K semileptonic form factors from lattice QCD

Elvira Gámiz

(with Lattice Fermilab and MILC Collaborations)

Universidad de Granada / CAFPE

(Spain)

· Universität Wien, 20 June 2012 ·

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1. Introduction

Searching for New Physics via precise measurements/SM predictions of flavor observables.

Constraining possible NP models.

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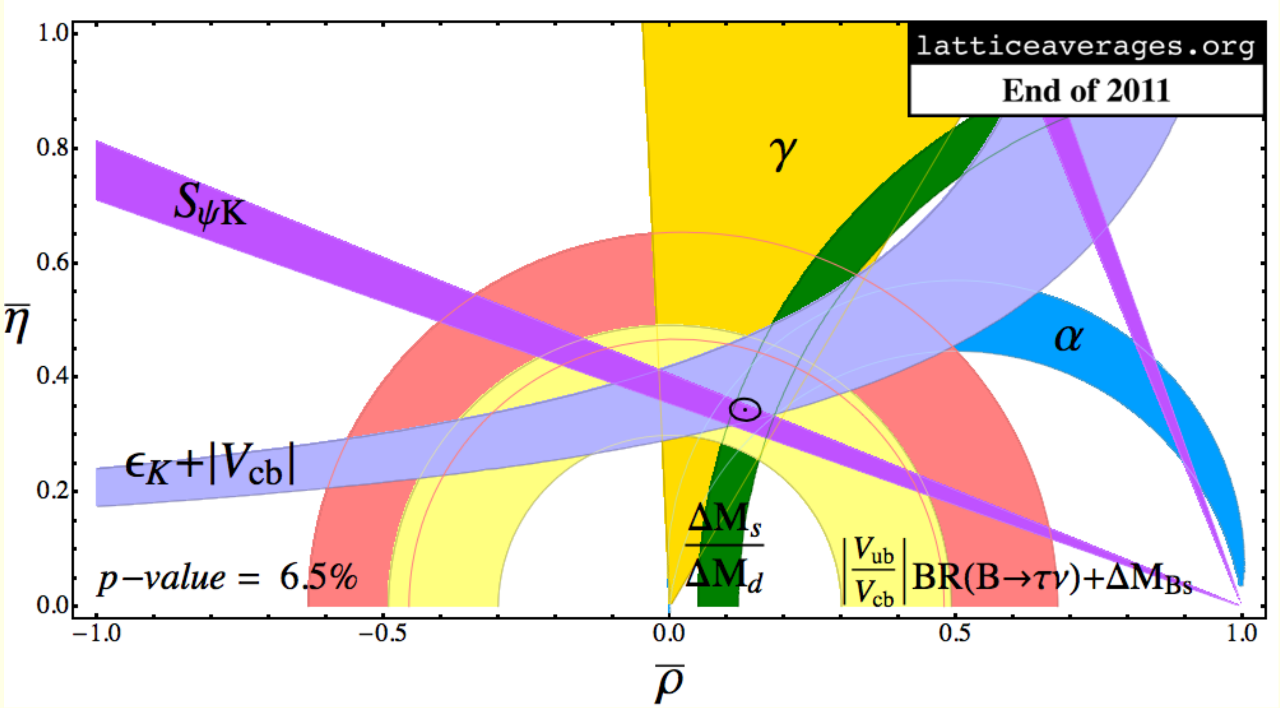
Laiho,Lunghi,Van de Water PRD81:034503 (2010)



Example: UT fits

Error bands are still dominated by theory errors, in particular due to hadronic matrix elements

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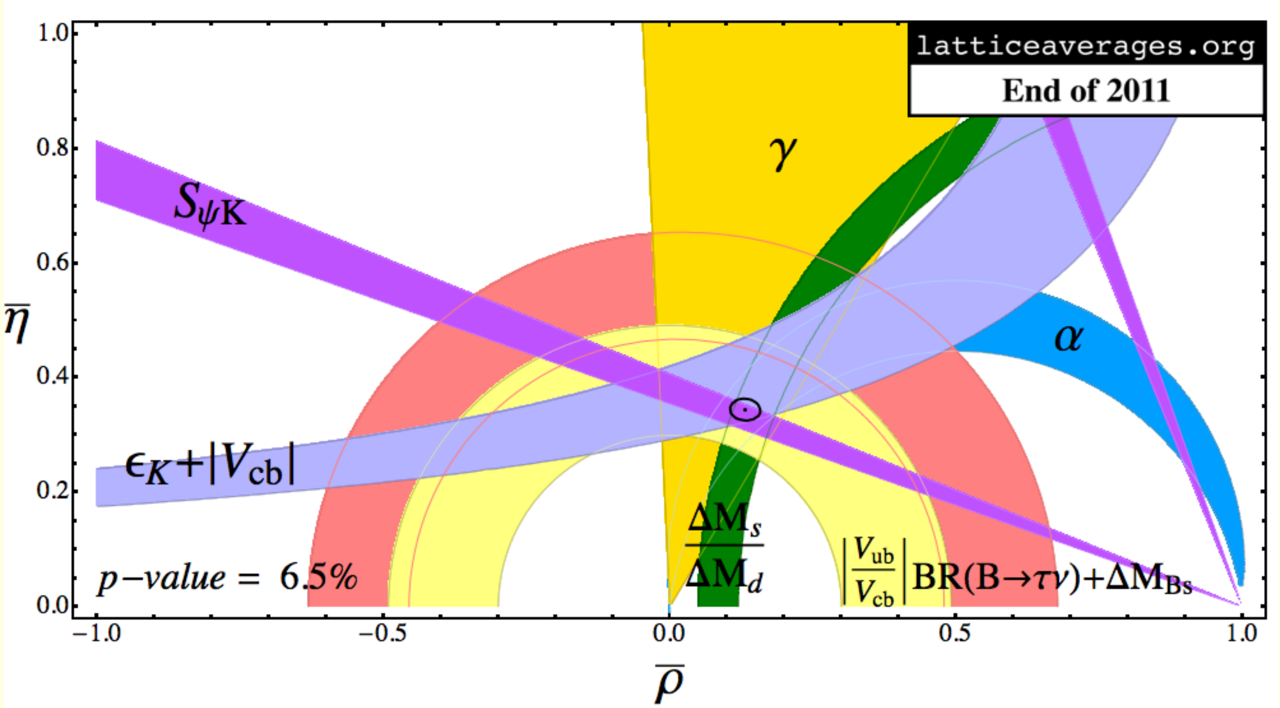

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Error bands are still dominated by theory errors, in particular due to hadronic matrix elements $\rightarrow$ use **lattice QCD**

1. Introduction: Lattice QCD

Lattice QCD: Numerical evaluation of QCD path integral (rely only on first principles) using Monte Carlo methods.

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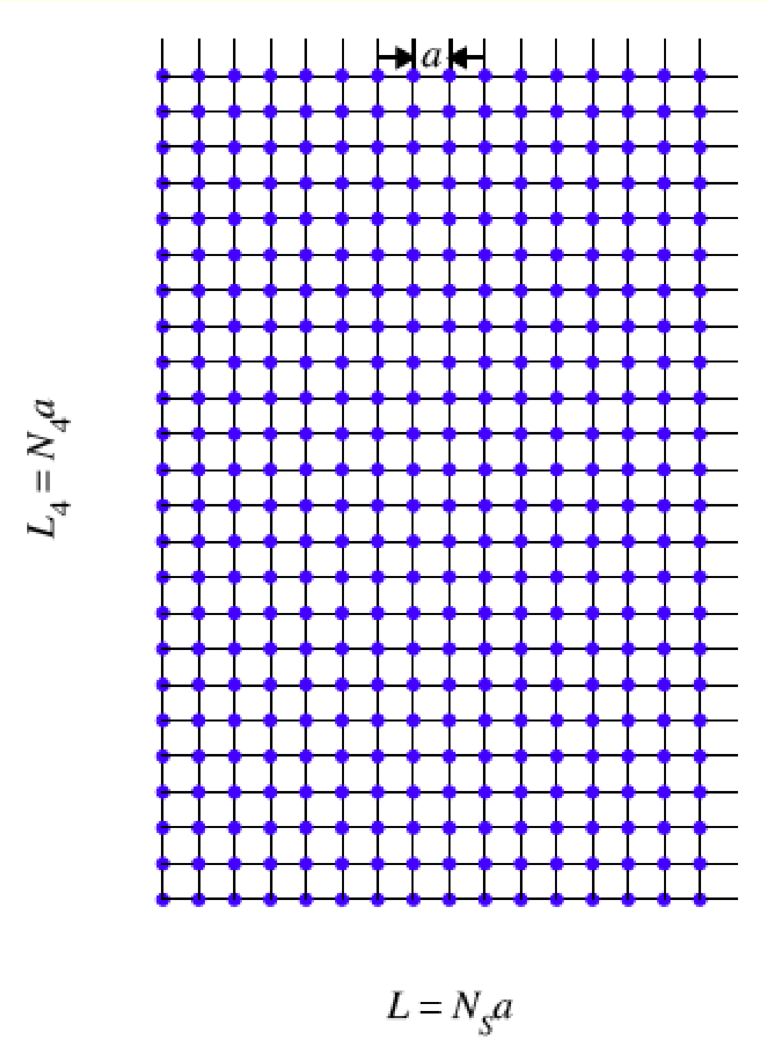

Define quantum field theory on an Euclidean spacetime lattice with length L (provides an IR cutoff in the path integral) and lattice spacing a (provides and UV cutoff in the path integral).

* Replace derivatives by discrete differences and integrals by sums

$$\partial\psi(x) \rightarrow \frac{\psi(x+a) - \psi(x-a)}{2a}$$

$$\psi(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\tilde{\psi}(k) \rightarrow \sum_k e^{-ik\cdot x}\tilde{\psi}(x)$$

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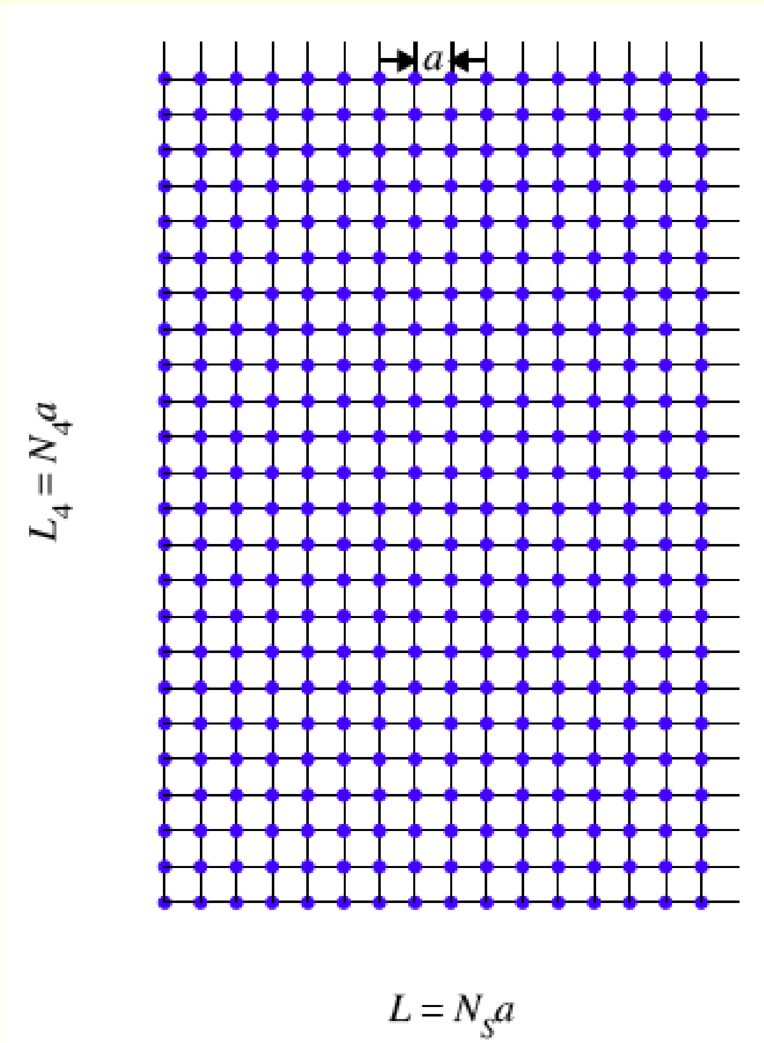

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* Recover continuum action when $a \rightarrow 0$ and $L, L_4 \rightarrow \infty$.

1. Introduction: Lattice QCD

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Control over systematic errors:

* Unquenched calculations: Incorporate the vacuum polarization effects (in a realistic way).

  ** Quenching the strange quark could have an error as large as 5% and need a $N_f = 2+1$ to have an estimate $\rightarrow$ want $N_f = 2+1$

  ** Neglecting sea charm has effects $\mathcal{O}(1\%)$ (can be estimated with HQET). **Starting to need sea charm effects.**

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* Discretization: improved actions (a^2 errors suppressed) + simulations at several $a's$ $\rightarrow$ continuum limit.

1. Introduction: Lattice QCD

Control over systematic errors:

* Chiral extrapolation: The lightest the quarks the most expensive to simulate $\rightarrow$ in most of the simulations $m_\pi^{\rm lat} > m_\pi^{\rm phys}$.

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* Renormalization: non-perturbative, perturbative.

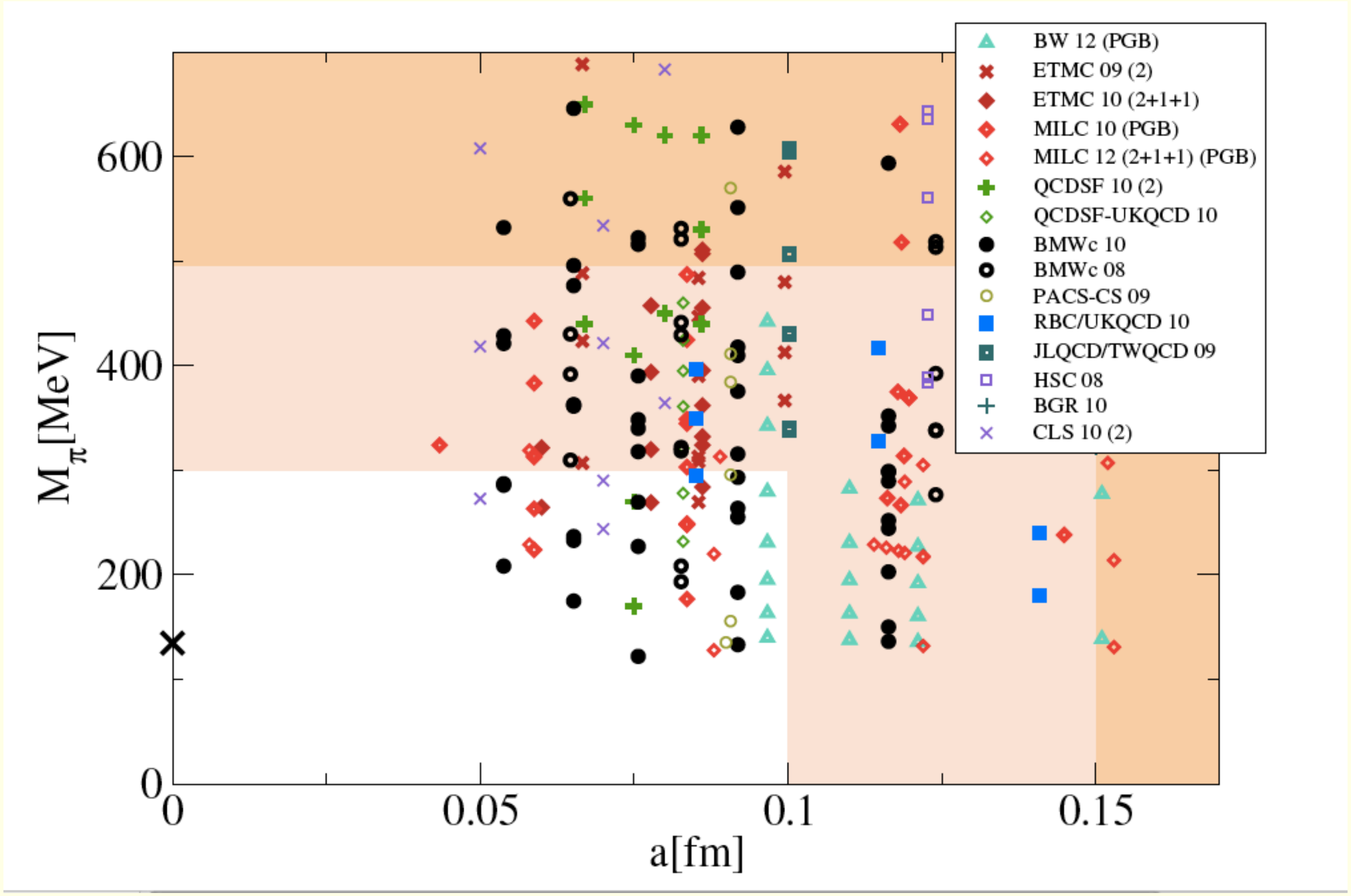
* Tuning lattice scale and quark masses (parameters of the lattice action)

* Finite volume, isospin effects, electromagnetic effects, ...

Systematically improvable

1. Introduction: Overview of simulations parameters

Several $N_f = 2+1$ and even $N_f = 2+1+1$, and **physical quark masses**.



First results with simulations with physical light quark masses starting to appear.

plot by **C. Hoelbling**,

1. Introduction: Averaging lattice QCD results

J. Laiho, E. Lunghi, and R. Van de Water (LLV)

Phys.Rev.D81:034503,2010, most updated results in www.latticeaverages.org

* Phenomenologically relevant light and heavy quantities + UT fits with lattice inputs.

* Include only $N_f = 2+1$.

* Only published results (including proceedings).

Flavianet Lattice Average group: (FLAG)

Eur. Phys. J. C71(2011)1695, updated results in http://itpwiki.unibe.ch/flag

* K and π physics, including LEC's.

Flavor Lattice Averaging Group (FLAG-2): 28 people representing all big lattice collaborations.

* Light and heavy quantities. **First review by summer 2013**

1. Introduction: Heavy quarks on the lattice

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

* **Effective theories:** Need to include multiple operators matched to full QCD `B-physics` $\surd$

** HQET (static,...): sytematic expansion in $1/m_h$.

** NRQCD: systematic (non-relativistic) expansion in (v_h/c).

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* **Relativistic (improved) formulations:**

  ** Allow accurate results for `charm` (especially twisted mass, HISQ (Highly improved staggered quarks)).

  ** Advantages of having the same formulation for light and heavy: ratios light/heavy, PCAC for heavy-light, ... Also simpler tuning of masses.

  ** Also for `bottom`: Results for $m_c \cdots \leq m_b$ and extrapolation to m_b (twisted mass, HISQ).

2. Highlights of flavour physics on the lattice

2.1. Decay constants

Decay constants come from simple matrix element $\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = i f_P p_\mu$ $\rightarrow$ precise calculations on the lattice

* Even higher precision for ratios due to cancellation of statistics and systematics uncertainties

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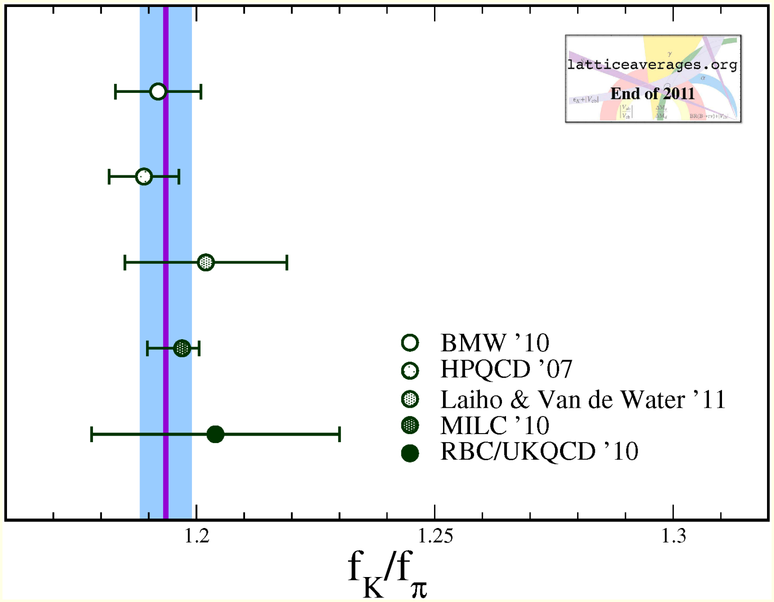
$\frac{f_K}{f_\pi}$: $0.6 - 2\%$ errors, 0.4% average

Many $N_f = 2+1$ lattice calculations $\rightarrow$ good test of lattice QCD



$$f_K/f_\pi^{\mathbf{LLV}} = 1.1936 \pm 0.0053$$

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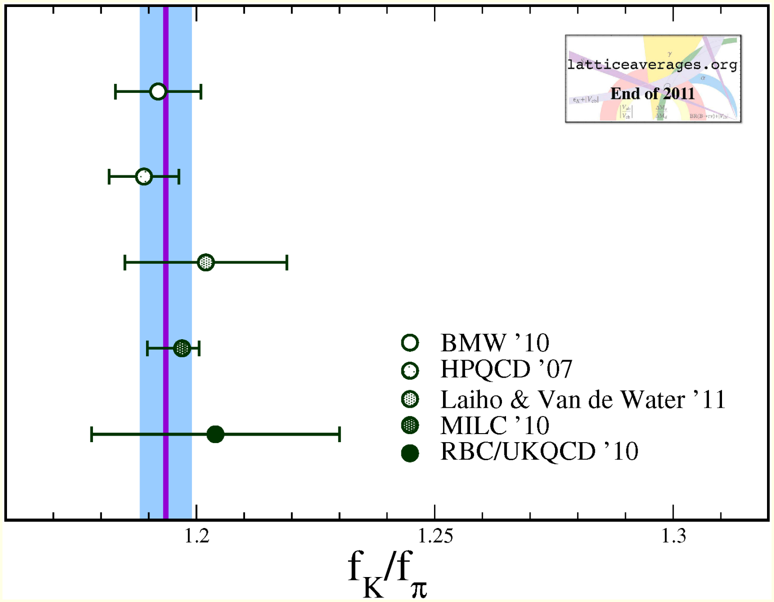


$f_K/f_\pi^{\mathbf{LLV}} = 1.1936 \pm 0.0053$

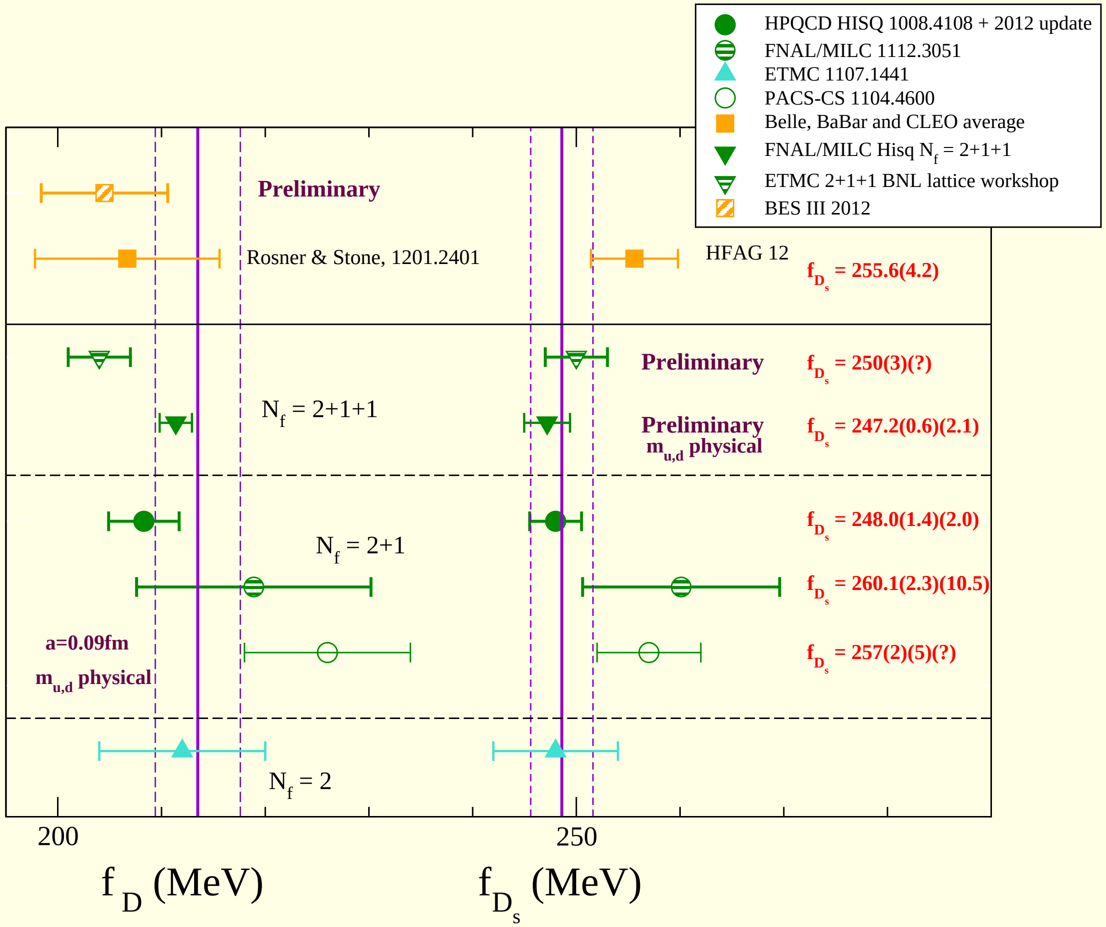
New: First calculation with physical quark masses

$$\frac{f_K}{f_\pi} = 1.1947(26)(37)$$

FNAL/MILC, 1301.5855

2.1.1. D and D_s decay constants

Reduction of errors in f_D and f_{D_s} due to the use of relativistic actions.



(experimental averages use $|V_{cs}| = 0.97345(22)$, $|V_{cd}| = 0.2245(12)$)

$$f_D^{\mathbf{LLV}} = (213.5 \pm 4.1)\ MeV \qquad f_{D_s}^{\mathbf{LLV}} = (248.6 \pm 3.0)\ MeV$$

$f_{D_s}^{\mathbf{exp}} = (255.6 \pm 4.2)\ MeV \rightarrow$ tension is now down to $\sim 2\sigma$.

2.1.2. B and B_s decay constants

Needed for processes potentially sensitive to NP: $B_{(s)} \to \mu^+\mu^-$.

Check agreement theory-experiment $Br(B^- \to \tau^-\bar{\nu}_\tau)$.

UT inputs.

2.1.2. B and B_s decay constants

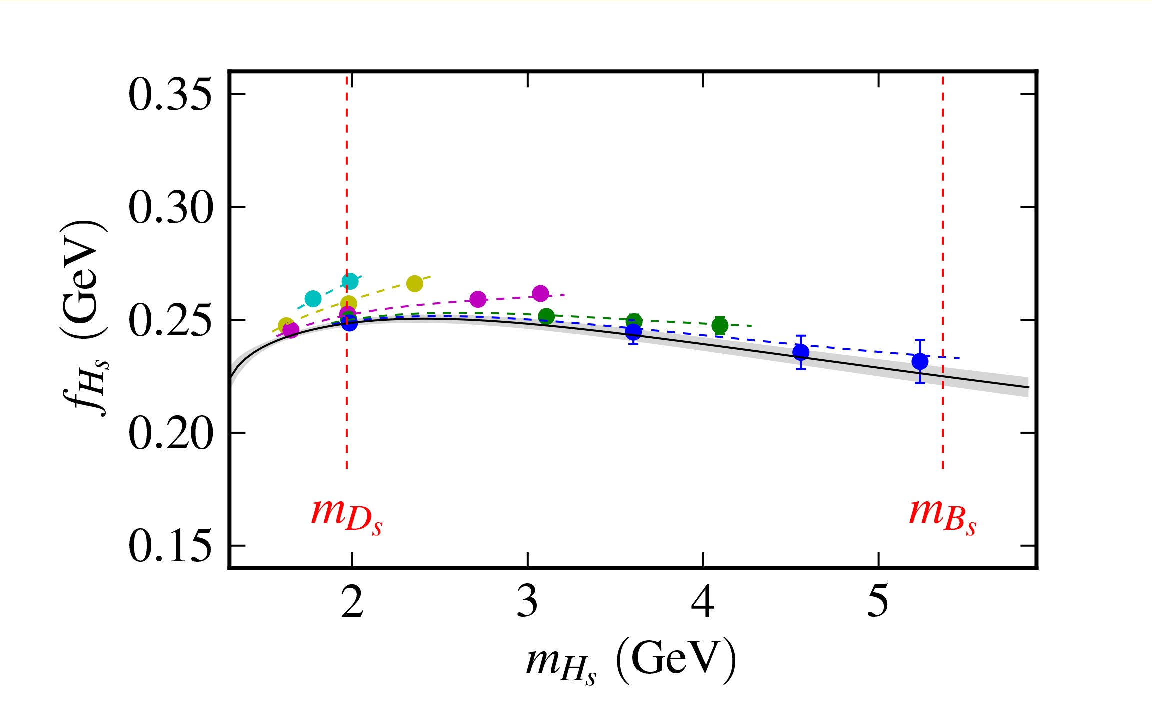
HPQCD **relativistic**, PRD 85 (2012) 031503: $N_f = 2+1$ with four a's.

* Using relativistic description (HISQ) for b reduce the error to 2%.

** No effective theory errors, no renormalization.

* Cross-checks: $m_b^{\overline{MS}}$, $m_{B_s} - m_{\eta_b}/2$, f_K, f_π.

* First empirical evidence for $1/\sqrt{m_{B_s}}$ depende predicted by HQET.



$$f_{B_s} = 224(4)\ \mathrm{MeV}$$

2.1.2. B and B_s decay constants

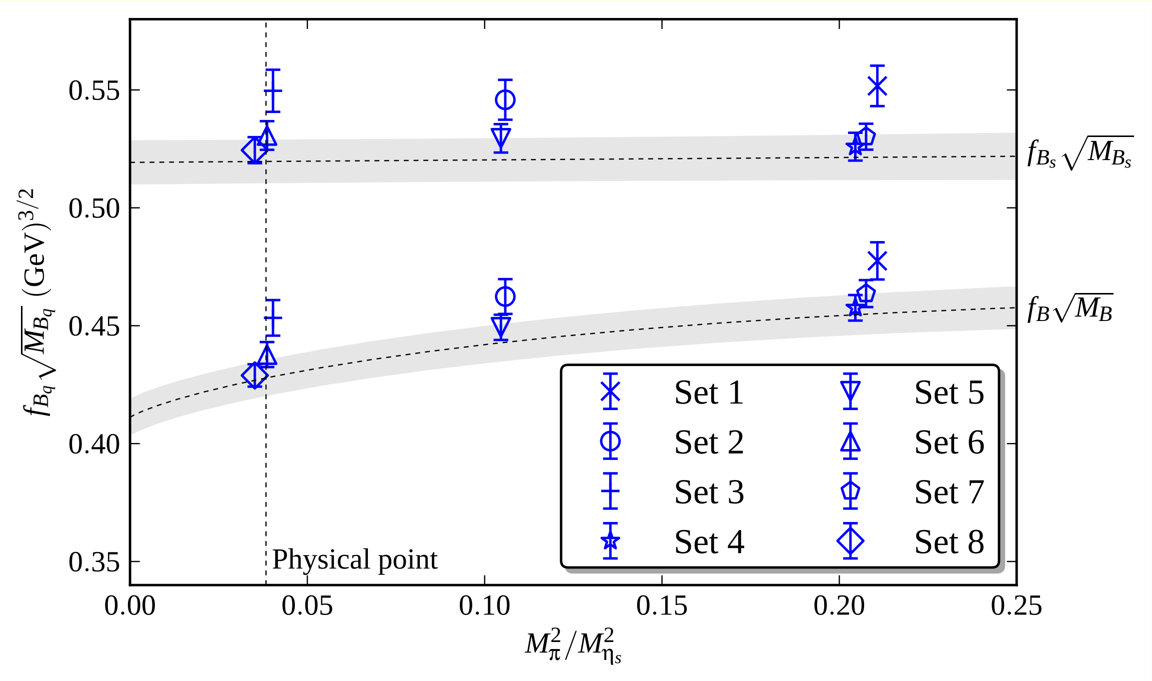
\# First calculation with **physical light quark masses**: **HPQCD**, 1302.2644

* $N_f = 2+1+1$ **MILC** configurations. Three a's.

* NRQCD description of b quarks.

* New estimate of matching errors:

fit α_s^2 terms instead of power counting.

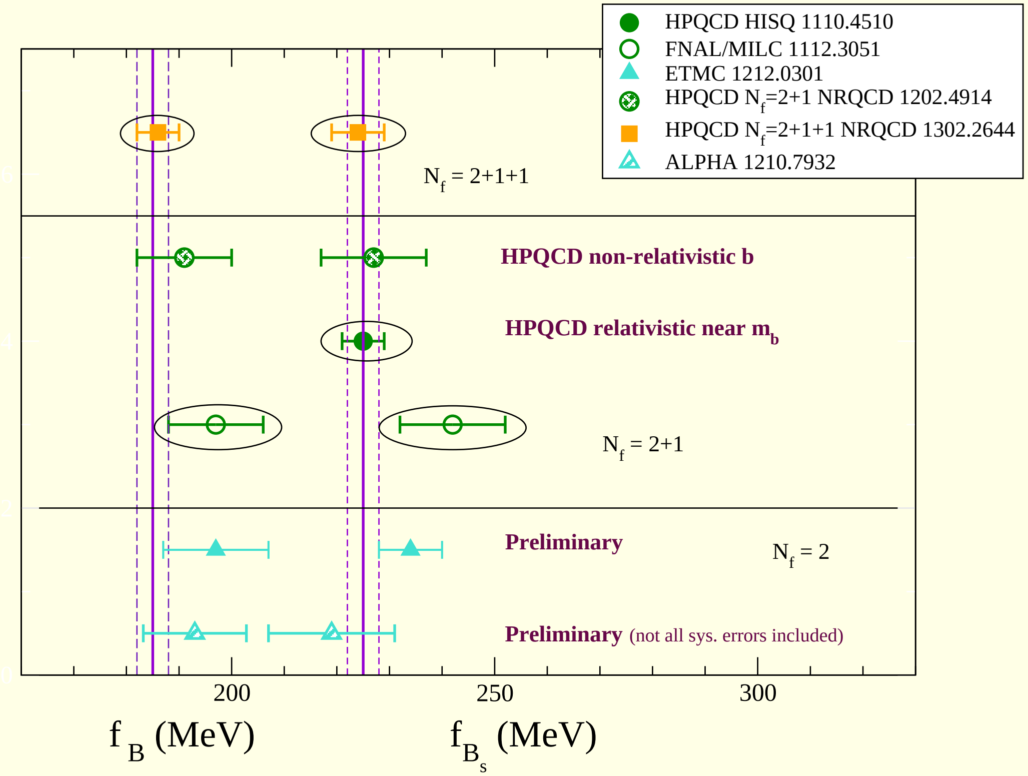


$$f_B = 186(4)\ \text{GeV}$$

$$f_{B_s} = 224(5)\ \text{GeV}$$

$$f_{B_s}/f_B = 1.205(7)$$

2.1.2. B and B_s decay constants



Averages from 1302.2644

$$f_B = (185 \pm 3)\ \ MeV$$

$$f_{B_s} = (225 \pm 3)\ \ MeV$$

$$f_{B_s}/f_B = 1.218(8)$$

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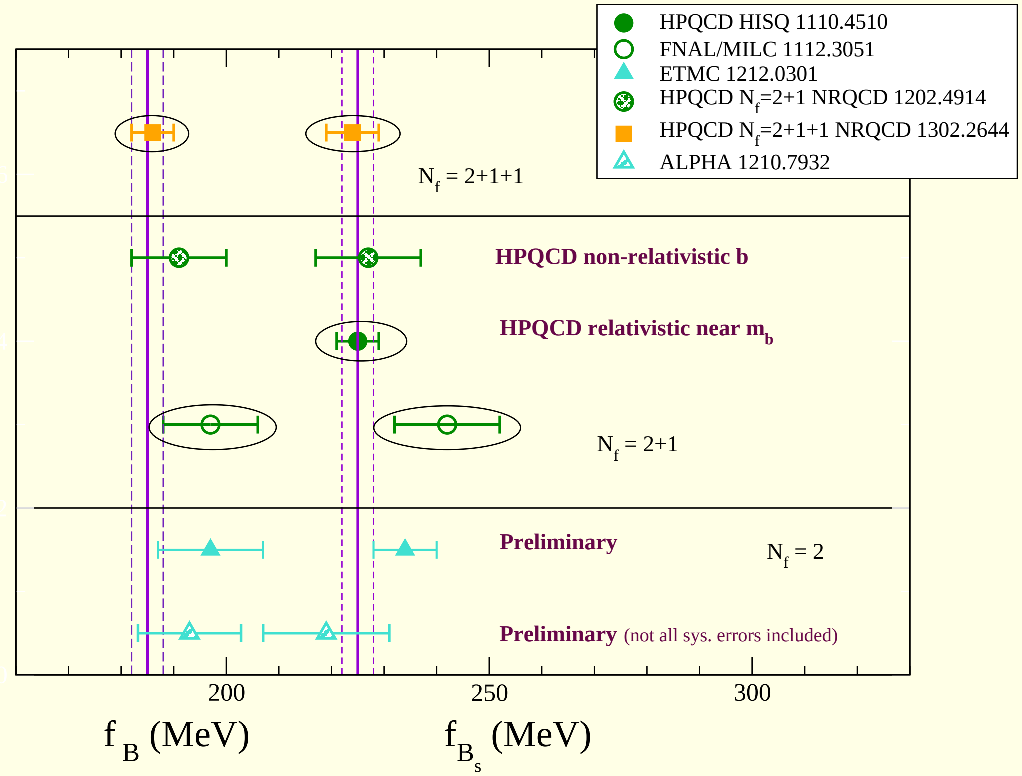

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Using f_B above: $Br(B^+ \to \tau\nu)/|V_{ub}|^2 = 6.05(20)$ 1302.2644

Belle, 1208.4678: $Br(B^+ \to \tau\nu)/|V_{ub}^{exc.}|^2 = 6.9 \pm 3.1$

$$Br(B^+ \to \tau\nu)/|V_{ub}^{inc.}|^2 = 3.9 \pm 1.7$$

Averages in, 1201.2401: $Br(B^+ \to \tau\nu)/|V_{ub}^{exc.}|^2 = 16.1 \pm 4.2$

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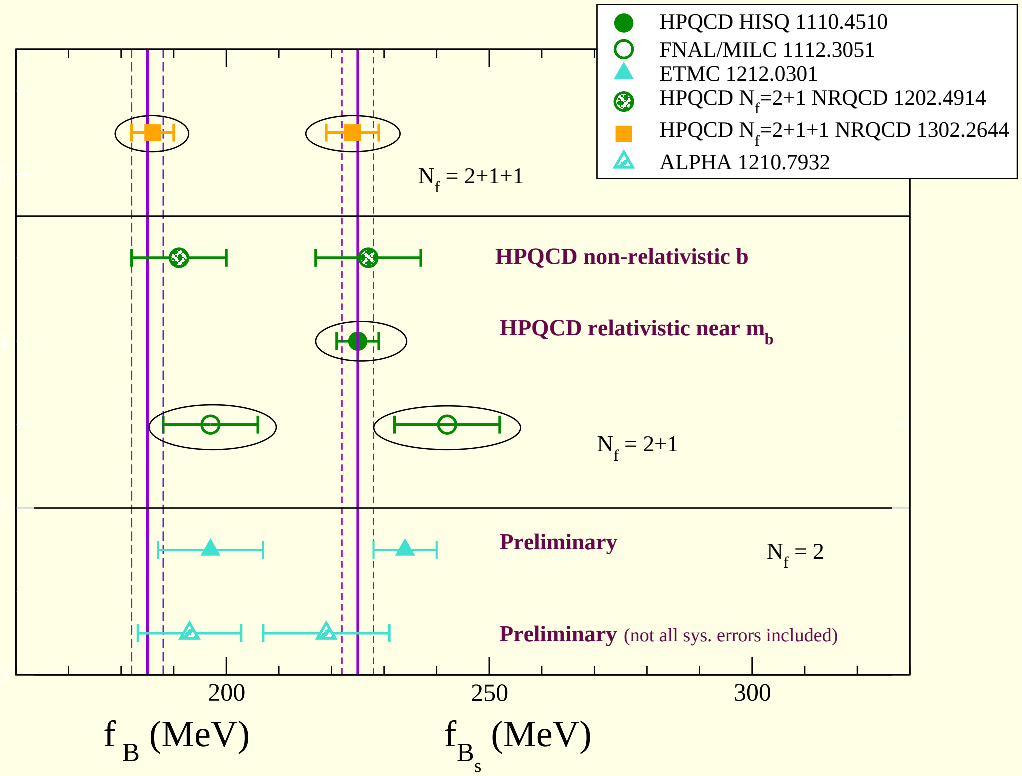

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\# In progress: **FNAL/MILC, ALPHA, ETMC, RBC/UKQCD**

2.2. Neutral meson mixing

2.2.1 $K^0 - \bar{K}^0$ mixing

One of the most stringent constraints in UT analyses comes from indirect CP violation in K decays.

$$|\epsilon_K| \quad = e^{i\phi_\epsilon}\kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \eta \left(|V_{cb}|^2 (1-\bar{\rho}) + \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \right)$$

* Lattice QCD techniques have reduced $\hat{B}_K$ errors to $\sim 1.3\%$:

$$\boxed{\hat{B}_K^{\mathbf{LLV}} = 0.7643 \pm 0.0097}$$

$\rightarrow \hat{B}_K$ is no longer the dominant source of uncertainty in neutral K mixing, but $|V_{cb}|$ and the NNLO pert. QCD coeficient η_{cc}

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First unquenched calculations of complete set of $\Delta S = 2$ effective operators describing $K - \bar{K}$ mixing

* $N_f = 2$: **ETMC**, 1207.1287
* $N_f = 2+1$: No extrapolation to the continuum **RBC/UKQCD**, 1206.5737
* In progress: $N_f = 2+1+1$ **ETMC**, $N_f = 2+1$ **SWME**

2.2.2. Neutral B-meson mixing

Hints of NP in neutral B-meson mixing: UTfit **1010.5089**, CKMfitter **1203.0238**, like-sign dimuon charge asymmetry **1106.6308** + UT tensions . . .

Not confirmed by recent analyses (B_s) **Lenz et al**, 1203.0238, **or by recent LHCb measurements.**

Still room for important effects in B mixing. **Lenz et al**, 1203.0238

SM predictions + BSM contributions = experiment $\rightarrow$ constraints on BSM building **Dobrescu and Krnjaic**, 1104.2893; **Altmannshofer and Carena**, 1110.0843; **Buras and Girrbach**, 1201.1302 ...

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Need matrix elements of all operators in $\Delta B = 2$ effective Hamiltonian

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = \left(\bar{b}^\alpha \gamma_\mu L\, q^\alpha\right)\left(\bar{b}^\beta \gamma^\mu L\, q^\beta\right) \qquad \textbf{SM}$$

$$Q_2^q = \left(\bar{b}^\alpha L\, q^\alpha\right)\left(\bar{b}^\beta L\, q^\beta\right) \qquad Q_3^q = \left(\bar{b}^\alpha L\, q^\beta\right)\left(\bar{b}^\beta L\, q^\alpha\right)$$

$$Q_4^q = \left(\bar{b}^\alpha L\, q^\alpha\right)\left(\bar{b}^\beta R\, q^\beta\right) \qquad Q_5^q = \left(\bar{b}^\alpha L\, q^\beta\right)\left(\bar{b}^\beta R\, q^\alpha\right)$$

$\tilde{Q}_{1,2,3} = Q_{1,2,3}$ with the replacement $L(R) \rightarrow R(L)$

2.2.2. Neutral B-meson mixing

There is not a complete unquenched lattice calculation of all the operators in $\mathcal{H}_{eff}^{\Delta B=2}$ yet.

* Only for $\langle \bar{B}_q^0 | O_1^q | B_q^0 \rangle(\mu) \equiv \frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2$:

$$f_{B_s}\sqrt{\hat{B}_{B_s}}^{\,\mathbf{LLV}} = 279(15)\mathrm{MeV} \qquad f_{B_d}\sqrt{\hat{B}_{B_d}}^{\,\mathbf{LLV}} = 227(19)\mathrm{MeV}$$

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which leads to the SM predictions:

$\Delta M_s^{SM} = (19.6 \pm 2.1) ps^{-1}$ **Lenz,Nierste** + above average

$\Delta M_s^{SM} = (16.9 \pm 1.2) ps^{-1}$ **Lenz,Nierste** + aver. f_{B_s} + B_{B_s} below

$\Delta M_s^{exp} = (17.768 \pm 0.023 \pm 0.006) ps^{-1}$ **LHCb** Moriond 2013 **preliminary**

(bag parameter, $B_{B_s}^{\overline{MS}}(m_b) = 0.86(4)$, from **HPQCD**, 0902.1815)

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* And the $SU(3)$ ratio $\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$:

$$\xi^{\text{aver. } \mathbf{N_f=2+1}} = 1.251 \pm 0.032$$

2.2.2 Neutral B-meson mixing

In progress: Among others **ETMC** ($N_f = 2+1+1$), **FNAL/MILC** ($N_f = 2+1$)

$[\mathrm{GeV}^2]$	B_d^0 BBGLN	B_d^0 BJU	B_s^0 BBGLN	B_s^0 BJU
$f_{B_q}^2 B_{B_q}^{(1)}$	0.0411(75)		0.0559(68)	
$f_{B_q}^2 B_{B_q}^{(2)}$	0.0574(92)	0.0538(87)	0.086(11)	0.080(10)
$f_{B_q}^2 B_{B_q}^{(3)}$	0.058(11)	0.058(11)	0.084(13)	0.084(13)
$f_{B_q}^2 B_{B_q}^{(4)}$	0.093(10)		0.135(15)	
$f_{B_q}^2 B_{B_q}^{(5)}$	0.127(15)		0.178(20)	

Preliminary results from

FNAL/MILC, 1112.5642

$N_f = 2+1$

* $\langle Q_1 \rangle, \langle Q_3 \rangle$ will also allow new prediction for $\Delta\Gamma_s$.

$\Delta\Gamma_s^{SM} = (0.075 \pm 0.020) ps^{-1}$ **Nierste**, CKM2012 using **preliminary** results above

$\Delta\Gamma_s^{exp} = (0.106 \pm 0.011 \pm 0.007) ps^{-1}$ **LHCb**, Moriond 2013

2.3 Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Bag parameters describing $B-\texttt{meson}$ mixing in the SM can be used for theoretical prediction of $\mathcal{B}r(B \to \mu^+\mu^-)$ **Buras**, hep-ph/0303060

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W}\right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Need to include the effects of a non-vanishing $\Delta\Gamma_s$ to compare with experiment **K. de Bruyn et al.**, 1204.1737

$$\mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \to \mathcal{B}r(B_q \to \mu^+\mu^-)_{y_s} \equiv \mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \times \frac{1}{1-y_s}$$

with $y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$.

2.3 Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Bag parameters describing $B-$**meson** mixing in the SM can be used for theoretical prediction of $\mathcal{B}r(B \to \mu^+\mu^-)$ **Buras**, hep-ph/0303060

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Need to include the effects of a non-vanishing $\Delta\Gamma_s$ to compare with experiment **K. de Bruyn et al.**, 1204.1737

$$\mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \to \mathcal{B}r(B_q \to \mu^+\mu^-)_{y_s} \equiv \mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \times \frac{1}{1-y_s}$$

with $y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$.

* Using $\hat{B}_{B_s} = 1.33(6)$, $\hat{B}_{B_d} = 1.26(11)$ **HPQCD**, 0902.1815, $y_s = 0.087 \pm 0.014$ **LHCb**,1212.4140

$$\mathcal{B}r(B_s \to \mu^+\mu^-)_{y_s} = (3.71 \pm 0.17) \times 10^{-9}$$ **Buras et al.** 1303.3820

$$\mathcal{B}r(B_d \to \mu^+\mu^-) = (1.03 \pm 0.09) \times 10^{-10}$$

Error dominated by uncertainty in the bag parameter **Buras et al.** 1303.3820

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Indirect determination

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Improved $f_{B_{s,d}}^{lattice}$ makes direct theoretical calculation competitive
Buras and Girrbach,1204.5064

* Using the lattice averages giving in 1302.2644: $f_B = (185 \pm 3)\ MeV$ and $f_{B_s} = (225 \pm 3)\ MeV$.

$$\mathcal{B}r(B_s \to \mu^+\mu^-)_{y_s} = (3.56 \pm 0.18) \times 10^{-9}$$ **Buras et al.** 1303.3820

Dominant errors: $|V_{tb}^* V_{ts}|$ 4%, f_{B_s} 2.7%

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Most stringent experimental bounds **LHCb** Moriond 2013:

$$\mathcal{B}r(B_s \to \mu^+\mu^-) = \left(3.2^{+1.4+0.5}_{-1.2-0.3}\right) \times 10^{-9}$$

$$\mathcal{B}r(B_d \to \mu^+\mu^-) < 9.4 \times 10^{-10} \text{ at } 95\% \text{ CL}$$

2.4 Exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$

\# $2 - 3\sigma$'s disagreement between exclusive and inclusive determinations of $|V_{ub}|$ and $|V_{cb}|$ **G. Ricciardi**, 1305.2844

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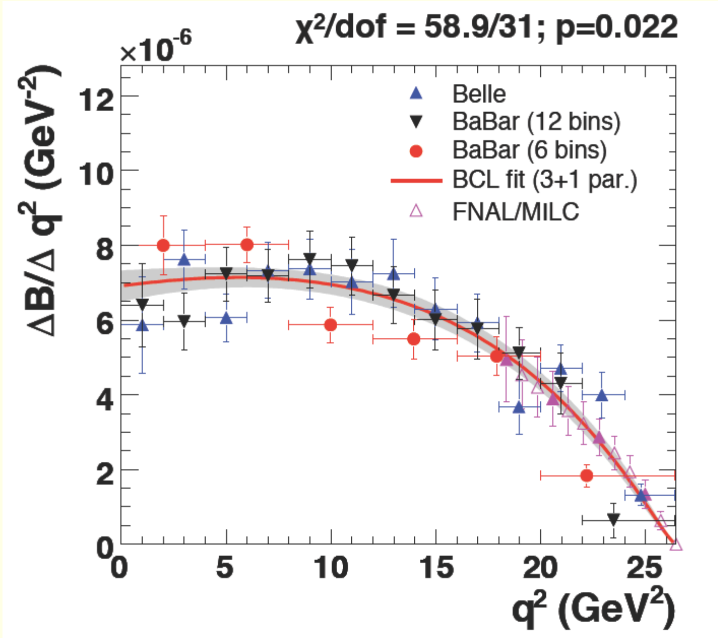
\# $2-3\sigma$'s disagreement between exclusive and inclusive determinations of $|V_{ub}|$ and $|V_{cb}|$ **G. Ricciardi**, 1305.2844

Exclusive $|V_{ub}|$: $B \to \pi l \nu$



Combined fit of lattice data

FNAL/MILC, 0811.3604

and experimental data

HFAG 2012, from BaBar and Belle data

from different q^2 regions using z-expansion.

$$|V_{ub}^{exc.}| = (3.23 \pm 0.30) \times 10^{-3}$$

* **In progress:** **FNAL/MILC**, **HPQCD**, **RBC/UKQCD**, **ALPHA**

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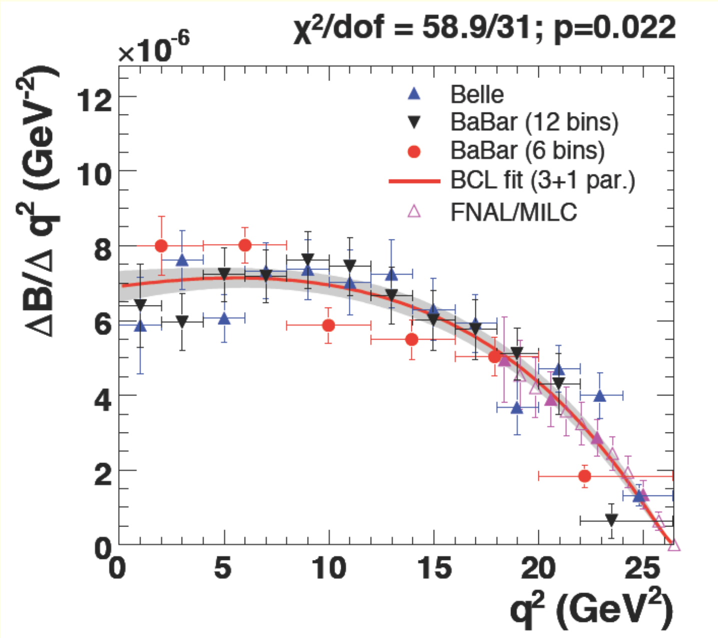

$$|V_{ub}^{exc.}| = (3.23 \pm 0.30) \times 10^{-3}$$

* **In progress: FNAL/MILC, HPQCD, RBC/UKQCD, ALPHA**

Alternative to $B \to \pi l \nu$ to extract $|V_{ub}|$: $B_s \to K l \nu$

* Experiment: Expect to be measured by **LHCb** and **Belle II**

* On the lattice: Corresponding form factors can be calculated with smaller errors (spectator quark is heavier (`strange`)

2.4 Exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$

Extraction of V_{cb} from exclusive B decays ($w = v \cdot v'$ is the velocity transfer):

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2-1)^{1/2} |\mathcal{F}(w)|^2$$

$$\frac{d\Gamma(B \to D l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2-1)^{3/2} |\mathcal{G}(w)|^2$$

State-of-the-art calculation: **FNAL/MILC** determination of $\mathcal{F}$ at zero recoil (blind anlysis based on HQ expasion and double ratio methods) + **BaBar** and **Belle**

$$|V_{cb}|_{excl} = (39.54 \pm 0.50_{exp} \pm 0.74_{LQCD}) \times 10^{-3}$$

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Calculation of non-zero recoil form factors $B \to D^{(*)} l\nu$ in progress **FNAL/MILC**, arXiv:1111.0677.

$\rightarrow$ will allow complementary extraction of $|V_{cb}|$.

2.5. $B \to D\tau\nu$ and NP hints?

BaBar recently measured the ratio of branching fractions

$$R(D) = \frac{\mathcal{B}r(B\to D\tau\nu)}{\mathcal{B}r(B\to Dl\nu)} = 0.440(72), \quad R(D^*) = 0.332 \pm 0.030$$ PRL109 (2012)101802

Using form factors in **Kamenik,Mescia**, 0802.3790 (quenched lattice)

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$N_f = 2+1$ form factor calculation by **FNAL/MILC**, PRL109 (2012)071802

$\boxed{R(D) = 0.316(12)(7)}$ $\to$ 1.7σ from experiment

Becirevic, Kosnik, Tayduganov, 1206.4977: $R(D) = 0.31(2)$

* In progress: Analysis in the complete $N_f = 2+1$ **FNAL/MILC** data set

$\to$ important reduction of errors in $R(D)$

* Another target: unquenched lattice calculation of $R(D^*)$

2.5. B rare decays: $B \to K l^+ l^-$

Potentially sensitive to NP effects.

Active effort to constraint NP with experimental results for $B \to K l^+ l^-$, usually in combination with other rare B decays

Becirevic et al, 1205.5811, **Bobeth et al**, 111.2558, 1212.2321,

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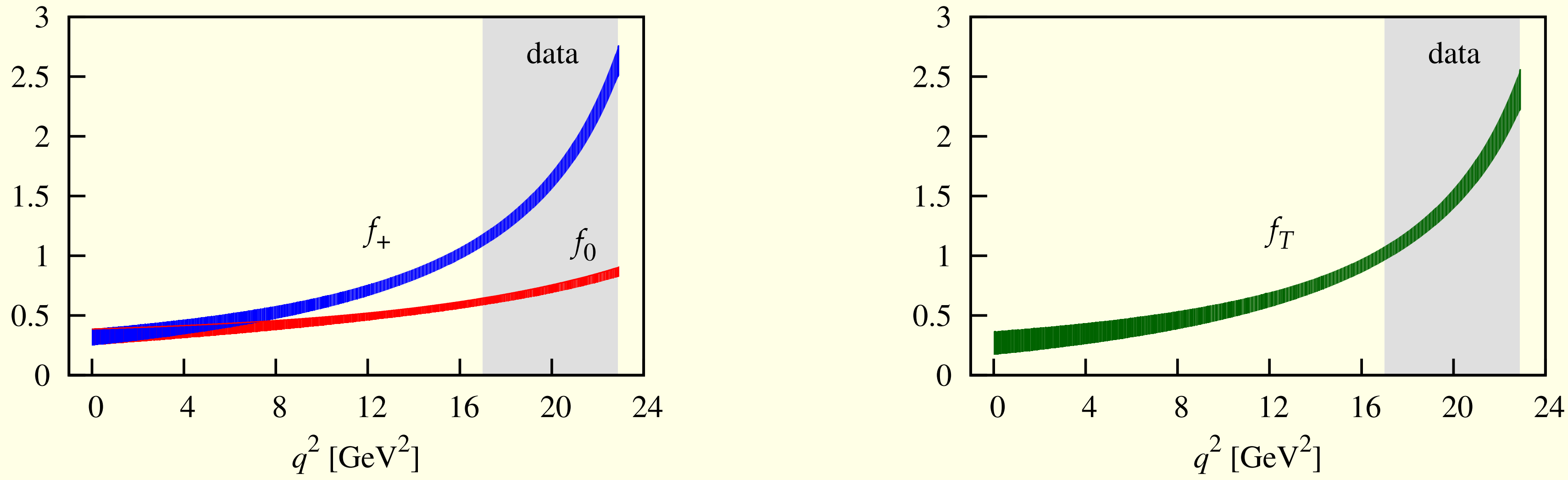
First unquenched determination of the form factors describing $B \to K l^+ l^-$ for $l = e, \mu, \tau$ **HPQCD**, 1306.0434, 1306.2384



NRQCD description for b quarks, two lattice spacings, `ChPT` for the chiral extrapolation, shape from `z-expansion` (data at $q^2 \geq 17\ \mathrm{GeV}^2$)

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* SM differential branching fractions $dB/dq^2(B \to Kll)$ for $l = e, \mu\tau$ obtained with these form factors agree with experiment.

* They calculate the ratio of branching fractions $R_e^\mu = 1.00029(69)$ and the flat term in the angular distribution of the differential decay rate $F_H^{e,\mu,\tau}$ in experimentally motivated q^2 bins.

$$\frac{1}{\Gamma_l}\frac{d\Gamma_l}{d\cos\theta_l} = \frac{1}{2}F_H^l + A_{FB}^l \cos\theta_l + \frac{3}{4}(1 - F_H^l)(1 - \cos^2\theta_l)$$

* They predict $B(B \to K\tau^+\tau^-) = (1.41 \pm 0.15) \cdot 10^{-7}$ and the ratio of branching fractions $R_l^\tau = 1.176(40)$, for $l = e, \mu$.

2.5. B rare decays: $B \to Kl^+l^-$

\# First unquenched determination of the form factors describing $B \to Kl^+l^-$ for $l = e, \mu, \tau$ **HPQCD**, 1306.0434, 1306.2384

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\# Similar results from **FNAL/MILC** soon.

\# Lattice studies of $B \to K^*l^+l^-$ in progress. Some preliminary results in **M. Wingate**, talk at Lattice2012

2.6 D semileptonic decays

At zero momentum transfer, $q^2 = 0$:
Extraction of the CKM matrix elements $|V_{cd(cs)}|$.

At non-zero momentum transfer, $q^2 \neq 0$:
Testing lattice QCD: shape of the form factors
$\rightarrow$ use same methodology for processes like $B \rightarrow \pi l \nu$ or $B \rightarrow K l \bar{l}$

Correlated signals of NP to those in leptonic decays.

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Correlated signals of NP to those in leptonic decays.

The erros on those studies are still dominated by errors in the calculation of the relevant form factors.

$$\frac{d}{dq^2}\Gamma(D \to K(\pi) l \nu) \quad \propto \quad |V_{cs(cd)}|^2 \, |f_+^{D \to K(\pi)}(q^2)|^2$$

where the vector form factor for any semileptonic decay $P_1 \to P_2 l \nu$ is defined by

$$\langle P_2 | V^\mu | P_1 \rangle = f_+^{P_1 P_2}(q^2) \left[p^\mu_{P_1} + p^\mu_{P_2} - \frac{m^2_{P_1} - m^2_{P_2}}{q^2} q^\mu \right] + f_0^{P_1 P_2}(q^2) \frac{m^2_{P_1} - m^2_{P_2}}{q^2} q^\mu$$

2.6 D semileptonic decays

Important reduction of errors in the lattice determination of the form factors $f_+^{D(K)}(0)$ by the **HPQCD Collaboration**, Phys.Rev.D82:114506(2010), due mainly to

* Use a relativistic action, HISQ, to describe light and charm quarks.

* Use the Ward identity ($S = \bar{a}b$)

$$q^\mu \langle P_2 | V_\mu^{cont.} | P_1 \rangle = (m_b - m_a) \langle P_2 | S^{cont} | P_1 \rangle$$

that relates matrix elements of vector and scalar currents. In the lattice

$$q^\mu \langle P_2 | V_\mu^{lat.} | P_1 \rangle Z = (m_b - m_a) \langle P_2 | S^{lat.} | P_1 \rangle$$

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$$q^\mu \langle P_2 | V_\mu^{lat.} | P_1 \rangle Z = (m_b - m_a) \langle P_2 | S^{lat.} | P_1 \rangle$$

$\rightarrow$ replace the V_μ with an S current in the 3-point function

$$f_0^{P_1 P_2}(q^2) = \frac{m_b - m_a}{m_{P_1}^2 - m_{P_2}^2} \langle P_2 | S | P_1 \rangle_{q^2} \Longrightarrow \boxed{f_+^{P_1 P_2}(0) = f_0^{P_1 P_2}(0) = \frac{m_b - m_a}{m_{P_1}^2 - m_{P_2}^2} \langle S \rangle_{q^2=0}}$$

2.6 D semileptonic decays

Advantages of the HPQCD method based on Ward identity:

* No need of renormalization factors Z.
* Need less inversions than the traditional double ratio method.
* S currents used are local.

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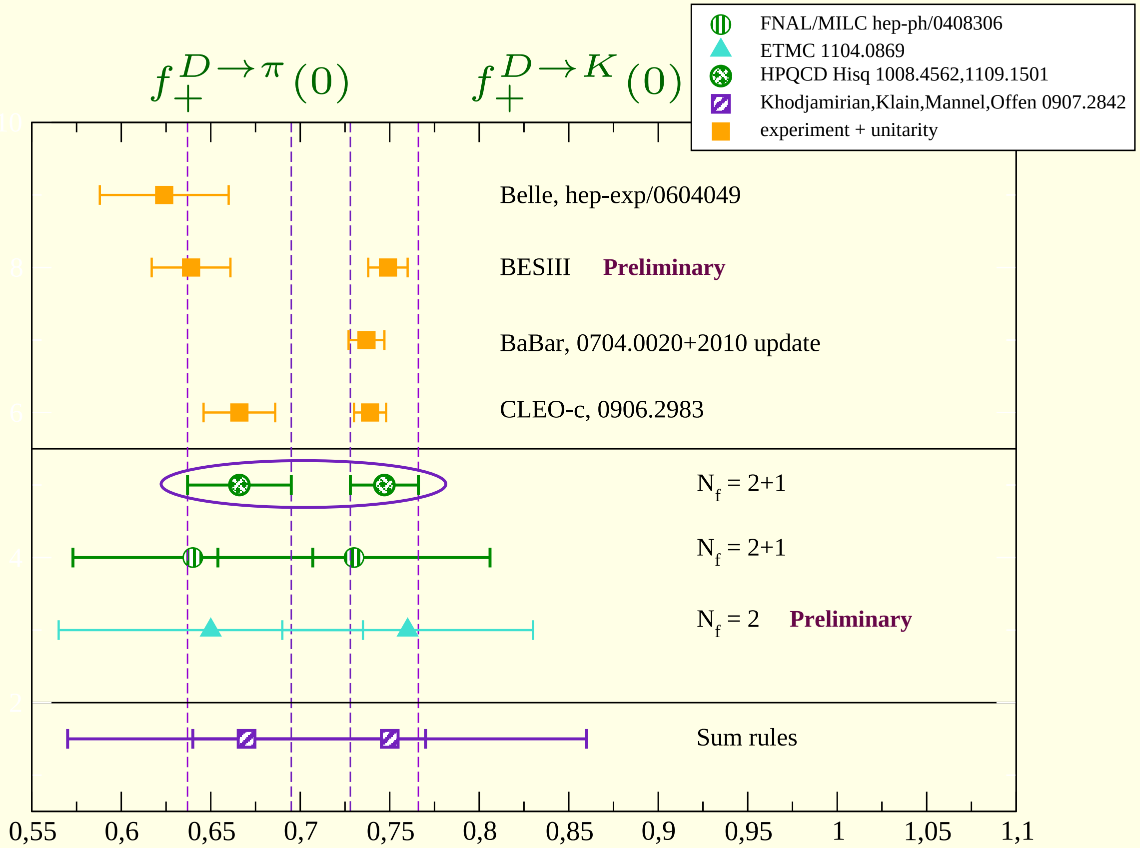
* No need of renormalization factors Z.
* Need less inversions than the traditional double ratio method.
* S currents used are local.

Downside: can get $f_+^{K\pi}(q^2)$ only at $q^2 = 0 \rightarrow$ concentrate on the calculation of $f_0(q^2 = 0)$ ($\equiv$ extraction of $|V_{cd,cs,us}|$)

2.6 D semileptonic decays



error $f_+^{D\to K}$: 11% $\to$ 2.5%.

error $f_+^{D\to \pi}$: 10% $\to$ 5%.

$|V_{cs}| = 0.961(11)_{exp}(24)_{lat}$ compatible with unitarity value $|V_{cs}|^{unit.} = 0.97345(16)$

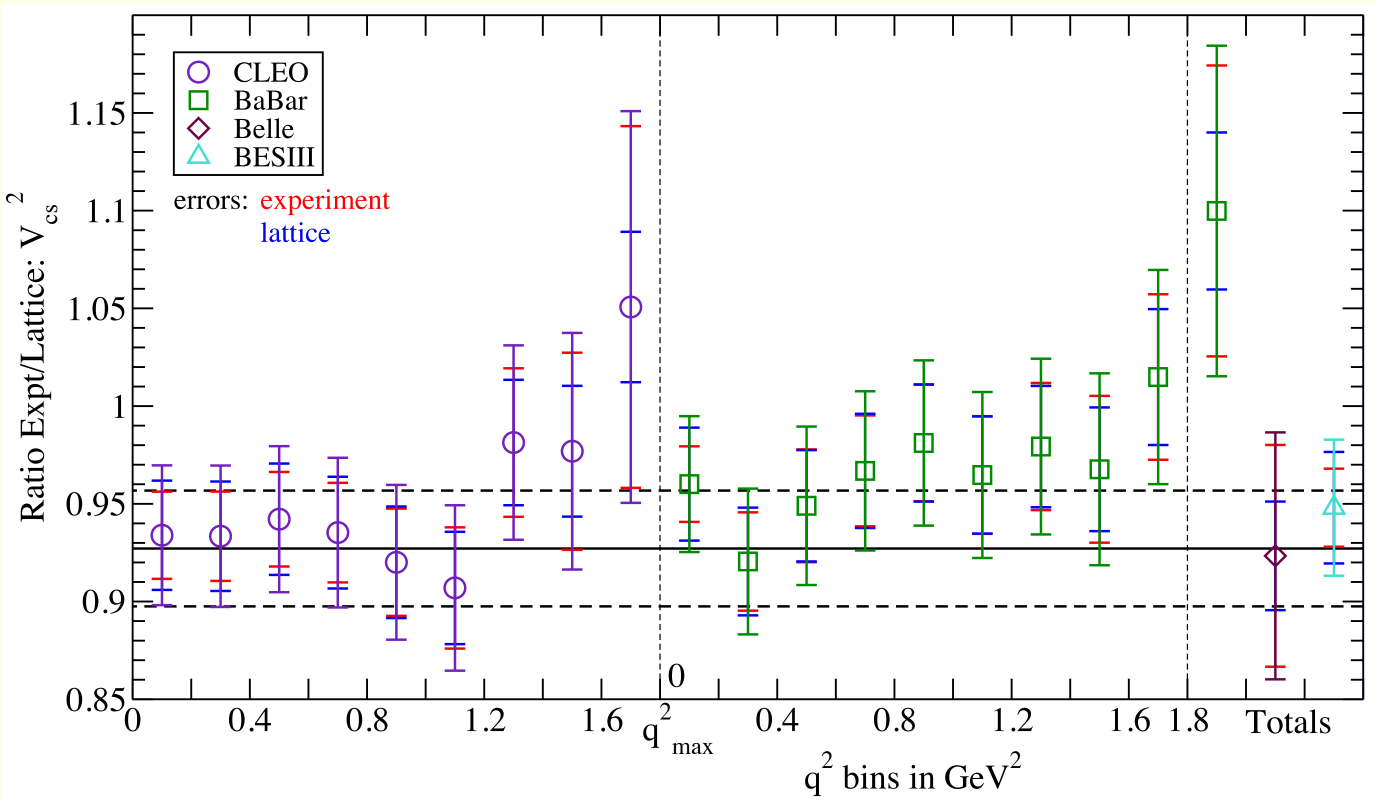
$|V_{cd}| = 0.225(6)_{exp}(10)_{lat}$ compatible with unitarity value $|V_{cd}|^{unit.} = 0.2252(7)$

* competitive with ν scattering determination $|V_{cd}|^{\nu} = 0.230(11)$

2.6 D semileptonic decays: Form factors at $q^2 \neq 0$

Calculation of $f_0^{DK}(q^2)$ (using Ward identity method) and $f_+^{DK}(q^2)$ (using definition, needs renormalization) **HPQCD**, 1305.1462

* Global fit to available experimental data $\rightarrow$ extraction of $|V_{cs}|$ using all experimental q^2 bins.



$$\boxed{|V_{cs}| = 0.963(5)_{exp}(14)_{lat}}$$

1.5% error

3. K semileptonic decays:

$f_+^{K\pi}(0)$ and extraction of $|V_{us}|$

FNAL/MILC, 1212.4993

3.1. Introduction

The photon-inclusive decay rate for all $K \to \pi l \nu$ decay modes can be related to $|V_{us}|$ via

$$\Gamma_{K_{l3(\gamma)}} = \frac{G_F^2 M_K^5 C_K^2}{128\pi^3} S_{\rm EW} |V_{us} f_+^{K^0\pi^-}(0)|^2 I_{Kl}^{(0)} \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

with $C_K = 1(1/\sqrt{2})$ for neutral (charged) K, $S_{EW} = 1.0223(5)$, $I_{Kl}^{(0)}$ a phase integral depending on shape of $f_\pm^{K\pi}$, and $\delta_{\rm EM}^{Kl}$, $\delta_{\rm SU(2)}^{K\pi}$ are long-distance em and strong isospin corrections respectively

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Experimental average, Moulson, 1209.3426

$$|V_{us}| f_+(0)^{K\to\pi} = 0.2163(\pm 0.23\%)$$

$f_+(0)^{K\to\pi}$: 0.4% error

FNAL/MILC, 1212.4993

* Check unitarity in the first row of CKM matrix.

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008(6)$$

fits to K_{l3}, K_{l2} exper. data and lattice results for $f_+(0)^{K\to\pi}$ and f_K/f_π

$\to \mathcal{O}(11\ \text{TeV})$ bound on the scale of new physics **Cirigliano et al**, 0908.1754

3.1. Introduction

* Look for new physics effects in the comparison of $|V_{us}|$ from helicity suppressed $K_{\mu 2}$ versus helicity allowed K_{l3}

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+^{K\pi}(0)} \right) \times \mathrm{experim.\,data\,on}\, K_{\mu 2}\pi_{\mu 2}\,\mathrm{and}\, K_{l3}$$

  * In the SM $R_{\mu 23} = 1$. Not true for some BSM theories (for example, charged Higgs)

  * With **FNAL/MILC** inputs: $R_{\mu 23} = 1.005(7)$. Limited by lattice inputs

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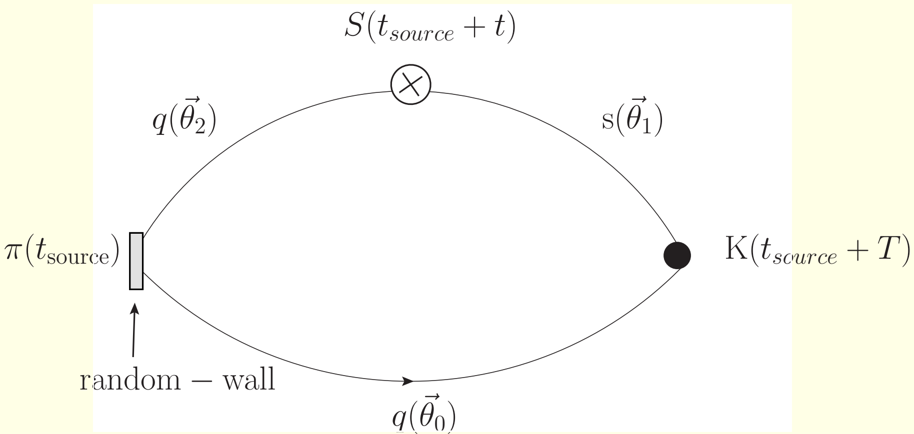
On the lattice: Calculate $f_+^{K^0\pi^-}$ (set mesons masses to phys. ones).

  * Follow HPQCD method developed for D semileptonic decays

$$f_+^{K\pi}(0) = f_0^{K\pi}(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2}\langle\pi|S|K\rangle_{q^2=0}$$

3.2. Methodology



* **Twisted boundary conditions** $\rightarrow$ allow generating correlation functions with non-zero external momentum such that $q^2 \simeq 0$ (or any other q^2)

Avoids extrapolation $q^2 \rightarrow 0$

Twisted boundary conditions: $\psi(x_k + L) = e^{i\theta_k}\psi(x_k)$
(with k a spatial direction and L the spatial length of the lattice).

$\rightarrow$ the propagator carries a momentum $p_k = \pi\frac{\theta_k}{L}$

* We inject momentum in either K (moving K data) or π (moving pion data).

3.3. Analysis on the asqtad $N_f = 2+1$ MILC ensembles

3.3.1 Simulation details

HISQ valence quarks on $N_f = 2 + 1$ Asqtad MILC configurations

(HISQ action has smaller a^2 errors, specially designed for `charm`)

$\approx a$ (fm)	am_l/am_s	`Volume`	N_{conf}	$N_{sources}$	N_T	$aM_{\pi,P}^{val}$
0.12	0.4	$20^3 \times 64$	2052	4	5	0.31315
	0.2	$20^3 \times 64$	2243	4	8	0.22587
	0.14	$20^3 \times 64$	2109	4	5	0.18907
	0.1	$24^3 \times 64$	2098	8	5	0.15657
0.09	0.4	$28^3 \times 96$	1996	4	5	0.20341
	0.2	$28^3 \times 96$	1946	4	5	0.14572

with N_T is the number of source-sink separations.

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(HISQ action has smaller a^2 errors, specially designed for `charm`)

$\approx a$ (fm)	am_l/am_s	`Volume`	N_{conf}	$N_{sources}$	N_T	$aM_{\pi,P}^{val}$
0.12	0.4	$20^3 \times 64$	2052	4	5	0.31315
	0.2	$20^3 \times 64$	2243	4	8	0.22587
	0.14	$20^3 \times 64$	2109	4	5	0.18907
	0.1	$24^3 \times 64$	2098	8	5	0.15657
0.09	0.4	$28^3 \times 96$	1996	4	5	0.20341
	0.2	$28^3 \times 96$	1946	4	5	0.14572

with N_T is the number of source-sink separations.

* Strange valence quark masses are tuned to their physical values

C.T.H. Davies et al, PRD81(2010)

3.3.1 Simulation details

HISQ valence quarks on $N_f = 2 + 1$ Asqtad MILC configurations

(HISQ action has smaller a^2 errors, specially designed for `charm`)

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with N_T is the number of source-sink separations.

* Strange valence quark masses are tuned to their physical values

C.T.H. Davies et al, PRD81(2010)

* Light valence quark masses: $\frac{m_l^{val}(HISQ)}{m_s^{phys}(HISQ)} = \frac{m_l^{sea}(Asqtad)}{m_s^{phys}(Asqtad)}$

3.3.2 Chiral and continuum extrapolation

The form factor $f_+(0)$ can be written in ChPT as

$$f_+(0) = 1 + f_2 + f_4 + f_6 + ... = 1 + f_2 + \Delta f$$

- # $f_+(0)$ goes to 1 in the $SU(3)$ limit due to vector current conservation
- # Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

3.3.2 Chiral and continuum extrapolation

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Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

* At finite lattice spacing systematic errors can enter due to violations of the dispersion relation needed to derive

$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2=0}$$

Dispersion relation violations in our data are $\leq 0.15\%$.

3.3.2 Chiral and continuum extrapolation

* One-loop (NLO) partially quenched `Staggered ChPT` +

  ** `Staggered ChPT`: logs are known non-analytical functions of $m_{K,\pi}$ containing dominant taste-breaking a^2 effects

  $\rightarrow$ remove the dominant light discretization errors (remain $a^2\alpha_s^2, a^4$)

$$f_+^{K\pi}(0) = 1 + f_2^{PQ\,,stag.}(a) + K_1^{(a)}\left(\frac{a}{r_1}\right)^2 +$$

3.3.2 Chiral and continuum extrapolation

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* Two-loop (NNLO) continuum ChPT by **Bijnens & Talavera**, arXiv:0303103.

$$f_+^{K\pi}(0) = 1 + f_2^{PQ,stag.}(a) + K_1^{(a)}\left(\frac{a}{r_1}\right)^2 + f_4^{cont.}(\mathrm{logs}) + f_4^{cont.}(L_i' s)$$
$$+ r_1^4(m_\pi^2 - m_K^2)^2\left[C_6'^{(1)} + K_2^a\left(\frac{a}{r_1}\right)^2\right]$$

where $C_6'^{(1)} \propto C_{12} + C_{34} - L_5^2$. L_5 is an $\mathcal{O}(p^4)$ LEC and $C_{12,34}$ are $\mathcal{O}(p^6)$ LECs

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* Free parameters of the fit: $C_6^{\prime(1)}$, $K_1^{(a)}$, $L_i's$ (priors equal to values in **Amoros et al**, 0101127, with enlarged errors), $\delta_A^{mix}, \delta_V^{mix}$ ($\mathcal{O}(a^2)$ SChPT param.)

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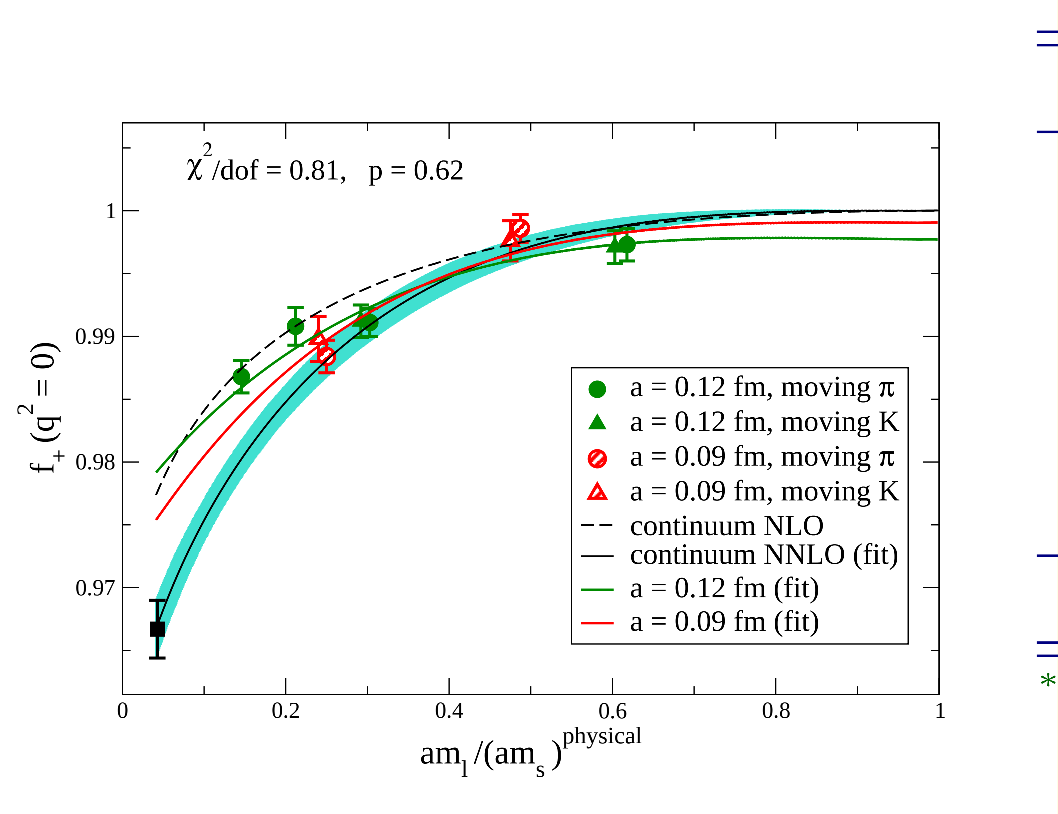
* Check: Use analytical parametrization for NNLO contribution

$\rightarrow$ central value changes by less than 0.2%

3.3.3 Results



Source of uncertainty	Error $f_+(0)$ (%)
Statistics	0.24
Chiral ext. & fitting*	0.3
Discretization	0.1
Scale	0.06
Finite volume	0.1
Total Error	0.42

* Difference between m_s^{sea} and m_s^{val} at two loops

$$\boxed{f_+(0) = 0.9667 \pm 0.0023 \pm 0.0033}$$

$$(C_{12}^r + C_{34}^r)(M_\rho) = (4.57 \pm 0.44 \pm 0.90) \cdot 10^{-6}$$

3.3.3 Results: Comparison with previous work and unitarity

this work	$0.9667(23)(33)$	$N_f = 2+1$
RBC/UKQCD 13	$0.9670(20)^{+(18)}_{-(46)}$	$N_f = 2+1$
RBC/UKQCD 10	$0.9599(34)\left(^{+31}_{-43}\right)$	$N_f = 2+1$
ETMC	$0.9560(57)(62)$	$N_f = 2$
Kastner & Neufeld	$0.986(8)$	ChPT
Cirigliano	$0.984(12)$	χPT
Jamin, Oller, & Pich	$0.974(11)$	ChPT
Bijnens & Talavera	$0.976(10)$	ChPT
Leutwyler & Roos	$0.961(8)$	Quark model

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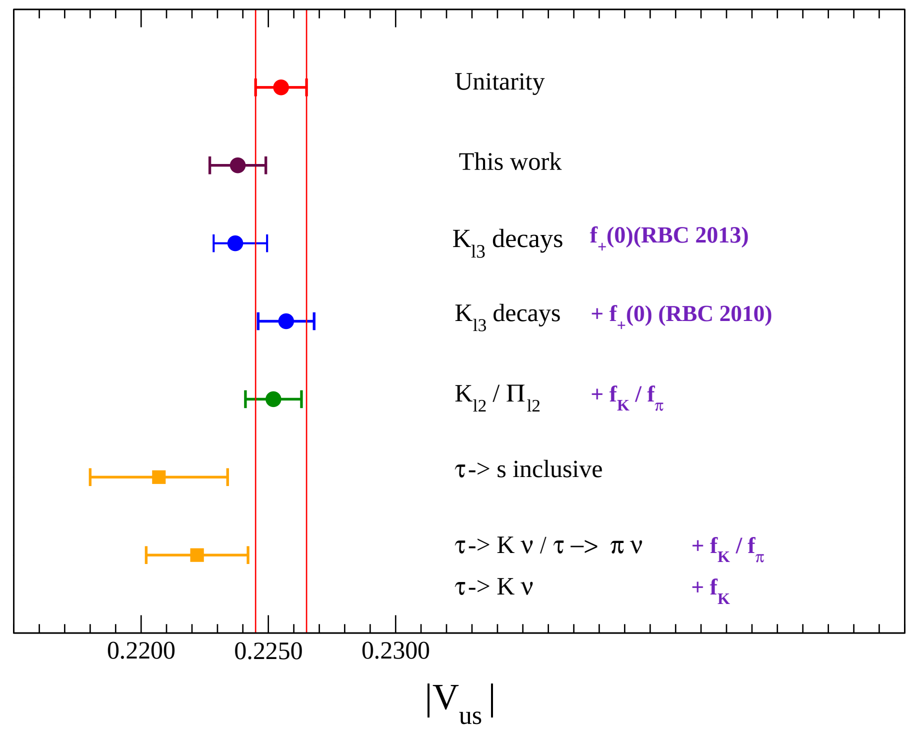


With this value of $f_+^{K\pi}(0)$ and latest experimental data ($|V_{us}|f_+(0) = 0.2163(5)$ **Moulson**, 1209.3426):

$$|V_{us}| = 0.2238 \pm 0.0009 \pm 0.0005$$

$$\rightarrow \Delta_{\mathrm{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008(6)$$

3.4. Analysis on the HISQ $N_f = 2 + 1 + 1$ MILC ensembles

3.4.1 Simulation parameters

$a(fm)$	m_l/m_s	Volume	$N_{conf.} \times N_{t_s}$	am_s^{sea}	am_s^{val}	
0.15	0.035	$32^3 \times 48$	1000×4	0.0647	0.0691	
0.12	0.200	$24^3 \times 64$	1053×8	0.0509	0.0535	
	0.100	$32^3 \times 64$	993×4	0.0507	0.053	
	0.100	$40^3 \times 64$	391×4	0.0507	0.053	FV check
	0.035	$48^3 \times 64$	945×8	0.0507	0.0531	
0.09	0.200	$32^3 \times 96$	775×4	0.037	0.038	
	0.100	$48^3 \times 96$	853×4	0.0363	0.038	
	0.035	$64^3 \times 96$	625×4	0.0363	0.0363	

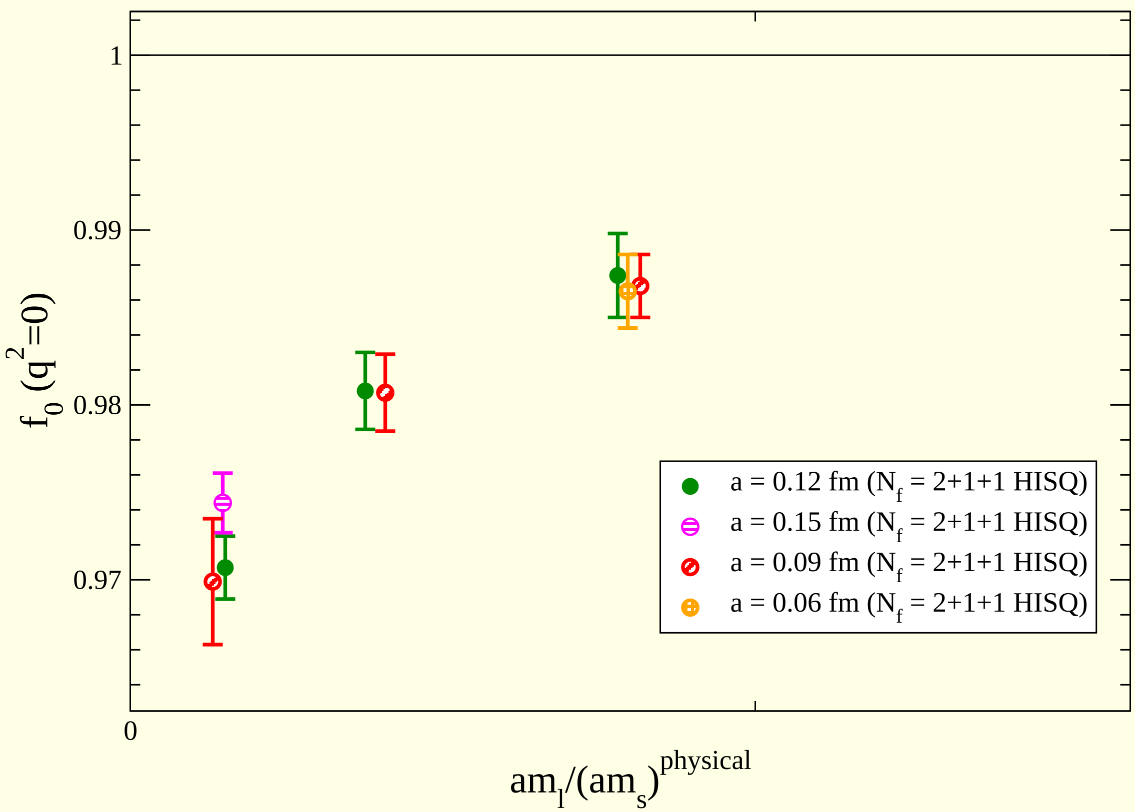
* Physical quark mass ensembles

* HISQ action on the sea: smaller discretization effects.

* Charm quarks on the sea.

* Better tuned strange quark mass on the sea.

3.4.2 Preliminary results

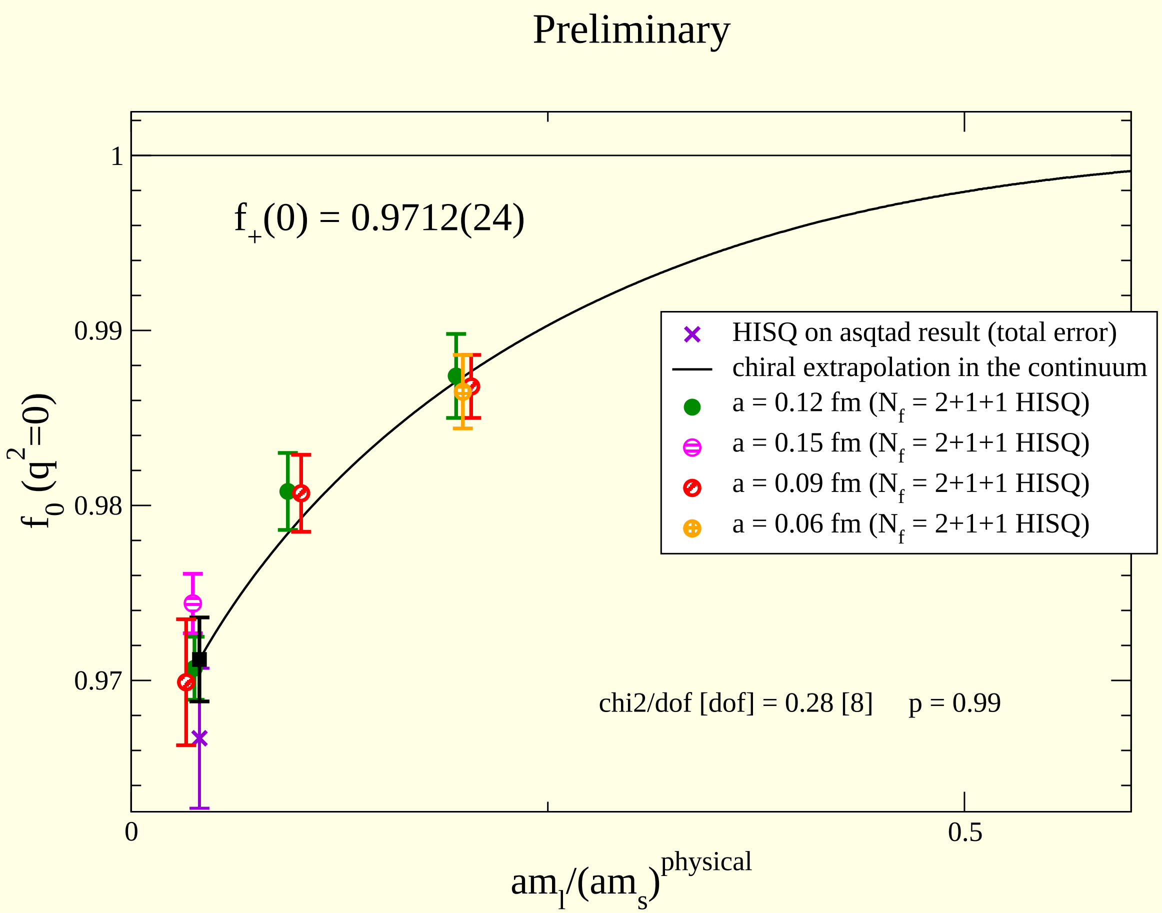


\# Statistical errors: 0.2-0.4%. Still larger than in the previous calculation (need more statistics).

\# We do not see discretization effects except in the $a \approx 0.15\ fm$ ensemble.

3.4.2 Preliminary results

Try the same chiral+continuum extrapolation strategy: one-loop partially quenched SChPT + two loops continuum ChPT.



Only statistical errors included in plot

In progress: Include finite volume corrections at one loop in the SChPT fit function, C. Bernard, J. Bijnens, E.G.

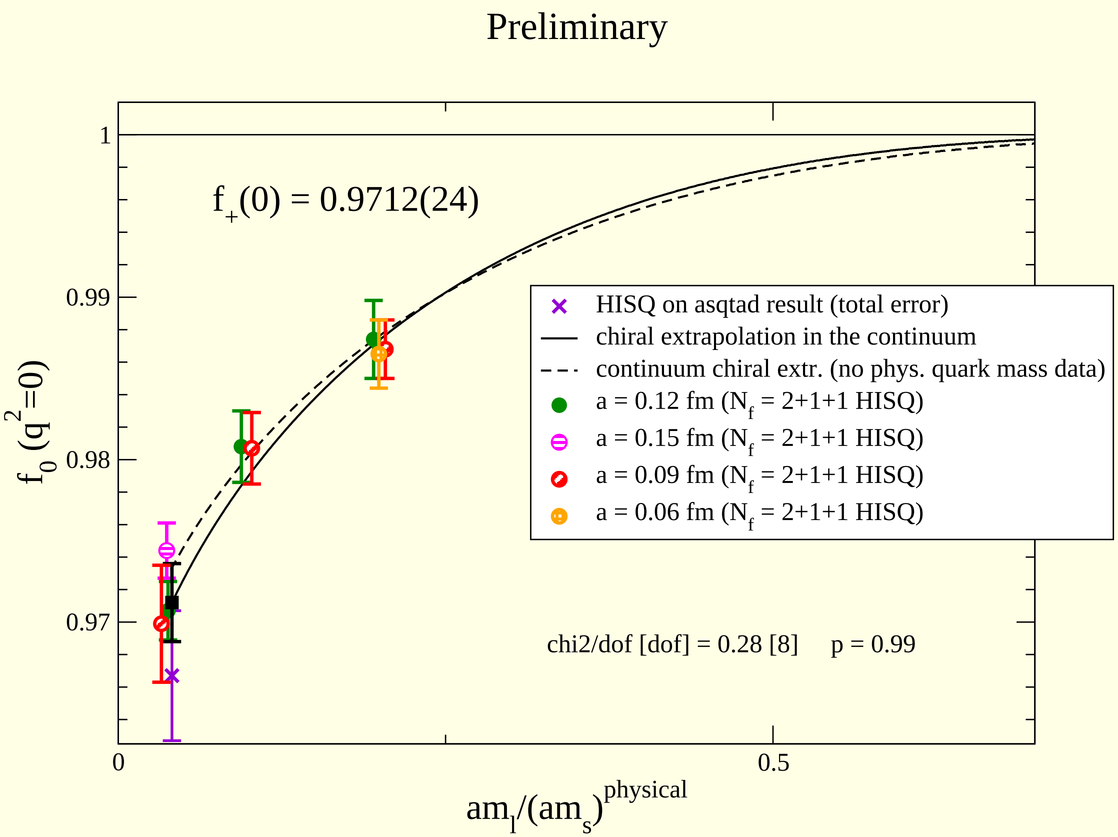
3.4.2 Preliminary results

Investigating the extrapolation strategy and systematic errors.

Some checks:

* Substituting two-loop ChPT by NNLO analytical param.: $\leq 0.15\%$ shift

* Non including physical quark mass ensembles in the chiral+cont. fit



$f_+(0) = 0.9734(30)$ stat. error only ...

4. Conclusions and outlook

State-of-the-art calculation of $f_+^{K\pi}(0)$:

$$f_+^{K\pi}(0) = 0.9667 \pm 0.0023 \pm 0.0033$$

(together with **RBC/UKQCD**, 1305.7217, $f_+^{K\pi}(0) = 0.9670 \pm 0.0020^{+18}_{-46}$)

* Keys of precision:

** $N_f = 2+1$ MILC ensembles (great statistics, variety of quark masses)

** HISQ action on the valence (small discretization error)

** one-loop `SChPT` + two-loop `ChPT` (controlled extrapolation to the continuum and physical point).

* With this value of $f_+^{K\pi}(0)$ and the latest experimental average for $|V_{us}|f_+^{K\pi}(0)$ we get:

$$|V_{us}| = 0.2238 \pm 0.0009_{lat.} \pm 0.0005_{exp.}$$

(1.5σ smaller than unitarity value)

** Form factor error still dominates the determination of $|V_{us}|$.

4. Conclusions and outlook

\# Working on a new determination to try to reduce previous dominant sources of error using MILC HISQ $N_f = 2+1+1$ ensembles

* Physical light quark masses: Reduce chiral extrapolation error.

* HISQ action on the sea: Smaller discretization errors.

* Better tunning of sea quark masses: Reduce chiral extrapolation error.

* Include sea charm quark effects

Very preliminary error budget

Source of uncertainty	Error $f_+(0)$ (%)
Statistics	$0.2 - 0.3$
Chiral ext. & fitting	≤ 0.15
Discretization	≤ 0.1
Scale	0.06
Finite volume	≤ 0.1
Total Error	0.3-0.37

4. Conclusions and outlook

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Total Error	0.3-0.37

* This is just the first calculation on the HISQ ensembles. We can improve in: statistics, discretization errors (smaller lattice spacings), finite volume uncertainty (ChPT calculation) ...

Goal: match experimental error 0.23%

4. Conclusions and outlook

Very preliminary error budget

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* This is just the first calculation on the HISQ ensembles. We can improve in: statistics, discretization errors (smaller lattice spacings), finite volume uncertainty (ChPT calculation) ...

Goal: match experimental error 0.23%

Study chiral behaviour of the vector and scalar form factors (at $q^2 = 0$ and $q^2 \neq 0$).

2.2.1. $K \to \pi\pi$ and $\varepsilon'_K/\varepsilon_K$

Going beyond gold-plated quantities.

$\Delta I = 3/2$ contribution:

* **RBC**: First quantitative results at the 20% level from a direct calculation at a small pion mass.

arXiv:1111.1699,1111.4889

* **Laiho and Van de Water**: New method developed based on combining ChPT (indirect) and direct methods.

arXiv:1011.4524

$\Delta I = 1/2$ contribution: * **RBC**: First calculation using the direct method on small volume
and large pion mass with a 25%. Feasibility study.

arXiv:1111.1699

asqtad and HISQ data