Charm and Bottom mass determination from relativistic QCD sum rules at four loops

Bahman Dehnadi

University of Vienna

23 - 05 - 2013

arXiv:1102.2264 [hep-ph]: A. H. Hoang – MPI & U. Vienna
V. Mateu – MIT & IFIC
B. Dehnadi – Shiraz University & U. Vienna
S. M. Zebarjad – Shiraz University

Outline

- **General remarks on heavy quark masses**
  - Concept of mass in QCD.
  - Motivations for a precise determination.
  - Recent results.
  - Relativistic QCD sum rules.
- **Experimental data**
  - Collecting experimental data.
  - How to combine data from different experiments? new
  - How to treat errors and correlations?
  - Experimental moments and examination of method.
- **Theoretical developments**
  - Analytic properties (Various expansions at four loops).
  - OPE and non-perturbative contributions.
  - Estimate of (theoretical) perturbative uncertainties. new
- **Results for Charm Mass**
- **Bottom Mass**

INTRODUCTION

Remarks on heavy quark masses

Parameter in QCD Lagrangian $\longrightarrow$ formal definition (as strong coupling)

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k \left(i\not{D} - m_k \right) q_k - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

Confinement $\longrightarrow$ m_k is not physical observable

Running mass: Observable and scheme dependent

$\overline{\mathrm{MS}}$ Scheme

- Short distance scheme.
- Standard mass for comparison: $\overline{m}_q\,(\overline{m}_q\,)$
- Free of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon ambiguity.

Impact of precision

Strong dependence in flavour processes

Constrains new physics

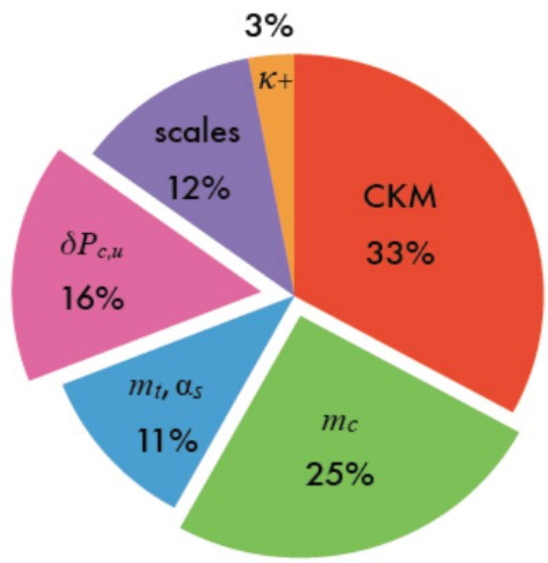
SM prediction(s) of $K^+ \to \pi^+ \nu\bar{\nu}$: error budget

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \{8.57 \pm 0.93, 8.51 \pm 0.73, 8.04 \pm 0.98\} \times 10^{-11}$$

$$m_c(m_c) = (1.30 \pm 0.05)\ \mathrm{GeV}$$

$$m_c(m_c) = (1.286 \pm 0.013)\ \mathrm{GeV}^{*}$$

$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)\ \mathrm{GeV}^{\dagger}$$



from U. Haisch

*Kühn et al. '07 †Hoang & Manohar '05

Determinations of m_c

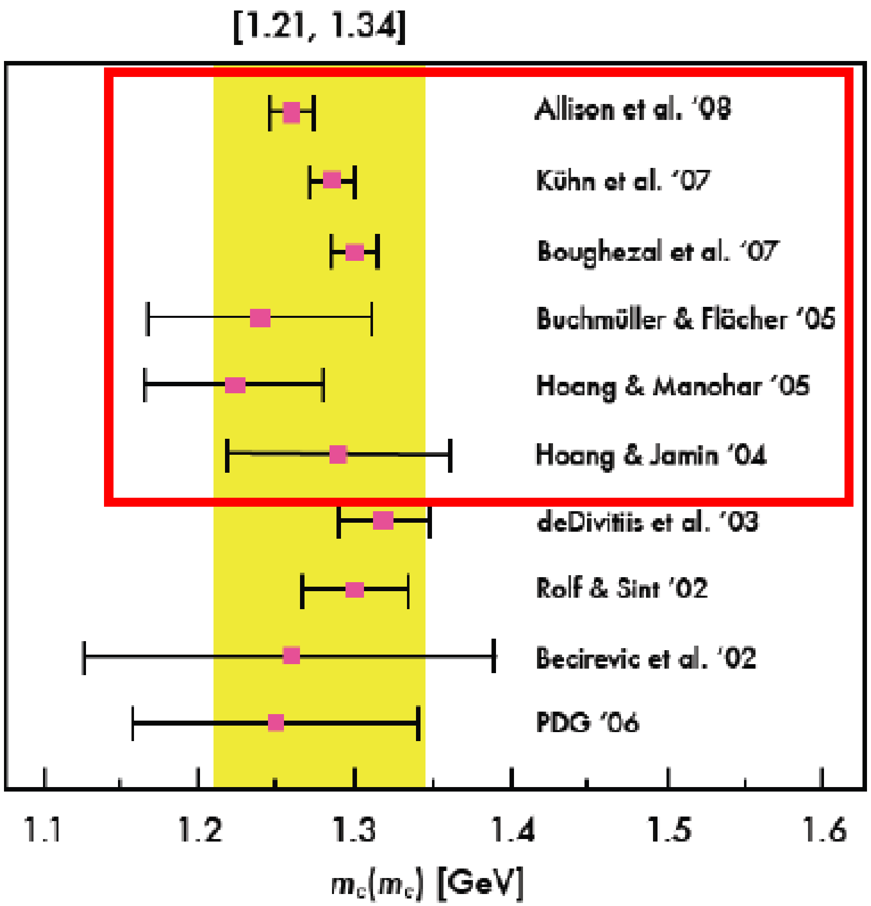
Spectral moments of inclusive B decays (nonrelativistic)

Charmominum sum rules (relativistic)

Lattice

Taken from A. Hoang

Flavor institute CERN 2008



$m_c(m_c)$ [GeV]	method
1.266 ± 0.014	lattice, unquenched, staggered
1.286 ± 0.013	low-momentum sum rules, N^3LO
1.295 ± 0.015	low-momentum sum rules, N^3LO
1.24 ± 0.07	fit to B-decay distribution, $\alpha_s^2\beta_0$
$1.224 \pm 0.017 \pm 0.054$	fit to B-decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
$1.26 \pm 0.04 \pm 0.12$	lattice, quenched
1.25 ± 0.09	PDG 2006

Relativistic sum rules

Total hadronic cross section

$$R_{e^+e^-\to c\bar{c}+X}(s) = \frac{\sigma_{e^+e^-\to c\bar{c}+X}(s)}{\sigma_{e^+e^-\to \mu^+\mu^-}(s)}$$

$\Longrightarrow$

Moments of the cross section

$$M_n = \int_{4m^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R_{e^+e^-\to c\bar{c}+X}(s)$$

Vacuum polarization function

$$(g_{\mu\nu}q^2 - q_\mu q_\nu)\ \Pi(q^2) = -i\int \mathrm{d}x\, e^{iqx}\langle 0\,|T\, j_\mu(x) j_\nu(0)|\,0\,\rangle$$

Vector current (electromagnetic)

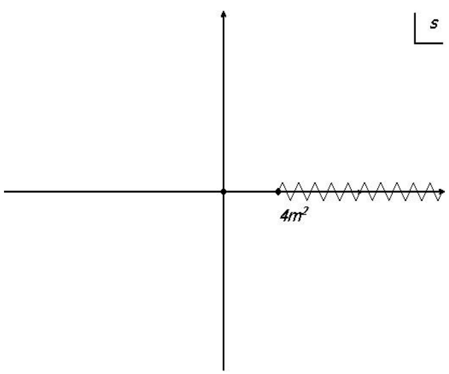
$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$$

electric charge

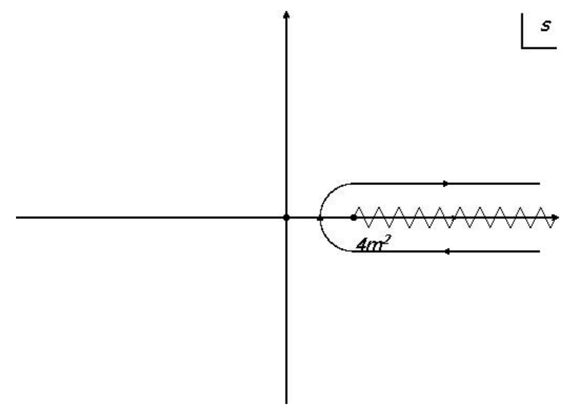
$$R(s) = 12\pi \boxed{Q^2}\, \mathrm{Im}\,\Pi(s+i\,0^+)$$

$\Longrightarrow$

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{12\pi^2 Q^2}\int_{4m^2}^{\infty} \mathrm{d}s\, \frac{R(s)}{s(q^2-s)}$$



$\Longrightarrow$



$$\Pi(q^2 \approx 0, m) = \frac{1}{12\pi^2 Q^2}\sum_{n=0}^{\infty} M_n\, q^{2n}$$

$\Longrightarrow$

$$M_n = 6\pi\, i\, Q^2 \oint \mathrm{d}s\, \frac{\Pi(s)}{s^{n+1}}$$

Relativistic sum rules

Effective energy smearing range: $\boxed{E_{eff} = \frac{m_c}{n}}$ (assimptotically correct for large n)

$$\frac{m_c}{n} \gg \Lambda_{QCD}$$

- Since we want to apply perturbation theory for Wilson coefficients.
- Otherwise the OPE converges badly.

n=1 is the cleanest moment, and we will focuss on it for the analyses presented in this seminar.

(n = 2 is also fine)

Determination of m_q from sum rules

Fixed order analysis
(correlated variation)
$\mu_\alpha = \mu_m$

charm

Kühn et al ('08)[3] $\bar{m}_c(\bar{m}_c) = 1.286 \pm \boxed{0.009_{\exp}} \pm 0.009_\alpha \pm 0.002_\mu$

Boughezal et al ('08) [4] $1.295 \pm 0.012_{\exp} \pm 0.009_\alpha \pm 0.003_\mu$

Maier et al (08) [5] $1.277 \pm \boxed{0.006_{\exp}} \pm 0.014_\alpha \pm 0.005_\mu$

Only for n = 1 **[3,4]**, 2 **[5]** 3-loops in pert. theory. Updated experimental data

Tiny errors! (underestimated ?)

Need for more general analysis

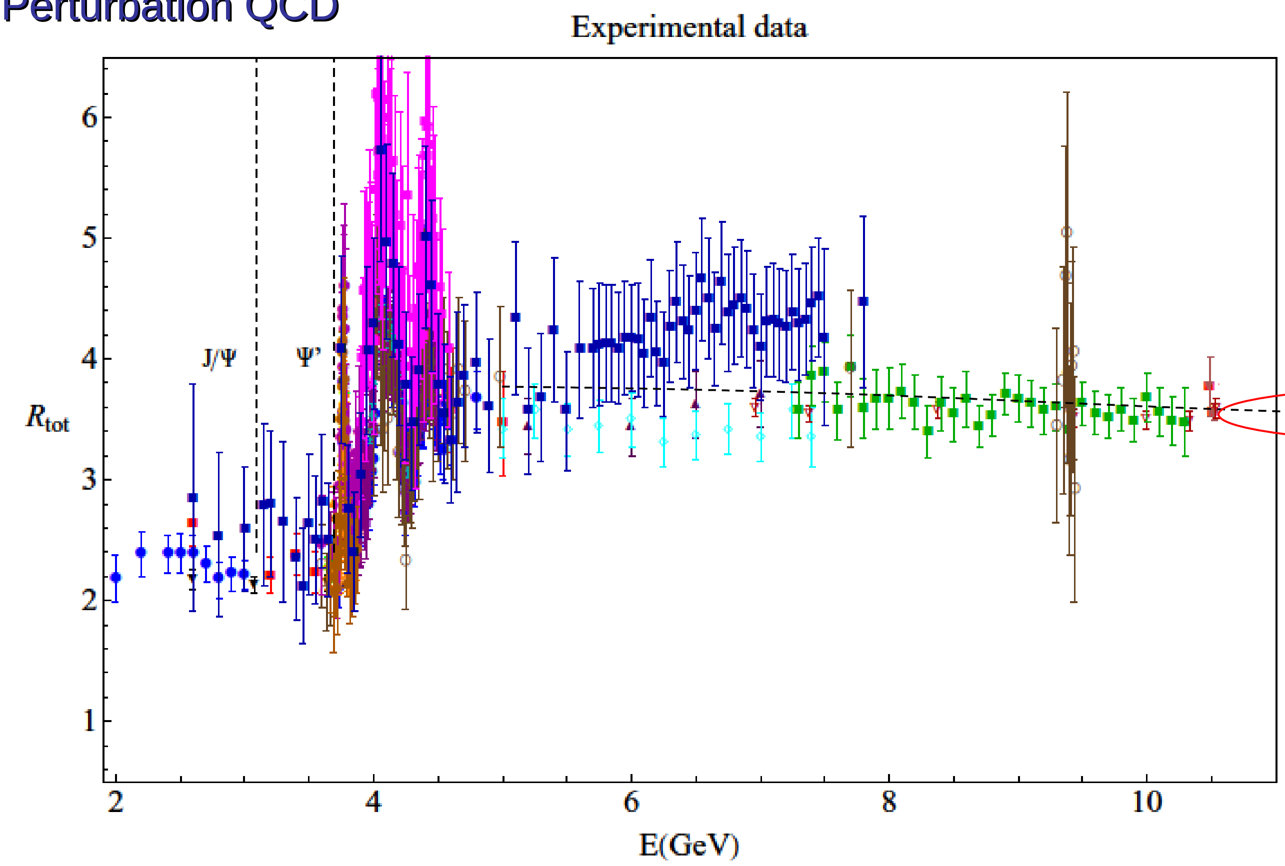
The aims of this work:

- An account of all available hadronic cross section data.
- A thorough analysis of perturbative uncertainities.

Experimental data

Experimental data: charm

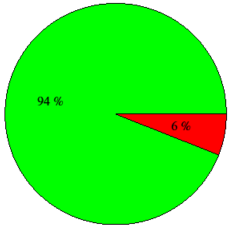
- Resonances
- Sub-Threshold and Threshold (3.73 - 4.8)
- Gap region and high energy region (4.8 - 10.538)
- Perturbation QCD



- Only where there is no data
- Assign a conservative 10% error to reduce model dependence

$M_1 \rightarrow$ 6%

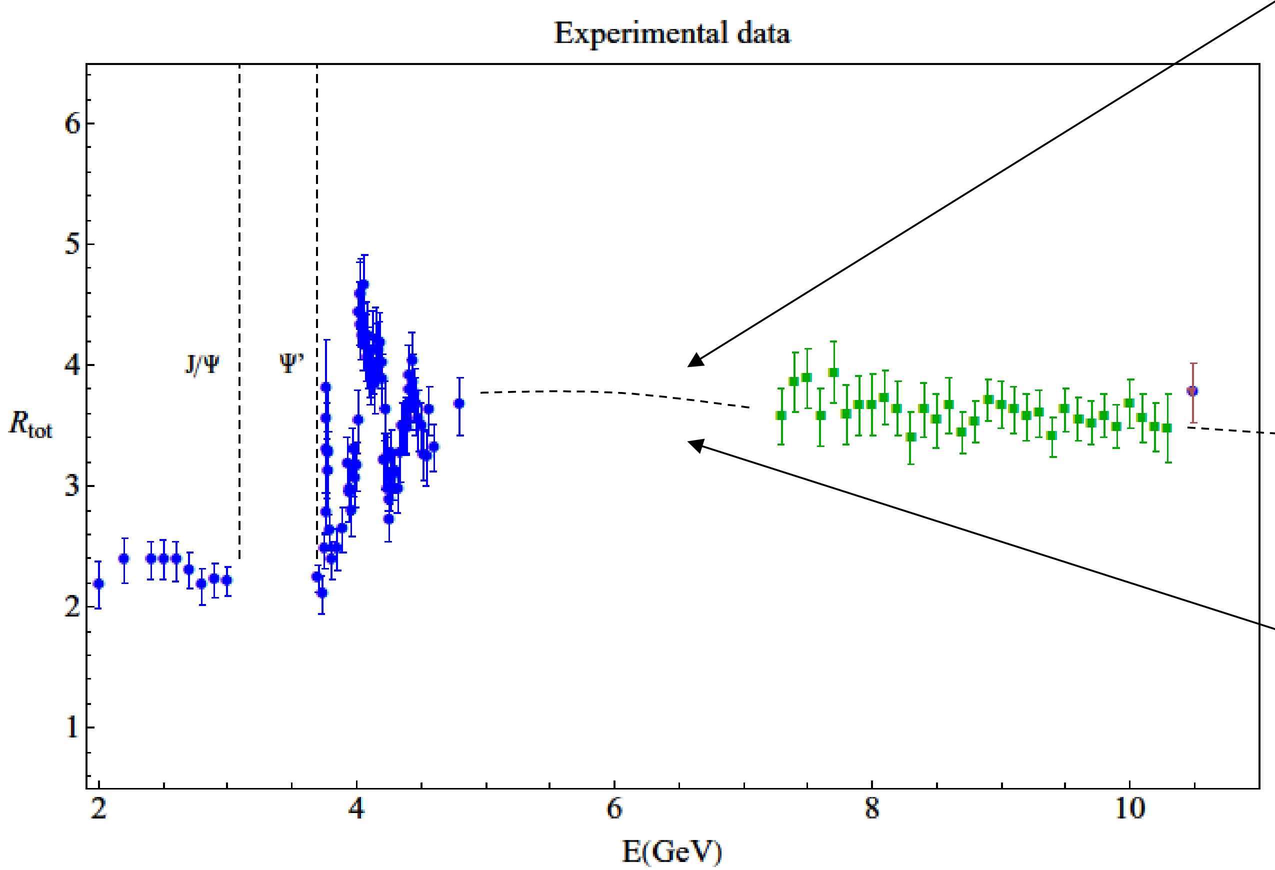
$M_{n>1} \rightarrow$ < 1%



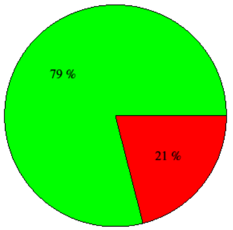
Experimental data: charm

Data used in Hoang and Jamin (2004)

- Perturbation theory only in gap and region with no data
- 10% error assigned as well

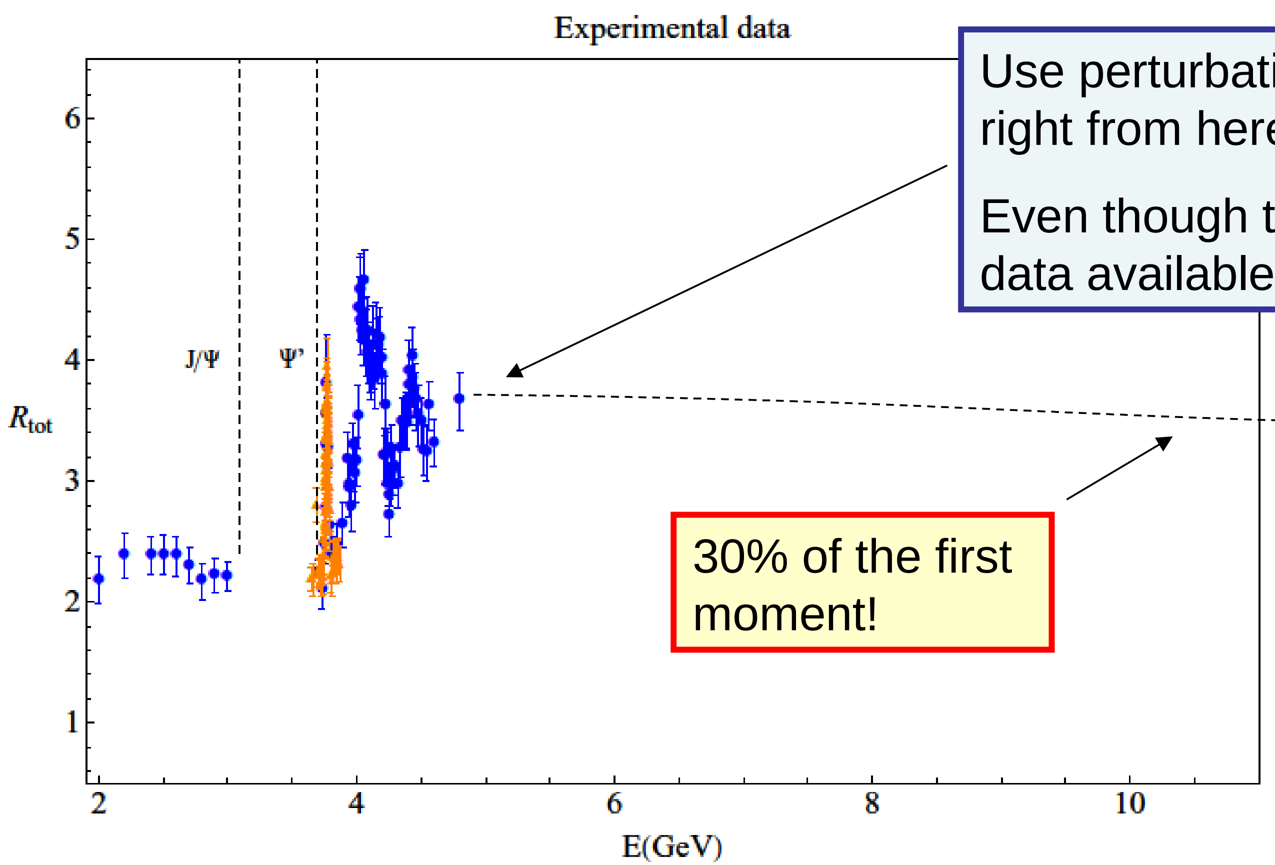


21% of the first moment



Experimental data: charm

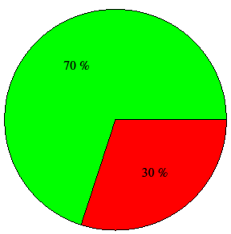
Data used in Kühn et al (2004, 2005, ...)



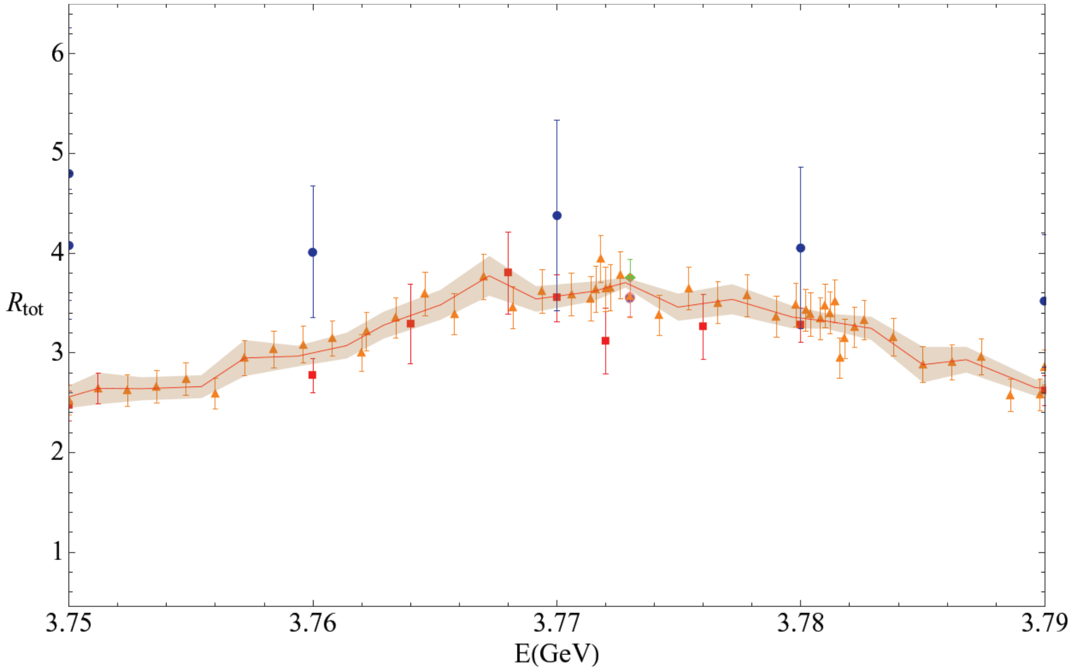
Use perturbation theory right from here!

Even though there is data available...

30% of the first moment!

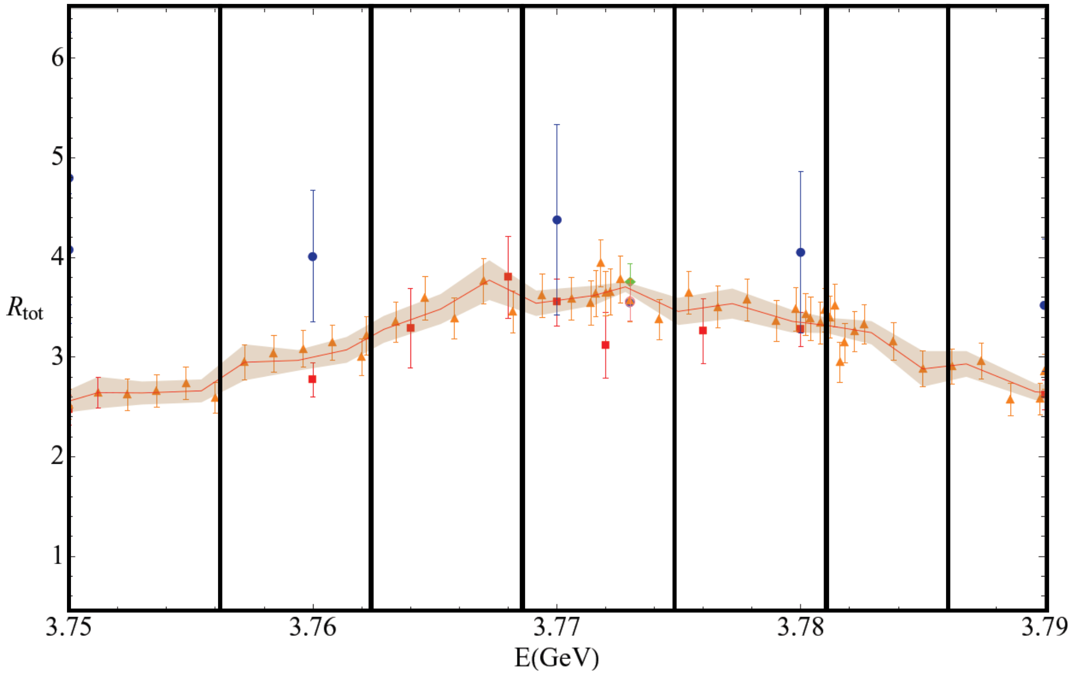


Fit procedure



Method inspired by a similar one in Hagiwara, Martin & Teubner.

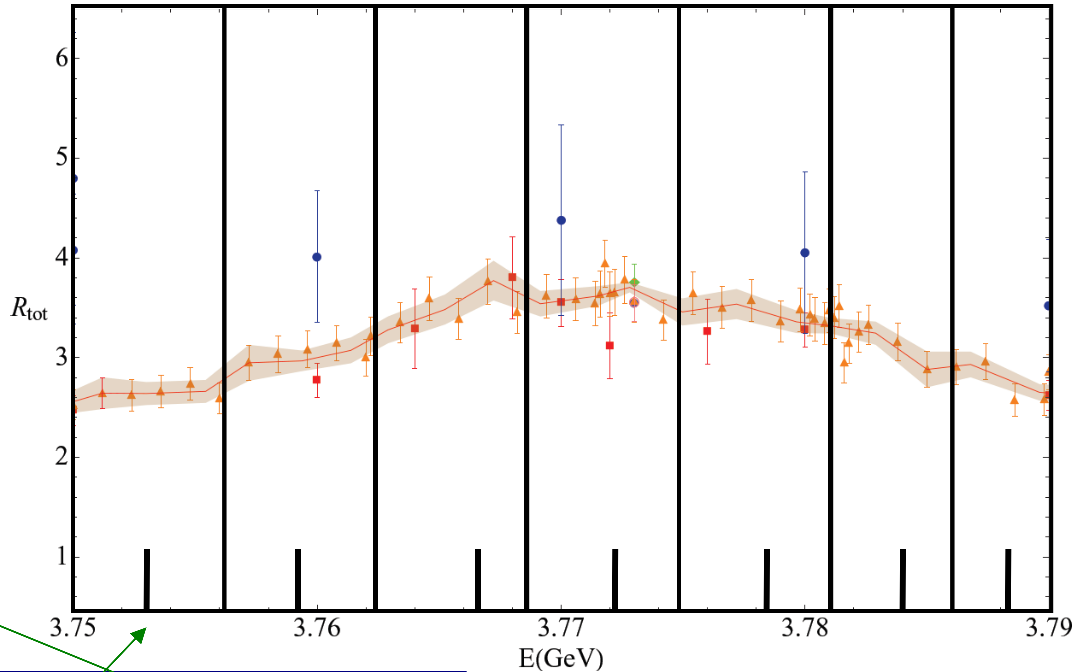
Fit procedure



1. Recluster data. Clusters not necessarily equally sized.

   Number of clusters and size of cluster according to the structure of the data

Fit procedure



Experimental energies

Correlated error

$$E_m = \frac{\sum_{k,i} \frac{E_i^{k,m}}{(\sigma_i^{k,m})^2 + (\Delta_i^{k,m})^2}}{\sum_{k,i} \frac{1}{(\sigma_i^{k,m})^2 + (\Delta_i^{k,m})^2}}$$

Cluster energy

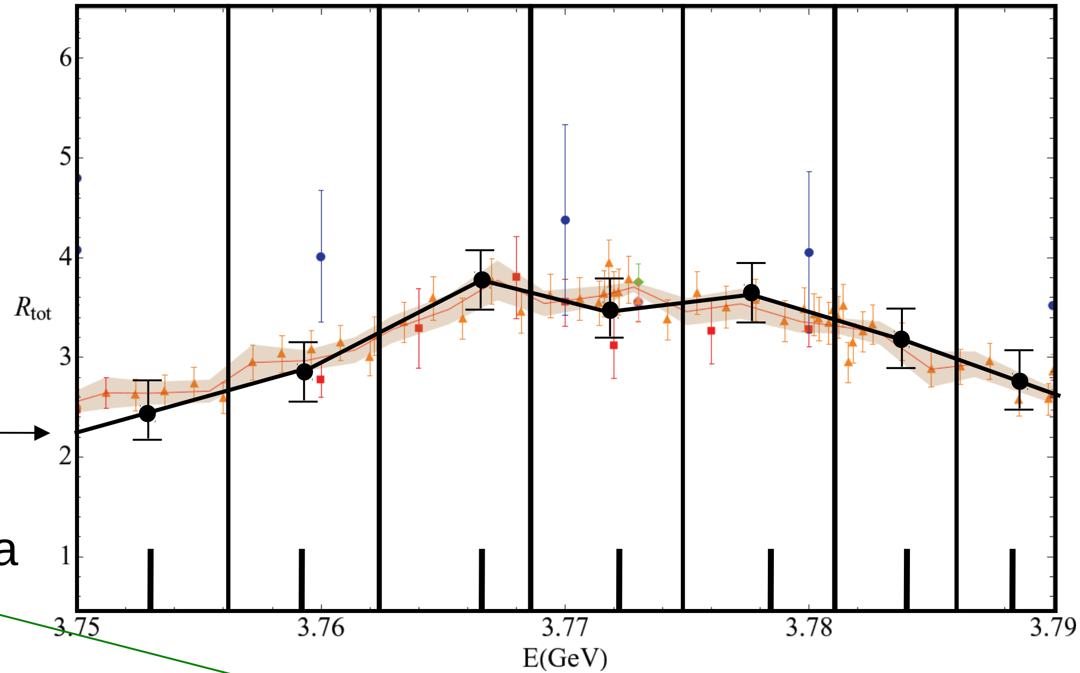
Uncorrelated error

- k : Label for experiments
- $N_{\rm exp}$: Number of experiments
- m : Label for clusters
- $N_{\rm clusters}$: Number of clusters
- i : Label for data points
- $N^{k,m}$: Number of data points for experiment k in cluster m

2. Calculate the energy of the cluster. One weights the energy of the data points inside the clusters with their errors.

Fit procedure

R_n



Experimental data

$$\chi^2 = \sum_{k=1}^{N_{\text{exp}}} \left[b_k^2 + \sum_{k=1}^{N_{\{k,m\}}} \left(\frac{R_{\text{uds}}^{\{k,m\}} - (1 + \Delta f_i^{\{k,m\}}\, b_k)\, n\, R_{had}(E_{uds}^{\{k,m\}})}{\sigma_i^{\{k,m\}}} \right)^2 + \sum_{i=2}^{N_{\text{clusters}}} \sum_{k=1}^{N_{\{k,m\}}} \left(\frac{R_i^{\{k,m\}} - (1 + \Delta f_i^{\{k,m\}}\, b_k)(R_i + n\, R_{had}(E_i^{\{k,m\}}))}{\sigma_i^{\{k,m\}}} \right)^2 \right]$$

Fit parameters

3. Fit the value of R for each cluster. Data is allowed to “move” within its systematic error. The method renders errors and correlations among various clusters. One can then calculate errors and correlations for the moments.

Fit procedure

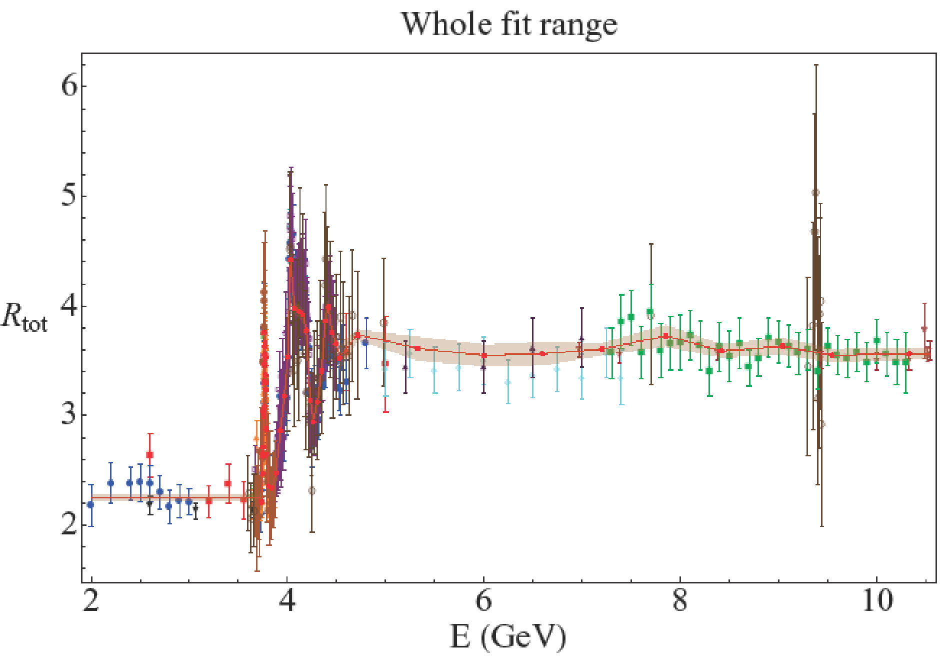
Prediction for moments $M_n = m_n 10^{n+1}\ \mathrm{GeV}^{n+1}$
$M_1 = 21.38 \pm 0.20_{stat} \pm 0.46_{sys}$ $M_2 = 14.91 \pm 0.18_{stat} \pm 0.29_{sys}$ $M_3 = 13.11 \pm 0.19_{stat} \pm 0.25_{sys}$ $M_4 = 12.49 \pm 0.19_{stat} \pm 0.23_{sys}$

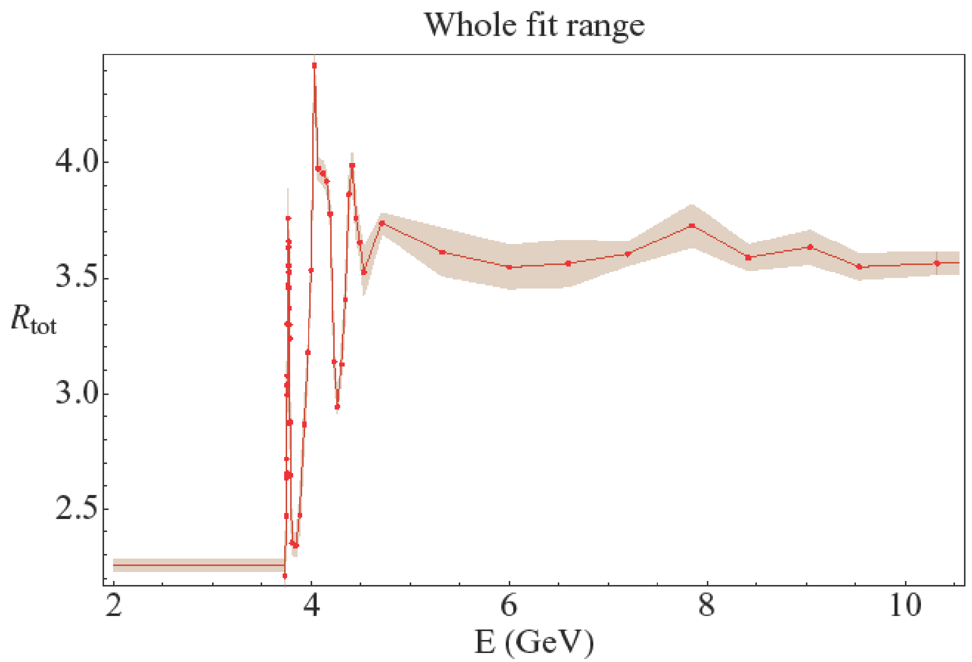
$$\frac{\chi^2}{\mathrm{d.o.f.}} = 1.89$$

We also predict correlations among the various moments, useful for simultaneous fits.

$$C^{\mathrm{exp}} = \begin{pmatrix} 0.250 & 0.167 & 0.147 & 0.142 \\ 0.167 & 0.120 & 0.107 & 0.103 \\ 0.147 & 0.107 & 0.095 & 0.092 \\ 0.142 & 0.103 & 0.092 & 0.090 \end{pmatrix} \quad C^{\mathrm{exp}}_{\mathrm{uc}} = \begin{pmatrix} 0.041 & 0.035 & 0.034 & 0.034 \\ 0.035 & 0.034 & 0.034 & 0.035 \\ 0.034 & 0.034 & 0.035 & 0.036 \\ 0.034 & 0.035 & 0.036 & 0.037 \end{pmatrix}$$

Fit results

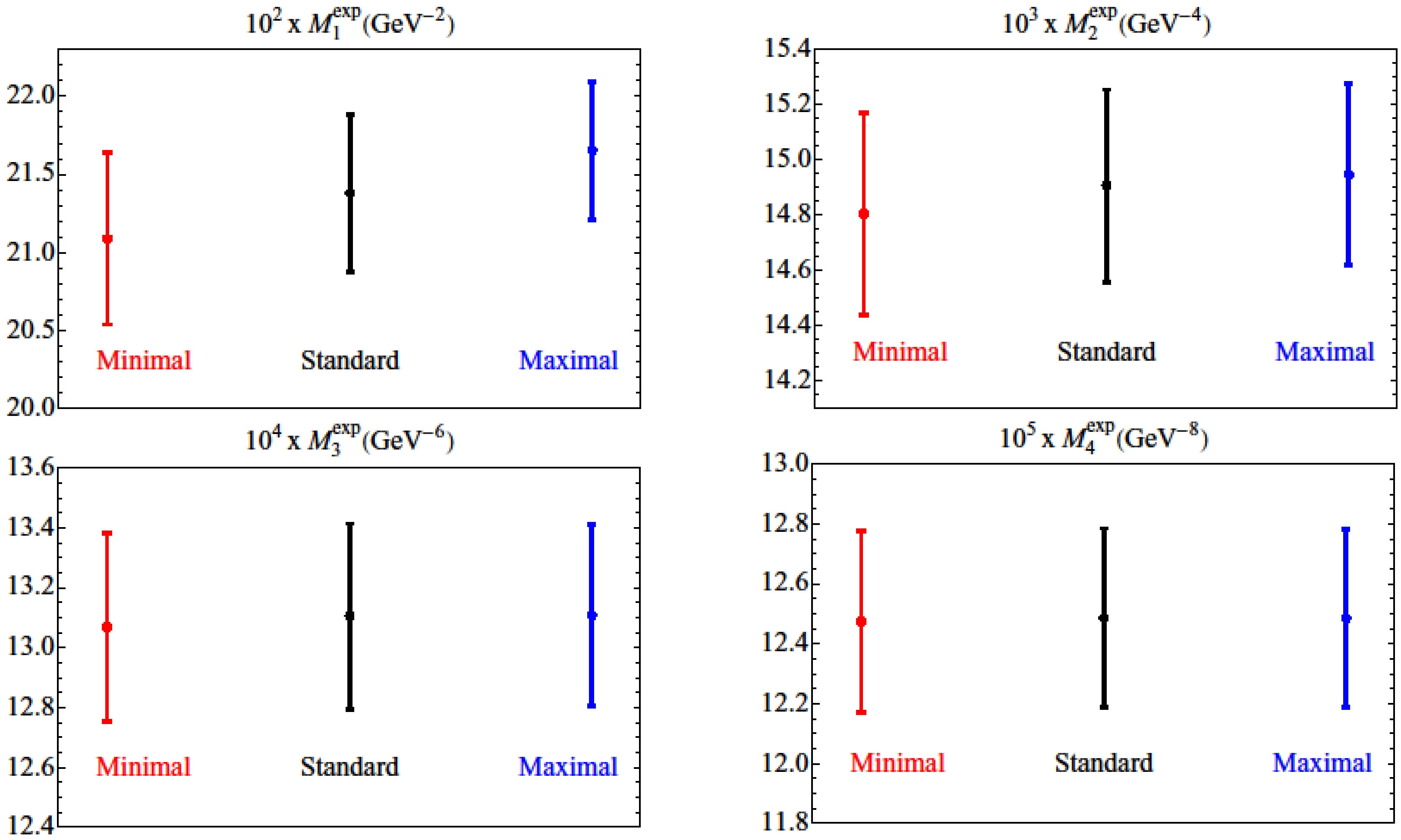




Our default fit assumptions:

I.One quadratic half of resonances partial width uncertainties uncorrelated / other half correlated.
II.Treating the entire systematic uncertainties of R-ratio as correlated when it is not specified.
III. Defining the cluster energies through the weighed average.
IV.Using cluster distribution (2,20,20,10) .
V. Default data set collection.

Comparison selections



Minimal selection: All data necessary to cover the whole energy region with the most accurate ones.

Standard selection: All data sets except three ones with the largest uncertainties.

Maximal selection: Contains all 19 data sets.

Stability of choices

Default: widths 50% correlated among themselves and with the continuous data sets.

For those sets with no information on correlations, assume a 100% correlation.

Different correlation between narrow resonances and data

Different correlation for some datasets

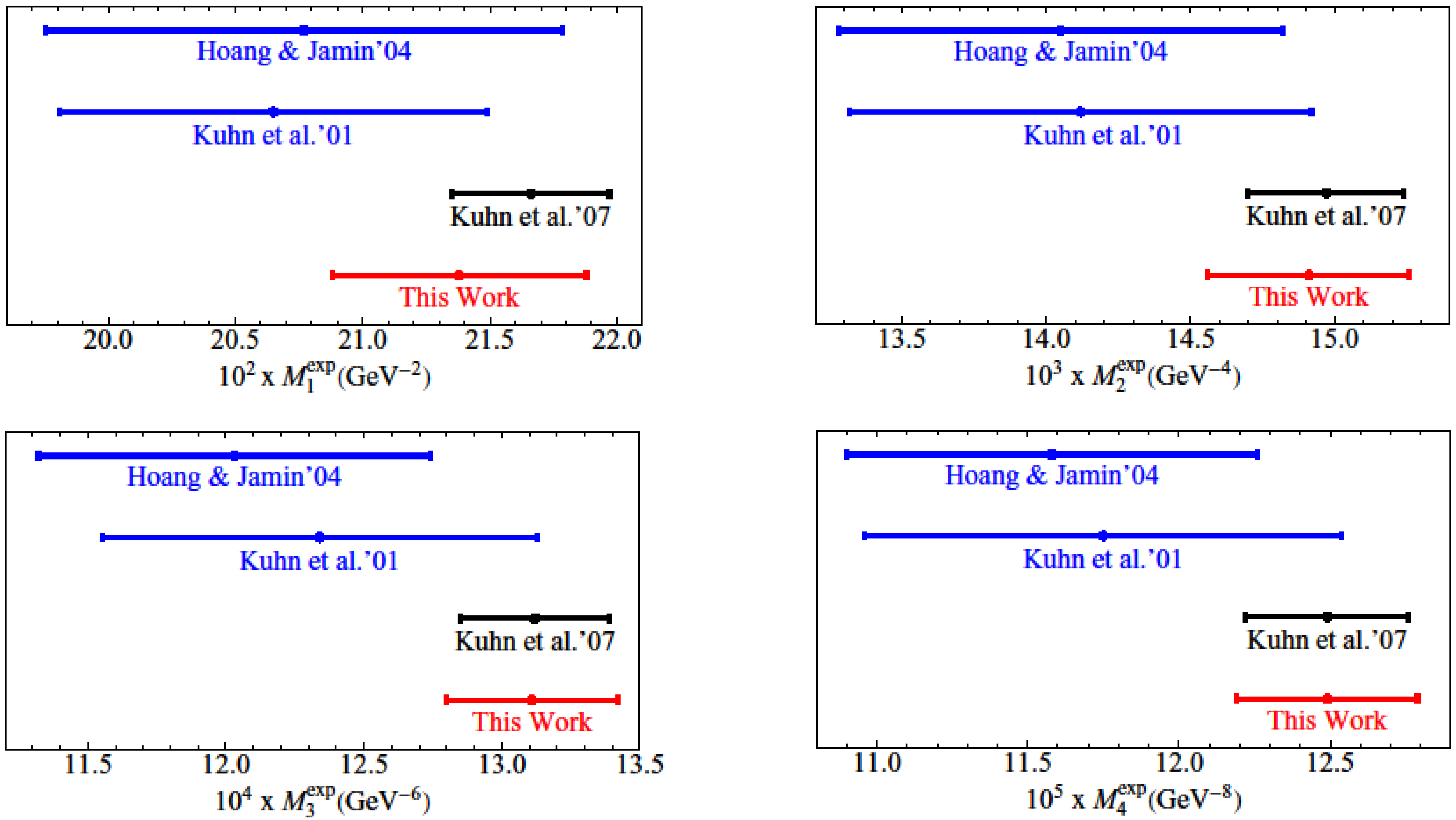
	Default	Minimal Overlap	No Correlation	50% Correlation	Uncorrelated
$n = 1$	21.38(20|46)	21.38(20|37)	21.38(20|30)	21.24(22|47)	21.09(28|25)
$n = 2$	14.91(18|29)	14.91(18|25)	14.91(18|22)	14.87(19|30)	14.84(20|21)
$n = 3$	13.10(19|25)	13.10(19|21)	13.10(19|22)	13.10(19|25)	13.10(19|21)
$n = 4$	12.49(19|23)	12.48(19|21)	12.49(19|21)	12.49(19|23)	12.49(19|21)

Different cluster energy definition

Different clustering

	Default	Regular average	Middle point	$(2, 20, 40, 10)$	$(2, 10, 20, 10)$	$(2, 20, 20, 20)$
$n = 1$	21.38(20|46)	21.37(20|46)	21.40(20|45)	21.38(20|46)	21.39(20|46)	21.34(20|46)
$n = 2$	14.91(18|29)	14.90(18|29)	14.91(18|29)	14.91(18|30)	14.91(18|29)	14.88(18|29)
$n = 3$	13.10(19|25)	13.10(19|25)	13.11(19|25)	13.11(19|25)	13.11(19|25)	13.08(19|25)
$n = 4$	12.49(19|23)	12.48(19|23)	12.49(19|23)	12.49(19|23)	12.49(19|23)	12.47(19|23)

Comparison with other analyses



- Blue lines use outdated experimental data for narrow resonances.
- Different analyses tend to agree better for large n → Narrow resonances dominate

Theoretical developments

Methods in perturbation theory

Fixed order $M_n^{\exp} = M_n^{\rm pert} = \dfrac{1}{(4\overline{m}_c^2(\mu_m))^n} \sum_{i,a,b} \left(\dfrac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b} \ln^a\left(\dfrac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\dfrac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$

Numerical solution for mass:

sometimes there is no solution

Expanded out $\left(M_n^{\exp}\right)^{1/2n} = \left(M_n^{\rm th,pert}\right)^{1/2n} = \dfrac{1}{2\overline{m}_c(\mu_m)} \sum_{i,a,b} \left(\dfrac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\dfrac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\dfrac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$

Analytic solution for mass

always has a solution!

Iterative

$$\overline{m}_c^{(0)}(\mu_m) = \frac{1}{2\left(\boldsymbol{M}_n^{\exp}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c^{(1)}(\mu_m) = \frac{1}{2\left(\boldsymbol{M}_n^{\exp}\right)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}(\mu_m)}{\mu_m^2}\right)\right]\right\} \quad \ldots \text{ etc}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Methods in perturbation theory

Fixed order

$$M_n^{\mathrm{exp}} = M_n^{\mathrm{pert}} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

μ_α and μ_m independent

residual μ_α and μ_m dependence due to truncation of α series

Expanded out

$$\left(M_n^{\mathrm{exp}}\right)^{1/2n} = \left(M_n^{\mathrm{th,pert}}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Iterative

$$\overline{m}_c^{(0)}(\mu_m) = \frac{1}{2\left(M_n^{\mathrm{exp}}\right)^{1/2n}}\tilde{C}_{n,0}^{0,0}$$

- residual μ_α dependence
- renders correct μ_m dependence to the order of truncation

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Contour improved analysis

Contour Improved First applied to hadronic tau decays Liberder & Pich ('92)

Now μ depends on s $\rightarrow$ rearrangement of higher order contributions

$$M_n^{\mathrm{c,pert}} = \frac{6\pi Q_c^2}{i}\int_c \frac{\mathrm{d}s}{s^{n+1}}\Pi(q^2,\alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)),\overline{m}_c(\mu_m),\mu_\alpha^c(s,\overline{m}_c^2),\mu_m)$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2\left(1-\frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

Reweights threshold versus continuum effects

$$\Pi^{\overline{\mathrm{MS}}}\left(q^2,\alpha_s(\mu_\alpha^c(q^2,\overline{m}_c^2)),\overline{m}_c(\mu_m),\mu_\alpha^c(q^2,\overline{m}_c^2),\mu_m\right) = \sum_{n=0}^{\infty} q^{2n}\, M_n^{\mathrm{c,pert}}$$

Contour improved methods are (perturbatively) sensitive to the value of $\Pi(0)$

Nonperturbative contribution

$$M_n^{\rm th} = \boxed{M_n^{\rm pert}} + \boxed{\Delta M_n^{\langle G^2\rangle}} + \ldots$$

already discussed

gluon condensate distribution

$$\Delta M_n^{\langle G^2\rangle} = \frac{1}{(4\overline{m}_c^2(\mu_m))^{n+2}} \boxed{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm RGI}} \left\{ a_n^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi} \left[a_n^{1,0} + a_n^{1,1} \log \frac{\overline{m}_c^2(\mu_m)}{\mu_m^2} \right] \right\}$$

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm RGI} = \underbrace{0.006 \pm 0.012 \ {\rm GeV}^4}_{}$$

200% error

Compatible with 0

Nth Moment	n=1	n=2	n=3	n=4
Contribution to the moments	0.2%	0.6%	2%	3 %
Correction in the mass (MeV)	1.11	1.17	1.21	1.24

State of the art of calculations

For n=1,2,3 the $C_n^{0,0}$ coefficients are known at O(α_s^3)

Kühn et al, Maier et al, Boughezal et al

For n ≥ 4, $C_n^{0,0}$ are known in a semianalytic aproach (Padé approximants)

this method renders a central value and an error

Hoang, Mateu & Zebarjad

Maier et al

Greynat et al

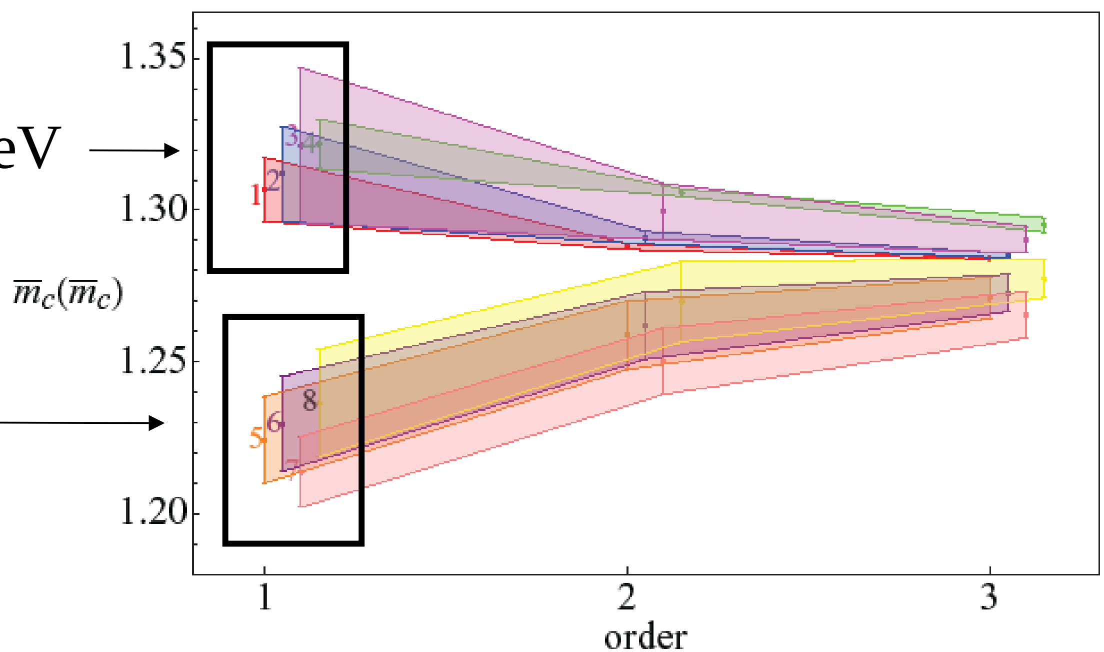
The rest of $C_n^{a,b}$ can be deduced by RGE evolution

A first look into the various methods

$$2\,\mathrm{GeV} \le \mu_m = \mu_\alpha \le 4\,\mathrm{GeV} \longrightarrow$$

$$\mu_m = \overline{m}_c(\overline{m}_c) \longrightarrow$$

$$2\,\mathrm{GeV} \le \mu_\alpha \le 4\,\mathrm{GeV}$$



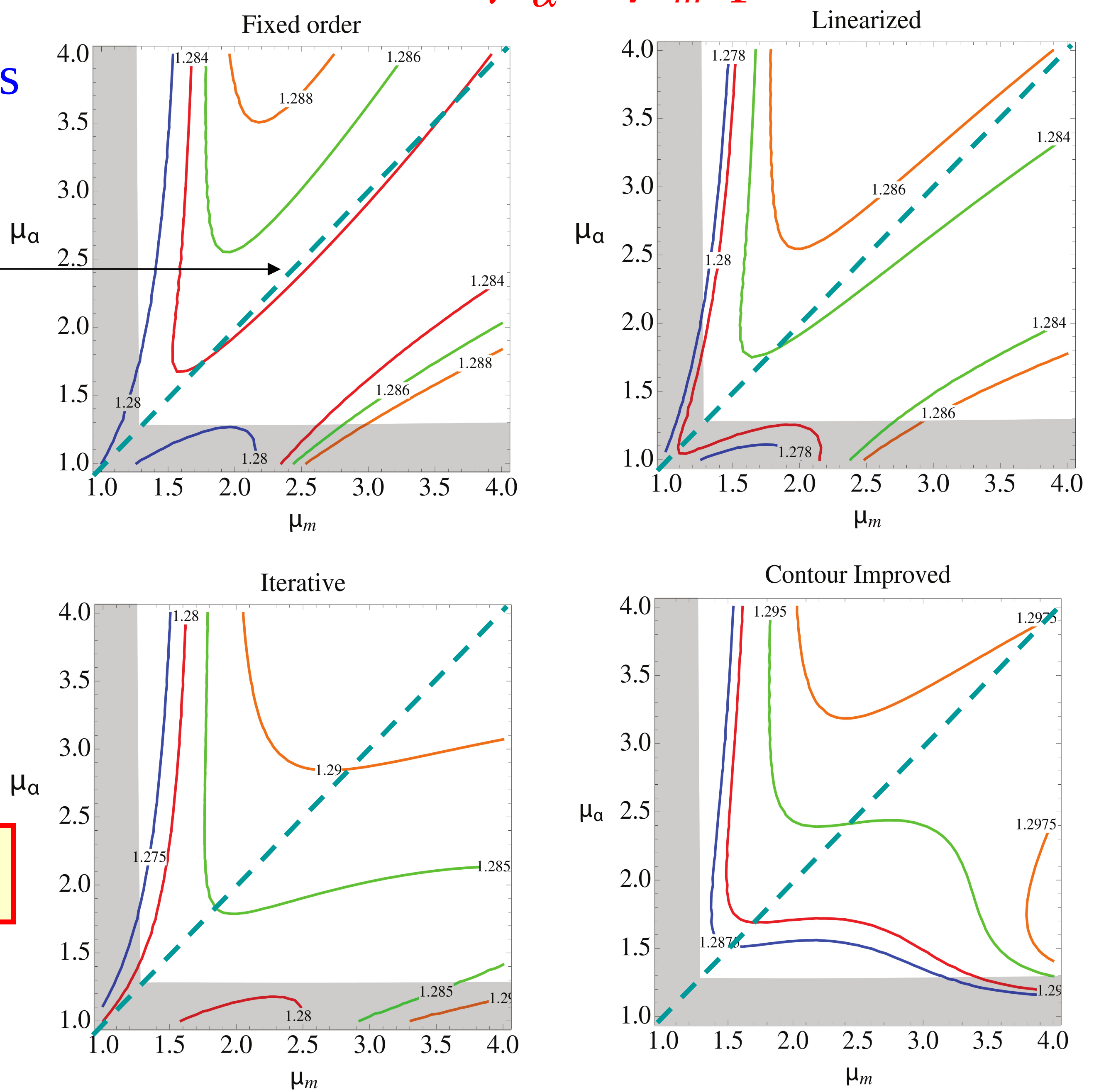
Contours in the $\mu_\alpha - \mu_m$ plane

$O(\alpha_s^3)$ analyses

first moment

Kühn et al path !

$\mu_\alpha = \mu_m$



$m(m) \le \mu_m, \mu_\alpha \le 4\,\mathrm{GeV}$

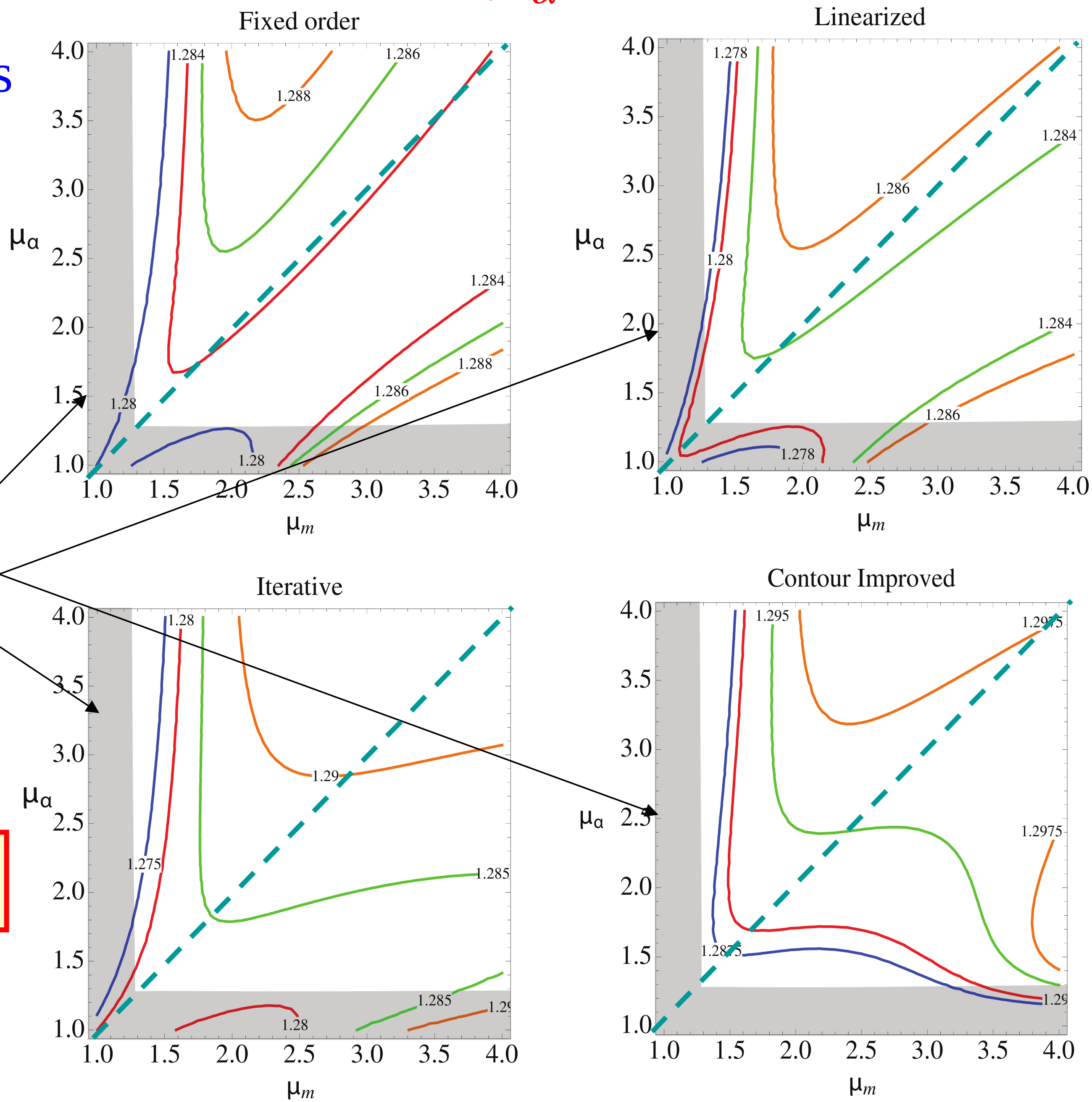
Contours in the $\mu_\alpha - \mu_m$ plane

$O(\alpha_s^3)$ analyses

first moment

Exclude regions with $\mu_m, \mu_\alpha < \bar{m}_c(\bar{m}_c)$

$m(m) \leq \mu_m, \mu_\alpha \leq 4\,\mathrm{GeV}$



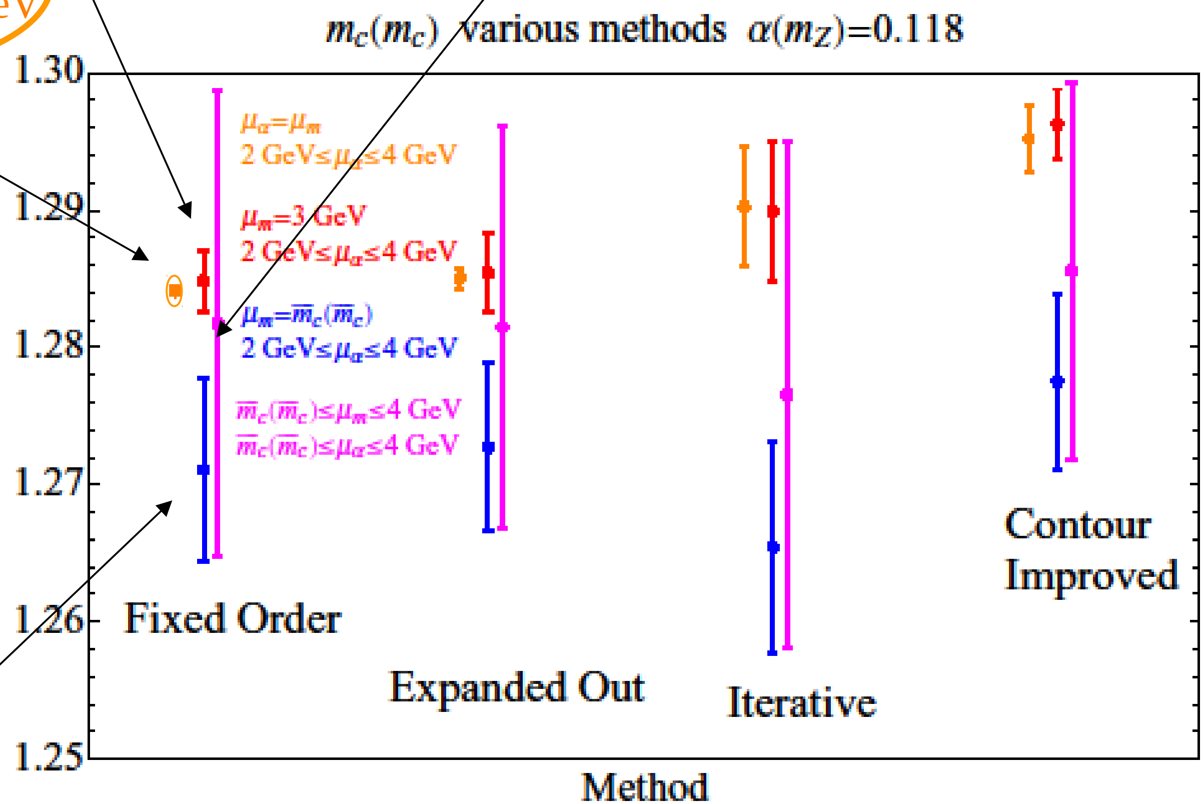
Various error estimates

$O(\alpha_s^3)$ analyses, first moment

$\mu_m = 3\,\text{GeV}$
$2\,\text{GeV} \le \mu_\alpha \le 4\,\text{GeV}$

Double variation $m(m) \le \mu_m, \mu_\alpha \le 4\,\text{GeV}$

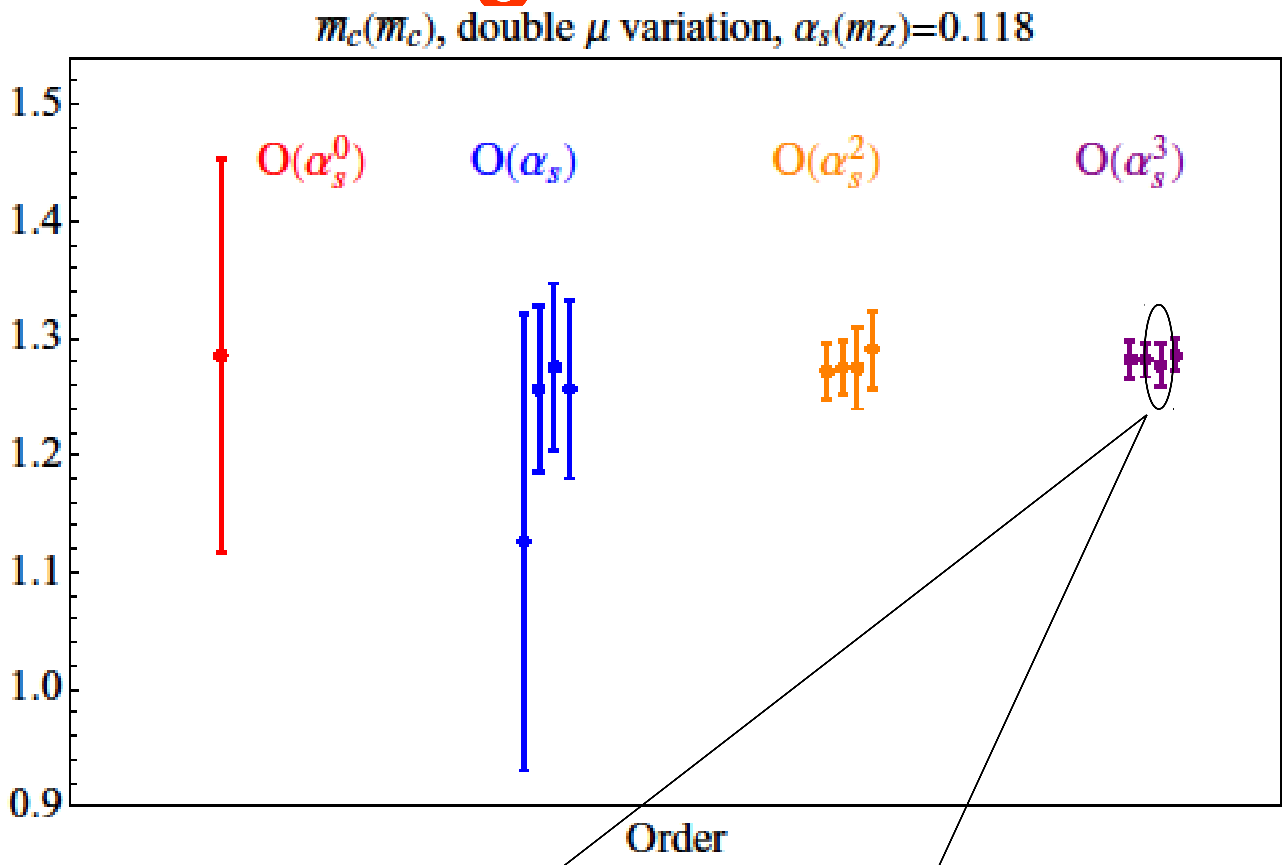
Kühn
$2\,\text{GeV} \le (\mu_\alpha = \mu_m) \le 4\,\text{GeV}$



$\mu_m = \overline{m}_c(\overline{m}_c)$
$2\,\text{GeV} \le \mu_\alpha \le 4\,\text{GeV}$

Results

Convergence of errors



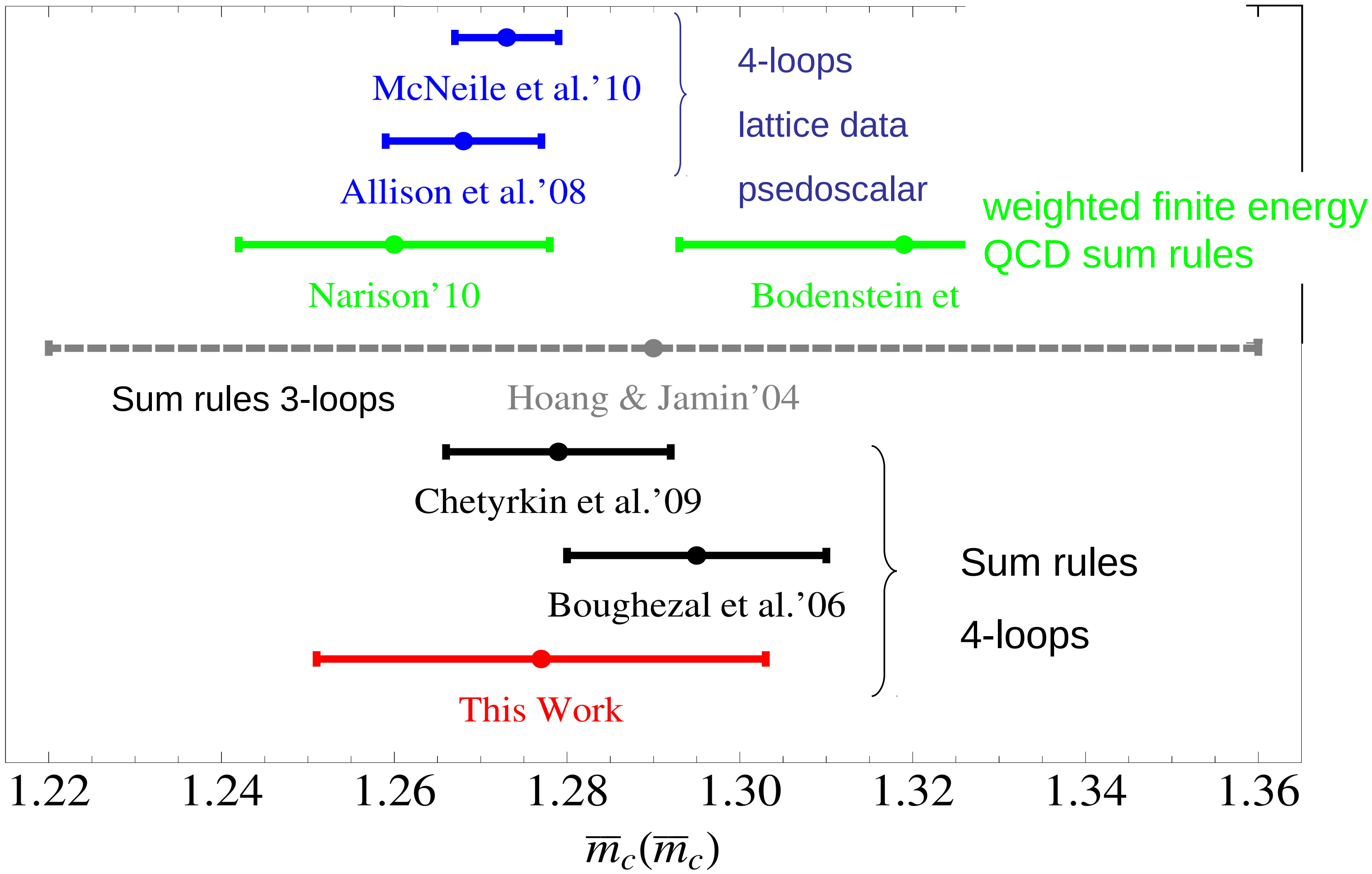
Using double variation all methods have similar values and errors

Result for $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

$$\bar{m}_c(\bar{m}_c) = 1.277 \pm 0.006_{\text{stat}} \pm 0.013_{\text{sys}} \pm 0.019_{\text{th}} \pm 0.009_{\alpha} \pm 0.002_{\langle GG \rangle}$$

$$= 1.277 \pm 0.025$$

Comparison to similar analyses



Bottom Mass

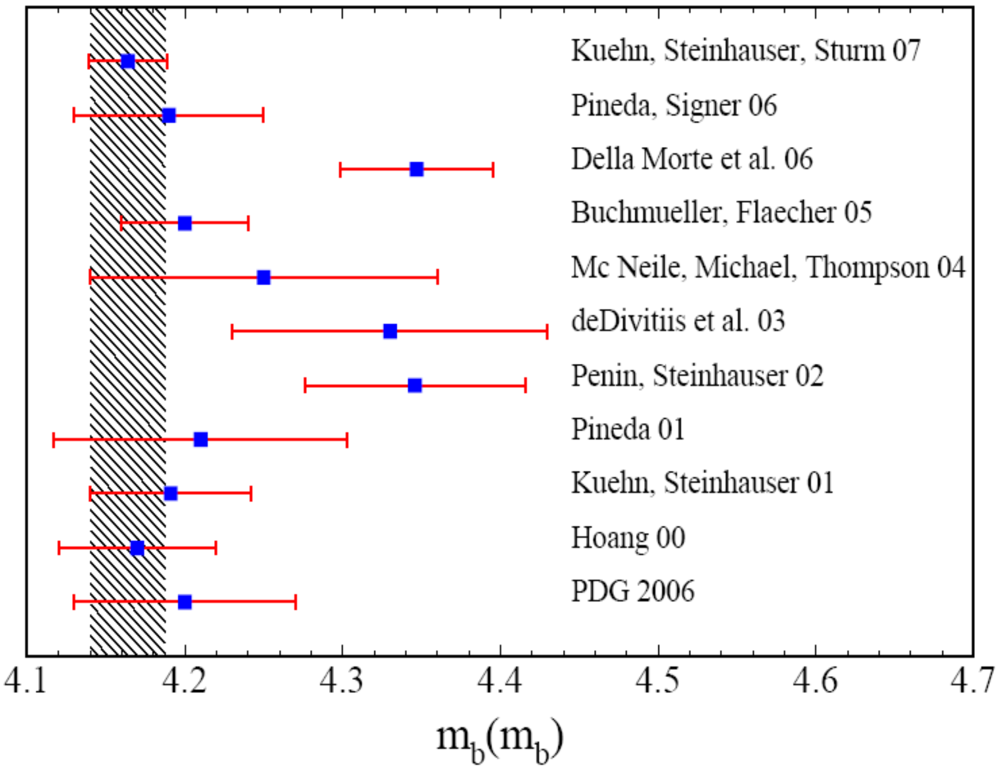
Determination of m_b

Spectral moments of inclusive B decays (nonrelativistic)

Bottomonium sum rules (relativistic)

Lattice

Taken from Kühn et al



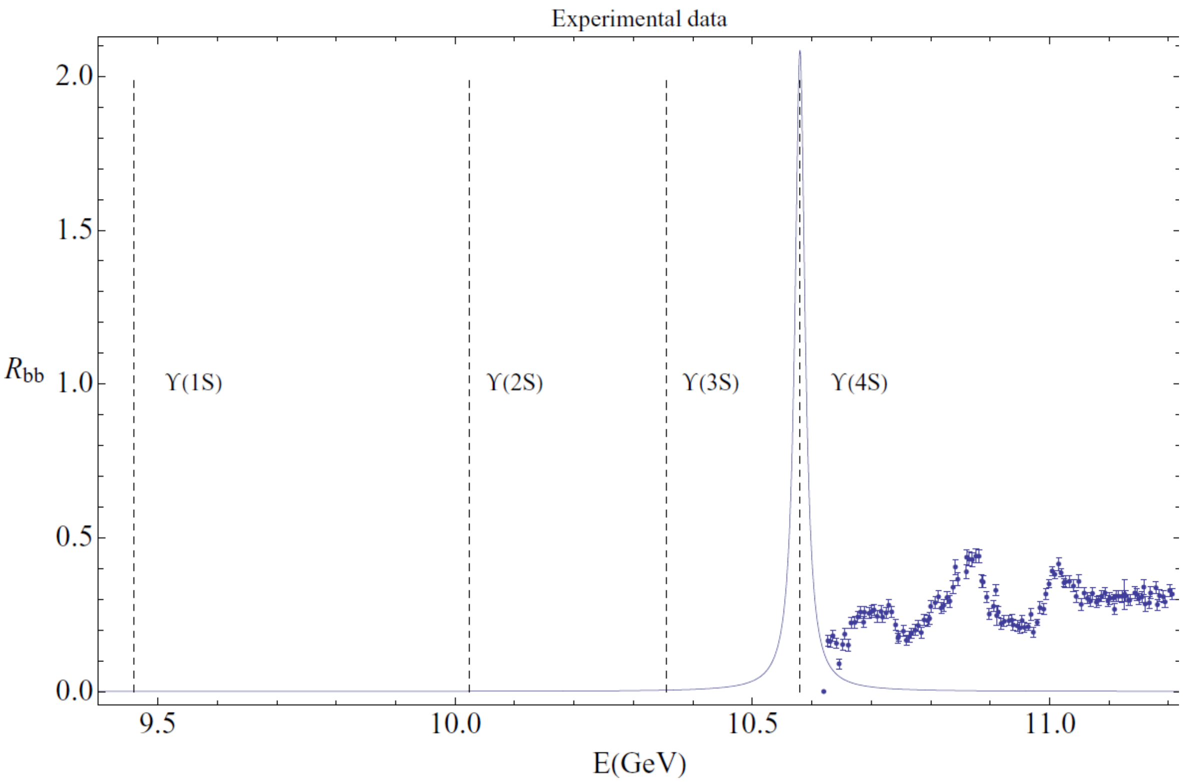
$m_b(m_b)$ (GeV)	Method
4.164 ± 0.025	low-moment sum rules, NNNLO
4.19 ± 0.06	Υ sum rules, NNLL (not complete)
4.347 ± 0.048	lattice (ALPHA), quenched
4.20 ± 0.04	fit to B decay distribution, $\alpha_s^2\beta_0$
$4.25 \pm 0.02 \pm 0.11$	lattice (UKQCD)
4.33 ± 0.10	lattice, quenched
4.346 ± 0.070	$\Upsilon(1S)$, NNNLO
$4.210 \pm 0.090 \pm 0.025$	$\Upsilon(1S)$, NNLO
4.191 ± 0.051	low-moment sum rules, NNLO
4.17 ± 0.05	Υ sum rules, NNLO
4.20 ± 0.07	PDG

Also low-moment sum rules N^3LO
Boughezal et al [4]

$$m_b(m_b) = 4.205 \pm 0.058$$

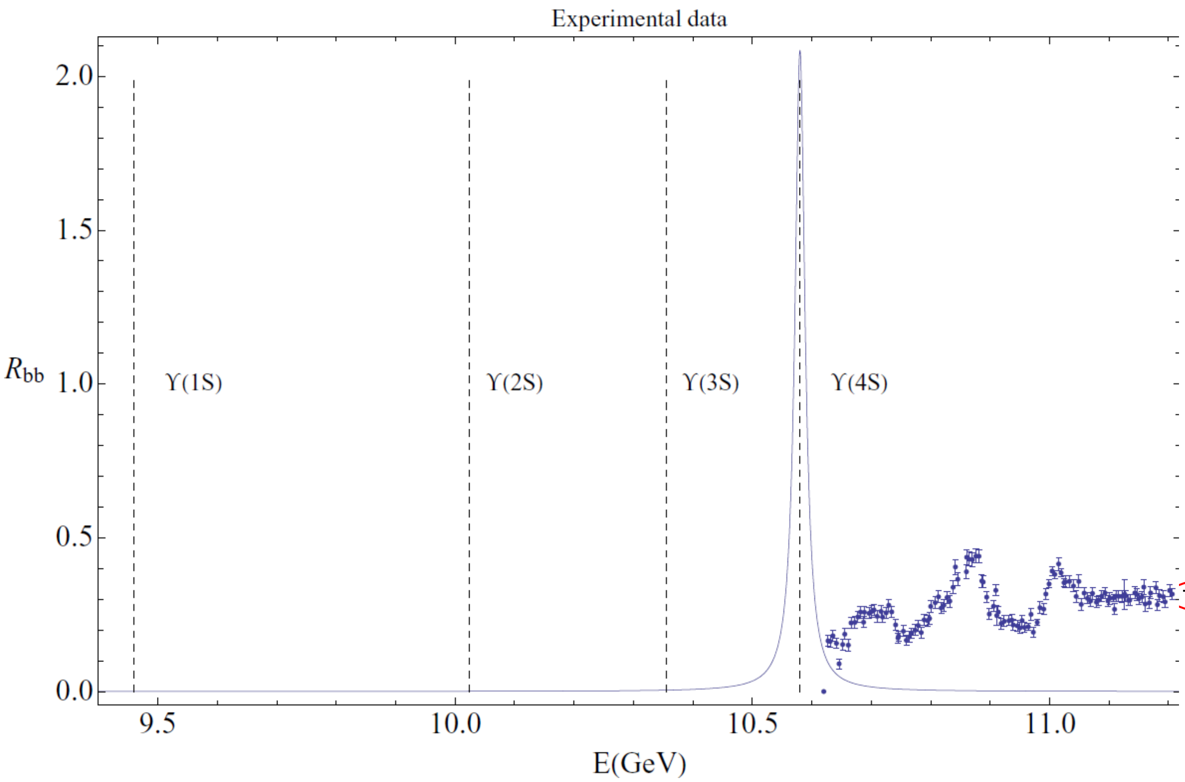
Experimental data: bottom

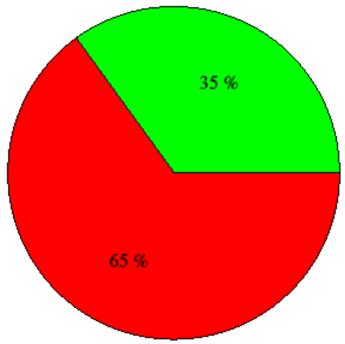
Babar data



Experimental data: bottom

Perturbation theory





Perturbative QCD

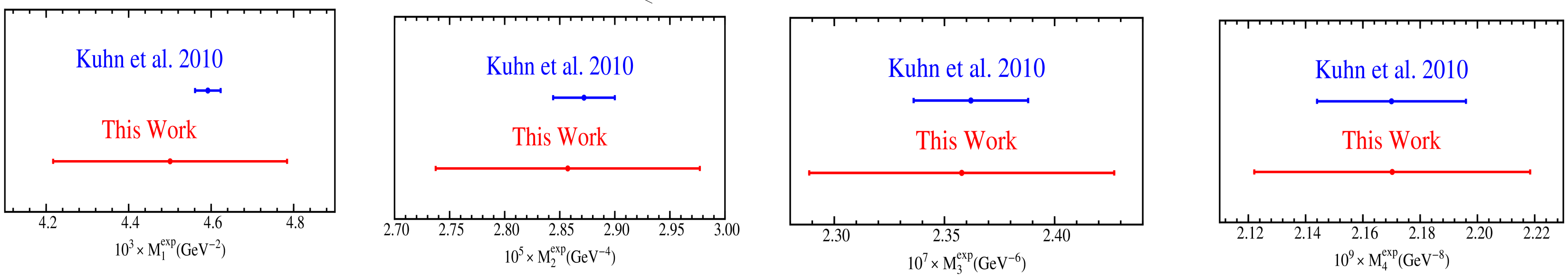
Aren't we comparing theory to theory?

10% error gives a huge error to the total moment

65% of the first moment for bottom sum rules !!

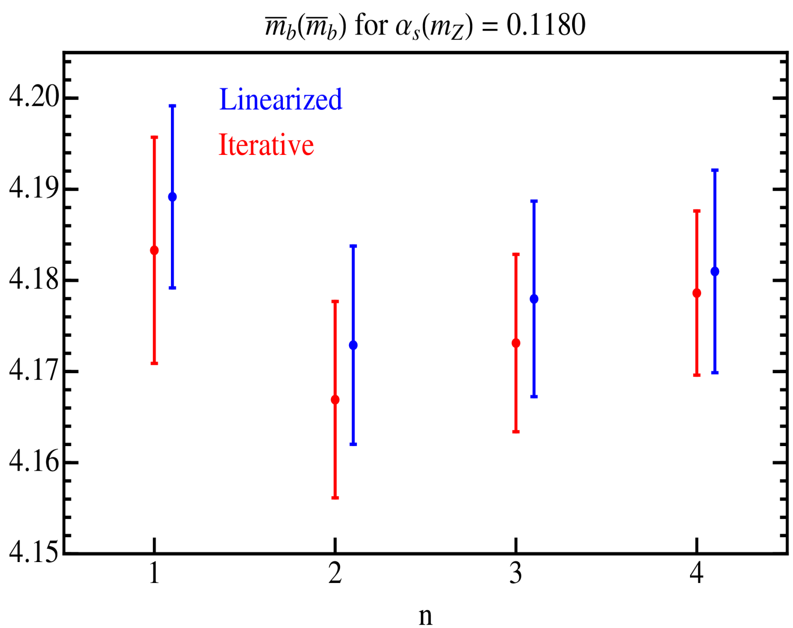
Comparison with other analyses

Kühn et al ('10)[3] $\bar{m}_b(\bar{m}_b) = 4.163 \pm 0.010_{exp} \pm 0.012_{\alpha} \pm 0.003_{\mu}$



Contribution of the perturbative approximation reduces in the higher moments.
Non-relativistic QCD sum rules (n > 4):

(A. H. Hoang, P. R. Femena & M. Stahlhofen.(JHEP, 2012)) $\overline{m}_b(\overline{m}_b) = 4.235 \pm 0.055_{\mathrm{pert}} \pm 0.003_{\mathrm{exp}}\,\mathrm{GeV}$



- Preliminary analyses:
- Convergent even at the higher moments.
- The third moment of relativistic sum rules:

$$\alpha_s(m_Z) = 0.1184 \pm 0.0021$$

$$\bar{m}_b(\bar{m}_b) = 4.178 \pm 0.011_{th} \pm 0.005_{\alpha} \pm 0.020_{sys} \pm 0.004_{stat}$$

Conclusions and outlook

- It is essential to have a reliable error estimate for charm and bottom masses.
- Concerning relativistic sum rules, a revision of perturbative errors was mandatory.
- Experimental input must be treated with care (combining various sets of data, correlations, systematic errors …)
- Perturbative QCD should be used only where there is no data, and assigning a conservative error.
  - For charm PQCD is only a small fraction of the moment → small impact.
  - For bottom PQCD is a sizeable fraction of the moment → big errors!
- The analysis can be easily extended to other correlators connection to lattice

Stay tuned for updated numbers on charm, and for results on bottom mass and pseudoscalar correlators.

Result for $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

$$\overline{m}_c(\overline{m}_c) = 1.277 \pm 0.006_{\rm stat} \pm 0.013_{\rm sys} \pm 0.019_{\rm th} \pm 0.009_{\alpha} \pm 0.002_{\langle GG \rangle}$$
$$= 1.277 \pm 0.025$$

Size of neglected terms

	Mass corrections	Secondary Radiation	Singlet	Z-boson
$i=1$	0.2	0.04	3×10^{-4}	0.007
$i=2$	0.01	0.002	2×10^{-5}	0.004
$i=3$	8×10^{-4}	3×10^{-4}	2×10^{-6}	0.003
$i=4$	5×10^{-5}	7×10^{-5}	1×10^{-7}	0.003