Strong coupling from τ lepton decays

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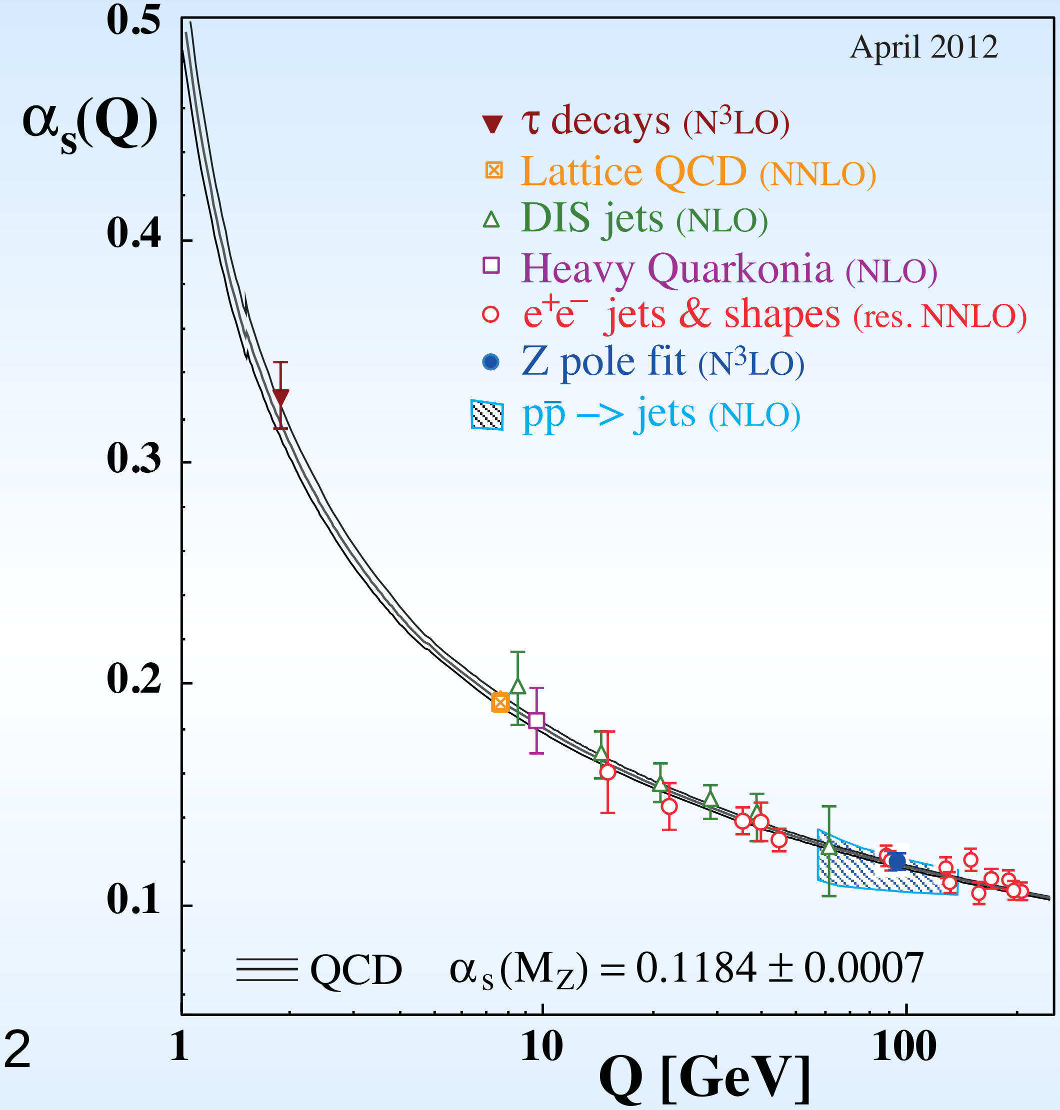
α_s measurements



PDG 2012

For 0.6 % precision at M_Z need "only" ≈ 2 % at M_τ.

Hadronic τ decay rate

Consider the physical quantity R_τ: (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau(\gamma))} = 3.6280(94)\,. \quad \text{(HFAG 2012)}$$

R_τ is related to the QCD correlators $\Pi^{(1,0)}(x)$: $(x \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int\limits_0^1 dx\,(1-x)^2 \Big[(1+2x)\,\mathrm{Im}\Pi^{(1)}(x) + \mathrm{Im}\Pi^{(0)}(x)\Big]\,,$$

with appropriate combinations of mesonic 2-point correlators

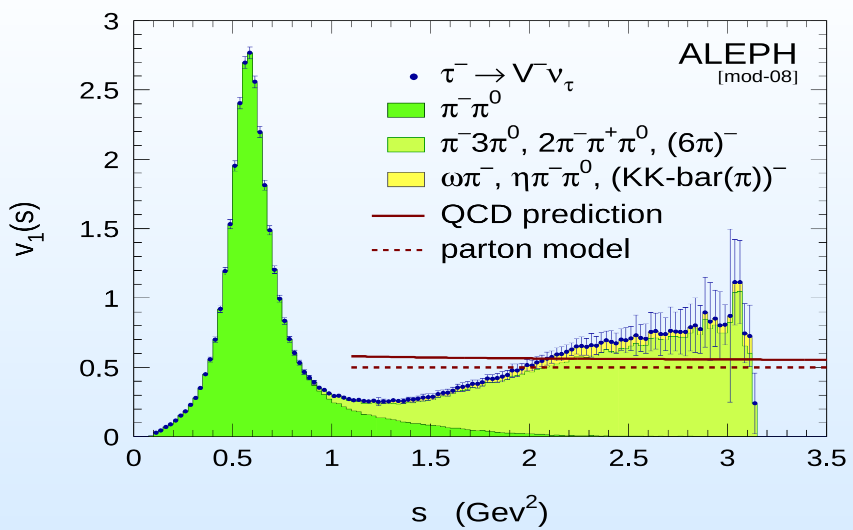
$$\Pi^{(J)}(x) = |V_{ud}|^2\Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2\Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

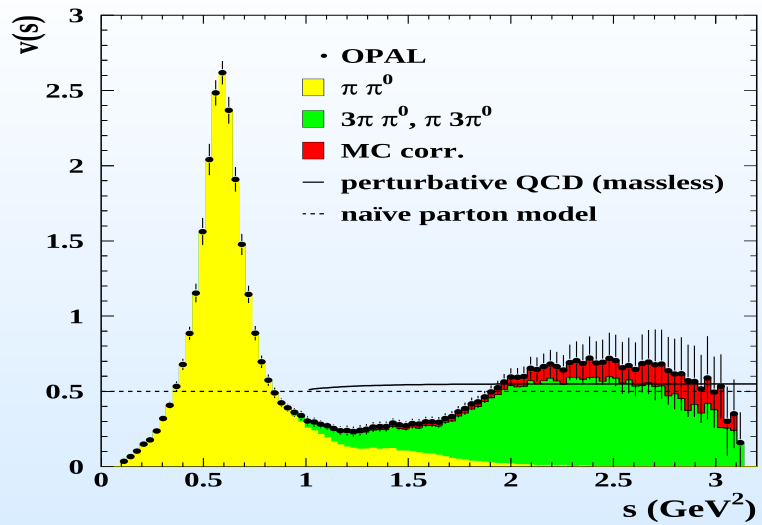
τ decay spectra

Experimental information can be inferred from the moments

$$R_\tau^w(s_0) \equiv \int_0^{s_0} ds\, w(s)\, \frac{d\widetilde{R}_\tau}{ds} = R_{\tau,V}^w + R_{\tau,A}^w + R_{\tau,S}^w\,.$$

$$w_\tau(s) = \left(1-\frac{s}{s_0}\right)^2\left(1+2\frac{s}{s_0}\right).$$





Hadronic τ decay moments

Theoretically, R_τ is calculated via the contour-integral:

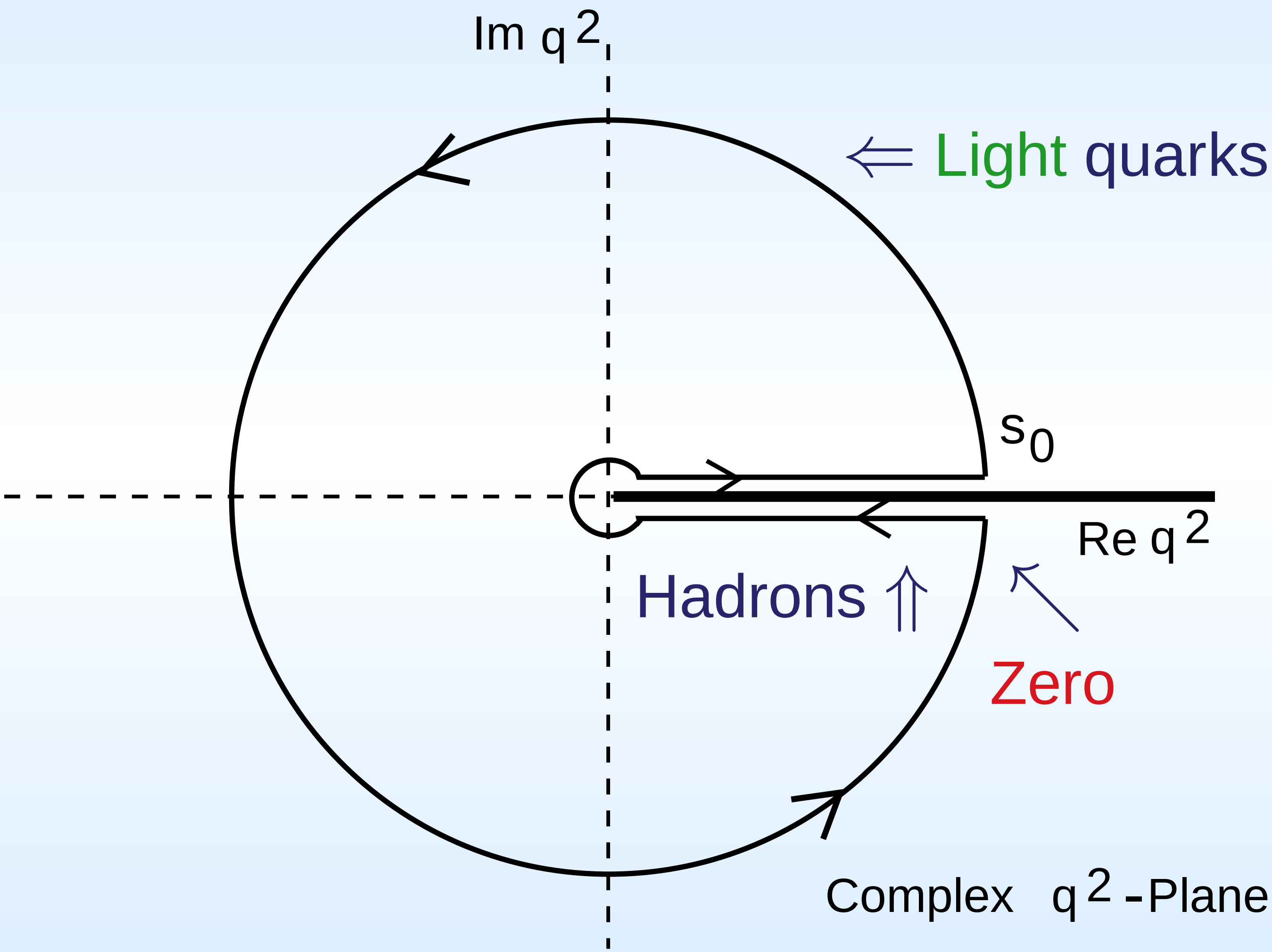
$$R^w_{V/A}(s_0) \equiv 6\pi i \oint\limits_{|s|=s_0} \frac{ds}{s_0}\, w(s) \left[\Pi^{(1+0)}_{V/A}(s) + \frac{2s}{(s_0+2s)}\Pi^{(0)}_{V/A}(s)\right].$$

Generally, R^w_τ takes the structure:

$$R^w_\tau = N_c\, S_{\rm EW} \Big\{ (|V_{ud}|^2+|V_{us}|^2)\left[1+\delta^{w(0)}\right] + \sum_{D\geq 2}\left[|V_{ud}|^2\delta^{w(D)}_{ud}+|V_{us}|^2\delta^{w(D)}_{us}\right]\Big\}.$$

$\delta^{w(D)}_{ud}$ and $\delta^{w(D)}_{us}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle\bar{q}q\rangle$.

Contour integral



Adler function

The perturbative part $\delta^{(0)}$ is related to the Adler function $D(s)$:

$$D(s) \equiv -\, s \, \frac{\mathrm{d}}{\mathrm{d}s} \Pi_V(s) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k \, c_{n,k} \ln^{k-1}\!\left(\frac{-s}{\mu^2}\right)$$

where $a_\mu \equiv \alpha_s(\mu)/\pi$.

Resumming the Log's with the scale choice $\mu^2 = -s \equiv Q^2$:

$$D(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} c_{n,1} \, a^n(Q^2)$$

As a consequence, only the coefficients $c_{n,1}$ are independent:

$$c_{0,1} = c_{11} = 1 \,, \quad c_{2,1} = 1.640 \,, \quad c_{3,1} = 6.371 \,,$$

$$c_{4,1} = 49.076 \,!!$$

(Baikov, Chetyrkin, Kühn 2008)

RG-improvement

Fixed-order perturbation theory amounts to choose $\mu^2 = M_\tau^2$:

$$\delta^{(0)}_{\rm FO} = \sum_{n=1}^{\infty} a^n(M_\tau^2) \sum_{k=1}^{n+1} k\, c_{n,k} J_{k-1} = \sum_{n=1}^{\infty} [c_{n,1} + g_n] a^n(M_\tau^2)$$

A given perturbative order n depends on all coefficients $c_{m,1}$ with $m \leq n$, and on the coefficients of the QCD β-function.

Contour-improved perturbation theory employs $\mu^2 = -M_\tau^2 x$:

(Pivovarov; Le Diberder, Pich 1992)

$$\delta^{(0)}_{\rm CI} = \sum_{n=1}^{\infty} c_{n,1} J_n^a(M_\tau^2) \qquad \text{with}$$

$$J_n^a(M_\tau^2) = \frac{1}{2\pi i} \oint\limits_{|x|=1} \frac{dx}{x} (1-x)^3 (1+x) a^n(-M_\tau^2 x)$$

FOPT vs CIPT problem

The purely perturbative contribution $\delta^{(0)}$ is plagued by differences for different RG-resummations. (FOPT vs CIPT.)

Using $\alpha_s(M_\tau) = 0.3186$, the numerical analysis results in:

$$
\begin{array}{lcccccl}
 & a^1 & a^2 & a^3 & a^4 & a^5 & \\
\delta^{(0)}_{\rm FO} = & 0.101 + & 0.054 + & 0.027 + & 0.013 & (+0.006) & = 0.196\,(0.202) \\
\delta^{(0)}_{\rm CI} = & 0.137 + & 0.026 + & 0.010 + & 0.007 & (+0.003) & = 0.181\,(0.185)
\end{array}
$$

Contour-improved PT appears to be better convergent.

The difference between both approaches is $0.015\,(0.017)$!

This problematic entails a ≈ 6 % difference for $\alpha_s(M_\tau)$.

Borel transform

To further investigate the difference between CI and FOPT, let us consider the Borel-transformed Adler function.

$$4\pi^2 D(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(s)^{n+1} \,,$$

where $r_n = c_{n+1,1}/\pi^{n+1}$. The Borel-transform reads:

$$\widehat{D}(\alpha_s) = \int_0^\infty \mathrm{d}t \, \mathrm{e}^{-t/\alpha_s} B[\widehat{D}](t) \,; \quad B[\widehat{D}](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!} \,.$$

Generally, the Borel-transform $B[\widehat{D}]$ developes poles and cuts at integer values p of $u \equiv \beta_1 t/(2\pi)$. (Except at $u=1$.)

The poles at negative p are called UV renormalon poles and the ones at positive p IR renormalons.

Borel model

To proceed, realistic model $B[\widehat{D}](u)$: (Beneke, MJ 2008)

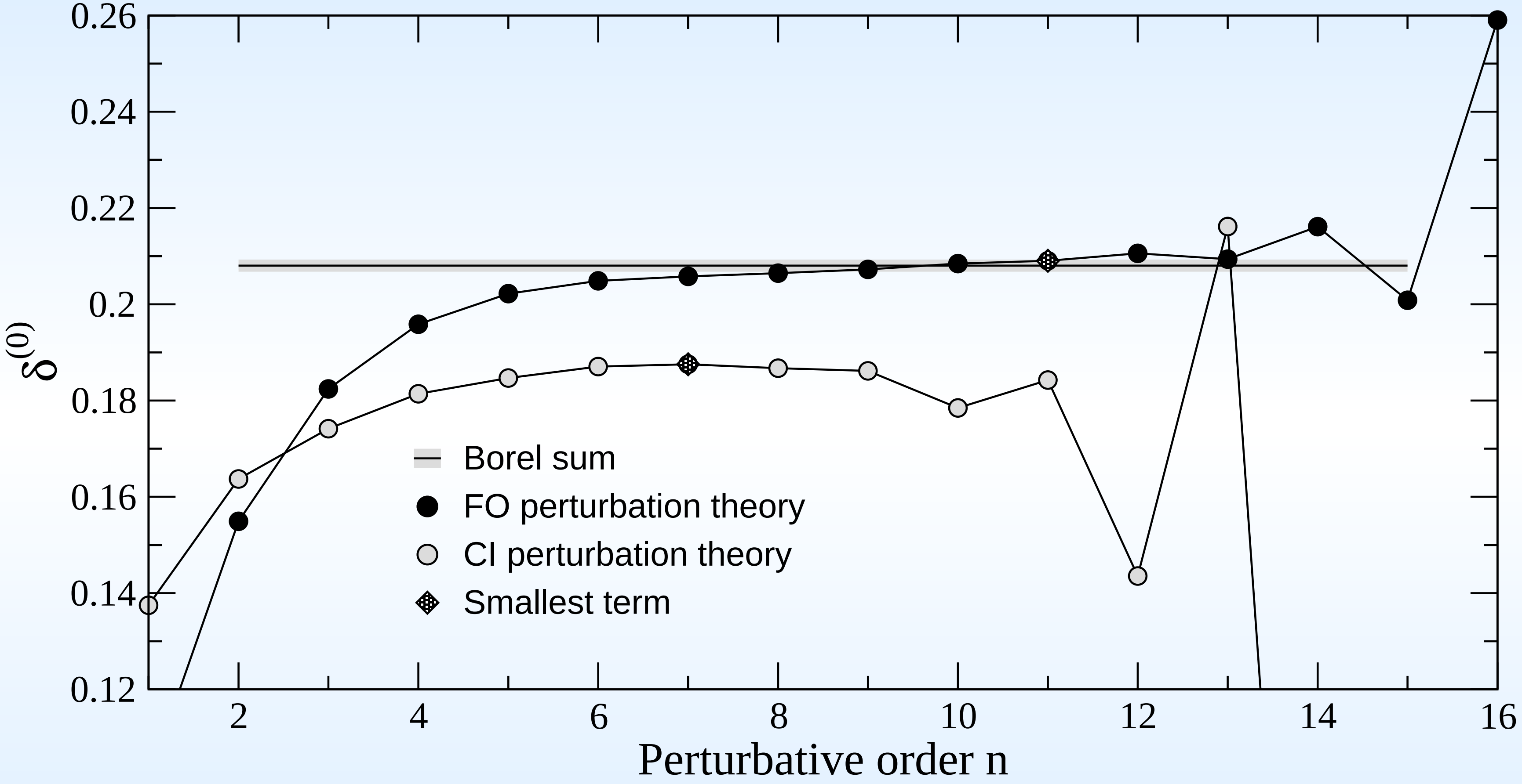
$$B[\widehat{D}](u) = B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_2^{\rm IR}](u) + B[\widehat{D}_3^{\rm IR}](u) + d_0^{\rm PO} + d_1^{\rm PO}\, u\,,$$

where

$$B[\widehat{D}_p](u) = \frac{d_p}{(p\pm u)^{1+\gamma}}\left[\,1 + b_1(p\pm u) + b_2(p\pm u)^2\,\right].$$

- Main model incorporates the leading UV pole ($u=-1$), as well as the two leading IR renormalons ($u=2,3$).
- It should reproduce the exactly known $c_{n,1}$, $n\leq 4$.
- For both UV and IR, the residues d_p are free while γ, $b_{1,2}$ depend on anomalous dimensions and β-coefficients.

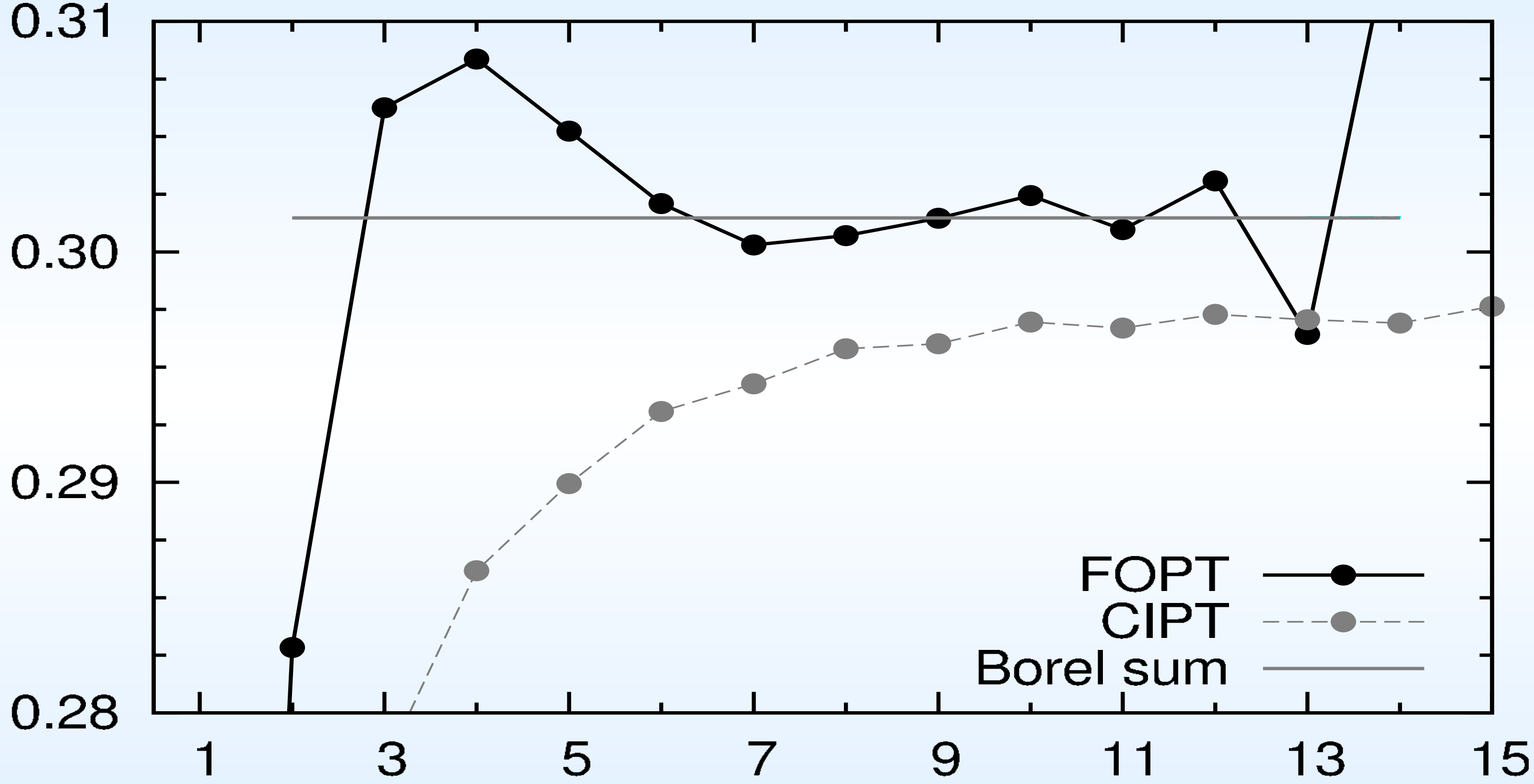
Central Borel model



$$\alpha_s(M_\tau) = 0.3186\,, \quad c_{5,1} = 283\,.$$ (Beneke, MJ 2008)

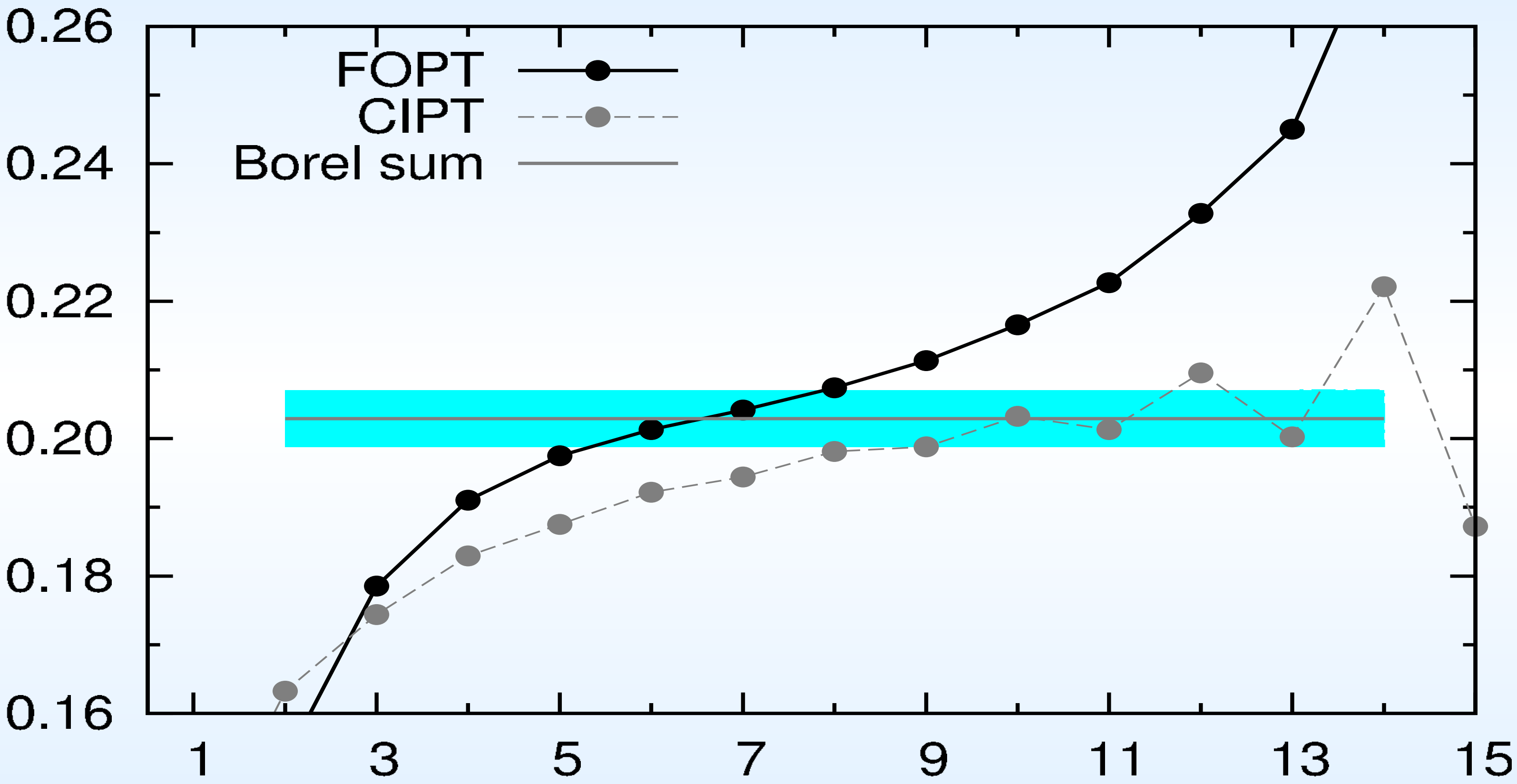
Other moments

$$w(x) = 1:$$



$\alpha_s(M_\tau) = 0.3186\,, \quad c_{5,1} = 283\,.$ (Beneke, Boito, MJ 2012)

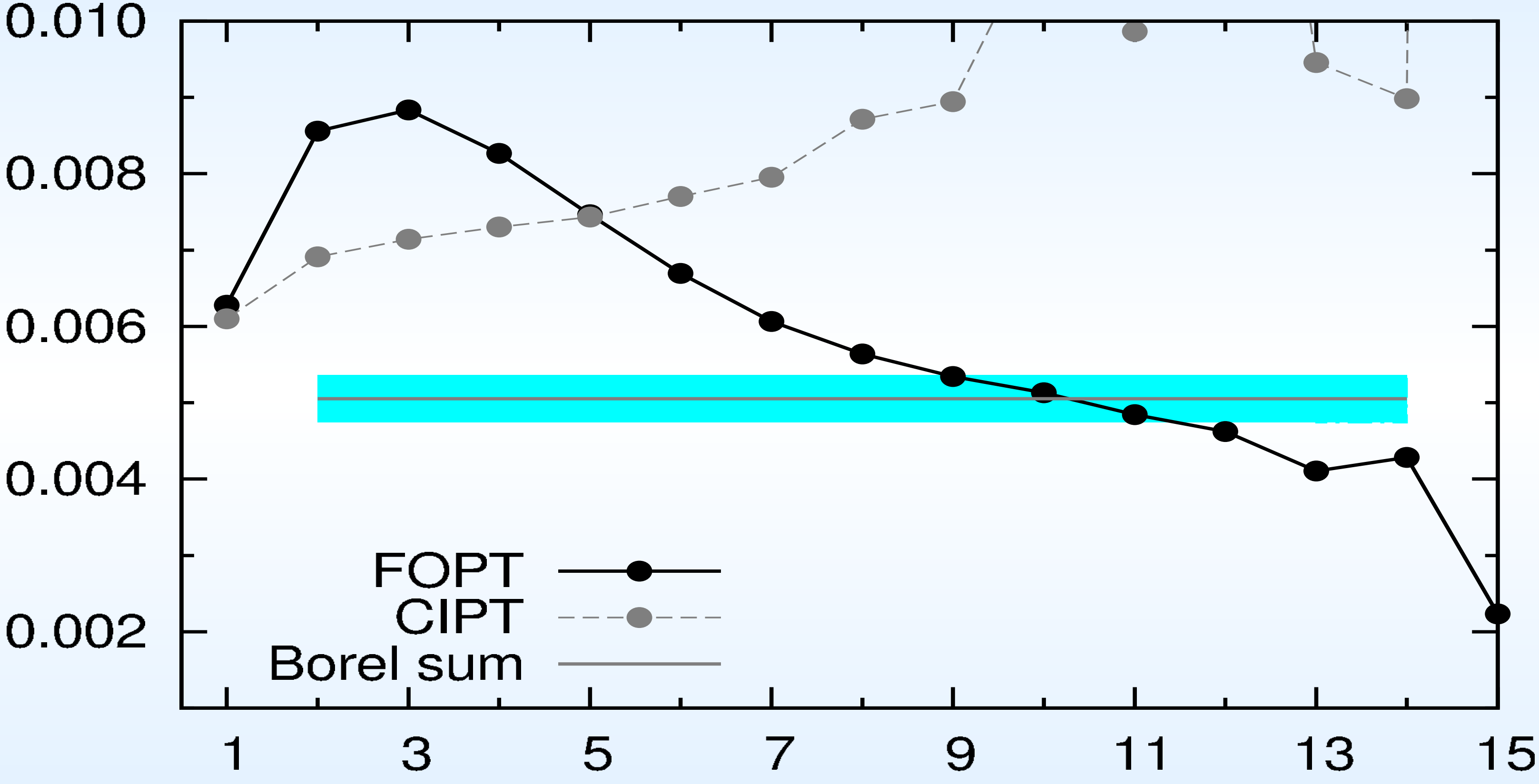
$w(x) = 1 - x$:



$\alpha_s(M_\tau) = 0.3186\,, \quad c_{5,1} = 283\,.$ (Beneke, Boito, MJ 2012)

Other moments

$$w(x) = (1-x)^3 x^2 (1+2x):$$



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Model dependence

The behaviour of the Borel model crucially depends on the residue of the gluon-condensate renormalon pole.

Assuming some sensitivity to the $u=2$ pole at intermediate orders (3-5), a fit to the known $c_{n,1}$ yields $d_2^{\rm IR} \approx 3.2$.

For small $d_2^{\rm IR}$, models can be constructed for which Contour-improved PT is the preferred resummation.

Hence, to make progress the value of $d_2^{\rm IR}$ should be corroborated. Two possible routes:

i) As the renormalon ambiguity is universal, employ PT series of other correlators to obtain additional information.

ii) Determine $d_2^{\rm IR}$ from the lattice. Not possible directly for the Adler function, but for the plaquette.

Duality violations

In the OPE, close to the Minkowskian axis ($s > 0$), so-called Duality Violations (DV's) can appear.

They can be studied on the basis of a toy-model:

(Shifman et al. 1998-2000)

(Catà, Golterman, Peris 2005/2008)

$$\Pi_V(s) = -\,\psi\left(\frac{M_V^2 + u(s)}{\Lambda^2}\right) + \text{const.}\,.$$

where

$$u(s) = \Lambda^2 \left(\frac{-s}{\Lambda^2}\right)^{\zeta} \quad \text{and} \quad \zeta = 1 - \frac{a}{\pi N_c}\,.$$

The model is based on large-N_c QCD and Regge-theory.

$$M_V = 770\,\mathrm{MeV}, \quad \Lambda = 1.2\,\mathrm{GeV}, \quad a = 0.4\,.$$

Duality violations

The OPE corresponds to the asymptotic expansion of the ψ-function for large s (large u).

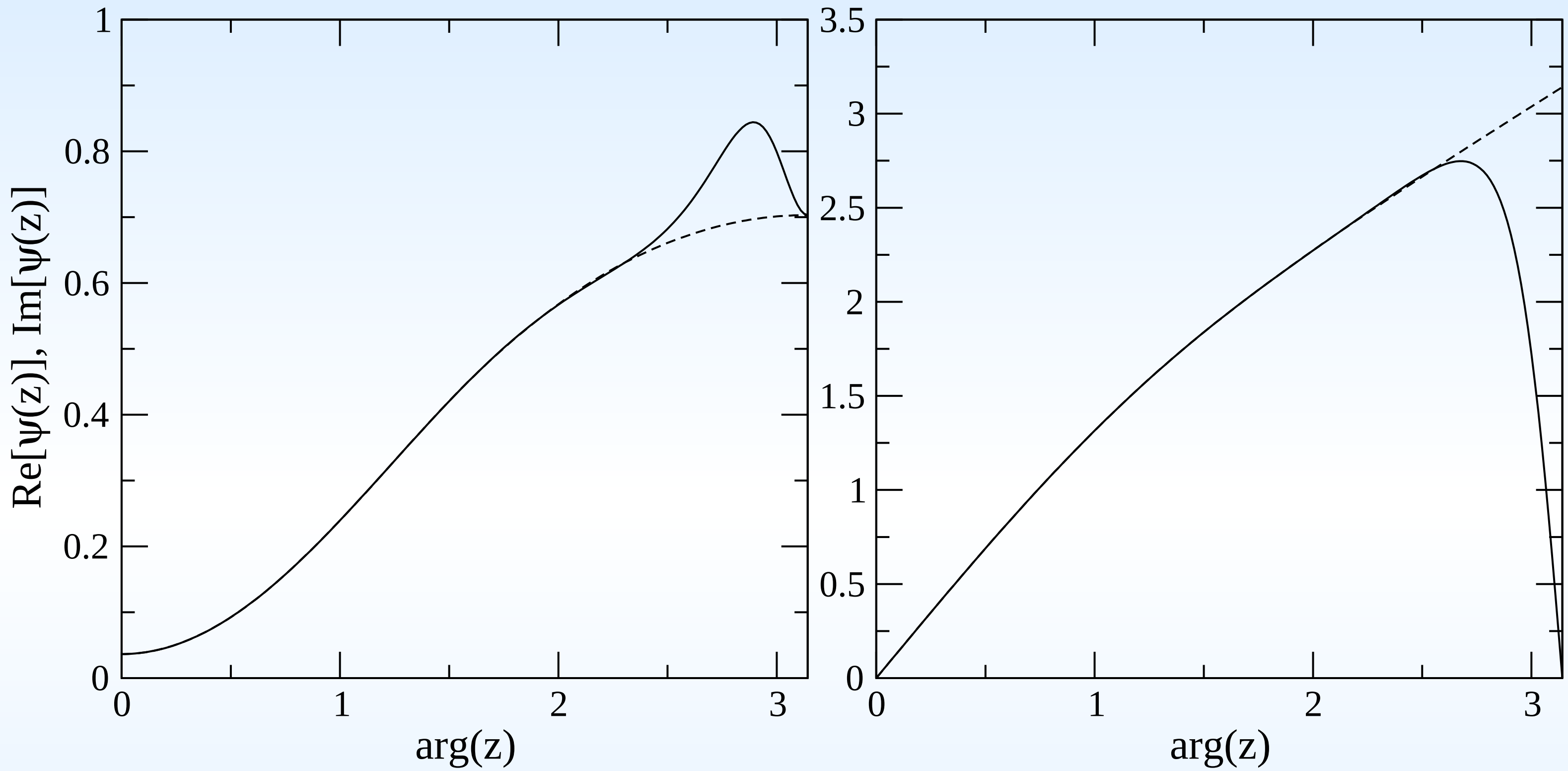
$$\psi(z) \sim \ln z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2n z^{2n}} \,, \qquad \mathrm{Re}\, z > 0 \,.$$

In the Minkowskian region, an additional term arises:

$$-\,\pi \left[\cot(\pi z) \pm i \right], \qquad \mathrm{Re}\, z < 0 \,, \;\; \mathrm{Im}\, z \gtrless 0 \,.$$

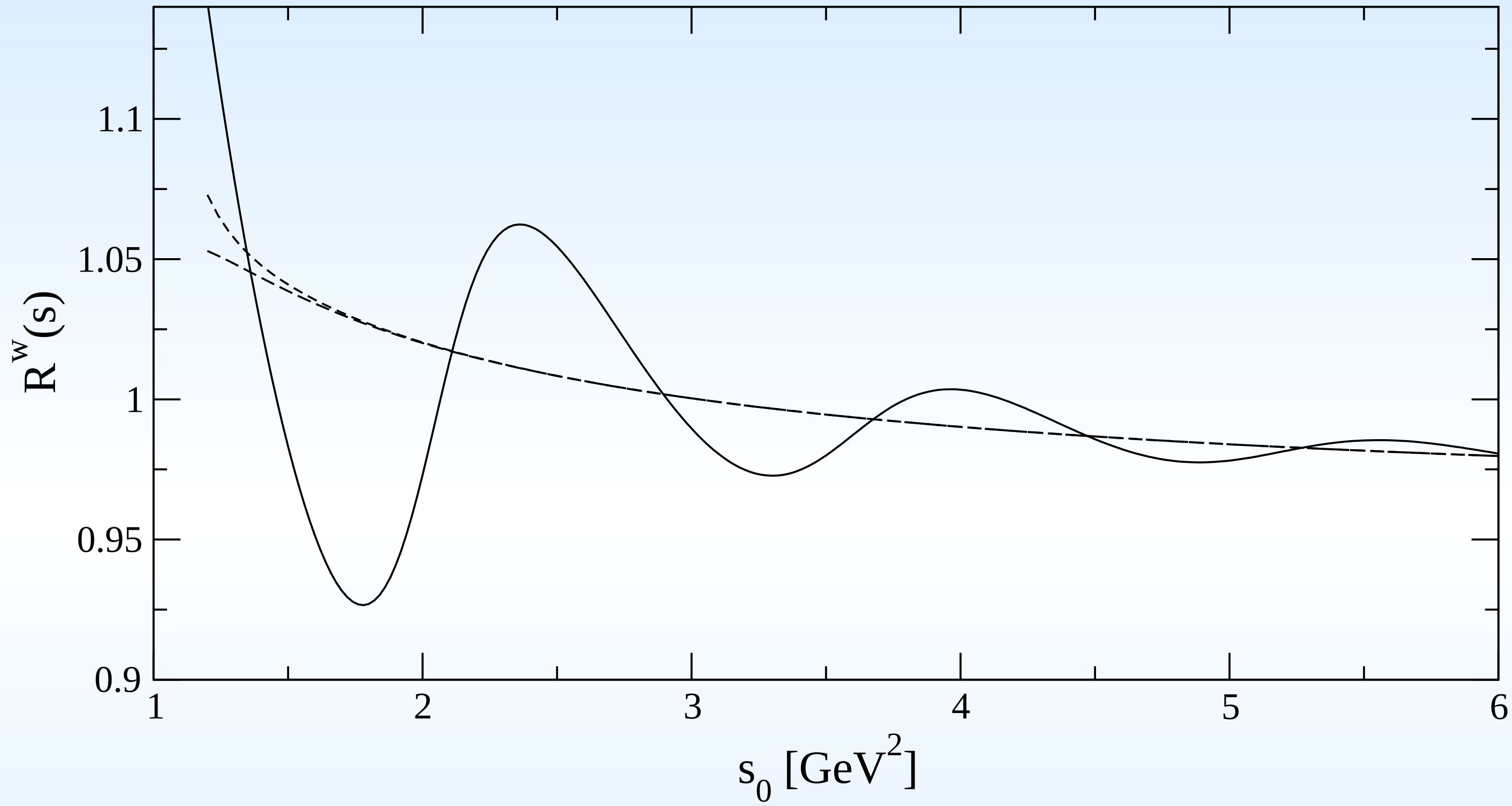
Formally, this term is exponentially suppressed, but it is enhanced by the poles of the ψ-function.

Asymptotic expansion



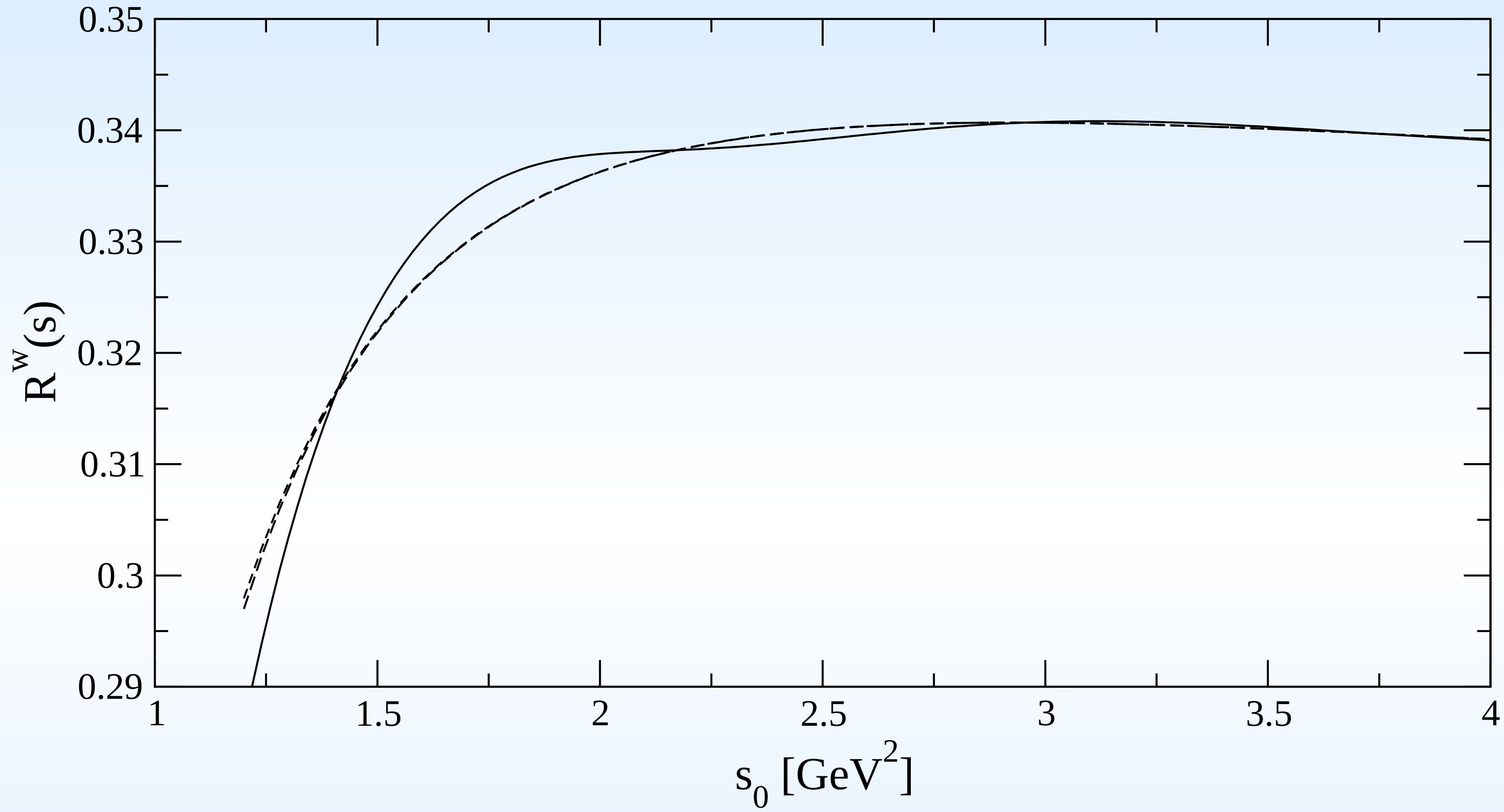
$$z = 1.5 \cdot \exp\left(i\varphi\right)$$

ψ-function moments



ψ-function moment for $w(z) = 1$.

ψ-function moments



ψ-function moment for $w(z) = (1 - z)^2$.

Fits to OPAL data

In fits to experimental data, a model for DV's should be included.

The ψ-function model suggests an oscillating, decaying exponential, which can be chosen of the form:

$$\rho^{\rm DV}_{V/A}(s) = \kappa_{V/A}\, {\rm e}^{\gamma_{V/A} s} \sin\left(\alpha_{V/A} + \beta_{V/A} s\right) .$$

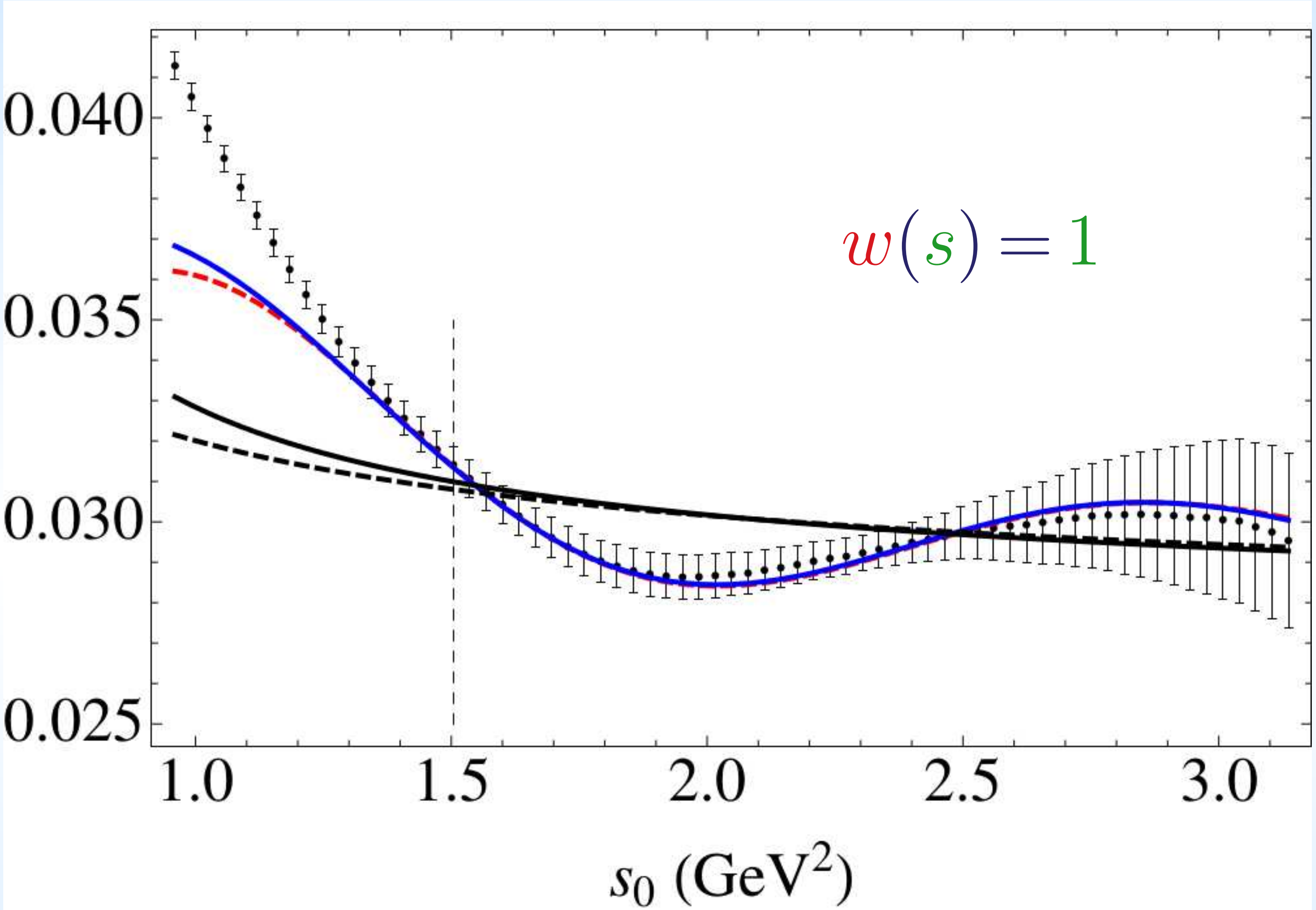
The fit quantities are the w-moments of the exp spectra.

$$R^w_{\tau,V/A}(s_0) \equiv \int\limits_0^{s_0} ds\, w(s)\, \rho_{V/A}(s) .$$

The cleanest moment turns out to be $w(s) = 1$.

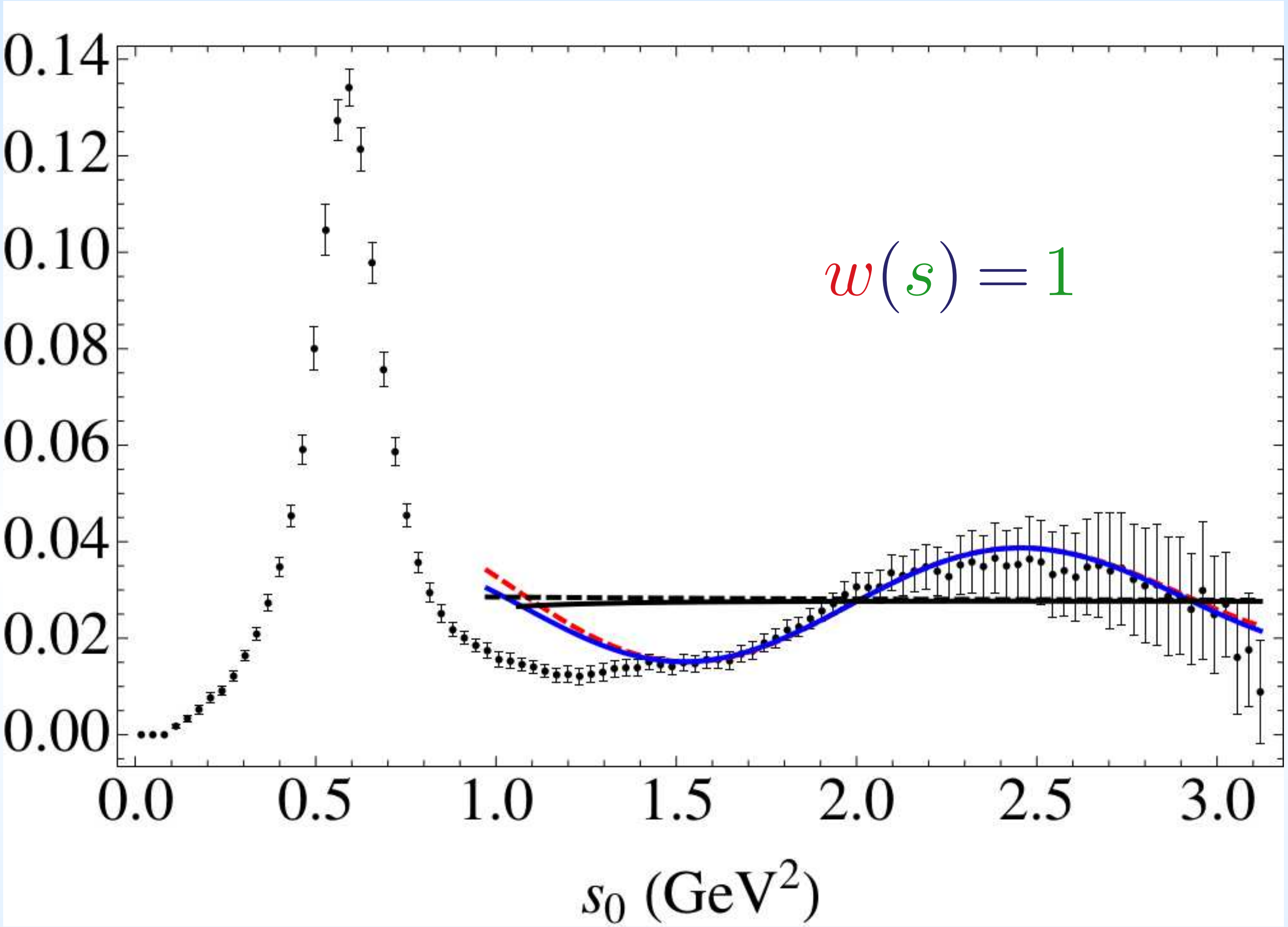
Fitting combinations of several moments is complicated by very strong correlations.

OPAL V-moment



(Boito, Golterman, MJ, Mahdavi, Maltman, Osborne, Peris 2012)

OPAL V-spectrum



(Boito, Golterman, MJ, Mahdavi, Maltman, Osborne, Peris 2012)

Summary

☞ Presently, the most reliable value of α_s from τ's including DV's comes from the trivial moment $w(s) = 1$.

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.325 \pm 0.016 \pm 0.007 \quad \text{(FOPT)}$$

$$\Rightarrow \quad \alpha_s(M_\tau) = 0.347 \pm 0.024 \pm 0.005 \quad \text{(CIPT)}$$

☞ This moment only has very small contaminations from QCD condensate contributions.

☞ These values should be compared to the World Average (PDG 2012): $\alpha_s(M_\tau) = 0.3186 \pm 0.0058$.

Main messages

- ☞ The ambiguity in the perturbative resummation should be overcome. Possible routes: dedicated moments and/or better understanding of Borel models.

- ☞ Use of several moments to fit α_s and condensates for consistency should include duality violations.

- ☞ Due to strong correlations in moment fits, better τ spectral data, e.g. from B-factories would be extremely helpful to resolve the theoretical issues.

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Thank You for Your attention !