JET BROADENING IN EFFECTIVE FIELD THEORY: WHEN DIMENSIONAL REGULARISATION FAILS

[GUIDO BELL]

based on: T. Becher, GB, M. Neubert, Phys. Lett. B 704 (2011) 276
T. Becher, GB, Phys. Lett. B 713 (2012) 41
T. Becher, GB, JHEP 1211 (2012) 126

OUTLINE

EVENT-SHAPE VARIABLES

FACTORISATION

REVIEW OF THRUST ANALYSIS

FACTORISATION BREAKDOWN FOR BROADENING

ANALYTIC REGULARISATION IN SCET

RESUMMATION

COLLINEAR ANOMALY

NNLL RESUMMATION

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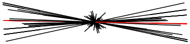
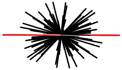
Canonical event shape

Thrust:

$$T = \frac{1}{Q} \max_{\vec{n}} \left(\sum_i |\vec{p}_i \cdot \vec{n}_T| \right)$$

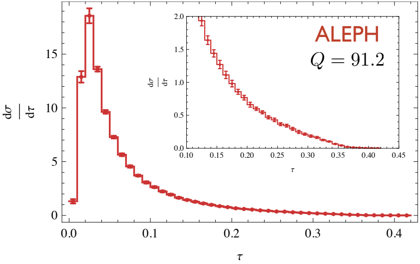


two-jet like: $T \simeq 1$



spherical: $T \simeq 1/2$

Thrust distribution precisely measured at LEP $\quad (\tau = 1 - T)$



in the two-jet region $\tau \simeq 0$

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\tau} \simeq \frac{\alpha_s C_F}{2\pi} \left[-\frac{4 \ln \tau + 3}{\tau} + \dots \right]$$

$\Rightarrow$ Sudakov logs require resummation

Motivation

Why e^+e^- event shapes in 2013?

- clean environment to test understanding of QCD

  perturbation theory $+$ resummation $+$ non-perturbative effects

  same methods are applied at the LHC: soft gluon resummation ($\Rightarrow$ thrust)

  p_T resummation ($\Rightarrow$ broadening)

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  p_T resummation ($\Rightarrow$ broadening)

- precision determination of α_s

  traditionally based on a fit to six event shapes $(T, \rho_H, B_T, B_W, C, y_3)$

  NLO + NLL: $\alpha_s(M_Z) = 0.1202 \pm 0.0003\,(\text{stat}) \pm 0.0009\,(\text{exp}) \pm 0.0013\,(\text{had}) \pm 0.0047\,(\text{theo})$

  [LEP QCD working group 04]

  NNLO + NLL: $\alpha_s(M_Z) = 0.1224 \pm 0.0009\,(\text{stat}) \pm 0.0009\,(\text{exp}) \pm 0.0012\,(\text{had}) \pm 0.0035\,(\text{theo})$

  [Dissertori et al 09]

  $\Rightarrow$ further improvements require to go beyond NLL resummation!

Beyond NLL?

Traditional resummations are based on the coherent branching algorithm

[Catani, Trentadue, Turnock, Webber 93]

- sums probabilities for independent gluon emissions
- apparently hard to extend beyond NLL

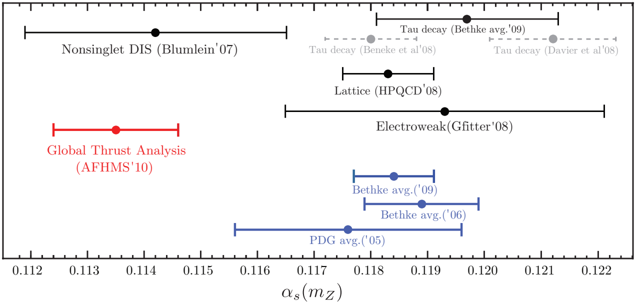
In SCET resummations are formulated in an operator language on the amplitude level

- extension to higher orders requires standard EFT techniques
- thrust analysis extended by two orders to N^3LL accuracy [Becher, Schwartz 08]
- field theoretical treatment of power corrections [Abbate, Fickinger, Hoang, Mateu, Stewart 10]

  two-dimensional fit to world thrust data

  NNLO + N^3LL: $\alpha_s(M_Z) = 0.1135 \pm 0.0002 \text{ (exp)} \pm 0.0005 \text{ (had)} \pm 0.0009 \text{ (pert)}$

Beyond NLL?



- field theoretical treatment of power corrections [Abbate, Fickinger, Hoang, Mateu, Stewart 10]

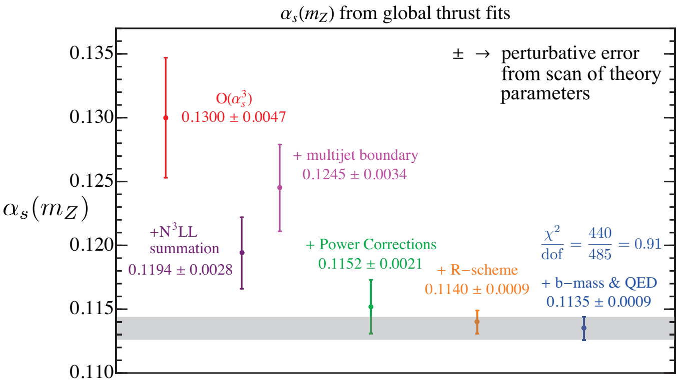
  two-dimensional fit to world thrust data

  NNLO + N^{3}LL: $\alpha_s(M_Z) = 0.1135 \pm 0.0002\,(\text{exp}) \pm 0.0005\,(\text{had}) \pm 0.0009\,(\text{pert})$

  world average: $\alpha_s(M_Z) = 0.1184 \pm 0.0007$

  almost 4σ below world average?

Precision thrust analysis



distribution: $\alpha_s(M_Z) = 0.1135 \pm 0.0002\,(\text{exp}) \pm 0.0005\,(\text{had}) \pm 0.0009\,(\text{pert})$ [Abbate et al 10]

moment: $\alpha_s(M_Z) = 0.1140 \pm 0.0004\,(\text{exp}) \pm 0.0013\,(\text{had}) \pm 0.0007\,(\text{pert})$ [Abbate et al 12]

NNLO + NNLL: $\alpha_s(M_Z) = 0.1131\,^{+0.0028}_{-0.0022}$ [Monni, Gehrmann, Luisoni 12]

Event shape studies in SCET

Heavy jet mass: [Chien, Schwartz 10, Abbate et al in progress]

$$\boxed{\rho_H = \frac{1}{Q^2} \max\left(M_L^2, M_R^2\right)} \qquad \text{hemisphere jet masses} \quad M_{L/R}^2 = \Big(\sum_{i \in L/R} p_i\Big)^2$$

- similar to thrust $\Rightarrow$ again N^{3}LL resummation
- non-perturbative effects more involved (hadron masses, ...) [Mateu, Stewart, Thaler 12]

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Total and wide jet broadening: [Chiu, Jain, Neill, Rothstein 11; Becher, GB, Neubert 11]

$$\boxed{\begin{aligned} b_T &= b_L + b_R \\ b_W &= \max\left(b_L, b_R\right) \end{aligned}}$$

hemisphere jet broadenings $b_{L/R} = \frac{1}{2}\sum_{i\in L/R} |\vec{p}_i \times \vec{n}_T|$

- orthogonal to thrust (measure transverse momentum distribution)
- different type of factorisation formula $\Rightarrow$ aim at NNLL resummation

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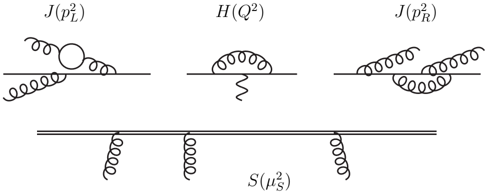
Thrust in SCET

In the two-jet limit $\tau \to 0$ the thrust distribution factorises as [Fleming, Hoang, Mantry, Stewart 07; Schwartz 07]

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \int dp_R^2 \; J(p_L^2,\mu) \; J(p_R^2,\mu) \; S\Big(\tau Q - \frac{p_L^2+p_R^2}{Q},\mu\Big)$$

multi-scale problem: $Q^2 \gg p_L^2 \sim p_R^2 \sim \tau Q^2 \gg \tau^2 Q^2$

hard — collinear — soft



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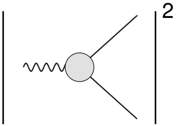
Hard function:

- on-shell vector form factor of a massless quark

$H(Q^2) =$



- known to three-loop accuracy [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser 09; Gehrmann, Glover, Huber, Ikizlerli, Studerus 10]
- also enters Drell-Yan and DIS in the endpoint region

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Jet function:

- imaginary part of quark propagator in light-cone gauge

$$J(p^2) \sim \text{Im}\left[\text{F.T.}\left\langle 0 \left| \frac{\not{n}\not{\bar{n}}}{4} W^\dagger(0)\psi(0)\;\bar{\psi}(x)W(x)\frac{\not{\bar{n}}\not{n}}{4}\right| 0\right\rangle\right] \qquad W(x) = \mathbf{P}\exp\left(ig_s\int_{-\infty}^0 ds\;\bar{n}\cdot A(x+s\bar{n})\right)$$

- known to two-loop accuracy (anomalous dimension to three-loop) [Becher, Neubert 06]
- also enters inclusive B decays and DIS in the endpoint region

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Soft function:

- matrix element of Wilson lines along the directions of energetic quarks

$$S(\omega) = \sum_X \left| \left\langle X \middle| S_n^\dagger(0)\, S_{\bar n}(0) \middle| 0 \right\rangle \right|^2 \; \delta(\omega - n\cdot p_{X_n} - \bar n \cdot p_{X_{\bar n}}) \qquad S_n(x) = \mathbf{P}\exp\left(ig_s \int_{-\infty}^0 ds\; n\cdot A_s(x+sn)\right)$$

- known to two-loop accuracy (anomalous dimension to three-loop)

[Kelley, Schwartz, Schabinger, Zhu 11; Monni, Gehrmann, Luisoni 11; Hornig, Lee, Stewart, Walsh Zuberi 11]

How does resummation work (roughly)?

Let us have a closer look at the one-loop expressions

$$H(Q^2,\mu) \;=\; 1 + \frac{\alpha_s C_F}{4\pi}\left[-2\ln^2\frac{Q^2}{\mu^2} + 6\ln\frac{Q^2}{\mu^2} - 16 + \frac{7\pi^2}{3}\right]$$

$$J(p^2,\mu) \;=\; \delta(p^2) + \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{4\ln(p^2/\mu^2)-3}{p^2}\right)_*^{[\mu^2]} + (7-\pi^2)\,\delta(p^2)\right]$$

$$S(\omega,\mu) \;=\; \delta(\omega) + \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{-16\ln(\omega/\mu)}{\omega}\right)_*^{[\mu]} + \frac{\pi^2}{3}\,\delta(\omega)\right]$$

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General structure:

- logarithms $\Leftrightarrow$ divergences

  anomalous dimensions of EFT operators $\Rightarrow$ resum logs via RG techniques

$$\frac{d}{d\ln\mu}H(Q^2,\mu) = \left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s)\right]H(Q^2,\mu)$$

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Notice: there is no large log when each function is evaluated at its natural scale!

Angularities

Interesting class of event shape variables [Berger, Kucs, Sterman 03]

$$\tau_a = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$$

- interpolates between thrust ($a = 0$) and broadening ($a = 1$)
- infrared safe for $a < 2$, but standard factorisation only for $a < 1$

SCET analysis [Hornig, Lee, Ovanesyan 09]

- relevant scales: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 \tau_a^{\frac{2}{2-a}} \gg \mu_S^2 \sim Q^2 \tau_a^2$

  thrust: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 \tau \gg \mu_S^2 \sim Q^2 \tau^2$ (SCET_I)

  broadening: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 B^2 \sim \mu_S^2 \sim Q^2 B^2$ (SCET_II)

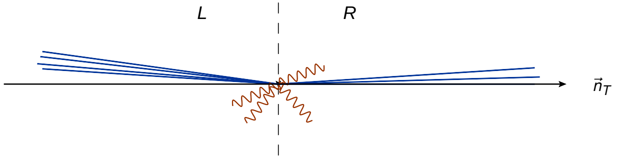
$\Rightarrow$ factorisation formula for broadening will be different (and more complicated)

Jet broadening

In the two-jet limit $b_L \sim b_R \to 0$ expect that the broadening distribution factorises as

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{db_L\,db_R} = H(Q^2,\mu)\,\int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp\; \mathcal{J}_L(b_L - b_L^s, p_L^\perp, \mu)\; \mathcal{J}_R(b_R - b_R^s, p_R^\perp, \mu)\; \mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp, \mu)$$

two-scale problem: $Q^2 \gg b_L^2 \sim b_R^2$



- relevant modes have $p_{\text{coll}}^\perp \sim p_{\text{soft}}^\perp \sim b_{L,R}$
- jet recoils against soft radiation

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Hard function:

- precisely the same object as for thrust
- recall the RG equation

$$\frac{d}{d\ln\mu} H(Q^2,\mu) = \left[2\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s)\right] H(Q^2,\mu)$$

$\Rightarrow$ there is a hidden Q-dependence in the second line!

thrust $\frac{\mu_J^2}{\mu_S} = \frac{\tau Q^2}{\tau Q} = Q \quad \Leftrightarrow \quad$ broadening $\frac{\mu_J^2}{\mu_S} = \frac{b^2}{b} = b$

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Some manipulations:

- Laplace transform $b_{L,R} \to \tau_{L,R}$
- Fourier transform $p_{L,R}^\perp \to x_{L,R}^\perp$
- define dimensionless variable $z_{L,R} = \frac{2|x_{L,R}^\perp|}{\tau_{L,R}}$

$\Rightarrow$ the naive factorisation theorem takes the form

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L \int_0^\infty dz_R\; \overline{\mathcal{J}}_L(\tau_L, z_L, \mu)\; \overline{\mathcal{J}}_R(\tau_R, z_R, \mu)\; \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)$$

Jet function

The quark jet function for broadening reads

$$\mathcal{J}(b, p^\perp) \sim \sum_X \delta(\bar{n} \cdot p_X - Q)\ \delta^{d-2}(p_X^\perp - p^\perp)\ \delta\Big(b - \tfrac{1}{2} \sum_{i \in X} |p_i^\perp|\Big)\ \Big|\Big\langle X \Big| \bar{\psi}(0) W(0) \tfrac{\not{\bar{n}} \not{n}}{4} \Big| 0 \Big\rangle\Big|^2$$

- delta-functions ensure that jet has given energy, $p^\perp$ and b
- tree level: $\mathcal{J}(b, p^\perp) = \delta\Big(b - \frac{1}{2}|p^\perp|\Big) \quad \Rightarrow \quad \overline{\mathcal{J}}(\tau, z) = \frac{z}{\big(1+z^2\big)^{3/2}} + \mathcal{O}(\epsilon)$

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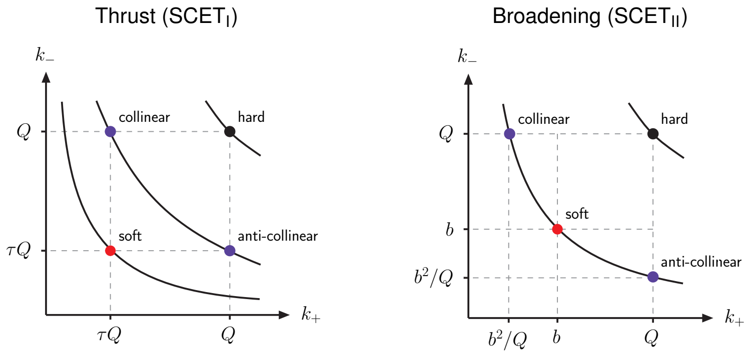
At one-loop the calculation involves

- Wilson-line diagrams are not well-defined in dimensional regularisation!

  $\displaystyle\int_0^Q \frac{dk_-}{k_-}$ diverges in the soft limit (DR regularises $d^{d-2}k_\perp$)

- this does not happen for thrust or any SCET_I problem

Momentum modes



- thrust: $p_s^2 \ll p_c^2$
- broadening: $p_s^2 \sim p_c^2$

$\Rightarrow$ cannot distinguish soft mode from collinear mode when radiated into jet direction

$\Rightarrow$ need additional regulator that distinguishes modes by their rapidities

Regularisation in SCET_{II}

The regularisation of individual diagrams is largely arbitrary, one could use e.g.

$$\frac{1}{p^2 + i\varepsilon} \quad \to \quad \frac{1}{p^2 - \Delta + i\varepsilon}, \quad \frac{(\nu^2)^\alpha}{(p^2 + i\varepsilon)^{1+\alpha}}, \quad \dots$$

- trivial for QCD, but regularises ill-defined EFT diagrams
- spoils gauge-invariance and eikonal structure of Wilson line emissions

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In a massless theory it is sufficient to regularise phase space integrals [Becher, GB 11]

$$\boxed{\int d^d k \;\delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^\alpha \delta(k^2)\,\theta(k^0)}$$

- does not modify SCET at all $\Rightarrow$ keeps gauge-invariance and eikonal structure
- analytic, minimal and adopted to the problem (LC propagators)

Why does it work?

Our new prescription amounts to

$$\int d^dk\ \delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^dk\ \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

- virtual corrections do not need regularisation

  matrix elements of Wilson lines in QCD $\Rightarrow$ the same for thrust and broadening

  technical reason: $\int d^{d-2}k_\perp\ f(k_\perp, k_+) \sim k_+^{-\epsilon}$

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  technical reason: $\int d^{d-2}k_\perp \;\; f(k_\perp, k_+) \;\sim\; k_+^{-\epsilon}$

- required for observables sensitive to transverse momenta

  $f(k_\perp, k_+) \sim \delta^{d-2}(k_\perp - p_\perp)$ $\Rightarrow$ factor $k_+^{-\epsilon}$ absent $\Rightarrow$ reinstalled as $k_+^{-\alpha}$

  can show that the prescription regularises all LC singularities in SCET [Becher, GB 11]

- not sufficient for cases where virtual corrections are ill-defined

  examples: electroweak Sudakov corrections, Regge limits

Jet function revisited

With the additional regulator in place, the jet functions can be evaluated

$$\mathcal{J}_L(b, p^\perp = 0) = \delta(b) + \frac{C_F \alpha_s}{2\pi} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \frac{1}{b} \left(\frac{\mu}{b}\right)^{2\epsilon} \left[1 - \epsilon + \frac{4\,\Gamma(2+\alpha)\,\Gamma(\alpha)}{\Gamma(2+2\alpha)} \left(\frac{Q\nu_+}{b^2}\right)^\alpha \right]$$

$$\mathcal{J}_R(b, p^\perp = 0) = \delta(b) + \frac{C_F \alpha_s}{2\pi} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \frac{1}{b} \left(\frac{\mu}{b}\right)^{2\epsilon} \left[1 - \epsilon + \frac{4\,\Gamma(-\alpha)}{\Gamma(2-\alpha)} \left(\frac{\nu_+}{Q}\right)^\alpha \right]$$

- ordered limit $\alpha \to 0,\ \varepsilon \to 0$ generates a pole in the analytic regulator
- note the characteristic scaling $\left(\frac{\nu_+}{k_+}\right)^\alpha$ in each region

For $p^\perp \neq 0$ the computation is considerably more involved $(\to$ later)

$$\overline{\mathcal{J}}_L(\tau, z) = \overline{\mathcal{J}}_L^{(0)}(\tau, z) \left[1 - \frac{C_F \alpha_s}{\pi} \frac{1}{\alpha} \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}^2\right) + 2 \ln \frac{\sqrt{1+z^2}+1}{4} \right) \left(Q\nu_+ \bar{\tau}^2\right)^\alpha + \dots \right]$$

- divergent term has non-trivial z-dependence

Soft function

The soft function for broadening reads

$$\mathcal{S}(b_L, b_R, p_L^\perp, p_R^\perp) \sim \sum_{X_L, X_R} \delta^{d-2}(p_{X_L}^\perp - p_L^\perp)\ \delta^{d-2}(p_{X_R}^\perp - p_R^\perp)$$

$$\delta\Big(b_L - \tfrac{1}{2}\sum_{i\in X_L} |p_{L,i}^\perp|\Big)\ \delta\Big(b_R - \tfrac{1}{2}\sum_{j\in X_R} |p_{R,j}^\perp|\Big)\ \Big|\Big\langle X_L X_R \Big| S_n^\dagger(0)\, S_{\bar n}(0) \Big| 0 \Big\rangle\Big|^2$$

- split final state into left and right-moving particles
- tree level: $\mathcal{S}(b_L, b_R, p_L^\perp, p_R^\perp) = \delta(b_L)\,\delta(b_R)\,\delta^{d-2}(p_L^\perp)\,\delta^{d-2}(p_R^\perp) \quad \Rightarrow \quad \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) = 1$

At one-loop the calculation involves

$$\Rightarrow\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) = 1 + \frac{C_F \alpha_s}{\pi} \left\{ \frac{1}{\alpha} \left(\frac{1}{\epsilon} + \ln(\mu^2 \bar\tau_L^2) + 2 \ln \frac{\sqrt{1+z_L^2}+1}{4} \right) \left(\nu_+ \bar\tau_L\right)^\alpha - (L \leftrightarrow R) + \dots \right\}$$

Anomalous Q dependence

Let us now put the jet and soft functions together

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \ = \ \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{(0)}(\tau_R, z_R)$$

$$\Bigg\{ 1 + \frac{C_F \alpha_s}{\pi} \Bigg[\Bigg(-\frac{1}{\alpha} - \ln\left(Q\nu_+ \bar{\tau}_L^2\right) + \frac{1}{\alpha} + \ln\left(\nu_+ \bar{\tau}_L\right) \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2 \ln \frac{\sqrt{1+z_L^2}+1}{4} \right)$$

$$+ \Bigg(+\frac{1}{\alpha} + \ln\left(\frac{\nu_+}{Q}\right) - \frac{1}{\alpha} - \ln\left(\nu_+ \bar{\tau}_R\right) \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2 \ln \frac{\sqrt{1+z_R^2}+1}{4} \right) + \dots \Bigg] \Bigg\}$$

- well-defined without additional regulators

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$$\Bigg\{1 + \frac{C_F \alpha_s}{\pi} \Bigg[\Bigg(\quad - \ln\left(Q\nu_+ \bar{\tau}_L^2\right) \quad + \ln\left(\nu_+ \bar{\tau}_L\right) \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2\ln \frac{\sqrt{1+z_L^2}+1}{4} \right)$$

$$+ \Bigg(\quad + \ln\left(\frac{\nu_+}{Q}\right) \quad - \ln\left(\nu_+ \bar{\tau}_R\right) \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2\ln \frac{\sqrt{1+z_R^2}+1}{4} \right) + \ldots \Bigg] \Bigg\}$$

- well-defined without additional regulators
- similarly the artificial scale ν_+ drops out

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Let us now put the jet and soft functions together

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$$\left\{1 + \frac{C_F\alpha_s}{\pi}\left[\left(\qquad - \ln\left(Q\bar{\tau}_L\right) \qquad \right)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_L^2\right) + 2\ln\frac{\sqrt{1+z_L^2}+1}{4}\right)\right.\right.$$

$$\left.\left. + \left(\qquad - \ln\left(Q\bar{\tau}_R\right) \qquad \right)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_R^2\right) + 2\ln\frac{\sqrt{1+z_R^2}+1}{4}\right) + \dots\right]\right\}$$

- well-defined without additional regulators
- similarly the artificial scale ν_+ drops out
- the hidden Q dependence shows up!

$\Rightarrow$ the naive factorisation formula does not achieve a proper scale separation

How can we resum a logarithm that appears in a matching calculation?

OUTLINE

EVENT-SHAPE VARIABLES

FACTORISATION

REVIEW OF THRUST ANALYSIS

FACTORISATION BREAKDOWN FOR BROADENING

ANALYTIC REGULARISATION IN SCET

RESUMMATION

COLLINEAR ANOMALY

NNLL RESUMMATION

Collinear anomaly

Can show that the Q dependence exponentiates using and extending arguments from

- electroweak Sudakov resummation [Chiu, Golf, Kelley, Manohar 07]
- p_T resummation in Drell-Yan production [Becher, Neubert 10]

Start from the logarithm of the product of jet and soft functions

$$\ln P = \ln \overline{\mathcal{J}}_L\Big(\ln\,(Q\nu_+\bar{\tau}_L^2);\ \tau_L, z_L\Big) + \ln \overline{\mathcal{J}}_R\Big(\ln\,(\tfrac{\nu_+}{Q});\ \tau_R, z_R\Big) + \ln \overline{\mathcal{S}}\Big(\ln\,(\nu_+\bar{\tau}_L);\ \tau_L, \tau_R, z_L, z_R\Big)$$

collinear: $k_+ \sim \frac{b^2}{Q}$ — anticollinear: $k_+ \sim Q$ — soft: $k_+ \sim b$

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collinear: $k_+ \sim \frac{b^2}{Q}$ anticollinear: $k_+ \sim Q$ soft: $k_+ \sim b$

- use that product does not depend on ν_+ and that it is LR symmetric

$$\Rightarrow \ln P = \frac{k_2(\mu)}{4} \ln^2(Q^2\,\bar\tau_L\bar\tau_R) - F_B(\tau_L, z_L, \mu)\ln(Q^2\bar\tau_L^2) - F_B(\tau_R, z_R, \mu)\ln(Q^2\bar\tau_R^2) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

- RG invariance implies $k_2(\mu) = 0$ to all orders

$$\Rightarrow \boxed{P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) = (Q^2\bar\tau_L^2)^{-F_B(\tau_L, z_L, \mu)}\ (Q^2\bar\tau_R^2)^{-F_B(\tau_R, z_R, \mu)}\ W(\tau_L, \tau_R, z_L, z_R, \mu)}$$

Final factorisation formula

The corrected all-order generalisation of the naive factorisation formula becomes

[Becher, GB, Neubert 11]

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L\int_0^\infty dz_R\;\left(Q^2\bar{\tau}_L^2\right)^{-F_B(\tau_L,z_L,\mu)}\;\left(Q^2\bar{\tau}_R^2\right)^{-F_B(\tau_R,z_R,\mu)}\;W(\tau_L,\tau_R,z_L,z_R,\mu)$$

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To NLL the Mellin inversion can be performed analytically

$$\frac{1}{\sigma_0}\frac{d\sigma}{db_T} = H(Q^2,\mu)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\frac{1}{b_T}\left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{db_W} = H(Q^2,\mu)\,\frac{2\eta\,e^{-2\gamma_E\eta}}{\Gamma^2(1+\eta)}\,\frac{1}{b_W}\left(\frac{b_W}{\mu}\right)^{2\eta} I^2(\eta)$$

$$\eta = \frac{C_F\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2} = \mathcal{O}(1)$$

The non-trivial z-dependence of the anomaly coefficient is encoded in

$$I(\eta) = \int_0^\infty dz\,\frac{z}{(1+z^2)^{3/2}}\left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta} = \frac{4^\eta}{1+\eta}\;{}_2F_1(\eta,1+\eta,2+\eta,-1)$$

Comparison with literature

Traditional resummation

- pioneering work missed quark recoil effects $\Rightarrow$ valid to LL [Catani, Turnock, Webber 92]
- first NLL resummation by Dokshitzer et al [Dokshitzer, Lucenti, Marchesini, Salam 98]

  we find complete analytical agreement with this work

Resummation using SCET [Chiu, Jain, Neill, Rothstein 11,12]

- start from same naive factorisation formula
- modify Wilson-line propagators to regularise rapidity divergences
- treat additional divergences in a "rapidity renormalization group"
- 2011 paper missed quark recoil effects $\Rightarrow$ valid only to LL

  2012 paper in agreement with Dokshitzer et al

Beyond NLL

The extension to NNLL requires three ingredients

- one-loop soft function
- one-loop jet function
- two-loop anomaly coefficient

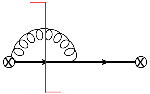
The calculation of the one-loop soft function is straight-forward [Becher, GB, Neubert 11]

$$\Rightarrow \qquad \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ (\mu^2 \bar{\tau}_L^2)^\varepsilon \; (\nu_+ \bar{\tau}_L)^\alpha \left[\frac{4}{\alpha} \left(\frac{1}{\varepsilon} + 2 \ln \left(\frac{1 + \sqrt{1 + z_L^2}}{4} \right) \right) - \frac{2}{\varepsilon^2} \right. \right.$$

$$\left. \left. + 8 \, \mathrm{Li}_2 \left(- \frac{\sqrt{1 + z_L^2} - 1}{\sqrt{1 + z_L^2} + 1} \right) + 4 \ln^2 \left(\frac{1 + \sqrt{1 + z_L^2}}{4} \right) + \frac{5\pi^2}{6} \right] - (L \leftrightarrow R) \right\}$$

One-loop jet function

The calculation of the one-loop jet function is surprisingly complicated



$$\sim \int d^d q \;\delta(q^2)\;\theta(q^0)\;\int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\;\theta(k^0)\;\frac{\bar{n}q\;(\bar{n}k+\bar{n}q)}{\bar{n}k\;(q+k)^2}$$

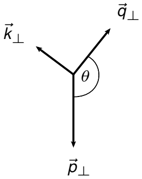
$$\times\;\delta\Big(Q-\bar{n}q-\bar{n}k\Big)\;\delta^{d-2}\Big(p_\perp-q_\perp-k_\perp\Big)\;\delta\Big(b-\tfrac{1}{2}|q_\perp|-\tfrac{1}{2}|k_\perp|\Big)$$

$$\sim \int_0^1 d\eta\,\eta\,(1-\eta)^{-1+\alpha}\int_{1-y}^{1+y} d\xi\;\frac{\xi(2-\xi)^{1-2\alpha}(\xi(2-\xi)-1+y^2)^{-\frac{1}{2}-\varepsilon}}{(\xi-2y\eta)^2+4\eta(1-y)(1+y-\xi)}$$

- non-trivial angle complicates calculation
- expansion in α and ϵ is subtle

  $\Rightarrow$ have to keep $(2b-p)^{-1-\epsilon}, (2b-p)^{-1-2\epsilon}, \ldots$ to all orders
- computed the integrals in closed form without expanding in ϵ

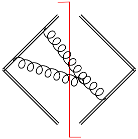
  $\Rightarrow$ hypergeometric functions of half-integer parameters
- performed Laplace + Fourier transformations analytically



Two-loop anomaly coefficient

Most easily extracted from the two-loop soft function

- again two particles in final state
- but requires to go one order higher in ϵ-expansion
- encounter Nielsen polylogs and elliptic integrals

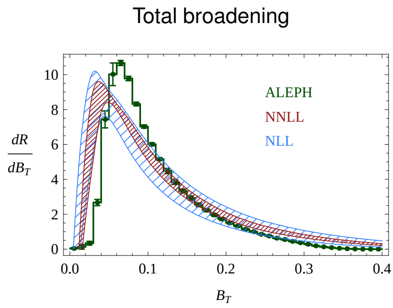


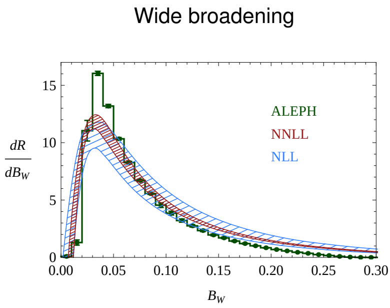
$$
\begin{aligned}
d_2^B(z) = C_A \Big\{ & -\frac{1+z^2}{9}\, h_1(z) + \frac{67+2z^2}{9}\, h_2(z) - 8\, h_3(z) + 32\, S_{1,2}\Big(-\frac{z_-}{z_+}\Big) - 8\,\mathrm{Li}_3\Big(-\frac{z_-}{z_+}\Big) \\
& + 8\, S_{1,2}(-w) - 24\,\mathrm{Li}_3(-w) - 24\, S_{1,2}(1-w) + 8\,\mathrm{Li}_3(1-w) + 24\, S_{1,2}\Big(\frac{1-w}{2}\Big) \\
& + \text{a few more lines} \Big\} \\
+ T_F n_f \Big\{ & \frac{2(1+z^2)}{9}\, h_1(z) - \frac{2(13+2z^2)}{9}\, h_2(z) - \frac{4}{3}\ln^2 z_+ - \frac{20}{9}\ln z_+ + \frac{4}{9} z^2 - \frac{82}{27} \\
& + \frac{4w(5-z^2)}{9} \ln\Big(\frac{1+w}{w}\Big) + \frac{2w(11+2z^2)}{9} \Big\}
\end{aligned}
$$

with $w = \sqrt{1+z^2}$ and $z_\pm = (w \pm 1)/4$

A glimpse at the data

[Becher, GB 12]



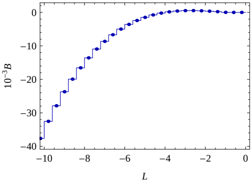


- theory uncertainty significantly reduced in fit region for α_s extraction
- without matching to fixed-order calculation
- without estimate of non-perturbative corrections

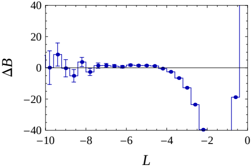
Compare with fixed order

Confront with output of fixed-order MC generators (EVENT2, EERAD3)

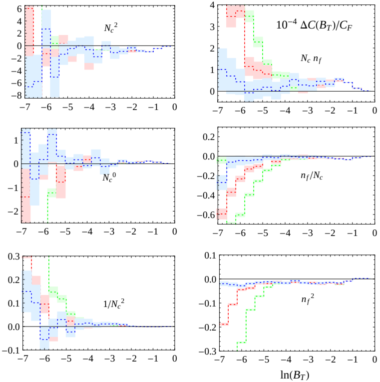
NLO



NLO—NNLL



NNLO—NNLL



$\Rightarrow$ we obtain the right logarithmic terms for small values of $L = \ln B_T$

Conclusions

Resummation beyond standard RG techniques via collinear anomaly

- we proposed an analytic phase space regularisation for SCET_{II} problems

$$\int d^d k \;\delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

- respects symmetries of EFT, well-suited for efficient calculations

We determined all ingredients to perform NNLL resummation for jet broadening

- allows for precision determinations of α_s from b_T and b_W distributions

The formalism is relevant for many interesting LHC observables

- Higgs production, $t\bar{t}$, jet vetoes, jet substructure, ...

Transverse momentum-dependent PDFs

Central ingredient for p_T resummation at hardon colliders

$$\mathcal{B}_{q/N}(z, x_T; \mu) = \frac{1}{2\pi} \int dt \, e^{-izt\bar{n}\cdot p} \sum\!\!\!\!\!\!\int_X \frac{\bar{n\!\!\!/}_{\alpha\beta}}{2} \langle N(p)| \, \bar{\chi}_\alpha(t\bar{n}+x_\perp) \, |X\rangle \langle X| \, \chi_\beta(0) \, |N(p)\rangle$$

$\Rightarrow$ ill-defined in DimReg because of unregularized rapidity divergences

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Many attempts to find an optimal definition, e.g. [Collins 11]

$$\mathcal{B}_{q/N}(z, x_T; \zeta_A; \mu) = \lim_{y_{1,2}\to\pm\infty} \mathcal{B}^{\text{unsub}}_{q/N}(z, x_T; y_p - y_2) \sqrt{\frac{\tilde{S}(x_T, y_1, y_n)}{\tilde{S}(x_T, y_1, y_2)\, \tilde{S}_{(0)}(x_T, y_n, y_2)}}$$

”This definition seems unexpectedly complicated”

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"This definition seems unexpectedly complicated"

We propose a minimal modification of the naive definition [Becher, GB 11]

$$\mathcal{B}_{q/N}(z, x_T; \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \sum\!\!\!\!\!\!\int_{X,reg} \frac{\bar{\not{n}}_{\alpha\beta}}{2} \langle N(p)|\, \bar{\chi}_\alpha(t\bar{n}+x_\perp)\, |X\rangle \langle X|\, \chi_\beta(0)\, |N(p)\rangle$$

$\Rightarrow$ the only definition that has shown to work at two-loop order [Gehrmann, Lübbert, Yang 12]