Thermodynamics of SU(2) quantum Yang-Mills theory and CMB anomalies

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Outline

- **motivation:** nonperturbative, analytical approach to YMTD

- **essentials, thermal ground state:**
  coarse-graining over nonpropagating
  (anti-)calorons of winding number unity, effective action
- **adjoint Higgs mechanism:**
  massive vector modes and kinematic constraints (1),
  coupling, deconf.-preconf. phase boundary, (anti-)caloron action,
- **radiative corrections:**
  kinematic constraints (2), polarisation tensor of massless mode,
  longitudinal and transverse thermal dispersion,
  „photon-photon“ scattering
- **SU(2) postulate for photon propagation:**
  Yang-Mills scale or critical temperature
  (radio-frequency CMB observations)
- **CMB large-angle anomalies (WMAP, Planck):**
  possible explanation via SU(2) dispersion,
  onset of dynamical breaking of statistical isotropy at redshift unity,
  SU(2) vector modes and cosmic neutrinos

motivation

- Andrei Linde (1980):
  „Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability)

  - no „convergence“ of series since kinetic and interaction energies comparable in this sector

- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

  - nonperturbative, lattice β function

nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst et Hofmann (2004), Hofmann (2005-2007), Giacosa et Hofmann (2006), Schwarz et al. (2007), Ludescher et Hofmann (2008), Falquez et al. (2010- 2011), Hofmann (2012)]

thermal ground state at high temperature:

- **Euclidean action:**

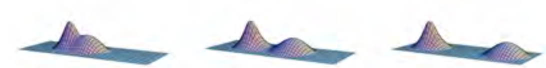
$$\boxed{S = \frac{\mathrm{tr}}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}\,,} \qquad (\,\beta \equiv 1/T\,)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ [Schafer et Shuryak (1996)]

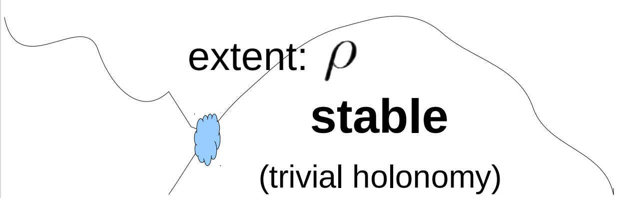
- **(anti)selfdual gauge fields:** $\boxed{F_{\mu\nu}[A] = \pm\tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.}$

field configs. stabilized by winding: $S_3 \to SU(2) = S_3$

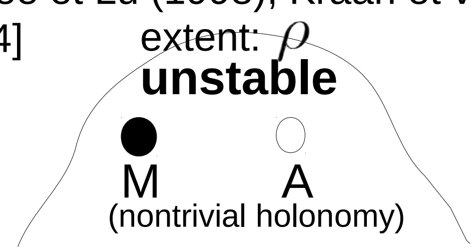
- **in particular:** (anti)calorons of winding number unity



[Harrington et Shepard (1977)



[Nahm (1981-84), Lee et Lu (1998), Kraan et v. Baal (1998), Diakonov et al. 2004]



spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ

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$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0}) \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} F_{\mu\nu}(\tau,\vec{x}) \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of **family of phases**, where

$$\left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right]$$ and

$$\left\{(\tau,\vec{x}),(\tau,\vec{0})\right\} \equiv \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\}^\dagger$$

- magnetic-magnetic correlations contribute only

- uniquely determined, annihilating operator: $$D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$$

-$\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration, later!

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field

- no explicit β dependence in ϕ field dynamics (caloron action!)

- absorb β dependence of operator D into potential V

(BPS and EL yield: $\frac{\partial V(|\phi|^2)}{\partial|\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2} \Rightarrow$

$$\boxed{V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}}$$

(Yang-Mills scale)

and

$$\boxed{|\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}}$$

- BPS equation:

$$\partial_\tau \phi = \pm 2i\,\Lambda^3\, t_3\, \phi^{-1} = \pm i\, V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity for V !

effective action (deconfining phase)

- $$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\left(\frac{1}{2}G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

  ((i) perturbative renormalizability
  (ii) ϕ 's inertness – no higher dim. operators to mediate
  4-momentum transfer between ϕ and a_μ
  (iii) gauge invariance)

- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

$$a_\mu^{\rm gs} = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}\,t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

interacting small-holonomy (anti)calorons (collapsing monopole-antimonopole pairs)

(vanishing entropy density!)

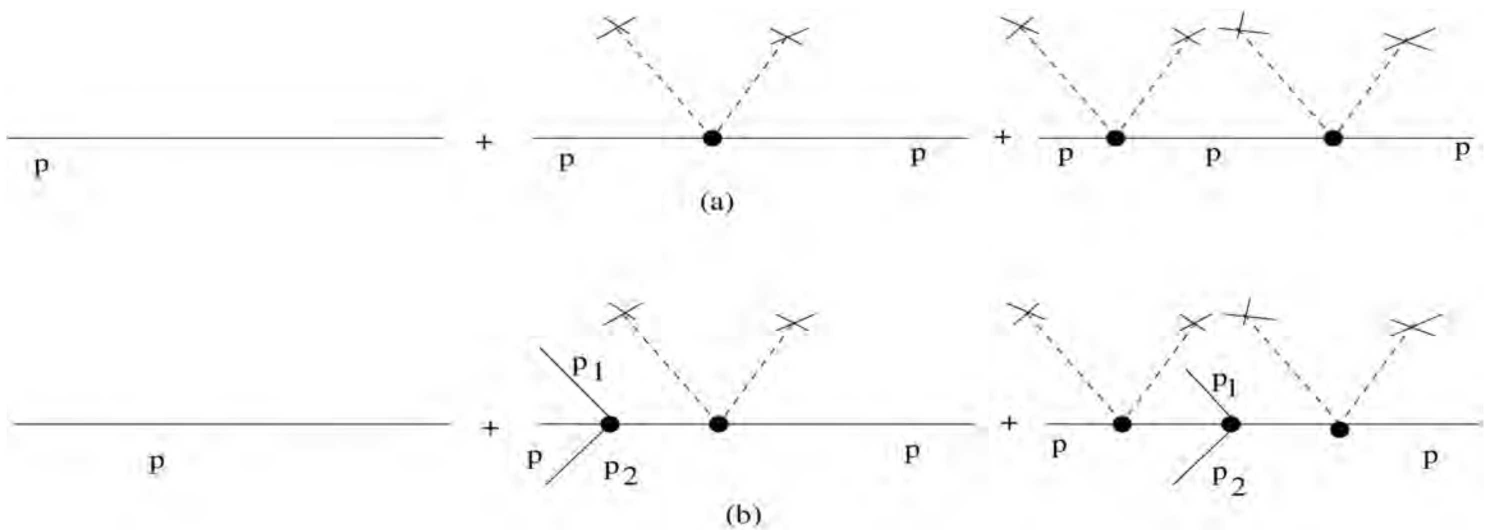
adjoint Higgs (deconfining phase)

(SU(2) → U(1))

- from effective action:

$$\boxed{m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a]} \xrightarrow{\text{unitary gauge}} \boxed{m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,, \;\; m_3 = 0}$$

- no momentum transfer to ϕ , **but this infinitely often** (Dyson series for mass generation):



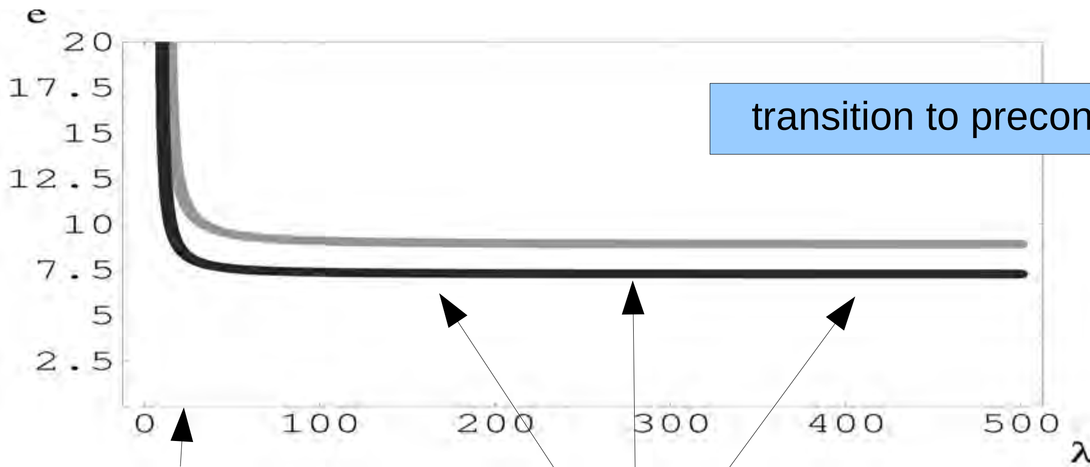
- no off-shell propagation of massive modes (otherwise: momentum transfer to ϕ !)

effective gauge coupling

- evolution of effective gauge coupling:

e

thermodynamical consistency



transition to preconfining phase

[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8\pi}$$

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

- restore $\hbar$

$$e = \frac{\sqrt{8\pi}}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[Brodsky et al. (2011); Kaviani et Hofmann 2012, Hofmann (2012,2013)]

electric-magnetically dual interpretation:

- if SU(2) something to do with photons (later!) then **electric-magnetically dual** interpretation required: in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\boxed{\alpha = \frac{Q^2}{4\pi\hbar}\,,}$$

for α to be unitless:

$$\boxed{Q \propto \frac{1}{e}\,.}$$

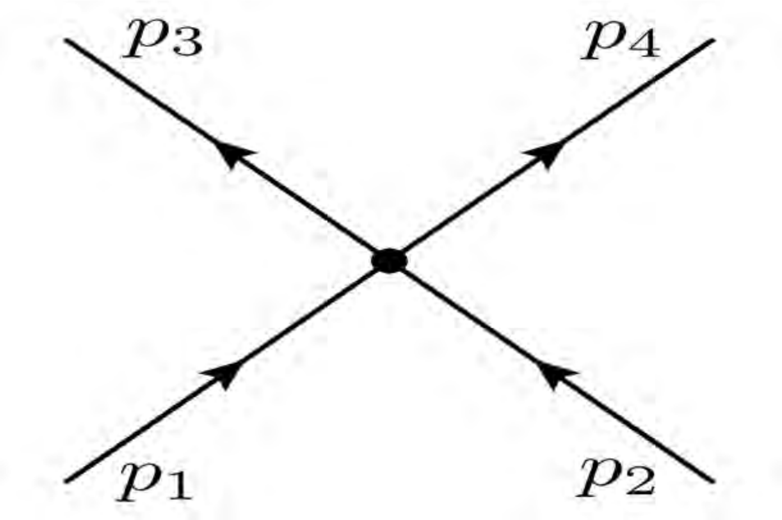
But: magnetic coupling in SU(2)

$$\boxed{g = \frac{4\pi}{e}\,.}$$

$\Rightarrow$ SU(2) is to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

radiative corrections (deconfining phase)

- constrained momentum transfer in effective 4-vertex (unitary-Coulomb gauge):



s-channel: $\boxed{|(p_1+p_2)^2| \le |\phi|^2}$

t-channel: $\boxed{|(p_1-p_3)^2| \le |\phi|^2}$

u-channel: $\boxed{|(p_1-p_4)^2| \le |\phi|^2}$

- coherent average over all three channels $\longrightarrow$ thermodynamical quanties: 2-loop/1-loop ($<10^{-3}$), 3-loop/1-loop ($<10^{-7}$),

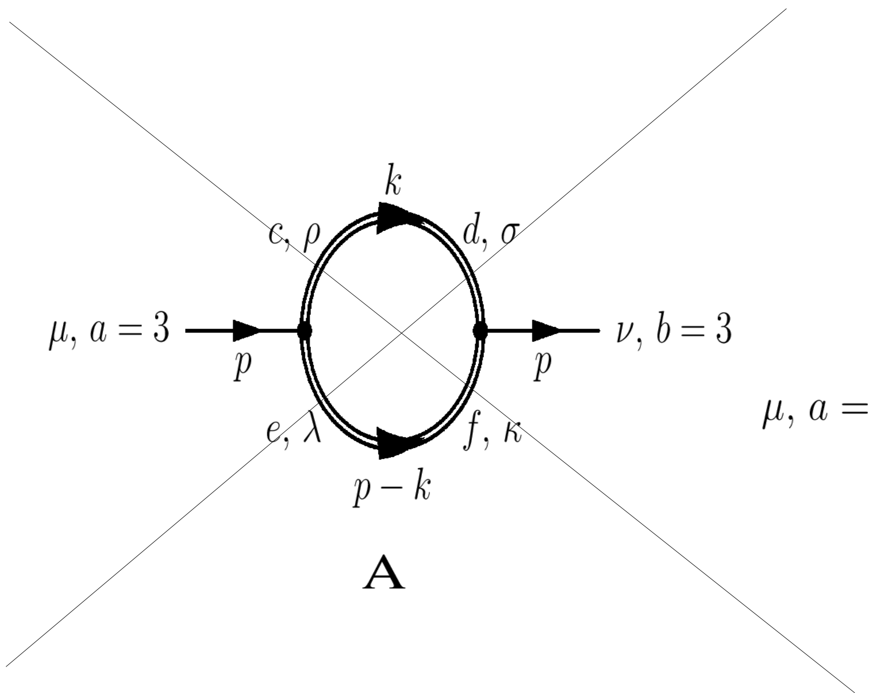
conjecture:

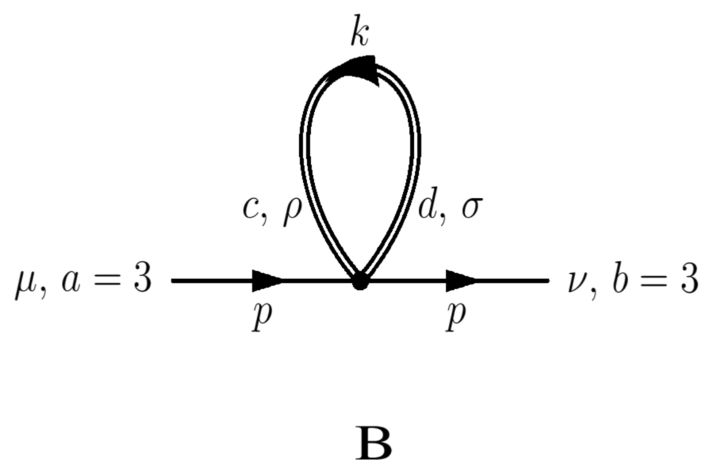
loop expansion into 1-PI diagrams probably terminates at finite order

[Hofmann 2006]

radiative corrections (deconfining phase)

- polarisation tensor of massless mode (Coulomb gauge):





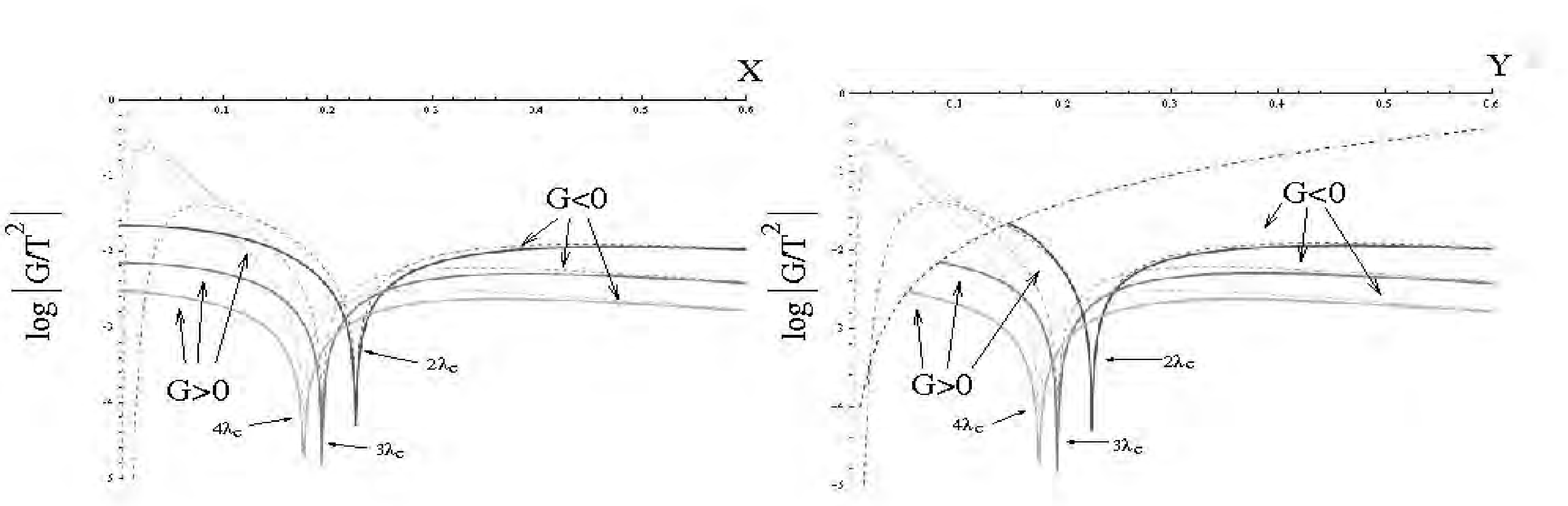
(excluded by kinematic constraints:
on-shellness of vector mode,
restricted off-shellness of massless mode)

screening functions G, F
as solutions of respective
gap equations

radiative corrections (deconfining phase)

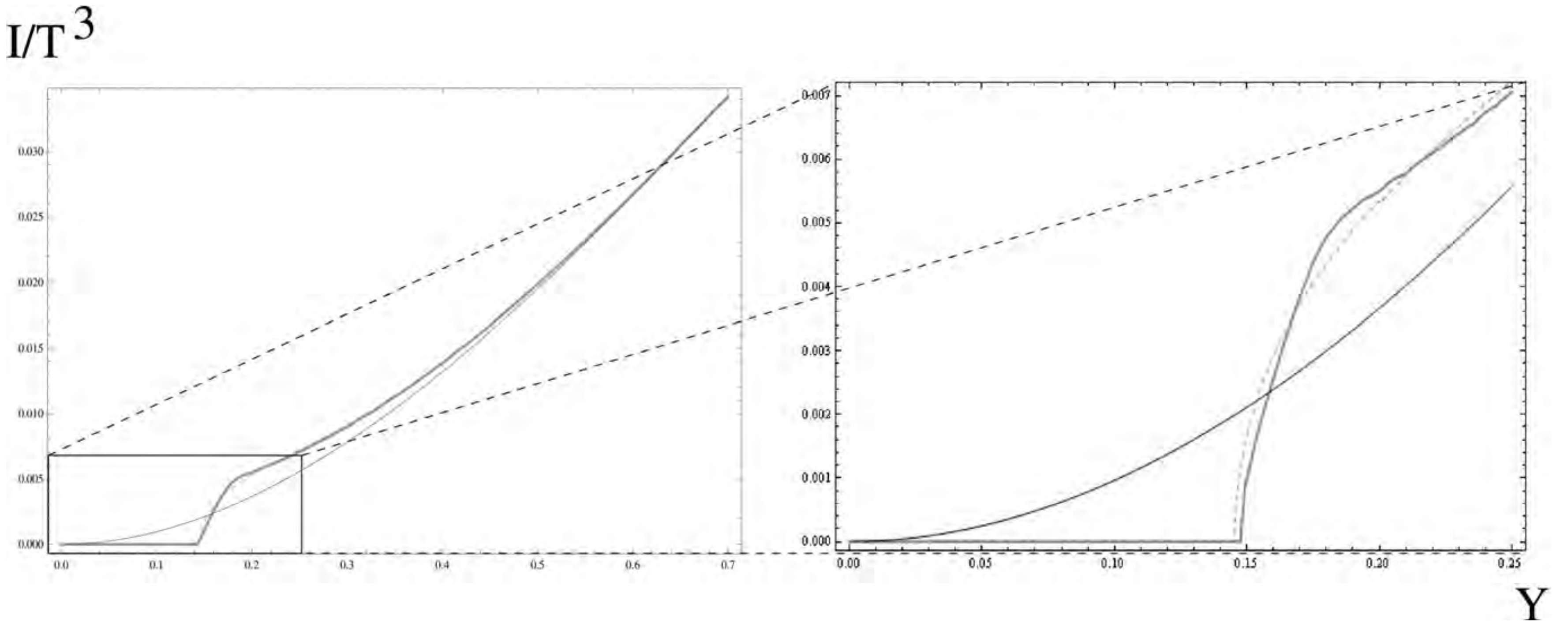
- transverse photons, screening function G :

[Schwarz et al. (2007), Ludescher et Hofmann (2008)]



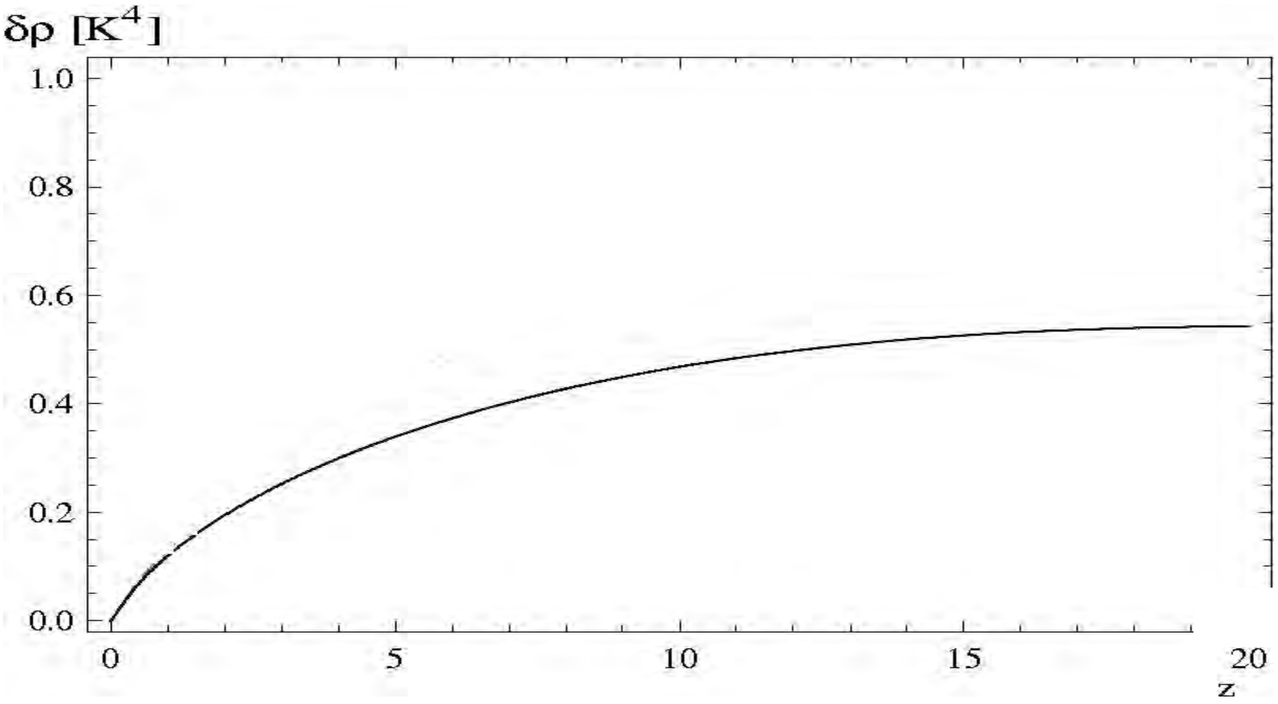
radiative corrections (deconfining phase)

- spectral distribution of energy density, massless mode – transverse propagation at $T = 2T_0$



radiative corrections (deconfining phase)

- difference between energy density of SU(2) and U(1), massless mode – transverse propagation

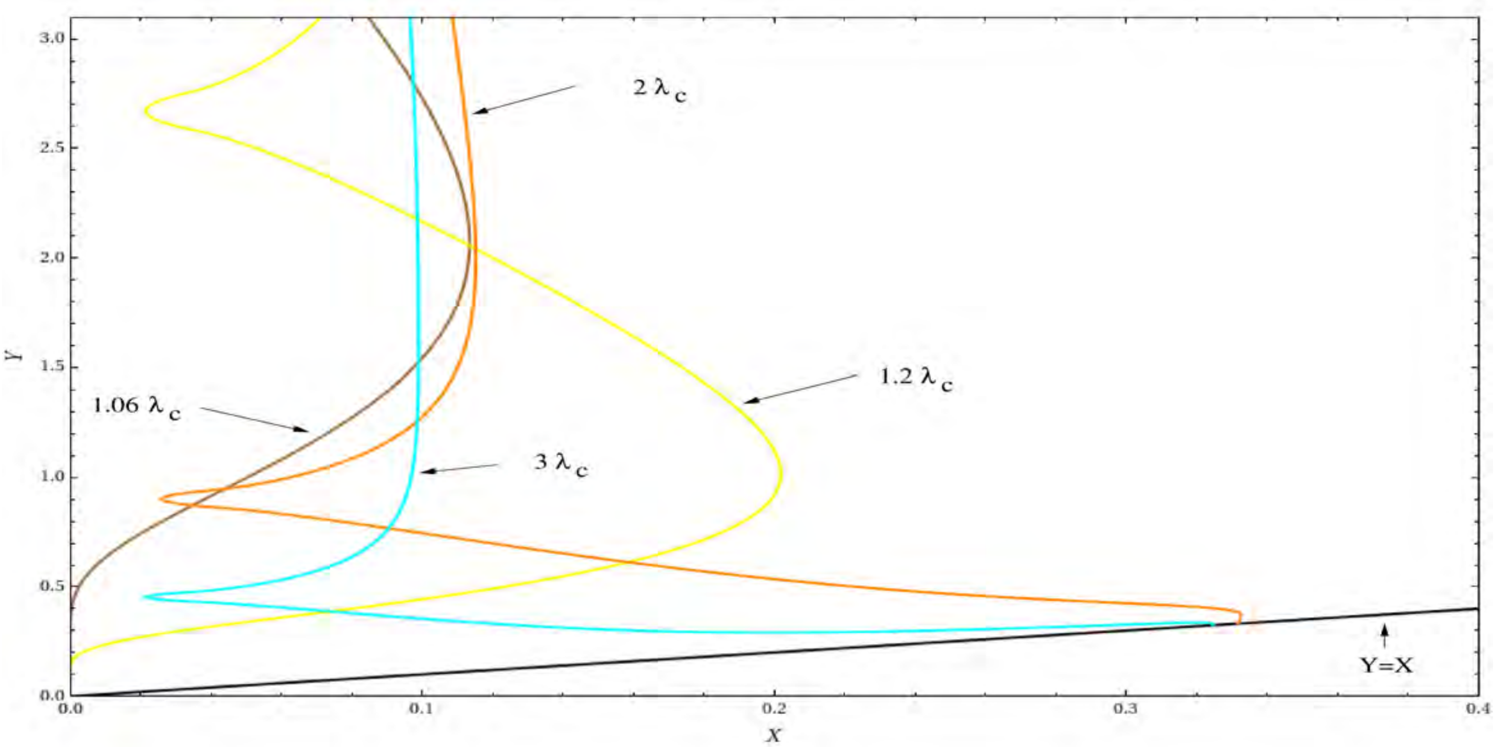


$T/T_c - 1$

(**positive** slope $\longleftrightarrow$ bias for **negative** temperature fluctuations, later!)

radiative corrections (deconfining phase)

- low-momentum-support dispersion law, massless mode - longitudinal propagation

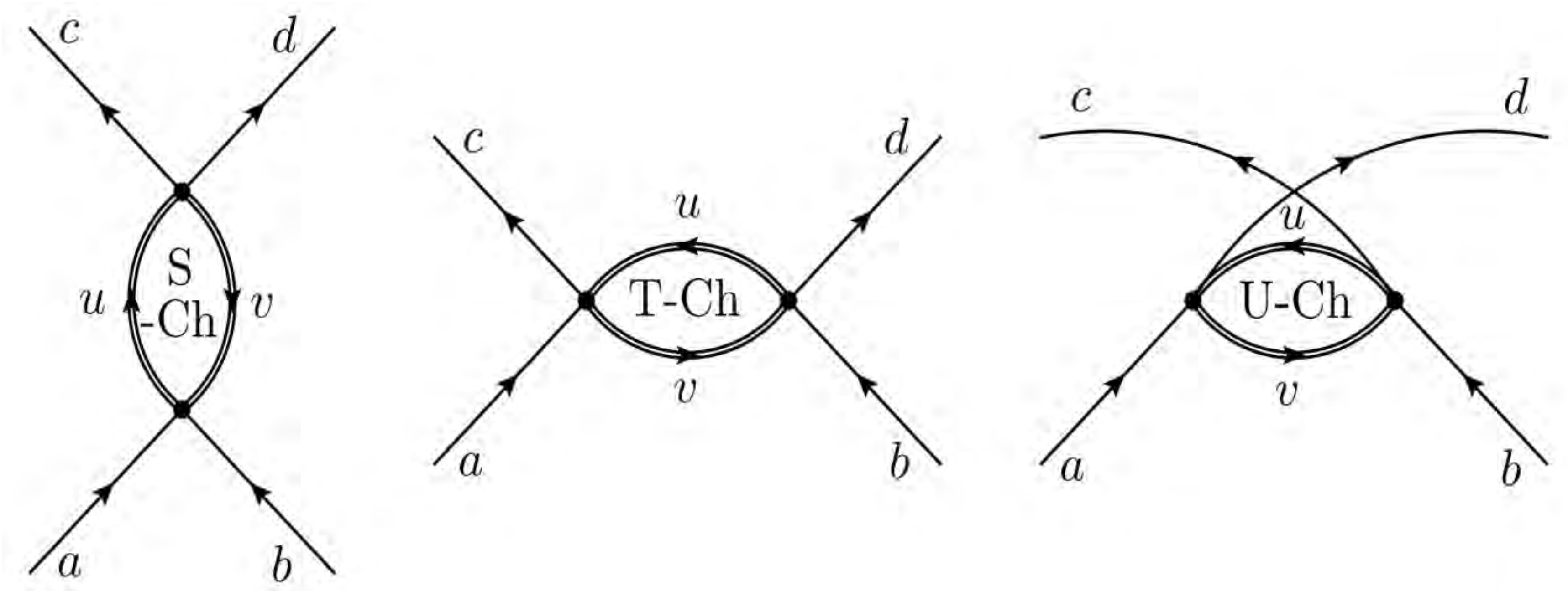


(charge-density waves: real-world magnetic modes, intergalactic magnetic fields [Falquez et al (2011)])

radiative corrections (deconfining phase)

- „photon-photon“ scattering [Krasowski et Hofmann (2013)]

  due to kinematic constraints only topology
  with two 4-vertices contributes



radiative corrections (deconfining phase)

- analysis of forbidden sign-combinations of u_0, v_0 leads to exclusion tables for each of overall S, T, or U channels

  for example:

Table 1
Forbidden combinations of energy flow (marked with a X) in all scattering channel combinations of vertex 1 and vertex 2 in the overall S-channel.

Vertex 1 / Vertex 2	s-ch.		t-ch.		u-ch.	
s- ch.		X	X	X	X	X
	X		X	X	X	X
t-ch.	X	X	X		X	
	X	X	X	X	X	X
u-ch.	X	X	X		X	
	X	X	X	X	X	X

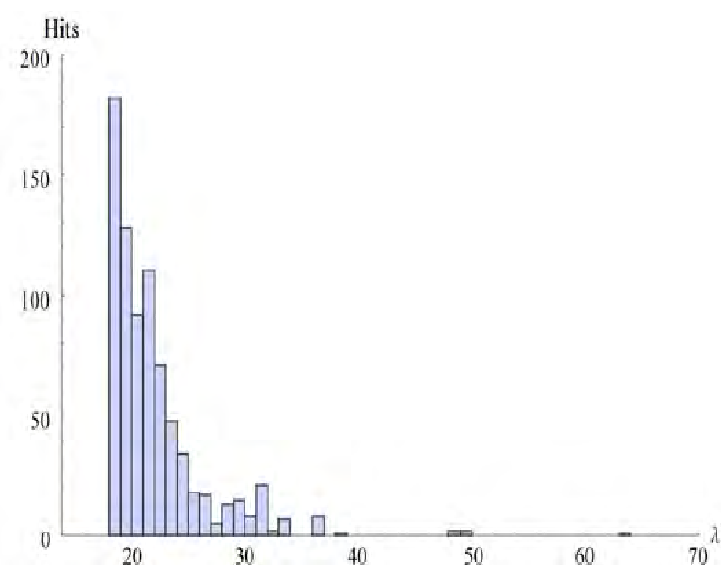
Tool for eventual proof of termination of loop expansion at finite irreducible loop order.

radiative corrections (deconfining phase)

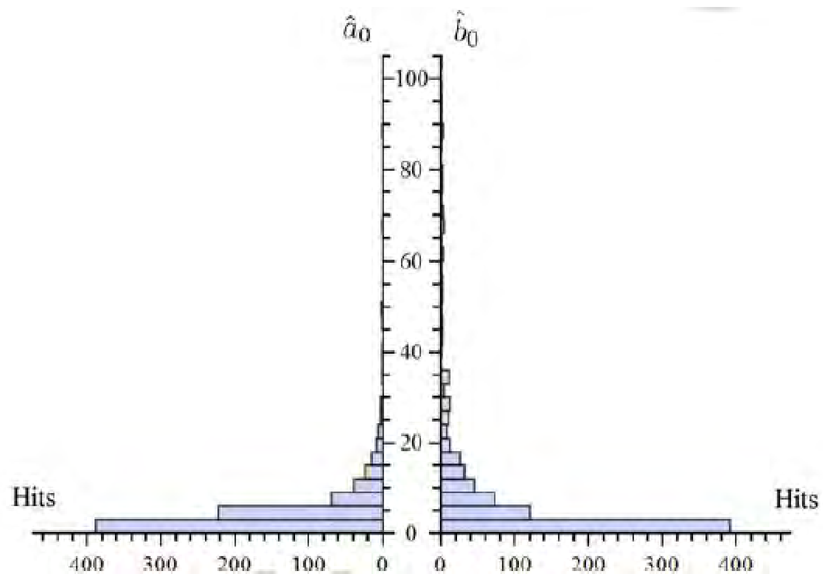
conclusions:

- practically no S-channel scattering
  (no pair creation or annihilation of massive modes out of / into massless ones)
- feeble contribution of Tor U channels
  (fraction 10^{-7} of unconstrained phase space)
  at low temperatures and low energies of massless modes,

Monte Carlo frequency distribution of:



temperature



energy

SU(2) postulate for photon propagation

- What is T_c ?

- strong increase of CMB line temperture below $\nu = 3$ GHz

$$T(\nu) = T_0 + T_R \left(\frac{\nu}{\nu_0}\right)^{\beta}$$

[Fixsen et al. (2009), Haslam et al. (1981), Reich et Reich (1986), Roger et al. (1999), Maeda et al. (1999)]

where: $T_0 = 2.725\,\mathrm{K}\,;\ \nu_0 = 1\,\mathrm{GHz}\,;$
$\beta = -2.62 \pm 0.04\,; T_R = (1.19 \pm 0.14)\,\mathrm{K}\,.$

(radio-frequency surveys of CMB yield line temperatures as:

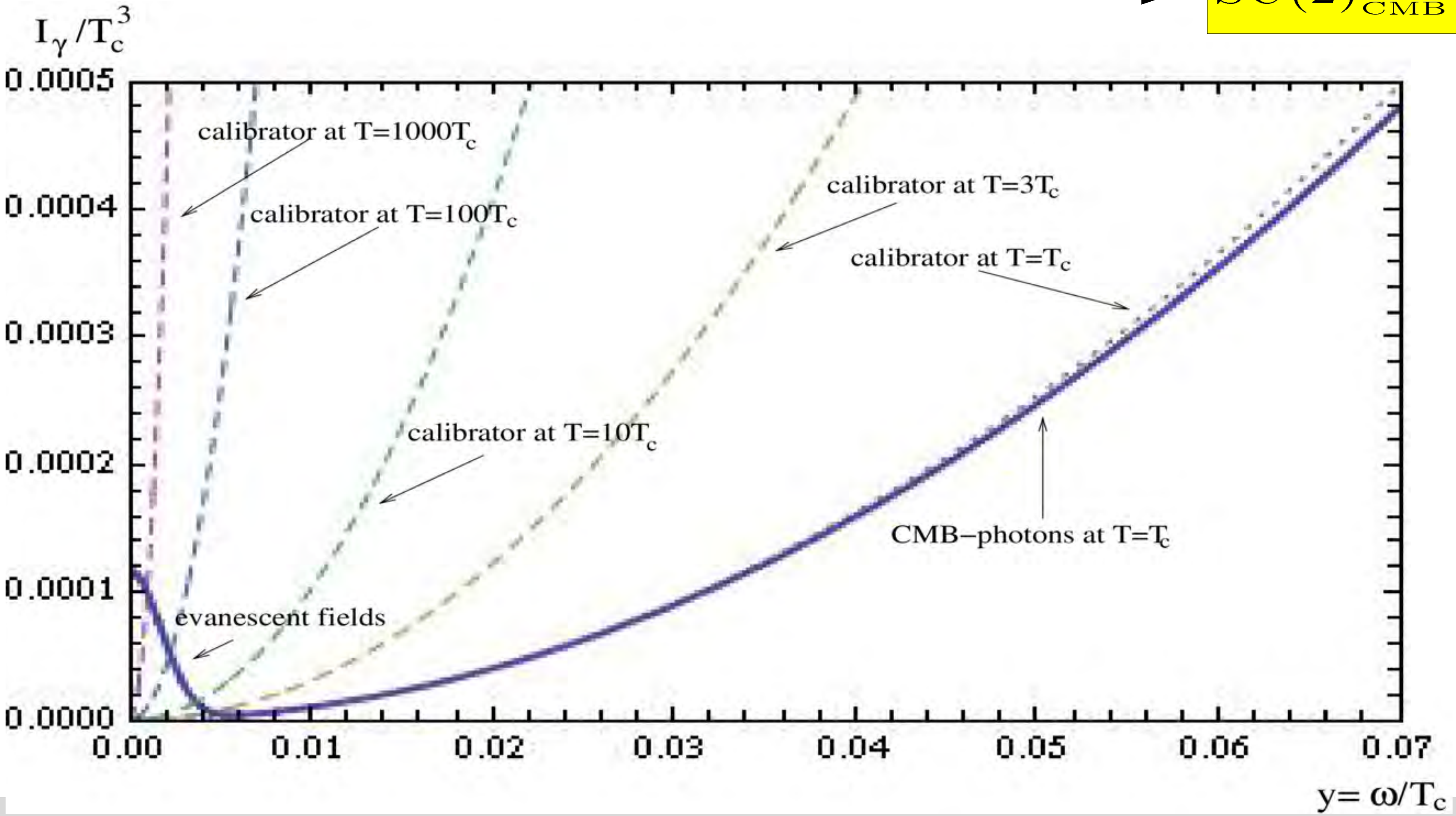
source	ν[GHz]	T[K]
Roger	0.022	21200 ± 5125
Maeda	0.045	4355 ± 520
Haslam	0.408	16.24 ± 3.4
Reich	1.42	3.213 ± 0.53
Arcade2	3.20	2.792 ± 0.010
Arcade2	3.41	2.771 ± 0.009 .

)

evanescent low-frequency modes

- bump from evanescent modes ($\omega < m_\gamma$),
  m_γ photon Meissner mass (condensation of electric monopoles)
- T_c very close to present CMB temperature T_0 (onset of dec.-prec. PT)
  [Hofmann (2009)]

$\longrightarrow$ $\mathrm{SU(2)}_{\mathrm{CMB}}$



Yang-Mills scale of $SU(2)_{CMB}$:

$$T_c = \frac{13.87}{2\pi}\Lambda_{\mathrm{CMB}} = 2.725\,\mathrm{Kelvin} \sim 2 \times 10^{-4}\,\mathrm{eV}$$

Dynamical breaking of statistical isotropy:
Temperature fluctuations in Cosmic Microwave Background

- CMB temperature fluctuations expanded into spherical harmonics

$$\delta T(\phi,\theta) = \sum_{l,m} a_{lm} Y_l^m(\phi,\theta)$$

- a_{lm} assumed to be independent Gaussian random variables

Is this really so for all l ?

some CMB large-angle anomalies: WMAP and Planck

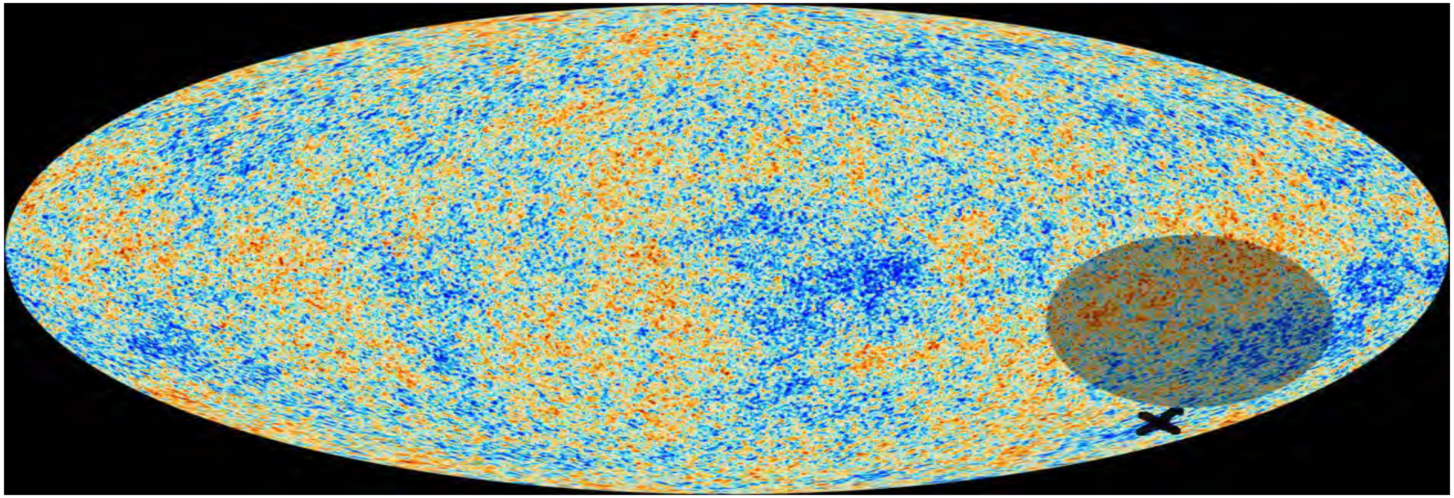
- dipolar power asymmetry (extends from $l = 2, \cdots, 600$ in blocks of $\Delta l = 100$)
  [Hansen et al. (2009), Ade et al. (2013), etc.]

- low variance on ecliptic North, associated with l=2,3
  [Monteserin et al. (2008), Cruz et al., (2011), Ade et al. (2013), etc.]

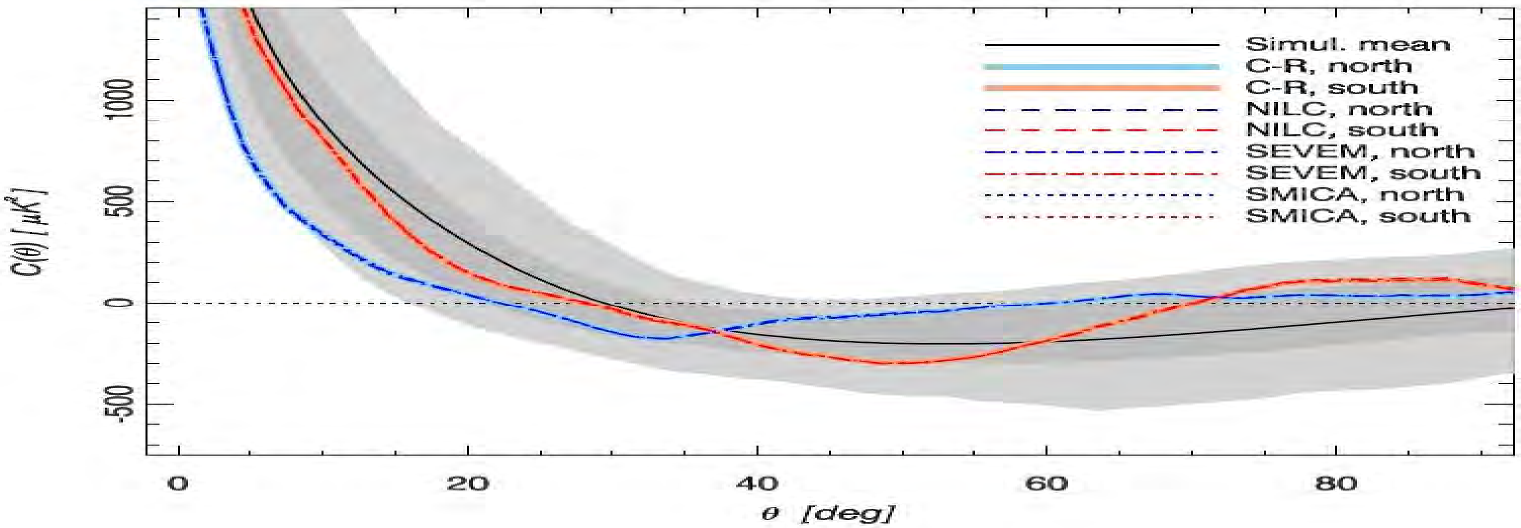
- anomalous alignment of l=2,3 (3°-9°)
  [Tegmark et al. (2003), de Oliveira-Costa et al. (2004), Ade et al. (2013), etc.
  (estimator of axis: maximum of angular momentum dispersion),
  Copi et al. (2004), Schwarz et al. (2004), Bielewicz et al. (2005,2009), Copi et al. (2010), etc.
  (multipole vector decomposition)]

- cold spot (-73μK@4°; -20μK@10°; l,b=207.8°,-56.3°)
  [Viela et al. (2004), Cruz et al. 2005, Rudnick et al. (2007), Ade et al. (2013), etc.]
- hemispherical asymmetry
  (for l=2-40 max. larger power on hemisphere l,b=237°,-20°)
  [Eriksen et al. (2004), Hansen et al. (2004), Park (2004), Ade et al. (2013), etc.]
- mirror parity violation (plane of max. antisymmetry: l,b=262°,-14°)
  [Finelli et al.(2012); Ben-David et al. (2012), etc.]
- suppression of $\langle TT\rangle(\theta) \equiv C(\theta)$ for $\theta \geq 60^\circ$ on ecliptic North
  [Spergel et al. (2003), Copi et al. (2004,2007,2009,2010), Ade et al. (2013), etc.]

cold spot



TT suppression on ecliptic North



successful phenomenological attempt at explanation: multiplicative, dipolar modulation model

[Gordon et al. (2005), Eriksen et al. (2007), Hoftuft et al. (2009), Ade et al. (2013)]

$$\vec{d}(\vec{n}) = (1 + A\vec{p} \cdot \vec{n})\vec{s}_{\mathrm{iso}} + \vec{n}$$

dipole amplitude   dipole direction   instrumental noise   isotropic CMB sky

maximum likelihood at: $A \sim 0.07; \;\; l_p \sim 220^\circ; b_p \sim -21^\circ$

- robust against change of foreground treatment and experiment (WMAP,Planck)

- comparison with CMB cold spot: $l_{cs} \sim 207.8^\circ; b_{cs} \sim -56.3^\circ$

$$\Rightarrow \angle \vec{p}, \vec{e}_{cs} \sim 36^\circ$$

dynamical breaking of statistical isotropy:

- integrated blackbody anomaly due to SU(2)$_{\mathrm{CMB}}$:

- $$\boxed{\delta\rho(T) \equiv \rho_{\mathrm{SU(2)_{CMB}}} - \rho_{\mathrm{U(1)}}}$$
- $$T = \bar{T}(t) + \delta T(t,\vec{x})$$
- SU(2)$_{\mathrm{CMB}}$ bias factor $F(\bar{T},\delta T)$ for δT in phys. voxel volume $\Delta V \sim \frac{(2\pi a)^3}{k_s^3}$. (Silk cutoff)

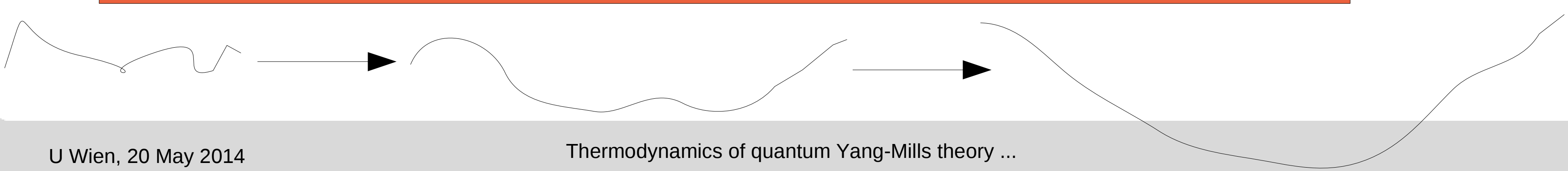
$$\boxed{F(\bar{T},\delta T) = \frac{P_{\mathrm{SU(2)}}}{P_{\mathrm{U(1)}}}}$$

where

$$\boxed{P = \frac{\exp(-\rho\Delta V/\bar{T})}{\int_{T_0}^{\infty} dT\,\exp(-\rho\Delta V/\bar{T})}}$$

(in comoving Fourier-space simulation:
use convolution $\boxed{\tilde{F} * \tilde{\delta T}}$ for conventionally evolved $\tilde{\delta T}$ at $\{z_n\}$,
then projection)

Since slope of $\delta\rho$ positive $\Longrightarrow$ negative δT favoured!



dynamical breaking of statistical isotropy:

- semiquantitative model: effective SU(2)$_{\mathrm{CMB}}$ evolution

$$\boxed{\sqrt{-g}\,\mathcal{L}_{\mathrm{CMB}} = \left(\frac{\bar{T}_0}{\bar{T}}\right)^3 \left(k\,\partial_\mu \delta T \partial^\mu \delta T - \delta\rho(T)\right)}$$

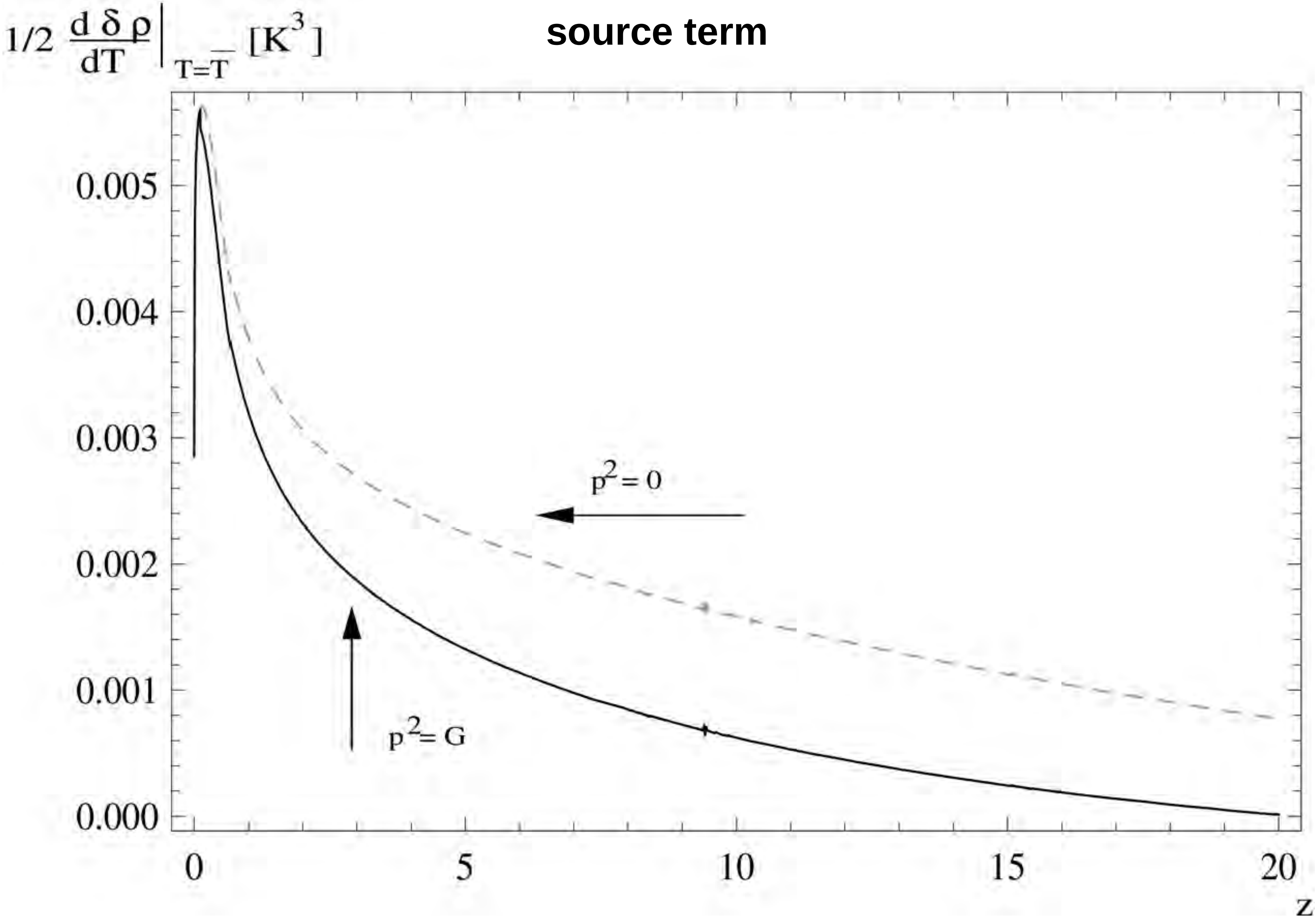
- assuming 3D spherical symmetry, causal boundary conditions

$$\boxed{0 = \partial_\tau\partial_\tau\delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^2 \left[\partial_\sigma\partial_\sigma\delta T + \frac{2}{\sigma}\partial_\sigma\delta T\right] - \frac{3}{\bar{T}}\partial_\tau\bar{T}\,\partial_\tau\delta T + \frac{T_0^2}{kH_0^2}\left[\frac{1}{2}\frac{\mathrm{d}^2\hat{\rho}}{\mathrm{d}T^2}\Big|_{T=\bar{T}}\delta T + \frac{1}{2}\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\Big|_{T=\bar{T}}\right]}$$

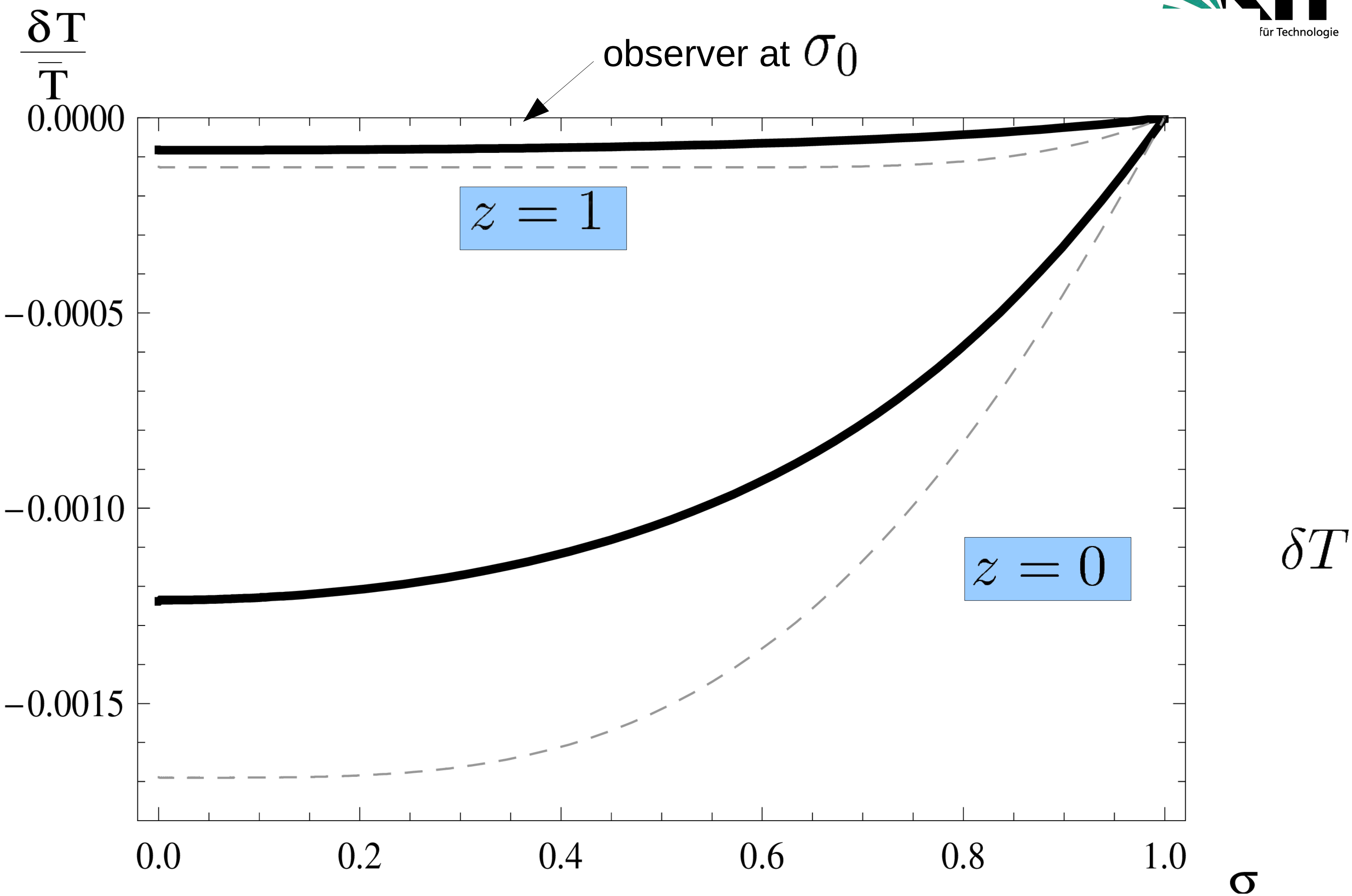
source term

dynamical breaking of statistical isotropy:

source term



dynamical breaking of statistical isotropy:



dynamical breaking of statistical isotropy:

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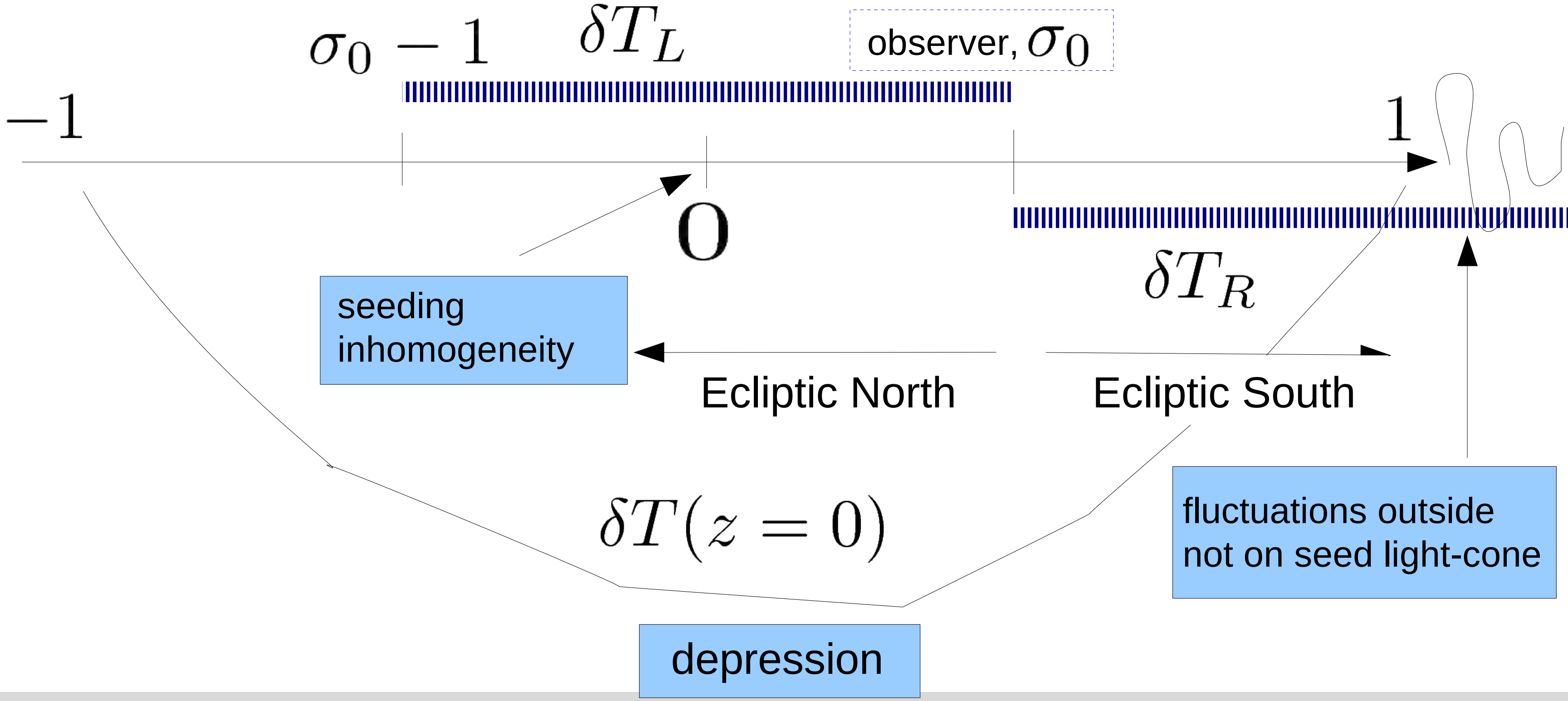
- **low variance, power asymmetry:**
  (simplified, instantaneous light propagation for projection)

$$\delta T_L \equiv \int_{\sigma_0}^{1} d\xi\, \delta T(z=0,\xi)\,, \quad \delta T_R \equiv \int_{\sigma_0-1}^{\sigma_0} d\xi\, \delta T(z=0,\xi)$$



dynamical breaking of statistical isotropy:

- **suppression of TT for** $\theta \geq 60^\circ$ **:**

  rapid build-up of profile for $z \leq 1$

- **dynamical contribution in measured (kinematically dominated) CMB dipole** →

$$\boxed{|\vec{D}_{dyn}| = \frac{1}{2}(\delta T_L - \delta T_R)}$$

- $\boxed{\text{offset} = \frac{1}{2}(\delta T_L + \delta T_R)}$ → **cold spot**

→ $\boxed{\vec{d}_{cs} || \vec{e}_{\text{mirror antisymm}}}$ $\boxed{\vec{d}_{cs} || \vec{e}_{\text{hemisph asymmetry}}}$

Planck results**:**

$$\angle \vec{e}_{\text{mirror antisym}}, \vec{e}_{cs} \sim 42^\circ - 56^\circ \,;$$

$$\angle \vec{e}_{\text{hemisph asym}}, \vec{e}_{cs} \sim 42^\circ \,.$$

SU(2) vector modes and cosmic neutrinos:

from Planck: $$\boxed{N_{\mathrm{eff}} = 3.30 \pm 0.27}$$

But have 2 x 3 ~ N_{eff} x 2 rel. d.o.f. from $SU(2)_{CMB}$ vector modes.

Too many rel. d.o.f ?
Do vector modes play role of cosmological neutrinos?
Neutrinos (luke-)warm dark matter?

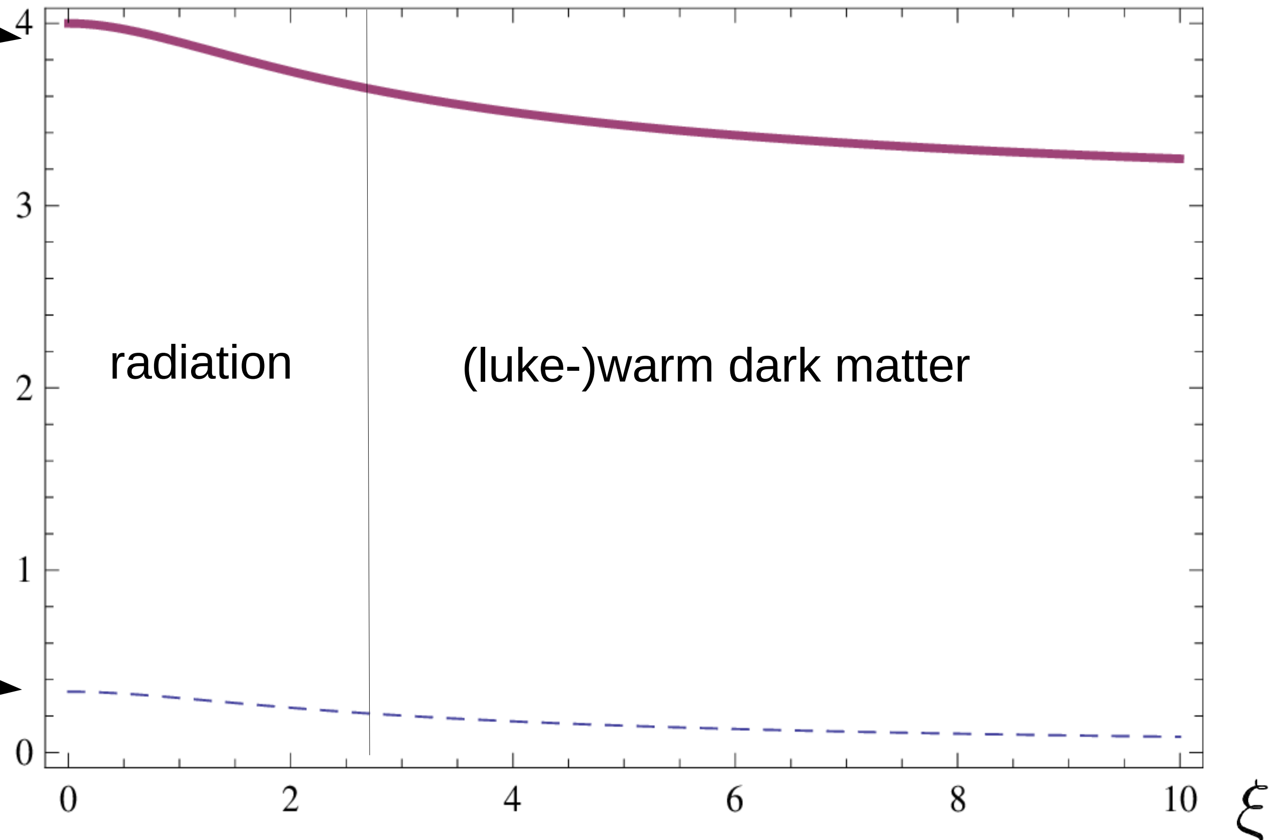
massive cosmic neutrino equation of state:

assume: $m_\nu = \xi T$

(neutrino single center-vortex loop of yet another confining-phase SU(2), neutrino mass induced by environment)

[Moosmann,Hofmann 2008]

$3(1+w(\xi))$



$w(\xi) = \dfrac{P(\xi)}{\rho(\xi)}$

(solar neutrino scale: $\sim 8 \times 10^{-3}\,\mathrm{eV}$, present CMB temperature: $\sim 2 \times 10^{-4}\,\mathrm{eV}$)

Summary

- SU(2) thermodynamics nonperturbatively:
  caloron, thermal ground state, adjoint Higgs mechanism, caloron action

- blackbody anomaly:
  thermal photon dispersion, critical temperature for dec.-prec. PT from low-frequency spectral anomaly (Arcade2, terrestial radio-frequency CMB observations)

- CMB large-angle anomalies (WMAP, Planck):
  Yang-Mills favours **negative temperature fluctuations**, semiquantitative model, cosmic neutrinos and relativistic vector modes

Thank you.