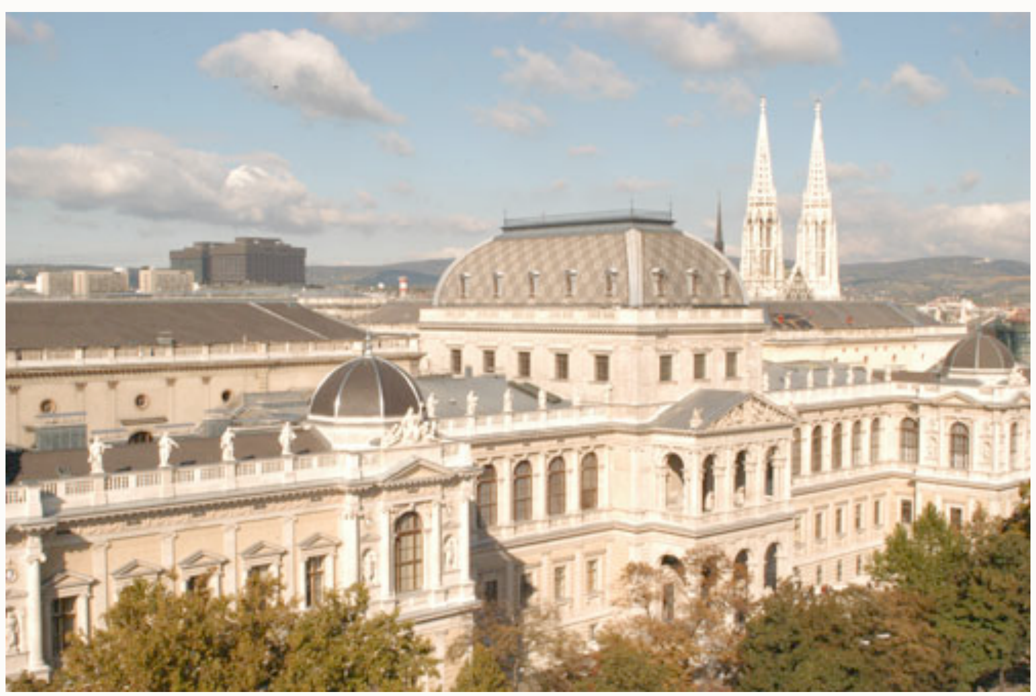
The Principle of Maximal Conformality

The Elimination of the QCD Renormalization Scale Ambiguity

Stan Brodsky



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with Leonardo Di Giustino , Xing-Gang Wu, and Matin Mojaza

Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

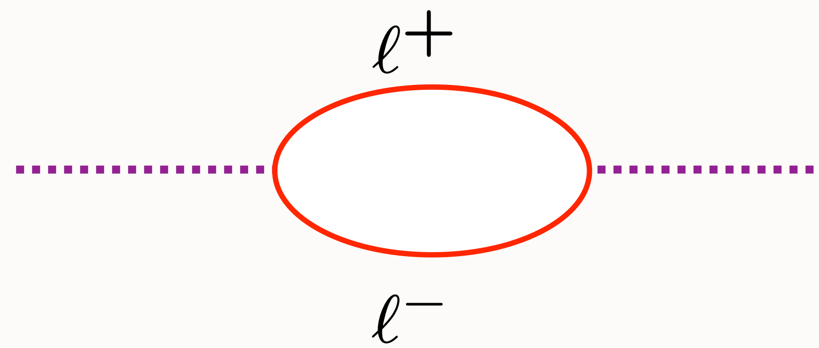
Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right)\sqrt{1+\frac{4m^2}{Q^2}}\,\log\frac{1+\sqrt{1+\frac{4m^2}{Q^2}}}{\left|1-\sqrt{1+\frac{4m^2}{Q^2}}\right|}\right]$$

Analytically continue to timelike t: Complex

$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi}\frac{Q^2}{m^2}$$

$Q^2 << 4M^2$ **Serber-Uehling**

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\log\frac{Q^2}{m^2}$$

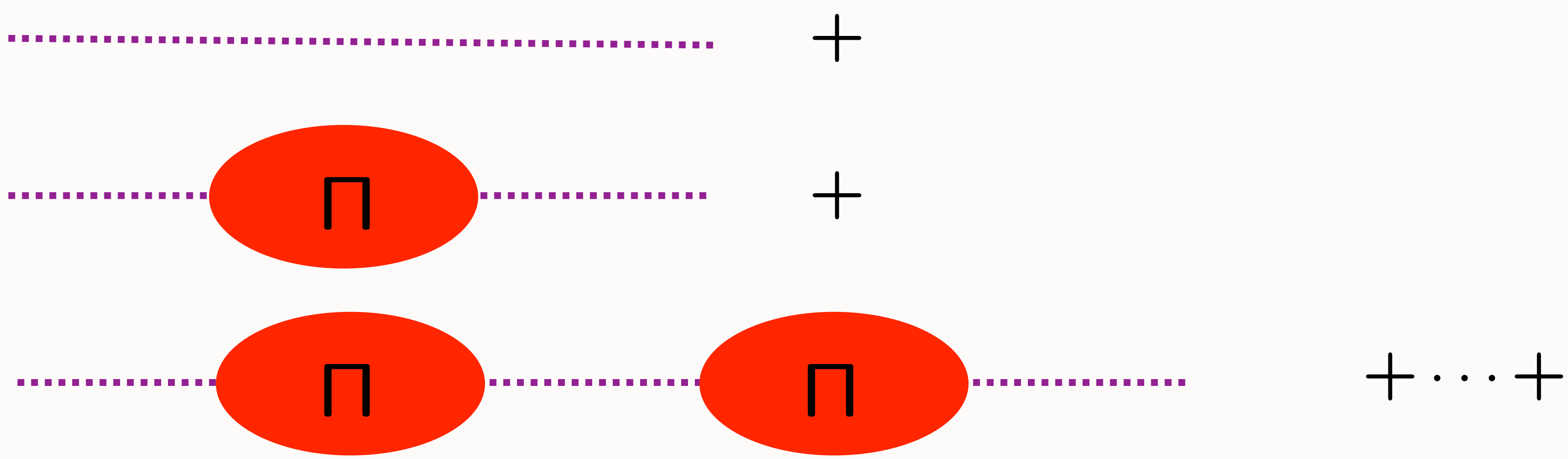
$Q^2 >> 4M^2$ **Potential Landau Pole**

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0$$

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

All-orders lepton-loop corrections to dressed photon propagator



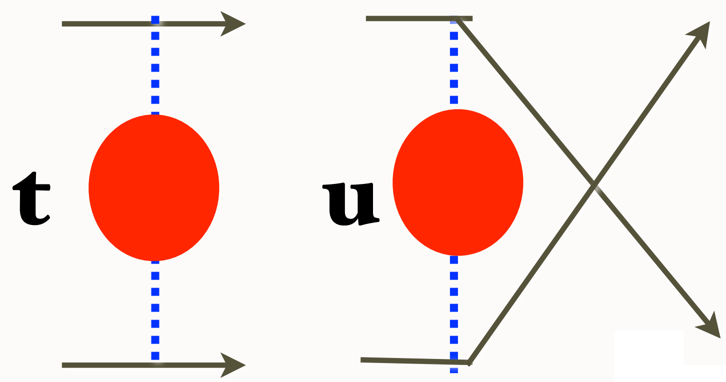
$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

Initial **scale t_o is arbitrary -- Variation gives RGE Equations**

Physical renormalization scale t not arbitrary!

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- Two separate physical scales: t, u = photon virtuality

- Gauge Invariant. Dressed photon propagator

- Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!

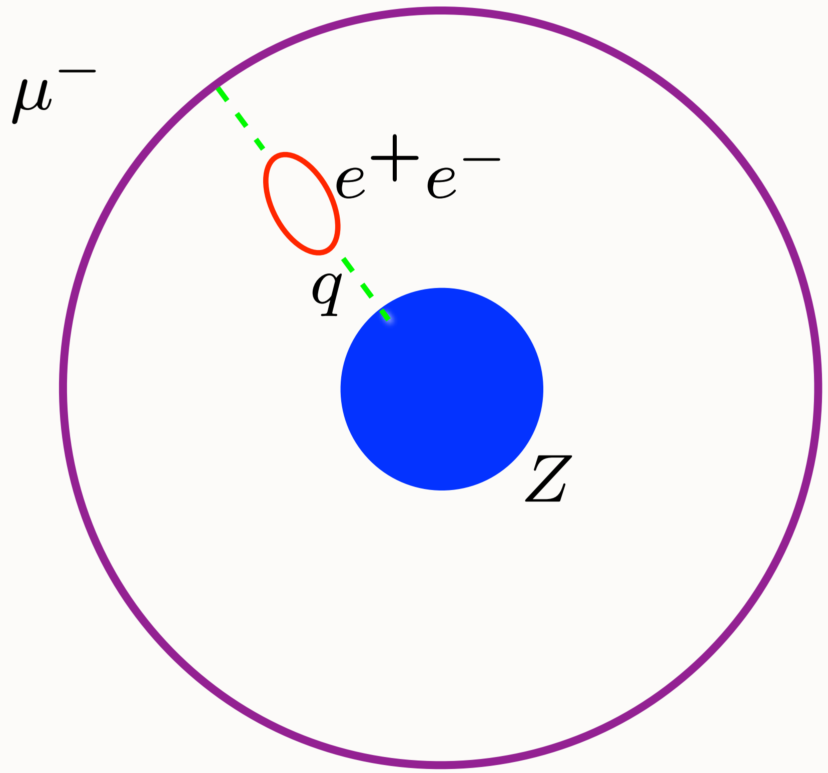
- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!

- Number of active leptons correctly set

- Analytic: reproduces correct behavior at lepton mass thresholds

- No renormalization scale ambiguity!

Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Relation between scales of the Gell-Mann--Low and $\overline{\text{MS}}$ schemes

$$\log \frac{\mu_0^2}{m_\ell^2} = 6 \int_0^1 x(1-x) \log \frac{m_\ell^2 + Q_0^2 x(1-x)}{m_\ell^2}$$

$$\log \frac{\mu_0^2}{m_\ell^2} = \log \frac{Q_0^2}{m_\ell^2} - 5/3$$

$$\boxed{\mu_0^2 = Q_0^2 \, e^{-5/3}} \quad \text{when } Q_0^2 >> m_\ell^2$$

D. S. Hwang, sjb

M. Binger

"Scale Displacement" between schemes

Can use $\overline{\text{MS}}$ scheme in QED; answers are scheme independent

Analytic extension: coupling is complex for timelike argument

The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Physical predictions cannot depend on renormalization scheme
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds -- number of active leptons set
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent!

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On the elimination of scale ambiguities in perturbative quantum chromodynamics

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(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Coupling at Initial Scale

Running Coupling Effects

Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

Light by Light Loops

BLM: Absorb β terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Principle of Maximal Conformality

BLM Scale Setting

$\beta_0 = 11 - \frac{2}{3} n_f$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2} \beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right] ,$$

where

Conformal coefficient - independent of β

$$Q^* = Q \exp(3 A_{VP}) ,$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33 A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion:
Gardi, Grunberg, Rathsman, sjb

Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- **All terms associated with nonzero beta function summed into running coupling**

- **BLM Scale Q* sets the number of active flavors**

- **Only n_f dependence (associated with renormalization) required to determine renormalization scale at NLO**

- **Result is scheme independent! Q* has exactly the correct dependence to compensate for change of scheme**

- **Correct Abelian limit**

- **Resulting series identical to conformal series!**

- **Renormalon n! growth of PQCD coefficients from beta function eliminated!**

- All non-conformal and scheme-dependent β-terms in the perturbative series are summed into the running coupling. The result is consistent with conformal theory (scheme-independent).
- Transitivity => proper scale-displacement or commensurate scale relations also ensure scheme-independence.

H.J. Lu and S.J. Brodsky, Phys.Rev. D**48**, 3310 (1993).

- The active flavors n_f in β-function is correctly determined.

S.J. Brodsky, M.S. Gill, M. Melles, J. Rathsman, Phys.Rev. D**58**, 116006 (1998).

- Renormalons growing as $(n!\ \beta^m\ \alpha_s^n)$ are avoided =>better convergence

S.J. Brodsky and X.G. Wu, arXiv:1203.5312, SLAC-PUB-14898.
S.J. Brodsky and X.G. Wu, arXiv:1204.1405, SLAC-PUB-14898.

- The PMC method agrees with the standard QED results in the Nc -> 0 limit.

S.J. Brodsky and P. Huet, Phys.Lett. B**417**, 145 (1998); A.L. Kataev, Phys.Lett. B**691**, 82 (2010).

- Higher-order calculation easier, we only need to calculate n_f-terms

Up to NNNLO PMC/BLM scale setting can be found in SJB

S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys.Rev. D**28**, 228(1983).
S.J. Brodsky and L.D. Giustino, arXiv: 1107.0338.
S.J. Brodsky and X.G. Wu, Phys.Rev. D**85**, 034038 (2012).

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Myths concerning scale setting

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- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

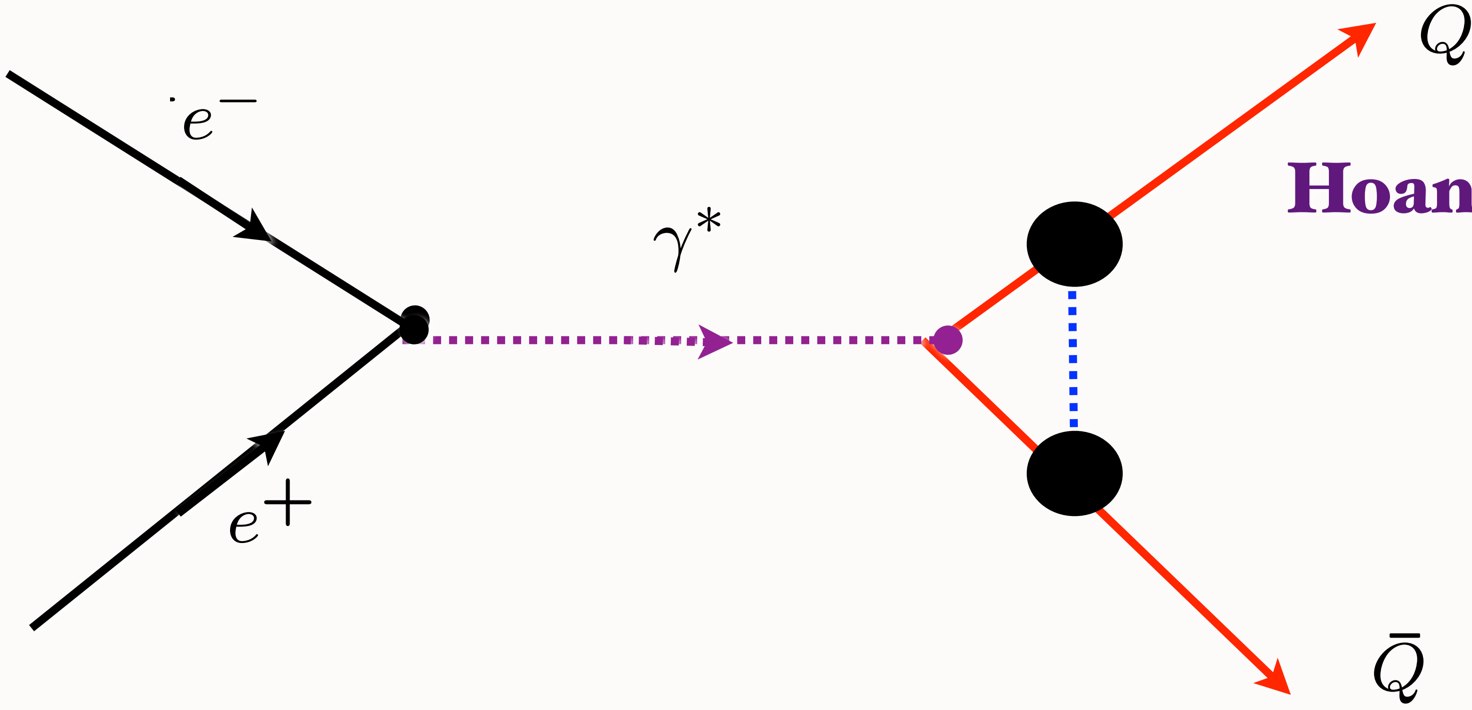
Huet, sjb

$$\boxed{\lim N_C \to 0 \text{ at fixed } \alpha = C_F\alpha_s, n_\ell = n_F/C_F}$$

$$\boxed{\text{QCD} \to \text{Abelian Gauge Theory}}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = \left[1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}\right] \times \left[1 + \frac{\pi\alpha_s(sv^2)}{4v}\right]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Application A: Scale setting in 3-jets events at LO

R. K. Ellis *et Al*, Nucl. Phys. B178, 421-456 (1981

S.J. Brodsky and L.D. Giustino, arXiv: 1107.0338.

$$\boxed{e^+e^- \to q + \bar{q} + g \text{ up to NLO}}$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{(s)} + d\sigma^3}{dy} = \int_y^{1-2y} dz \int_y^{1-y-z} dx \, T[1-x-z, x, z]\alpha_s(s)$$

y: the maximum virtuality of the jet

$$\left[1 - \frac{\alpha_s(s)}{\pi}\left(\frac{\beta_0}{4}\left(\ln[x] + \ln[z] - \frac{5}{3}\right) + \cdots\right)\right]$$

$$= \alpha_s(s)\left[T(y) - \frac{\alpha_s(s)}{\pi}\left(\left(C(y) - \frac{5}{3}T(y)\right)\frac{\beta_0}{4} + \cdots\right)\right]$$

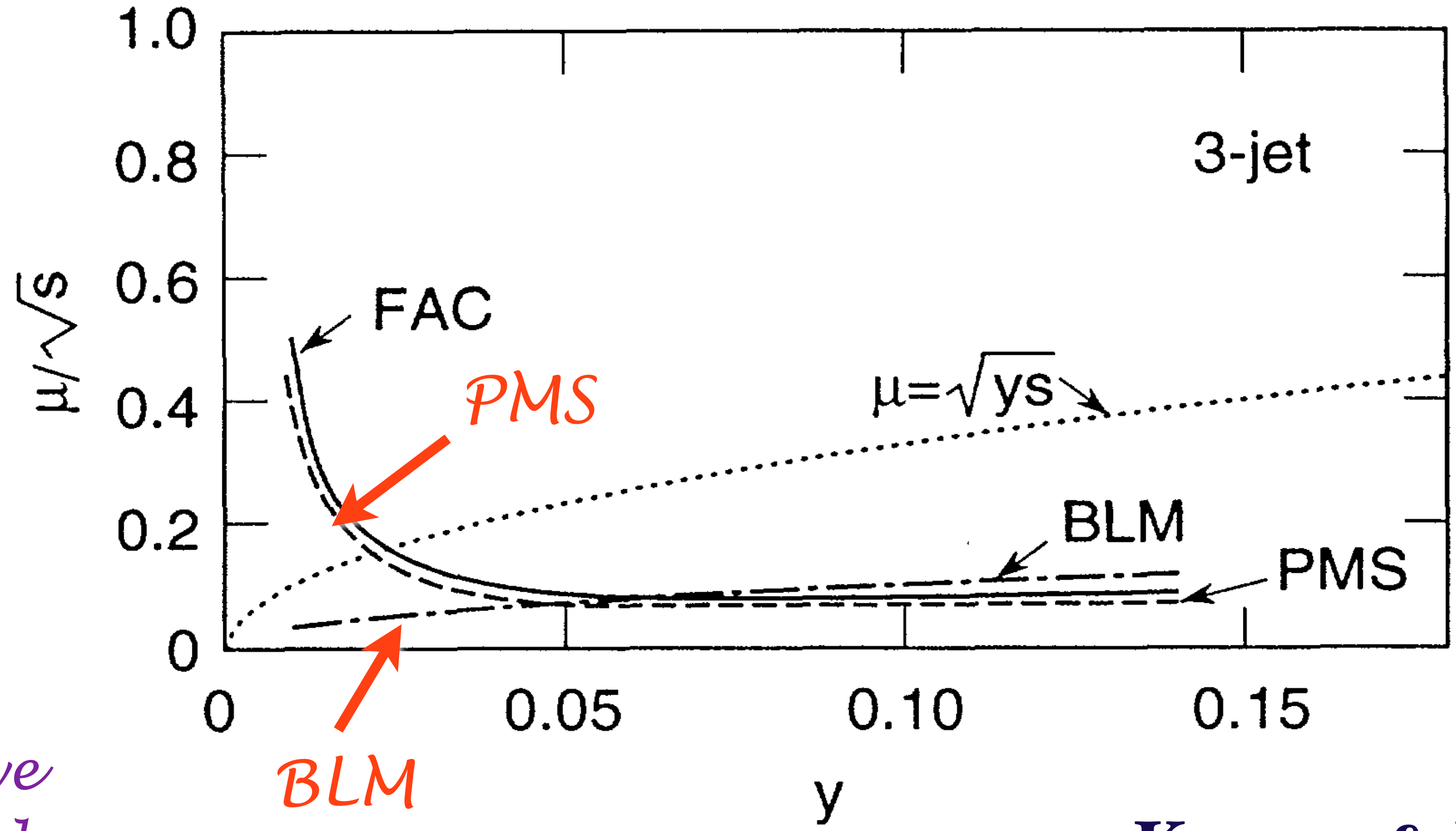
$$= T(y)\alpha_s(s)\left[1 - \frac{\alpha_s(s)}{\pi}\left(\frac{1}{4}\left(\frac{C(y)}{T(y)} - \frac{5}{3}\right)\beta_0 + \cdots\right)\right]$$

$$= T(y)\alpha_s(\mu^2_{BLM}) + \cdots$$

$$\boxed{\mu^2_{BLM} = s \times \exp\left(-\frac{5}{3} + \frac{C(y)}{T(y)}\right)}$$

LO-BLM/PMC scale

Three-jet production in electron-positron annihilation



Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

BLM scale is gluon jet virtuality

PMS & FAC have wrong physical dependence!

Kramer & Lampe

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Other Jet Observables using BLM: Rathsman

PMS vs. PMC

- **PMS/FAC - incorrectly sums conformal terms -- even minimizes physical asymmetries!   PMS violates transitivity**

- **PMC/BLM: exposes conformal series - no renormalons**

- **Conformal series has new physics -- not associated with renormalization**

- **PMC: No need to analyze diagrams or codes -- simply identify non-conformal logarithms -- then shift scale**

- **PMC: Applies to subprocesses with multiple final particles- recursive procedure**

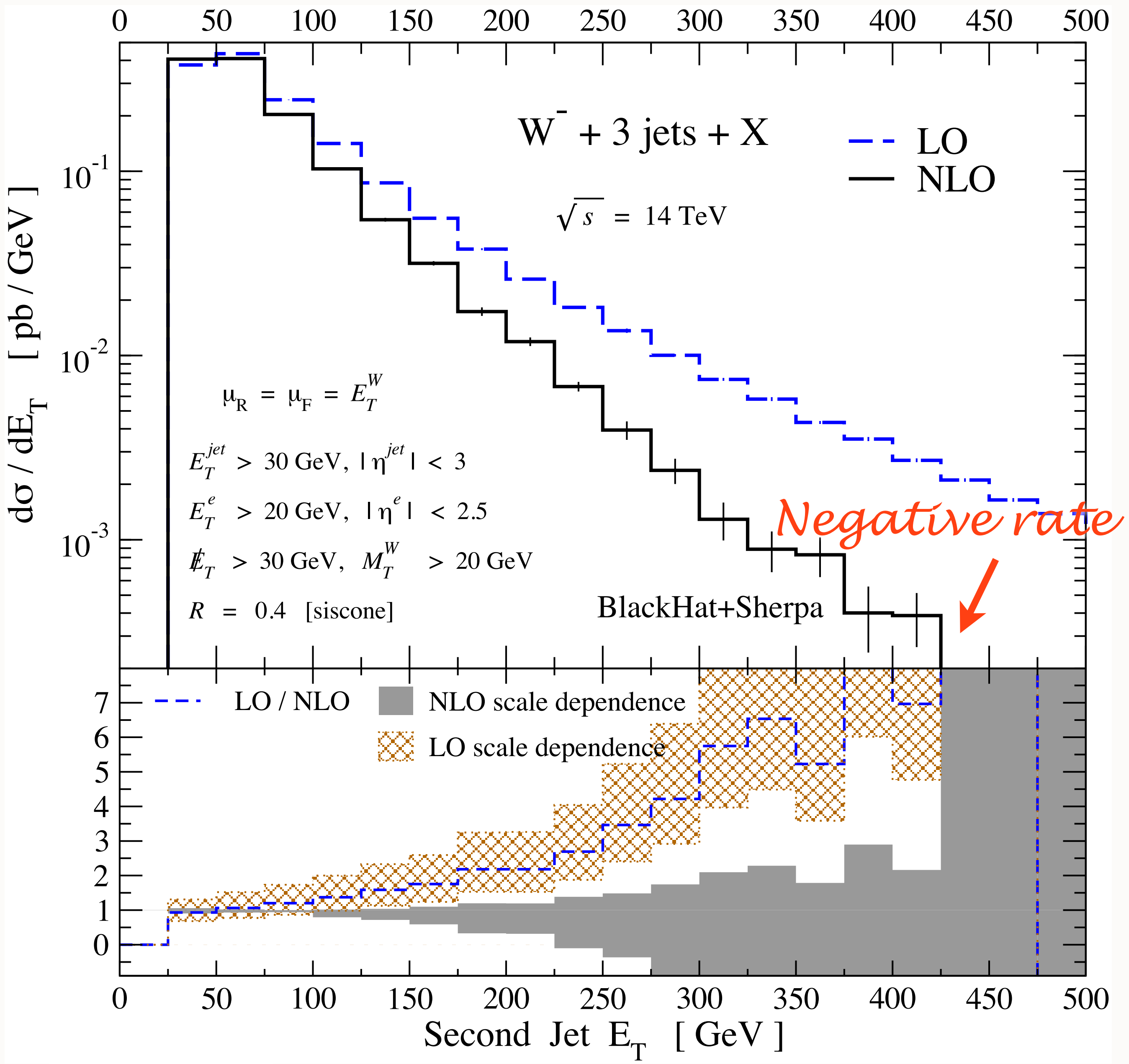
- **PMC/BLM: Agrees with QED in Abelian limit**

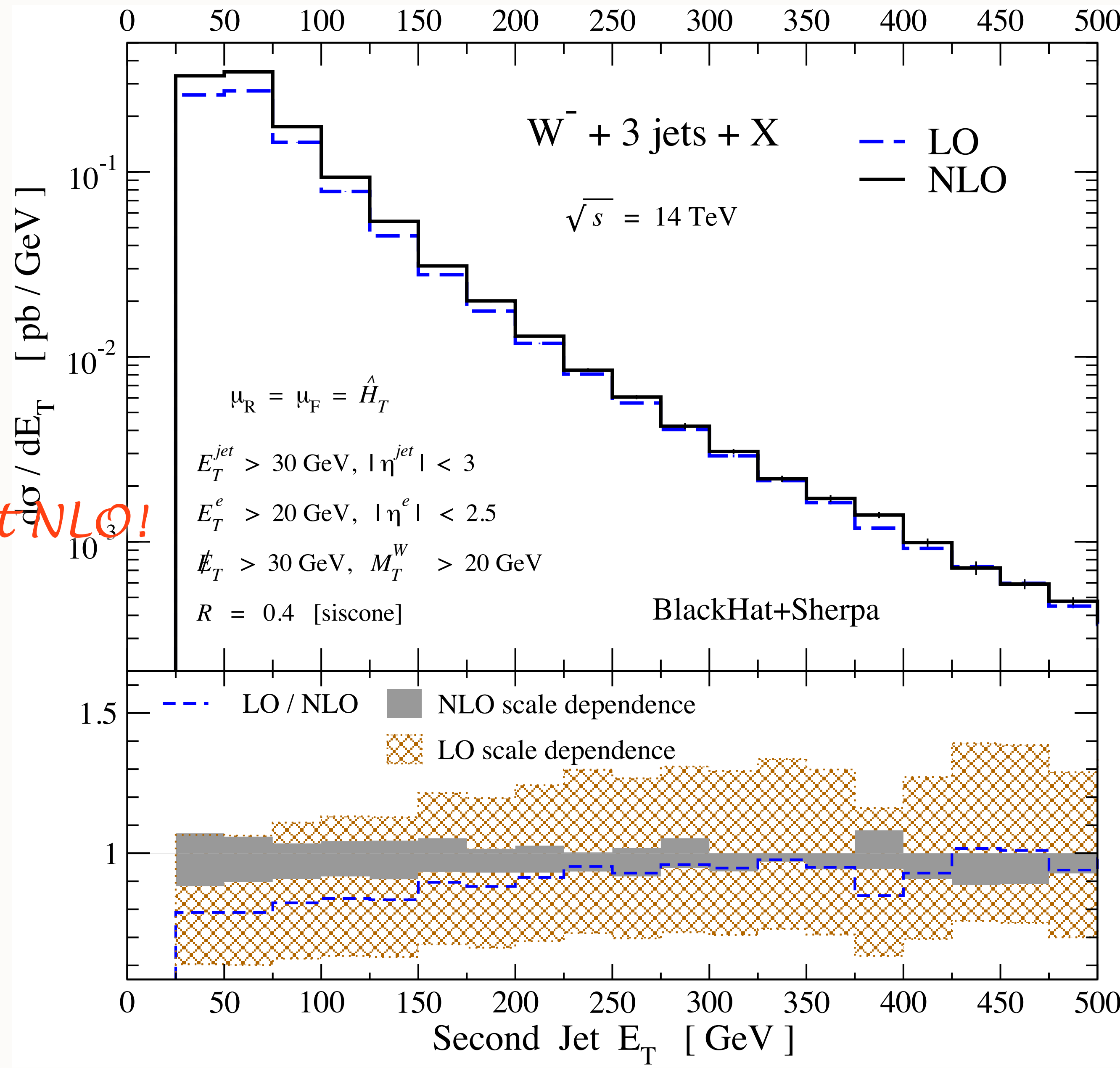
- **PMC/BLM: Result is independent of scheme and initial scale choice**

Next-to-Leading Order QCD Predictions for W + 3-Jet Distributions at Hadron Colliders

Black Hat

$$\mu_R = \mu_F = E_T^W \qquad\qquad \mu_R = \mu_F = \hat{H}_T$$





. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maıtre

Single PMC Renormalization Scale

- BLM: Set scale in each skeleton graph to absorb all nonzero beta terms.
- In practice easier to set a single global scale
- Consider general hard subprocess: $a + b \to c + d + e + \cdots$

$$p_{ij}^2 = p_i \cdot p_j$$

e.g., $pp \to W + 3 \text{ jets} + X$

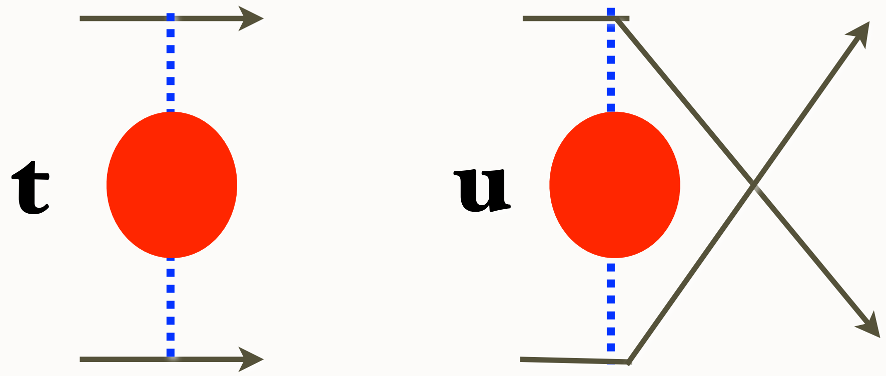
$$\hat{\mu}^2 = C \times \Pi_{ij} \; [p_{ij}^2]^{w_{ij}} \qquad \log \hat{\mu}^2 = \sum_{i \neq j} w_{ij} \log p_{ij}^2 + \log C$$

$$w_{ij} = \frac{f_{ij}}{\sum_{i \neq j} f_{ij}}.$$

C is the scheme displacement

$C = e^{-5/3}$ for $\overline{MS}$

Example: Spinless electron-electron scattering



$$M = \frac{s-t}{t}\alpha(t) + \frac{s-u}{u}\alpha(u)$$

Scales sum VP to all orders

$$M \simeq [\tfrac{s-t}{t} + \tfrac{s-u}{u}]\alpha(\mu_0^2)$$
$$+[\tfrac{s-t}{t}]\frac{\alpha^2(\mu_0^2)}{3\pi}n_\ell \log(\tfrac{t}{\mu_0^2}) + [\tfrac{s-u}{u}]\frac{\alpha^2(\mu_0^2)}{3\pi}n_\ell \log(\tfrac{u}{\mu_0^2})$$

$$M = [\frac{s-t}{t} + \frac{s-u}{u}]\alpha(\hat{\mu}^2)$$

$$\boxed{\hat{\mu}^2 = t^{w_t} \times u^{w_u}}$$

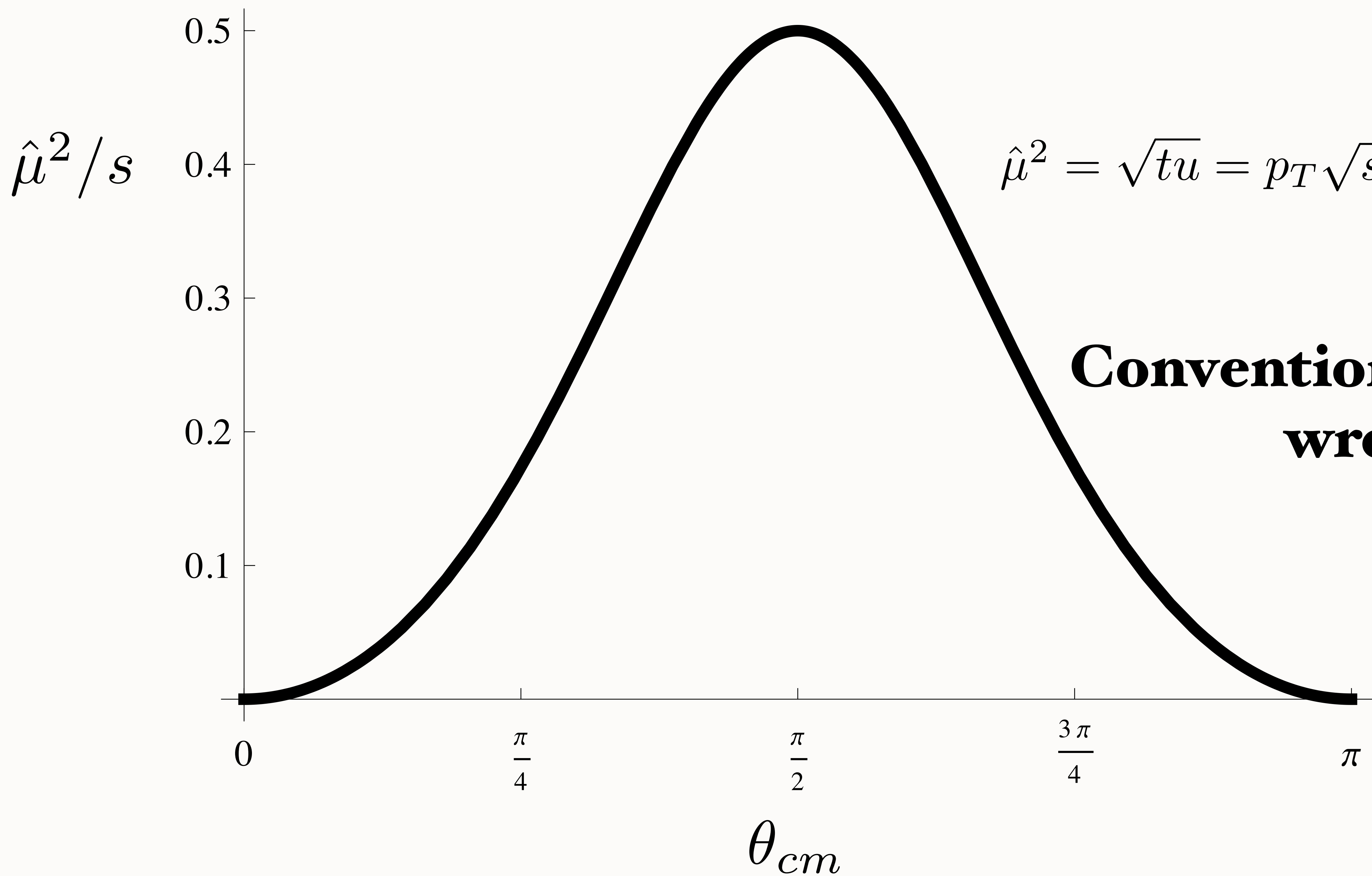
$$w_t = \frac{\frac{s-t}{t}}{\frac{s-t}{t} + \frac{s-u}{u}} \qquad w_u = \frac{\frac{s-u}{u}}{\frac{s-t}{t} + \frac{s-u}{u}}$$

Identify w_t from $\frac{dM}{d\log t}$

Remaining $\mathcal{O}(\alpha^2)$ correction is conformal

Spinless electron-electron scattering

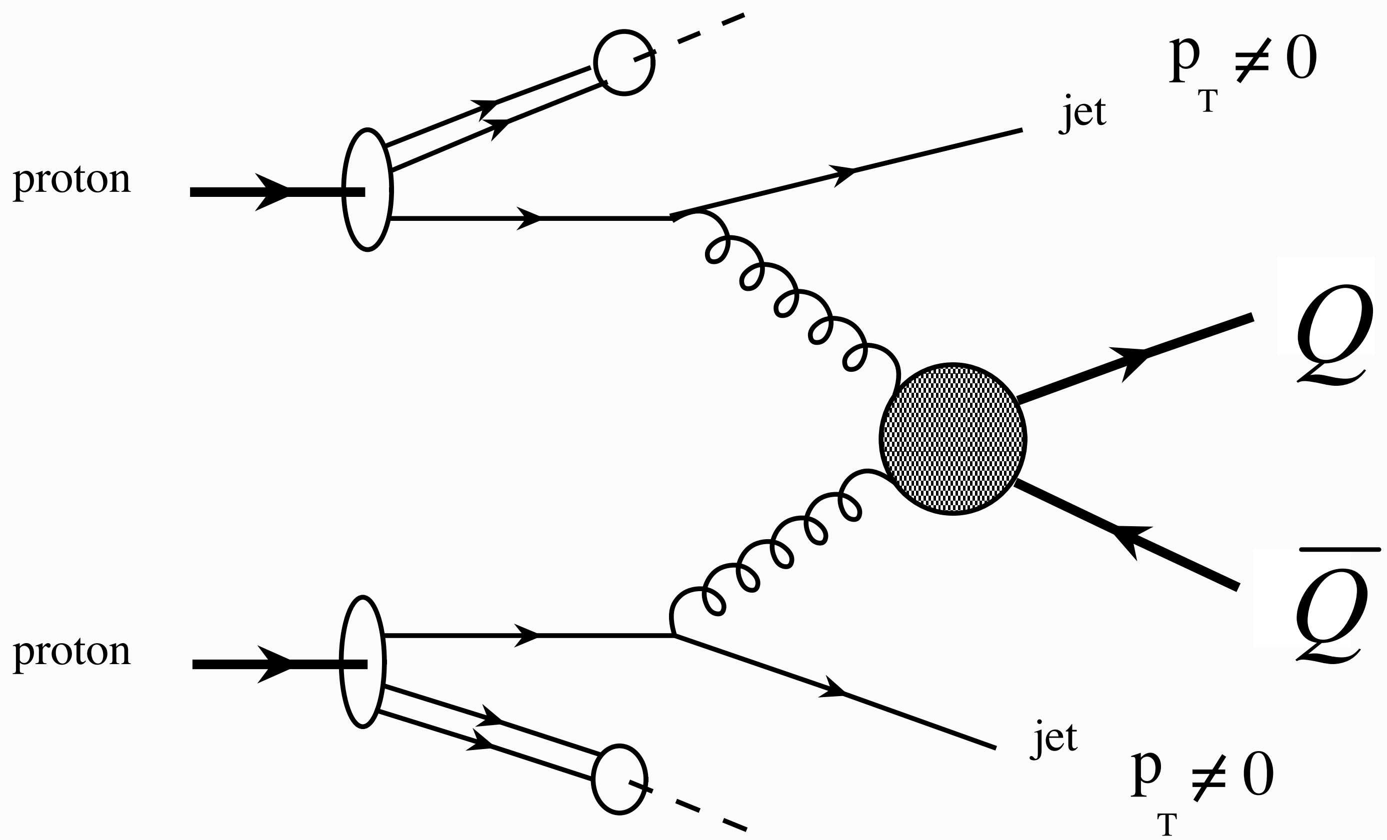
$$\hat{\mu}^2 = t^{w_t} \times u^{w_u}$$



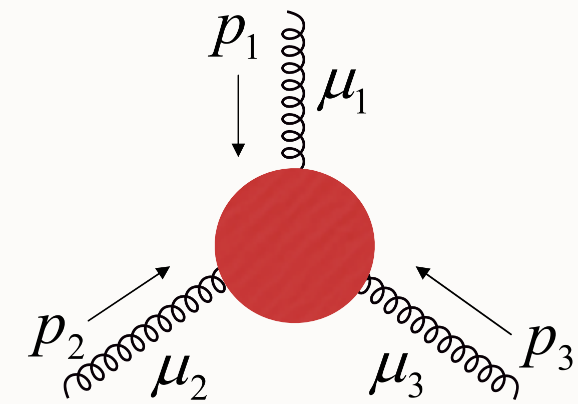
$\hat{\mu}^2 = \sqrt{tu} = p_T\sqrt{s}$ at $\theta_{cm} = \pi/2$

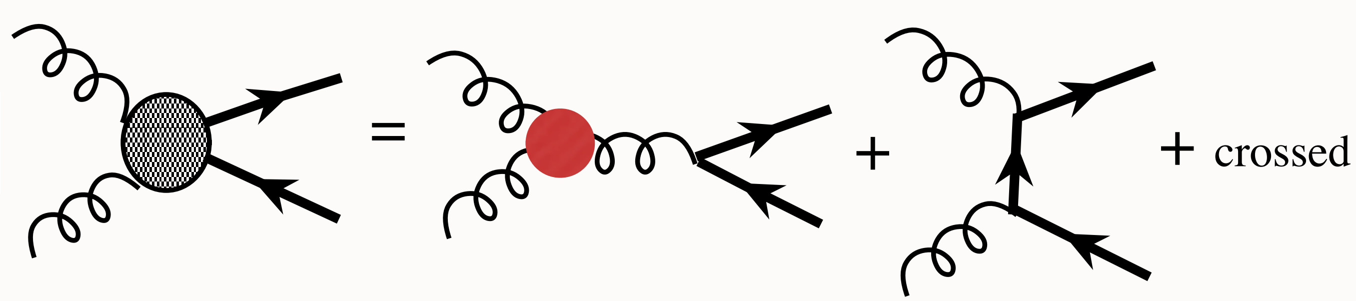
Conventional guess is wrong

Heavy Quark Hadroproduction



3-gluon coupling depends on 3 physical scales





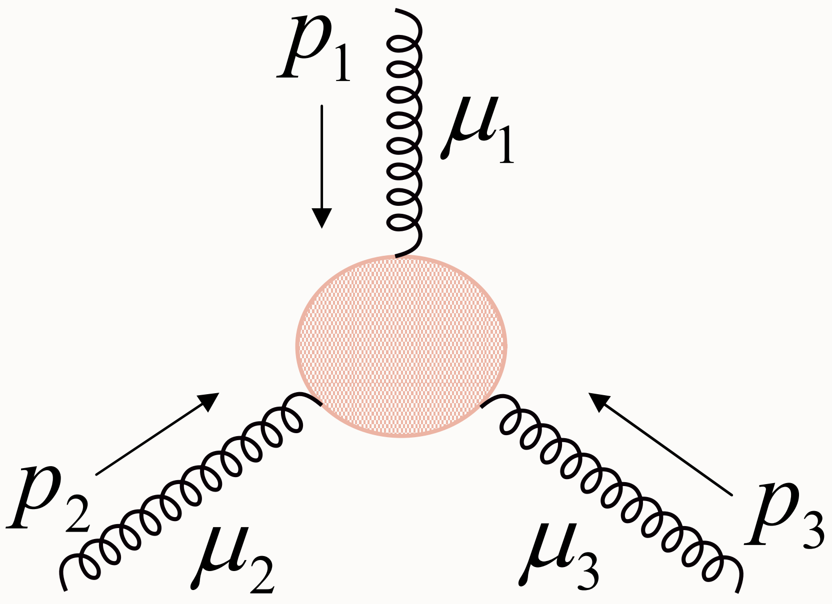
The Renormalization Scale Problem

$$\rho(Q^2) = C_0 + C_1\alpha_s(\mu_R) + C_2\alpha_s^2(\mu_R) + \cdots$$

$$\mu_R^2 = CQ^2$$

Is there a way to set the renormalization scale μ_R?

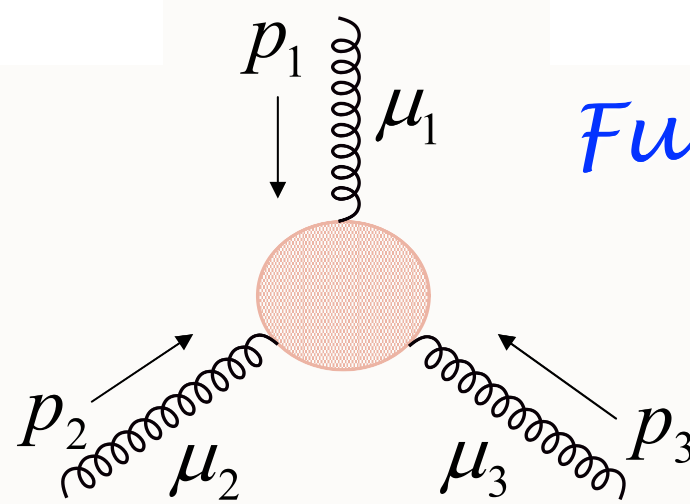
What happens if there are multiple physical scales ?



Binger, sjb
H. J. Lu

General Structure of the Three-Gluon Vertex

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Full analytic calculation, general masses, spin Pinch Scheme

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3

with $p_1 + p_2 + p_3 = 0$

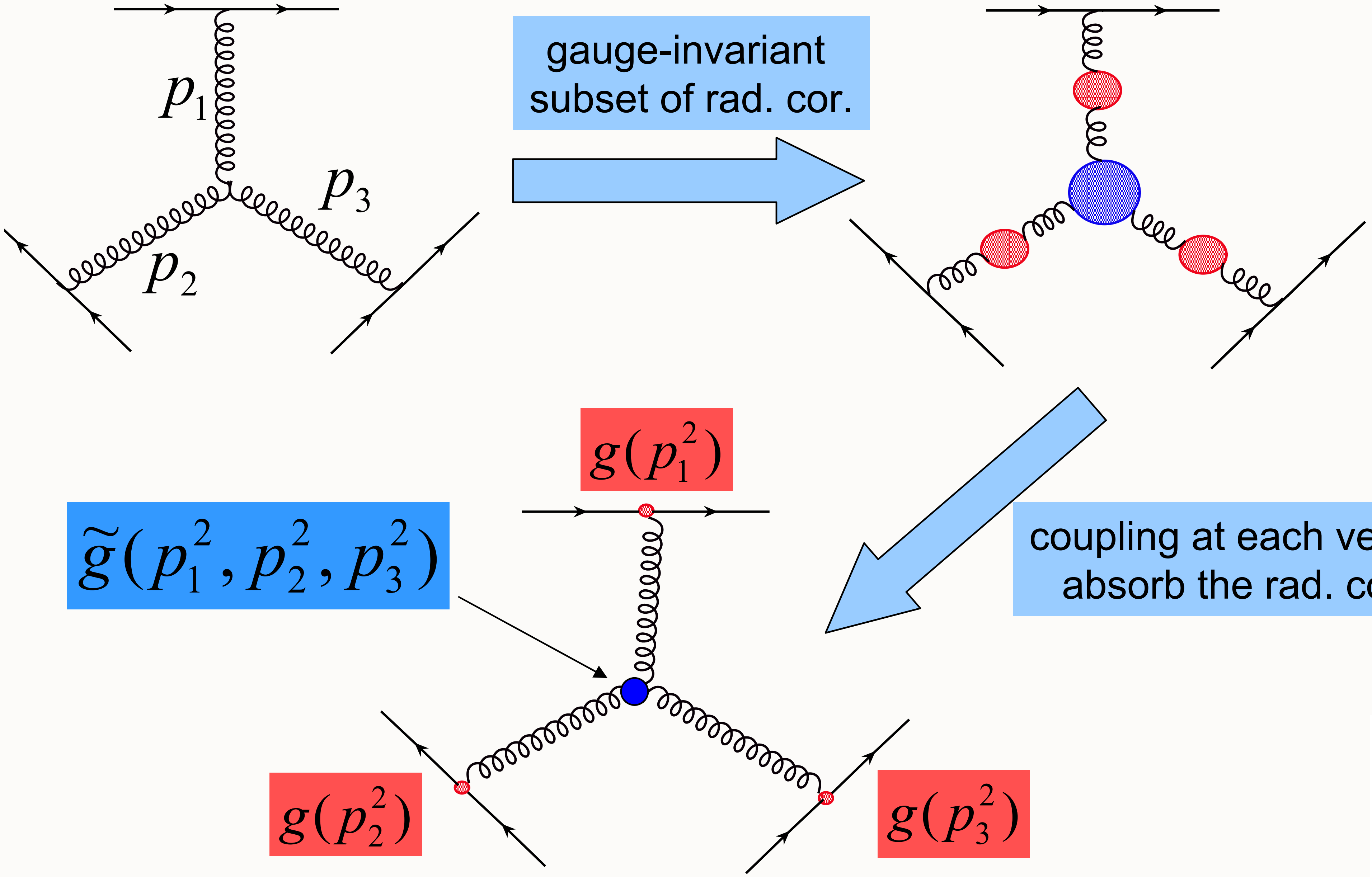
⇒ 14 basis tensors and form factors

PHYSICAL REVIEW D **74**, 054016 (2006)

Form factors of the gauge-invariant three-gluon vertex

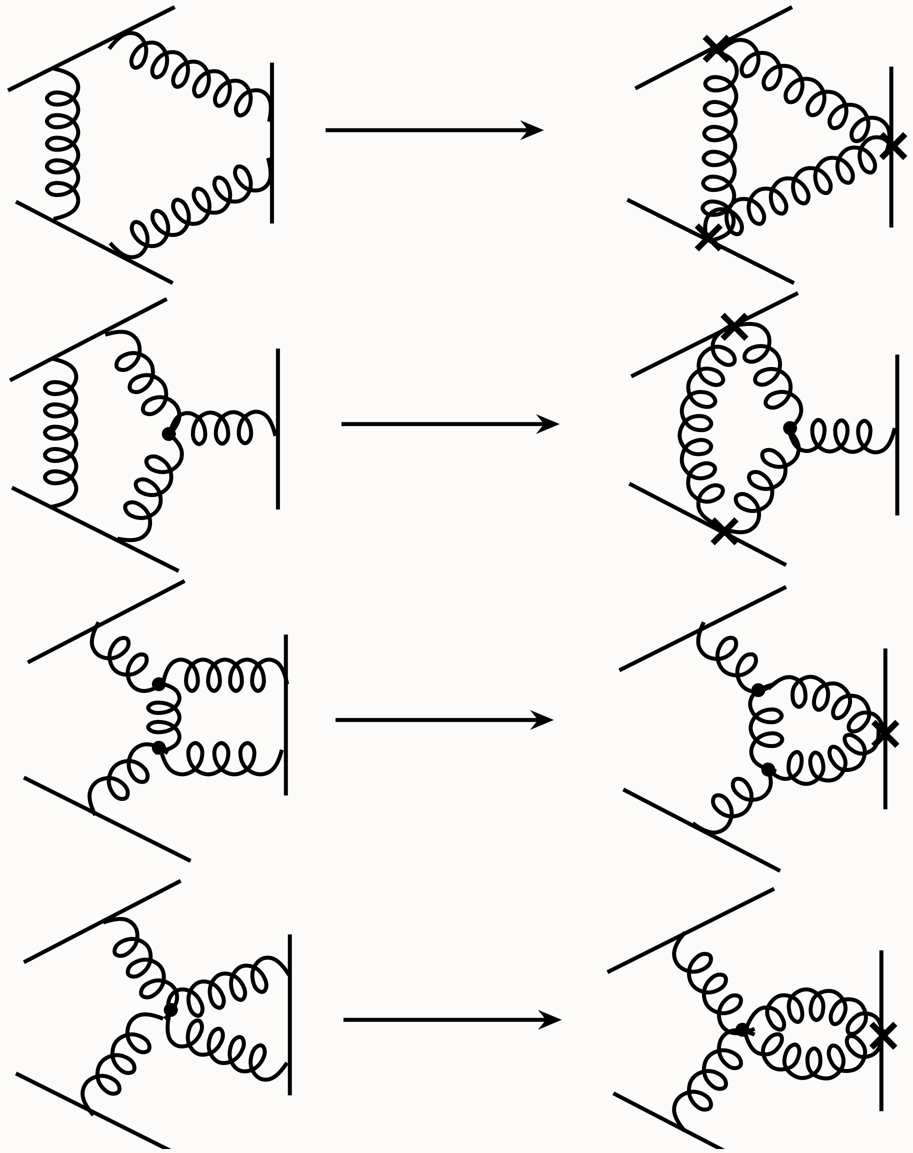
Michael Binger* and Stanley J. Brodsky†

Multi-scale Renormalization of the Three-Gluon Vertex

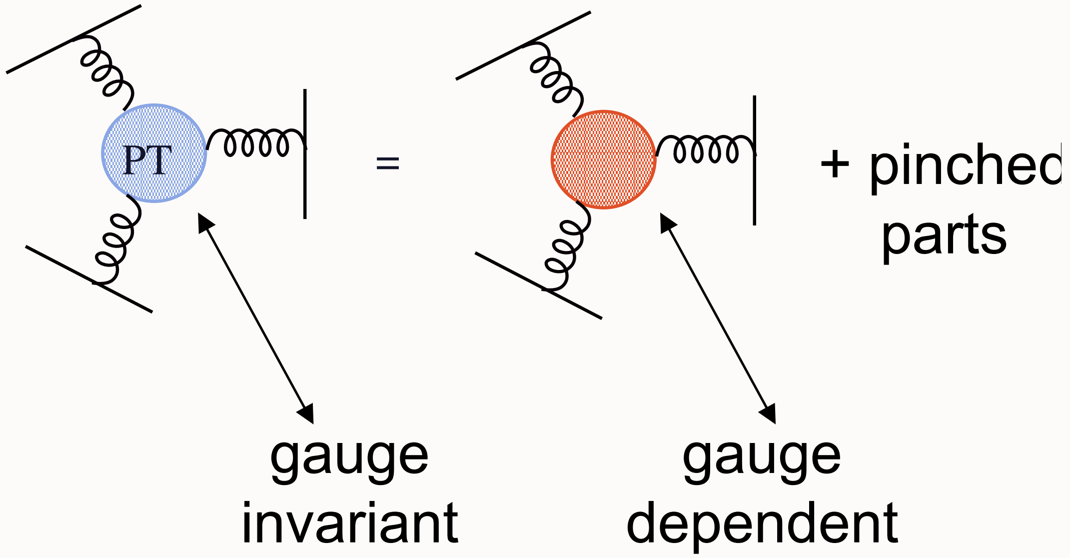


The Gauge Invariant Three Gluon Vertex

Cornwall and Papavassiliou performed the PT construction :

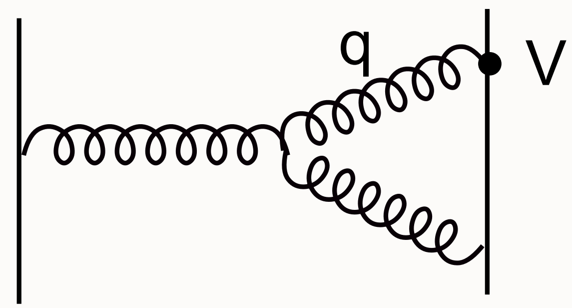


The “pinched” parts are added to the “regular” 3 gluon vertex

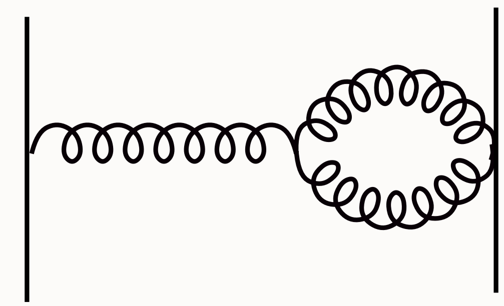


The Pinch Technique

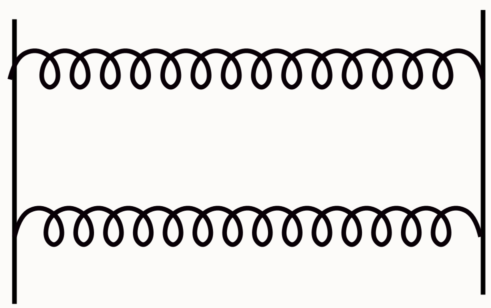
(Cornwall, Papavassiliou)



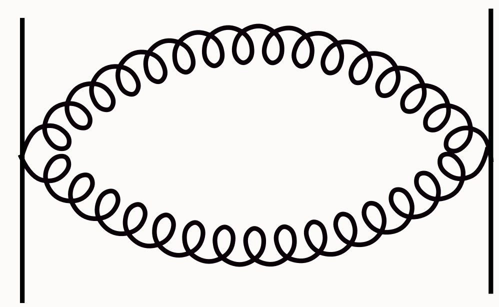
self–energy–like projection



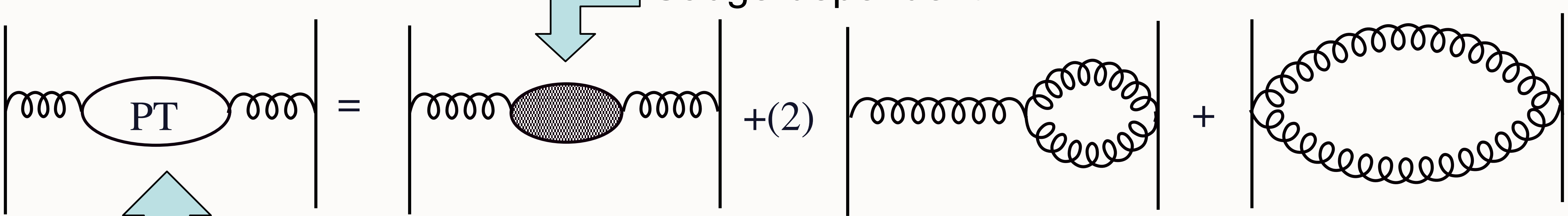
$$q \cdot V(p,k) = S^{-1}(p) - S^{-1}(k)$$



self–energy–like projection



Gauge-dependent



Gauge-invariant gluon self-energy!

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natural generalization of QED charge

3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

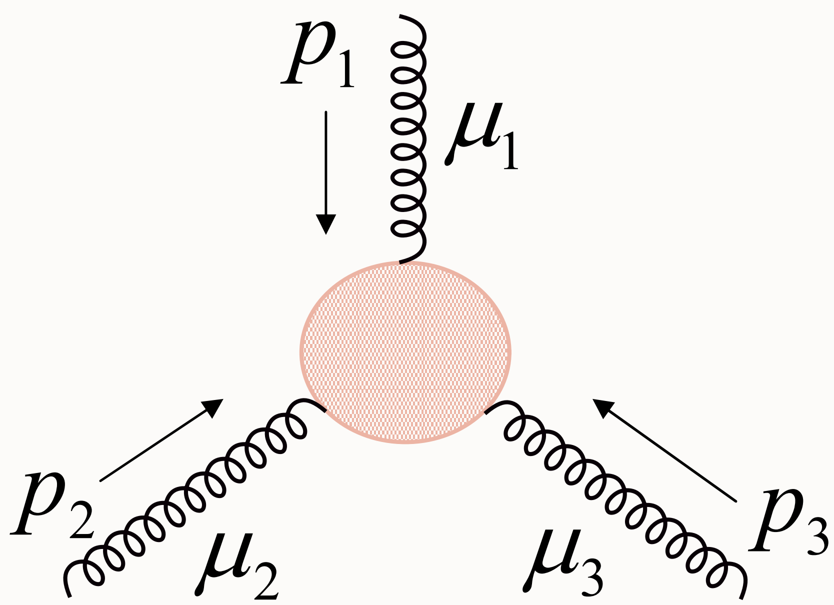
$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots\right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

L(a,b,c) = 3-scale “log-like” function

L(a,a,a) = log(a)

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

H. J. Lu

Scale determines effective number of flavors

3 Scale Effective Scale

$$L(a,b,c) \equiv \log\left(Q^2_{eff}(a,b,c)\right) + i\,\mathrm{Im}\,L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\tilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54 |a|$$

$$Q^2_{eff}(a,a,c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,-a,c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Pinch Scheme

Surprising dependence on Invariants

Grunberg

Define QCD Coupling from Observables

Effective Charges: analytic at quark mass thresholds, finite at small momenta

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations:

Relate observable to observable at commensurate scales

H.Lu, Rathsman, sjb

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32} C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32} C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2\right\}.$$

Eliminate MS
Find Amazing Simplification

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

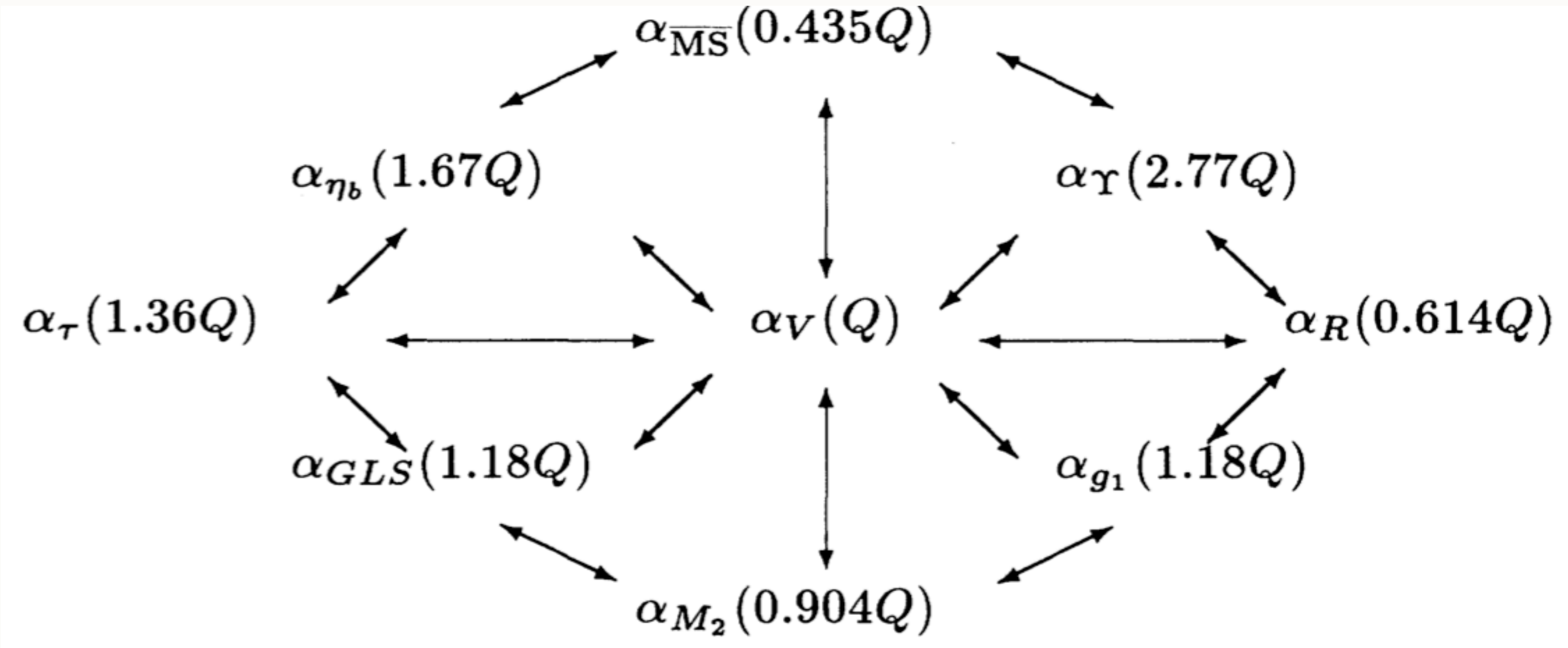
No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

H. J. Lu, sjb

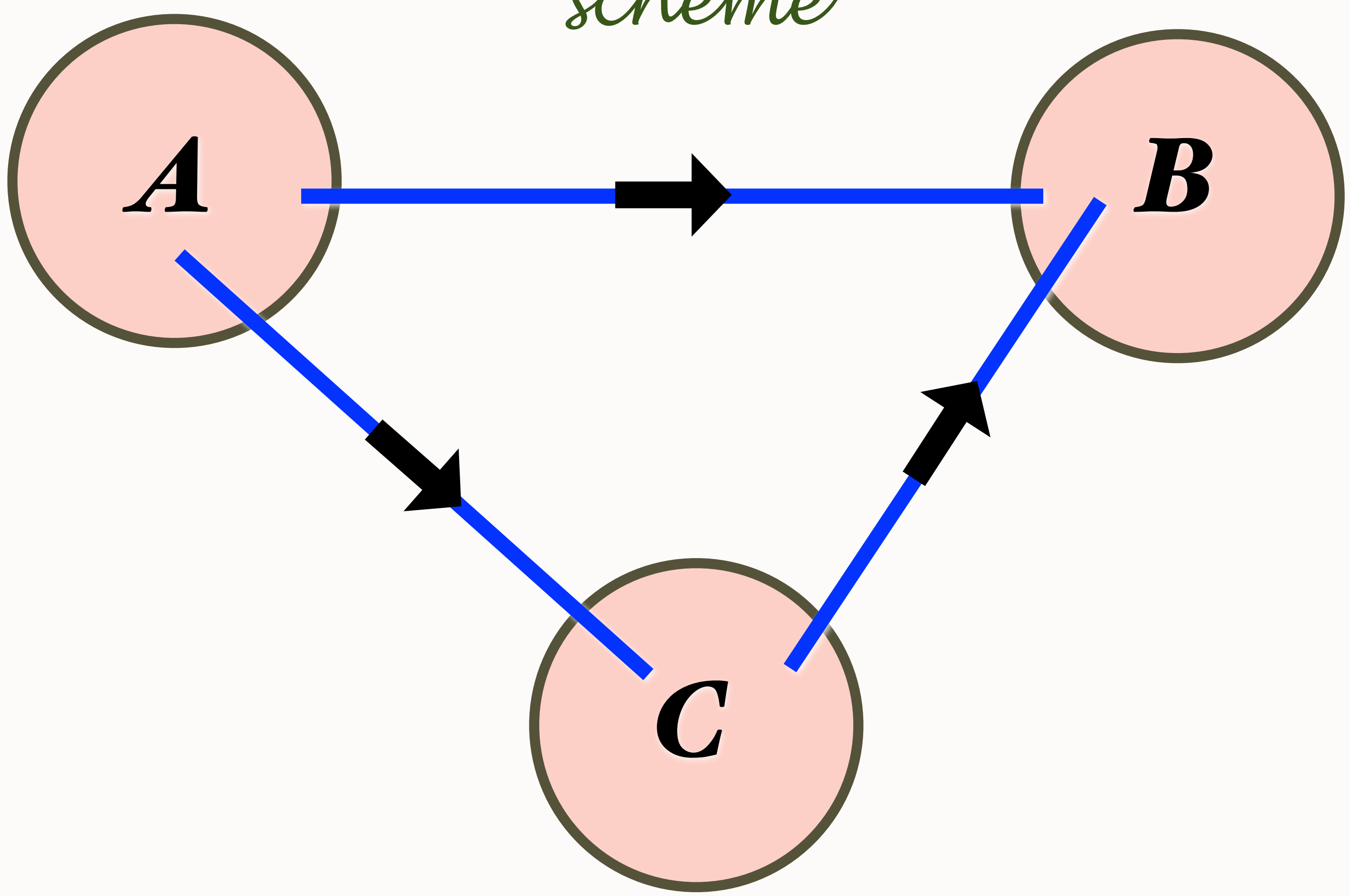

$\alpha_{\overline{\mathrm{MS}}}(0.435Q)$
$\alpha_{\eta_b}(1.67Q)$
$\alpha_{\Upsilon}(2.77Q)$
$\alpha_{\tau}(1.36Q)$
$\alpha_V(Q)$
$\alpha_R(0.614Q)$
$\alpha_{GLS}(1.18Q)$
$\alpha_{g_1}(1.18Q)$
$\alpha_{M_2}(0.904Q)$

$$\frac{\alpha_\tau(M_\tau)}{\pi} = \frac{\alpha_R(Q^*)}{\pi},$$

$$Q^* = M_\tau \exp\left[-\frac{19}{24} - \frac{169}{128}\frac{\alpha_R(M_\tau)}{\pi}\right]$$

Transitivity Property of Renormalization Group

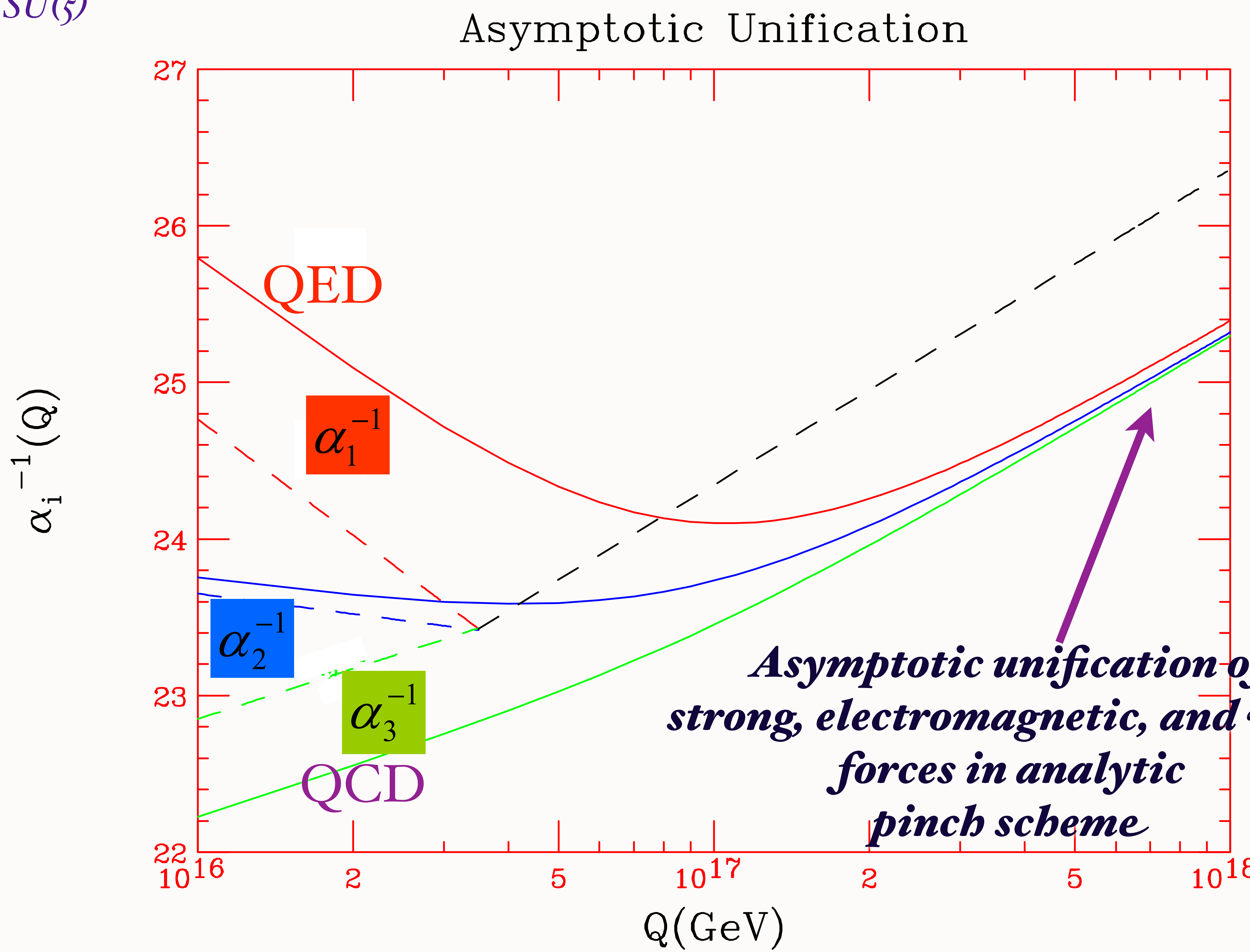
Relations between observables must be independent of intermediate scheme



H. J. Lu, sjb

A ➡ ***C*** ***C*** ➡ ***B*** ***identical to*** ***A*** ➡ ***B***

Violated by PMS!



Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme

Unification in Physical Schemes

- Smooth analytic threshold behavior with automatic decoupling
- More directly reflects the unification of the forces
- Higher "unification" scale than usual

Conformal symmetry: Template for QCD

- **Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses**
- **Eigensolutions of ERBL evolution equation for distribution amplitudes**

  Frishman, Lepage, Mackenzie, Sachrajda, sjb, Gardi, Braun
- **Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation**
- **Fix Renormalization Scale (BLM, Effective Charges)**

  Gardi, Grunberg, Rathsman,Gabadadze, Kataev, Lepage, Lu, Mackenzie, sjb
- **The BFKL QCD Pomeron with Optimal Renormalization**

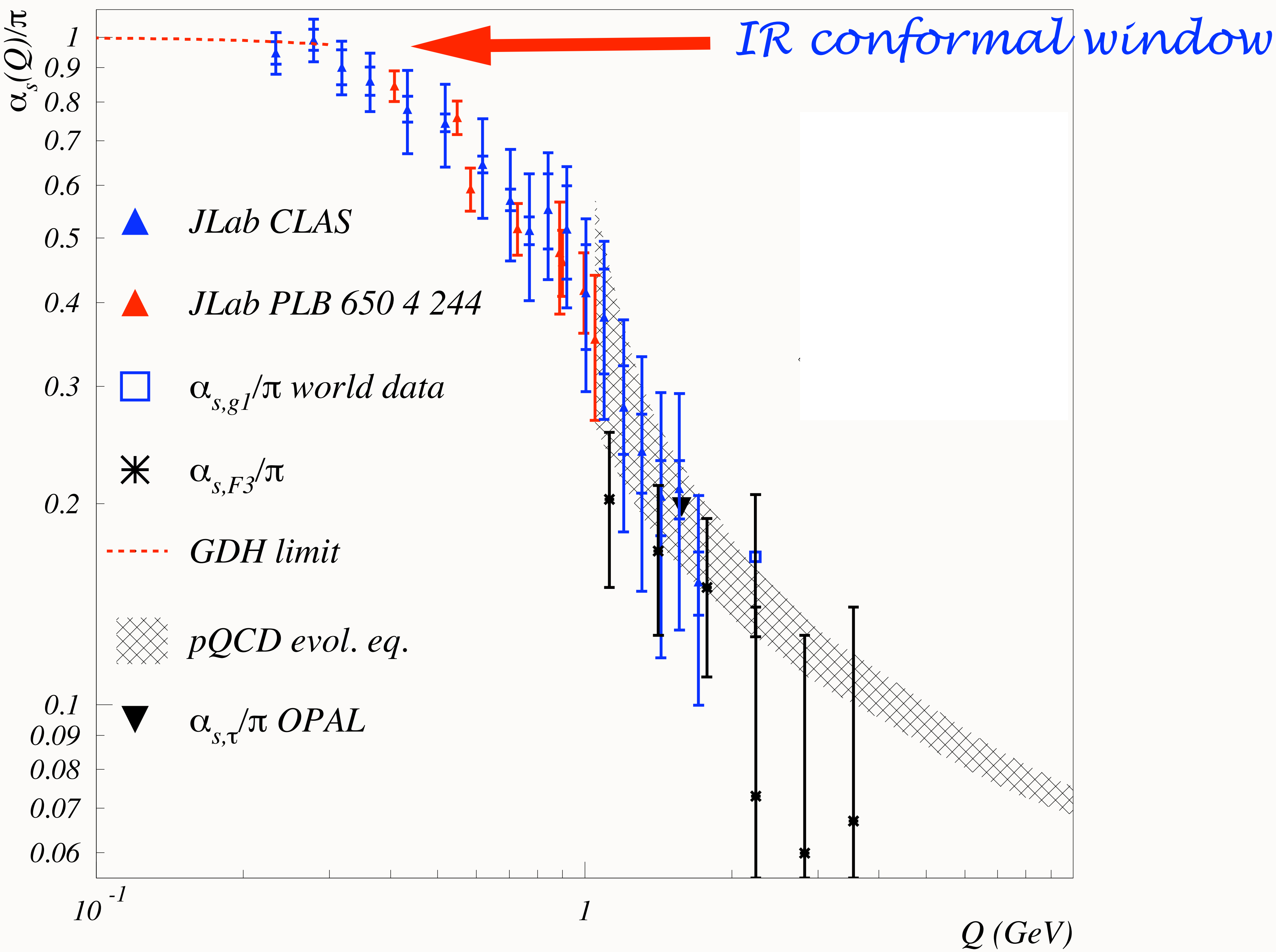
  Kim, Fadin, Lipatov, Pivovarov, sjb
- **IR Fixed Point -- A Conformal Domain**
- **Use AdS/CFT**

IR Fixed Point for QCD?

- Effective Gluon Mass Cornwall
- Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point! Alkofer, Fischer, von Smekal et al.
- Lattice Gauge Theory **Furui and Nakajima**
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2
- Justifies application of AdS/CFT in strong-coupling conformal window

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g}\; e^{\phi(z)} \frac{1}{g_5^2} G^2 \quad \text{where } \sqrt{g} = \left(\frac{R}{z}\right)^5 \text{ and } \phi(z) = +\kappa^2 z^2$$

Define an effective coupling $g_5(z)$

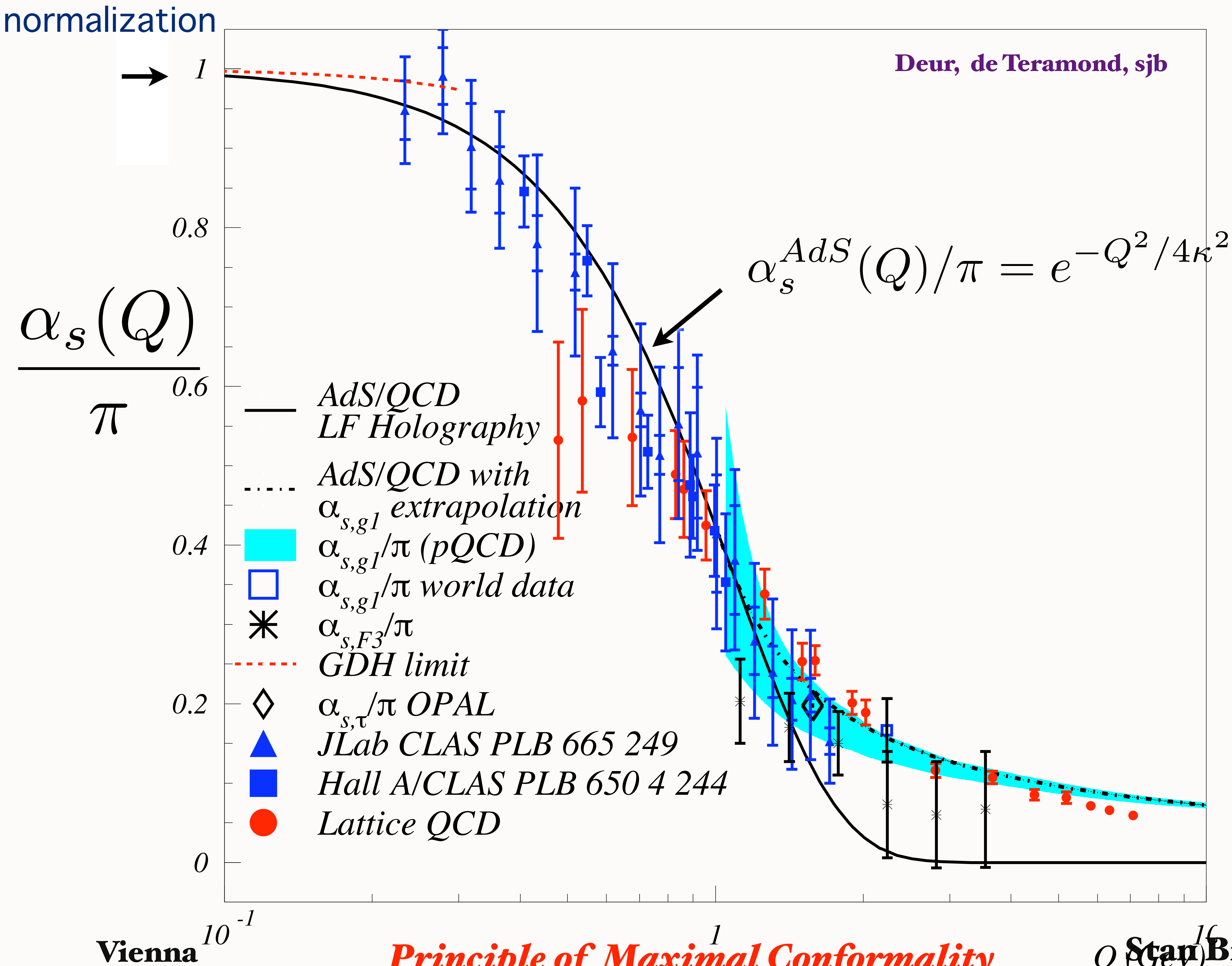
$$S = -\frac{1}{4}\int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2$$

$$\text{Thus } \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

Light-Front Holography: $z \to \zeta = b_\perp \sqrt{x(1-x)}$

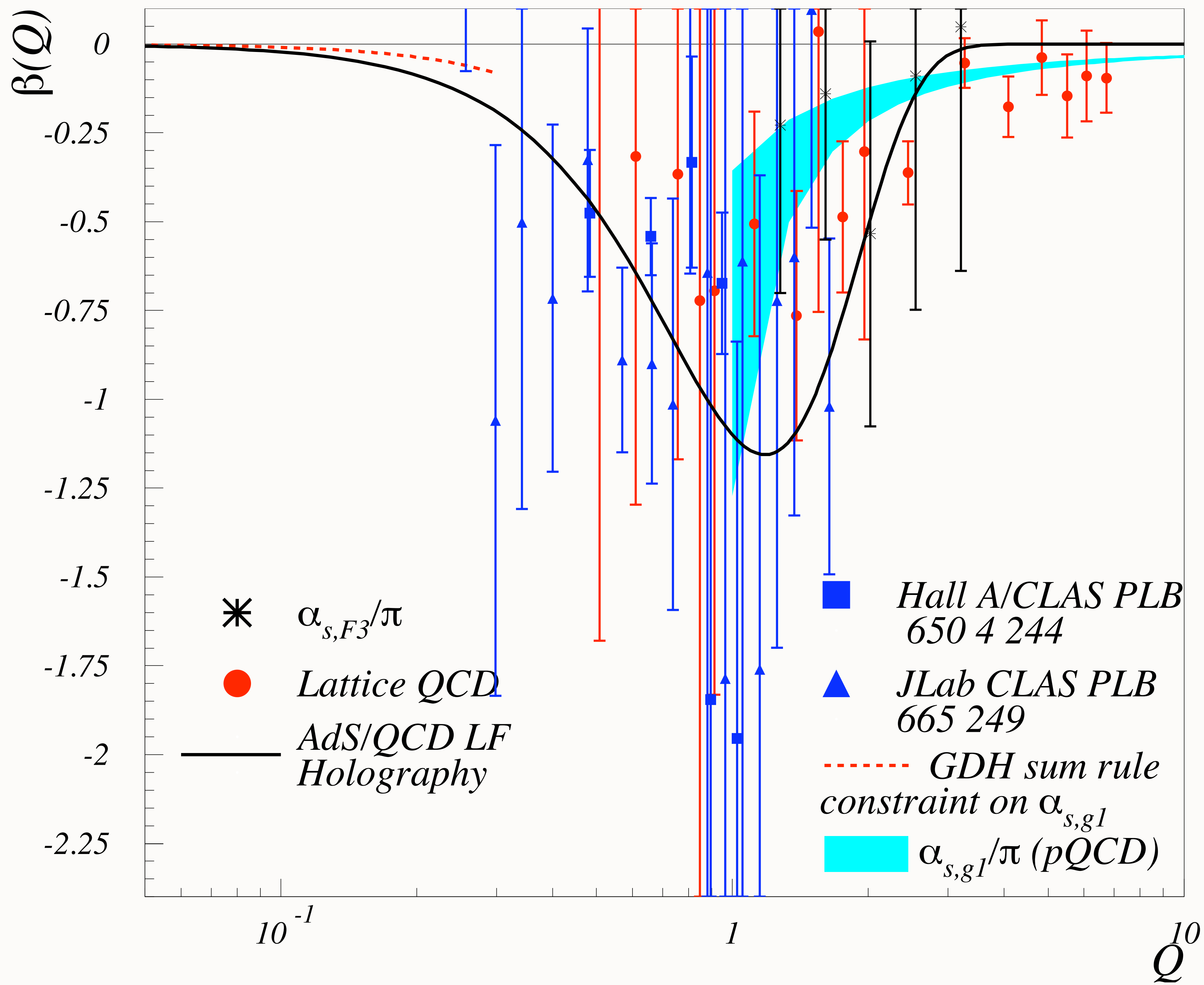
$$\alpha_s(Q^2) \propto \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \propto e^{-Q^2/4\kappa^2}$$

Running Coupling from AdS/QCD



Principle of Maximal Conformality

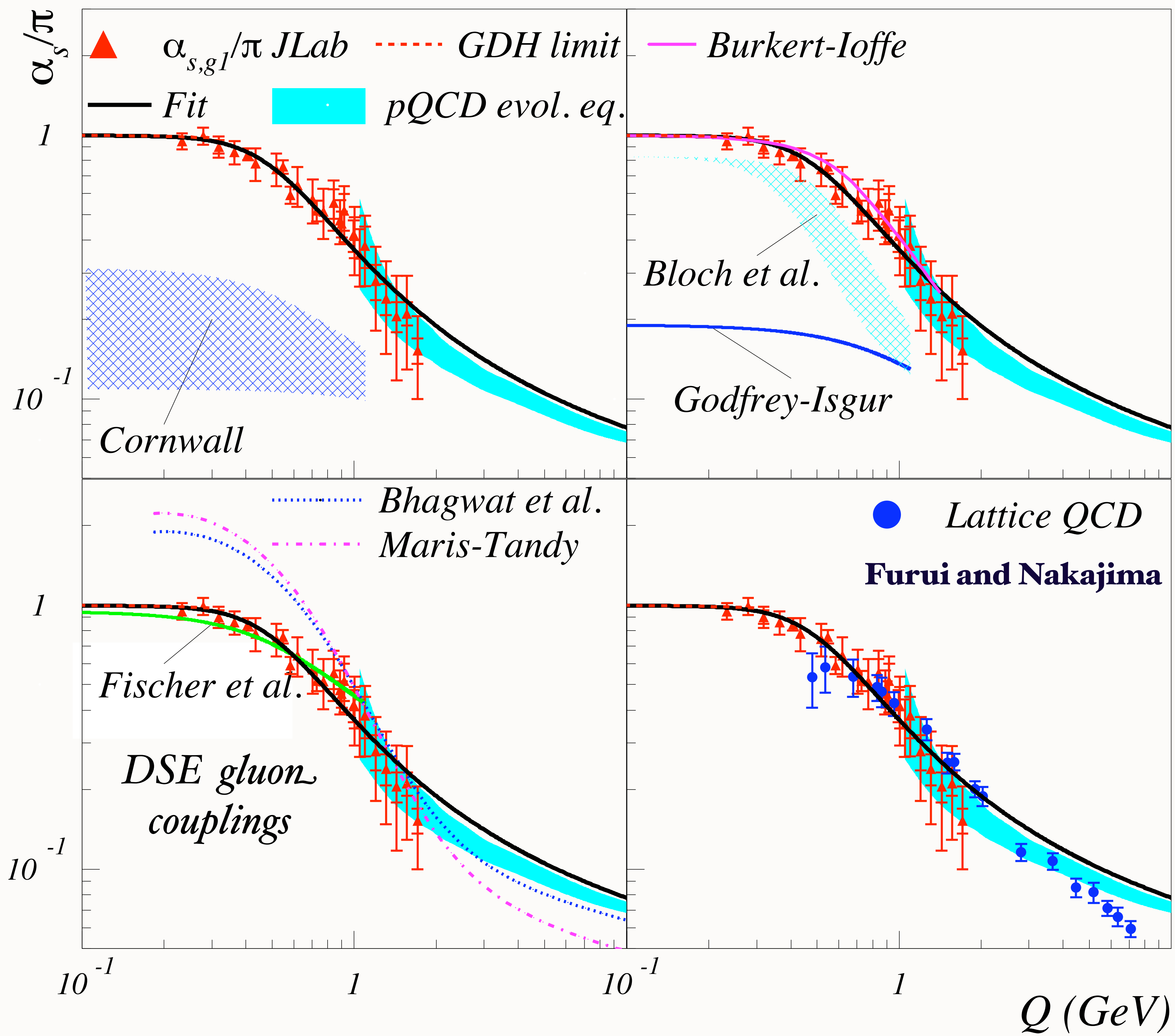
β - Function of AdS/QCD Coupling



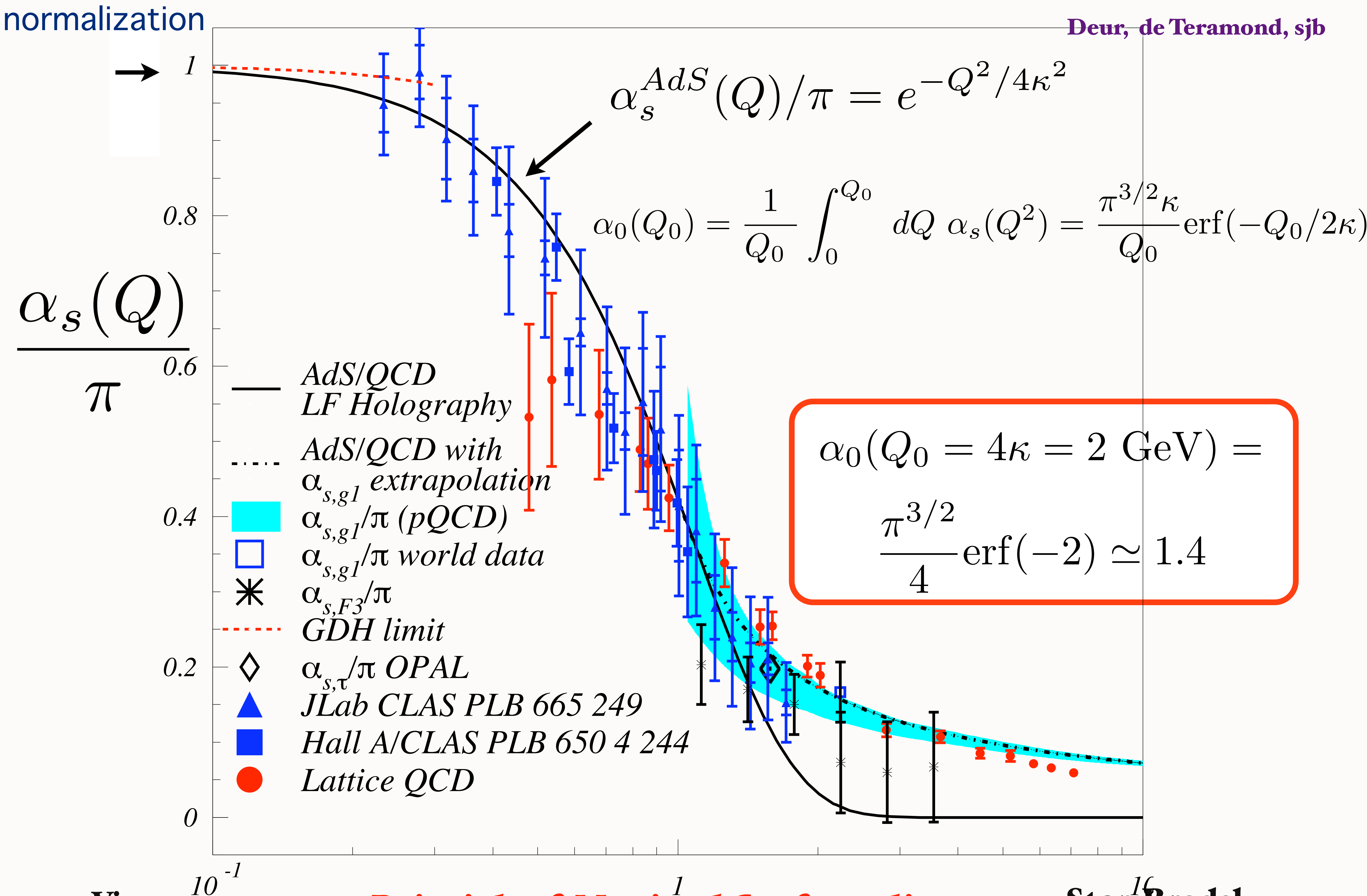
Deur, de Teramond, sjb

Deur, Korsch, et al.

Evidence for IR Fixed Point



Running Coupling from AdS/QCD



$$\alpha_0(Q_0 = 4\kappa = 2\ \mathrm{GeV}) = \frac{\pi^{3/2}}{4}\mathrm{erf}(-2) \simeq 1.4$$

Principle of Maximal Conformality

Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR** $(x-y)^2 < \Lambda^{-2}_{QCD}$
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**
- **Similar in spirit to Cornwall's Effective Gluon mass**

Quark and Gluon vacuum polarization insertions decouple: IR Fixed-Point

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken, SLAC-PUB 1053 Cargese Lectures 1989

Myths concerning scale setting

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb — **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling; scheme independent
- Standard procedure in QED
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scales depend on all invariants

Conformal Template

- **BLM scale-setting: Retain conformal series; nonzero β-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.**

- **Principle of Maximal Conformality: Single Effective Scale**

- **Commensurate Scale Relations based on conformal template; scheme-independent**

- **Pinch Scheme -- provides analytic, gauge invariant, 3-g form factors**

- **Analytic scheme for coupling unification**

- **IR Fixed point -- conformal symmetry motivation for AdS/CFT**

- **Light-Front Schrödinger Equation: analytic first approximation to QCD**

- **Dilaton-modified AdS5: Predict Hadron Spectrum, Form Factors, α_s, β**

- **Light-Front Wave Functions from Holography: Hadronization at the amplitude level**

The reason (why BLM/PMC is useful) is clear

Basic features of BLM/PMC

- It satisfies all the above properties: Existence, Unitary, Transitivity, Reflexivity.
- All non-conformal and scheme-dependent β-terms in the perturbative series are summed into the running coupling. The result is conformal, and it is scheme-independent due to the proper scale-displacement in α_s.
- The active flavors n_f in β-function is correctly determined.
- Renormalons growing as ($n!\ \beta^m\ \alpha_s^n$) are avoided.
- The PMC method agrees with the standard QED results in the Nc -> 0 limit.

The BLM – PMC correspondence

X-G Wu, sjb

PMC, **dealing with the β-series**, provides the principle underlying BLM scale setting.

However to find what's the β-expansion series like ?

1) There are few cases, people have calculated the β-terms directly, since it is more convenient to calculate the n_f-terms (light-quark loops). So usually, we need to transform the n_f-terms into β-terms for PMC.

2) The relation between β and n_f is not in a simple way, i.e. β_2 include the 2-quark -loop, 1-quark-loop and 0-quark-loop contributions. So to get the same n_f-series, the combination of β-term is not unique, which is more adaptable ?

$$\beta_0 = 11 - \frac{2}{3} n_f$$

$$\beta_1 = 102 - \frac{38}{3} n_f$$

$$\beta_2^{\overline{MS}} = \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2$$

$$\beta_3^{\overline{MS}} \simeq 29243.0 - 6964.30 n_f + 405.089 n_f^2 + 1.49931 n_f^3$$

O.V. Tarasov, A.A. Vladimirov and A. Yu Zharkov, Phys.Lett. B**93**, 429(1980); T. van Ritbergen, J.A.M. Vermaseren and S.A. Larin, Phys.Lett. B**400**, 379(1997); M. Czakon, Nucl.Phys. B**710**, 485(2005).

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standard procedures for PMC

To set the BLM scales up to NNLO, the starting point

$$\rho = r_0 \Big[a_s^n(Q) + (A_1 + A_2 n_f) a_s^{n+1}(Q) + (B_1 + B_2 n_f + B_3 n_f^2) a_s^{n+2}(Q) + (C_1 + C_2 n_f + C_3 n_f^2 + C_4 n_f^3) a_s^{n+3}(Q) + \cdots \Big]$$

free of $a_s = \left(\frac{\alpha_s}{\pi}\right)$

first

to set the effective scale Q^* at LO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^*) + (\widetilde{B}_1 + \widetilde{B}_2 n_f) a_s^{n+2}(Q^*) + (\widetilde{C}_1 + \widetilde{C}_2 n_f + \widetilde{C}_3 n_f^2) a_s^{n+3}(Q^*) + \cdots \Big] . \quad (11)$$

second

The second step is to set the effective scale Q^{**} at NLO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{**}) + (\widetilde{\widetilde{C}}_1 + \widetilde{\widetilde{C}}_2 n_f) a_s^{n+3}(Q^{**}) + \cdots \Big] , \quad (12)$$

standard procedures for PMC

LO

NLO

NNLO

final

and the final step is to set the effective scale Q^{***} at NNLO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{***}) + \widetilde{\widetilde{\widetilde{C}}}_1 a_s^{n+3}(Q^{***}) + \cdots \Big] . \quad (13)$$

Teach a robot to compute the PMC scales

M. Mojaza, Xing-Gang Wu, sjb

Generalize $\overline{MS}$ Scheme by subtracting $\log 4\pi - \gamma_E - \delta$

Call this the $\mathcal{R}_\delta$ renormalization scheme

$$\mathcal{R}_0 = \overline{\text{MS}} \ ,$$
$$\mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \ .$$

All $\mathcal{R}_\delta$ renormalization schemes have same β-function

$$\mu_{\delta_2} = \mu_{\delta_1} e^{\frac{\delta_1 - \delta_2}{2}} \ .$$

In particular:

$$\mu_{\overline{\text{MS}}} = \mu_{\text{MS}} \ e^{(\ln 4\pi - \gamma_E)/2},$$
$$\mu_\delta = \mu_{\overline{\text{MS}}} \ e^{-\delta/2} \ .$$

Since ρ is a physical observable, it must be independent of the arbitrary renormalization scheme and scale. That is,

initial

$$\boxed{\frac{\partial \rho_\delta}{\partial \mu_\delta} = 0 \,, \qquad \frac{\partial \rho_\delta}{\partial \delta} = 0 \,,} \tag{16}$$

Generalization: use δ_n at n-loops.

$$\begin{aligned}\rho_\delta(Q^2) =& r_0 + r_1 a_1(Q) + (r_2 - \beta_0 r_1 \delta_1) a_2(Q)^2 \\ &+ [r_3 - \beta_1 r_1 \delta_1 - 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(Q)^3 \\ &+ [r_4 - \beta_2 r_1 \delta_1 - 2\beta_1 r_2 \delta_2 - 3\beta_0 r_3 \delta_3 + 3\beta_0^2 r_2 \delta_2^2 \\ &- \beta_0^3 r_1 \delta_1^3 + \frac{5}{2}\beta_1 \beta_0 r_1 \delta_1^2] a(Q)^4 + \mathcal{O}(a^5)\end{aligned} \tag{20}$$

Shows the general way that nonconformal terms enter an observable

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5) \qquad (19)\end{aligned}$$

PMC scales thus satisfy

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_3)^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

X-G Wu, sjb

BLM/PMC Scale-Setting for R(Q)

$$R_{e^+e^-}(Q) = 3\sum_q e_q^2\left[1 + \left(a^{\overline{MS}}(Q)\right) + (1.9857 - 0.1152 n_f)\left(a^{\overline{MS}}(Q)\right)^2 + \left(-6.63694 - 1.20013 n_f - 0.00518 n_f^2 - 1.240\frac{(\sum_q e_q)^2}{3\sum_q e_q^2}\right)\left(a^{\overline{MS}}(Q)\right)^3 + \left(-156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.0215 n_f^3 + C\frac{(\sum_q e_q)^2}{3\sum_q e_q^2}\right)\left(a^{\overline{MS}}(Q)\right)^4\right]$$

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q)$$

C is for singlet contribution and is small

As usual, we set *C*=0

P.A. Baikov, K.G. Chetyrkin and J.H. Kuhn, Phys.Rev.Lett.**101**, 012002(2008); arXiv:0906.2987[hep-ph]; K. Nakamura et al. (Particle Data Group), J.Phys. G**37**, 075021 (2010).

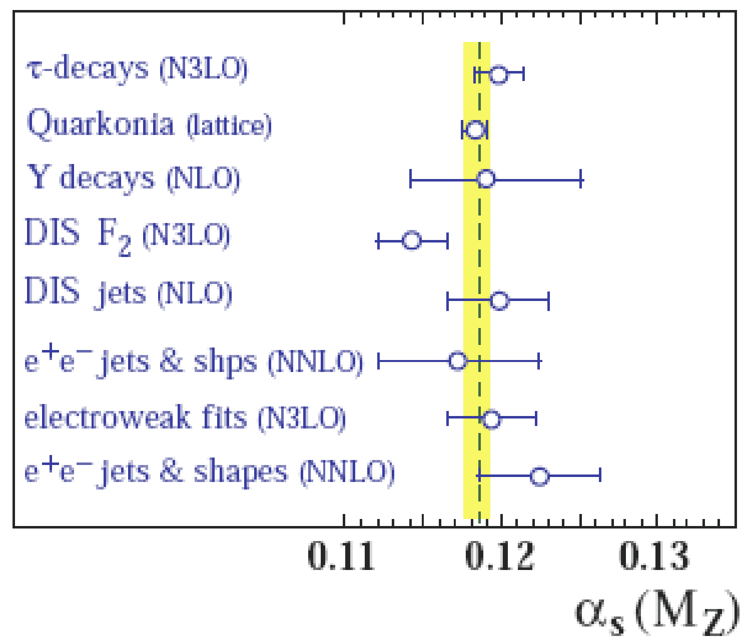
$$R_{e^+e^-}(Q) = 3\sum_q e_q^2\left[1 + \left(a_s^{\overline{MS}}(Q^*)\right) + \widetilde{A}\left(a_s^{\overline{MS}}(Q^{**})\right)^2 + \widetilde{\widetilde{B}}\left(a_s^{\overline{MS}}(Q^{***})\right)^3 + \widetilde{\widetilde{\widetilde{C}}}\left(a_s^{\overline{MS}}(Q^{***})\right)^4\right], \quad (44)$$

taking the experimental results for R(Q)

From the experimental value, $r_{e^+e^-}(31.6GeV) = \frac{3}{11} R_{e^+e^-}(31.6GeV) = 1.0527 \pm 0.0050$ [26], we obtain

$$\Lambda^{'tH}_{\overline{MS}} = 412^{+206}_{-161}\text{MeV}$$
$$\Lambda_{\overline{MS}} = 359^{+181}_{-140}\text{MeV}$$

$$\alpha_s^{\overline{MS}}(M_Z) = 0.129^{+0.009}_{-0.010}$$



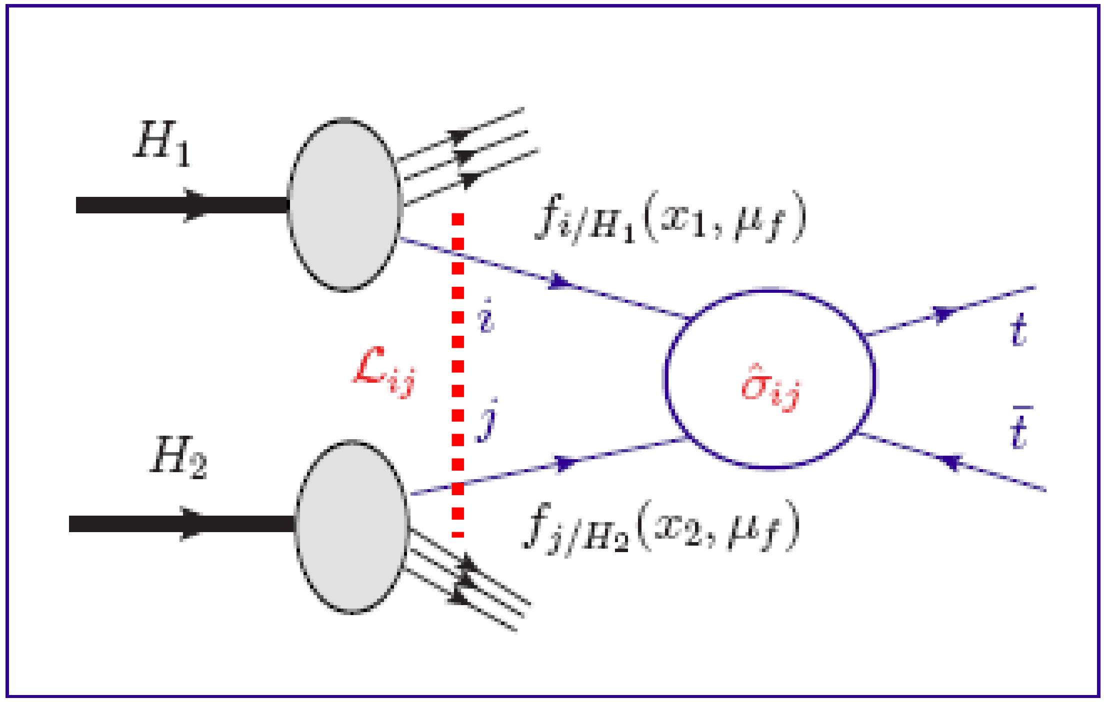
Application D: Scale setting for top-pair production at NLO - needs NNLO n_f-terms

(Rough results; To Be Continued)

$$p + p(\bar{p}) \rightarrow Q + \bar{Q} + X$$
$$q + \bar{q} \rightarrow Q + \bar{Q} + X$$
$$g + g \rightarrow Q + \bar{Q} + X$$
$$g + q \rightarrow Q + \bar{Q} + X$$
$$\sigma(S, m^2) = \sum_{ij} \int dx_1 dx_2 \hat{\sigma}_{ij}(s, m^2, \mu^2) f_i(x_1, \mu^2) f_j(x_2, \mu^2)$$

Main Point: The expansion coefficients also include factorization scale-dependence, heavy-quark mass dependence, how to improve the PMC/BLM procedure?

Hadronic Cross-Section



$$\sigma_{H_1H_2\to t\bar{t}X}=\sum_{i,j}\int_{4m_t^2}^{S} ds\,\mathcal{L}_{ij}(s,S,\mu_f)\hat{\sigma}_{ij}(s,\alpha_s(\mu_r),\mu_r,\mu_f),$$

Parton luminosity

$$\mathcal{L}_{ij}=\frac{1}{S}\int_{s}^{S}\frac{d\hat{s}}{\hat{s}}f_{i/H_1}(x_1,\mu_f)\,f_{j/H_2}(x_2,\mu_f)$$

Subprocess Cross-Section NNLO

$$\hat{\sigma}_{ij}=\frac{1}{m_t^2}\left\{\underbrace{f^0_{ij}(\rho,Q)a_s^2(Q)}_{\text{LO}}+\underbrace{f^1_{ij}(\rho,Q)a_s^3(Q)}_{\text{NLO}}+\underbrace{f^2_{ij}(\rho,Q)a_s^4(Q)}_{\text{NNLO}}\right\},$$

X-G Wu, sjb

NLO

$$f^1_{ij}(\rho,Q) = [A_{1ij} + B_{1ij} n_f] + D_{1ij}\left(\frac{\pi}{v}\right)$$

$$\boxed{A_{0ij} = f^0_{ij}(\rho,Q)}$$

NNLO

$$f^2_{ij}(\rho,Q) = [A_{2ij} + B_{2ij} n_f + C_{2ij} n_f^2] + [D_{2ij} + E_{2ij} n_f]\left(\frac{\pi}{v}\right) + F_{2ij}\left(\frac{\pi}{v}\right)^2$$

$$\begin{aligned} m_t^2 \hat{\sigma}_{ij} = & A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^*) + \\ & \left[\tilde{A}_{2ij} + \tilde{B}_{2ij} n_f\right] a_s^4(Q_1^*) + D_{1ij}\left(\frac{\pi}{v}\right) a_s^3(Q_2^*) + \\ & \left[\tilde{D}_{2ij}\right]\left(\frac{\pi}{v}\right) a_s^4(Q_2^*) + F_{2ij}\left(\frac{\pi}{v}\right)^2 a_s^4(Q_2^*). \end{aligned}$$

first step

second step

$$\begin{aligned} m_t^2 \hat{\sigma}_{ij} = & A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^{**}) + \\ & \left[\tilde{\tilde{A}}_{2ij}\right] a_s^4(Q_1^{**}) + D_{1ij}\left(\frac{\pi}{v}\right) a_s^3(Q_2^*) + \\ & \left[\tilde{D}_{2ij}\left(\frac{\pi}{v}\right) + F_{2ij}\left(\frac{\pi}{v}\right)^2\right] a_s^4(Q_2^*) \\ = & A_{0ij} a_s^2(Q_1^*) + \left[\tilde{A}_{1ij}\right] a_s^3(Q_1^{**}) + \left[\tilde{\tilde{A}}_{2ij}\right] a_s^4(Q_1^{**}) \\ & + \left(\frac{\pi}{v}\right) D_{1ij}\left[\frac{2\kappa}{1-\exp(-2\kappa)}\right] a_s^3(Q_2^*), \end{aligned}$$

Sommerfeld rescattering

X-G Wu, sjb

LO PMC scale

$$\ln\frac{Q_1^{*2}}{Q^2} = \ln\frac{Q_{10}^{*2}}{Q^2} + \frac{\chi}{4}\beta_0 \ln\frac{Q_{10}^{*2}}{Q^2} a_s(Q)$$

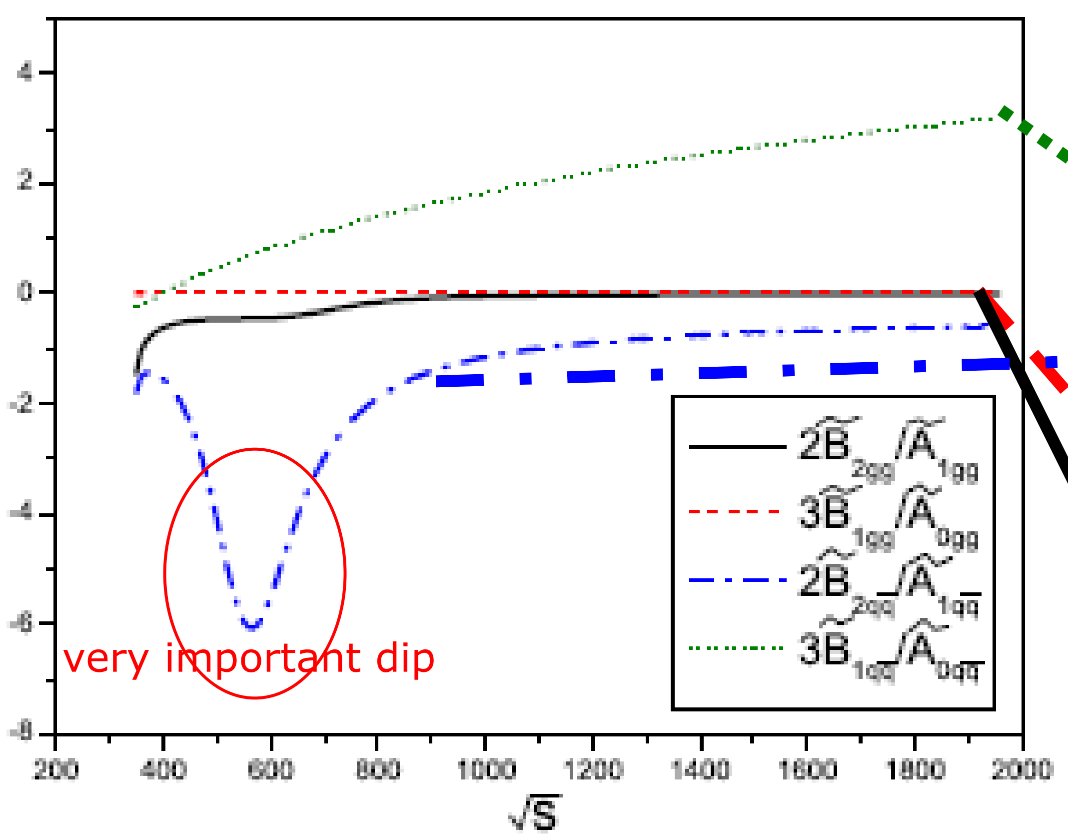
$$\ln\frac{Q_{10}^{*2}}{Q^2} = \frac{3B_{1ij}}{A_{0ij}}\ , \quad \chi = \frac{9B_{1ij}^2 - 12A_{0ij}C_{2ij}}{2A_{0ij}B_{1ij}}$$

perturbative series

NLO PMC scale

$$\boxed{\ln\frac{Q_1^{**2}}{Q_1^{*2}} = \frac{2\tilde{B}_{2ij}}{\tilde{A}_{1ij}}}$$

most important greatly improve the NLO estimation



$$\text{determines} \Rightarrow q\bar{q} - channel \ \text{LO} \ scale$$

$$\text{determines} \Rightarrow q\bar{q} - channel \ \text{NLO} \ scale$$

$$\text{determines} \Rightarrow gg - channel \ \text{LO} \ scale$$

$$\text{determines} \Rightarrow gg - channel \ \text{NLO} \ scale$$

slight change for gg-channel

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	4.989	0.975	0.489	6.453	4.841	1.756	-0.063	6.489
(gg)-channel	0.522	0.425	0.155	1.102	0.520	0.506	0.148	1.200
(gq)-channel	0.000	-0.0366	0.0050	-0.0316	0.000	-0.0367	0.0050	-0.0315
$(g\bar{q})$-channel	0.000	-0.0367	0.0050	-0.0315	0.000	-0.0366	0.0050	-0.0316
sum	5.511	1.326	0.654	7.489	5.3613	2.188	0.095	7.626

TABLE I. Total cross-sections (in unit: pb) for the top-quark pair production at the Tevatron with $\sqrt{S} = 1.96$ TeV. For the conventional scale-setting, we set the renormalization scale $\mu_r \equiv Q$. For the PMC scale-setting, we set the initial renormalization scale $\mu_r^{\rm init} = Q$. Here $Q = m_t = 172.9$ GeV and the central CT10 as the PDF [51].

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	23.283	3.374	1.842	28.527	22.244	7.127	-0.765	28.429
(gg)-channel	78.692	45.918	10.637	135.113	78.399	53.570	8.539	142.548
(gq)-channel	0.000	-0.401	1.404	1.025	0.000	-0.408	1.403	1.006
$(g\bar{q})$-channel	0.000	-0.420	0.235	-0.186	0.000	-0.424	0.235	-0.188
sum	101.975	48.471	14.118	164.594	100.643	59.865	9.414	171.796

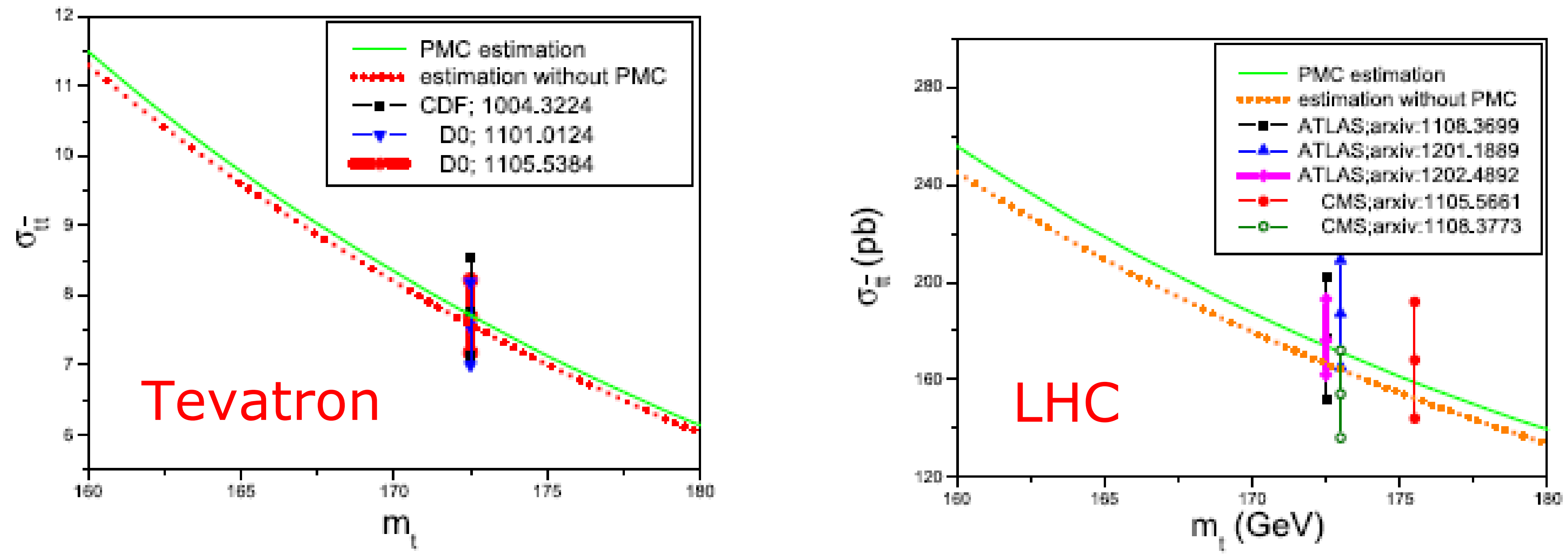
TABLE II. Total cross-sections (in unit: pb) for the top-quark pair production at the LHC with $\sqrt{S} = 7$ TeV. For the conventional scale-setting, we set the renormalization scale $\mu_r \equiv Q$. For the PMC scale-setting, we set the initial renormalization scale $\mu_r^{\rm init} = Q$. Here $Q = m_t = 172.9$ GeV and the central CT10 as the PDF [51].

A proper NLO scale is clearly very important !

especially to understand the ttbar-asymmetry

Conventional scale choice: m_t
"Lucky guess" for total rate

X-G Wu, sjb



$$m_t = 172.9 \pm 1.1 \text{ GeV}$$

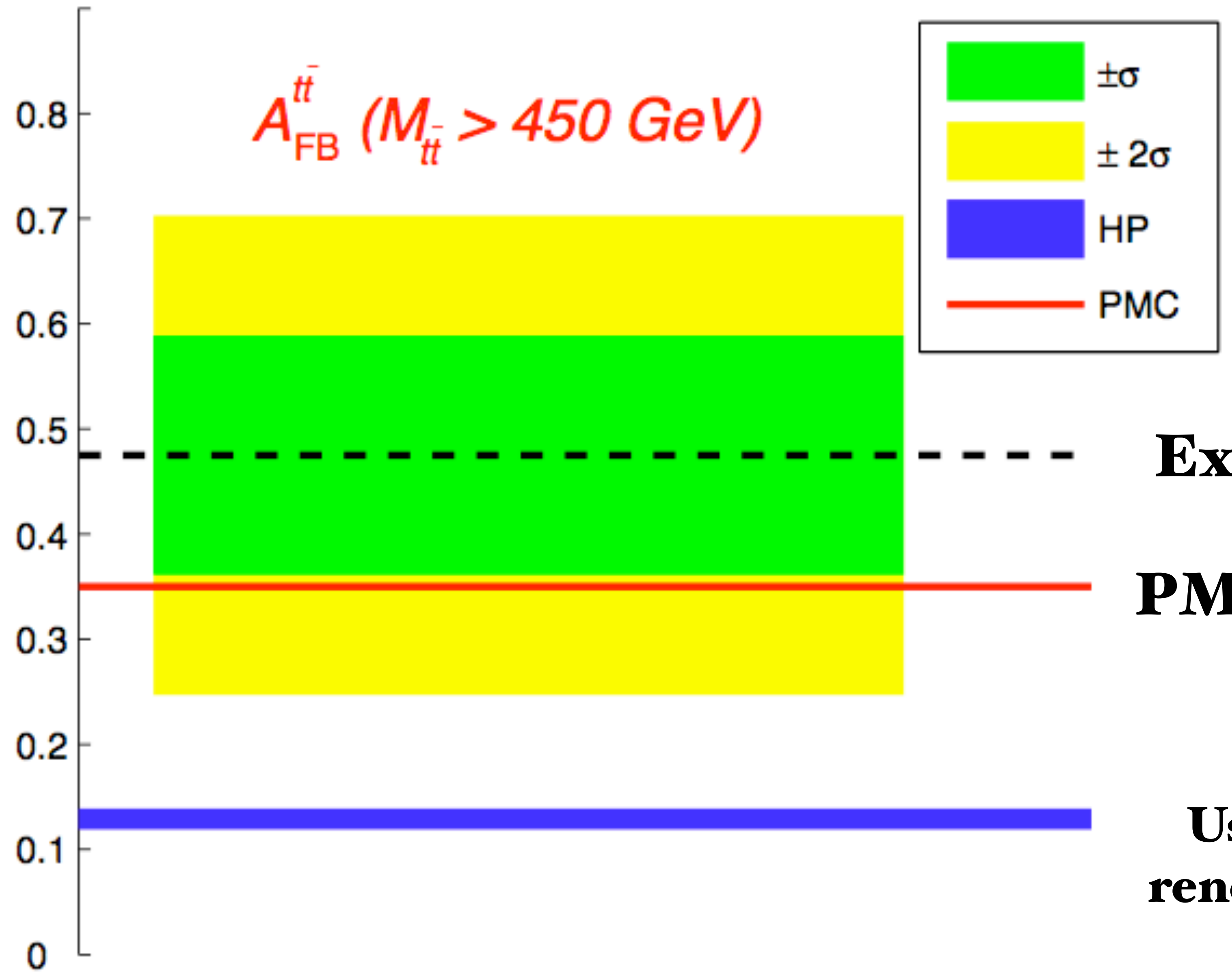
$$\sigma_{\text{Tevatron, 1.96 TeV}} = 7.626^{+0.265}_{-0.257} \text{ pb}$$
$$\sigma_{\text{LHC, 7 TeV}} = 171.8^{+5.8}_{-5.6} \text{ pb}$$
$$\sigma_{\text{LHC, 14 TeV}} = 941.3^{+28.4}_{-26.5} \text{ pb}$$

PDF+α_s error

$$\alpha_s(m_z) = 0.118 \pm 0.001$$

$$\sigma_{\text{Tevatron, 1.96 TeV}} = 7.626^{+0.143}_{-0.130} \text{ pb}$$
$$\sigma_{\text{LHC, 7 TeV}} = 171.8^{+3.8}_{-3.5} \text{ pb}$$
$$\sigma_{\text{LHC, 14 TeV}} = 941.3^{+14.6}_{-15.6} \text{ pb}$$

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

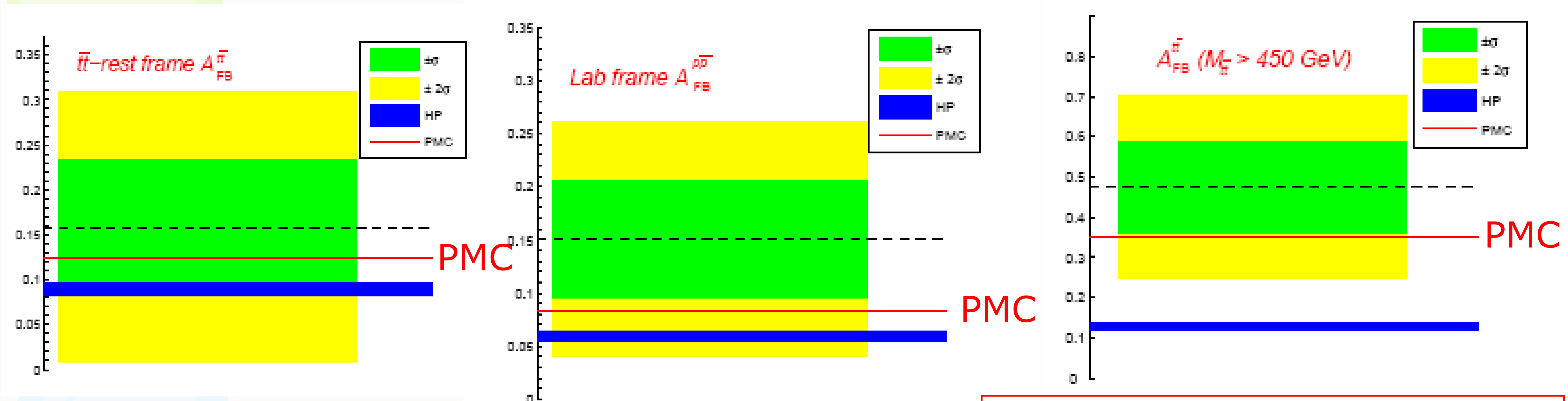
PMC Prediction

Using conventional guess for renormalization scale and range

$t\bar{t}$ asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

PMC: ~1 sigma from experiment

X-G Wu, sjb



$$A_{FB}^{t\bar{t},\mathrm{PMC}} \simeq 12.7\% \; ; \quad A_{FB}^{p\bar{p},\mathrm{PMC}} \simeq 8.39\%$$

$$\overline{\alpha}_s\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right) = 0.1460$$

$$\overline{\mu}_R^{\mathrm{PMC,NLO}} \sim \exp(-19/10)m_t \simeq 26 \text{ GeV}.$$

$$A_{FB}^{t\bar{t},PMC}(M_{t\bar{t}} > 450 \text{ GeV}) \simeq 35.0\%$$

HP

$$A_{FB}^{t\bar{t}} = (9.7, 8.9, 8.3)\%, \qquad A_{FB}^{p\bar{p}} = (6.4, 5.9, 5.4)\%.$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (13.9, 12.8, 11.9)\%,$$

CDF

$$A_{FB}^{t\bar{t},\mathrm{CDF}} = (15.8 \pm 7.5)\% \quad A_{FB}^{p\bar{p},\mathrm{CDF}} = (15.0 \pm 5.5)\%$$

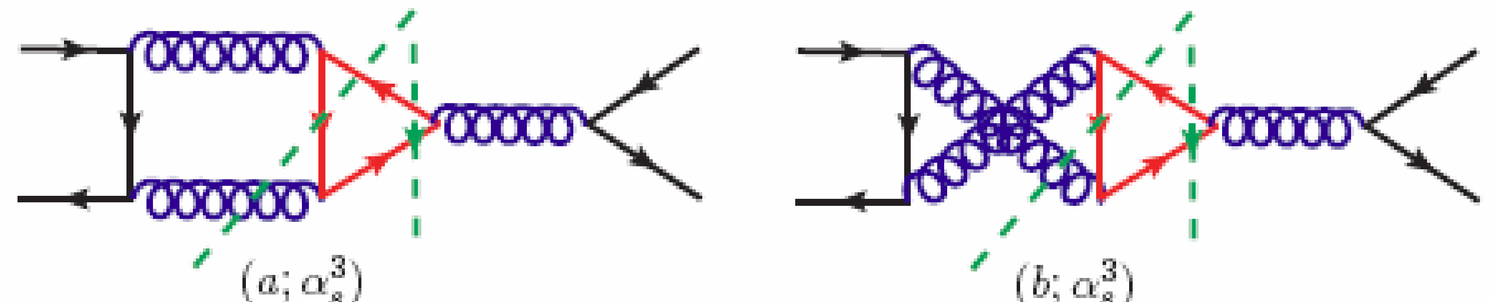
$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (47.5 \pm 11.4)\%$$

T. Aaltonen *et al.*, CDF Collaboration, Phys.Rev. D**83**, 112003 (2011).

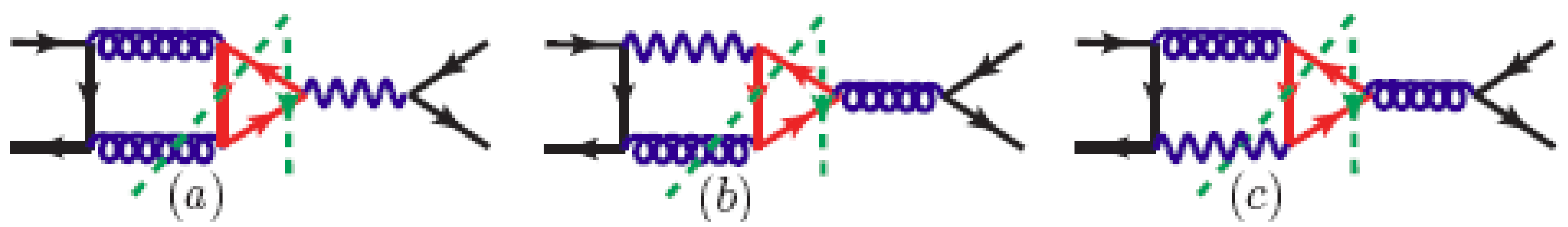
why the conventional scale-setting gives small asymmetry ?

X-G Wu, sjb

dominant asymmetric $q\bar{q}$ – *channel*



NLO



~20%

	Conventional scale-setting				PMC scale-setting			
	LO	NLO	NNLO	*total*	LO	NLO	NNLO	*total*
$(q\bar{q})$-channel	4.890	0.963	0.483	6.336	4.748	1.727	-0.058	6.417
(gg)-channel	0.526	0.440	0.166	1.132	0.524	0.525	0.160	1.208
(gq)-channel	0.000	-0.0381	0.0049	-0.0332	0.000	-0.0381	0.0049	-0.0332
$(g\bar{q})$-channel	0.000	-0.0381	0.0049	-0.0332	0.000	-0.0381	0.0049	-0.0332
sum	5.416	0.985	0.659	7.402	5.272	2.176	0.112	7.559

TABLE I. Total cross-sections (in unit: pb) for the top-quark pair production at the Tevatron with $p\bar{p}$-collision energy $\sqrt{S} = 1.96$ TeV. For conventional scale-setting, we set the renormalization scale $\mu_R \equiv Q$. For PMC scale-setting, we set the initial renormalization scale $\mu_R^{\rm init} = Q$. Here we take $Q = m_t = 172.9$ GeV and use the MSRT 2004-QED parton distributions [54] as the PDF.

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\boxed{\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F}$$

$$\boxed{\text{QCD} \to \text{Abelian Gauge Theory}}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

Need to set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\text{init}})$

↓

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

↓

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

$through\ the\ PMC - BLM\ correspondence\ principle$

↓

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\text{PMC}}\ to\ eliminate\ \{\beta_i^R\} - terms$

↓

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\text{init}}\ and\ scheme\ at\ fixed\ order$

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

Same as QED Scale Setting

Apply to Evolution kernels, hard subprocesses

Eliminates unnecessary systematic uncertainty

Xing-Gang Wu
Leonardo di Giustino, SJB

Principle of Maximum Conformality

Conventional scale-setting method

=> **Guess** a renormalization scale Q
Q being the typical momentum transfer of the process
=> **Keep its form fixed** during the calculation
=> **Vary** it over the range [Q/2, 2 Q] to discuss its uncertainty

Under such method, the scale-error is an important systematic error

Problems

I) Its estimation is **scheme-dependent** at fixed order.

II) Its estimation **strongly depends on the choice of Q**. Why just a factor of ½ or 2 and not 10 or 20 ? Which is the typical momentum transfer ?

III) It gives wrong result for QED ! **GM-L-scheme**.

IV) The **convergence** of the perturbative series is problematic. There are large **renormalon** terms in higher-orders which make the prediction unreliable or even wrong.

Greatly depresses the predictive power of pQCD !

Summary and Outlook

I. **BLM/PMC** provides self-consistent way to set the effective scales, which leads to **scheme-independent result**. QCD is not confromal, however one can use the PMC to convert a PQCD series with the corresponding conformal QCD series.

II. After PMC/BLM, one can use the **Mellin-space analysis** to compute the conformal QCD correlators and amplitudes. To be considered.

Fitzpatrick
JHEP 1111 (2011) 095

III. A combination of BLM/PMC to Extended-RGE can be used to derive a **precise QCD estimation**.

IV. Top-pair production total cross-section agree with exp data.

V. Top-pair asymmetries are within 1σ-error. **SM is OK ?**

VI. More applications, **such as Higgs production**, will appear.

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

Gauge Principle:

Physical Observables cannot depend on the choice of gauge

Renormalization Group Principle:

Physical Observables cannot depend on the choice of scheme

PMC Scale-Setting Satisfies all Principles of the Renormalization Group

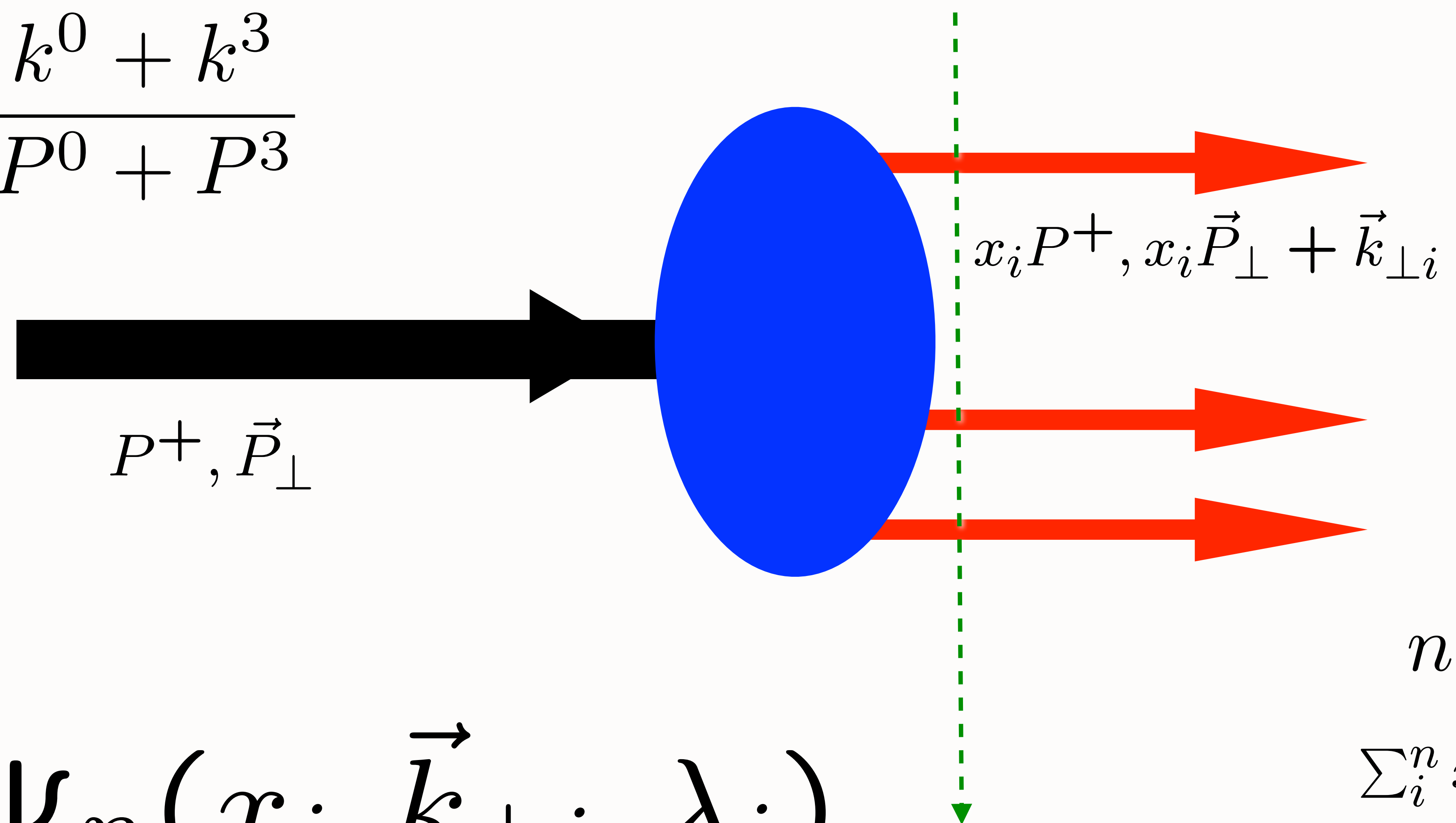
The conventional method of guessing the renormalization scale and range cannot be justified: It introduces an unnecessary uncertainty in QCD predictions

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$k^{\pm} = k^0 \pm k^z$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$n = 3$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Bethe-Salpeter WF integrated over k^-

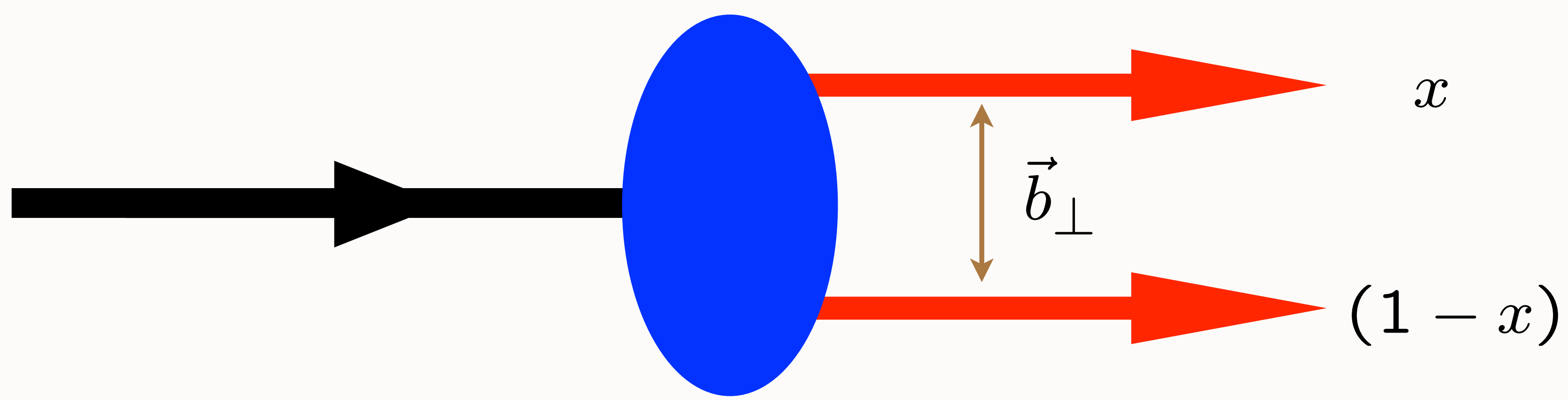
Square: Structure Functions Measured in DIS

LF(3+1) $\longleftrightarrow$ AdS_5

Light-Front Holography

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors

Light-Front Schrödinger Equation

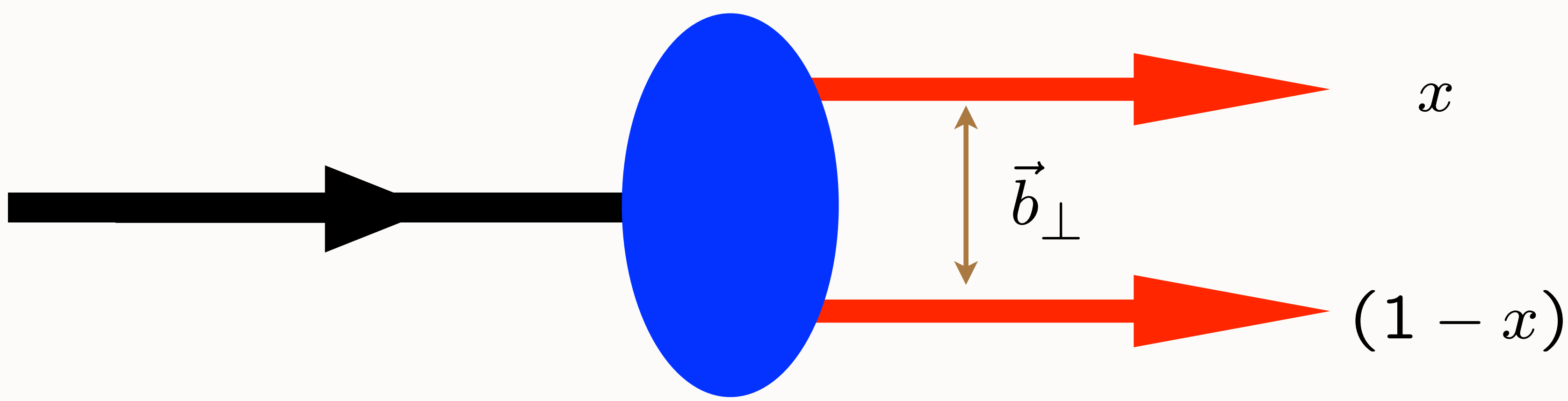
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

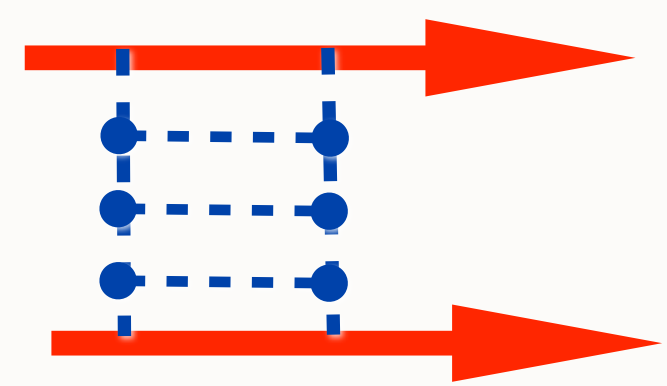
$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2, J, L, M^2)\right]\Psi_{J,L}(\zeta^2) = M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$

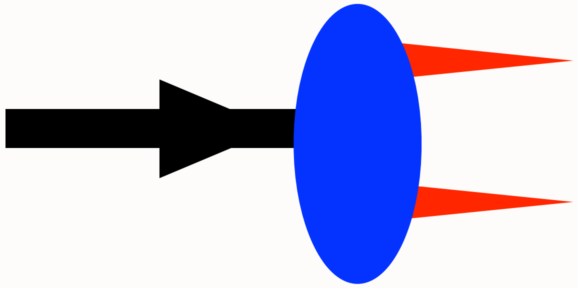


U is the exact QCD potential
Conjecture: 'H'-diagrams generate

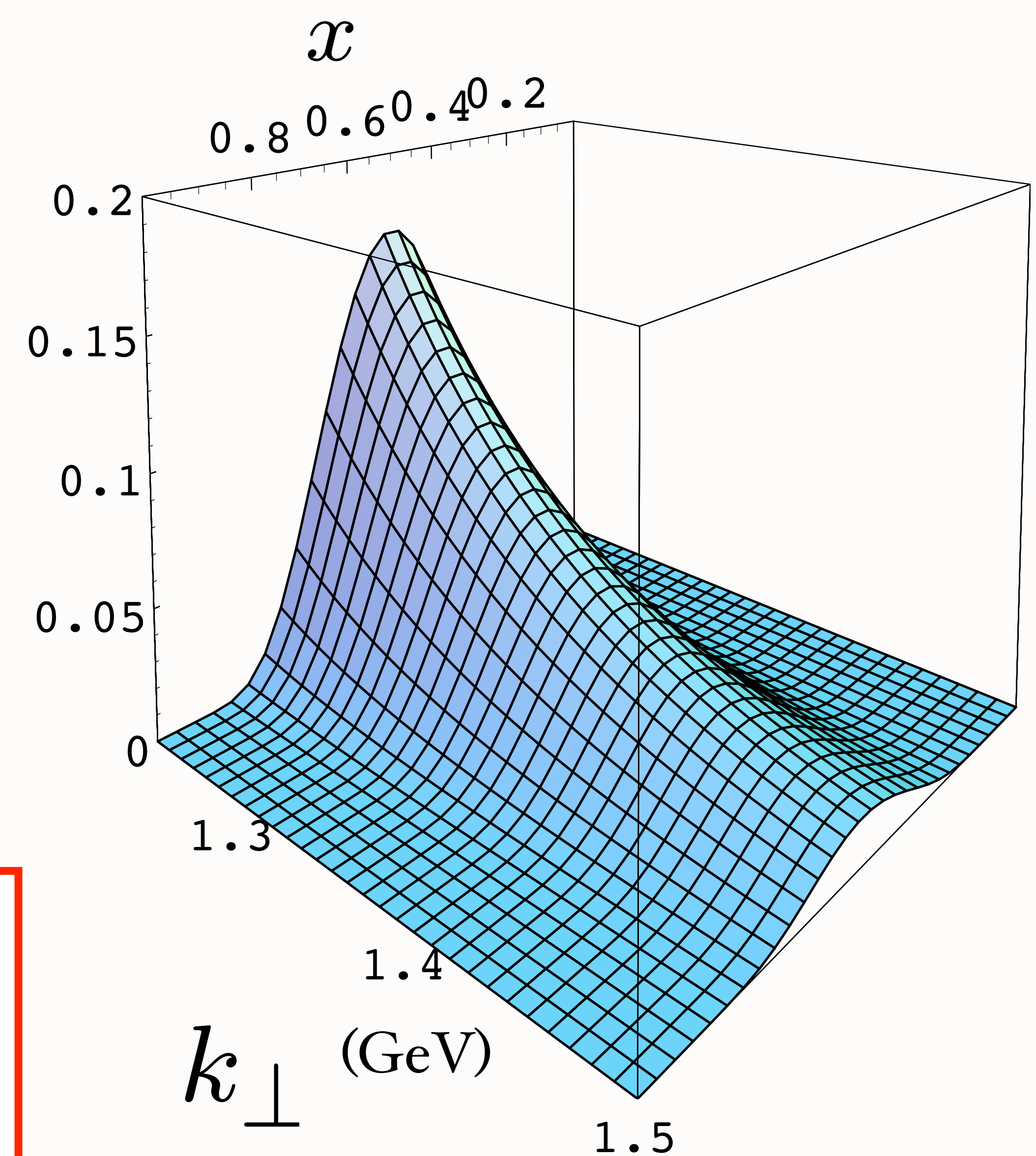
$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$



Prediction from AdS/QCD: Meson LFWF



$$\psi_M(x, k_\perp^2)$$



de Teramond, sjb

"Soft Wall" model

massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

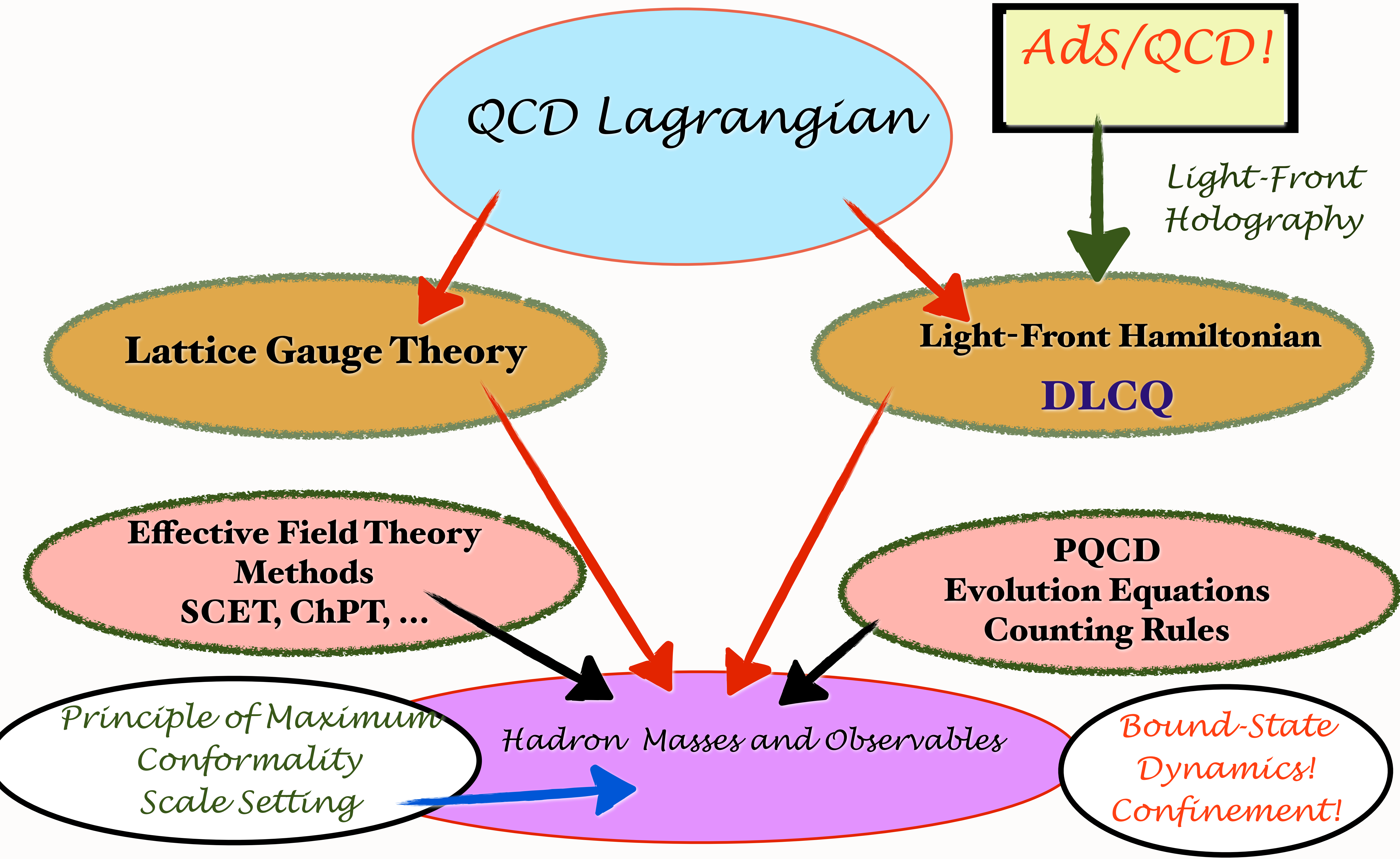
J. Day, sjb

Use Light-Front Wavefunctions from AdS/QCD to eliminate the Factorization-Scale Ambiguity

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

Predict Hadron Properties from First Principles!

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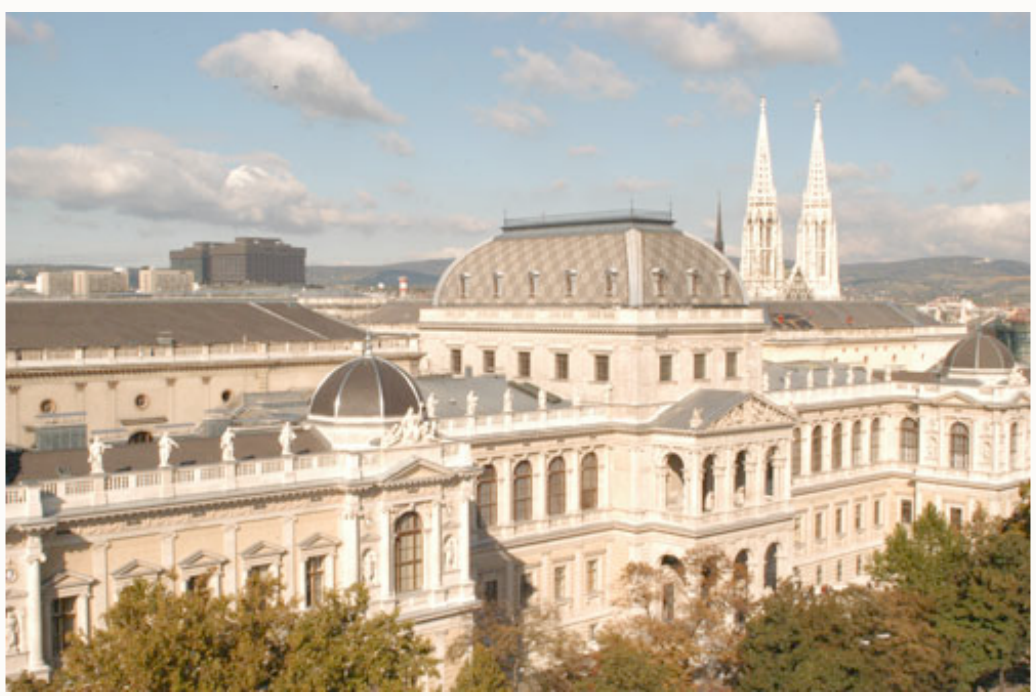
The Principle of Maximal Conformality

The Elimination of the QCD Renormalization Scale Ambiguity

Stan Brodsky

Thanks for an outstanding visit to Vienna!!



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with Leonardo Di Giustino , Xing-Gang Wu, and Matin Mojaza