The Principle of Maximal Conformality

The Elimination of the QCD Renormalization Scale Ambiguity

Stan Brodsky

with Leonardo Di Giustino, Xing-Gang Wu, and Matín Mojaza
Goals

• Test QCD to maximum precision

• High precision determination of \( \alpha_s(Q^2) \) at all scales

• Relate observable to observable --no scheme or scale ambiguity

• Eliminate renormalization scale ambiguity in a scheme-independent manner

• Relate renormalization schemes without ambiguity

• Maximize sensitivity to new physics at the colliders
Myths concerning scale setting

• Renormalization scale “unphysical”: No optimal physical scale

• Can ignore possibility of multiple physical scales

• Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$

• Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!
Electron-Electron Scattering in QED

\[ M_{ee\to ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

Gell-Mann--Low Effective Charge
QED One-Loop Vacuum Polarization

\[ \Pi(Q^2) = \frac{\alpha(0)}{3\pi} \left[ \frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2}) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|} \right] \]

Analytically continue to timelike \( t \): Complex

\[ \Pi(Q^2) = \frac{\alpha(0)}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4M^2 \quad \text{Serber-Uehling} \]

\[ \Pi(Q^2) = \frac{\alpha(0)}{3\pi} \log \frac{Q^2}{m^2} \quad Q^2 \gg 4M^2 \quad \text{Potential Landau Pole} \]

\[ \beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3} \left( \frac{\alpha}{4\pi} \right)^2 n_\ell > 0 \]
QED Effective Charge

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

All-orders lepton-loop corrections to dressed photon propagator

\[ \alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t_0)} \]
\[ \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)} \]

Initial scale \( t_0 \) is arbitrary -- Variation gives RGE Equations
Physical renormalization scale \( t \) not arbitrary!
Electron-Electron Scattering in QED

\[ M_{ee\rightarrow ee}(++; ++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

- Two separate physical scales: \( t, u \) = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling. **This is the purpose of the running coupling!**
- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- **No renormalization scale ambiguity!**
Another Example in QED: Muonic Atoms

$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1 - \Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in $\mu$ Pb

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Relation between scales of the Gell-Mann--Low and $\overline{\text{MS}}$ schemes

\[ \log \frac{\mu_0^2}{m^2_\ell} = 6 \int_0^1 x(1 - x) \log \frac{m^2_\ell + Q^2_0 x(1 - x)}{m^2_\ell} \]

\[ \log \frac{\mu_0^2}{m^2_\ell} = \log \frac{Q^2_0}{m^2_\ell} - \frac{5}{3} \]

\[ \mu_0^2 = Q_0^2 e^{-5/3} \quad \text{when } Q_0^2 \gg m^2_\ell \]

D. S. Hwang, sjb

M. Binger

“Scale Displacement” between schemes

Can use $\overline{\text{MS}}$ scheme in QED; answers are scheme independent
Analytic extension: coupling is complex for timelike argument
The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Physical predictions cannot depend on renormalization scheme
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds -- number of active leptons set
- Examples: muonic atoms, $g-2$, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent!
On the elimination of scale ambiguities in perturbative quantum chromodynamics

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We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the $\gamma$. Our analysis calls into question recent determinations of the QCD coupling constant based upon $\gamma$ decay.
**QCD Observables**

\[ O = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2}) \]

**BLM: Absorb \( \beta \) terms into running coupling**

\[ O = C(\alpha_s(Q^*2)) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2}) \]

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**Principle of Maximal Conformality**
BLM Scale Setting

\[ \beta_0 = 11 - \frac{2}{3} n_f \]

\[ \rho = C_0 \alpha_{\text{MS}}(Q) \left[ 1 + \frac{\alpha_{\text{MS}}(Q)}{\pi} \left( -\frac{3}{2} \beta_0 A_{\text{VP}} + \frac{33}{2} A_{\text{VP}} + B \right) + \ldots \right] \]

by

\[ \rho = C_0 \alpha_{\text{MS}}(Q^*) \left[ 1 + \frac{\alpha_{\text{MS}}(Q^*)}{\pi} C_1^* + \ldots \right], \]

where

Conformal coefficient - independent of \( \beta \)

\[ Q^* = Q \exp(3A_{\text{VP}}), \]

\[ C_1^* = \frac{33}{2} A_{\text{VP}} + B. \]

The term \( 33A_{\text{VP}}/2 \) in \( C_1^* \) serves to remove that part of the constant \( B \) which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by \( \beta_0 = 11 - \frac{2}{3} n_f. \)

Use skeleton expansion:

Gardi, Grunberg, Rathsman, sjb

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Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.  
Lepage, Mackenzie, sjb  

• All terms associated with nonzero beta function summed into running coupling

• BLM Scale $Q^*$ sets the number of active flavors

• Only $n_f$ dependence (associated with renormalization) required to determine renormalization scale at NLO

• Result is scheme independent! $Q^*$ has exactly the correct dependence to compensate for change of scheme

• Correct Abelian limit

• Resulting series identical to conformal series!

• Renormalon $n!$ growth of PQCD coefficients from beta function eliminated!
• All non-conformal and scheme-dependent $\beta$-terms in the perturbative series are summed into the running coupling. The result is consistent with conformal theory (scheme-independent).

• Transitivity $\Rightarrow$ proper scale-displacement or commensurate scale relations also ensure scheme-independence.

• The active flavors $n_f$ in $\beta$-function is correctly determined.

• Renormalons growing as $(n! \beta^m \alpha_s^n)$ are avoided $\Rightarrow$ better convergence

• The PMC method agrees with the standard QED results in the $N_c \rightarrow 0$ limit.

• Higher-order calculation easier, we only need to calculate $n_f$-terms

Up to NNNLO PMC/BLM scale setting can be found in SJB
Myths concerning scale setting

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!
\[ C_F = \frac{N_C^2 - 1}{2N_C} \]

\[ \lim_{N_C \to 0} \text{ at fixed } \alpha = C_F \alpha_s, \quad n_\ell = n_F/C_F \]

QCD \rightarrow \text{Abelian Gauge Theory}

**Analytic Feature of SU(N_c) Gauge Theory**

**Scale-Setting procedure for QCD must be applicable to QED**
Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v
Application A: Scale setting in 3-jets events at LO


$$e^+e^- \rightarrow q + \bar{q} + g \text{ up to NLO}$$

$$\frac{1}{\sigma_0} \frac{d\sigma(s) + d\sigma^3}{dy} = \int_{1-2y}^{1-y-z} dx \int_{1-y-z}^{1-y-z} dx \ T[1 - x - z, x, z] \alpha_s(s) \left[ 1 - \frac{\alpha_s(s)}{\pi} \left( \frac{\beta_0}{4} \left( \ln[x] + \ln[z] - \frac{5}{3} \right) + \ldots \right) \right]$$

$$= \alpha_s(s) \left[ T(y) - \frac{\alpha_s(s)}{\pi} \left( \frac{C(y) - \frac{5}{3} T(y)}{T(y)} \frac{\beta_0}{4} + \ldots \right) \right]$$

$$= T(y) \alpha_s(s) \left[ 1 - \frac{\alpha_s(s)}{\pi} \left( \frac{1}{4} \left( \frac{C(y)}{T(y)} - \frac{5}{3} \right) \beta_0 + \ldots \right) \right]$$

$$= T(y) \alpha_s(\mu_{BLM}^2) + \ldots$$

$$\mu_{BLM}^2 = s \times \exp \left( -\frac{5}{3} + \frac{C(y)}{T(y)} \right)$$

LO-BLM/PMC scale
The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in $e^+e^-$ annihilation, as computed by Kramer and Lampe. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low $y$. In particular, the latter two methods predict increasing values of $\mu$ as the jet invariant mass $M < \sqrt{(ys)}$ decreases.

**Other Jet Observables using BLM:** Rathsman

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PMS vs. PMC

- PMS/FAC - incorrectly sums conformal terms -- even minimizes physical asymmetries! PMS violates transitivity

- PMC/BLM: exposes conformal series - no renormalons

- Conformal series has new physics -- not associated with renormalization

- PMC: No need to analyze diagrams or codes -- simply identify non-conformal logarithms -- then shift scale

- PMC: Applies to subprocesses with multiple final particles - recursive procedure

- PMC/BLM: Agrees with QED in Abelian limit

- PMC/BLM: Result is independent of scheme and initial scale choice
Next-to-Leading Order QCD Predictions for W + 3-Jet Distributions at Hadron Colliders

$\mu_R = \mu_F = E_T^W$

$\mu_R = \mu_F = \hat{H}_T$

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Single PMC Renormalization Scale

- BLM: Set scale in each skeleton graph to absorb all nonzero beta terms.
- In practice easier to set a single global scale
- Consider general hard subprocess: \( a + b \rightarrow c + d + e + \cdots \)

\[
\hat{\mu}^2 = C \times \prod_{ij} [p_{ij}^2]^{w_{ij}} \quad \log \hat{\mu}^2 = \sum_{i \neq j} w_{ij} \log p_{ij}^2 + \log C
\]

\[
w_{ij} = \frac{f_{ij}}{\sum_{i \neq j} f_{ij}}. \quad C \text{ is the scheme displacement}
\]

\[
C = e^{-5/3} \text{ for } \overline{MS}
\]

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Example: Spinless electron-electron scattering

\[ M = \frac{s-t}{t} \alpha(t) + \frac{s-u}{u} \alpha(u) \]

Scales sum VP to all orders

\[ M \approx \left[ \frac{s-t}{t} + \frac{s-u}{u} \right] \alpha(\mu_0^2) \]
\[ \quad + \left[ \frac{s-t}{t} \right] \frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log \left( \frac{t}{\mu_0^2} \right) + \left[ \frac{s-u}{u} \right] \frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log \left( \frac{u}{\mu_0^2} \right) \]

\[ M = \left[ \frac{s-t}{t} + \frac{s-u}{u} \right] \alpha(\hat{\mu}^2) \]

\[ \hat{\mu}^2 = t^{w_t} \times u^{w_u} \]

\[ w_t = \frac{s-t}{s-t/u} \quad w_u = \frac{s-u}{s-t/u} \]

Identify \( w_t \) from \( \frac{dM}{d \log t} \)

Remaining \( \mathcal{O}(\alpha^2) \) correction is conformal
Spinless electron-electron scattering

\[ \hat{\mu}^2 = t^{w_t} \times u^{w_u} \]

Conventional guess is wrong

\[ \hat{\mu}^2 = \sqrt{tu} = p_T \sqrt{s} \text{ at } \theta_{cm} = \pi/2 \]
Heavy Quark Hadroproduction

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3-gluon coupling depends on 3 physical scales
The Renormalization Scale Problem

\[ \rho(Q^2) = C_0 + C_1 \alpha_s(\mu_R) + C_2 \alpha_s^2(\mu_R) + \cdots \]

\[ \mu_R^2 = C Q^2 \]

Is there a way to set the renormalization scale \( \mu_R \)?

What happens if there are multiple physical scales?
General Structure of the Three-Gluon Vertex

\[ \hat{\Gamma}_{\mu_1\mu_2\mu_3} = \]

3 index tensor \( \hat{\Gamma}_{\mu_1\mu_2\mu_3} \) built out of \( g_{\mu\nu} \) and \( p_1, p_2, p_3 \)

with \( p_1 + p_2 + p_3 = 0 \)

\[ \Rightarrow \text{14 basis tensors and form factors} \]

Full analytic calculation, general masses, spin

Pinch Scheme

Form factors of the gauge-invariant three-gluon vertex

Michael Binger* and Stanley J. Brodsky†

PHYSICAL REVIEW D 74, 054016 (2006)
Multi-scale Renormalization of the Three-Gluon Vertex

\[ \tilde{g}(p_1^2, p_2^2, p_3^2) \]

- Gauge-invariant subset of rad. cor.
- Coupling at each vertex absorb the rad. cor.

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The Gauge Invariant
Three Gluon Vertex

Cornwall and Papavassiliou performed the PT construction:

The “pinched” parts are added to the “regular” 3 gluon vertex

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The Pinch Technique

(q \cdot V(p, k) = S^{-1}(p) - S^{-1}(k))

Gauge-dependent

Gauge-invariant gluon self-energy!

natural generalization of QED charge
3 Scale Effective Charge

\[ \tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi} \]

(First suggested by H.J. Lu)

\[ \frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi} \beta_0 \left( L(a,b,c) - \frac{1}{\varepsilon} + \cdots \right) \]

\[ \frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi} \beta_0 \left[ L(a,b,c) - L(a_0,b_0,c_0) \right] \]

\[ L(a,b,c) = 3\text{-scale “log-like” function} \]

\[ L(a,a,a) = \log(a) \]
\[ \hat{\Gamma}_{\mu_1 \mu_2 \mu_3} = \]

\[ \mu^2 R \simeq \frac{p^2_{\text{min}} p^2_{\text{med}}}{p^2_{\text{max}}} \]

*Scale determines effective number of flavors*

H. J. Lu

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3 Scale Effective Scale

\[ L(a, b, c) \equiv \log(Q_{eff}^2(a, b, c)) + i \Im L(a, b, c) \]

Governed strength of the three-gluon vertex

\[ \frac{1}{\tilde{\alpha}(a, b, c)} = \frac{1}{\tilde{\alpha}(a_0, b_0, c_0)} + \frac{1}{4\pi} \beta_0 \left[ L(a, b, c) - L(a_0, b_0, c_0) \right] \]

\[ \hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\tilde{\alpha}(a, b, c)} \]

Generalization of BLM Scale to 3-Gluon Vertex

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SLAC National Accelerator Laboratory
Properties of the Effective Scale

\[ Q_{\text{eff}}^2 (a, b, c) = Q_{\text{eff}}^2 (-a, -b, -c) \]

\[ Q_{\text{eff}}^2 (\lambda a, \lambda b, \lambda c) = |\lambda| Q_{\text{eff}}^2 (a, b, c) \]

\[ Q_{\text{eff}}^2 (a, a, a) = |a| \]

\[ Q_{\text{eff}}^2 (a, -a, -a) \approx 5.54 |a| \]

\[ Q_{\text{eff}}^2 (a, a, c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2 (a, -a, c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2 (a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c| \]

Surprising dependence on Invariants

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Define QCD Coupling from Observables

Effective Charges: analytic at quark mass thresholds, finite at small momenta

\[ R_{e^+e^- \rightarrow X(s)} \equiv 3 \sum_q e_q^2 \left[ 1 + \frac{\alpha R(s)}{\pi} \right] \]

\[ \Gamma(\tau \rightarrow Xe\nu)(m_\tau^2) \equiv \Gamma_0(\tau \rightarrow u\bar{d}e\nu) \times \left[ 1 + \frac{\alpha_T(m_\tau^2)}{\pi} \right] \]

Commensurate scale relations:

Relate observable to observable

at commensurate scales

H. Lu, Rathsman, sjb

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Relate Observables to Each Other

• Eliminate intermediate scheme

• No scale ambiguity

• Transitive!

• Commensurate Scale Relations

• Conformal Template

• Example: Generalized Crewther Relation

\[
R_{e^+ e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left( 1 + \frac{\alpha_R(Q)}{\pi} \right).
\]

\[
\int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha g_1(Q)}{\pi} \right]
\]
\[ \frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\text{MS}}(Q)}{\pi} + \left( \frac{\alpha_{\text{MS}}(Q)}{\pi} \right)^2 \left[ \left( \frac{41}{8} - \frac{11}{3} \zeta_3 \right) C_A - \frac{1}{8} C_F + \left( -\frac{11}{12} + \frac{2}{3} \zeta_3 \right) f \right] \\
+ \left( \frac{\alpha_{\text{MS}}(Q)}{\pi} \right)^3 \left\{ \left( \frac{90445}{2592} - 2737 \frac{432}{108} \zeta_5 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2 \right) C_A^2 + \left( -\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5 \right) C_A C_F - \frac{23}{32} C_F^2 \\
+ \left[ \left( -\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2 \right) C_A + \left( -\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5 \right) C_F \right] f \\
+ \left( \frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2 \right) f^2 + \left( \frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F d(R)} \left( \sum_f Q_f \right)^2 \right\}. \]

\[ \frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\text{MS}}(Q)}{\pi} + \left( \frac{\alpha_{\text{MS}}(Q)}{\pi} \right)^2 \left[ \frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right] \\
+ \left( \frac{\alpha_{\text{MS}}(Q)}{\pi} \right)^3 \left\{ \left( \frac{5437}{648} - \frac{55}{18} \zeta_5 \right) C_A^2 + \left( -\frac{1241}{432} + \frac{11}{9} \zeta_3 \right) C_A C_F + \frac{1}{32} C_F^2 \\
+ \left[ \left( -\frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5 \right) C_A + \left( \frac{133}{864} + \frac{5}{18} \zeta_3 \right) C_F \right] f + \frac{115}{648} f^2 \right\}. \]

Eliminate MS
Find Amazing Simplification

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\[ R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right] . \]

\[
\int_0^1 dx \left[ g_{1p}^{ep}(x, Q^2) - g_{1n}^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]
\]

\[
\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3
\]

**Geometric Series in Conformal QCD**

**Generalized Crewther Relation**

Lu, Kataev, Gabadadze, Sjb
**Generalized Crewther Relation.**

\[
[1 + \frac{\alpha_R(s^*)}{\pi}] [1 - \frac{\alpha g_1(q^2)}{\pi}] = 1
\]

\[
\sqrt{s^*} \approx 0.52Q
\]

Conformal relation true to all orders in perturbation theory.

No radiative corrections to axial anomaly.

Nonconformal terms set relative scales (BLM).

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

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**Principle of Maximal Conformality**
Principle of Maximal Conformality

\[ \alpha_{\text{MS}}(0.435Q) \]

\[ \alpha_{\eta_b}(1.67Q) \]

\[ \alpha_{\tau}(1.36Q) \]

\[ \alpha_{GLS}(1.18Q) \]

\[ \alpha_{M_2}(0.904Q) \]

\[ \alpha_V(Q) \]

\[ \alpha_{\tau}(2.77Q) \]

\[ \alpha_{g_1}(1.18Q) \]

\[ \alpha_R(0.614Q) \]
\[
\frac{\alpha_{\tau}(M_{\tau})}{\pi} = \frac{\alpha_R(Q^*)}{\pi},
\]
\[
Q^* = M_{\tau} \exp \left[ -\frac{19}{24} - \frac{169}{128} \frac{\alpha_R(M_{\tau})}{\pi} \right]
\]
Transitivity Property of Renormalization Group

Relations between observables must be independent of intermediate scheme

A → C  C → B  identical to  A → B

Violated by PMS!

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Press and Media : SLAC National Accelerator Laboratory

http://www6.slac.stanford.edu/Press.aspx

Stanford Menlo Accelerator discovered pulsar, corpse of a stellar "bottomonium"...
Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme.
Unification in Physical Schemes

- Smooth analytic threshold behavior with automatic decoupling
- More directly reflects the unification of the forces
- Higher “unification” scale than usual
Conformal symmetry: Template for QCD

• Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

• Eigensolutions of ERBL evolution equation for distribution amplitudes
  Frishman, Lepage, Mackenzie, Sachrajda, sjb, Gardi, Braun

• Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

• Fix Renormalization Scale (BLM, Effective Charges)
  Gardi, Grunberg, Rathsman,Gabadadze, Kataev, Lepage, Lu, Mackenzie, sjb

• The BFKL QCD Pomeron with Optimal Renormalization
  Kim, Fadin, Lipatov, Pivovarov, sjb

• IR Fixed Point -- A Conformal Domain

• Use AdS/CFT
IR Fixed Point for QCD?

- Effective Gluon Mass Cornwall
- Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point! Alkofer, Fischer, von Smekal et al.
- Lattice Gauge Theory Furui and Nakajima
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small $Q^2$
- Justifies application of AdS/CFT in strong-coupling conformal window

Principle of Maximal Conformality

Vienna November 8, 2012

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Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

\[ \Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s g_1(Q^2)}{\pi} \right] \]

**Principle of Maximal Conformality**

Stan Brodsky

Vienna
November 8, 2012
Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

\[ S = -\frac{1}{4} \int d^4x dz \sqrt{g} \, e^\phi(z) \frac{1}{g_5^2} G^2 \quad \text{where} \quad \sqrt{g} = \left(\frac{R}{z}\right)^5 \quad \text{and} \quad \phi(z) = +\kappa^2 z^2 \]

Define an effective coupling \( g_5(z) \)

\[ S = -\frac{1}{4} \int d^4x dz \sqrt{g} \frac{1}{g^2_5(z)} G^2 \]

Thus \( \frac{1}{g^2_5(z)} = e^\phi(z) \frac{1}{g^2_5(0)} \) or \( g^2_5(z) = e^{-\kappa^2 z^2} g^2_5(0) \)

Light-Front Holography: \( z \rightarrow \zeta = b_\perp \sqrt{x(1-x)} \)

\[ \alpha_s(Q^2) \propto \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \propto e^{-Q^2/4\kappa^2} \]
Running Coupling from AdS/QCD

![Graph showing running coupling and AdS/QCD extrapolation](image)

\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

Normalization

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Deur, de Teramond, sjb
Evidence for IR Fixed Point

\[ \frac{\alpha_s g_1}{\pi} \]

\( Q \) (GeV)

Fischer et al.

DSE gluon couplings

Cornwall

Bhagwat et al.

Maris-Tandy

Bloch et al.

Godfrey-Isgur

Burkert-Ioffe

Fit

pQCD evol. eq.

GDH limit
Running Coupling from AdS/QCD

\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

\[ \alpha_s(Q) = \frac{1}{Q} \int_0^Q dQ \alpha_s(Q^2) = \frac{\pi^{3/2}\kappa}{Q_0} \text{erf}(-Q_0/2\kappa) \]

\[ \alpha_0(Q_0 = 4\kappa = 2 \text{ GeV}) = \frac{\pi^{3/2}}{4} \text{erf}(-2) \approx 1.4 \]
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain
- All perturbative calculations regulated in IR
  \[(x - y)^2 < \Lambda_{QCD}^{-2}\]
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation
- Similar in spirit to Cornwall’s Effective Gluon mass

Quark and Gluon vacuum polarization insertions decouple: IR Fixed-Point

\[QCD\]

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken, SLAC-PUB 1053
Cargese Lectures 1989
• Renormalization scale “unphysical”: No optimal physical scale

• Can ignore possibility of multiple physical scales

• Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$

• Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!
Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

- All terms associated with nonzero beta function summed into running coupling; scheme independent
- Standard procedure in QED
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scales depend on all invariants

Lepage, Mackenzie, sjb

Conformal Template

- **BLM scale-setting**: Retain conformal series; nonzero $\beta$-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.

- **Principle of Maximal Conformality**: Single Effective Scale

- **Commensurate Scale Relations** based on conformal template; scheme-independent

- **Pinch Scheme** -- provides analytic, gauge invariant, 3-g form factors

- **Analytic scheme for coupling unification**

- **IR Fixed point** -- conformal symmetry motivation for AdS/CFT

- **Light-Front Schrödinger Equation**: analytic first approximation to QCD

- **Dilaton-modified AdS5**: Predict Hadron Spectrum, Form Factors, $\alpha_s, \beta$

- **Light-Front Wave Functions from Holography**: Hadronization at the amplitude level
The reason (why BLM/PMC is useful) is clear

Basic features of BLM/PMC

• It satisfies all the above properties: Existence, Unitary, Transitivity, Reflexivity.

• All non-conformal and scheme-dependent $\beta$-terms in the perturbative series are summed into the running coupling. The result is conformal, and it is scheme-independent due to the proper scale-displacement in $\alpha_s$.

• The active flavors $n_f$ in $\beta$-function is correctly determined.

• Renormalons growing as $(n! \beta^m \alpha_s^n)$ are avoided.

• The PMC method agrees with the standard QED results in the $N_c \rightarrow 0$ limit.
PMC, **dealing with the β-series**, provides the principle underlying BLM scale setting.

**However to find what’s the β-expansion series like?**

1) There are few cases, people have calculated the β-terms directly, since it is more convenient to calculate the $n_f$-terms (light-quark loops). So usually, we need to transform the $n_f$-terms into β-terms for PMC.

2) The relation between β and $n_f$ is not in a simple way, i.e. β₂ include the 2-quark-loop, 1-quark-loop and 0-quark-loop contributions. So to get the same $n_f$-series, the combination of β-term is not unique, **which is more adaptable**?

\[
\begin{align*}
\beta_0 &= 11 - \frac{2}{3} n_f \\
\beta_1 &= 102 - \frac{38}{3} n_f \\
\beta_2^{\overline{MS}} &= \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2 \\
\beta_3^{\overline{MS}} &\approx 29243.0 - 6964.30 n_f + 405.089 n_f^2 + 1.49931 n_f^3
\end{align*}
\]

To set the BLM scales up to NNLO, the starting point is to set the effective scale $Q^*$ at LO

$$
\rho = r_0 \left[ a_s^n(Q^*) + \tilde{A}_1 a_s^{n+1}(Q^*) + (\tilde{B}_1 + \tilde{B}_2 n_f) a_s^{n+2}(Q^*) + (\tilde{C}_1 + \tilde{C}_2 n_f + \tilde{C}_3 n_f^2) a_s^{n+3}(Q^*) + \cdots \right]. \quad (11)
$$

The second step is to set the effective scale $Q^{**}$ at NLO

$$
\rho = r_0 \left[ a_s^n(Q^*) + \tilde{A}_1 a_s^{n+1}(Q^{**}) + \tilde{B}_1 a_s^{n+2}(Q^{**}) + (\tilde{C}_1 + \tilde{C}_2 n_f + \tilde{C}_3 n_f^2) a_s^{n+3}(Q^{**}) + \cdots \right]. \quad (12)
$$

Free of $a_s = \left( \frac{\alpha_s}{\pi} \right)$
and the final step is to set the effective scale $Q^{***}$ at NNLO

$$\rho = \rho_0 \left[ a_s^n(Q^*) + \tilde{A}_1 a_s^{n+1}(Q^{**}) + \tilde{B}_1 a_s^{n+2}(Q^{***}) 
+ C_1 a_s^{n+3}(Q^{***}) + \cdots \right].$$  \hspace{1cm} (13)
Generalize $\overline{MS}$ Scheme by subtracting $\log 4\pi - \gamma_E - \delta$

Call this the $\mathcal{R}_\delta$ renormalization scheme

$$\mathcal{R}_0 = \overline{MS},$$
$$\mathcal{R}_{\ln 4\pi - \gamma_E} = MS.$$

All $\mathcal{R}_\delta$ renormalization schemes have same $\beta$-function

$$\mu_{\delta_2} = \mu_{\delta_1} e^{\frac{\delta_1 - \delta_2}{2}}.$$

In particular:

$$\mu_{\overline{MS}} = \mu_{MS} e^{(\ln 4\pi - \gamma_E)/2},$$
$$\mu_\delta = \mu_{\overline{MS}} e^{-\delta/2}.$$
Since $\rho$ is a physical observable, it must be independent of the arbitrary renormalization scheme and scale. That is,

$$\frac{\partial \rho_\delta}{\partial \mu_\delta} = 0, \quad \frac{\partial \rho_\delta}{\partial \delta} = 0,$$

(16)

Generalization: use $\delta_n$ at $n$-loops.

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(Q) + (r_2 - \beta_0 r_1 \delta_1) a_2(Q)^2$$
$$+ [r_3 - \beta_1 r_1 \delta_1 - 2 \beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(Q)^3$$
$$+ [r_4 - \beta_2 r_1 \delta_1 - 2 \beta_1 r_2 \delta_2 - 3 \beta_0 r_3 \delta_3 + 3 \beta_0^2 r_2 \delta_2^2$$
$$- \beta_0^3 r_1 \delta_1^3 + \frac{5}{2} \beta_1 \beta_0 r_1 \delta_1^2] a(Q)^4 + \mathcal{O}(a^5)$$

\(20\)

**Shows the general way that nonconformal terms enter an observable**
Preliminary

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2$$
$$+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1 \beta_0 r_{3,2} + 3\beta_0 r_{4,1}$$
$$+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5)$$  (19)

**PMC scales thus satisfy**

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$
$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$
$$r_{3,0}a(Q_3)^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$
Application C: Scale setting for $R(Q)$

$$R_{e^+e^-}(Q) = 3 \sum_q e_q^2 \left[ 1 + \left( \alpha_{\overline{MS}}(Q) \right) + (1.9857 - 0.1152n_f) \left( \alpha_{\overline{MS}}(Q) \right)^2 
+ \left( -6.63694 - 1.20013n_f - 0.00518n_f^2 - 1.240 \frac{\left( \sum_q e_q \right)^2}{3 \sum_q e_q^2} \right) \left( \alpha_{\overline{MS}}(Q) \right)^3 
+ \left( -156.61 + 18.77n_f - 0.7974n_f^2 + 0.0215n_f^3 + C \frac{\left( \sum_q e_q \right)^2}{3 \sum_q e_q^2} \right) \left( \alpha_{\overline{MS}}(Q) \right)^4 \right]$$

$C$ is for singlet contribution and is small

As usual, we set $C=0$

---

taking the experimental results for $R(Q)$

From the experimental value, $r_{e^+e^-} (31.6\text{GeV}) = \frac{3}{11} R_{e^+e^-} (31.6\text{GeV}) = 1.0527 \pm 0.0050$ [26], we obtain

\[
\begin{align*}
\Lambda_H^{tH \over MS} &= 412^{+206}_{-161} \text{MeV} \\
\Lambda^{MS} &= 359^{+181}_{-140} \text{MeV}
\end{align*}
\]

\[
\alpha_s^{MS}(M_Z) = 0.129^{+0.009}_{-0.010}
\]
Application D: Scale setting for top-pair production at NLO - needs NNLO $n_f$-terms

(Rough results; To Be Continued)

\[ p + p(\vec{p}) \rightarrow Q + \bar{Q} + X \]
\[ q + \bar{q} \rightarrow Q + \bar{Q} + X \]
\[ g + g \rightarrow Q + \bar{Q} + X \]
\[ g + q \rightarrow Q + \bar{Q} + X \]

\[ \sigma(S, m^2) = \sum_{ij} \int dx_1 dx_2 \hat{\sigma}_{ij}(s, m^2, \mu^2) f_i(x_1, \mu^2) f_j(x_2, \mu^2) \]

Main Point: The expansion coefficients also include factorization scale-dependence, heavy-quark mass dependence, how to improve the PMC/BLM procedure?

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Hadronic Cross-Section

\[ \sigma_{H_1 H_2 \rightarrow t\bar{t}X} = \sum_{i,j} \int_{4m_t^2}^{S} ds \ \mathcal{L}_{ij}(s, S, \mu_f) \hat{\sigma}_{ij}(s, \alpha_s(\mu_r), \mu_r, \mu_f) \]

Parton luminosity

\[ \mathcal{L}_{ij} = \frac{1}{S} \int_{s}^{S} \frac{d\hat{s}}{\hat{s}} f_{i/H_1}(x_1, \mu_f) f_{j/H_2}(x_2, \mu_f) \]

Subprocess Cross-Section

NNLO

\[ \hat{\sigma}_{ij} = \frac{1}{m_t^2} \left\{ f_{ij}^0(\rho, Q)a_s^2(Q) + f_{ij}^1(\rho, Q)a_s^3(Q) + f_{ij}^2(\rho, Q)a_s^4(Q) \right\} \]
\[ f_{ij}^1(\rho, Q) = [A_{1ij} + B_{1ij}n_f] + D_{1ij} \left( \frac{\pi}{v} \right) \]

\[ A_{0ij} = f_{ij}^0(\rho, Q) \]

\[ f_{ij}^2(\rho, Q) = [A_{2ij} + B_{2ij}n_f + C_{2ij}n_f^2] + [D_{2ij} + E_{2ij}n_f]\left( \frac{\pi}{v} \right) + F_{2ij}\left( \frac{\pi}{v} \right)^2 \]

\[ m_t^2 \tilde{\sigma}_{ij} = A_{0ij}a_s^2(Q_1^*) + \left[ \tilde{A}_{1ij} \right] a_s^3(Q_1^*) + \left[ \tilde{A}_{2ij} + \tilde{B}_{2ij}n_f \right] a_s^4(Q_1^*) + D_{1ij} \left( \frac{\pi}{v} \right) a_s^3(Q_2^*) + \left[ \tilde{D}_{2ij} \right] \left( \frac{\pi}{v} \right) a_s^4(Q_2^*) + F_{2ij} \left( \frac{\pi}{v} \right)^2 a_s^4(Q_2^*) \]

**first step**

**second step**

Sommerfeld rescattering
perturbative series

\[
\ln \frac{Q_1^{*2}}{Q^2} = \ln \frac{Q_{10}^{*2}}{Q^2} + \frac{\chi}{4} \beta_0 \ln \frac{Q_{10}^{*2}}{Q^2} a_s(Q)
\]
\[
\ln \frac{Q_{10}^{*2}}{Q^2} = \frac{3B_{1ij}}{A_{0ij}}, \quad \chi = \frac{9B_{1ij}^2 - 12A_{0ij}C_{2ij}}{2A_{0ij}B_{1ij}}
\]

most important greatly improve the NLO estimation

\[
\ln \frac{Q_1^{**2}}{Q_1^{*2}} = \frac{2\tilde{B}_{2ij}}{\tilde{A}_{1ij}}
\]

determines \(q\bar{q}\) – channel LO scale
determines \(q\bar{q}\) – channel NLO scale
determines \(gg\) – channel LO scale
determines \(gg\) – channel NLO scale

slight change for gg-channel

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A proper NLO scale is clearly very important! especially to understand the ttbar-asymmetry

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Conventional scale choice: $m_t$
"Lucky guess" for total rate

$m_t = 172.9 \pm 1.1$ GeV

PDF+$\alpha_s$ error

$\alpha_s(m_Z) = 0.118 \pm 0.001$

$\sigma_{\text{Tevatron}, 1.96\,\text{TeV}} = 7.626^{+0.265}_{-0.257}\,\text{pb}$

$\sigma_{\text{LHC}, 7\,\text{TeV}} = 171.8^{+5.8}_{-5.6}\,\text{pb}$

$\sigma_{\text{LHC}, 14\,\text{TeV}} = 941.3^{+28.4}_{-26.5}\,\text{pb}$

$\sigma_{\text{Tevatron}, 1.96\,\text{TeV}} = 7.626^{+0.143}_{-0.130}\,\text{pb}$

$\sigma_{\text{LHC}, 7\,\text{TeV}} = 171.8^{+3.8}_{-3.5}\,\text{pb}$

$\sigma_{\text{LHC}, 14\,\text{TeV}} = 941.3^{+14.6}_{-15.6}\,\text{pb}$
Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the ‘Principle of Maximum Conformality’ (PMC)

\[ A_{FB}^{\tilde{t}\tilde{t}} (M_{\tilde{t}\tilde{t}} > 450 \text{ GeV}) \]

Experimental asymmetry
PMC Prediction

Using conventional guess for renormalization scale and range

\( t\bar{t} \) asymmetry predicted by pQCD NNLO within 1 \( \sigma \) of CDF/D0 measurements using PMC/BLM scale setting

Xing-Gang Wu
SJB
PMC: ~1 sigma from experiment

\[ A_{FB}^{t \bar{t}} \sim 12.7\% \; ; \; A_{FB}^{p \bar{p}} \sim 8.39\% \]

\[ A_{FB}^{t \bar{t}} = (9.7, 8.9, 8.3)\% , \quad A_{FB}^{p \bar{p}} = (6.4, 5.9, 5.4)\% . \]

\[ A_{FB}^{t \bar{t}}(M_{t \bar{t}} > 450 \text{ GeV}) = (13.9, 12.8, 11.9)\% , \]

\[ \bar{\alpha}_s(\mu_R^{PMC,NLO}) = 0.1460 \]

\[ \mu_R^{PMC,NLO} \sim \exp(-19/10)m_t \simeq 26 \text{ GeV} . \]

\[ A_{FB}^{t \bar{t}, PMC}(M_{t \bar{t}} > 450 \text{ GeV}) \simeq 35.0\% \]

\[ A_{FB}^{t \bar{t}, CDF} = (15.8 \pm 7.5)\% \; ; \; A_{FB}^{p \bar{p}, CDF} = (15.0 \pm 5.5)\% \]

\[ A_{FB}^{t \bar{t}}(M_{t \bar{t}} > 450 \text{ GeV}) = (47.5 \pm 11.4)\% \]

why the conventional scale-setting gives small asymmetry?

**dominant asymmetric $q\bar{q} - channel$**

**NLO**

<table>
<thead>
<tr>
<th>Channel</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>Total</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(q\bar{q})$-channel</td>
<td>4.890</td>
<td>0.963</td>
<td>0.483</td>
<td>6.336</td>
<td>4.748</td>
<td>1.727</td>
<td>-0.058</td>
<td>6.417</td>
</tr>
<tr>
<td>$(gq)$-channel</td>
<td>0.526</td>
<td>0.440</td>
<td>0.166</td>
<td>1.132</td>
<td>0.524</td>
<td>0.525</td>
<td>0.160</td>
<td>1.208</td>
</tr>
<tr>
<td>$(gg)$-channel</td>
<td>0.000</td>
<td>-0.0381</td>
<td>0.0049</td>
<td>-0.0332</td>
<td>0.000</td>
<td>-0.0381</td>
<td>0.0049</td>
<td>-0.0332</td>
</tr>
<tr>
<td>$(g\bar{q})$-channel</td>
<td>0.000</td>
<td>-0.0381</td>
<td>0.0049</td>
<td>-0.0332</td>
<td>0.000</td>
<td>-0.0381</td>
<td>0.0049</td>
<td>-0.0332</td>
</tr>
<tr>
<td>Sum</td>
<td>5.416</td>
<td>0.985</td>
<td>0.659</td>
<td>7.402</td>
<td>5.272</td>
<td>2.176</td>
<td>0.112</td>
<td>7.559</td>
</tr>
</tbody>
</table>

TABLE I. Total cross-sections (in unit: pb) for the top-quark pair production at the Tevatron with $p\bar{p}$-collision energy $\sqrt{s} = 1.96$ TeV. For conventional scale-setting, we set the renormalization scale $\mu_R = Q$. For PMC scale-setting, we set the initial renormalization scale $\mu_R^{\text{init}} = Q$. Here we take $Q = m_t = 172.9$ GeV and use the MSRT 2004-QED parton distributions [54] as the PDF.

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**Principle of Maximal Conformality**

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\[ C_F = \frac{N_C^2 - 1}{2N_C} \]

\[ \lim_{N_C \to 0 \text{ at fixed } \alpha} \alpha = C_F \alpha_s, \ n_\ell = n_F/C_F \]

QCD $\rightarrow$ Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED
Need to set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

**PMC/BLM**

No renormalization scale ambiguity!

- Result is independent of Renormalization scheme and initial scale!
- Same as QED Scale Setting
- Apply to Evolution kernels, hard subprocesses
- Eliminates unnecessary systematic uncertainty

**Principle of Maximum Conformality**

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

Choose $\mu_R^{\text{init}}$; arbitrary initial renormalization scale

Identify $\{\beta_i^R\}$ -- terms using $n_f$ -- terms through the PMC -- BLM correspondence principle

Shift scale of $\alpha_s$ to $\mu_R^{\text{PMC}}$ to eliminate $\{\beta_i^R\}$ -- terms

Conformal Series

Result is independent of $\mu_R^{\text{init}}$ and scheme at fixed order

---

**Principle of Maximal Conformality**

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November 8, 2012

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Leonardo di Giustino, SJB

Xing-Gang Wu

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Conventional scale-setting method

- **Guess** a renormalization scale \( Q \)
  - \( Q \) being the typical momentum transfer of the process
- **Keep its form fixed** during the calculation
- **Vary** it over the range \([Q/2, 2Q]\) to discuss its uncertainty

Under such method, the scale-error is an important systematic error

I) Its estimation is **scheme-dependent** at fixed order.
II) Its estimation **strongly depends on the choice of** \( Q \). Why just a factor of \( \frac{1}{2} \) or 2 and not 10 or 20? Which is the typical momentum transfer?
III) It gives wrong result for QED! **GM-L-scheme**.
IV) The **convergence** of the perturbative series is problematic. There are large **renormalon** terms in higher-orders which make the prediction unreliable or even wrong.

**Greatly depresses the predictive power of pQCD!**
I. **BLM/PMC** provides self-consistent way to set the effective scales, which leads to **scheme-independent result**. QCD is not conformal, however one can use the PMC to convert a PQCD series with the corresponding conformal QCD series.

II. After PMC/BLM, one can use the **Mellin-space analysis** to compute the conformal QCD correlators and amplitudes. To be considered.

III. A combination of BLM/PMC to Extended-RGE can be used to derive a **precise QCD estimation**.

IV. Top-pair production total cross-section agree with exp data.

V. Top-pair asymmetries are within $1\sigma$-error. **SM is OK ?**

VI. More applications, **such as Higgs production**, will appear.
**Generalized Crewther Relation**

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha g_1(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

**Conformal relation true to all orders in perturbation theory**

**No radiative corrections to axial anomaly**

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

**Both observables go through new quark thresholds at commensurate scales!**
Gauge Principle: Physical Observables cannot depend on the choice of gauge

Renormalization Group Principle: Physical Observables cannot depend on the choice of scheme

PMC Scale-Setting Satisfies all Principles of the Renormalization Group

The conventional method of guessing the renormalization scale and range cannot be justified: It introduces an unnecessary uncertainty in QCD predictions
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ k^\pm = k^0 \pm k^z \]

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

\[ P^+, \vec{P}_\perp \]

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \]

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp \]

Invariant under boosts! Independent of \( P^\mu \)

Bethe-Salpeter WF integrated over \( k^- \)

Square: Structure Functions Measured in DIS
\[ \psi(x, b_\perp) \leftrightarrow \phi(z) \]

\[ \zeta = \sqrt{x(1-x)b_\perp^2} \leftrightarrow z \]

\[ \psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta) \]

**Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors**

Vienna 
November 8, 2012

**Principle of Maximal Conformality**

Stan Brodsky
Relativistic LF single-variable radial equation for QCD & QED

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U(\zeta^2, J, L, M^2) \right] \Psi_{J,L}(\zeta^2) = M^2 \Psi_{J,L}(\zeta^2) \]

\[ \zeta^2 = x(1 - x)b_{\perp}^2. \]

U is the exact QCD potential
Conjecture: ‘H’-diagrams generate

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]
Use Light-Front Wavefunctions from AdS/QCD to eliminate the Factorization-Scale Ambiguity

\[ \psi_M(x, k^2_\perp) \]

Note coupling \( k^2_\perp, x \)

J. Day, sjb

de Teramond, sjb

“Soft Wall” model

massless quarks

Use Light-Front Wavefunctions from AdS/QCD to eliminate the Factorization-Scale Ambiguity
QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary
Predict Hadron Properties from First Principles!

QCD Lagrangian

Lattice Gauge Theory

Light-Front Hamiltonian

DLCQ

Effective Field Theory Methods
SCET, ChPT, ...

PQCD Evolution Equations
Counting Rules

AdS/QCD!

Light-Front Holography

Hadron Masses and Observables

Principle of Maximum Conformality
Scale Setting

Bound-State Dynamics! Confinement!
The Principle of Maximal Conformality

The Elimination of the QCD Renormalization Scale Ambiguity

Stan Brodsky

Thanks for an outstanding visit to Vienna!!

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