Three Types of Dark Matter and Their Unification

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Dark Matter from Supersymmetry

Physics Beyond the Standard Model (SM) should include neutrino mass and dark matter (DM).
Independent of the former, the best known candidate for DM is in the context of supersymmetry (SUSY). In the MSSM, the lightest neutral particle having odd R parity is a DM candidate. It is usually assumed to be a fermion, i.e. the lightest neutralino. The lightest neutral boson, presumably a scalar neutrino, is excluded by direct search experiments because the elastic cross section for $\tilde{\nu} q \to \tilde{\nu} q$ via Z exchange is too big by 9 to 10 orders of magnitude.

For many years, the lightest neutralino as DM dominated the thinking in this field. At present, faced with the absence of any hint of SUSY from the LHC and the increasing narrowing of the parameter space for the simplest version of SUSY DM, we should consider two possibilities:
(1) the SUSY breaking scale is higher than expected, and
(2) there may be more than just one type of DM.

The generic idea of multipartite dark matter was first considered by Cao/Ma/Wudka/Yuan(2007). There were then only two specific models, but now there are many.

For example, if a second scalar doublet (η^+, η^0) is added to the SM, and is assigned odd under an exactly conserved Z_2 [Deshpande/Ma(1978)] with all SM particles even, a viable DM scenario may be realized.

(η^+, η^0) differs from the scalar MSSM $(\tilde{\nu}, \tilde{l})$ doublet, because η_R^0 and η_I^0 are split in mass by the Z_2 conserving term $(\lambda_5/2)(\Phi^\dagger \eta)^2 + H.c.$ which is absent in the MSSM.

Since $(\eta^0)^* \partial_\mu \eta^0 - \eta^0 \partial_\mu (\eta^0)^* = i(\eta_R^0 \partial_\mu \eta_I^0 - \eta_I^0 \partial_\mu \eta_R^0)$, the interaction $\eta_R^0 q \to \eta_I^0 q$ via Z exchange is forbidden by phase space if η_I^0 is heavier than η_R^0 by about 1 MeV.

Dark Matter from Radiative Neutrino Mass

The motivation for adding the (η^+, η^0) doublet for DM was to connect it with neutrino mass, which first appeared in [Ma, Phys. Rev. D 73, 077301 (2006)]. Let three neutral fermion singlets N_i odd under Z_2 be added to the SM, then the interaction $(\nu\eta^0 - l\eta^+)N$ is allowed but not $(\nu\phi^0 - l\phi^+)N$. Thus N interacts with ν, but they are not Dirac mass partners. Note that the same $(\lambda_5/2)(\Phi^\dagger\eta)^2$ term which splits η^0_R and η^0_I is essential for a nonzero radiative Majorana neutrino mass. This is the so-called scotogenic model of neutrino mass.

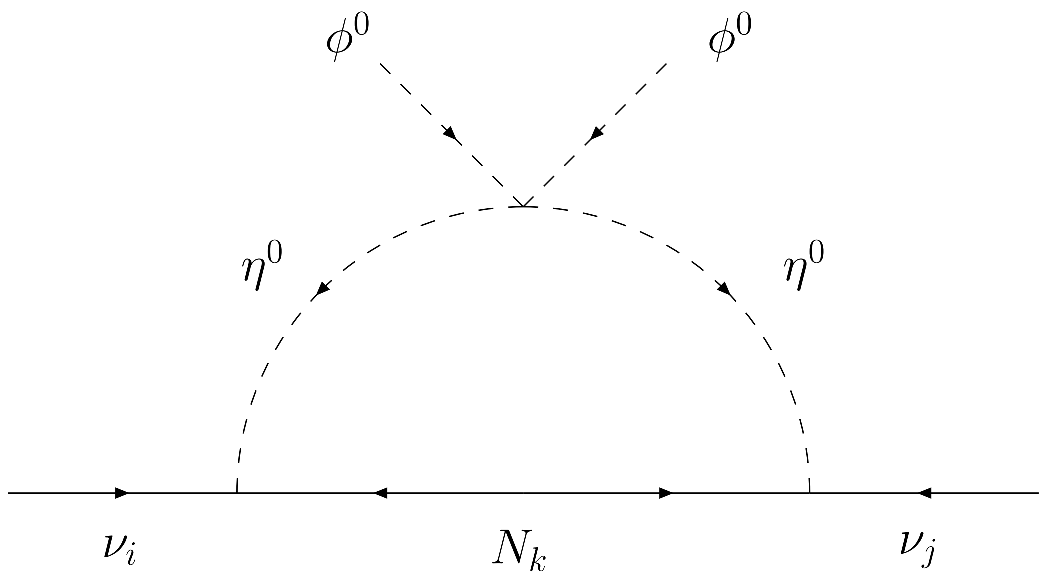

Figure 1: One-loop generation of neutrino mass with Z_2 dark matter.

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)],$$

where $f(x) = -\ln x/(1-x)$.

Let $m_R^2 - m_I^2 = 2\lambda_5 v^2 << m_0^2 = (m_R^2 + m_I^2)/2$, then

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} I(M_i^2/m_0^2),$$

$$I(x) = \frac{\lambda_5 v^2}{8\pi^2} \left(\frac{x}{1-x} \right) \left[1 + \frac{x \ln x}{1-x} \right].$$

For $x_i >> 1$, i.e. N_i very heavy,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_5 v^2}{8\pi^2} \sum_i \frac{h_{\alpha i} h_{\beta i}}{M_i} [\ln x_i - 1]$$

instead of the canonical seesaw $v^2 \sum_i h_{\alpha i} h_{\beta i}/M_i$.

In leptogenesis, the lightest M_i may then be much below the Davidson-Ibarra bound of about 10^9 GeV, thus avoiding a potential conflict of gravitino overproduction and thermal leptogenesis if SUSY is considered.

Ma(2006): The SUSY extension of the scotogenic model implies at least 2 coexisting dark-matter particles.

Dark Matter from Left-Right Symmetry

If the SM is extended to accommodate
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$,
then the conventional assignment of
$(\nu, l)_L \sim (1,2,1,-1/2), \; (\nu, l)_R \sim (1,1,2,-1/2)$,
$(u, d)_L \sim (3,2,1,1/6), \; (u, d)_R \sim (3,1,2,1/6)$,
shows that $X = (B-L)/2$ and $Y = T_{3R} + (B-L)/2$.
There must then be Higgs bidoublets:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \; \tilde{\Phi} = \begin{pmatrix} \bar{\phi}_2^0 & -\phi_1^+ \\ -\phi_2^- & \bar{\phi}_1^0 \end{pmatrix} \sim (1,2,2,0),$$

with Dirac fermion mass terms $m_l = f_l\langle\phi_2^0\rangle + f_l'\langle\bar{\phi}_1^0\rangle$ and $m_\nu = f_l\langle\phi_1^0\rangle + f_l'\langle\bar{\phi}_2^0\rangle$, and similarly in the quark sector, resulting in the appearance of undesirable tree-level flavor-changing neutral currents, as well as $W_L - W_R$ mixing. If supersymmetry is imposed, then $\tilde{\Phi}$ can be eliminated, but then $(m_\nu)_{ij} \propto (m_l)_{ij}$ as well as $(m_u)_{ij} \propto (m_d)_{ij}$. Hence the prevalent thinking is that $SU(2)_R \times U(1)_{B-L}$ is actually broken down to $U(1)_Y$ at a very high scale from an $SU(2)_R$ Higgs triplet $(\Delta^{++}, \Delta^{+}, \Delta^{0}) \sim (1,1,3,1)$ which provides ν_R at the same time with a large Majorana mass from $\langle\Delta^0\rangle$.

The Type I seesaw mechanism is thus implemented and everyone should be happy. But wait, no remnant of the $SU(2)_R$ gauge symmetry is detectable at the TeV scale and we will not know if ν_R really exists.

Is there a natural way to lower the $SU(2)_R \times U(1)_{B-L}$ breaking scale?

The answer was already provided 26 years ago!
Ma, Phys. Rev. D 36, 274 (1987):
In the superstring-inspired supersymmetric E_6 model, the $\underline{27}$ is decomposed under $[(SO(10), SU(5)]$ as
$(16, 10) + (16, 5^*) + (16, 1) + (10, 5) + (10, 5^*) + (1, 1)$.

Under its maximum subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$, the $\underline{27}$ of E_6 is given by

$$\begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} + \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & n^c \end{pmatrix} + \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}.$$

There are then two left-right options:

Option (A) is to go from E_6 to the conventional $SO(10)$, using $(16, 10) + (16, 5^*) + (16, 1)$, which then leads to the left-right model which everybody knows.

Option (B) switches $(10, 5^*)$ with $(16, 5^*)$ and $(1, 1)$ with $(16, 1)$, i.e. the first and third rows of $(3^*, 1, 3)$ and the first and third columns of $(1, 3, 3^*)$. Thus $(\nu, e)_R$ becomes $(n, e)_R$ and n_R is NOT the mass partner of ν_L. This is referred to in the literature as ALRM. Here the usual left-handed doublet is part of a bidoublet

$$\begin{pmatrix} \nu & E^c \\ e & N^c \end{pmatrix}_L \sim (1, 2, 2, 0).$$

However it was not realized that n_R would be a good dark-matter candidate.

Simpler nonsupersymmetric versions of the ALRM with n_R as dark matter have now been proposed.
Khalil/Lee/Ma(2009,2010): DLRM I, DLRM II

Consider $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times S$, with $Q = T_{3L} + T_{3R} + X$, where $X = (B - L)/2$ for the known quarks and leptons, and S is chosen so that $L = S - T_{3R}$ (DLRM I) or $L = S + T_{3R}$ (DLRM II) is conserved.

The resulting dark-matter fermion n_R has $L = 0$ (Majorana) in DLRM I and $L = 2$ (Dirac) in (DLRM II).

Particle Content of DLRM II

$$\psi_L = (\nu, e)_L \sim (1,2,1,-1/2;1),\ \nu_R \sim (1,1,1,0;1),$$
$$\psi_R = (n, e)_R \sim (1,1,2,-1/2;3/2),\ n_L \sim (1,1,1,0;2),$$

$$Q_L = (u, d)_L \sim (3,2,1,1/6;0),\ d_R \sim (3,1,1,-1/3;0),$$
$$Q_R = (u, h)_R \sim (3,1,2,1/6;-1/2),$$
$$h_L \sim (3,1,1,-1/3;-1),$$

$$\Phi \sim (1,2,2,0;-1/2),\ \tilde{\Phi} \sim (1,2,2,0;1/2),$$
$$\Phi_L = (\phi_L^+, \phi_L^0) \sim (1,2,1,1/2;0),$$
$$\Phi_R = (\phi_R^+, \phi_R^0) \sim (1,1,2,1/2;1/2).$$

Allowed Yukawa terms:

$\bar{\psi}_L \Phi \psi_R$, $\bar{\psi}_L \tilde{\Phi}_L \nu_R$, $\bar{\psi}_R \tilde{\Phi}_R n_L$, $\bar{Q}_L \tilde{\Phi} Q_R$, $\bar{Q}_L \Phi_L d_R$, $\bar{Q}_R \Phi_R h_L$.

Forbidden Yukawa terms:

$\bar{\psi}_L \tilde{\Phi} \psi_R$, $\bar{n}_L \nu_R$, $\bar{Q}_L \Phi Q_R$, $\bar{h}_L d_R$.

Hence m_e, m_u come from $v_2 = \langle \phi_2^0 \rangle$; m_ν, m_d from $v_3 = \langle \phi_L^0 \rangle$; and m_n, m_h from $v_4 = \langle \phi_R^0 \rangle$.

Note that $\langle \phi_1^0 \rangle = 0$ because it has $L = -1$, and so do W_R^-, ϕ_R^-, ϕ_1^-, and h.

This structure guarantees the absence of tree-level flavor changing neutral currents.

Let $e/g_L = s_L = \sin\theta_W$, $e/g_R = s_R$,
$e/g_X = \sqrt{1 - s_L^2 - s_R^2} = \sqrt{c_L^2 - s_R^2}$, then

$$A = s_L W_L^0 + s_R W_R^0 + \sqrt{c_L^2 - s_R^2} X,$$

$$Z = c_L W_L^0 - (s_L s_R / c_L) W_R^0 - (s_L \sqrt{c_L^2 - s_R^2} / c_L) X,$$

$$Z' = (\sqrt{c_L^2 - s_R^2} / c_L) W_R^0 - (s_R / c_L) X.$$

$$g_Z = e / s_L c_L, \quad J_Z = J_{3L} - s_L^2 J_{em},$$

$$g_{Z'} = e / s_R c_L \sqrt{c_L^2 - s_R^2}, \quad J_{Z'} = s_R^2 J_{3L} + c_L^2 J_{3R} - s_R^2 J_{em}.$$

No $Z - Z'$ mixing implies $v_2^2/(v_2^2 + v_3^2) = s_R^2/c_L^2$.

$$M_{W_R} \simeq (\sqrt{c_L^2 - s_R^2} / c_L) M_{Z'}.$$

Distinguishing this Z' from others: [Godfrey/Martin, PRL 101, 151803 (2008)]

$$\frac{\Gamma(Z' \to t\bar{t})}{\Gamma(Z' \to \mu^-\mu^+)} = \frac{(9 - 24r + 17r^2)}{3(1 - 4r + 5r^2)} = 4.44 \ (g_L = g_R),$$

$$\frac{\Gamma(Z' \to b\bar{b})}{\Gamma(Z' \to \mu^-\mu^+)} = \frac{5r^2}{3(1 - 4r + 5r^2)} = 0.60 \ (g_L = g_R),$$

where $r = s_R^2/c_L^2$. In the conventional LR model, change the numerator for $b\bar{b}$ to $(9 - 12r + 8r^2)$, i.e. 13.6 bigger $(g_L = g_R)$. In the ALRM, change the denominator for both to $3(2 - 6r + 5r^2)$, i.e. 2.6 bigger $(g_L = g_R)$.

Unification of All Three

Ma(2010): DLRMII may be supersymmetrized, but would not lead to gauge-coupling unification, unless there are additional superfields.

Bhattacharya/Ma/Wegman(2013):

Superfield	$SU(3)$ $SU(2)$ $SU(2)$ $U(1)$	S	M	H
$\psi = (\nu, e)$	$(1, 2, 1, -1/2)$	0	–	$+$
$\psi^c = (e^c, n^c)$	$(1, 1, 2, 1/2)$	–1/2	–	$+$
n	(1,1,1,0)	1	–	$+$
N	(1,1,1,0)	0	–	$-$

Superfield	$SU(3)\ SU(2)\ SU(2)\ U(1)$	S	M	H
$Q = (u, d)$	(3,2,1,1/6)	0	–	+
$Q^c = (h^c, u^c)$	$(3^*, 1, 2, -1/6)$	1/2	–	+
d^c	$(3^*, 1, 1, 1/3)$	0	–	+
h	(3,1,1,–1/3)	–1	–	+
Δ_1	(1,2,2,0)	1/2	+	+
Δ_2	(1,2,2,0)	–1/2	+	+
Φ_{L1}	(1,2,1,–1/2)	0	+	+
Φ_{L2}	(1,2,1,1/2)	0	+	+
Φ_{R1}	(1,1,2,–1/2)	–1/2	+	+
Φ_{R2}	(1,1,2,1/2)	1/2	+	+

Superfield	$SU(3)\ SU(2)\ SU(2)\ U(1)$	S	M	H
η_{L1}	(1,2,1,–1/2)	0	$+$	$-$
η_{L2}	(1,2,1,1/2)	0	$+$	$-$
η_{R1}	(1,1,2,–1/2)	–1/2	$+$	$-$
η_{R2}	(1,1,2,1/2)	1/2	$+$	$-$
ζ_1	(1,1,1,–1)	0	$+$	$-$
ζ_2	(1,1,1,1)	0	$+$	$-$
ζ_3	(1,1,1,0)	0	$+$	$-$

The symmetry $S \times M \times H$ is used to distinguish $(\psi, \Phi_{L1}, \eta_{L1})$, $(\psi^c, \Phi_{R2}, \eta_{R2})$, and (N, n, η_3).

Gauge coupling unification:

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1},$$

where $\alpha_i = g_i^2/4\pi$ and b_i are determined by the particle content between M_1 and M_2. Between M_Z and M_S (the SUSY scale), we have

$SU(3)_C: \; b_C = -11 + (4/3)N_f = -7,$

$SU(2)_L: \; b_L = -22/3 + (4/3)N_f + 2(1/6) = -3,$

$U(1)_Y: \; (3/5)b_Y = (4/3)N_f + (3/5)2(1/6) = 21/5,$

where 2 Higgs doublets have been assumed.

For the MSSM, between M_S and M_U, we then have
$SU(3)_C: \; b_C = -9 + (2)N_f = -3$,
$SU(2)_L: \; b_L = -6 + (2)N_f + (1)(2)(1/2) = 1$,
$U(1)_Y: \; (3/5)b_Y = (2)N_f + (3/5)(1)(4)(1/4) = 33/5$.

Using $\alpha_C(M_U) = \alpha_L(M_U) = (5/3)\alpha_Y(M_U) = \alpha_U$, we have the constraints

$$\ln \frac{M_U}{M_Z} = \frac{\pi}{2}\left(\frac{1}{\alpha_L(M_Z)} - \frac{1}{\alpha_C(M_Z)}\right),$$

$$\ln \frac{M_S}{M_Z} = \frac{\pi}{4}\left(\frac{3}{\alpha_Y(M_Z)} - \frac{12}{\alpha_L(M_Z)} + \frac{7}{\alpha_C(M_Z)}\right).$$

In this model, the boundary condition at M_R is

$$\frac{1}{\alpha_Y(M_R)} = \frac{1}{\alpha_{L,R}(M_R)} + \frac{1}{\alpha_X(M_R)}.$$

Assuming that $M_S = M_R$, we then have
$b_C = -9 + (2+1)N_f = 0$,
$b_{L,R} = -6 + (2)N_f + (1)(8)(1/2) = 4$,
$(3/2)b_X = (2+1)N_f + (1+1/2)[(16)(1/4) + (2)(1)] = 18$.

Remarkably, the resulting solutions for M_U/M_Z and M_S/M_Z are exactly as in the MSSM.

Actually, the one-loop MSSM RG equations are not quite consistent with precision electroweak data. The usual way out is to use two-loop equations plus threshold corrections. Suppose there exists another much larger scale m_X, such that the singlet $\zeta_{1,2,3}$ fields become massive, then the above equation for M_S/M_Z is the same if M_S/M_Z is replaced by $M_R^7/M_X^3 M_Z^4$. Now the one-loop equations are fine if $M_R^{7/4} M_X^{-3/4} \simeq 14.7$ GeV. From what the LHC has not seen so far, we set $M_R = 1$ TeV, then $M_X \simeq 280$ TeV. As a result, the $\zeta_{1,2,3}$ interactions may be ignored in studies of dark matter.

Illustrative Example

As an example, we assume the following three coexisting stable particles in ascending order of mass:
(1) the lightest neutralino $\tilde{\chi}^0_1$ $(S' = 0, H = +, R = -)$,
(2) the lightest scotino n_1 $(S' = 1, H = +, R = +)$, and
(3) the exotic $\tilde{\eta}^0_R$ fermion $(S' = 1, H = -, R = +)$,
where $R = MH(-1)^{2j}$ is the usual R parity.

Suppose n_1 annihilates through Z' to only SM particles, and $\tilde{\eta}^0_R$ does the same (plus of course to $n_1\bar{n}_1$), then $\langle \sigma v \rangle \simeq 2.59 \times 10^{-3} M^2/(4M^2 - m^2_{Z'})^2$ for both.

Since $\langle \sigma v \rangle$ is roughly inversely proportional to the relic abundance, we assume both to be about 3 pb. In this case, together they form about 2/3 of the dark matter of the Universe. Numerically, this may be achieved by $m(\tilde{\eta}_R^0) = 1.075$ TeV and $m(n_1) = 0.930$ TeV, assuming $m_{Z'} = 2$ TeV.

The remaining 1/3 of the dark-matter relic abundance may then be provided by the usual LSP of the MSSM. This allows it to be in a different region of parameter space, away from the part being squeezed by the collider data. More detailed numerical analysis is in progress.

Conclusion

The nature of dark matter is unknown, but the naive expectation that it is just one heavy particle may not be correct. There are three motivated origins of dark matter: from supersymmetry, from radiative neutrino masses, and from unconventional left-right gauge symmetry. It is shown how all three may be woven together into a model with gauge-coupling unification. A rich spectrum of new particles at the TeV scale may exist, with possible verification in the near future.