Mass modes and secondary massive quark radiation

Based on our talks presented at SCET2013

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Outline

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Use of SCET

- SCET: **S**oft **C**ollinear **E**ffective **T**heory (top-down EFT)
- disentangling QCD vs other interactions requires a separation of scales $\rightarrow$ natural framework for EFTs
- for energetic hadrons $E_{\mathrm{H}} \simeq Q \gg \Lambda_{\mathrm{QCD}} \simeq m_{\mathrm{H}}$
- for energetic jets $E_{\mathrm{J}} \simeq Q \gg m_{\mathrm{J}}$
- for (up to now) massless hard-soft-collinear interactions
- most of what can be done with SCET can be done with other techniques too. SCET can give new perspective on factorization, log-resummation, power corrections. . .

$B-$decays

$$B \to X_s\gamma, \quad B \to \pi\ell\nu, \quad B \to \pi\pi, \quad B \to D\pi \quad \ldots$$

QCD processes with large E transfer

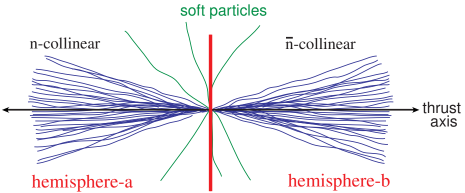
$$e^- p \to e^- X, \quad p\bar{p} \to X\ell^+\ell^-, \quad e^+e^- \to \text{jets} \quad \ldots$$

SCET construction

Bauer, Fleming, Pirjol, Stewart, 2001
Stewart: SCET lectures

1. determine relevant scales for particles in initial and final states $\Rightarrow$ expansion parameter λ is a ratio of such scales
2. construct momenta from these scales such that $p^2 \leq Q^2\lambda^2$ $\Rightarrow$ fields constructed from these sets of momenta
3. particles with $p^2 \gg Q^2\lambda^2$ are integrated out
4. power counting: assign to the SCET fields a scaling in λ
5. construct Lagrangian (ensuring SCET gauge invariance)

Degrees of freedom: $e^+e^- \to$ jets



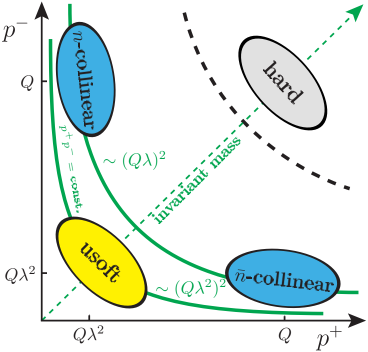
Relevant scales: $Q^2 \gg m_{\rm jet}^2 \gg \Lambda_{\rm QCD}^2 \Rightarrow \lambda \equiv m_{\rm jet}/Q$

Light-cone coordinates

$$n^\mu = (1,0,0,-1), \quad \bar{n}^\mu = (1,0,0,1), \quad n^2 = \bar{n}^2 = 0$$

$$p^+ = n \cdot p, \quad p^- = \bar{n} \cdot p, \quad p_\perp^i = (p_x, p_y), \quad p^\mu = p^+ \frac{\bar{n}^\mu}{2} + p^- \frac{n^\mu}{2} + p_\perp^\mu$$

- retain particles with $p^2 \leq Q^2\lambda^2 \Rightarrow$ collinear and ultrasoft
- jet constituents $p^\mu \simeq (m_{\rm jet}^2/Q, Q, m_{\rm jet}) = Q(\lambda^2, 1, \lambda)$
- ultrasoft particles $p^\mu \simeq (m_{\rm jet}^2/Q, m_{\rm jet}^2/Q, m_{\rm jet}^2/Q) = Q(\lambda^2, \lambda^2, \lambda^2)$ and $p^2 \simeq Q^2\lambda^4$
- any other with $p^2 \gg Q^2\lambda^2$ integrated out $\Rightarrow$ their dynamics present through matching coefficient $C(Q, \mu)$



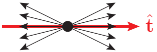
mode	$p^\mu = (+, -, \perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	–
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

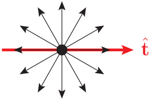
- a field for each mode, but several fields for each particle
- always integrate out all modes above given hyperbola

Outline

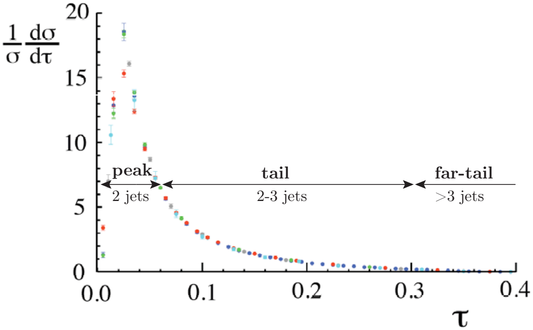
Thrust distribution

- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \in [0, \frac{1}{2}]$

back-to-back: $\tau \to 0$ 

isotropic: $\tau \to \frac{1}{2}$ 

- thrust distribution from LEP data ($e^+e^- \to jets$)



→ peak region ($\tau \sim \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\Lambda_{QCD}/Q}$
→ tail region ($\tau \gg \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\tau}$

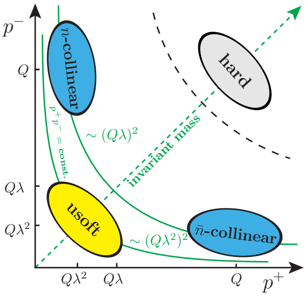
Factorization theorem for thrust

Fleming, Hoang, Mantry, Stewart (2007)
Bauer, Fleming, Lee, Sterman (2008)

SCET result for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau - \ell, \mu)$$

$Q \to$ CM energy, $\sigma_0 \to$ cross-section at LO



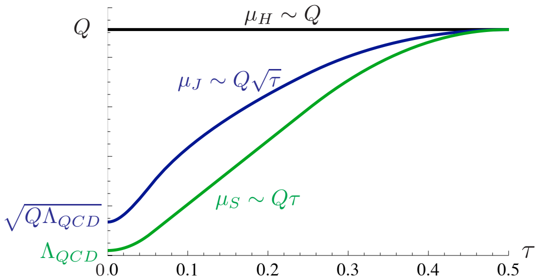
Ingredients:

- hard function: $H_0(Q,\mu) = |C_V(Q,\mu)|^2$
- thrust jet function: $J_0(s,\mu) = \int ds'\, J_n(s',\mu) J_{\bar{n}}(s-s',\mu)$
- thrust soft function: $S_0(\ell,\mu) \equiv \int dk_R\, dk_L\, \delta(\ell - k_R - k_L)\, S_0^{\text{hemi}}(k_R,k_L,\mu)$
  $\to \mu_S \sim Q\lambda^2 \sim \Lambda_{QCD}$: $S_0 = S_0^{\text{model}}$: non-perturbative model
  $\to \mu_S \sim Q\lambda^2 \gg \Lambda_{QCD}$: $S_0 = S_0^{\text{part}} \otimes S_0^{\text{model}}$: S_0^{part} partonic piece (perturbative)

Profile functions

Profile functions: Parametrization of renormalization scales in terms of thrust
$\rightarrow$ continuous transition between peak, tail and far-tail region
Abbate, Fickinger, Hoang, Mateu, Stewart (2011)

region		μ_H	μ_J	μ_S
peak	$\tau \sim \Lambda_{\text{QCD}}/Q$	Q	$\sqrt{Q\Lambda_{\text{QCD}}}$	Λ_{QCD}
tail	$\Lambda_{\text{QCD}}/Q \ll \tau \leq 1/3$	Q	$Q\sqrt{\tau}$	$Q\tau$
far-tail	$1/3 \leq \tau \leq 1/2$	Q	Q	Q



Outline

Mass effects in QCD

in QCD: e.g. DIS

- mass effects via different schemes (FFNS, VFNS, ...)
  $\rightarrow$ correct limiting behavior, continuous description?
- ACOT scheme (VFNS): heavy quark production (3 scales: Q, m, Λ_{QCD})
  Aivazis, Collins, Olness, Tung (1994)
  factorization theorem interpolating between the regions
  $\rightarrow m \gg Q$: full decoupling
  $\rightarrow m \sim Q$: exact kinematics, mass effects in Wilson coefficients (FFNS)
  $\rightarrow m \ll Q$: Log-resummation, massless kinematics, mass effects in pdf's

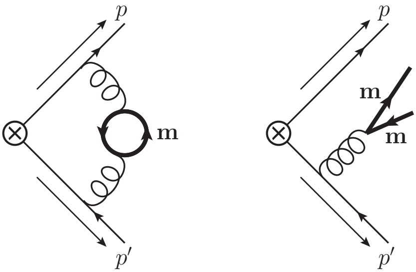
$\Rightarrow$ setup for additional scales? e.g. in endpoint region $x \rightarrow 1$: $Q^2(1-x)$

Mass effects in SCET

in SCET: event shapes for $e^+e^- \to jets$

- factorization and resummation for production of primary massive $t\bar{t}$-pairs
  Fleming, Hoang, Mantry, Stewart (2008)

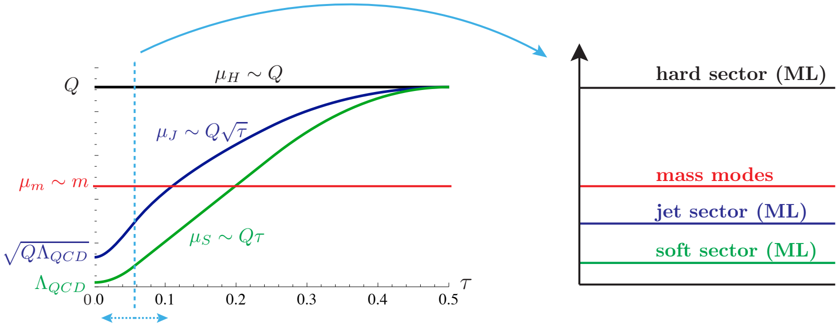
$\Rightarrow$ still missing: systematic treatment of secondary massive quarks
$\to$ nontrivial setup with dynamical thresholds



Scale hierarchies with a mass

introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

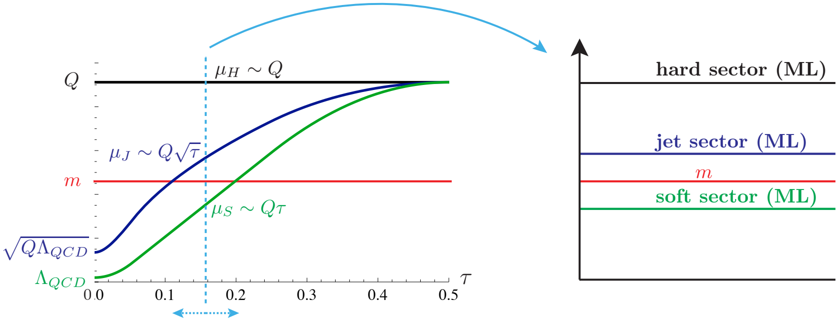
$$\mu_H > \mu_m > \mu_J > \mu_S$$



Scale hierarchies with a mass

introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

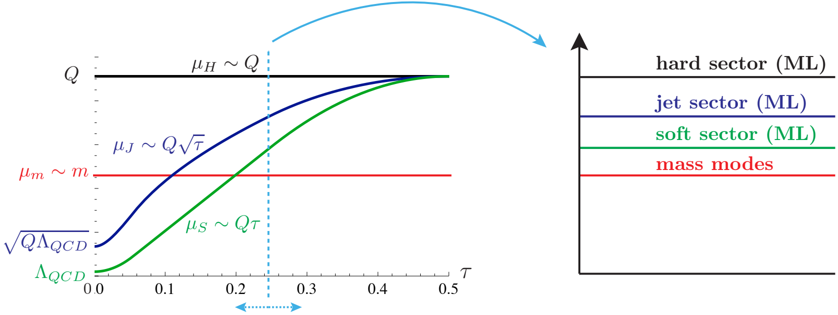
Scenario III: $\mu_H > \mu_J > \mu_m > \mu_S$



Scale hierarchies with a mass

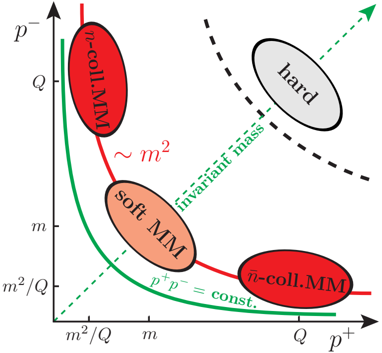
introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

Scenario IV: $\mu_H > \mu_J > \mu_S > \mu_m$



Mass modes

- new degrees of freedom: "mass modes"
- additional scaling parameter: $\lambda_m = m/Q$



mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2

$\Rightarrow$ How to include mass modes into massless setup?

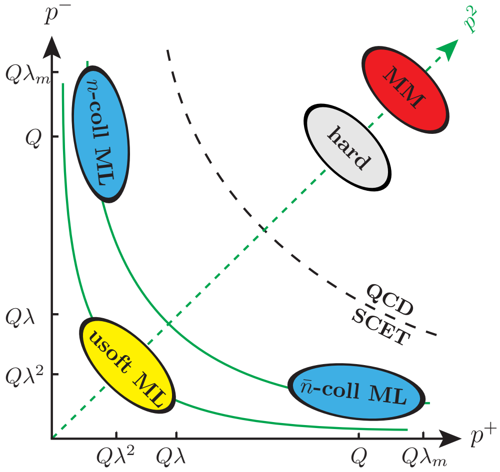
Mass mode setup

- construct a sequence of EFTs depending on $\lambda_m \leftrightarrow \lambda$

$$\begin{array}{c} \text{I. } \lambda_m > 1 > \lambda > \lambda^2 \\ \text{II. } 1 > \lambda_m > \lambda > \lambda^2 \\ \text{III. } 1 > \lambda > \lambda_m > \lambda^2 \\ \text{IV. } 1 > \lambda > \lambda^2 > \lambda_m \end{array}$$

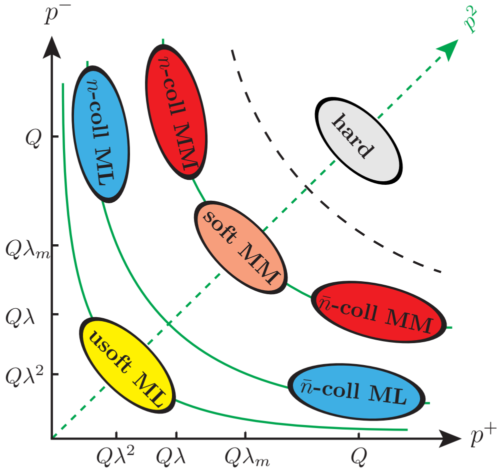
- aims:
  $\rightarrow$ continuity between scaling situations ("scenarios")
  $\rightarrow$ mass-independent UV divergences
  $\rightarrow$ decoupling for $m \rightarrow \infty$
  $\rightarrow$ correct IR-finite massless limit for $m \rightarrow 0$

I. $\lambda_m > 1 > \lambda > \lambda^2$



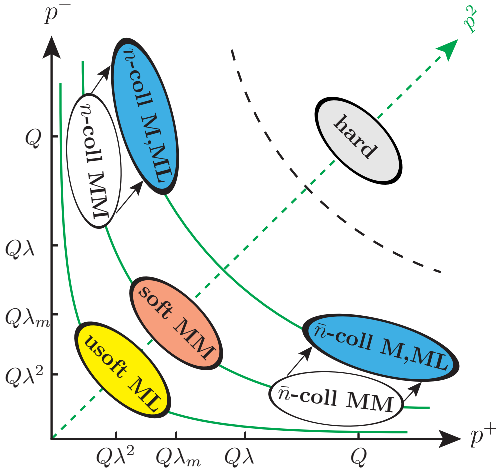
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

II. $1 > \lambda_m > \lambda > \lambda^2$



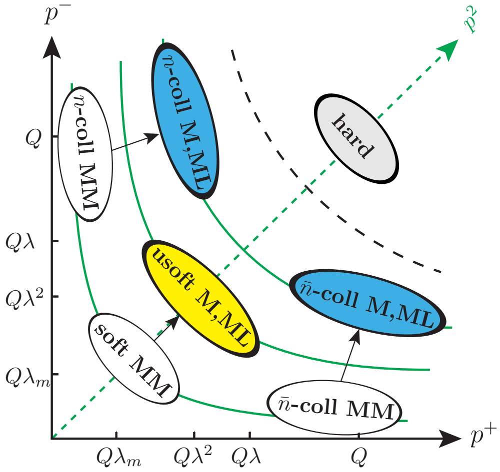
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

III. $1 > \lambda > \lambda_m > \lambda^2$



mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

IV. $1 > \lambda > \lambda^2 > \lambda_m$

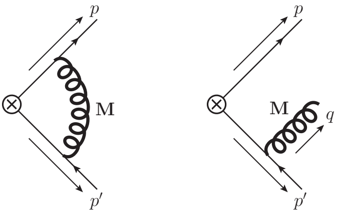


mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft M	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

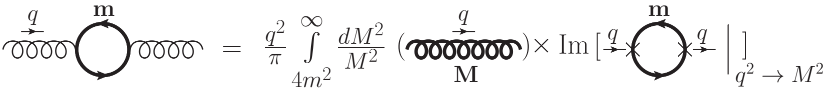
Outline

Motivation

Setup for massive gauge boson (mass M) with vector coupling
(group factors denoted as in $SU(3)$)

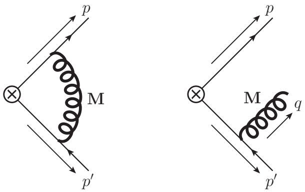


- dispersive technique:



- separate mass mode concept from technical issues at two-loop
- in principle applicable for EW effects, ...

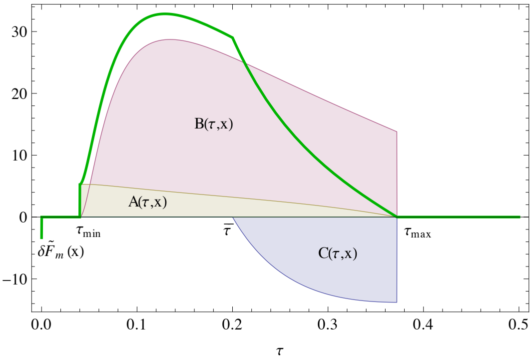
Full theory result



$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \frac{\alpha_s C_F}{4\pi}\Big\{\delta(\tau)\,\delta\tilde{F}_m^{\rm QCD}\left(\frac{M^2}{Q^2}\right) + \theta(\tau-\tau_{\rm min})\left[A\left(\tau,\frac{M^2}{Q^2}\right)+B\left(\tau,\frac{M^2}{Q^2}\right)\right] + \theta(\tau-\bar\tau)\,C\left(\tau,\frac{M^2}{Q^2}\right)\Big\}$$

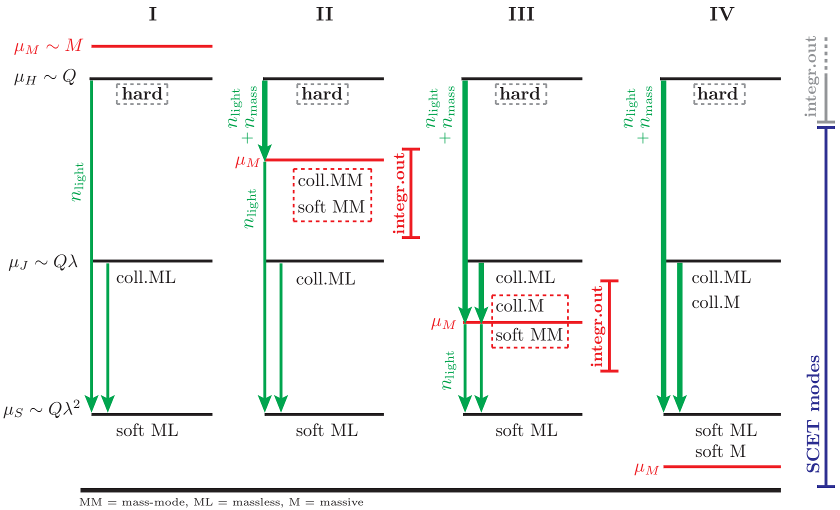
$\tau_{\rm min} = \frac{M^2}{Q^2} \rightarrow$ threshold for real jet radiation

$\bar\tau = \frac{M}{Q} \rightarrow$ threshold for real soft radiation

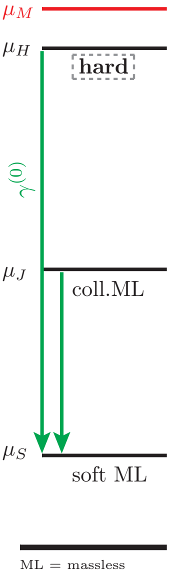


Mass mode setup: RG evolution

Top-down evolution: evolve to $\mu \sim \mu_S$



Scenario I: $\lambda_M > 1 > \lambda > \lambda^2$



integrate out mass modes at QCD level
$\rightarrow$ modification of hard matching coefficient,
otherwise like in massless SCET

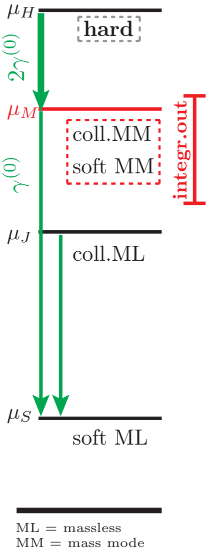
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^I(\mu_H)\right|^2 U_H^{(0)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0(Q\tau - \ell, \mu_S)$$

$U_H^{(0)}, U_J^{(0)}$: massless evolution factors

$$\mathcal{C}^I(\mu_H) = C_0(\mu_H) + \delta F_m^{\text{QCD}}$$

C_0: massless matching coefficient
δF_m^{QCD}: massive full theory contribution (OS)
$\rightarrow$ decoupling for $M/Q \rightarrow \infty$

Scenario II : $1 > \lambda_M > \lambda > \lambda^2$



mass modes enter SCET, but integrated out before the jet scale
→ modification of the matching coefficient at μ_H
→ additional matching contribution at μ_M
→ massless jet & soft function

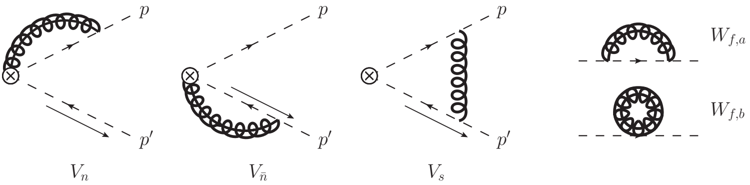
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_M) \left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M, \mu_S) \times \int d\ell \int ds\, J_0(s, \mu_J)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0(Q\tau - \ell, \mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H)$$

δF_m^{eff}: massive SCET contribution

Hard function contribution δF_m^{eff}



$$\delta F_m^{\text{eff}}(Q,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\ln\left(\frac{M^2}{\mu^2}\right)\left[2\ln\left(\frac{-Q^2}{\mu^2}\right) - \ln\left(\frac{M^2}{\mu^2}\right) - 3\right] - \frac{5\pi^2}{6} + \frac{9}{2}\right\}$$

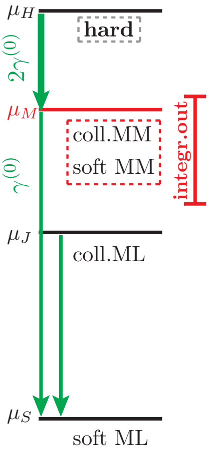
Chiu, Golf, Kelley, Manohar (2008)
Chiu, Fuhrer, Hoang, Kelley, Manohar (2009)

double counting of mass mode effects!
$\rightarrow$ subtraction of collinear diagrams with soft scaling necessary = soft-bin subtraction
$\rightarrow$ allows to obtain regulator-independent, gauge-invariant result

correct massless limit for matching coefficient:

$$\mathcal{C}^{II}(Q,M,\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H) \xrightarrow{M\to 0} 2C_0(Q,\mu_H)$$

Scenario II : $1 > \lambda_M > \lambda > \lambda^2$



ML = massless
MM = mass mode

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_M) \left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M, \mu_S)$$
$$\times \int d\ell \int ds\, J_0(s, \mu_J)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0(Q\tau - \ell, \mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H)$$

$$\mathcal{M}_H(\mu_M) = 1 + \delta F_m^{\text{eff}}(\mu_M)$$

δF_m^{eff}: massive SCET contribution

continuity to scenario I for $\mu_M = \mu_H$:

$$\left|\mathcal{C}^{II}(\mu_H)\right|^2 \left|\mathcal{M}_H(\mu_H)\right|^2 = \left|\mathcal{C}^{I}(\mu_H)\right|^2$$

Scenario II: Matching to full theory

- expansion of the most singular terms, i.e. for $\tau \sim M^2/Q^2 \ll 1$ and $\tau \sim M/Q \ll 1$

$$\frac{1}{\sigma_0}\,\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\text{FO}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M^2}{Q^2}\right)\right]\,\delta(\tau) + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\text{real}}(Q^2\tau) + Q\,\theta\left(\tau - \frac{M}{Q}\right)\delta S_m^{\text{real}}(Q\tau)$$

Scenario II: Matching to full theory

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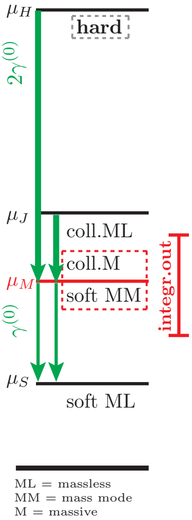
$$\frac{1}{\sigma_0}\,\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\text{FO}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M^2}{Q^2}\right)\right]\,\delta(\tau)$$
$$+\,Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\text{real}}(Q^2\tau) + Q\,\theta\left(\tau - \frac{M}{Q}\right)\delta S_m^{\text{real}}(Q\tau)$$

- matching with SCET result at fixed order gives mass matching functions in scenario II, $\tau < M^2/Q^2$:

$$\frac{1}{\sigma_0}\,\frac{d\sigma^{\text{SCET}}}{d\tau}\bigg|^{\text{II}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M}{Q}\right)\right]\delta(\tau)$$
$$+\,2\,\text{Re}\left[\delta F_m^{\text{eff}}(Q,M,\mu) + \mathcal{M}_H^{(1)}(Q,M,\mu)\right]\delta(\tau)$$

$$\rightarrow \text{Re}\left[\mathcal{M}_H^{(1)}(Q,M,\mu)\right] = -\text{Re}\left[\delta F_m^{\text{eff}}(Q,M,\mu)\right]$$

Scenario III: $1 > \lambda > \lambda_M > \lambda^2$



massive and massless collinear modes fluctuate over comparable scales ($\lambda_M \leq \lambda$)

$\rightarrow$ assign collinear massless scaling (keep $M \neq 0$)

$\rightarrow$ modification of the jet function at μ_J

$\rightarrow$ additional jet matching contribution at μ_M

$\rightarrow$ massless soft function

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_m) \left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M, \mu_S) \times \int d\ell \int ds \int ds' \int ds'' J_{0+m}(s, \mu_J)\, U_J^{(1)}(s'-s, \mu_M, \mu_J) \times \mathcal{M}_J(s''-s', \mu_M)\, U_J^{(0)}(s''-Q\ell, \mu_S, \mu_M)\, S_0(Q\tau - \ell, \mu_S)$$

$$J_{0+m}(s, \mu_J) = J_0(s, \mu_J) + \delta J_m^{\text{virt}}(s, \mu_J) + \theta(s - M^2)\, \delta J_m^{\text{real}}(s)$$

δJ_m^{virt}: virtual piece of jet function (distributive structure)

δJ_m^{real}: real radiation piece of jet function (function)

Jet function

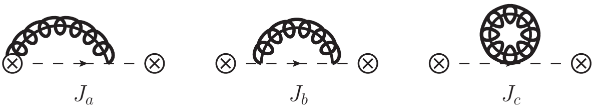


diagram J_a individually not well-defined $\rightarrow$ soft-bin subtractions are crucial!

$$J_{0+m}(s,M,\mu) = J_0(s,\mu) + \delta J_m^{\mathrm{virt}}(s,M,\mu) + \theta(s-M^2)\,\delta J_m^{\mathrm{real}}(s,M)$$

$$\mu^2 \delta J_m^{\mathrm{virt}}(s,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\delta(\bar{s})\left[-4\ln^2\left(\frac{M^2}{\mu^2}\right) - 6\ln\left(\frac{M^2}{\mu^2}\right) + 9 - 2\pi^2\right]\right.$$
$$\left. + 8\ln\left(\frac{M^2}{\mu^2}\right)\left[\frac{\theta(\bar{s})}{\bar{s}}\right]_+\right\}$$

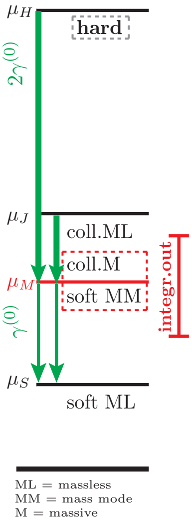
$$\delta J_m^{\mathrm{real}}(s,M) = \frac{\alpha_s C_F}{4\pi}\left\{\frac{2(M^2-s)(3s+M^2)}{s^3} + \frac{8}{s}\ln\left(\frac{s}{M^2}\right)\right\}$$

$\rightarrow$ $\delta J_m^{\mathrm{virt}}$ = virtual radiation ($\bar{s} \equiv s/\mu^2$)

$\rightarrow$ $\delta J_m^{\mathrm{real}}$ = real radiation for $s > M^2$, continuous: $\delta J_m^{\mathrm{real}}(s = M^2, M) = 0$

$\rightarrow$ correct massless limit: $J_{0+m}(s,M,\mu_J) \xrightarrow{M\to 0} 2J_0(s,\mu_J)$

Scenario III: $1 > \lambda > \lambda_M > \lambda^2$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_m)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M, \mu_S)$$
$$\times \int d\ell \int ds \int ds' \int ds'' J_{0+m}(s, \mu_J)\, U_J^{(1)}(s' - s, \mu_M, \mu_J)$$
$$\times\, \mathcal{M}_J(s'' - s', \mu_M)\, U_J^{(0)}(s'' - Q\ell, \mu_S, \mu_M)\, S_0(Q\tau - \ell, \mu_S)$$

$$J_{0+m}(s, \mu_J) = J_0(s, \mu_J) + \delta J_m^{\mathrm{virt}}(s, \mu_J) + \theta(s - M^2)\, \delta J_m^{\mathrm{real}}(s)$$

$$\mathcal{M}_J(s, \mu_M) = \delta(s) - \delta J_m^{\mathrm{virt}}(s, \mu_M)$$

$\delta J_m^{\mathrm{virt}}$: virtual piece of jet function (distributive structure)
$\delta J_m^{\mathrm{real}}$: real radiation piece of jet function (function)

continuity to scenario II for $\mu_M = \mu_J$ ($\mu_M \leq M$):

$$J_{0+m}(s, \mu_J)\, \mathcal{M}_J(s, \mu_J) = J_0(s, \mu_J)$$

Scenario III: Matching to full theory

- expansion of the most singular terms, i.e. for $\tau \sim M^2/Q^2 \ll 1$ and $\tau \sim M/Q \ll 1$

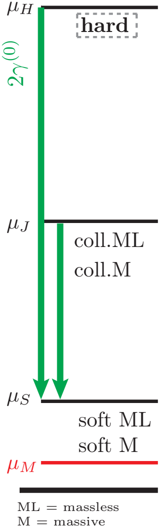
$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\text{FO}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M^2}{Q^2}\right)\right]\delta(\tau) + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\text{real}}(Q^2\tau) + Q\,\theta\left(\tau - \frac{M}{Q}\right)\delta S_m^{\text{real}}(Q\tau)$$

- matching with SCET result at fixed order gives mass matching functions
  e.g. in scenario III, $M/Q > \tau > M^2/Q^2$:

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{SCET}}}{d\tau}\bigg|^{\text{III}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M}{Q}\right)\right]\delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{eff}}(Q,M,\mu) + \mathcal{M}_H^{(1)}(Q,M,\mu)\right]\delta(\tau) + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\text{real}}(Q^2\tau, M) + Q^2\left[\delta J_m^{\text{virt}}(Q^2\tau,M,\mu) + \mathcal{M}_J^{(1)}(Q^2\tau,M,\mu)\right]$$

$\rightarrow \mathcal{M}_J^{(1)}(Q^2\tau,M,\mu) = -\delta J_m^{\text{virt}}(Q^2\tau,M,\mu)$ (integrate out virtual contributions)
$\rightarrow$ no real radiation appearing in mass matching functions

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_M$



massive soft and massless usoft modes fluctuate over comparable scales ($\lambda_M \leq \lambda^2$)

$\rightarrow$ assign usoft massless scaling (keep $M \neq 0$)!

$\rightarrow$ all structures get massive contributions

$\rightarrow$ massive modes stay in the game to the end

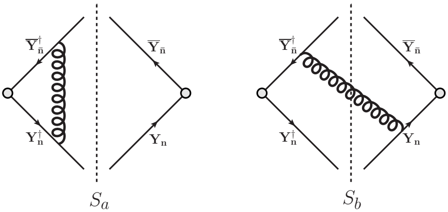
$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_S, \mu_J)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\text{virt}}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\text{real}}(\ell)$$

δS_m^{virt}: virtual piece of massive soft function (distributive structure)

δS_m^{real}: real radiation piece of massive soft function (function)

Soft function



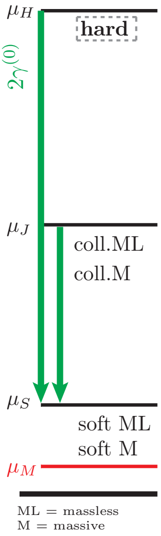
$$S_{0+m}(\ell, M, \mu) = S_0(\ell, \mu) + \delta S_m^{\text{virt}}(\ell, M, \mu) + \theta(\ell - M)\, \delta S_m^{\text{real}}(\ell, M)$$

$$\mu\, \delta S_m^{\text{virt}}(\ell, M, \mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \delta(\bar{\ell}) \left[2 \ln^2 \left(\frac{M^2}{\mu^2} \right) + \frac{\pi^2}{3} \right] - 8 \ln \left(\frac{M^2}{\mu^2} \right) \left[\frac{\theta(\bar{\ell})}{\bar{\ell}} \right]_+ \right\}$$

$$\delta S_m^{\text{real}}(\ell, M) = \frac{\alpha_s C_F}{4\pi} \left\{ -\frac{8}{\ell} \ln \left(\frac{\ell^2}{M^2} \right) \right\}$$

$\rightarrow$ δS_m^{virt} = virtual radiation ($\bar{\ell} \equiv \ell/\mu$)
$\rightarrow$ δS_m^{real} = real radiation for $\ell > M$, continuous: $\delta S_m^{\text{real}}(\ell = M, M) = 0$
$\rightarrow$ correct massless limit: $S_{0+m}(\ell, M, \mu_S) \xrightarrow{M \to 0} 2 S_0(\ell, \mu_S)$

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_M$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_S, \mu_J)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\mathrm{virt}}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\mathrm{real}}(\ell)$$

$\delta S_m^{\mathrm{virt}}$: virtual piece of massive soft function (distributive structure)
$\delta S_m^{\mathrm{real}}$: real radiation piece of massive soft function (function)

agreement with expanded full theory result at fixed order

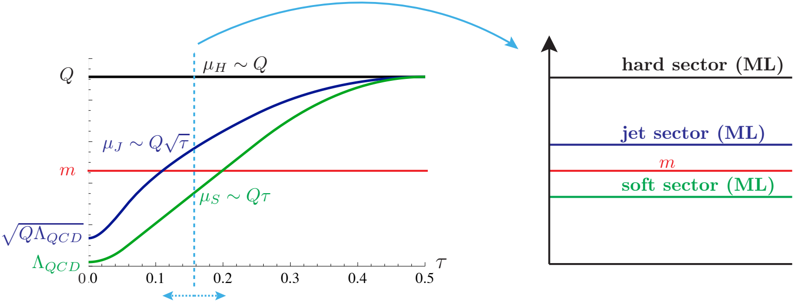
continuity to scenario III for $\mu_M = \mu_J$:
$\rightarrow$ consistency relation between virtual modes :

$$2\,\mathrm{Re}\left[\delta F_m^{\mathrm{eff}}(Q, \mu)\right] \delta(\tau) - Q^2 \delta J_m^{\mathrm{virt}}(Q^2\tau, \mu) - Q\, \delta S_m^{\mathrm{virt}}(Q\tau, \mu) = 0$$

Summary so far

√ EFT setup for massive gauge boson radiation in thrust distribution for different scale hierarchies

√ patch four scenarios **continuously** (up to $\mathcal{O}(\alpha_s^2)$)



several scenarios needed for one single thrust distribution

Summary so far

- $\surd$ EFT setup for massive gauge boson radiation in thrust distribution for different scale hierarchies
- $\surd$ patch four scenarios **continuously** (up to $\mathcal{O}(\alpha_s^2)$)
- $\surd$ same anomalous dimension as in $M \to 0$ limit $\Rightarrow$ evolution factors affected only by number of massive gauge bosons
- $\surd$ decoupling limit: $\quad \mathcal{C}^I(Q, M, \mu_H) \stackrel{M\to\infty}{\longrightarrow} C_0(Q, \mu_H)$
- $\surd$ massless limit for hard, jet and soft functions:

$$\begin{aligned} \mathcal{C}^{II}(Q, M, \mu_H) &\stackrel{M\to 0}{\longrightarrow} 2C_0(Q, \mu_H) \\ J_{0+m}(s, M, \mu_J) &\stackrel{M\to 0}{\longrightarrow} 2J_0(s, \mu_J) \\ S_{0+m}(\ell, M, \mu_S) &\stackrel{M\to 0}{\longrightarrow} 2S_0(\ell, \mu_S) \end{aligned}$$

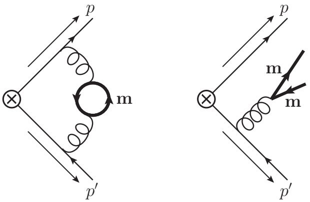
- $\surd$ interesting consistency condition $\to$ possible to rearrange mass-mode contributions into different functions depending on the choice of μ

$$2\,\mathrm{Re}\left[\delta F_{\mathrm{eff}}(Q, M, \mu)\right]\delta(\tau) - Q^2 \delta J_m^{\mathrm{virt}}(Q^2\tau, M, \mu) - Q\delta S_m^{\mathrm{virt}}(Q\tau, M, \mu) = 0$$

Outline

Aim

- **PART I**: technique for mass modes in SCET, massive gauge boson example
  arxiv:1302.4743
- **PART II** : two-loop extension $\Rightarrow$ *secondary* heavy quark radiation

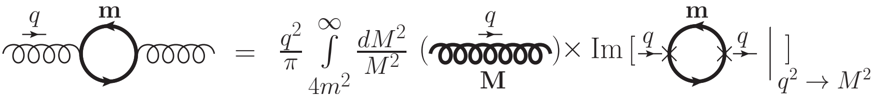


- *primary* quarks massless
- to be specific $n_m = 1$
- factorization theorems at two-loop (same structure as massive gauge boson case, but some complications)
- quark bubble $(C_F T_F n_f)$ contributions to $\mathcal{C}(Q, m, \mu)$, $J(s, m, \mu)$ and $S(\ell, m, \mu)$
  massless case $\rightarrow$ Moch et al. (2005), Becher et al. (2007), Kelley et al. (2011), Hornig et al. (2011)

Dispersion relation

Observation: interpret fermion bubble insertion as “gluon” with massive propagator

Hoang (1995), PhD thesis :)



$$\Pi^{\rm eff}_{\mu\nu}(q^2) \equiv \frac{(-i)^2 g_{\mu\rho}\Pi^{\rho\sigma}(q^2) g_{\sigma\nu}}{(q^2+i\epsilon)^2} = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{dM^2}{M^2}\,\mathrm{Im}\left[\Pi(M^2)\right]\frac{-i\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)}{q^2-M^2+i\epsilon}$$

- calculate one-loop massive gauge boson result
- convolute with Im $\left[\Pi(M^2)\right]$
- use same dispersive integral for full theory, collinear and soft sector
- convolution in $d = 4 - 2\epsilon$!!! (but for finite parts $d = 4$)

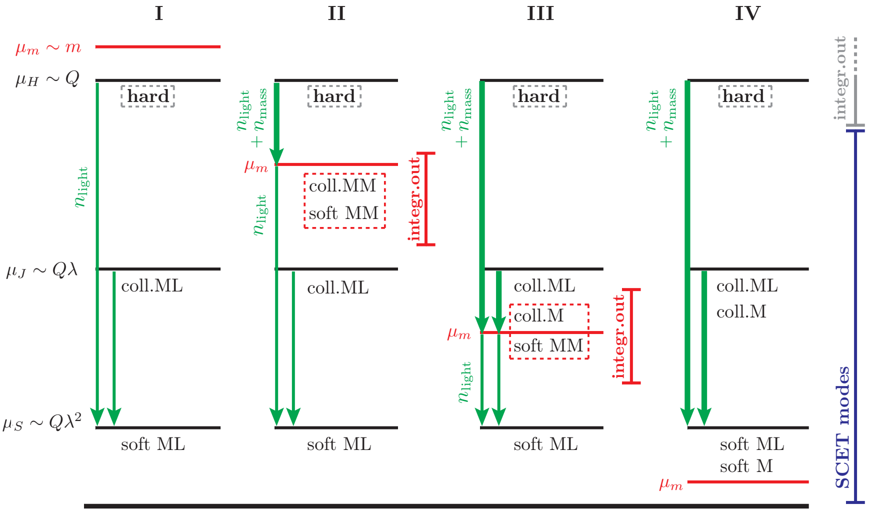
- Dispersive technique allows to address difficulties separately:
  1. separation of dynamical modes depending on m vs μ_H, μ_J, μ_S $\Rightarrow$ treated in massive “gluon” context
  2. secondary quark radiation and its influence on RG evolutions $\Rightarrow$ taken care of after final convolution
- **Part I**: Soft-bin subtraction can be carried out completely in the massive “gluon” context at $\mathcal{O}(\alpha_s)$.

  $\Downarrow$

  No further soft-bin subtraction needed at $\mathcal{O}(\alpha_s^2)$. Convolution already regulated in dim. reg.
- **Beware:** setup is general, but dispersive treatment suitable for observables depending on invariant mass of secondary fermion pair

Setup

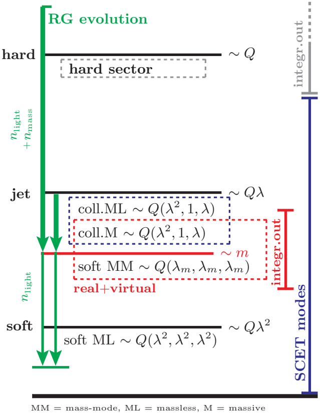
fermion mass ⇒ additional scale parameter, distinguishes four scenarios



MM = mass-mode, ML = massless, M = massive

Issues at two-loop

- Decoupling of α_s in evolution



- at scale μ_m integrate out massive flavours $\Rightarrow \alpha_s^{n_{\text{light}}+1} \to \alpha_s^{n_{\text{light}}}$
- terms of the form

$$\frac{4}{3}\frac{T_F}{4\pi} \log\left(m^2/\mu_m^2\right) \times \text{one loop}$$

appear in factorization theorem

- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet

Issues at two-loop

- Decoupling of α_s in evolution
  $\Downarrow$
- Subtracted vs unsubtracted dispersive relation
  - unsubtracted and unrenormalized ($\overline{MS}$ scheme)

$$\Pi(q^2) \quad = \quad \frac{1}{\pi}\int_{4m^2}^{\infty} dM^2 \frac{1}{M^2 - q^2 - i\epsilon} \,\text{Im}\left[\Pi(M^2)\right]$$

$$\text{use: } \Pi(q^2) - \frac{4}{3}\frac{1}{\epsilon}$$

    - heavy quark contributes to RGE
  - subtracted (on-shell scheme)

$$\Pi^{\text{os}}(q^2) = \Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_{4m^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{M^2 - q^2 - i\epsilon} \,\text{Im}\left[\Pi(M^2)\right]$$

    - heavy quark does not contribute to any RGE

  Use of unsubtracted relations crucial to cancel unwanted mass terms
- fixed order full QCD results not calculated yet
- Soft function with hemisphere (thrust) prescription is complicated

Issues at two-loop

- Decoupling of α_s in evolution
- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet
  - virtual terms given by vertex function $F_{QCD}(Q^2, m^2)$

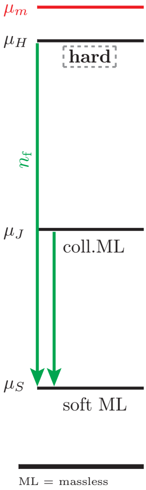
    Kniehl (1990), Hoang (1995)
  - and SCET gives all the singular terms
- Soft function with hemisphere (thrust) prescription is complicated

Issues at two-loop

- Decoupling of α_s in evolution
- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet
- Soft function with hemisphere (thrust) prescription is complicated
  - $S(k_L, k_R, m, \mu)$ not constrained by secondary quark invariant mass
  - $\Rightarrow$ dispersive approach only give correct UV and log structure

Scenario I : $\lambda_m > 1 > \lambda > \lambda^2$

heavy quark integrated out at QCD level $\Rightarrow \mathcal{C}^I(Q, m, \mu_H)$



$$\frac{d\sigma}{d\tau} = Q\sigma_0 \left|\mathcal{C}^I(Q, m, \mu_H)\right|^2 U_H^{(n_f)}(Q, \mu_H, \mu_S) \int ds \int ds' \times U_J^{(n_f)}(s - s', \mu_S, \mu_J) J_0(s', \mu_J) S_0\left(Q\tau - \frac{s}{Q}, \mu_S\right)$$

$$U_i^{(n_f)}(Q, \mu_H, \mu_S) = \text{massless RG factors, } n_f \text{ light flavours}$$

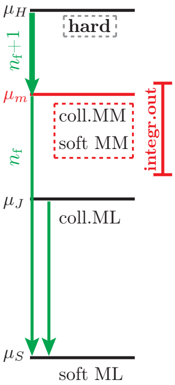
$$\mathcal{C}^I(Q, m, \mu) = 1 + \alpha_s^{(n_f)} C_0^{(1)} + (\alpha_s^{(n_f)})^2 \left(C_0^{(2)} + F_{\text{QCD}}^{(2)}(Q, m)\Big|^{\text{OS}}\right)$$

Hoang (1995)

- $F_{\text{QCD}}^{(2)}(Q, m \to \infty)\Big|^{\text{OS}} \to 0$ (decoupling)
- $F_{\text{QCD}}^{(2)}(Q, m \to 0)\Big|^{\text{OS}} \to a_1 \ln^3(-x) + a_2 \ln^2(-x) + a_3 \ln(-x) + \ldots$

$$x = m^2/Q^2$$

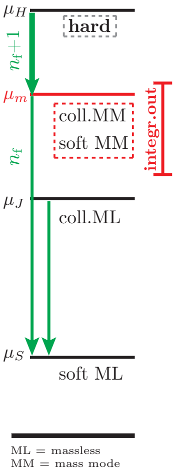
Scenario II : $1 > \lambda_m > \lambda > \lambda^2$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_m)\left|\mathcal{M}_{H_Q}(Q,m,\mu_m)\right|^2 \times U_{H_Q}^{(n_f)}(Q,\mu_m,\mu_S)\int ds\int ds'\, U_J^{(n_f)}(s-s',\mu_S,\mu_J) \times J_{0,\tau}(s',\mu_J) S_{0,\tau}\left(Q\tau-\frac{s}{Q},\mu_S\right)$$

ML = massless
MM = mass mode

Scenario II : $1 > \lambda_m > \lambda > \lambda^2$

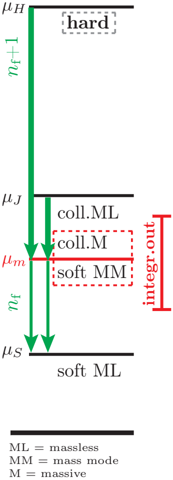


$$\mathcal{C}^{II}(Q,m,\mu) = 1 + C_0^{(1)}\alpha_s^{(n_f+1)} + (\alpha_s^{(n_f+1)})^2 \times \left(C_0^{(2)} + C_m^{(2)}(Q,m,\mu)\right)$$

$$\mathcal{M}_{H_Q}(Q,m,\mu_m) = 1 + (\alpha_s^{(n_f+1)})^2 \left[F_{\text{SCET}}^{(2)}(Q,m,\mu_m)\Big|^{\overline{\text{MS}}} + \alpha_s \text{ dec.}\right]$$

- $C_m^{(2)}(Q,m,\mu) \to$ mass modes contributions
- in $C_m^{(2)}(Q,m,\mu)$ cancellation of divergences takes place
- sum and subtract same terms at different scales (μ_H vs μ_m)
- continuous transition scenario I $\leftrightarrow$ II

Scenario III : $1 > \lambda > \lambda_m > \lambda^2$

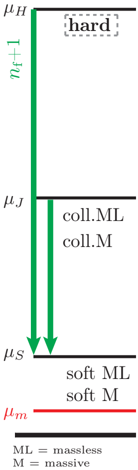


$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_m)\left|\mathcal{M}_{H_Q}(Q,m,\mu_m)\right|^2$$
$$\times U_{H_Q}^{(n_f)}(Q,\mu_m,\mu_S)\int ds\, ds'\, ds''\, dt\, J_{0+m}(s'',m,\mu_J)$$
$$\times U_J^{(n_f+1)}(s'-s'',\mu_m,\mu_J)\mathcal{M}_J(t,m,\mu_m)U_J^{(n_f)}(s-s',\mu_S,\mu_m)$$
$$\times S_0\left(Q\tau - s/Q - t/Q,\mu_S\right)$$

$$J_{0+m}(s,m,\mu) = 1 + \alpha_s^{(n_f+1)}J_0^{(1)} + (\alpha_s^{(n_f+1)})^2\Big(J_0^{(2)} + J_m^{\mathrm{virt},(2)}(s,m,\mu)\Big|^{\overline{\mathrm{MS}}} + \theta(s-4m^2)J_m^{\mathrm{real},(2)}(s,m^2)\Big)$$
$$\mathcal{M}_J(s,m,\mu_m) = \delta(s) + (\alpha_s^{(n_f+1)})^2\left[-J_m^{\mathrm{virt},(2)}(s,m,\mu_m)\Big|^{\overline{\mathrm{MS}}} + \alpha_s\text{ dec.}\right]$$

- match before real radiation sets in (i.e. $\tau_m < 4m^2/Q^2$)
- $J_m^{\mathrm{real},(2)}(s,m^2) = 0$ at threshold

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_m$



$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_S)\int ds\,ds'$$
$$U_J^{(n_f+1)}(s-s',\mu_S,\mu_J)J_{0+m}(s',m,\mu_J)$$
$$\times S_{0+m}\left(Q\tau-\frac{s}{Q},m,\mu_S\right)$$

$$S_m^{(2)}(\ell,m,\mu) = S_{\rm m}^{\text{virt},(2)}(\ell,m,\mu)+\theta(\ell-2m)S^{\text{real},(2)}(\ell,m)$$

- consistency relation:

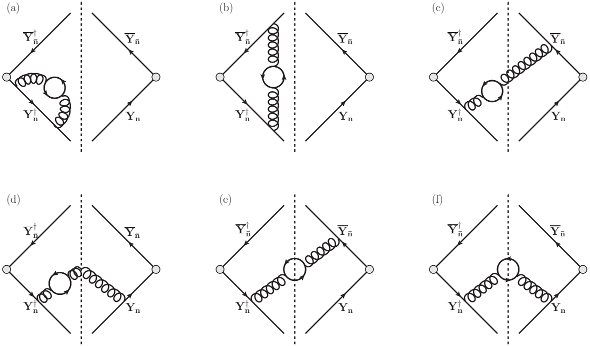
$$0 = 2\text{Re}\left[F_{\text{SCET}}\Big|^{\overline{\text{MS}}}\right]\delta(\tau) - Q^2 J_m^{\text{virt},(2)}\Big|^{\overline{\text{MS}}} - Q S_m^{\text{virt},(2)}\Big|^{\overline{\text{MS}}}$$
$$+\frac{4}{3}\ln\left(\frac{m^2}{\mu_m^2}\right)\left\{2\text{Re}\left[C_0^{(1)}\right]\delta(\tau)+Q^2J_0^{(1)}+QS_0^{(1)}\right\}$$

- $\Rightarrow$ continuous transition III $\leftrightarrow$ IV

Outline

Calculation for the soft function

- $m = 0$ calculation Kelley et al. (2011), Hornig et al. (2011)

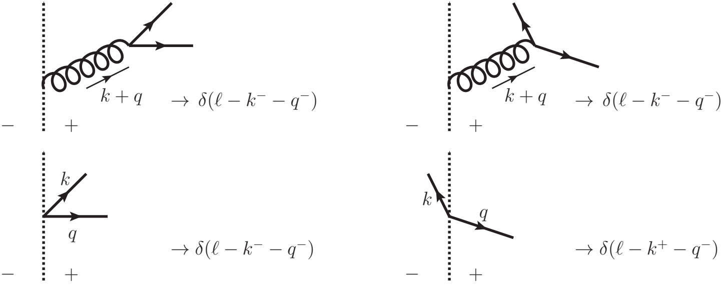


Remember: dispersive technique suitable for observables depending on fermion pair invariant mass $(k+q)^2$

"*quark hemisphere*" prescription $\Rightarrow$ k and q treated separately if both are in different hemispheres: $S^{(QQ)}(k_L, k_R, m, \mu)$

Prescriptions

First approximation: "*gluon hemisphere*" prescription $\Rightarrow k + q$: $S^{(g)}(k_L, k_R, m, \mu)$



same hemisphere: $k^+ > k^-$, $q^+ > q^-$

opposite hemisphere: $k^- > k^+$, $q^+ > q^-$

$$k^+ + q^+ > k^- + q^-$$

- quarks in same hemisphere $\Rightarrow S^{(g)}(k_L, k_R, m, \mu)\big|_{\text{same}} = S^{(QQ)}(k_L, k_R, m, \mu)\big|_{\text{same}}$
- quarks in opposite hemispheres $\Rightarrow S^{(g)}(k_L, k_R, m, \mu)\big|_{\text{opp}} \neq S^{(QQ)}(k_L, k_R, m, \mu)\big|_{\text{opp}}$

Method

- $S^{(g)}(k_L, k_R, m, \mu)$: same UV divergences of $S^{(QQ)}(k_L, k_R, m, \mu)$ (RG consistency)
-

$$S^{(QQ)}(k_L, k_R, m, \mu) = S^{(g)}(k_L, k_R, m, \mu) + \underbrace{\left(S^{(QQ)}(k_L, k_R, m, \mu)\Big|_{\text{opp}} - S^{(g)}(k_L, k_R, m, \mu)\Big|_{\text{opp}} \right)}_{\text{finite}=\Delta S(k_L, k_R, m)}$$

- $\Delta S(k_L, k_R, m)$ not easy analytically
- $\Delta S(k_L, k_R, m)$ finite! so numerically doable in 4 dimensions

Thrust soft function at two-loop

$$S_\tau(\ell,m,\mu) = \int dk_L\, dk_R\; S(k_L,k_R,m,\mu)\,\delta(\ell-k_L-k_R)$$

$$\begin{aligned} S_\tau^{(QQ),(2)}(\ell,m,\mu) \sim \alpha_s^2 \Bigg\{ & \delta(\ell)\left[\frac{1}{18}L_m^3 - \frac{5}{18}L_m^2 + \left(\frac{28}{27}-\frac{\pi^2}{18}\right)L_m + \dots\right] \\ & + \frac{1}{\mu}\left[\frac{\mu\,\theta(\ell)}{\ell}\right]_+ \left(\frac{1}{3}L_m^2 - \frac{10}{9}L_m + \frac{28}{27}\right) \\ & + \frac{1}{\mu}\left[\frac{\mu\,\theta(\ell)\ln(\ell/\mu)}{\ell}\right]_+ \frac{4}{3}L_m \Bigg\} \\ & + \theta(\ell-2m)S^{\text{real},(2)}(\ell,m) + \Delta S_\tau(\ell,m) \end{aligned}$$

- $L_m \equiv \ln(m/\mu)$

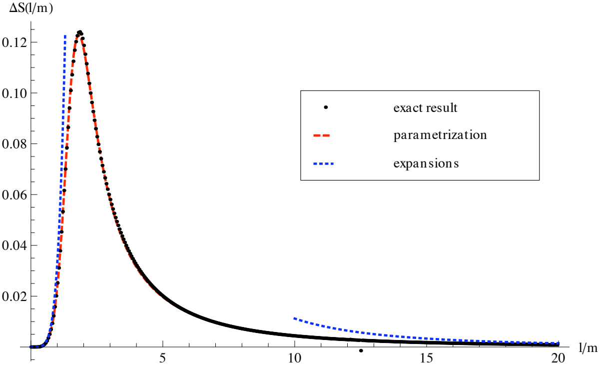
- $S_\tau^{(QQ),(2)}(\ell,m,\mu)$ reaches the massless limit
- same anomalous dimension as massless function
- massive quark radiation described by $\theta(\ell-2m)S^{\text{real},(2)}(\ell,m)$ (as in jet function)
- also $\Delta S_\tau(\ell,m)$ corresponds to real radiation

The correction $\Delta S_\tau(\ell, m)$

- crucial to find correct massless limit

$$\Delta S_\tau(\ell, m) \overset{m\to 0}{\to} \delta(\ell) \times \left\{ -\frac{4}{9} + \frac{13\pi^2}{54} - \frac{4\zeta(3)}{3} \right\}$$

  checked numerically and analytically!
- contains no thresholds even though describes real radiation
- numerically small $< 5\%$ of $S_\tau(\ell, m, \mu)$
- analytical asymptotic expansions



- fit to a Breit Wigner type function (4 free parameters)
- 7 parameters constrained with asymptotic expansions and normalization

$$\Delta S_\tau(x) \sim \frac{1}{m}\frac{(ax)^\alpha}{(1+(ax)^\beta)^{\gamma/\beta}}\left[b\,\log^2\left(1+Ax+Bx^2\right)+c\,\log\left(1+Cx+Dx^2\right)+d\right]$$

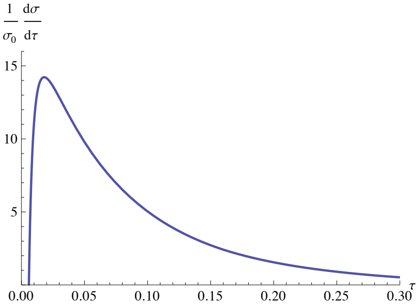
$$x \equiv \ell/m$$

Outline

Plots for $Q = 14, 35$ GeV

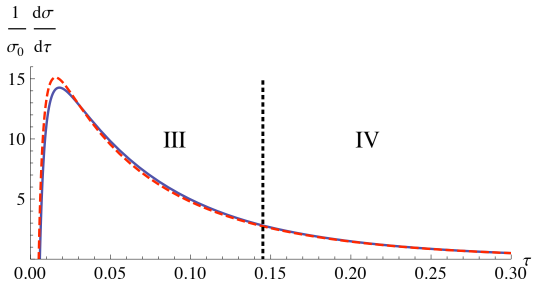
- numerical code for thrust distribution at N^{3}LL
- **preliminary**, so far no non-perturbative physics!
- $Q = 14, 35$ GeV $\leftrightarrow$ data from PETRA
- determination of $\alpha_s(M_z)$: $Q = 35 \dots 207$ GeV — Abbate et al. (2011, 2012)
- massless: 5 light flavours vs. massive: 4 light + 1 massive b ($m_b = 4.2$ GeV)

Thrust distribution: massive ($Q = 14$ GeV, $\alpha_s(M_z) = 0.118$)

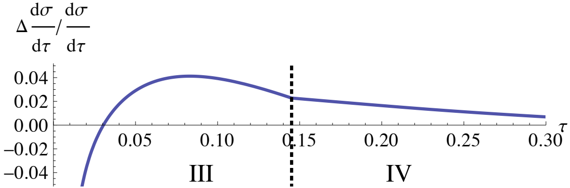


$Q = 14$ GeV

Thrust distribution: massive vs. massless

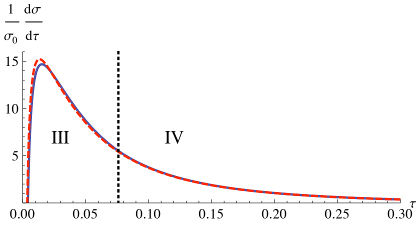


relative mass effects

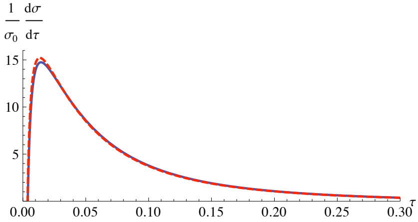


$Q = 35$ GeV: possible effect on α_s

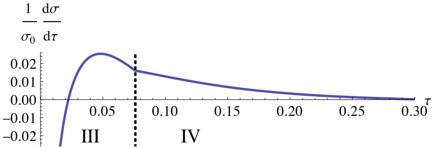
massive vs. massless



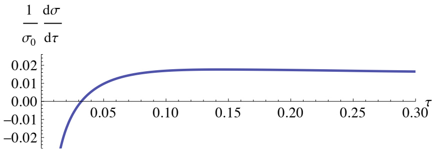
$\alpha_s(M_z) = 0.119$ vs. $\alpha_s(M_z) = 0.118$



relative mass effects



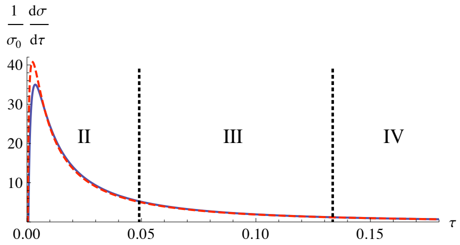
relative correction

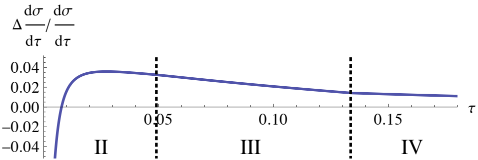


$Q = 500$ GeV

- $Q = 500$ GeV $\leftrightarrow$ ILC
- massless: 6 light flavours vs. massive: 5 light + 1 massive t ($m_t = 175$ GeV)

Thrust distribution: massive vs. massless

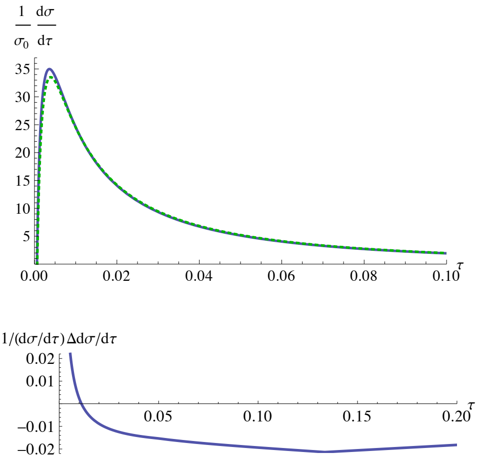




$Q = 500$ GeV (decoupling)

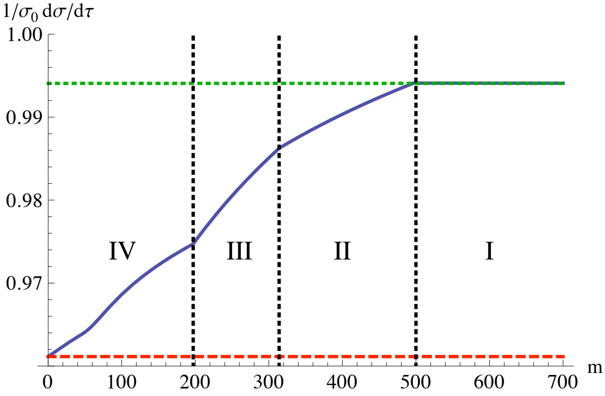
- $Q = 500$ GeV $\leftrightarrow$ ILC
- massless: 5 light flavours vs. massive: 5 light + 1 massive t ($m_t = 175$ GeV)

Thrust distribution: massive vs. massless



$Q = 500$ GeV: Consistency check

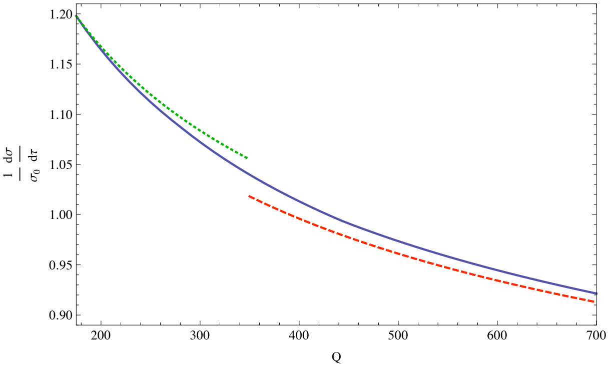
Thrust distribution: $\tau = 0.15$ fixed, vary mass m
massless limit (6 flavours): dashed
decoupling limit (5 flavours): dotted



continuous transitions and correct limiting behaviour!

Extreme approach

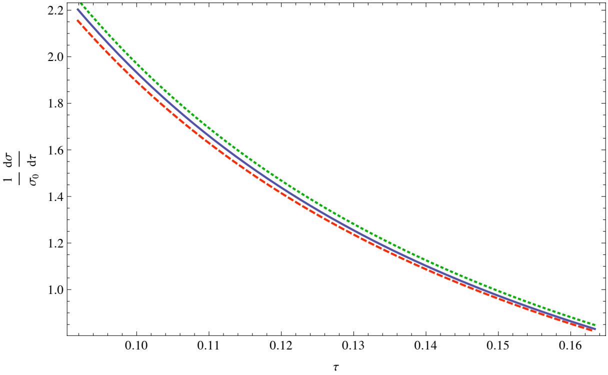
Thrust distribution: $\tau = 0.15$, $m_t = 175$ GeV fixed, vary Q
massless limit (6 flavours): dashed
decoupling limit (5 flavours): dotted



Extreme approach vs ours

- $Q = 500$ GeV
- $\tau \approx m_t^2/Q^2$

Thrust distribution: $m_t = 175$ GeV fixed, vary Q
massless limit (6 flavours): dashed
decoupling limit (5 flavours): dotted



deviation from extreme approach can be up to $\approx 4\%$

Outline

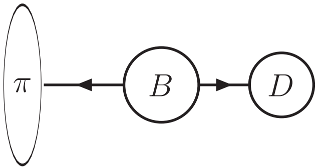
Summary & Outlook

- inclusion of heavy quark masses important for high precision collider physics
- setup for secondary massive quarks in terms of mass modes
- calculation of all ingredients for thrust distribution at N^3LL

NEXT:

- double hemisphere soft function $S^{(QQ)}(k_L, k_R, m, \mu)$
- renormalon analysis
- possible applications
  1. bottom mass effects in α_s determination at N^3LL
  2. analysis of LEP data (at $Q = 14$ GeV) and data from B factories
  3. gluino bounds Kaplan & Schwartz (2008)
  4. massive effects for parton distribution functions in heavy quark production
  5. hard photoproduction with a heavy quark jet
  6. ...

Degrees of freedom: $B \to D\pi$

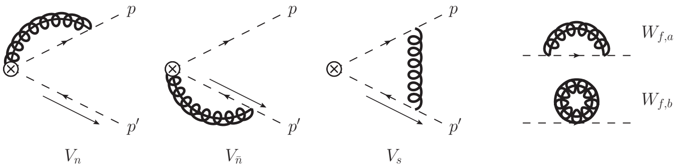


$$p_\pi^\mu \simeq (2\text{GeV}, 0, 0, -2\text{GeV}) = Qn^\mu \quad (E_\pi \simeq Q \gg \Lambda_{\text{QCD}} \simeq m_\pi)$$

- π rest frame: π constituents $p^\mu \simeq (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}})$
- B rest frame: π boosted along z, $p^\mu \simeq (\Lambda_{\text{QCD}}^2/Q, Q, \Lambda_{\text{QCD}})$ collinear π
- B and D constituents $p^\mu \simeq (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}})$
- $\lambda \equiv \Lambda_{\text{QCD}}/Q \Rightarrow$ collinear $p^\mu = Q(\lambda^2, 1, \lambda)$, soft $p^\mu = Q(\lambda, \lambda, \lambda)$

Soft-bin subtractions

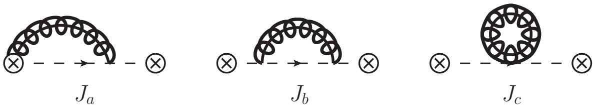
- hard function



$$\delta F_m^{\text{eff}} \sim V_n - V_{n,0M} + V_{\bar{n}} - V_{\bar{n},0M} + V_s - W_f$$
$$\sim V_n + V_{\bar{n}} - V_s - W_f$$

Idilbi, Mehen (2007)
no regulator required if suitable combinations of integrals taken

- jet function



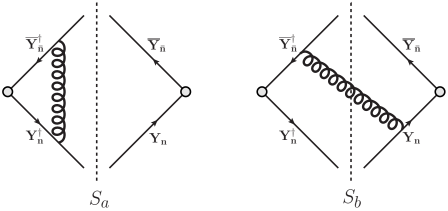
$$\delta J_m \sim J_a - J_{a,0M} + J_b + J_c$$

→ remark: no soft diagrams appearing here

Soft-bin subtractions to self-energy diagrams

- SCET reproduces QCD wave-function renormalization
- in scenario II: $Q < M < Q\lambda$
  $\rightarrow$ off-shellness for interactions with collinear mass mode gauge bosons:
  $s = (p+k)^2 \sim M^2$
  $\rightarrow$ off-shellness for interactions with soft mass mode gauge bosons: $s = (p+k)^2 \sim QM$
  $\Rightarrow$ soft-bin subtractions to (collinear) self-energy power-suppressed by M/Q

Soft function calculation



additional divergences in the lightcone components (not regularized by DIMREG)
$\rightarrow S_a$ and S_b individually ill-defined
$\rightarrow$ we use analytic α-regulator (DIMREG in lightcone components)
Becher, Bell (2011)

$$\int dk^- \int dk^+ \rightarrow \int dk^- \left(\frac{\nu_1}{k^-}\right)^{\alpha_1} \int dk^+ \left(\frac{\nu_2}{k^+}\right)^{\alpha_2}$$

$\rightarrow S_a = 0, \delta S_m = S_b$

A note on QCD form factor

Use of two different dispersive relations $\Rightarrow F_{\mathrm{QCD}}(Q,m,\mu)\Big|^{\overline{\mathrm{MS}}}$ vs $F_{\mathrm{QCD}}(Q,m,\mu)\Big|^{\mathrm{OS}}$

$$F^{(2)}_{\mathrm{QCD}}(Q,m,\mu)\Big|^{\overline{\mathrm{MS}}} = F^{(2)}_{\mathrm{QCD}}(Q,m)\Big|^{\mathrm{OS}} - \underbrace{\left(\Pi(0) - \frac{4}{3}\frac{1}{\epsilon}\right) \times (\text{1-loop QCD (in d-dim!)})}_{(\mathrm{OS}\leftrightarrow\overline{\mathrm{MS}})}$$

- $(\mathrm{OS} \leftrightarrow \overline{\mathrm{MS}})$ contains IR divergences which exactly cancel those from SCET diagrams

- "quark prescription" :

$$F(k_R,k_L) = (-2\pi i)^2\,\delta(k^2)\,\delta(q^2)\,\Theta(k^+ + k^-)\,\Theta(q^+ + q^-)$$
$$\times\big[\Theta(k^+ - k^-)\,\Theta(q^- - q^+)\,\delta(q^+ - k_R)\,\delta(k^- - k_L)$$
$$+\,\Theta(k^- - k^+)\,\Theta(q^+ - q^-)\,\delta(k^+ - k_R)\,\delta(q^- - k_L)$$
$$+\,\Theta(k^- - k^+)\,\Theta(q^- - q^+)\,\delta(k^+ + q^+ - k_R)\,\delta(k_L)$$
$$+\,\Theta(k^+ - k^-)\,\Theta(q^+ - q^-)\,\delta(k^- + q^- - k_L)\,\delta(k_R)\big]$$

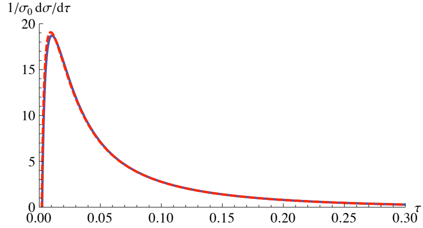
- "gluon prescription"

$$F^{(g)}(k_R,k_L) = (-2\pi i)^2\,\delta(k^2)\,\delta(q^2)\,\Theta(k^+ + k^-)\,\Theta(q^+ + q^-)$$
$$\times\big[\Theta(k^+ + q^+ - k^- - q^-)\,\delta(k_R)\,\delta(k^- + q^- - k_L)$$
$$+\,\Theta(k^- + q^- - k^+ - q^+)\,\delta(k_L)\,\delta(k^+ + q^+ - k_R)\big]$$

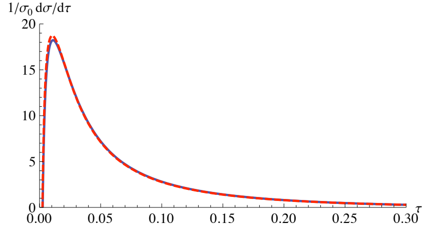
- $k_L, k_R =$ light cone components of left, right hemispheres momenta

$Q = 91$ GeV: possible effect on α_s

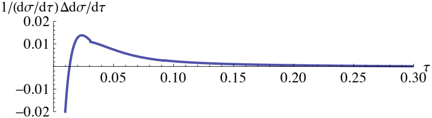
massive vs. massless



$\alpha_s(M_z) = 0.119$ vs. $\alpha_s(M_z) = 0.118$



relative mass effects



relative correction