24	1.90	$24^3 \times 48$	0.0040	0.15	0.19	150		
A60.24			0.0060			150		
A80.24			0.0080			150		
A100.24			0.0100			150		
B25.32	1.95	$32^3 \times 64$	0.0025	0.135	0.170	150	0.0141,	0.1750, 0.2140,
B35.32			0.0035			150	0.0180,	0.2530, 0.2920,
B55.32			0.0055			150	0.0219	0.3510, 0.4290
B75.32			0.0075			75		
B85.24	1.95	$24^3 \times 48$	0.0085	0.135	0.170	150		
D15.48	2.10	$48^3 \times 96$	0.0015	0.12	0.1385	60	0.0118,	0.1470, 0.1795,
D20.48			0.0020			90	0.0151,	0.2120, 0.2450,
D30.48			0.0030			90	0.0184	0.2945, 0.3595





A recent lattice calculation with Nf=2+1+1





$\overline{m}_{ud} = 3.6 \pm 0.2$ MeV [Nf=2]

$\overline{m}_{ud} = 3.42 \pm 0.09$ MeV [Nf=2+1]

$\overline{m}_s = 101 \pm 3$ MeV [Nf=2]

$\overline{m}_s = 93.8 \pm 2.4$ MeV [Nf=2+1]

$\overline{m}_{ud} = 3.70 \pm 0.17$ MeV [Nf=2+1+1]

$\overline{m}_s = 99.6 \pm 4.1$ MeV [Nf=2+1+1]

ETMC 2014

The lattice accuracy on light quark masses is at the few per cent level

② THE CKM MATRIX AND THE 1st ROW UNITARITY TEST

V_{us}, V_{ud} from f_K/f_π, $f_+(0)$

The determination of V_{us} and V_{ud} provides the most stringent CKM unitarity test

THE 1st ROW UNITARITY TEST

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1}$$

Central value:	$(0.974)^2$	$(0.225)^2$	$(3.75\cdot10^{-3})^2$
Error:	$3.9\cdot10^{-4}$	$4.5\cdot10^{-4}$	$\sim10^{-6}$

$\boxed{\simeq \cos\theta_C}$ $\boxed{\simeq \sin\theta_C}$

$$\begin{bmatrix} \boxed{V_{ud} \quad V_{us} \quad V_{ub}} \\ V_{cd} \quad V_{cs} \quad V_{cb} \\ V_{td} \quad V_{ts} \quad V_{tb} \end{bmatrix}$$

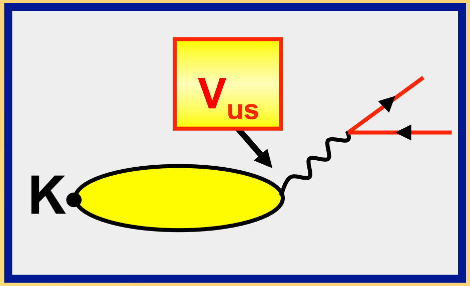
Processes: $K \to l\nu$, $K \to \pi l\nu$

Theory input: f_K/f_π , $f_+(0)$

Vus/Vud from Kμ2/πμ2 decays

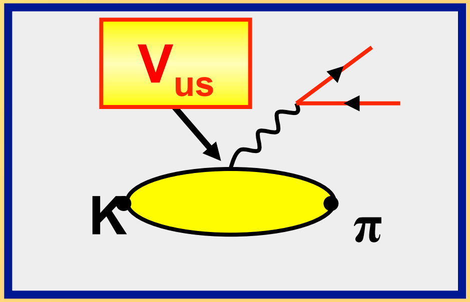
$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)} \times 0.9930(35)$$

[Marciano 04]



Vus from Kl3 decays

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\mathrm{EW}}[1+\Delta_{SU(2)}+2\Delta_{\mathrm{EM}}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$



$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_k}{f_\pi} = 0.2758(5)$$

0.2%

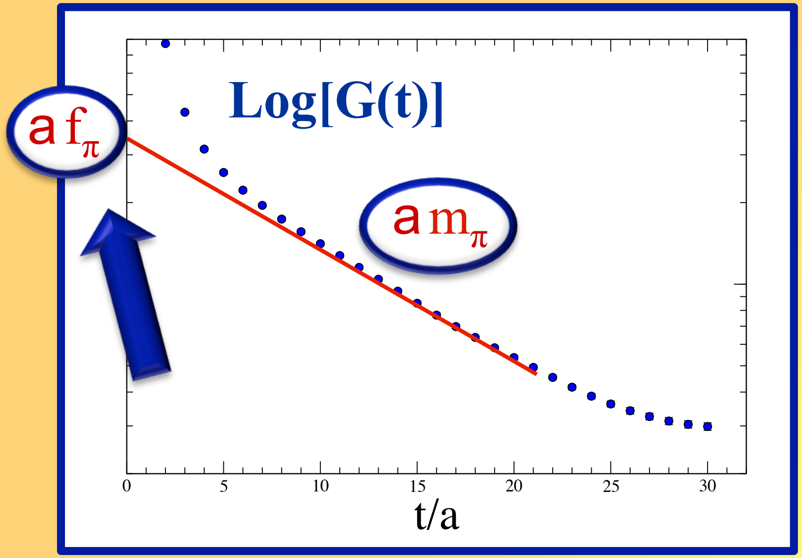
FlaviA net Kaon WG

arXiv:1005.2323 [hep-ph]

$$|V_{us}|\, f_+(0) = 0.2163(5)$$

0.2%

Lattice calculation of f_K/f_π



$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t) A_0^\dagger(\mathbf{0},0) \rangle \xrightarrow{t\to\infty} \frac{f_\pi^2 m_\pi^2}{2} \exp(-m_\pi t)$$

$f_{K^+}/f_{\pi^+} = 1.183(17)$

$f_{K^+} = 154.4(2.1)$ MeV

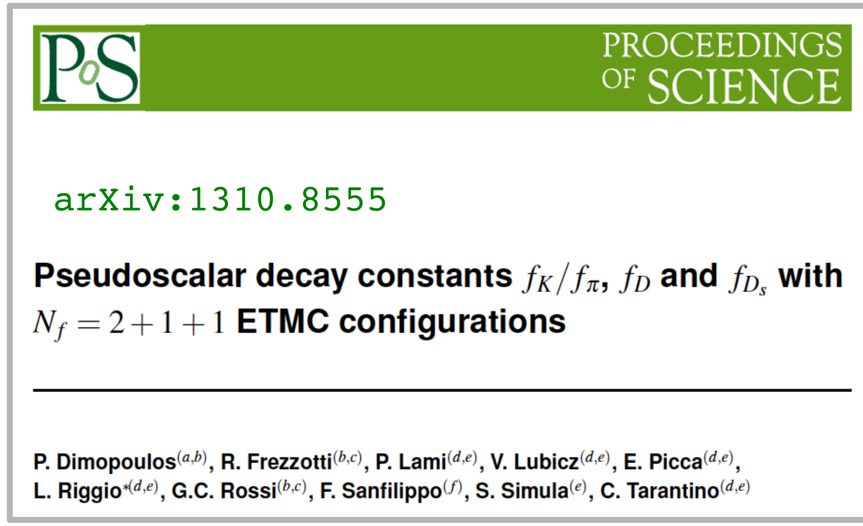
ETMC 13 (Nf=2+1+1) [preliminary]

Two other Nf=2+1+1 results

$f_{K^+}/f_{\pi^+} = 1.192(2)$ HPQCD 13

$f_{K^+}/f_{\pi^+} = 1.195(5)$ MILC 13

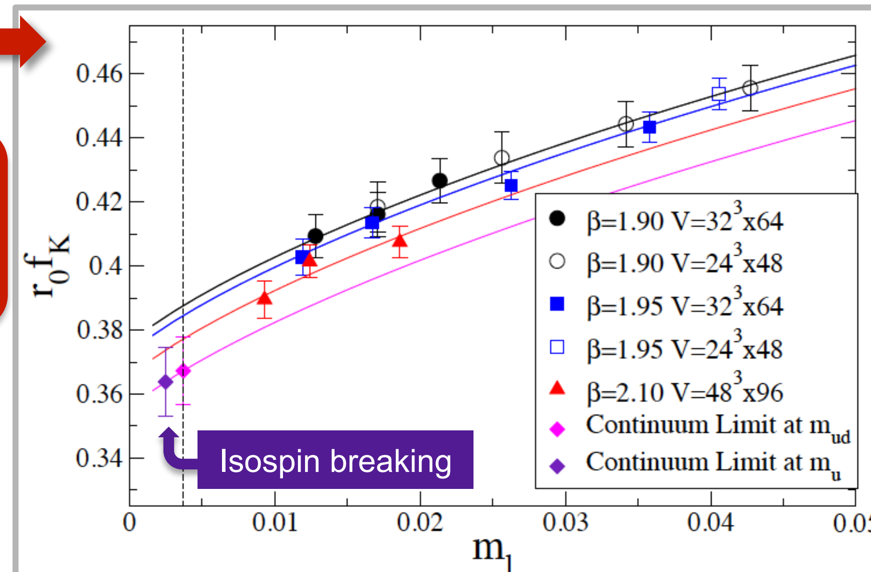
Simulations at the physical point !


PROCEEDINGS OF SCIENCE

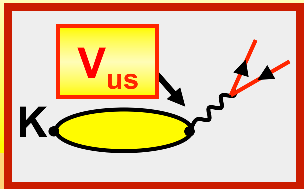
arXiv:1310.8555

Pseudoscalar decay constants f_K/f_π, f_D and f_{D_s} with $N_f = 2+1+1$ ETMC configurations

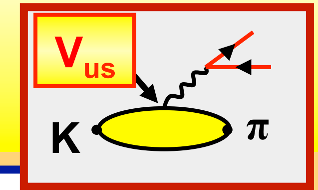
P. Dimopoulos[(a,b)], R. Frezzotti[(b,c)], P. Lami[(d,e)], V. Lubicz[(d,e)], E. Picca[(d,e)], L. Riggio*[(d,e)], G.C. Rossi[(b,c)], F. Sanfilippo[(f)], S. Simula[(e)], C. Tarantino[(d,e)]

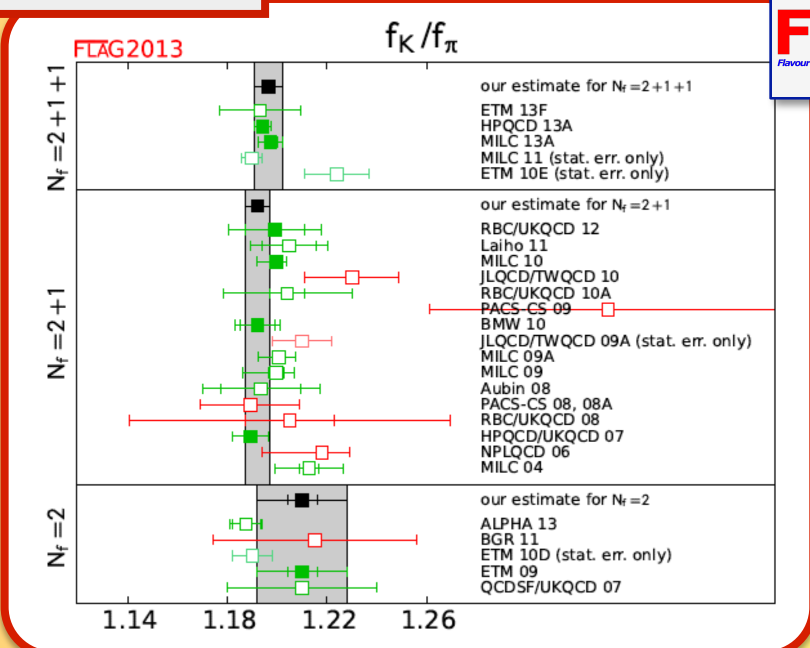


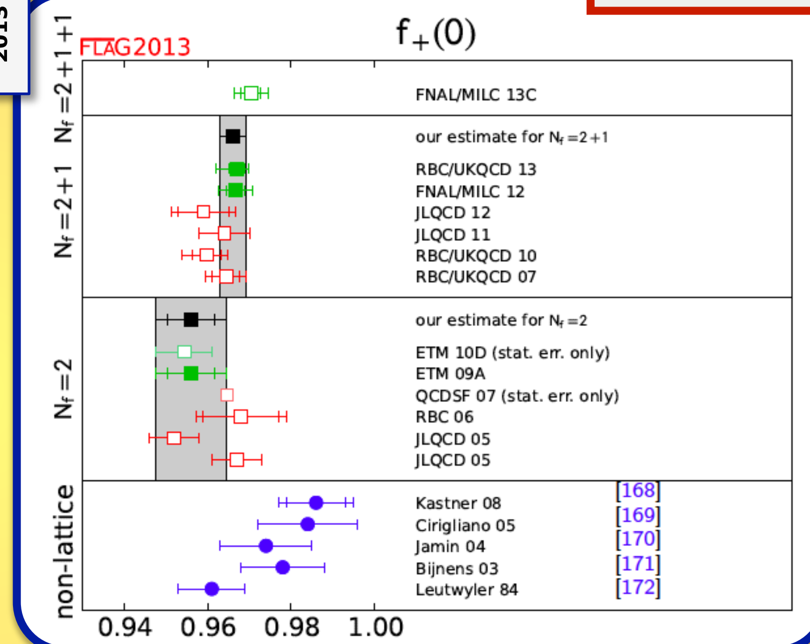
Lattice results for Vud and Vus:



f_K/f_π and $f_+(0)$







$f_{K^+}/f_{\pi^+} = 1.194(5)$ Nf=2+1+1

$f_{K^+}/f_{\pi^+} = 1.192(5)$ Nf=2+1 **0.4%**

$f_{K^+}/f_{\pi^+} = 1.205(18)$ Nf=2

$f_+(0) = 0.970(3)$ Nf=2+1+1

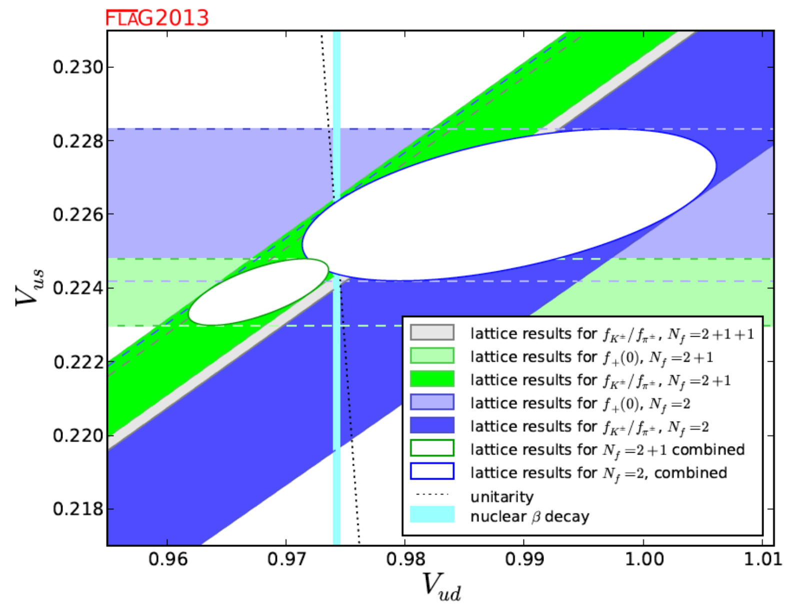
$f_+(0) = 0.966(3)$ Nf=2+1 **0.3%**

$f_+(0) = 0.956(8)$ Nf=2

State of the art LQCD calculations are Nf=2+1+1 at the physical point

Predictions of analytical models tends to be larger than lattice results

The 1st row unitarity test



The unitarity plot

From lattice results only

- $|V_{us}| = 0.2239(9)$
- $|V_{us}|/|V_{ud}| = 0.2314(11)$
- $|V_{ud}| = 0.968(6)$

From nuclear β-decays:

$|V_{ud}|_\beta = 0.97425(22)$

The unitarity test: $\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$

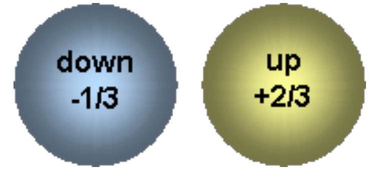
- From lattice only: $\Delta_u = (-14 \pm 11) \times 10^{-3}$
- From lattice Kl3 + $|V_{ud}|_\beta$: $\Delta_u = (-7 \pm 6) \times 10^{-4}$
- From lattice Kl2 + $|V_{ud}|_\beta$: $\Delta_u = (0 \pm 6) \times 10^{-4}$

③ ISOSPIN BREAKING EFFECTS ON THE LATTICE:

m_u/m_d, $M_{\pi+} - M_{\pi0}$, $M_{K+} - M_{K0}$, $M_n - M_p$, $[f_K/f_\pi]^{QCD}$

ISOSPIN BREAKING EFFECTS

Isospin symmetry is an almost exact property of the strong interactions



Isospin breaking effects are induced by:

$m_u \neq m_d$: $O[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"

$Q_u \neq Q_d$: $O(\alpha_{e.m.}) \approx 1/100$ "Electromagnetic"

Since electromagnetic interactions renormalize quark masses the two corrections are intrinsically related

Though small, IB effects play often a very important role

- The knowledge of mu and md (besides m_{ud}) is important for our understanding of flavor physics at the fundamental level

mu ≃ 2.5 MeV   md ≃ 5 MeV

mc ≃ 1.2 GeV   ms ≃ 100 MeV

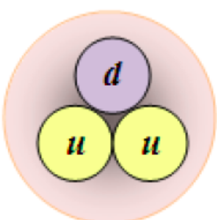
mt ≃ 175 GeV   mb ≃ 4.3 GeV

A remarkable relation:

$$\left(\frac{m_d}{m_s}\right)^{1/2} \simeq \left(\frac{m_u}{m_c}\right)^{1/4} \simeq V_{us} \simeq 0.22$$

- The actual values of the mass difference md-mu and quark charges Qd, Qu implies Mn > Mp and guarantees the stability of matter

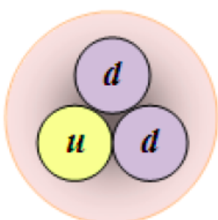
Proton



938.27 MeV

<

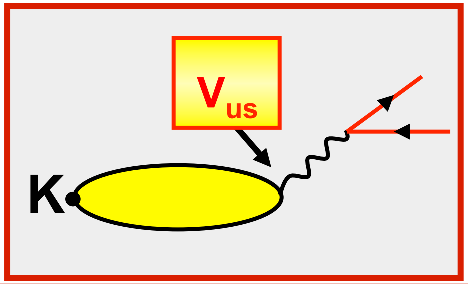
Neutron



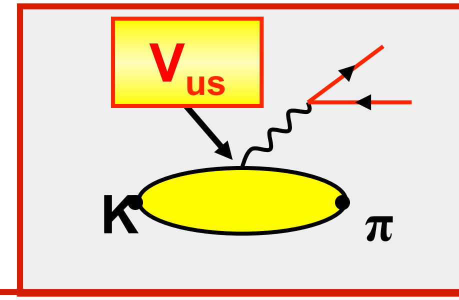
939.57 MeV

M(n) – M(p) = 1.3 MeV = 0.14%

EXPERIMENTS



FlaviA net Kaon WG

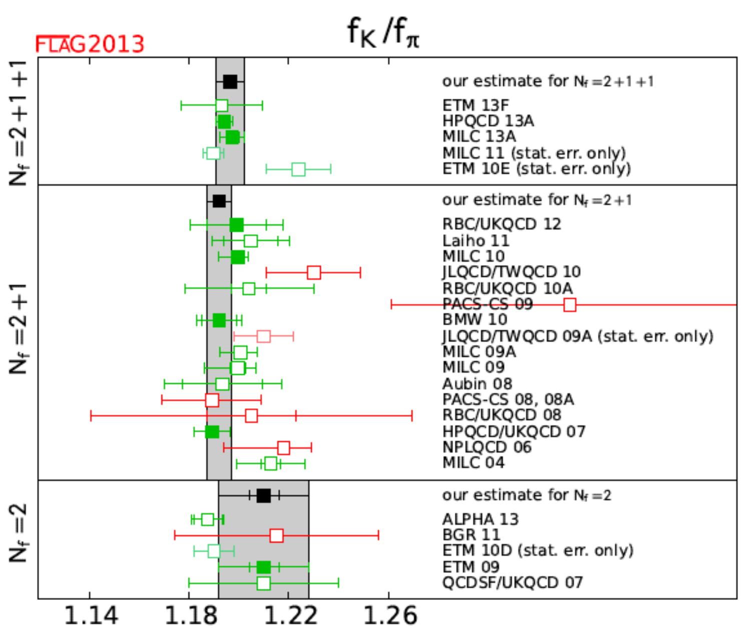


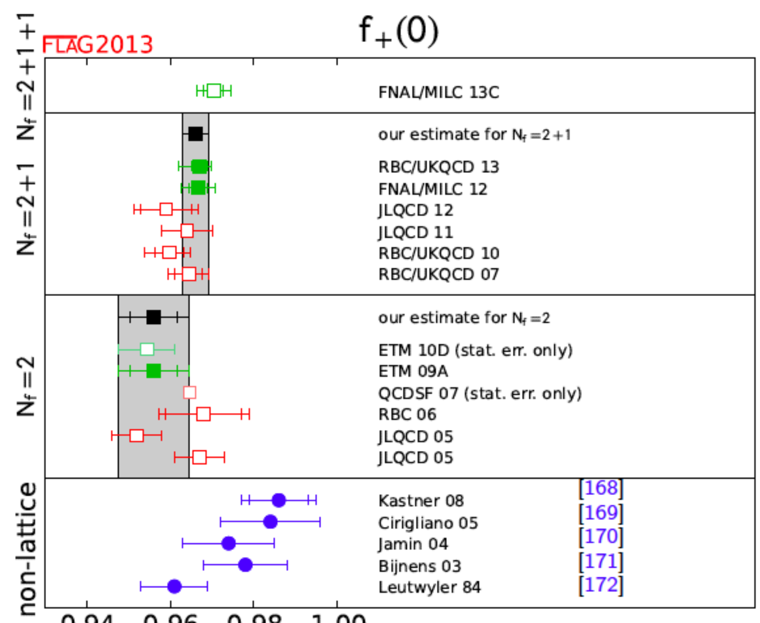
$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.2758(5)$$

0.2%

$$|V_{us}| f_+(0) = 0.2163(5)$$

THEORY: LATTICE QCD





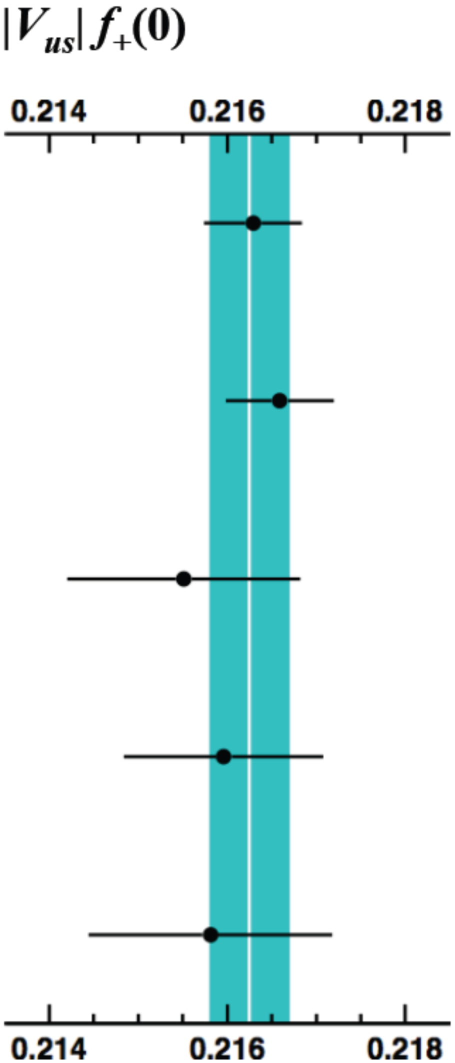
$f_K / f_\pi = 1.192(5)$

0.4%

$f_+(0) = 0.966(3)$

0.3%

$|V_{us}|\,f_+(0)$ from world data: 2012



Approx. contrib. to % err from:

		% err	BR	τ	$\delta_{SU,EM}$	Int
K_Le3	0.2163(5)	0.26	0.09	0.20	0.11	0.05
$K_L\mu3$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
K_Se3	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K^{\pm}e3$	0.2160(11)	0.52	0.31	0.09	0.41	0.04
$K^{\pm}\mu3$	0.2158(13)	0.63	0.47	0.08	0.41	0.06

Average: $|V_{us}|\,f_+(0) = 0.2163(5)$ $\chi^2/\mathrm{ndf} = 0.84/4$ (93%)

A strategy for Lattice QCD:

The isospin breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

G.M. de Divitiis,[a,b] P. Dimopoulos,[c,d] R. Frezzotti,[a,b] V. Lubicz,[e,f] G. Martinelli,[g,d] R. Petronzio,[a,b] G.C. Rossi,[a,b] F. Sanfilippo,[c,d] S. Simula,[f] N. Tantalo[a,b] and C. Tarantino[e,f]

PHYSICAL REVIEW D **87,** 114505 (2013)

Leading isospin breaking effects on the lattice

G. M. de Divitiis,[1,2] R. Frezzotti,[1,2] V. Lubicz,[3,4] G. Martinelli,[5,6] R. Petronzio,[1,2] G. C. Rossi,[1,2] F. Sanfilippo,[7] S. Simula,[4] and N. Tantalo[1,2]

(RM123 Collaboratio

arXiv:1303.4896

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(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

① The (md-mu) expansion

- Identify the isospin breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}\left(m_u + m_d\right)\left(\bar{u}u + \bar{d}d\right) - \frac{1}{2}\left(m_d - m_u\right)\left(\bar{u}u - \bar{d}d\right) \right] =$$

$$= \sum_x \left[m_{ud}\left(\bar{u}u + \bar{d}d\right) - \Delta m\left(\bar{u}u - \bar{d}d\right) \right] = S_0 - \Delta m\, \hat{S}$$

$\hat{S} = \Sigma_x(\bar{u}u - \bar{d}d)$

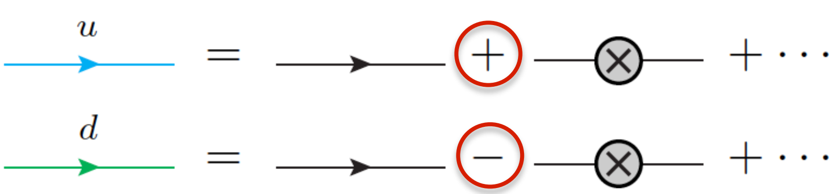
- Expand the functional integral in powers of Δm

$$\langle O \rangle = \frac{\int D\phi\, O\, e^{-S_0 + \Delta m\, \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m\, \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O\, e^{-S_0}\left(1 + \Delta m\, \hat{S}\right)}{\int D\phi\, e^{-S_0}\left(1 + \Delta m\, \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0}{1 + \cancel{\Delta m \langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0$$

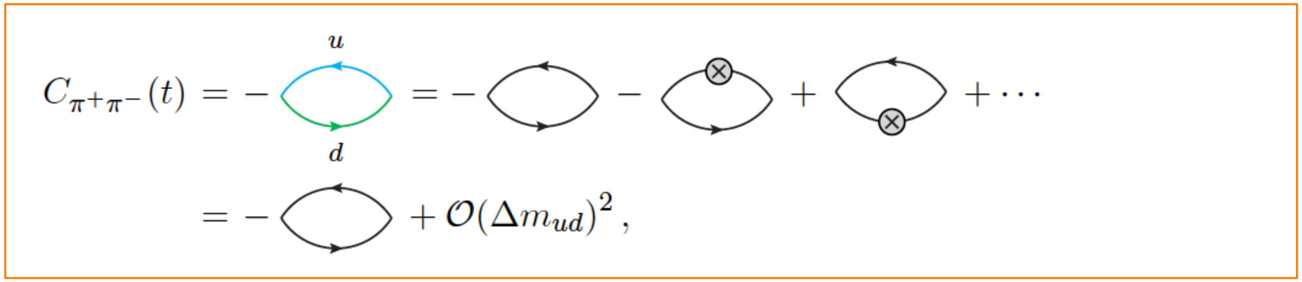
for isospin symmetry

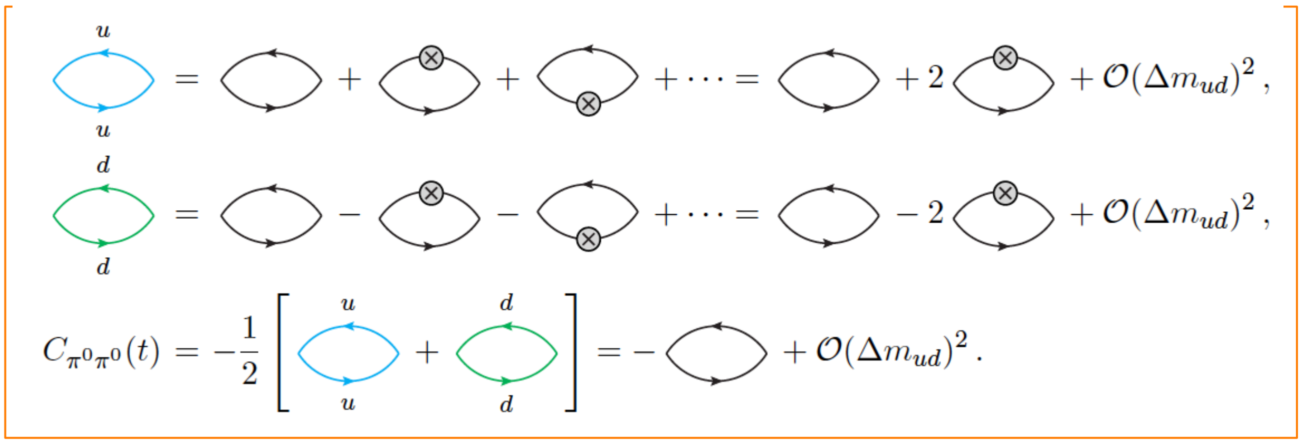
- At leading order in Δm the corrections only appear in the valence quark propagators:

(disconnected contractions of $\bar{u}u$ and $\bar{d}d$ vanish due to isospin symmetry)



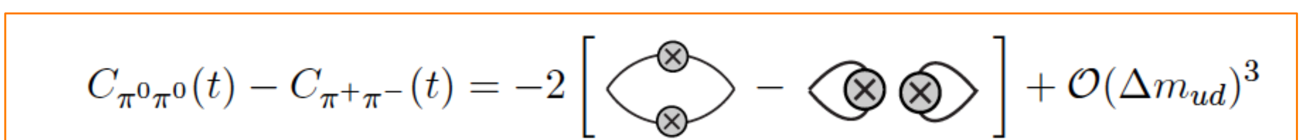
An example: the charged and neutral pions





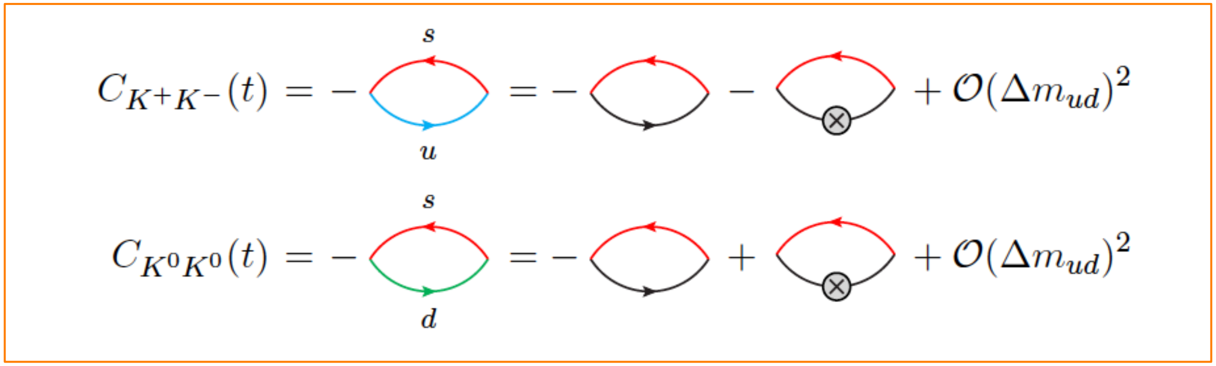
Because of the u $\longleftrightarrow$d symmetry, the corrections cancel at 1st order

This is certainly not the case at 2nd order:



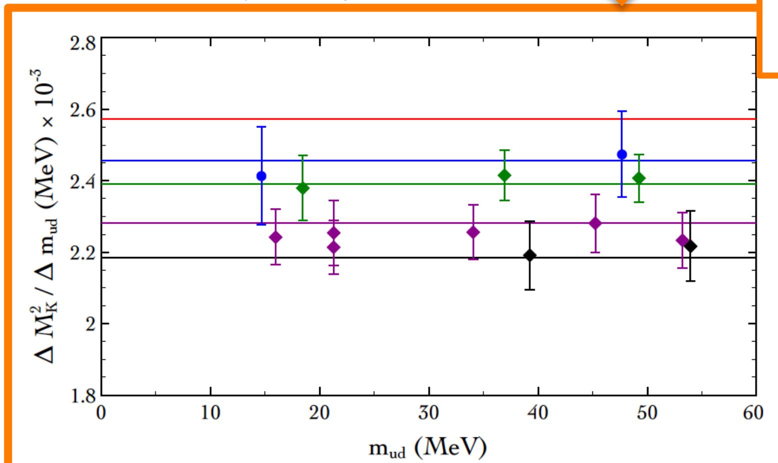
The charged and neutral kaons

Corrections to the charged and neutral kaons are equal and opposite at 1st order:



We compute the slope in Δm: $S_m = S_0 - \Delta m \hat{S}$

$$\frac{\left[M^2_{K^0} - M^2_{K^+}\right]^{QCD}}{m_d - m_u}\left(\overline{MS},\, 2\ \text{GeV}\right) = 2.57(8)\times 10^3\ \text{MeV}$$



But in order to get a determination of md-mu we must evaluate also the QED contribution to the mass splitting

A strategy for Lattice QCD:

The isospin breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

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2

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(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

QED ON THE LATTICE

- **Non-compact QED**: the **dynamical variable** is the gauge potential $A_\mu(x)$ in a fixed covariant gauge ($\nabla^-_\mu A_\mu(x) = 0$)

$$S_{QED} = \frac{1}{2}\sum_{x;\mu\nu} A_\nu(x)\left(-\nabla^-_\mu \nabla^+_\mu\right) A_\nu(x) \overset{(p.b.c.)}{=} \frac{1}{2}\sum_{k;\mu\nu} \tilde{A}^*_\nu(k)\left(2\sin(k_\mu/2)\right)^2 \tilde{A}_\nu(k)$$

- The photon propagator is IR divergent → subtract the zero momentum mode

- **Full covariant derivatives** are defined introducing **QED** and **QCD** links:

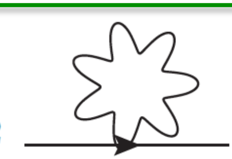
$$A_\mu(x) \to E_\mu(x) = e^{-iaeA_\mu(x)}$$

$$D^+_\mu q_f(x) = \left[E_\mu(x)\right]^{e_f} U_\mu(x)\, q_f(x+\hat{\mu}) - q_f(x)$$

QED QCD

- Since $E_\mu(x) = e^{-ieA_\mu(x)} = 1 - ieA_\mu(x) - 1/2\, e^2 A^2_\mu(x) + \ldots$ the expansion of the lattice action up to $O(e^2)$ contains 2 contributions:

$(e_f e)^2$ 

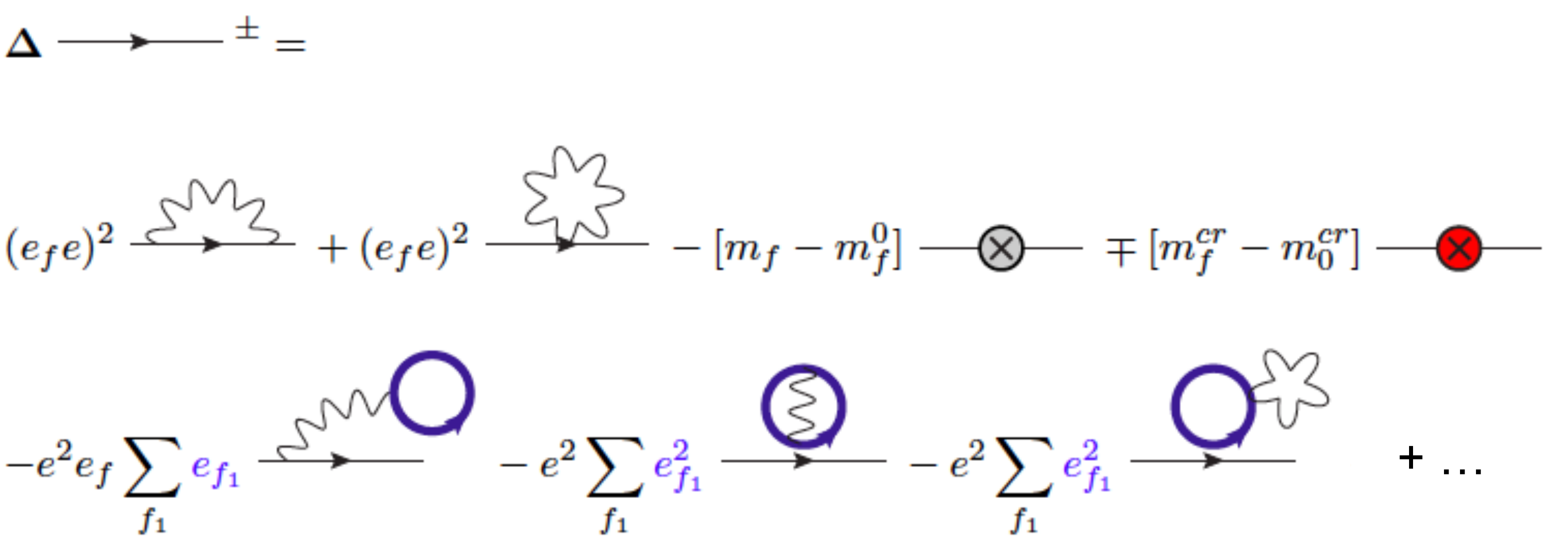
$(e_f e)^2$ 

- Switching on the e.m. interactions requires the introduction of new counterterms which renormalize the couplings of the theory:

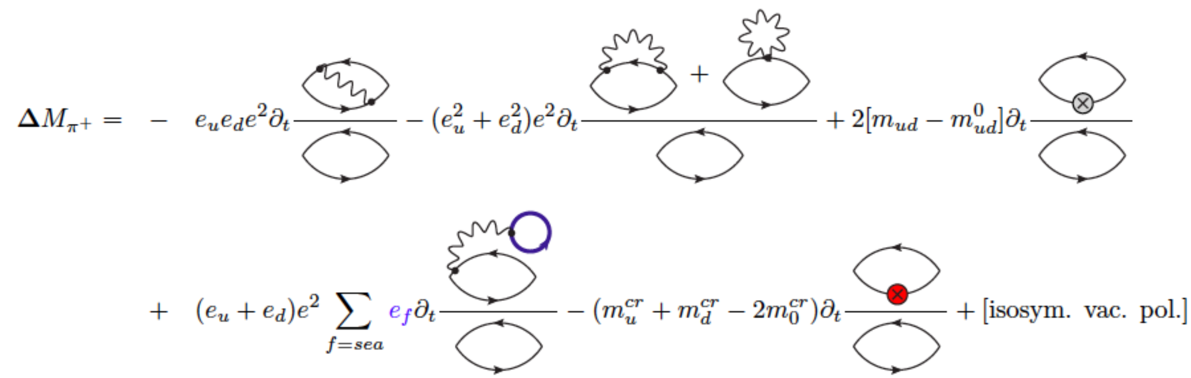
$$\vec{g}^{\,0} = \left(0, g_s^0, m_u^0, m_d^0, m_s^0, \dots\right) \;\rightarrow\; \vec{g} = \left(e^2, g_s, m_u, m_d, m_s, \dots\right)$$

- For any observable, the leading isospin breaking expansion reads,

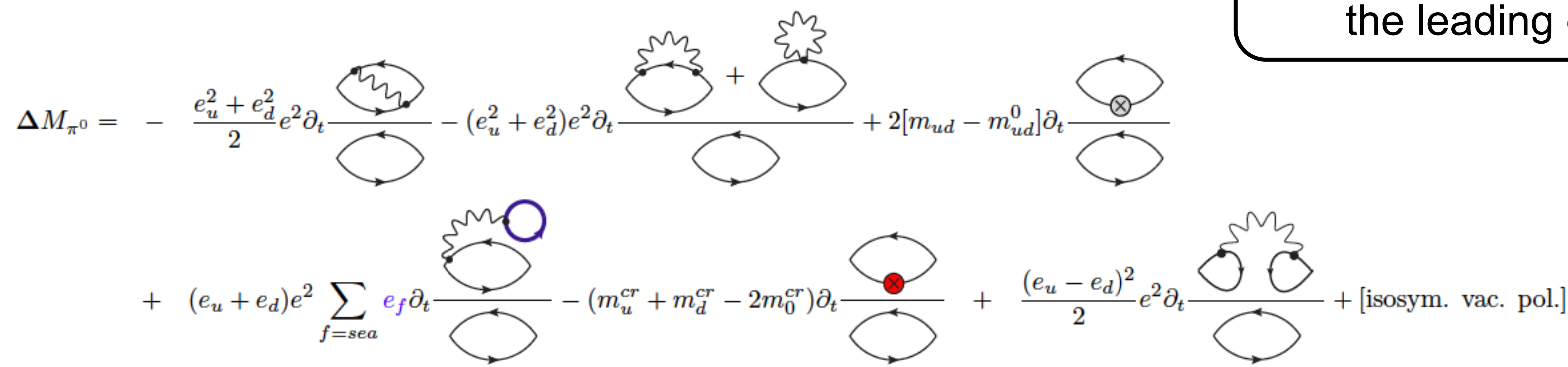
$$O(\vec{g}) = O(\vec{g}^{\,0}) + \left[e^2 \frac{\partial}{\partial e^2} + \left(g_s^2 - (g_s^0)^2\right)\frac{\partial}{\partial g_s^2} + \left(m_f - m_f^0\right)\frac{\partial}{\partial m_f} + \dots \right] O(\vec{g})\Big|_{\vec{g}=\vec{g}^{\,0}}$$

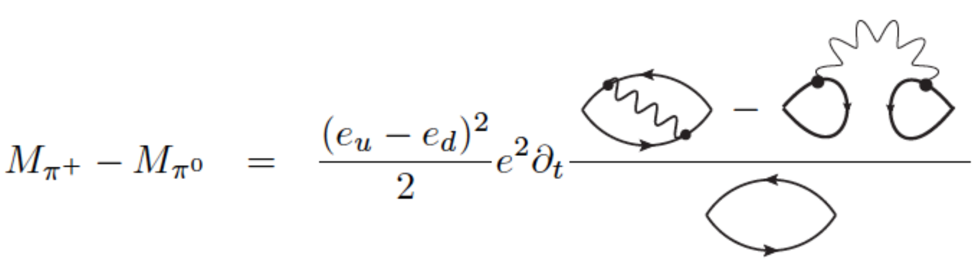


The charged-neutral pion mass splitting



Since $e_u \neq e_d$, sea quark contributions now enter at the leading order

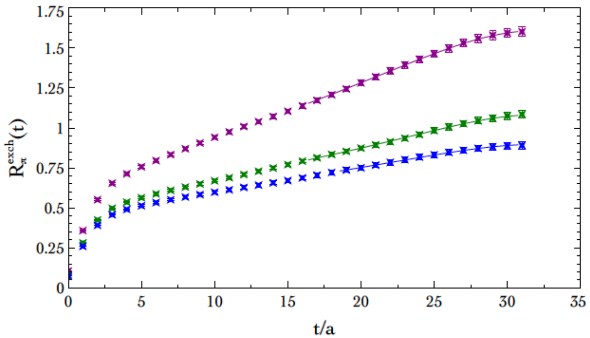


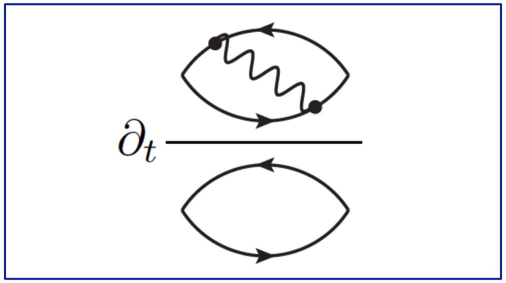


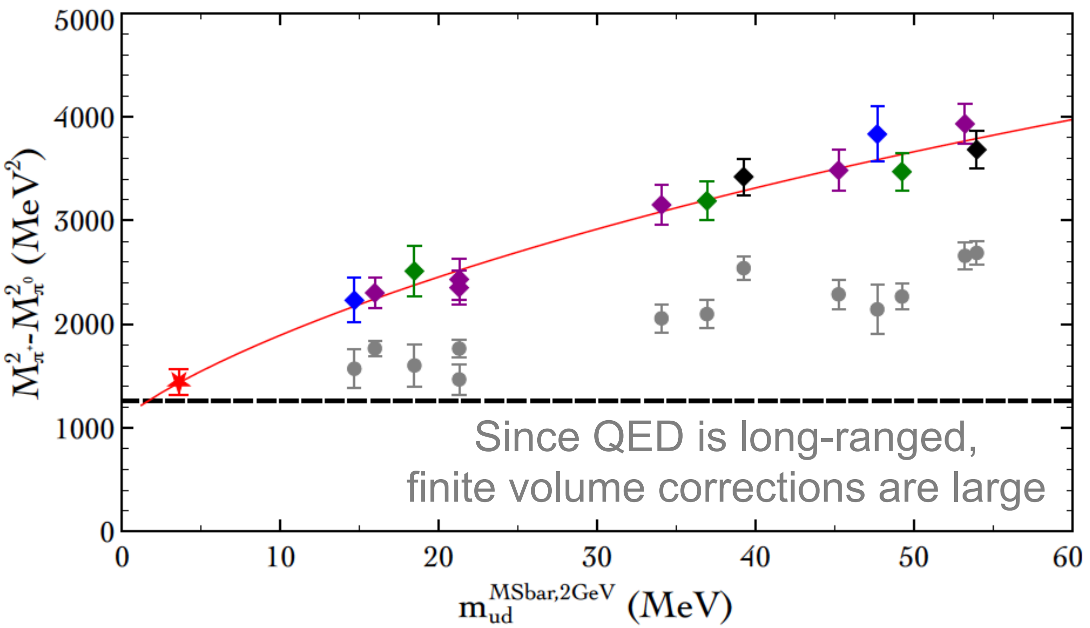
Only 2 diagrams contribute to the pion mass splitting.

The disconnected diagram, of $O(\alpha_{em} m_{ud})$, has been neglected in the present calculation

The charged-neutral pion mass splitting



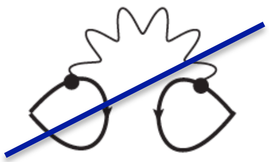




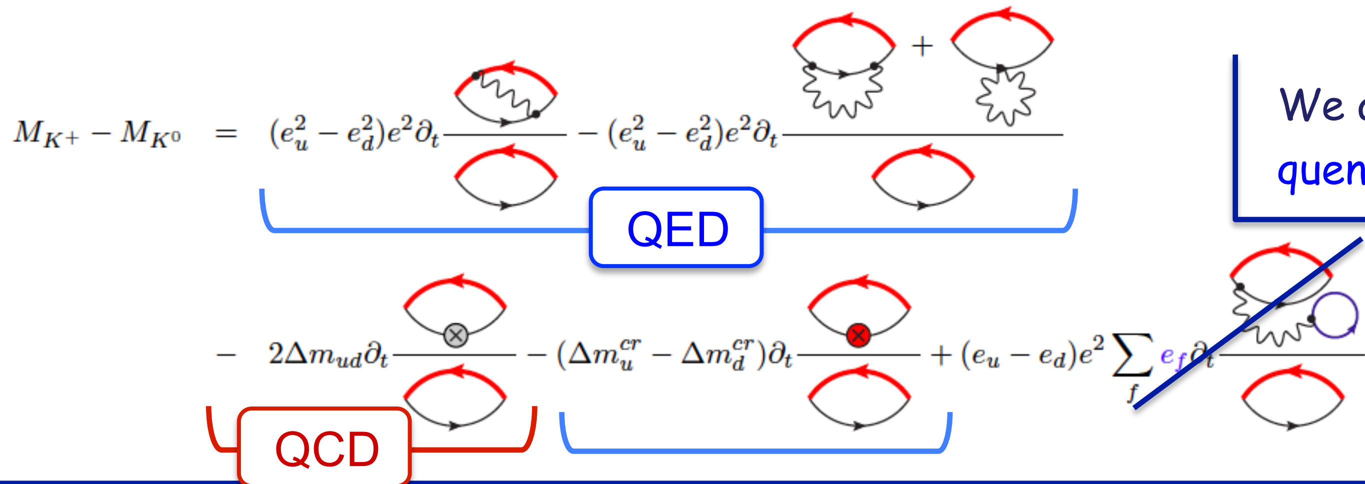
We obtain $\boxed{M_{\pi^+} - M_{\pi^0} = 5.2(5)(6)\ \text{MeV}}$ in good agreement

with the experimental value $\left[M_{\pi^+} - M_{\pi^0}\right]^{\exp} = 4.6\ \text{MeV}$

[It suggests, a posteriori, that the effect of having neglected the disconnected contribution of $O(\alpha_{em} m_{ud})$ is small]



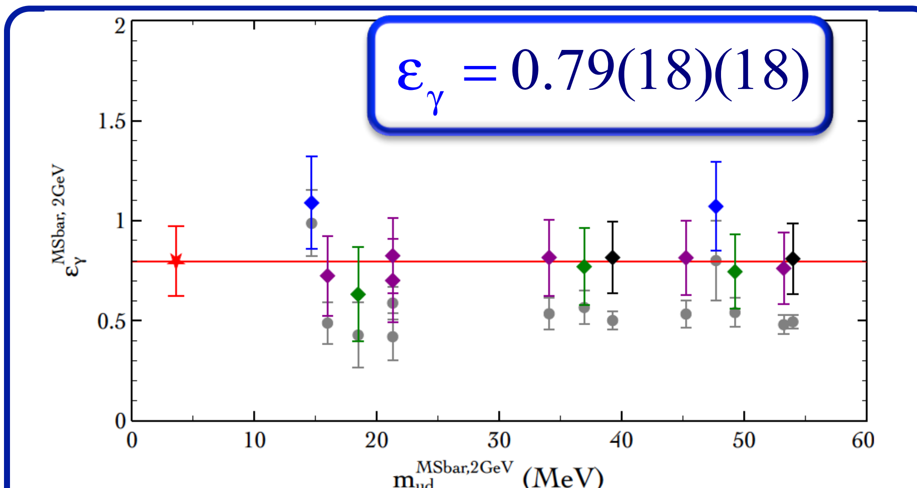
The charged and neutral kaon masses



We adopted the electro-quenched approximation (to be addressed in a future calculation)

The result can be expressed in terms of the violation of the Dashen's theorem:

$$\varepsilon_\gamma = \frac{\left[M_{K^+}^2 - M_{K^0}^2\right]^{QED} - \left[M_{\pi^+}^2 - M_{\pi^0}^2\right]^{QED}}{M_{\pi^+}^2 - M_{\pi^0}^2}$$

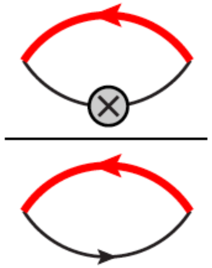


$$\left[M_{K^+} - M_{K^0}\right]^{QED} = 2.3(2)(2)\ \mathrm{MeV} \quad , \quad \left[M_{K^+} - M_{K^0}\right]^{QCD} = -6.2(2)(2)\ \mathrm{MeV}$$

The up and down quark masses

From: $$\frac{\left[M^2_{K^0} - M^2_{K^+}\right]^{QCD}}{m_d - m_u} = 2.57(8)\times 10^3 \text{ MeV}$$

$2\Delta m_{ud}\partial_t$



(all masses in MSbar at 2 GeV)

$(\bar{m}_d - \bar{m}_u) = 2.39(8)(17)$ MeV

$\bar{m}_u / \bar{m}_d = 0.50(2)(3)$

$\bar{m}_u = 2.40(23)$ MeV

$\bar{m}_d = 4.80(23)$ MeV

$\bar{m}_{ud} = 3.6(2)$ MeV

from isosymmetric LQCD

RM123 2013 (Nf=2)

$\bar{m}_u = 2.36(24)$ MeV

$\bar{m}_d = 5.03(26)$ MeV

$\bar{m}_{ud} = 3.70(17)$ MeV

$\bar{m}_u / \bar{m}_d = 0.470(56)$

ETMC 2014 (Nf=2+1+1)

Comparison with other approaches/results

Other lattice studies of QCD + QED have been /are being performed

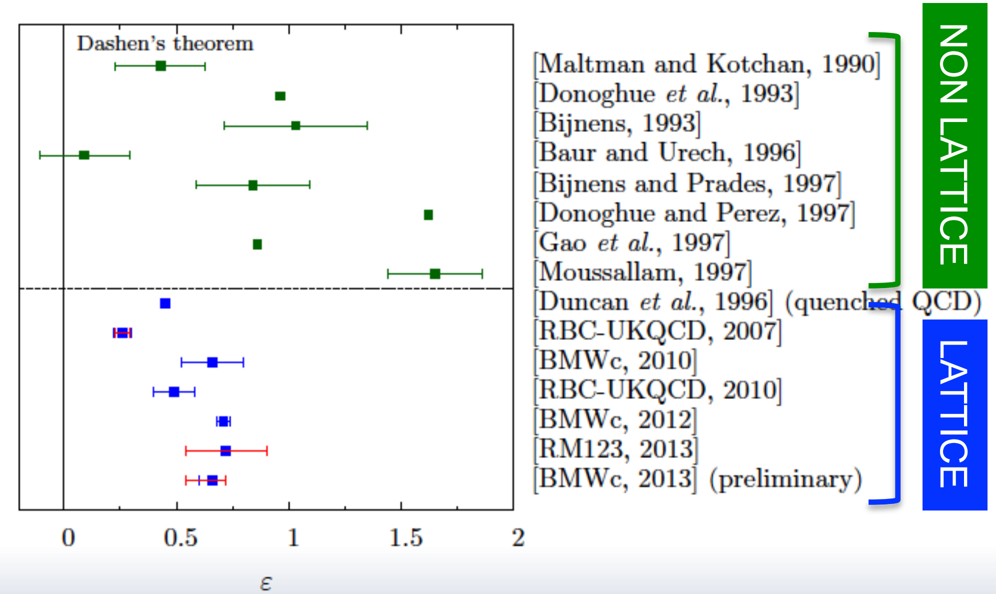
They are based on the "standard" approach: QED is introduced directly in the MC simulation, like QCD.

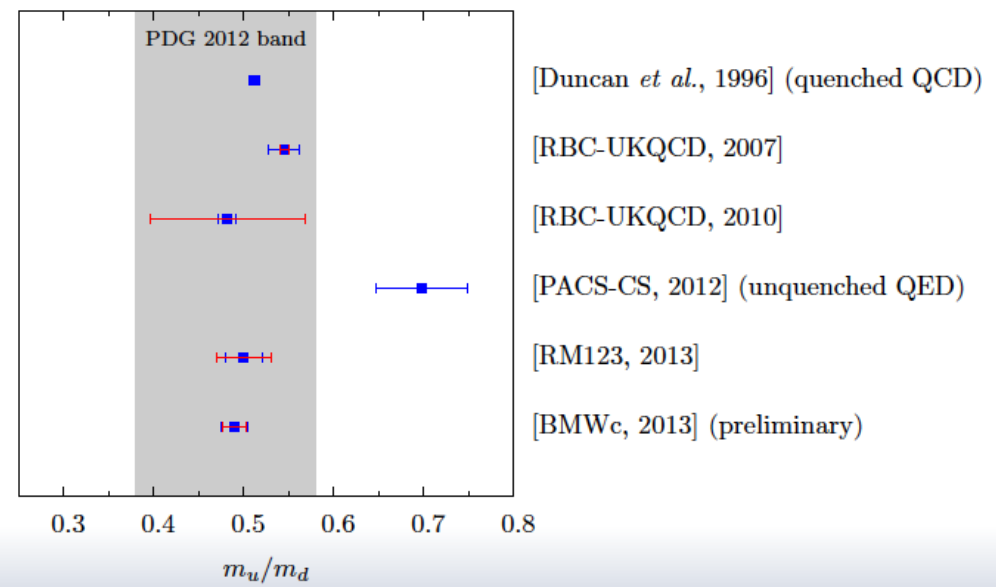
Advantages of our approach:

- The small parameters Δm and e are factorized in the expansion
- No need to generate new gauge configurations

Disadvantages:

- More vertices and correlations functions to be computed

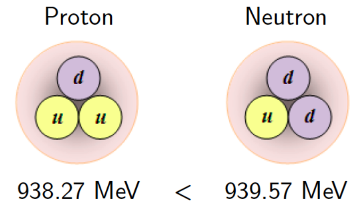




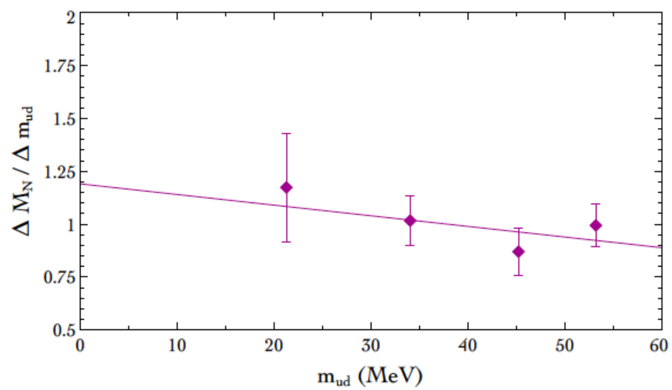
Antonin Portelli, talk at KAON13

The neutron-proton mass splitting

The up-down mass difference (QCD) and electromagnetic interactions have opposite effect on the neutron-proton mass splitting



- We have only evaluated so far the QCD contribution:



$$\left[M_N - M_P\right]^{QCD} = 2.9(6)\,\text{MeV}$$

$$\left[M_N - M_P\right]^{QED} = -1.6(6)\,\text{MeV}$$

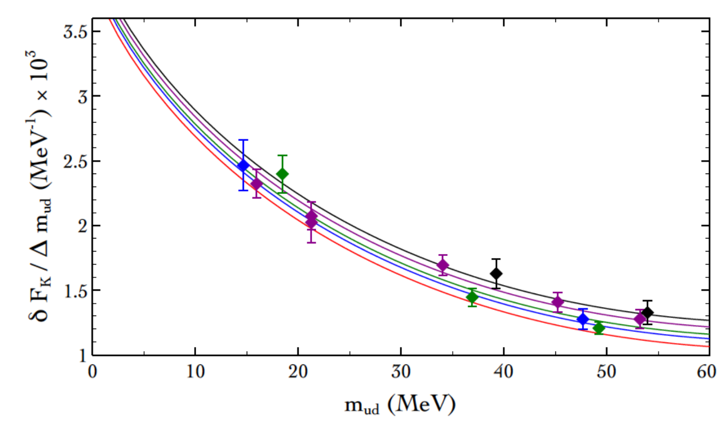
A study of both QCD and QED IB effects for the whole baryon octet is in progress

LQCD calculations of QCD+QED:	BMW Collab. arXiv:1306.2287	T. Blum et al. arXiv::1006.1311
$[\,M_N - M_p\,]^{QCD}$ (MeV)	2.28(25)(7)	2.51(14)(…)
$[\,M_N - M_p\,]^{QED}$ (MeV)	-1.59(30)(35)	-0.38(7)(…)

Isospin breaking effects in the ratio f_K/f_π

- We find that the QCD isospin breaking correction to the ratio f_K/f_π is rather small:

$$\delta_{SU(2)} = \frac{1}{f_K}\frac{\partial f_K}{\partial \Delta m_{ud}}\Delta m_{ud} = -0.40(3)(2)\,\%$$

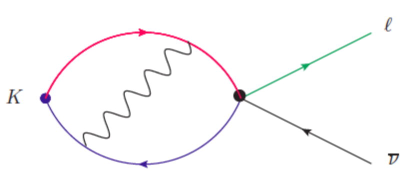


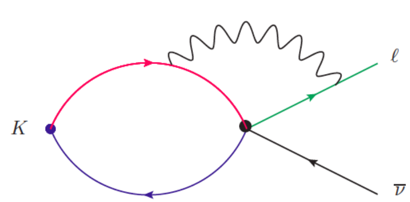
- The result is nevertheless larger than the prediction of SU(3) ChPT at NLO

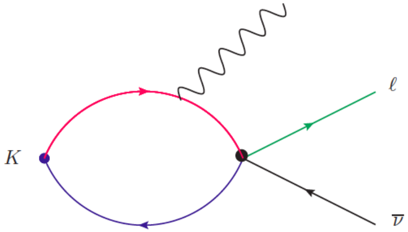
$$\delta_{SU(2)} = -\frac{1}{2}\frac{m_d - m_u}{m_s - m_{ud}}\left[\frac{f_K}{f_\pi} - 1 - \frac{M_K^2 - M_\pi^2 - M_\pi^2 \ln(M_K^2/M_\pi^2)}{64\pi^2 F_0^2}\right] = -0.21(6)\,\%$$

[Gasser, Leutwyler 1985; Cirigliano, Neufeld, 2011]

Lattice QCD evaluation of δ_{EM}: a challenging project







+ ...

1 LQCD CALULATIONS ARE RAPIDLY EXTENDING THEIR DOMAIN OF APPLICABILITY AND IMPROVING THEIR ACCURACY

2 FOR SEVERAL QUANTITIES IN FLAVOUR PHYSICS THE ACCURACY IS AT THE PERCENT LEVEL

3 STATE OF THE ART LQCD CALCULATIONS ARE Nf=2+1+1 SIMULATIONS AT PHYSICAL QUARK MASSES

4 ISOSPIN BREAKING EFFECTS ARE PHENOMENOLOGICALLY RELEVANT AND THEY ARE NOW BEING STUDIED ON THE LATTICE