Description of Exclusive Hadronic τ Decays

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Single-meson modes

Single meson modes have been calculated long ago:

(Marciano, Sirlin 1988)

Decay $\tau^- \to \pi^- \nu_\tau$:

$$B_{\tau\to\pi} = 12\pi^2 |V_{ud}|^2 S_{\rm EW} \frac{f_\pi^2}{M_\tau^2} \left(1 - \frac{M_\pi^2}{M_\tau^2}\right)^2 \cdot B_{\tau\to e} \approx 0.61 \cdot B_{\tau\to e} = 10.87\,\%$$

Decay $\tau^- \to K^- \nu_\tau$:

$$B_{\tau\to K} = 12\pi^2 |V_{us}|^2 S_{\rm EW} \frac{f_K^2}{M_\tau^2} \left(1 - \frac{M_K^2}{M_\tau^2}\right)^2 \cdot B_{\tau\to e} \approx 0.04 \cdot B_{\tau\to e} = 0.72\,\%$$

Employing $\pi^- \to \mu^- \nu_\mu$ and $K^- \to \mu^- \nu_\mu$, precise predictions can be made for the branching fractions $B_{\tau\to\pi}$ and $B_{\tau\to K}$.

Two-meson decays

Viable information can be obtained from the decay spectra for exclusive τ-decay channels.

The procedure can be exemplified through the description of the $\tau \to K\pi\nu_\tau$ decay spectrum: (MJ, Pich, Portolés 2006/08) (Boito, Escribano, MJ 2008) (Passemar et al. 2006-11)

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} \left(1 - \frac{s}{M_\tau^2}\right)^2 \times$$

$$\left[\left(1 + 2\,\frac{s}{M_\tau^2}\right) q_{K\pi}^3 \, |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s}\, q_{K\pi} |F_0^{K\pi}(s)|^2 \right].$$

To this end the $K\pi$ vector and scalar form factors $F_+^{K\pi}(s)$ and $F_0^{K\pi}(s)$ are required as an input.

$K\pi$ form factors

The $K\pi$ vector and scalar form factors are defined by:

$$\langle K^+(p')|\bar{u}\gamma_\mu s|\pi^0(p)\rangle \equiv \frac{1}{\sqrt{2}}\Big[(p'_\mu+p_\mu)F_+^{K\pi}(t)+(p'_\mu-p_\mu)F_-^{K\pi}(t)\Big]\,.$$

The scalar form factor results from the S-wave projection:

$$F_0^{K\pi}(t) \equiv F_+^{K\pi}(t)+\frac{t}{(M_K^2-M_\pi^2)}\,F_-^{K\pi}(t)\,.$$

A description of the $K\pi$ vector form factor can be obtained within chiral perturbation theory with resonances (RχPT):

$$F_+^{K\pi}(s) = \frac{m_{K^*}^2}{m_{K^*}^2-s-\kappa\,\widetilde{H}_{K\pi}(s)}\,.$$

The functional form of $F_+^{K\pi}(s)$ resembles a Breit-Wigner shape.

The imaginary part of the denominator is linked to the s-dependent width of the resonance:

$$\kappa \, \mathrm{Im}\widetilde{H}_{K\pi}(s) = m_{K^*} \gamma_{K^*}(s) \,,$$

where

$$\mathrm{Im}\widetilde{H}_{K\pi}(s) = \frac{s}{192\pi} \, \sigma_{K\pi}^3(s) \,,$$

and

$$\sigma_{K\pi}(s) = \frac{1}{s}\sqrt{[s-(M_K+M_\pi)^2][s-(M_K-M_\pi)^2]} \,.$$

Hence, it follows that:

$$\kappa = \frac{192\pi \, \gamma_{K^*}(m_{K^*}^2)}{m_{K^*} \, \sigma_{K\pi}^3(m_{K^*}^2)} \,; \qquad \gamma_{K^*} \equiv \gamma_{K^*}(m_{K^*}^2) \,.$$

The parameters of this model, namely m_{K^*} and γ_{K^*}, can be fitted from experimental data for p-wave $K\pi$ scattering, or from the τ data.

The physical parameters M_{K^*} and Γ_{K^*} can be inferred from the pole of $F_+^{K\pi}(s)$ in the complex s-plane.

Also a second resonance contribution can easily be included.

Define the reduced form factor:

$$\widetilde{F}_+^{K\pi}(s) \equiv \frac{F_+^{K\pi}(s)}{F_+^{K\pi}(0)}\,.$$

Then

$$\widetilde{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\widetilde{H}_{K\pi}(0) + \gamma s}{m_{K^*}^2 - s - \kappa_{K^*}\widetilde{H}_{K\pi}(s)} - \frac{\gamma s}{m_{K^{*\prime}}^2 - s - \kappa_{K^{*\prime}}\widetilde{H}_{K\pi}(s)}\,.$$

In the elastic, single-channel case a subtracted dispersive representation is available which is related to the Omnès solution:

$$\widetilde{F}_+^{K\pi}(s) = \exp\left(\frac{\alpha_1 s}{M_\pi^2} + \frac{\alpha_2 s^2}{2M_\pi^4} + \frac{s^3}{\pi} \int\limits_{s_{K\pi}}^{s_{\rm cut}} \frac{\delta_{K\pi}(s')}{(s')^3(s'-s-i0)}\, ds' \right)$$

where $\delta_{K\pi}(s)$ is the elastic P-wave $K\pi$ phase shift.

$$\tan\delta_{K\pi}(s) = \frac{{\rm Im} F_+^{K\pi}(s)}{{\rm Re} F_+^{K\pi}(s)}\,.$$

The two subtraction constants α_1 and α_2 are related to slope and curvature of the vector form factor:

$$\lambda_+' = \alpha_1\,, \qquad \lambda_+'' = \alpha_2 + \alpha_1^2\,.$$

Scalar form factor

The scalar form factor $F_0^{K\pi}(s)$ can be obtained from a dispersion relation analysis of S-wave $K\pi$ scattering data.

(MJ, Oller, Pich 2000/02)

The scalar form factors are defined by:

$$i\,\langle\Omega|\,\partial^\mu(\bar{s}\gamma_\mu u)\,|\Gamma\rangle = \frac{\Delta_{K\pi}}{\sqrt{2}}\,C_\Gamma\,F_0^\Gamma(s)$$

where the C_Γ are Clebsch-Gordan coefficients and

$$\Delta_{K\pi} = M_K^2 - M_\pi^2\,.$$

The form factors $F_\Gamma \equiv F_0^\Gamma$ are difficult to measured directly. An indirect determination is more appropriate.

From unitarity we have the following relation:

$$\mathrm{Im} F_k(s) = \sum_i \sigma_i(s)\, F_i(s)\, t_0^{ik}(s)^*$$

with $t_0^{ik}(s)$ the S-wave $I{=}1/2$ scattering amplitudes.

The $F_i(s)$ also satisfy dispersion relations.
In the 2-channel case with $F_1 \equiv F_{K\pi}$ and $F_3 \equiv F_{K\eta'}$:

$$F_1(s) = \frac{1}{\pi}\int_{s_1}^{\infty} \frac{\sigma_1 F_1\, t_0^{11}(s')^*}{(s'-s-i0)}\, ds' + \frac{1}{\pi}\int_{s_3}^{\infty} \frac{\sigma_3 F_3\, t_0^{13}(s')^*}{(s'-s-i0)}\, ds'$$

$$F_3(s) = \frac{1}{\pi}\int_{s_1}^{\infty} \frac{\sigma_1 F_1\, t_0^{13}(s')^*}{(s'-s-i0)}\, ds' + \frac{1}{\pi}\int_{s_3}^{\infty} \frac{\sigma_3 F_3\, t_0^{33}(s')^*}{(s'-s-i0)}\, ds'$$

Numerical analysis

Several ingredients are required for solving the set of coupled integral equations:

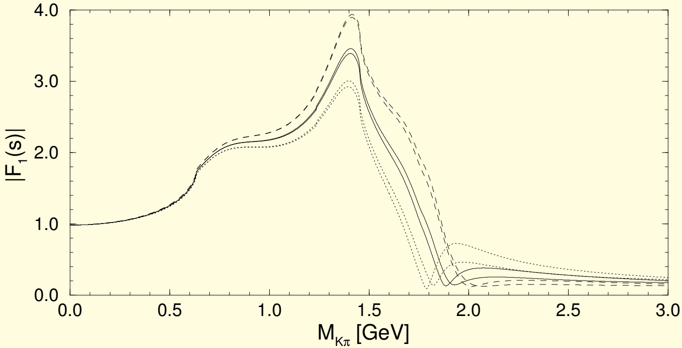
- An input for the scattering amplitudes is obtained by fitting an Ansatz from resonance ChPT to experimental data for S-wave $K\pi$ scattering.

  (MJ, Oller, Pich 2000/02)

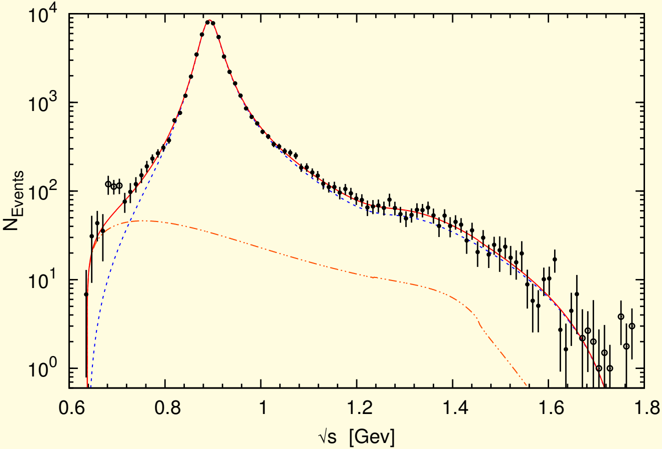
- Two integration constants are also required. These can be chosen to be: $F_{K\pi}(0) = 0.972 \pm 0.012$ and

$$\frac{F_{K\pi}(\Delta_{K\pi})}{F_{K\pi}(0)} = \frac{F_K}{F_\pi F_{K\pi}(0)} + \frac{\Delta_{\rm CT}}{F_{K\pi}(0)} = 1.2346(53)$$

- The last relation follows from the ratio of leptonic K and π decays, as well as $|V_{us}|F_{K\pi}(0)$ from K_{l3} decays.



4.0
3.0
2.0
1.0
0.0
$|F_1(s)|$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$M_{K\pi}$ [GeV]



Decay $\tau^- \to K_S \pi^- \nu_\tau$: (MJ, Pich, Portolés 2006/08)

Belle decay distribution. (Boito, Escribano, MJ 2009/10)

$$M_{K^*} = 892.0 \pm 0.5\,\text{MeV}, \qquad \Gamma_{K^*} = 46.5 \pm 1.1\,\text{MeV}$$

Slope parameters

As a prediction of the model, we also obtain slope and the curvature of the vector form factor $F_+^{K\pi}(s)$:

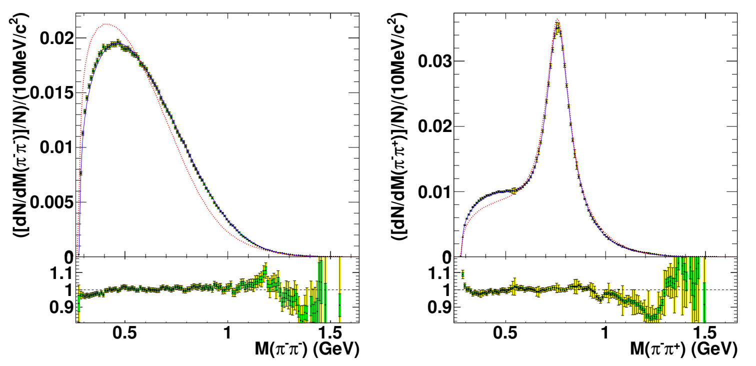
$$\lambda'_+ = (25.49 \pm 0.31) \cdot 10^{-3}, \qquad \lambda''_+ = (12.22 \pm 0.14) \cdot 10^{-4}.$$

Can be compared to earlier experimental determinations:

Collaboration	λ'_+ [10^{-3}]	λ''_+ [10^{-3}]
ISTRA 04	24.9 ± 1.6	0.84 ± 0.41
KTEV 04	20.64 ± 1.75	3.20 ± 0.69
NA48 04	28.0 ± 2.4	0.2 ± 0.5
KLOE 06	25.5 ± 1.8	1.4 ± 0.8

Multi meson modes

Decay $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$:



BaBar decay distribution. (Nugent et al. 2013)

Work in progress for $\tau \to K\pi\pi$ modes.

Conclusions

- Employing a dispersive representation of the $K\pi$ form factors, a satisfactory description of the $\tau \to \nu_\tau K\pi$ decay spectrum can be obtained.
- While a coupled-channel analysis is available for the scalar $K\pi$ form factors, our model for the vector form factors is purely elastic.
- To my mind, fits to experimental data should be done in a two-way approach: on the one hand, experimentalists can try to fit theoretical models provided by theorists.
- On the other hand, it would be very useful if unfolded distributions with correlations would be made available.

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Thank You!