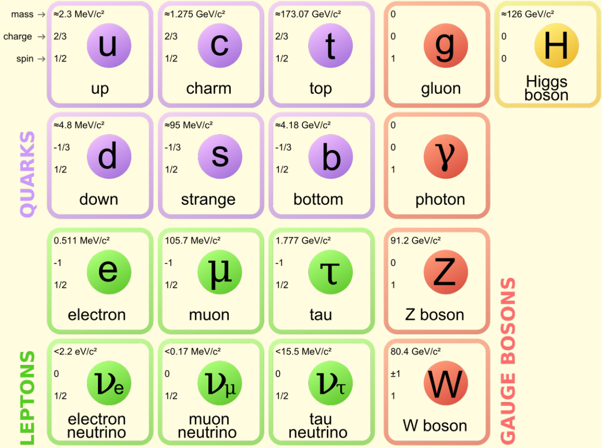
QCD Studies in Tau Decays

Matthias Jamin
ICREA & IFAE
Universitat Autònoma de Barcelona

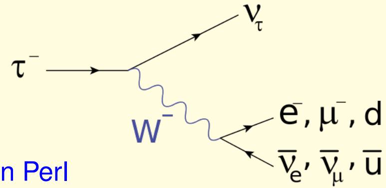
Standard Model Particles



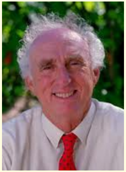
With $M_\tau = 1.777\,$GeV, only τ leptons decay into hadrons.

Tau lifetime: $\tau_\tau = 290.6 \cdot 10^{-15}\,$sec $\Rightarrow \Gamma_\tau = \hbar/\tau_\tau = 2.27 \cdot 10^{-3}\,$eV.

Tau Lepton



Discovered 1975 by Martin Perl
and the SLAC group in e^+e^- collisions.
$\Rightarrow$ Nobel prize in 1995.



Two important experimental observables:

Leptonic branching fractions:

$$B_{\tau\to l} = \mathrm{Br}[\tau^- \to l^- \bar{\nu}_l \nu_\tau] \approx \frac{1}{2+N_C} = \frac{1}{5} = 20\,\%$$

Total hadronic decay rate:

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \mathrm{hadrons}]}{\Gamma[\tau^- \to e^- \bar{\nu}_e \nu_\tau]} = \frac{(1 - B_{\tau\to e} - B_{\tau\to\mu})}{B_{\tau\to e}} \approx N_C = 3$$

Tau Decay Modes

(HFAG 2013)

Leptonic modes:	$\tau^- \to e^- \bar{\nu}_e \nu_\tau$	$(17.82 \pm 0.04)\%$
	$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$	$(17.39 \pm 0.04)\%$
Non-Strange modes:	$\tau^- \to \pi^- \nu_\tau$	$(10.81 \pm 0.05)\%$
	$\tau^- \to \pi^- \pi^0 \nu_\tau$	$(25.50 \pm 0.09)\%$
	$\tau^- \to \pi^- \pi^0 \pi^0 \nu_\tau$	$(9.24 \pm 0.10)\%$
	$\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$	$(9.00 \pm 0.05)\%$
	$\tau^- \to \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$	$(4.62 \pm 0.06)\%$
	$\tau^- \to K^- K^0 \nu_\tau$	$(0.16 \pm 0.02)\%$
Strange modes:	$\tau^- \to K^- \nu_\tau$	$(0.70 \pm 0.01)\%$
	$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$	$(0.82 \pm 0.02)\%$
	$\tau^- \to K^- \pi^0 \nu_\tau$	$(0.43 \pm 0.01)\%$

Covers $>$ 96% of the total τ decay width.

Single meson modes

Single meson modes have been calculated long ago:

(Marciano, Sirlin 1988)

Decay $\tau^- \to \pi^- \nu_\tau$:

$$B_{\tau\to\pi} = 12\pi^2 |V_{ud}|^2 S_{\rm EW} \frac{f_\pi^2}{M_\tau^2}\left(1 - \frac{M_\pi^2}{M_\tau^2}\right)^2 \cdot B_{\tau\to e} \approx 0.61 \cdot B_{\tau\to e} = 10.87\,\%$$

Decay $\tau^- \to K^- \nu_\tau$:

$$B_{\tau\to K} = 12\pi^2 |V_{us}|^2 S_{\rm EW} \frac{f_K^2}{M_\tau^2}\left(1 - \frac{M_K^2}{M_\tau^2}\right)^2 \cdot B_{\tau\to e} \approx 0.04 \cdot B_{\tau\to e} = 0.72\,\%$$

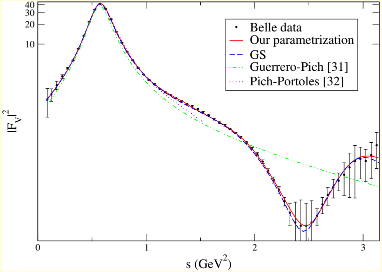
Employing $\pi^- \to \mu^- \nu_\mu$ and $K^- \to \mu^- \nu_\mu$, precise predictions can be made for the branching fractions $B_{\tau\to\pi}$ and $B_{\tau\to K}$.

Two meson modes

Decay $\tau^- \to \pi^-\pi^0\nu_\tau$:

Differential decay distribution

(Gómez Dumm, Roig 2013)

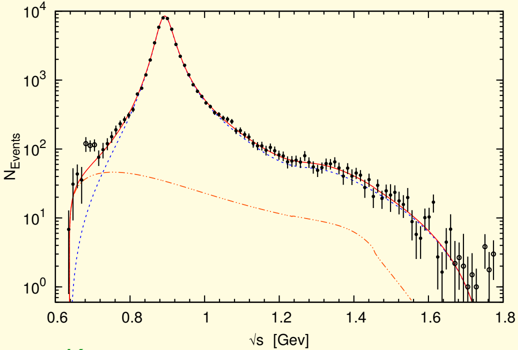


$$\frac{d\Gamma_{\pi\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2 M_\tau^3}{32\pi^3 s}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right) q_\pi^3(s)\,|F_V^\pi(s)|^2$$

Model for vector form factor $F_V^\pi(s)$ required.

Starting point: dispersive representation.

- Several hadronic resonances can be included.
- Low-energy behaviour is implemented to match χPT.
- Constraints from high-energy can be taken into account.
- Main fit parameters: Masses and width of resonances;
  Form factor slopes.

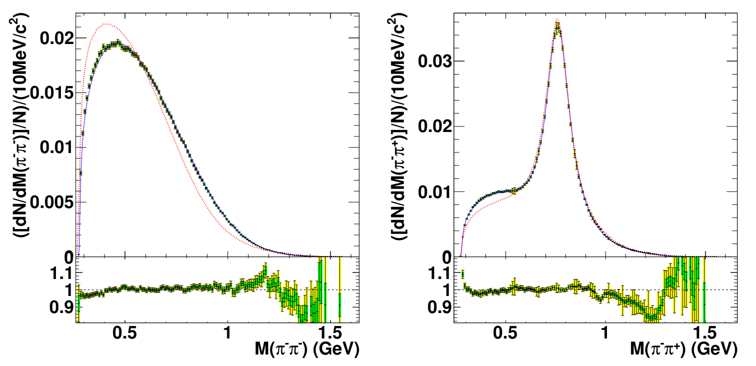


Decay $\tau^- \to K_S \pi^- \nu_\tau$: (MJ, Pich, Portolés 2006/08)

Belle decay distribution. (Boito, Escribano, MJ 2009/10)

Multi meson modes

Decay $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$:



BaBar decay distribution. (Nugent et al. 2013)

Work in progress for $\tau \to K\pi\pi$ modes.

Inclusive τ decays

Total hadronic τ decay rate:

$$R_\tau^{\rm exp} = \frac{\Gamma[\tau^- \to \nu_\tau + {\rm hadrons}]}{\Gamma[\tau^- \to e^- \bar\nu_e \nu_\tau]} = 3.6280(94) \qquad \text{(HFAG 2013)}$$

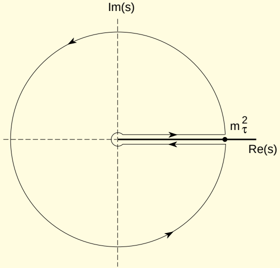
R_τ can be calculated as a spectral integral over basic QCD meson correlation functions:

$$R_\tau = 12\pi \int\limits_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left\{ \left(1 + 2\frac{s}{M_\tau^2}\right) {\rm Im}\,\Pi_\tau^{(T)}(s) + {\rm Im}\,\Pi_\tau^{(L)}(s) \right\}$$

$\Pi_\tau^{(J)}(s)$ corresponds to the combination:

$$\Pi_\tau^{(J)}(s) = |V_{ud}|^2 \left[\Pi_{ud}^{(V,J)}(s) + \Pi_{ud}^{(A,J)}(s)\right] + |V_{us}|^2 \left[\Pi_{us}^{(V,J)}(s) + \Pi_{us}^{(A,J)}(s)\right]$$

In QCD, the integral over the physical, Minkowskian region ($s > 0$) cannot be calculated.



Way out:

(Braaten, Narison, Pich 1992)

$$R_\tau = 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left\{\left(1+2\frac{s}{M_\tau^2}\right)\Pi_\tau^{(T+L)}(s)-2\frac{s}{M_\tau^2}\Pi_\tau^{(L)}(s)\right\}$$

Generally, R_τ then assumes the structure:

$$R_\tau = N_C\, S_{\mathrm{EW}}\left\{(|V_{ud}|^2+|V_{us}|^2)\left[1+\delta^{(0)}\right] + \sum_{D\geq 2}\left[|V_{ud}|^2\delta_{ud}^{(D)}+|V_{us}|^2\delta_{us}^{(D)}\right]\right\}$$

Decay spectra

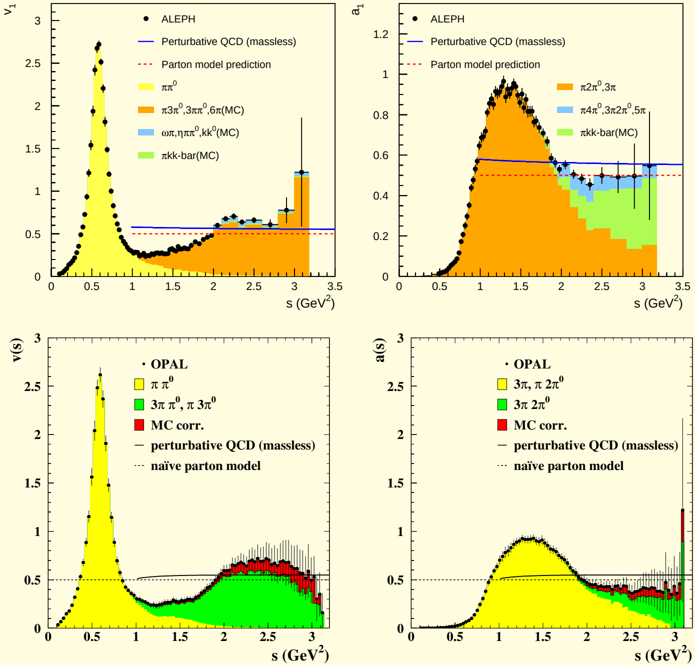
Additional experimental information:

- Inclusive differential decay distributions;
- Separation into Vector, Axialvector and Strange modes.

$$R_\tau = \int_0^{M_\tau^2} ds \, \frac{dR_\tau}{ds} = R_\tau^V + R_\tau^A + R_\tau^S$$

Dominant non-perturbative OPE corrections arise in the strange channel proportional to m_s^2 and $m_s\langle\bar{q}q\rangle$.

For α_s analysis only consider R_τ^V and R_τ^A.


ALEPH
Perturbative QCD (massless)
Parton model prediction
$\pi\pi^0$
$\pi 3\pi^0, 3\pi\pi^0, 6\pi$(MC)
$\omega\pi, \eta\pi\pi^0, kk^0$(MC)
πkk-bar(MC)
v_1
s (GeV^2)
ALEPH
Perturbative QCD (massless)
Parton model prediction
$\pi 2\pi^0, 3\pi$
$\pi 4\pi^0, 3\pi 2\pi^0, 5\pi$
πkk-bar(MC)
a_1
s (GeV^2)
OPAL
$\pi\,\pi^0$
$3\pi\,\pi^0, \pi\,3\pi^0$
MC corr.
perturbative QCD (massless)
naïve parton model
v(s)
s (GeV^2)
OPAL
$3\pi, \pi\,2\pi^0$
$3\pi\,2\pi^0$
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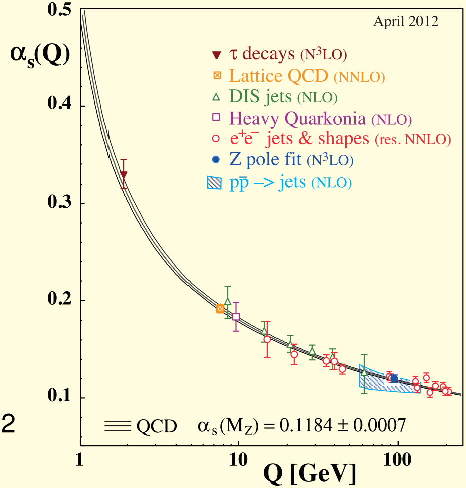
a(s)
s (GeV^2)

Strong coupling



PDG 2012

$$\Rightarrow \alpha_s(M_\tau) = 0.3186(58)$$

Strong coupling

Phenomenologically, R_τ^{V+A} can be expressed as follows:

$$R_\tau^{V+A} = 3\,|V_{ud}|^2\, S_{\rm EW}\Big[\,1+\delta^{(0)}+\delta_{V+A}^{\rm NP}\Big]$$

Purely perturbative QCD correction $\delta^{(0)}$ know to order α_s^4.

Depends on renormalisation-group resummation:

- Fixed-order perturbation theory (FOPT)
- Contour-improved perturbation theory (CIPT)

$$\delta_{\rm FO}^{(0)} \;=\; 0.2022 \pm 0.0069 \pm 0.0030 = 0.2022(75)$$
$$\delta_{\rm CI}^{(0)} \;=\; 0.1847 \pm 0.0048 \pm 0.0033 = 0.1847(58)$$

First error from uncertainty in $\alpha_s(M_\tau)$.
Second error from estimate of $\mathcal{O}(\alpha_s^5)$ contribution.

Scale resummation induces $\approx$ 2% difference.

Employing the experimental measurement $R_{V+A}^{\rm exp} = 3.4671(82)$:

$$\delta_{V+A,\rm FO}^{\rm NP} = -0.0086(80)\,, \qquad \delta_{V+A,\rm CI}^{\rm NP} = 0.0089(65)$$

$\delta_{V+A}^{\rm NP} = \delta_{V+A}^{\rm OPE} + \delta_{V+A}^{\rm DV}$ comprises a small ($\approx 1\%$) correction due to quark masses as well as OPE and duality violations.

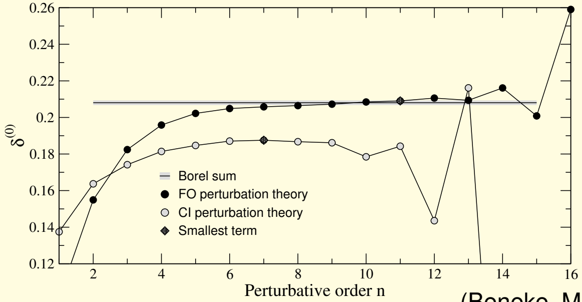
To disentangle OPE and DV contributions, additional experimental information is required:

- Moments with different spectral weight functions;
- Moments with upper integration limit $s_0 < M_\tau^2$.

Still, results of this program will depend on the question of renormalisation-group resummation (FOPT versus CIPT).

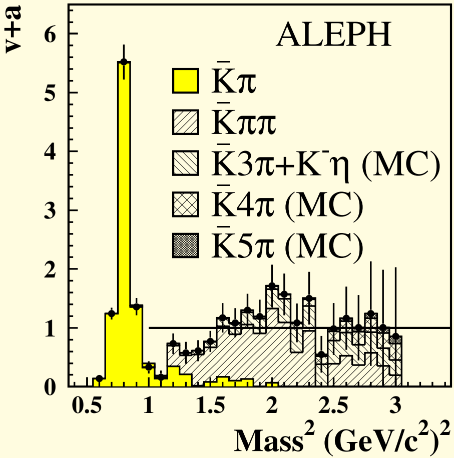
Difference between FOPT and CIPT can be understood on the basis of a model for the behaviour of higher orders in α_s:

- Model for Borel transform of Adler function;
- Incorporates renormalon structure known from RGE;
- Reproduces the known lowest-order coefficients;
- Allows resummation of the perturbative series.



(Beneke, MJ 2008)

Strange channel



To enhance sensitivity for strange quark effects, consider the flavour-SU(3) breaking difference:

(Prades, Pich; ALEPH 1998)

$$\delta R_\tau = \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2} = 3\, S_{\rm EW} \sum_{D\geq 2} \left(\delta_{ud}^{(D)} - \delta_{us}^{(D)} \right)$$

Flavour independent uncertainties drop out in the difference.

Leading contribution is proportional to m_s^2 whose series is not very well behaved.

Again RG resummation issue of FOPT versus CIPT.

Even more badly behaved scalar/pseudoscalar correlators from phenomenology.

(Gámiz, MJ, Pich, Prades, Schwab 2003/05)

Theoretically, δR_τ can be split into three contributions:

$$\delta R_\tau^{\rm th} = \delta R_\tau^{m^2} + \delta R_\tau^{D\geq 4} + \delta R_\tau^{S+P}$$

Last term known from phenomenology: $\delta R_\tau^{S+P} = 0.1544(37)$.

Estimates for $D \geq 4$ contribution small: $\delta R_\tau^{D\geq 4} = 0.0034(28)$.

Largest uncertainty: $\delta R_\tau^{m^2} = 9.3(3.4)\cdot m_s^2 = 0.082(31)$.

$$\delta R_\tau^{\rm th} = 0.240 \pm 0.031$$

On the other hand from $R_{\tau,S} = 0.1612(28)$ (HFAG 2013):

$$\delta R_\tau^{\rm exp} = 0.482 \pm 0.060$$

Hence, we face a serious discrepancy.

Outlook

- Lecture 2: perturbative series for R_τ, renormalon based Borel model, resummation dependence.
- Lecture 3: Multi-moment analysis for α_s, duality violation.
- Lecture 4: Description of exclusive τ decay distributions.

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Thank You!