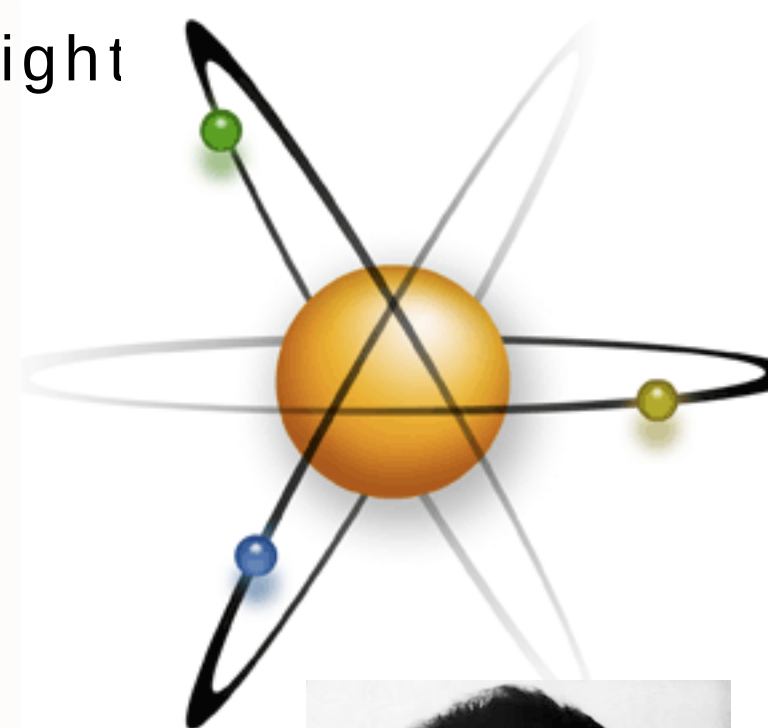
The remarkable connections between atomic and hadronic physics

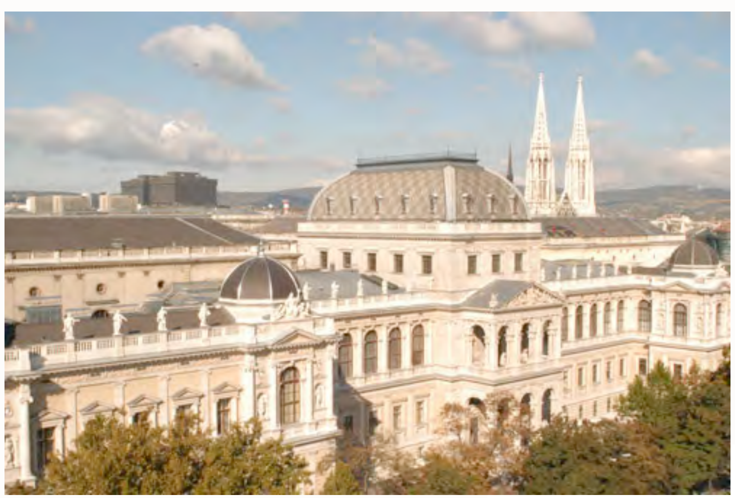
and Exotic Atoms in Flight



Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

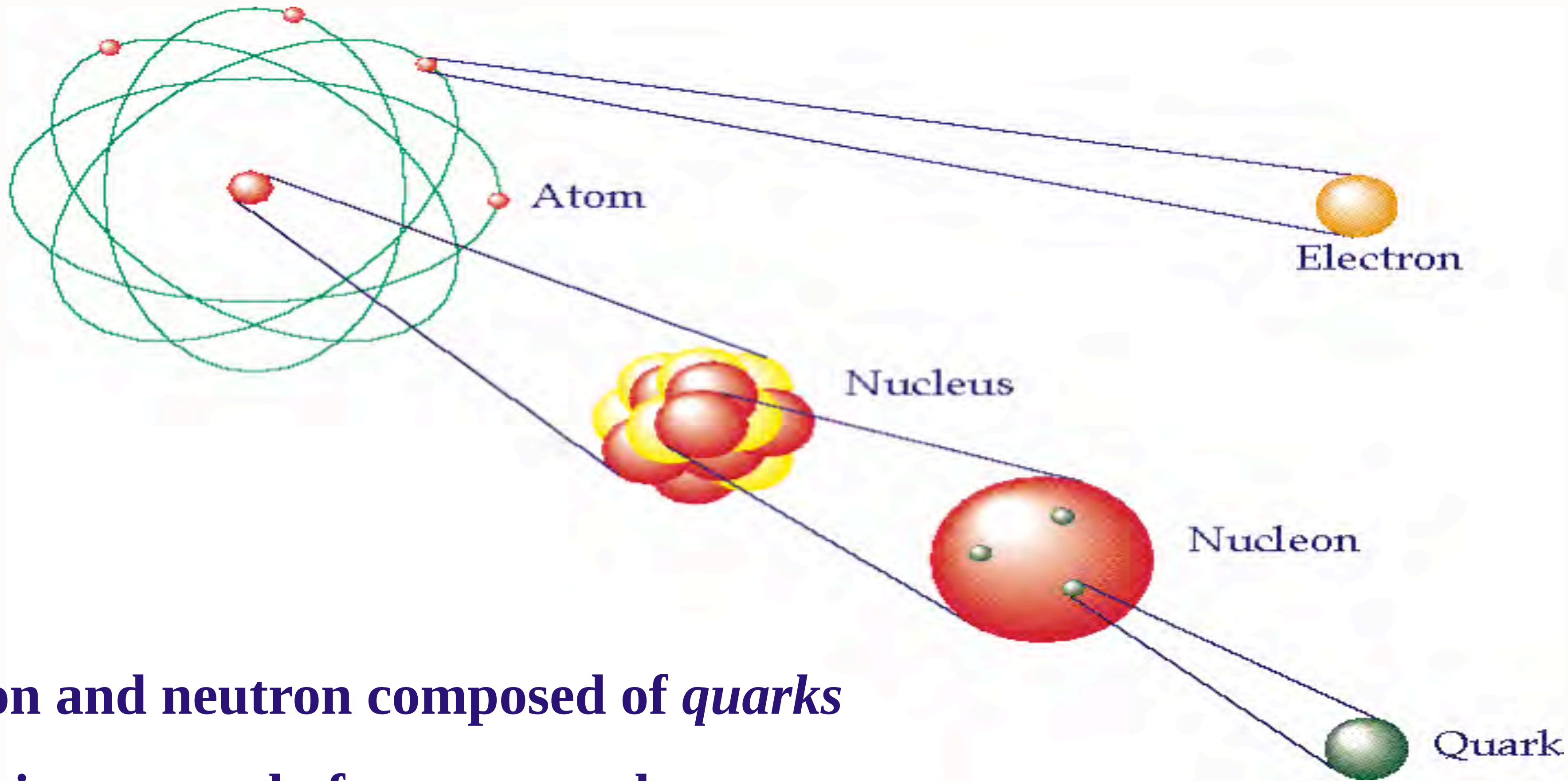
Stanford University



universität wien

Goal of Science:

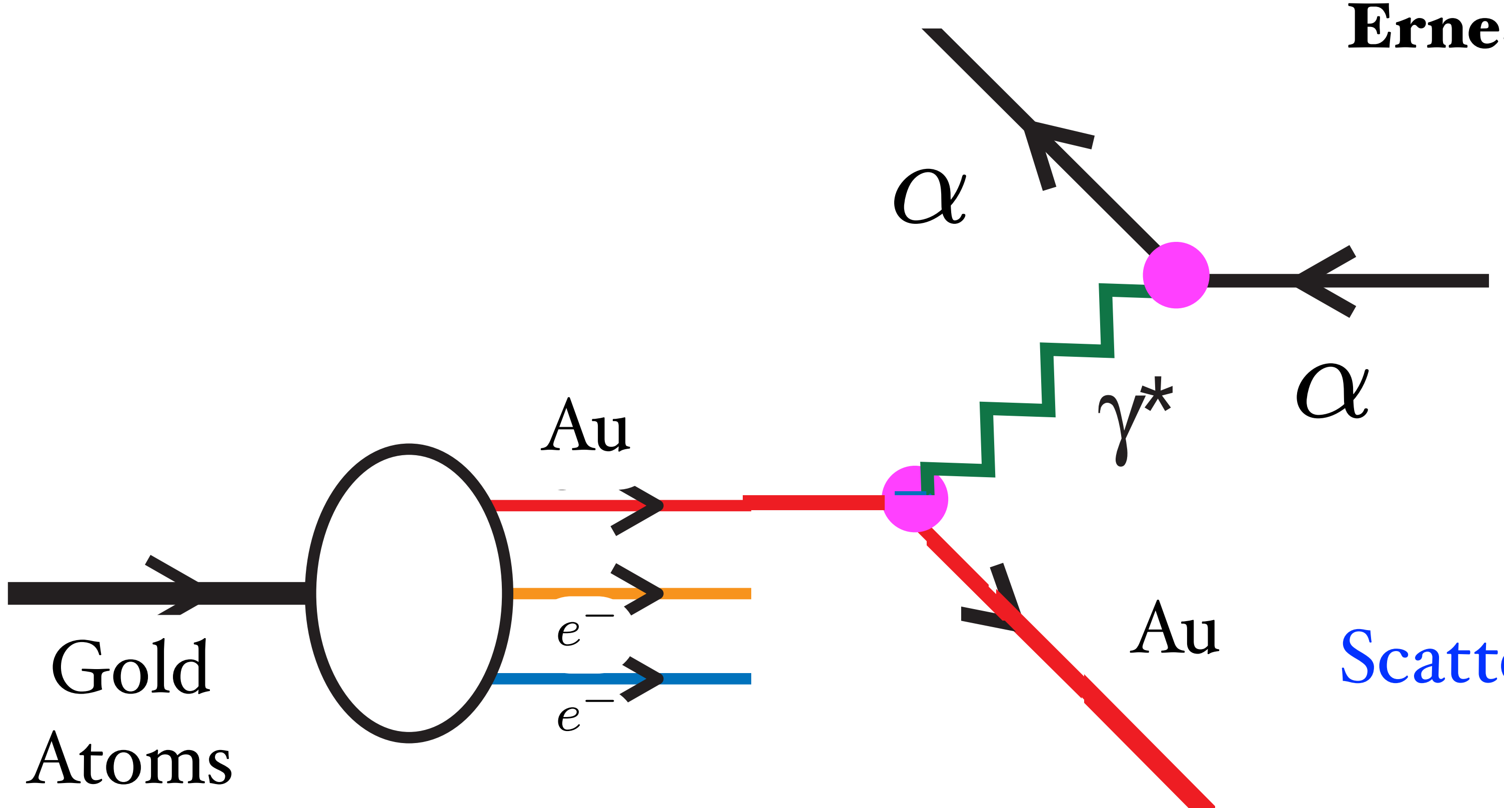
To understand the laws of physics and the fundamental composition of matter at the shortest possible distances.



- **Proton and neutron composed of *quarks***
- **Nuclei composed of protons and neutrons**
- **Atoms composed of nuclei and electrons …**

First Evidence for Nuclear Structure of Atoms

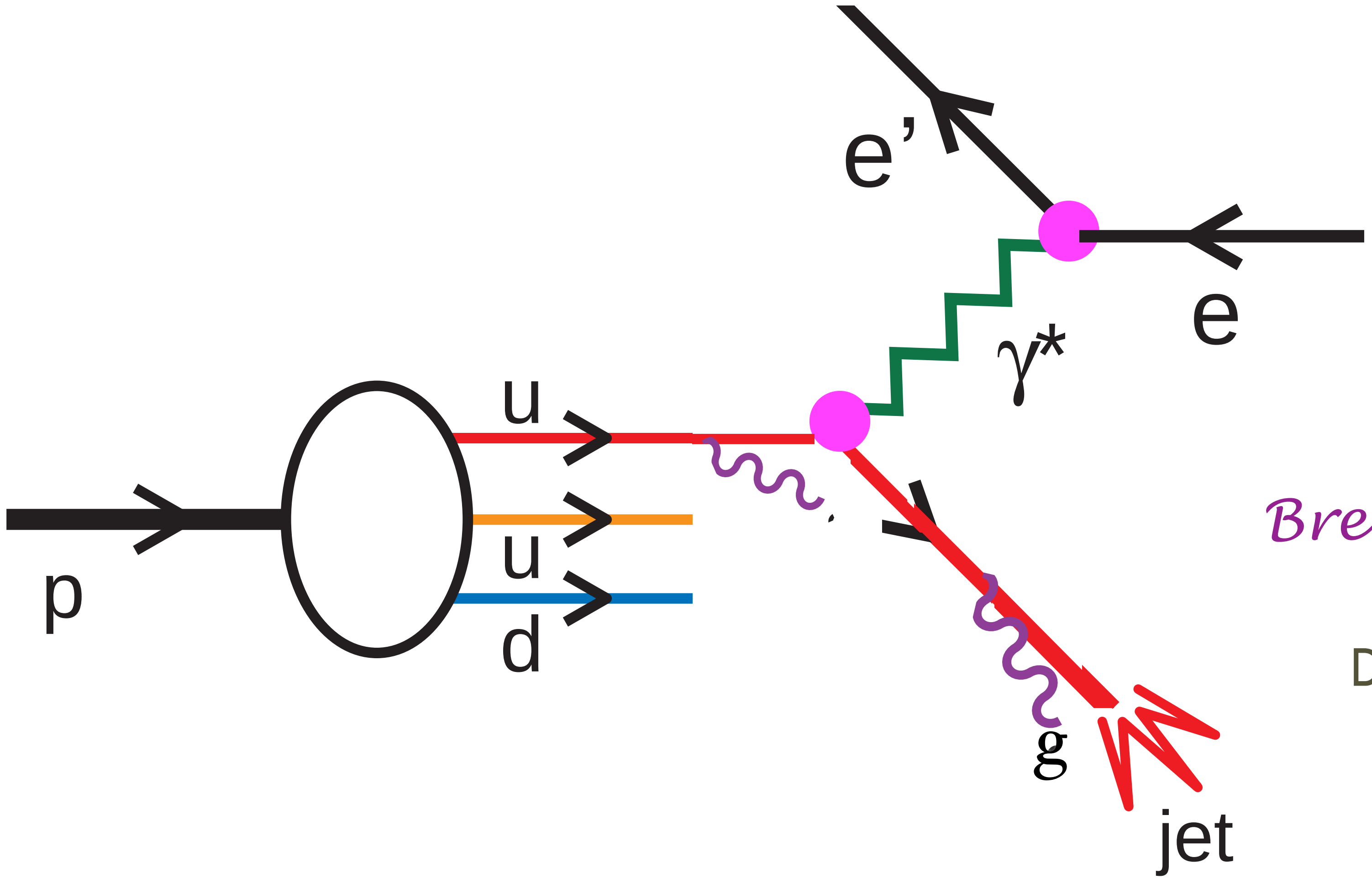
Ernest Rutherford
1911



Scattering at Large Angles!
"Point-like" Nucleus

Rutherford Scattering

First Evidence for Quark Structure of Matter



Gluonic Bremmstrahlung

DGLAP Evolution

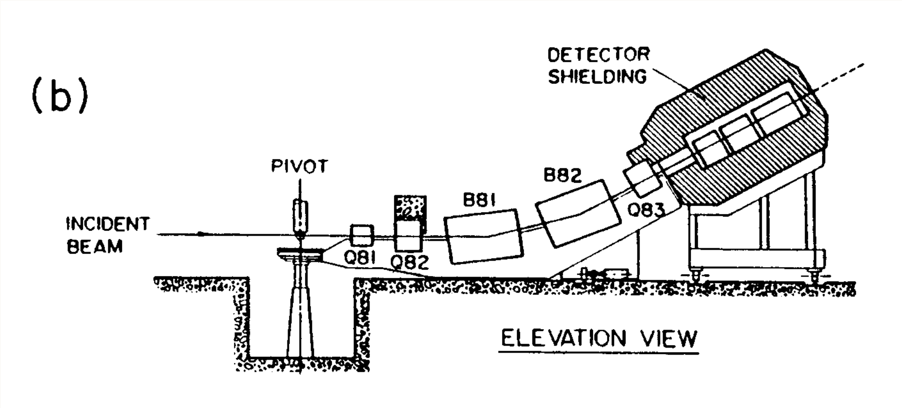
Deep Inelastic Electron-Proton Scattering

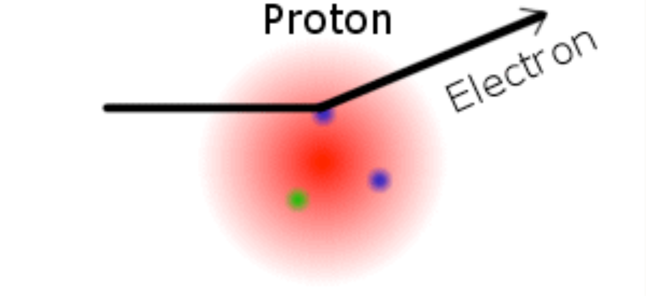
1967 SLAC Experiment:

Scatter 20 GeV/c Electrons on protons in a Hydrogen Target

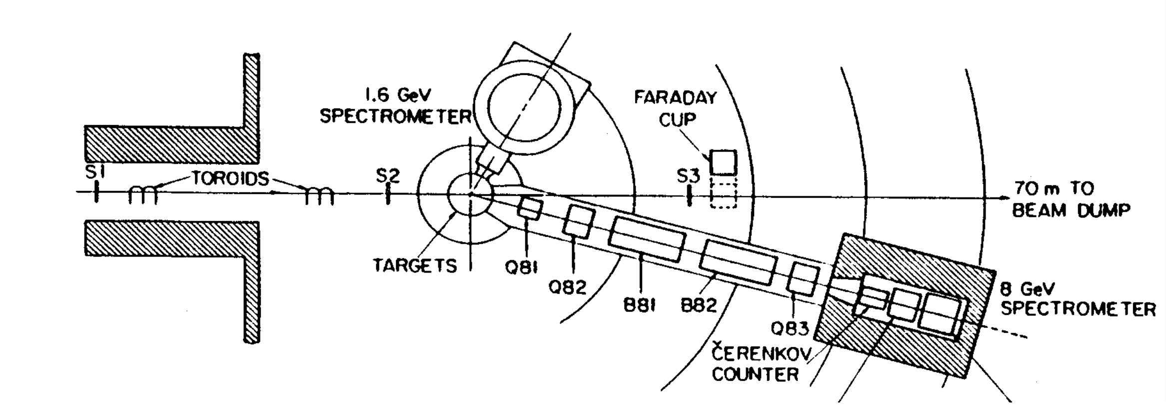
Discovery of the Quark Structure of Matter

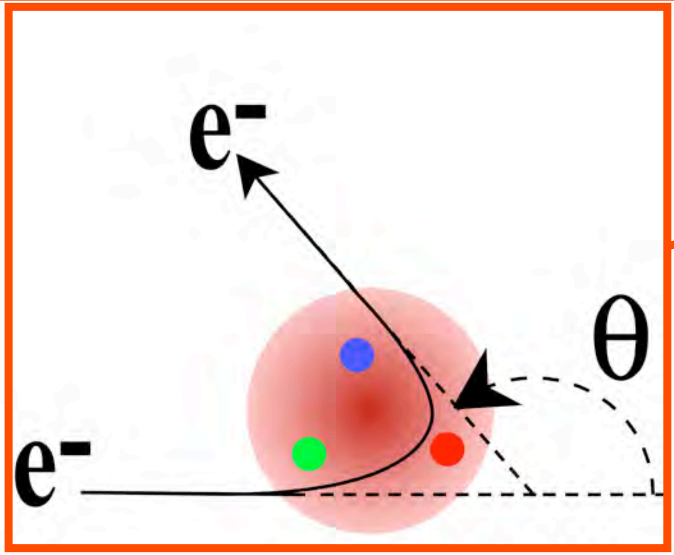
$$ep \to e' X$$





Discovery of quarks!

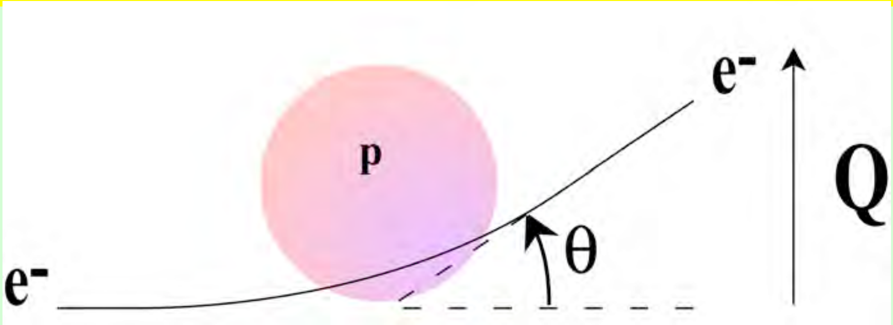




Deep inelastic scattering: Experiments on the proton and the observation of scaling*

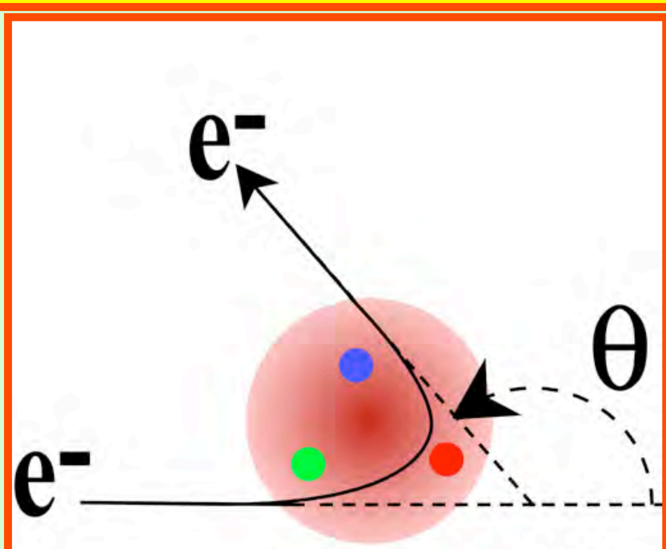
Friedman, Kendall, Taylor: Nobel Prize

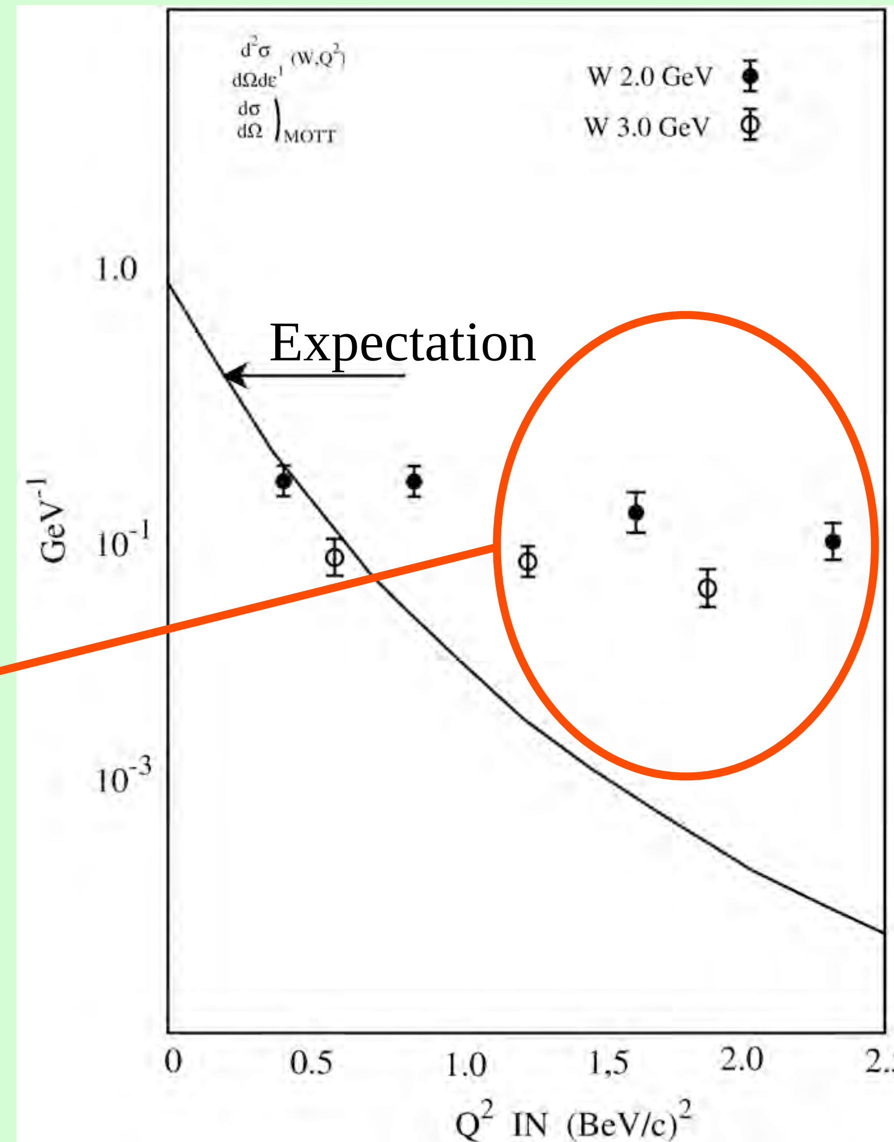
Deep inelastic electron-proton scattering



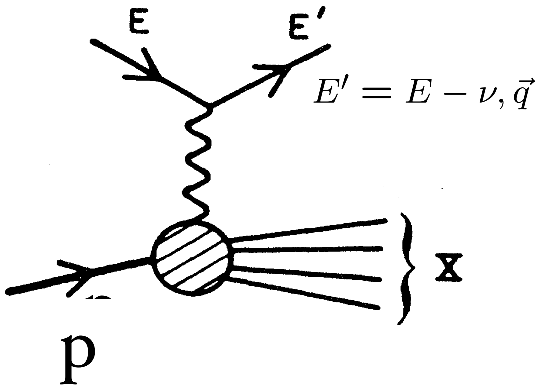
- Rutherford scattering using *very* high-energy electrons striking protons

Discovery of quarks!

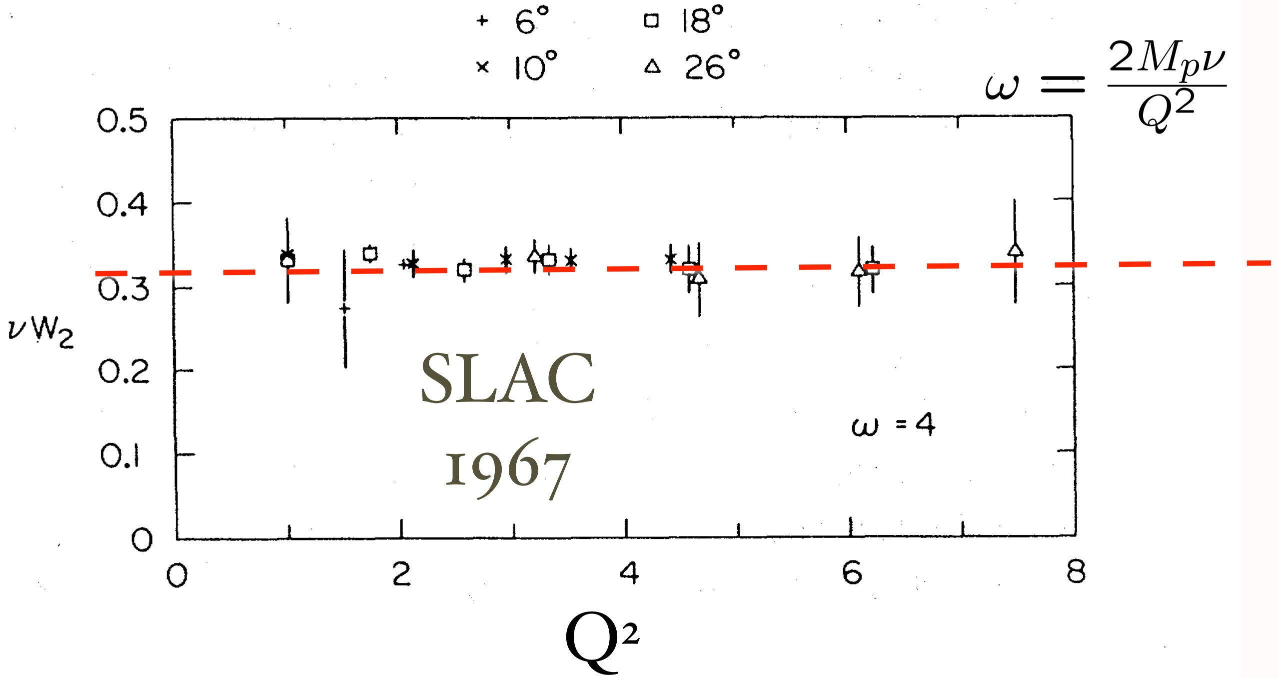




$$ep \to e'X$$



$$Q^2 = \vec{q}^2 - \nu^2$$



No intrinsic length scale !

Measure rate as a function of energy loss ν and momentum transfer Q

Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!

QED Lagrangian

$$\boxed{\mathcal{L}_{QED} = -\frac{1}{4}Tr(F^{\mu\nu}F_{\mu\nu}) + \sum_{\ell=1}^{n_\ell} i\bar{\Psi}_\ell D_\mu \gamma^\mu \Psi_\ell + \sum_{\ell=1}^{n_\ell} m_\ell \bar{\Psi}_\ell \Psi_\ell}$$

$$iD^\mu = i\partial^\mu - eA^\mu \qquad F^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu$$

Yang Mills Gauge Principle:
Phase Invariance at Every Point
of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Landau Pole

QCD Lagrangian

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

mass term

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

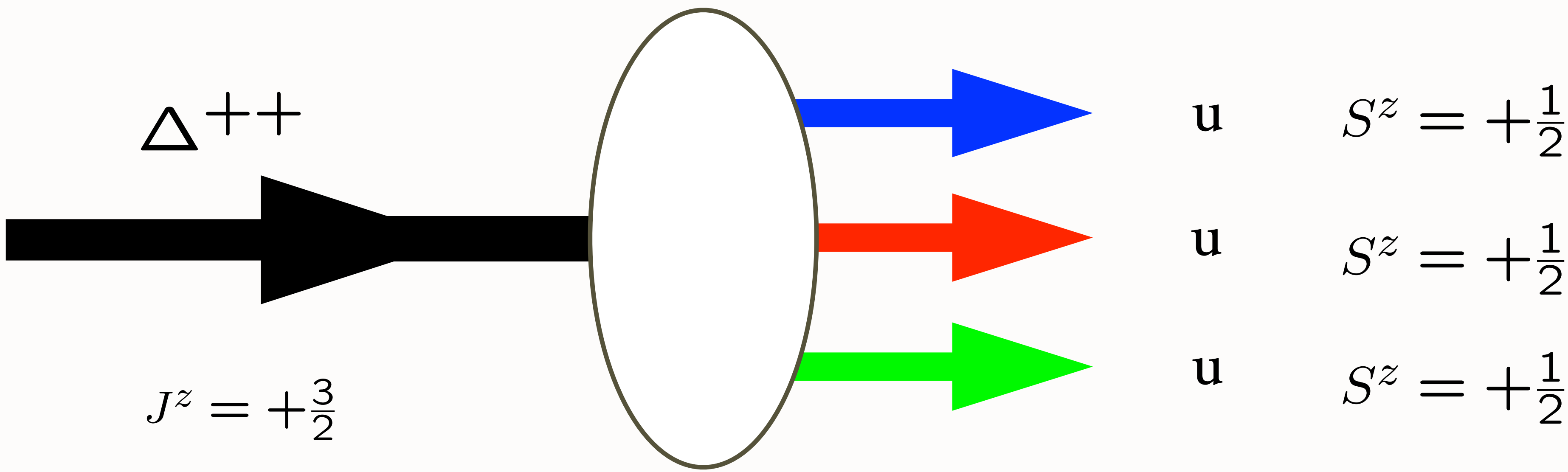
Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

Why are there three colors of quarks?

Greenberg

Pauli Exclusion Principle!

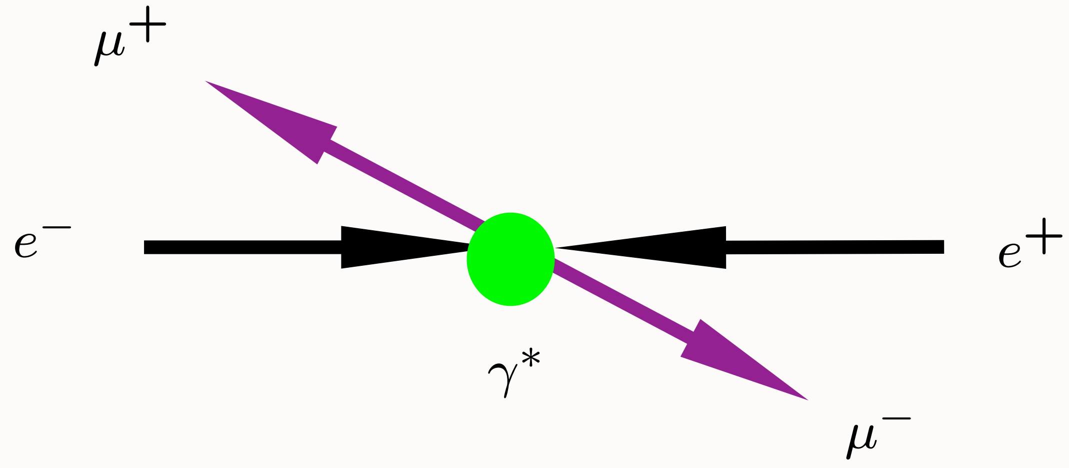
spin-half quarks cannot be in same quantum state !



Three Colors (Parastatistics) Solves Paradox

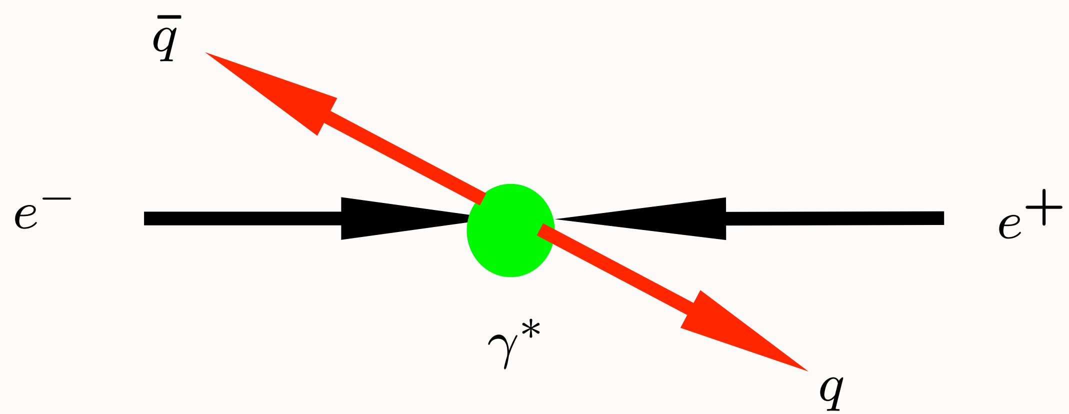
3 Colors Combine : WHITE $\quad SU(N_C), N_C = 3$

Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to \mu^+\mu^-$$

Electron-Positron Annihilation

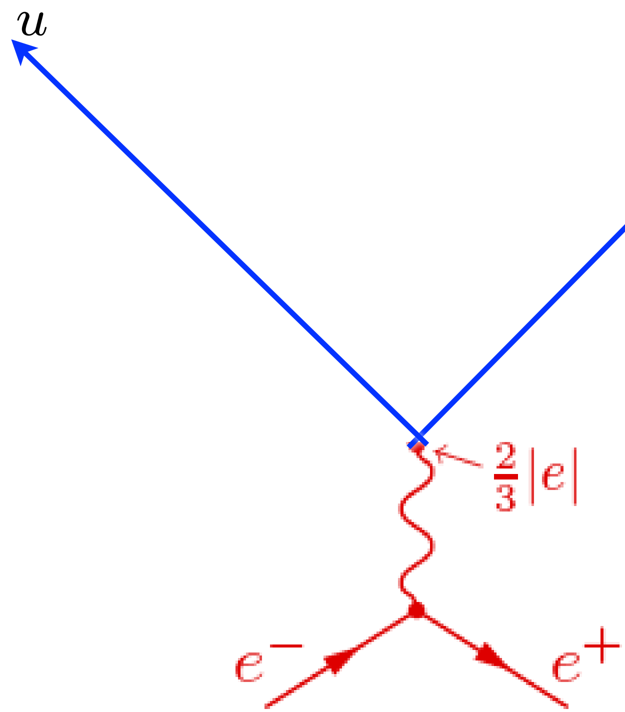


$$e^+e^- \to \gamma^* \to q\bar{q}$$

Rate proportional to quark charge squared
and the number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

How to Count Quarks



Color-triplet quark representation

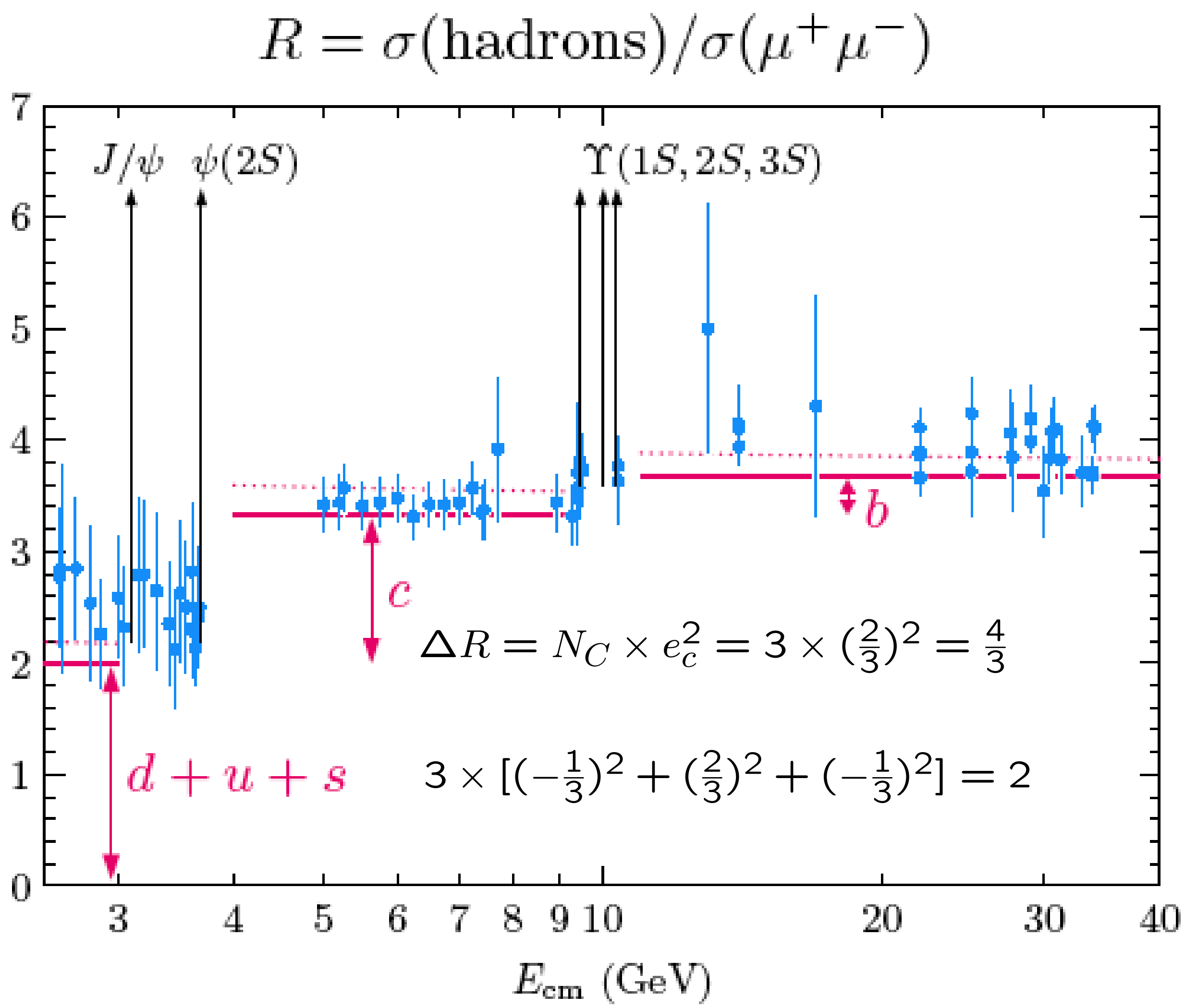
For $10 \text{ GeV} < E_{\rm cm} < 40 \text{ GeV}$,

$$\frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = \underset{\substack{\uparrow \\ \text{colors}}}{3} \times \left[\underset{\substack{\uparrow \\ d}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ u}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ s}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ c}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ b}}{\left(\frac{1}{3}\right)^2} \right]$$

How to Count Quarks

$J/\psi = (c\bar{c})_{1S}$

$\Upsilon = (b\bar{b})_{1S}$



$3 \times (-\frac{1}{3})^2 = \frac{1}{3}$

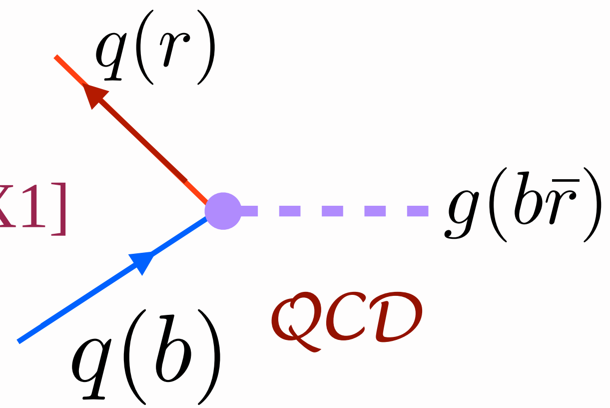
$\boxed{N_C = 3}$

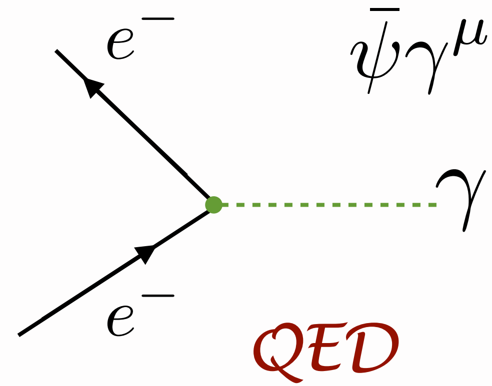
$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

Fundamental Couplings of QCD and QED

$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

[1X3] [3X3] [3X1]



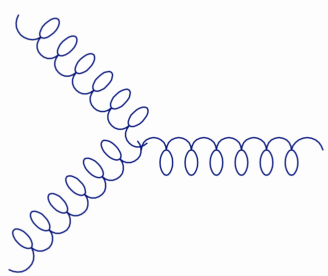


$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

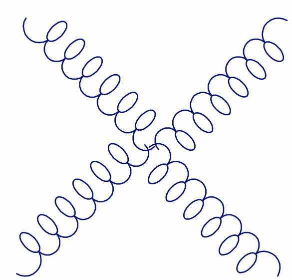
$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_{\mu}\gamma^{\mu}\Psi_f + \sum_{f=1}^{n_f} m_f\bar{\Psi}_f\Psi_f$$

$$G^{\mu\nu} = \partial^{\mu}A^{\mu} - \partial^{\nu}A^{\mu} - g[A^{\mu}, A^{\nu}]$$

Gluon vertices



QCD



$G^{\mu\nu}G_{\mu\nu}$

gluon self couplings

QED: Underlies Atomic Physics, Molecular Physics, Chemistry, Electromagnetic Interactions ...

QCD: Underlies Hadron Physics, Nuclear Physics, Strong Interactions, Jets

Theoretical Tools

- **Feynman diagrams and perturbation theory**
- **Bethe Salpeter Equation, Dyson-Schwinger Equations**
- **Lattice Gauge Theory,**
- **Discretized Light-Front Quantization**
- **AdS/QCD !**

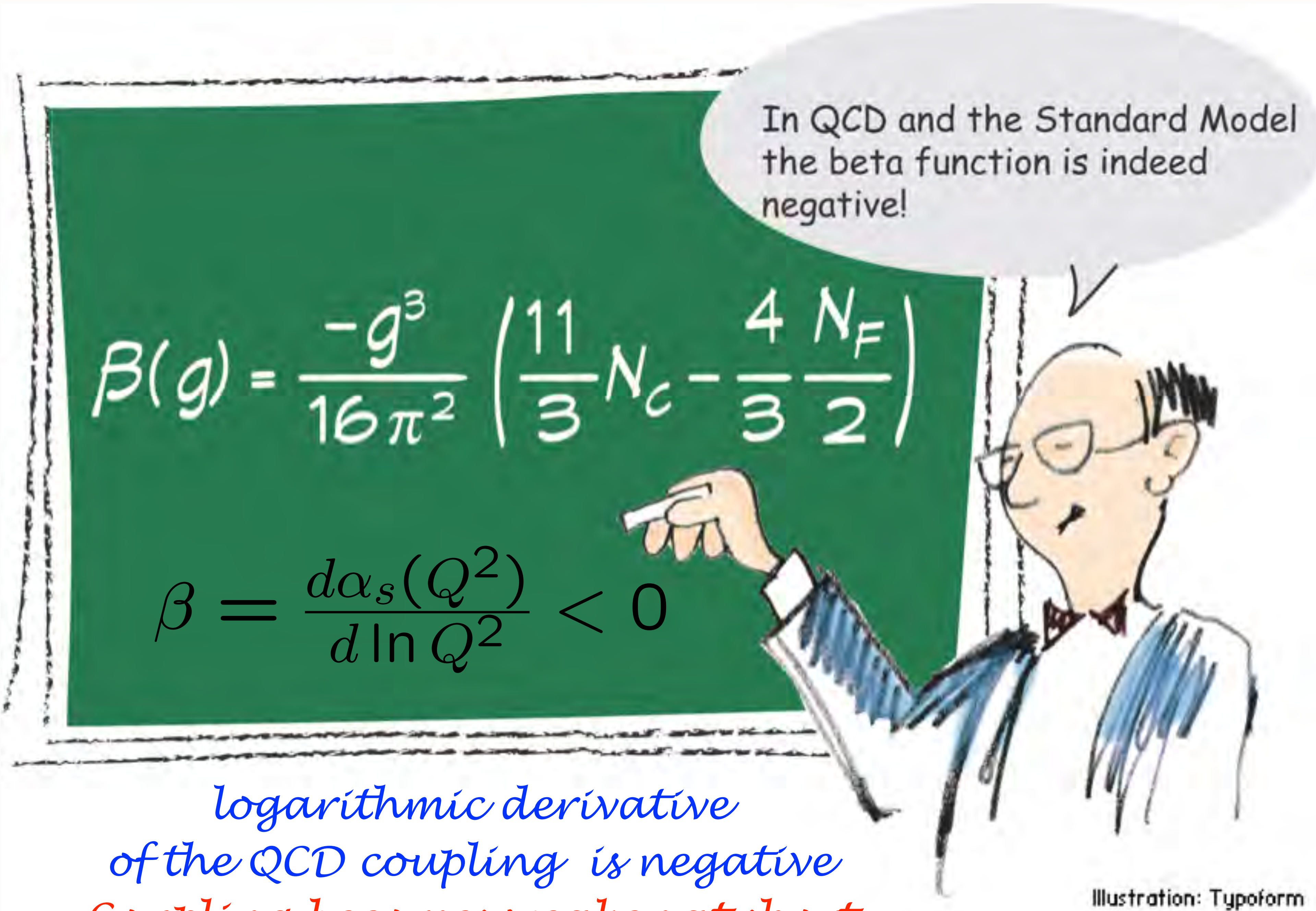


Illustration: Typoform

$$\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} < 0$$

logarithmic derivative of the QCD coupling is negative

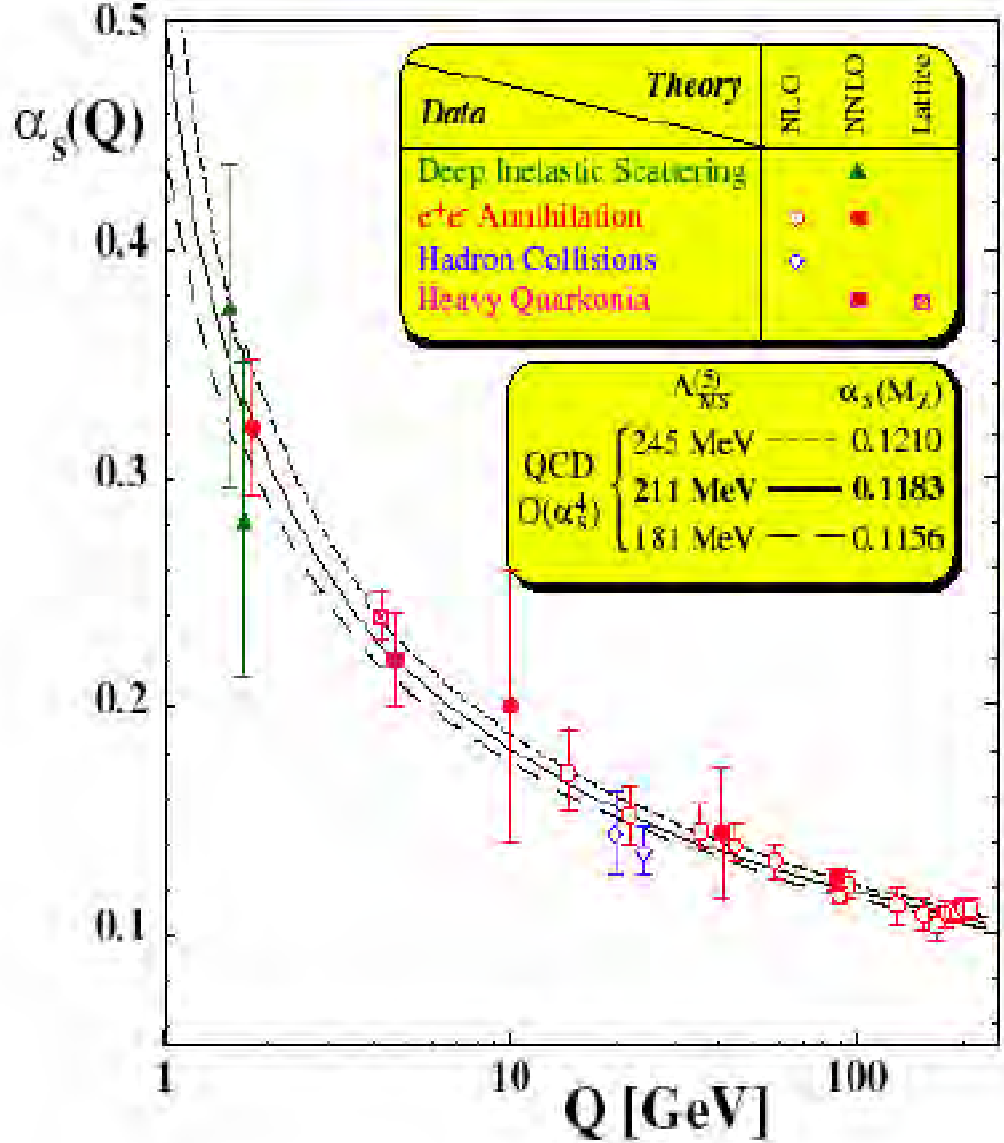
Coupling becomes weaker at short distances = high momentum transfer

Verification of Asymptotic Freedom

$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$

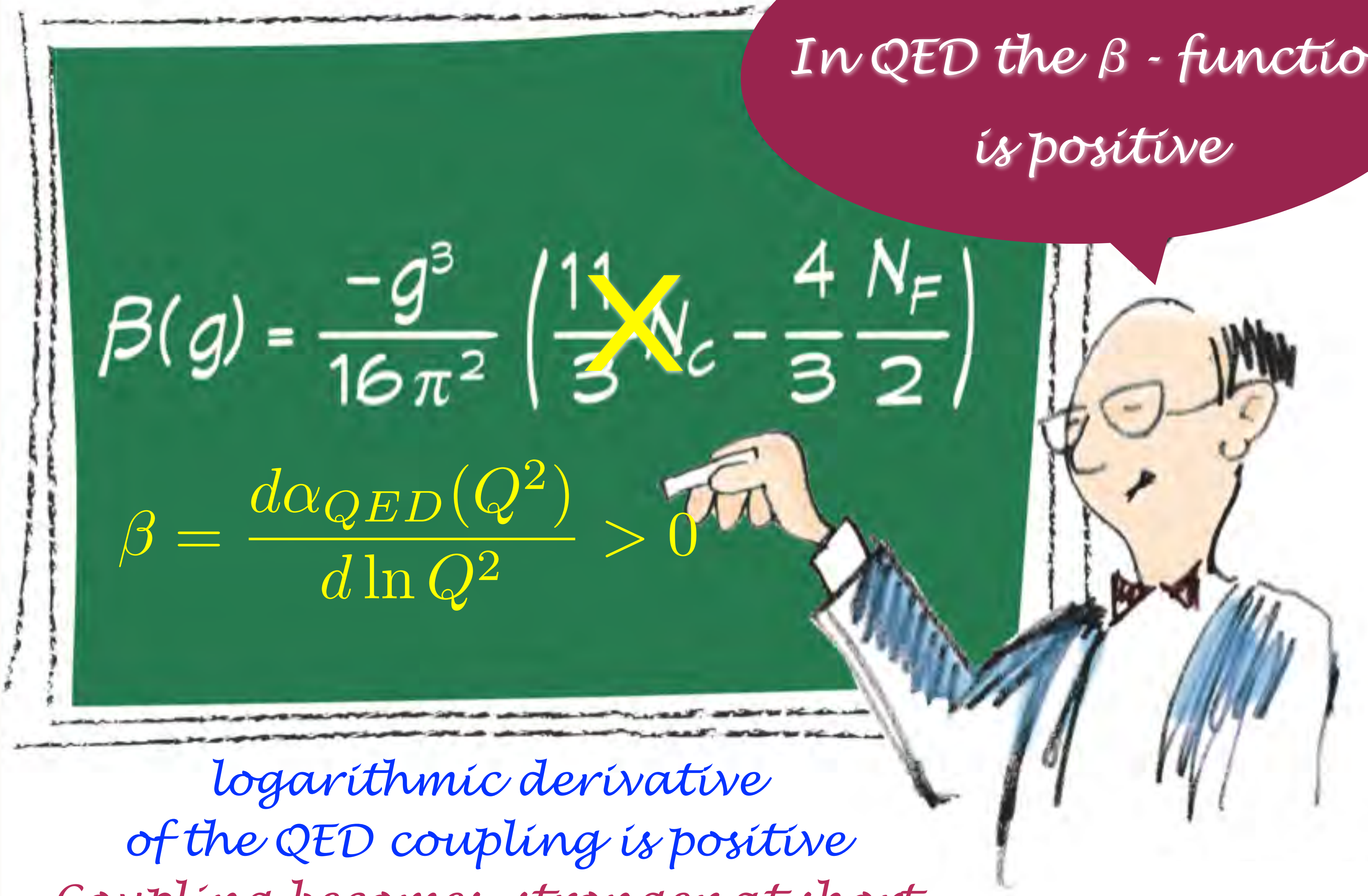
$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

proportional to $\alpha_s(Q)$



Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$

In QED the β - function is positive



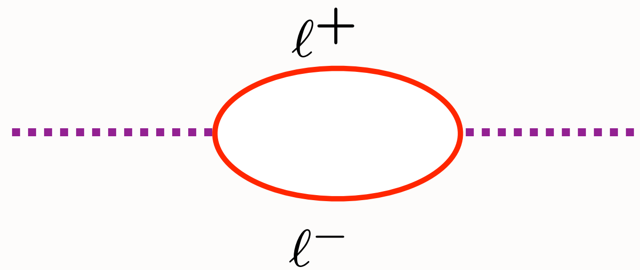
$$\beta = \frac{d\alpha_{QED}(Q^2)}{d\ln Q^2} > 0$$

logarithmic derivative
of the QED coupling is positive

Coupling becomes stronger at short distances = high momentum transfer

Landau Pole!

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right)\sqrt{1 + \frac{4m^2}{Q^2}}\,\log\frac{1+\sqrt{1+\frac{4m^2}{Q^2}}}{\left|1-\sqrt{1+\frac{4m^2}{Q^2}}\right|}\right]$$

Analytically continue to timelike t: Complex

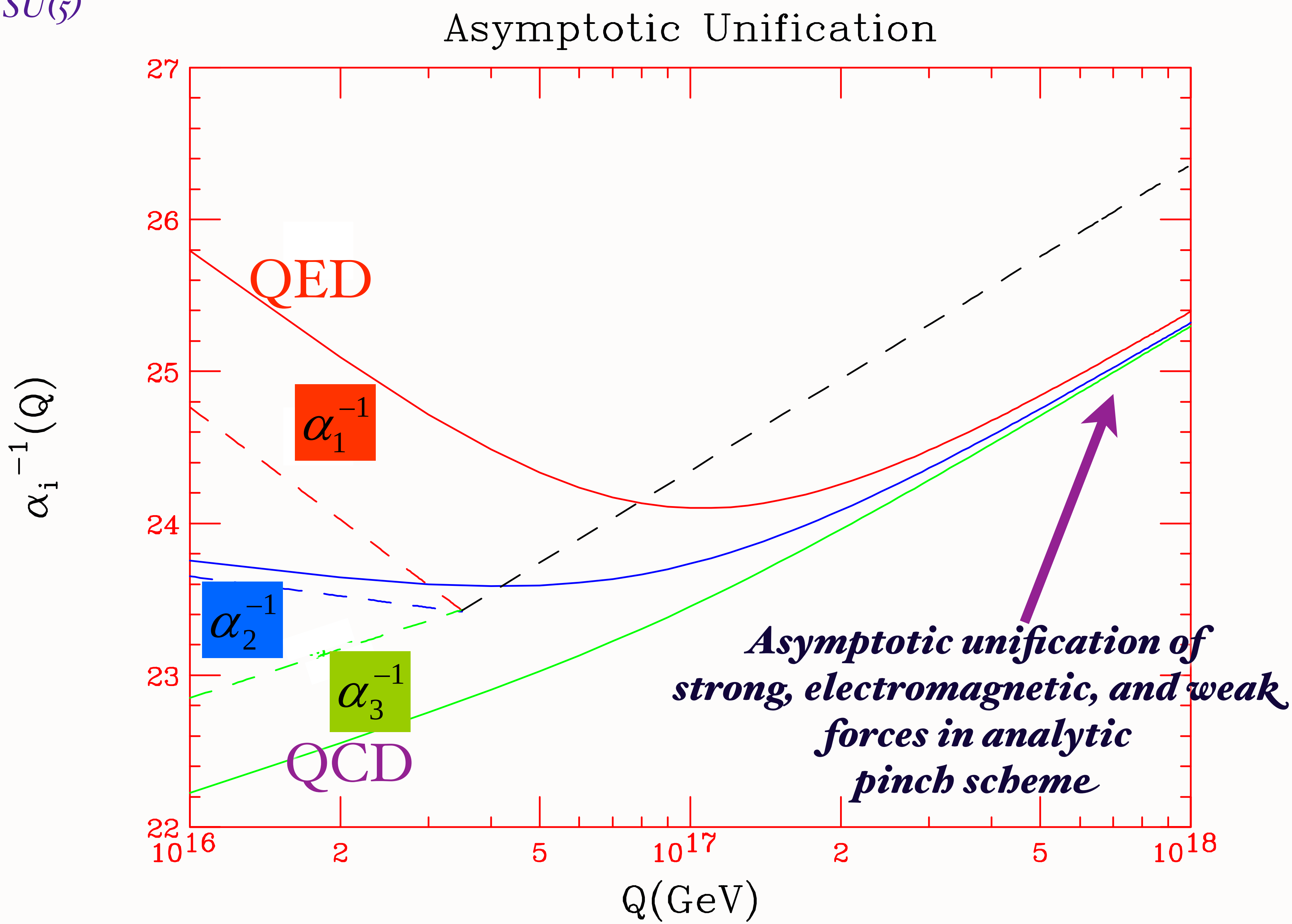
$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi}\frac{Q^2}{m^2}$$ $Q^2 << 4M^2$ **Serber-Uehling**

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}\frac{\log Q^2}{m^2}$$ $Q^2 >> 4M^2$ **Landau Pole**

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0$$

Supersymmetric SU(5)

Binger, sjb



QCD Lagrangian

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$\boxed{L_{QCD} = -\frac{1}{4g^2}\,\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{nf} i\,\overline{\psi_f}\,D_\mu\,\gamma^\mu\,\psi_f + \sum_{f=1}^{nf} m_f\,\overline{\psi_f}\,\psi_f}$$

QCD color charge — field strength tensor — covariant derivative — quark field

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Analytic limit of QCD: Abelian Gauge Theory

QCD $\longrightarrow$ **QED**

P. Huet, sjb

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

All analyses for Quantum Chromodynamics must be applicable to Quantum Electrodynamics

Given the elementary gauge theory interactions, all fundamental processes described in principle!

Example from QED:

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

$$\tfrac{1}{2}g_e = 1.001\ 159\ 652\ 201(30)$$ **QED prediction (Kinoshita, et al.)**

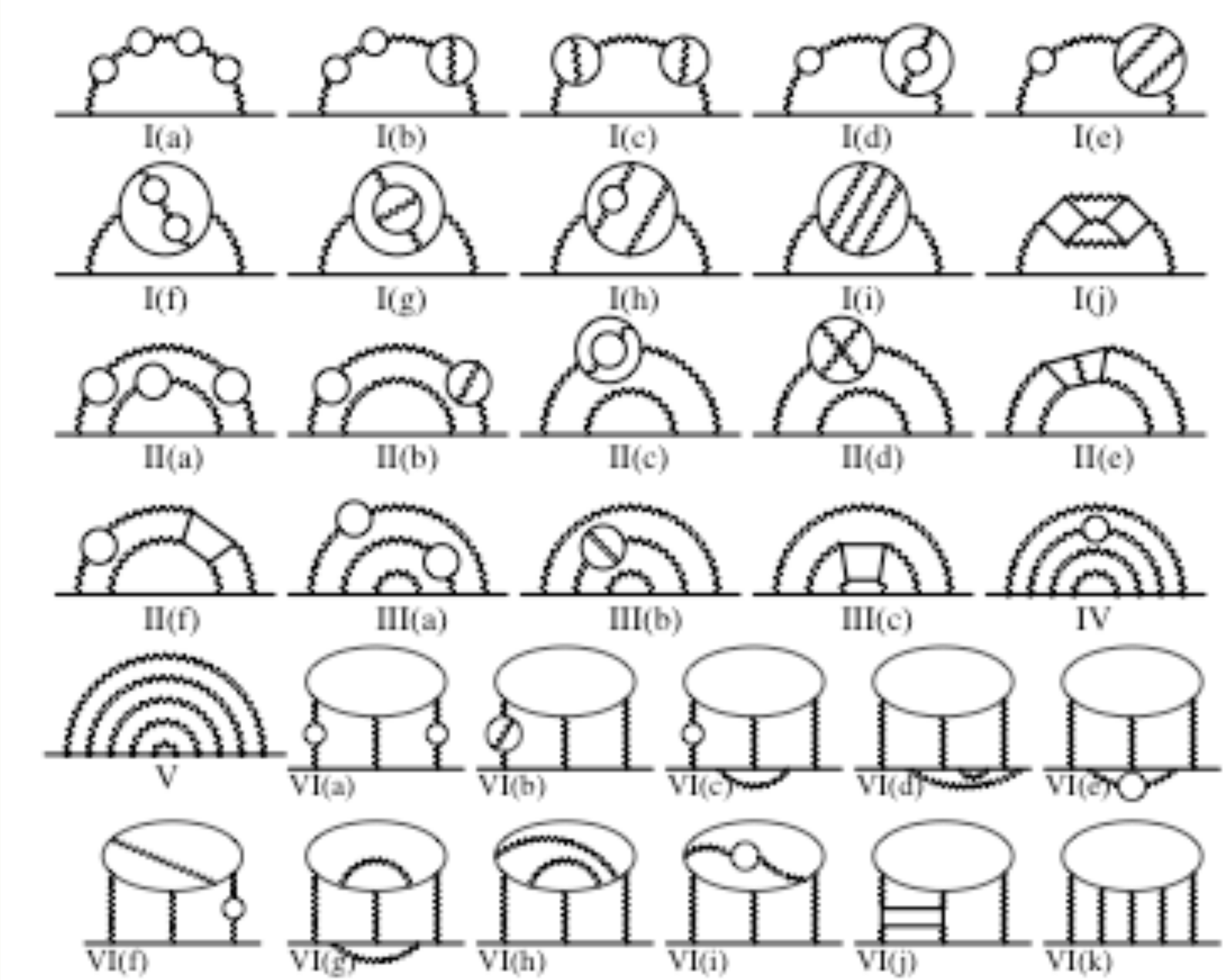
$$\tfrac{1}{2}g_e = 1.001\ 159\ 652\ 193(10)$$ **Measurement (Dehmelt, et al.)**

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 180\ 85\ [0.76\ ppt]$$

Dirac: $g_e \equiv 2$ **Measurement (Gabrielse, et al.)**

Tenth-Order QED Contribution to the Electron g-2 and an Improved Value of the Fine Structure Constant

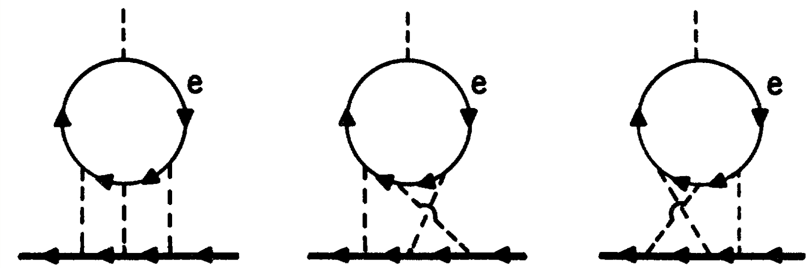
Tatsumi Aoyama, Masashi Hayakawa, Toichiro Kinoshita, and Makiko Nio



QED provides an asymptotic series relating g and α,

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$
$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

Light-by-Light Scattering Contribution to C_6



e

Aldins, Dufner, Kinoshita, sjb

$$\alpha^{-1} = 137.035999174(35)[0.25\text{ppb}]$$

Tenth-Order QED Contribution to the Electron g-2 and an Improved Value of the Fine Structure Constant

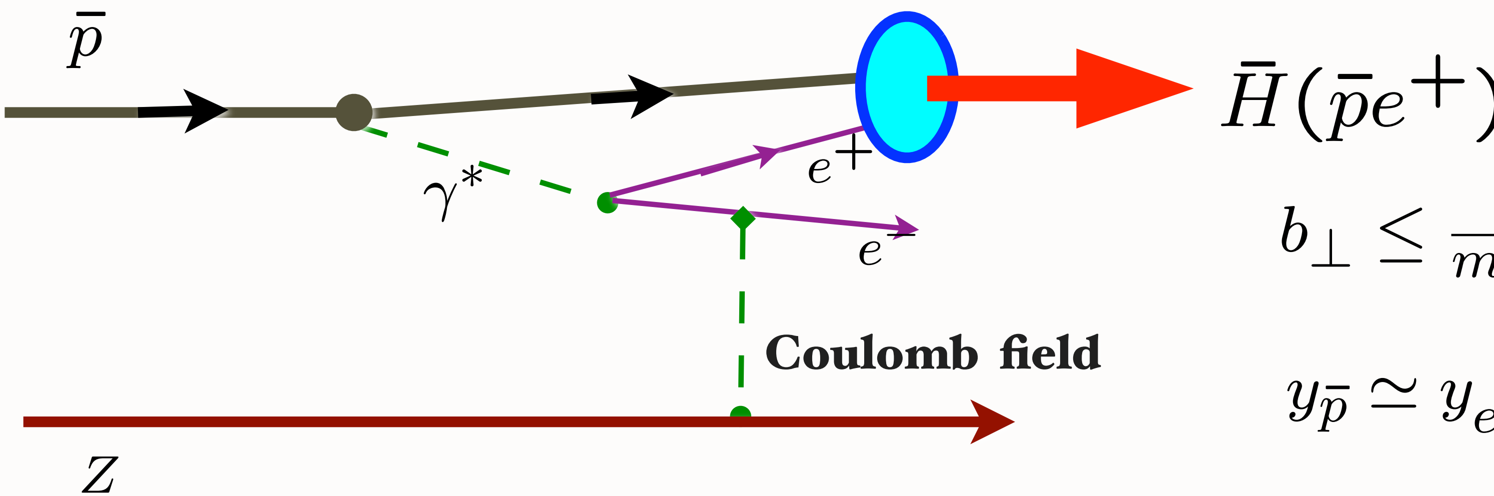
Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

$C_{10} \simeq 500$ **for muon g-2**

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

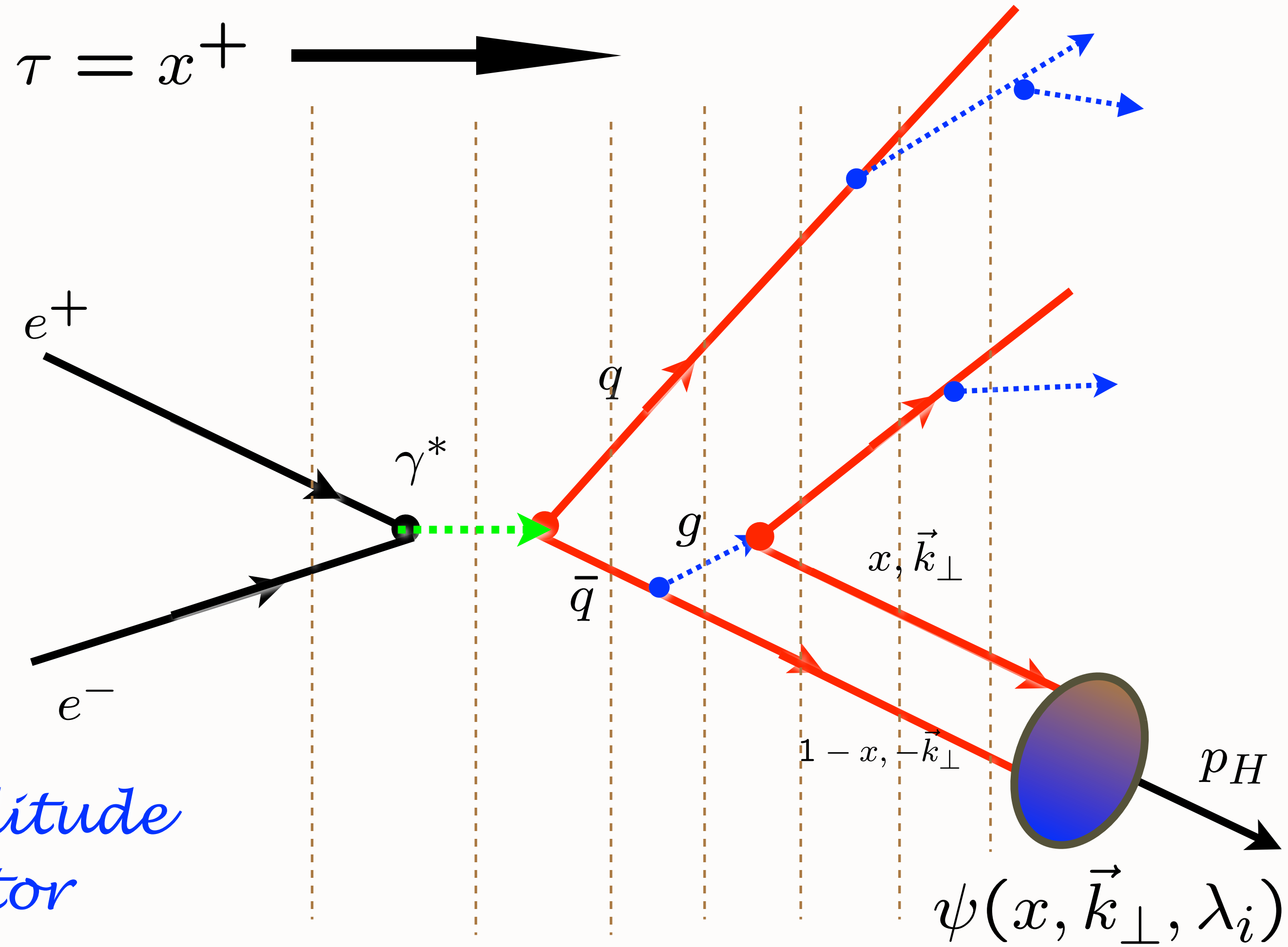
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Hadronization at the Amplitude Level



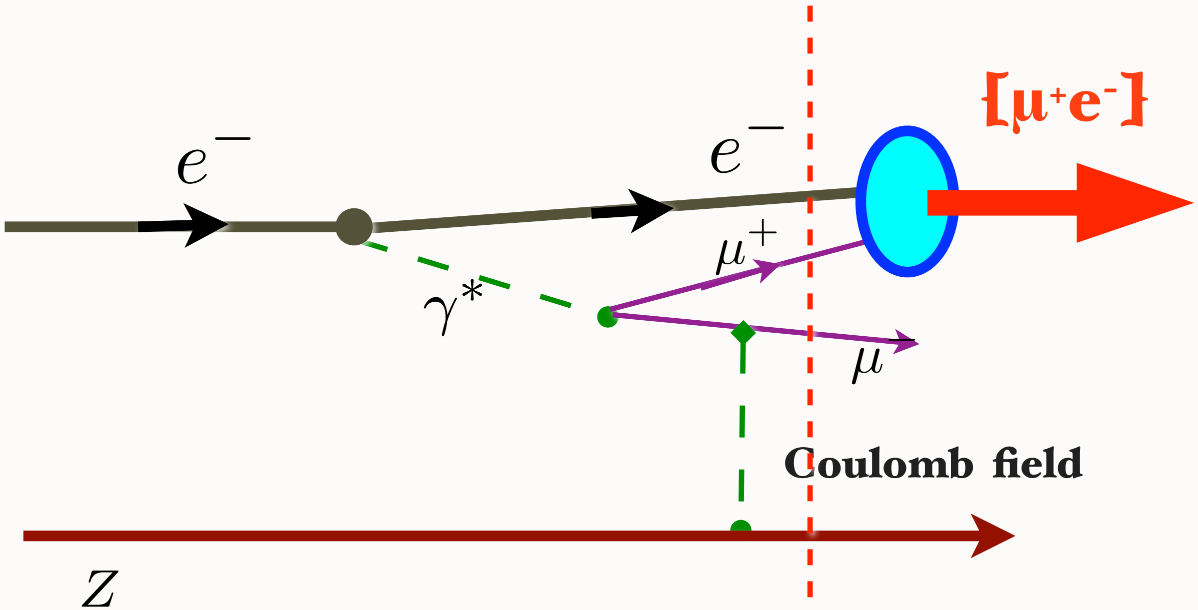
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Production of Relativistic Muonium $[\mu^+ e^-]$

- Never Observed Before?
- Measure Lamb Shift of Muonium by Robiscoe Method (Level Crossing by Induced Magnetic Field)
- Precision Tests of Time Dilation
- Dissociate to **muon and electron** with foils
- Flying Atoms

- **Production of Relativistic Muonium [μ+e-]**



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

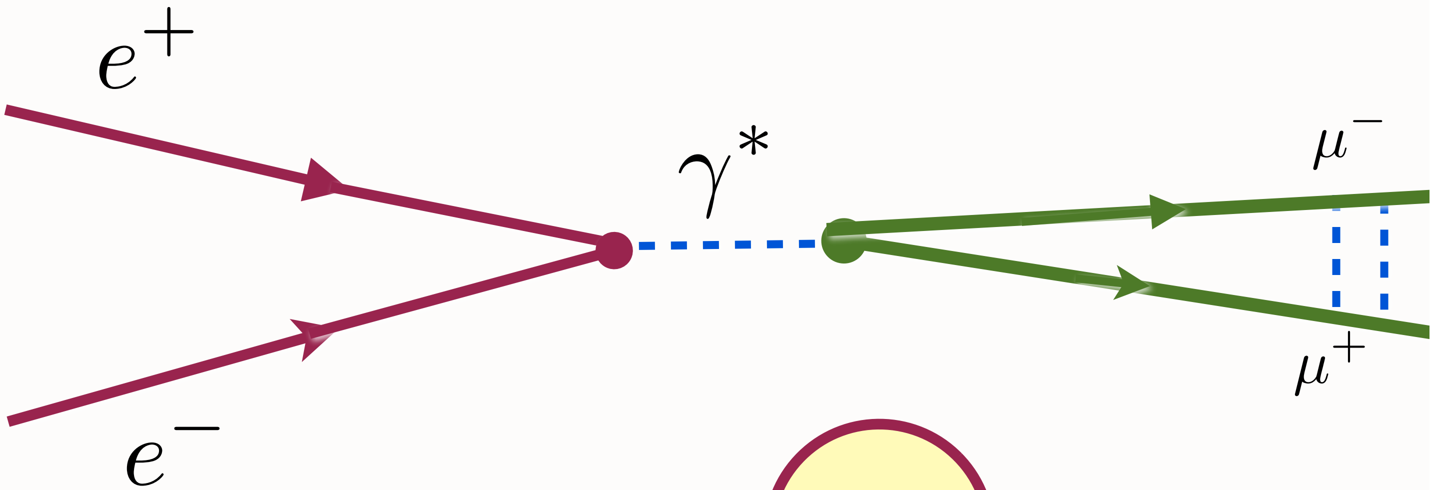
Coalescence of off-shell co-moving electron and muon

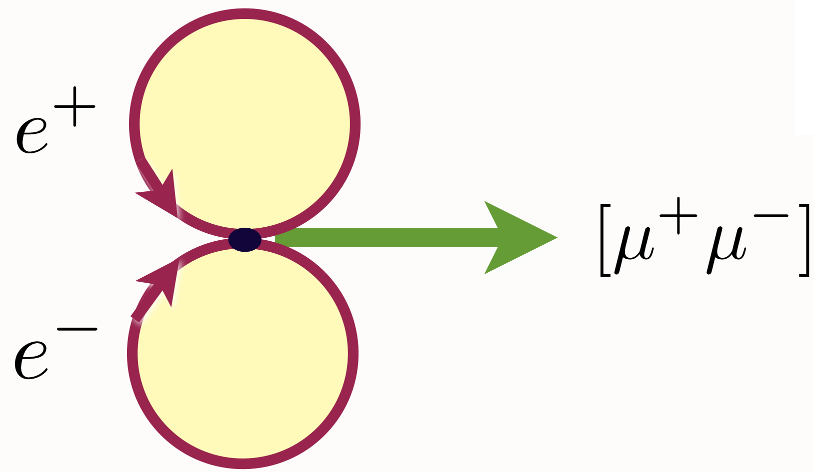
Wavefunction maximal at small impact separation and equal rapidity

"Atom Formation" at the Amplitude Level

True Muonium

Lebed, sjb





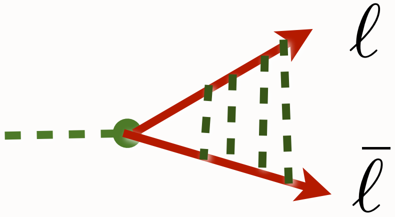
Electron-Positron Collider:

Bj: FISR (Fool's Intersecting Storage Ring) Frame

Coulomb Enhancement of Pair Production at Threshold

$$\sigma \longrightarrow \sigma S(\beta)$$

$$\beta = \sqrt{1 - \frac{4m_\ell^2}{s}}$$



$$X(\beta) = \frac{\pi\alpha\sqrt{1-\beta^2}}{\beta}$$

$$S(\beta) = \frac{X(\beta)}{1 - e^{-X(\beta)}}$$

Sommerfeld-Schwinger-Sakharov Effect

Bjorken: Analytical Connection to Rydberg Levels below Threshold

$$QCD : \pi\alpha \rightarrow \frac{4}{3}\alpha_s(\beta^2 s)$$

Kühn, Hoang, sjb

Production of True Muonium [$\mu^+\mu^-$]

PRL **102,** 213401 (2009) PHYSICAL REVIEW LETTERS week ending 29 MAY 2009

Production of the Smallest QED Atom: True Muonium ($\mu^+\mu^-$)

Stanley J. Brodsky*
SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA

Richard F. Lebed†
Department of Physics, Arizona State University, Tempe, Arizona 85287-1504, USA

(Received 22 April 2009; published 26 May 2009)

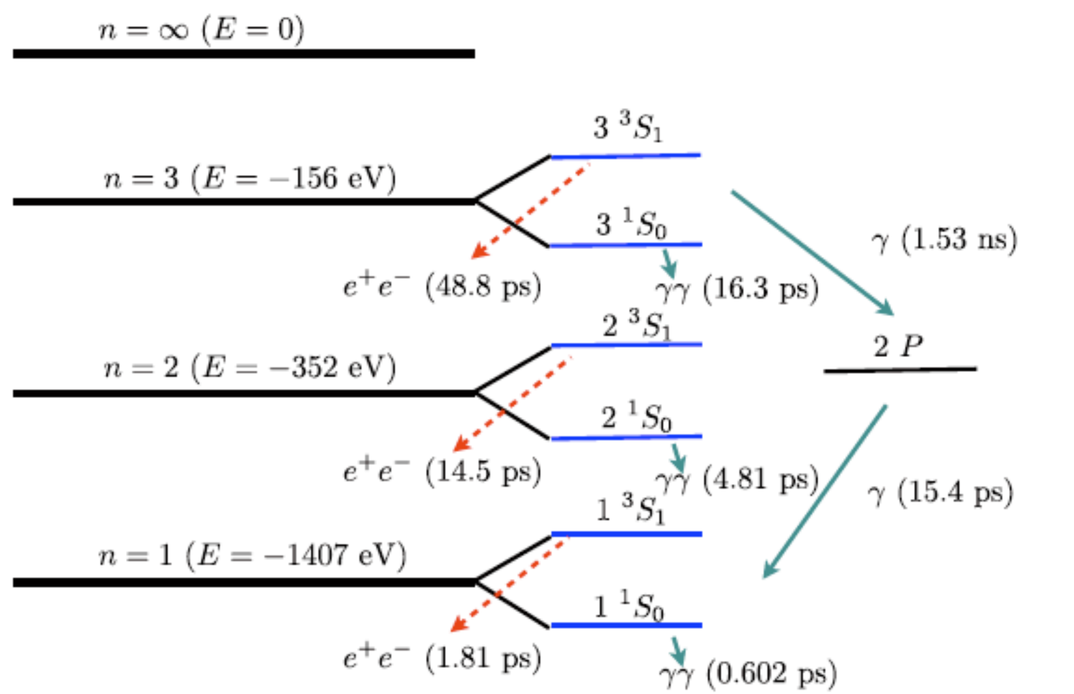
Rydberg Levels and Decays

$$\tau(n^3S_1 \to e^+e^-) = \frac{6\hbar n^3}{\alpha^5 mc^2}, \qquad \tau(n^1S_0 \to \gamma\gamma) = \frac{2\hbar n^3}{\alpha^5 mc^2},$$

$$\tau(2P \to 1S) = \left(\tfrac{3}{2}\right)^8 \frac{2\hbar}{\alpha^5 mc^2}, \qquad \tau(3S \to 2P) = \left(\tfrac{5}{2}\right)^9 \frac{4\hbar}{3\alpha^5 mc^2},$$

$$\frac{\tau(n^3S_1 \to e^+e^-)}{\tau(n^1S_0 \to \gamma\gamma)} = 3, \qquad \frac{\tau(2P \to 1S)}{\tau(n^1S_0 \to \gamma\gamma)} = \left(\tfrac{3}{2}\right)^8 \frac{1}{n^3} = \frac{25.6}{n^3},$$

$$\frac{\tau(3S \to 2P)}{\tau(2P \to 1S)} = \left(\tfrac{5}{3}\right)^9 = 99.2.$$

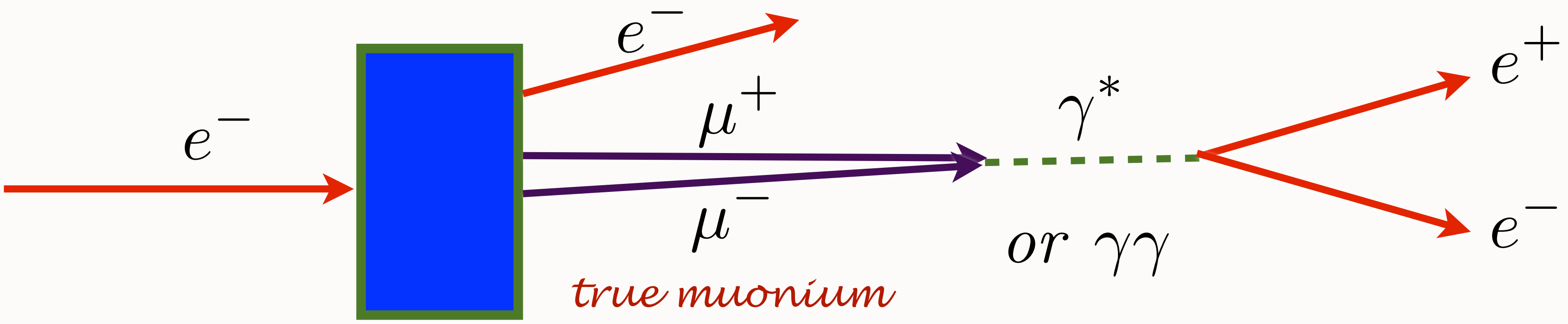


Production of bound triplet mu+ mu- system in collisions of electrons with atoms.
N. Arteaga-Romero, C. Carimalo, (Paris U., VI-VII) , V.G. Serbo, (Paris U., VI-VII & Novosibirsk State U.) . Jan 2000. 10pp.
Published in **Phys.Rev. A62:032501, 2000**.
e-Print: **hep-ph/0001278**

• Production of True Muonium [μ⁺μ⁻]

Use JLab Intense Electron Beam

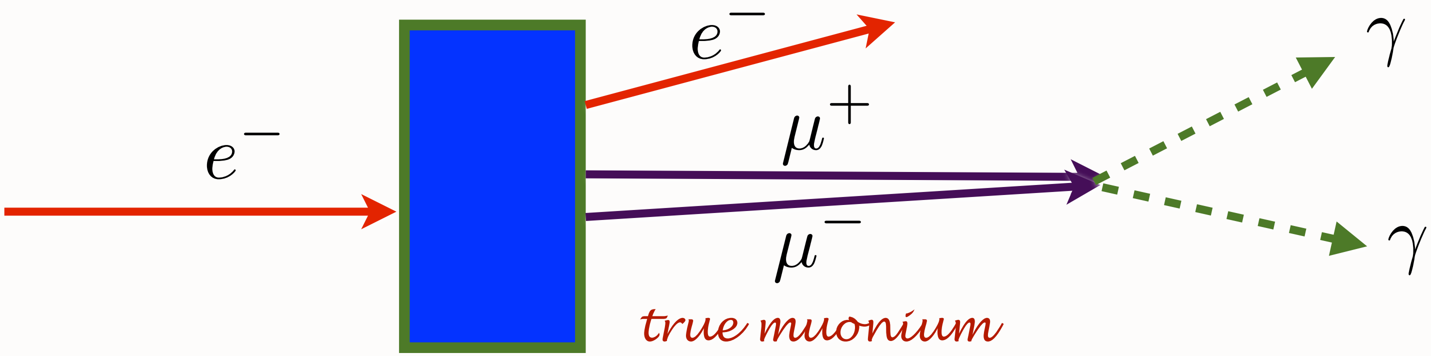
$$eZ \to eZ[\mu^+\mu^-]_{nS} \qquad q_{min} \simeq \frac{M^2_{\mu^+\mu^-}}{\nu} \sim 10 \text{ MeV}$$



- Produces all Rydberg Levels
- Analytic connection to continuum production -- enhanced by SSS at threshold
- Gap extends in cm multiplied by Lorentz boost
- Excite/De-excite levels with external fields, lasers

Production of True Muonium [μ⁺μ⁻]

$$eZ \to eZ[\mu^+\mu^-]_{nS} \qquad\qquad q_{min} \simeq \frac{M^2_{\mu^+\mu^-}}{\nu} \sim 10 \text{ MeV}$$



$C = +$

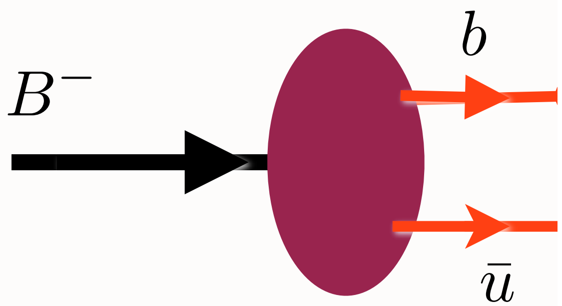
- Produces all Rydberg Levels
- Analytic connection to continuum production -- enhanced by SSS at threshold
- Gap extends in cm multiplied by Lorentz boost
- Excite/De-excite levels with external fields, lasers
- Measure Lamb Shift!

Novel Lepton Physics Studies in electron-nucleus reactions

Use JLab Intense Electron Beam

- **Production of True Muonium $[\mu^+\mu^-]$**
- **Production of Relativistic Muonium $[\mu^+e^-]$**
- **Test All-Orders Bethe-Maximon Formula for Pair Production**
- **Lepton Charge Asymmetry**
- **Test Landau-Pomeranchuk-Migdal (LPM)**

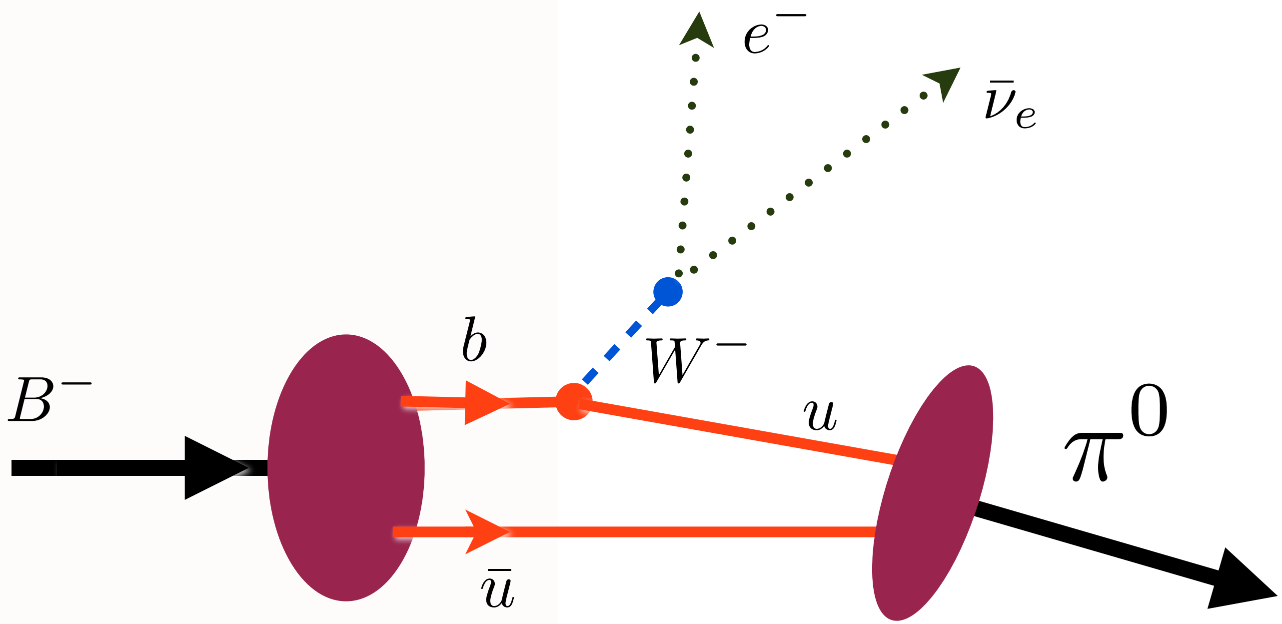
Exclusive B decay



$$\vec{p}_{\pi^0} = \vec{p}_{B^-} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{e^-}$$

Decay of the B meson to the pion plus electron and neutrino

Exclusive B decay

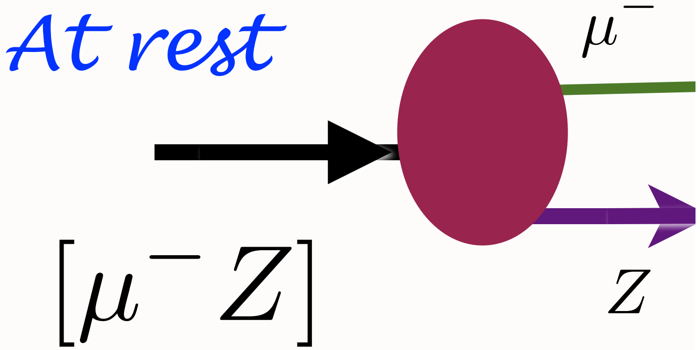


$$\vec{p}_{\pi^0} = \vec{p}_{B^-} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{e^-}$$

Decay of the B meson to the pion plus electron and neutrino

Atomic Alchemy

Greub, Munger, Wyler, sjb



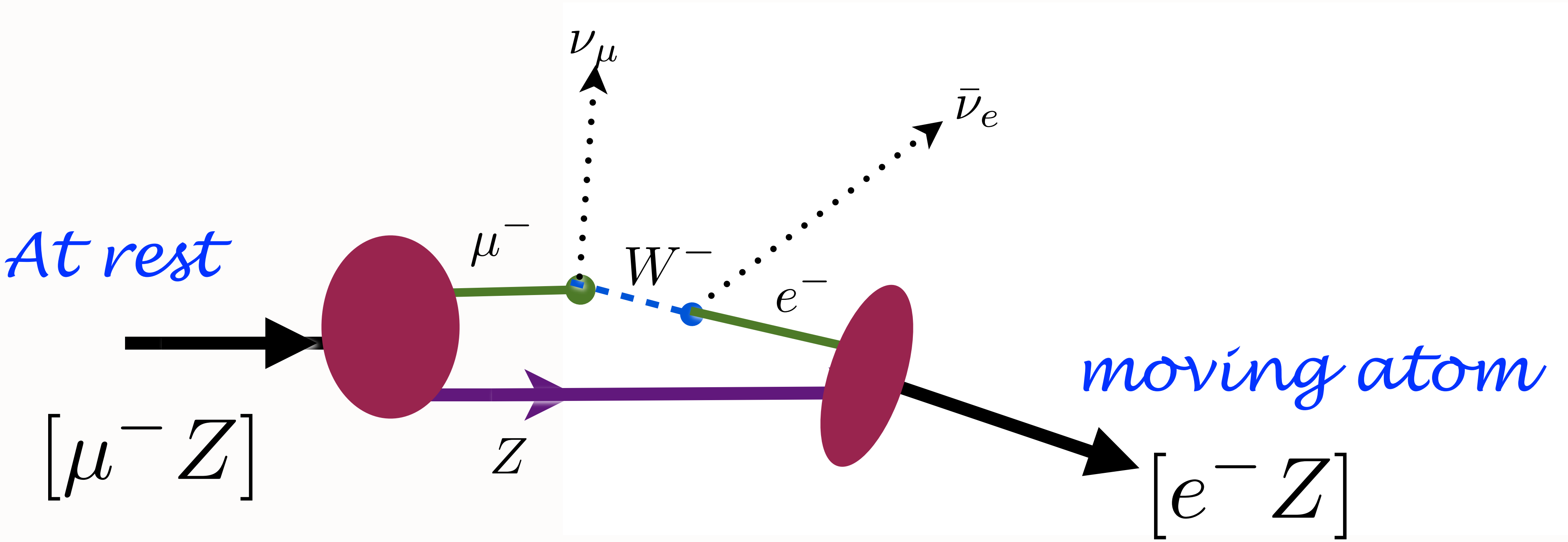
$$\vec{p}_{[e^- Z]} = \vec{p}_{[\mu^- Z]} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu} = -\vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu}$$

Decay of a muonic atom to a moving electronic atom plus two neutrinos

Measures very high momentum tail of atomic wavefunction

Atomic Alchemy

Greub, Munger, Wyler, sjb



$$\vec{p}_{[e^- Z]} = \vec{p}_{[\mu^- Z]} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu} = -\vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu}$$

Decay of a muonic atom to a moving electronic atom plus two neutrinos

Measures very high momentum tail of atomic wavefunction

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

C. Greub and D. Wyler
Institut für Theoretische Physik, Universität Zürich, Zürich, Switzerland

S. J. Brodsky and C. T. Munger
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 9 May 1994)

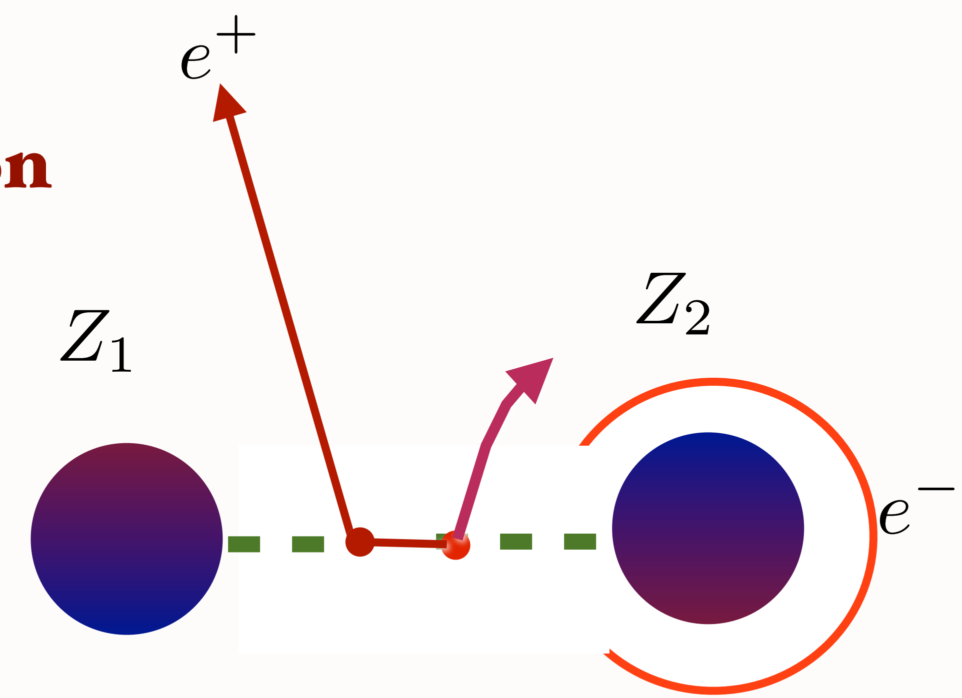
The rates of weak transitions between electromagnetic bound states, for example, $(\pi^+e^-) \to (\mu^+e^-)\nu_\mu$, and the exclusive weak decay of a muonic atom into an electronic atom, $(Z\mu^-) \to (Ze^-)\nu_\mu\bar{\nu}_e$, are calculated. For $Z = 80$, relativistic effects are shown to increase the latter rate by a factor of 50 compared to the results of a nonrelativistic calculation. It is argued that the conditions for producing the muonic decay in neon gas ($Z = 10$), where the branching ratio for the decay per captured muon is 1.7×10^{-9}, can be realized using cyclotron traps, though the prospect for a practical experiment seems remote. In lead the same ratio would be approximately $\sim 1 \times 10^{-6}$. In addition to providing detailed information on the high momentum tail of the wave functions in atomic physics, these decays of QED bound states provide a simple toy model for investigating kinematically analogous situations in exclusive heavy hadronic decays in quantum chromodynamics, such as $B \to K^*\gamma$ or $B \to \pi e\nu$.

PACS number(s): 36.10.Dr, 11.10.St, 13.20.He, 13.35.−r

Spontaneous Positron Production in Strong-Field QED

Adiabatic Collision

Greiner, Peiper
Rafelski
Popov, Marinov



$$Z_1 + Z_2 > 172$$

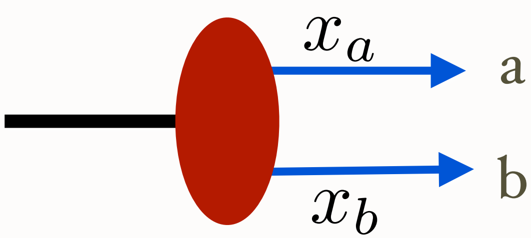
Gribov: Model for Quark Confinement

Many Analogs: QED/QCD

- Diffractive Dissociation of Atoms/Hadrons
- Atomic/Color Transparency
- Light-Front Wavefunctions
- Atomic Alchemy/B decay
- Atom Formation/Hadronization
- Spontaneous pair production/ Confinement
- Intrinsic heavy leptons/Intrinsic Charm
- True Muonium/Quarkonium

Bethe-Salpeter Equation for Hydrogenic Atoms

$$(\not{p}_a - m_a)(\not{p}_b - m_b)|N >= G|N >$$



$$p_a^\mu + p_b^\mu = P^\mu = (E_N, \vec{P})$$

an eigenvalue problem for $P^0 = E_N = \sqrt{M_N^2 + \vec{P}^2}$

$$(i\not{\partial}_a - m_a)(i\not{\partial}_b - m_b)\chi_N(x_a, x_b) = (G\chi_N)(x_a, x_b)$$

In momentum space: $P = p_a + p_b$ $\quad p = \tau_b p_a - \tau_a p_b$

$$\boxed{\begin{aligned}&[\gamma^{(a)} \cdot (\tau_a P + p) - m_a][\gamma^{(b)} \cdot (\tau_b P - p) - m_b]\Psi_N(p, P) \\ &\qquad = \int d^4p' G(p, p'; P)\Psi_N(p', P)\end{aligned}}$$

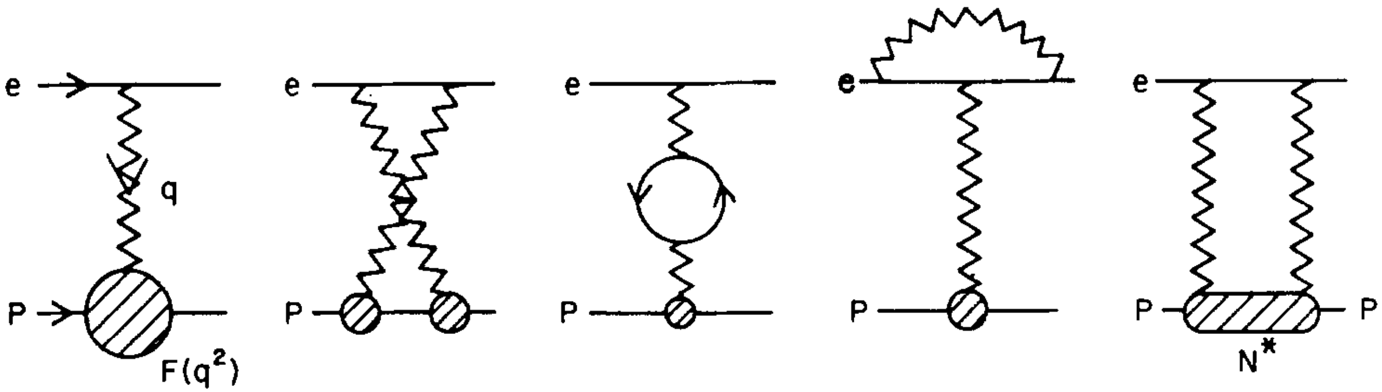
$$\tau_a = \frac{m_a}{m_a + m_b} \qquad \tau_b = \frac{m_b}{m_a + m_b}$$

Bethe-Salpeter Theory of Hydrogenic Atoms

Bethe–Salpeter Equation

$$(\not{p}_e - m_e)(\not{p}_p - m_p)\,\chi = G\chi$$

$$G = G_{1\gamma} \quad + \quad G_{\text{CROSSED}} \quad + \quad G_{\text{VAC.POL.}} \quad + \quad G_{\text{SELF ENERGY}} \quad + \quad G_{\text{NUC-POL}} + \cdots$$



$$G_{1\gamma} = G_{\text{COULOMB}} + G_{\text{TRANSVERSE}}$$

$$-\epsilon_\mu \frac{1}{q^2}\epsilon^\mu = \epsilon_0 \frac{1}{q^2}\epsilon_0 + \sum_{\substack{\text{TRAN}\\ i=1,2}} \epsilon_i \frac{1}{q^2}\epsilon_i$$

G_{COULOMB} $\rightarrow$ Schrödinger equation, proton finite size correction

$+\ G_{\text{TRANS}}$ $\rightarrow$ reduced mass corrections, HFS splittings

$+\ G^{(\text{all})}_{\text{CROSSED}}$ $\rightarrow$ Dirac equation, relativistic reduced mass correction

$+\ G_{\text{VAC-POL}} + G_{\text{SELF ENERGY}}$ $\rightarrow$ Lamb shift, radiative corrections to HFS

$+\ G_{\text{NUC-POL}}$ $\rightarrow$ correction to HFS

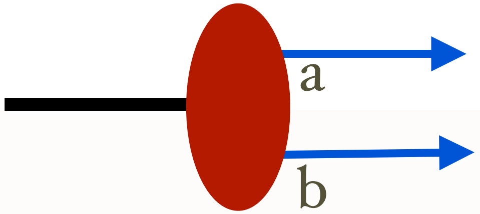
Features of Bethe-Salpeter Equation

- **Exact Bound-State Formalism for QED if one includes all 2PI kernels**
- **Eigenvalues give complete spectrum, bound state and continuum**
- **Relativistic, Frame Independent**
- **Feynman virtualities:** $p_i^2 \neq m_i^2$
- **Reduces to Dirac Coulomb Equation if one includes all crossed graph 2PI kernels**
- **Matrix Elements of electromagnetic current from sum of all 2PI contributions**
- **Normalization of Bethe-Salpeter Wavefunctions also requires sum of all 2PI kernels**
- **n-body formulation difficult**
- **No cluster decomposition theorem**

Solution to Salpeter Equation in CM frame

Total spin S,
Projection S^z = M

$$\varphi_{\mathscr{M}}(\mathbf{x}_a, \mathbf{x}_b, X^0)_{SM} = \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a{}^0 + m_a}{2p_a{}^0} \frac{p_b{}^0 + m_b}{2p_b{}^0}\right)^{1/2} \left(\begin{array}{c} 1 \\ \dfrac{\boldsymbol{\sigma}_a \cdot \mathbf{p}}{2m_a + k_a} \end{array}\right) \otimes \left(\begin{array}{c} 1 \\ -\dfrac{\boldsymbol{\sigma}_b \cdot \mathbf{p}}{2m_b + k_b} \end{array}\right) \times \phi_{\mathscr{M}}(\mathbf{p})\, \chi_{SM} e^{i\mathbf{p}\cdot\mathbf{x} - i\mathscr{M}X^0}$$



$$k_{a,b} \equiv -\tau_{b,a}(U + W)$$

$$\int d^3x_a\, d^3x_b\, \varphi_{\mathscr{M}}(\mathbf{x}_a, \mathbf{x}_b)^\dagger (\Lambda_{++} - \Lambda_{--})\, \varphi_{\mathscr{M}}(\mathbf{x}_a, \mathbf{x}_b) = 1$$

$$\int d^3p\, |\phi_{\mathscr{M}}(\mathbf{p})|^2 = 1.$$

$$\chi_{\mathscr{M}}^{\alpha\beta}(x_a, x_b)_{SM} = \langle 0 \,|\, T(\psi_a{}^\alpha(x_a)\, \psi_b{}^\beta(x_b)) \,|\, 0\mathscr{M}SM \rangle$$

Lorentz Boost

$$\Phi_{\mathcal{M}}(x_a, x_b)_{SM} =< 0|T(\psi(x_a)\psi_b(x_b)|\vec{P} = \vec{0}, \mathcal{M}, S, M >$$

$$\Phi_{E,\vec{P}}(x'_a, x'_b)_{SM} = S_a(\Lambda)S_b(\Lambda)\Phi_{\mathcal{M}}(x_a, x_b)_{SM}$$

$$S_a(\Lambda) = \sqrt{\frac{E+\mathcal{M}}{2\mathcal{M}}}\left(1 + \frac{\vec{\alpha}_a \cdot \vec{P}}{\mathcal{M}+E}\right)$$

$$S_a(\Lambda)u(0) = u(p) = \sqrt{\frac{p^0+m}{2m}}\left(\begin{array}{c} 1 \\ \frac{\sigma \cdot p}{p^0+m} \end{array}\right)\chi$$

Single particle wave-packet

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}}\, u(p)\, \phi(p)\, e^{-ip.x}$$

$$u(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \chi.$$

Guess wavefunction for moving bound state

Wrong!!

$$\varphi_{E\mathbf{P}}(\mathbf{x}_a \quad \mathbf{x}_b\,,\, X^0)_{SM}$$

$$= \frac{E+\mathcal{M}}{2\mathcal{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a{}^0+m_a}{2p_a{}^0}\,\frac{p_b{}^0+m_b}{2p_b{}^0}\right)^{1/2}$$

$$\times \begin{pmatrix} 1 \\ \boldsymbol{\sigma}_a \cdot \left(\dfrac{\mathbf{P}}{\mathcal{M}+E} + \dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{pmatrix} \otimes \begin{pmatrix} 1 \\ \boldsymbol{\sigma}_b \cdot \left(\dfrac{\mathbf{P}}{\mathcal{M}+E} - \dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{pmatrix}$$

$$\times\ \phi_{\mathcal{M}}(\mathbf{p})\, \chi_{SM} \exp[i\mathbf{p}\cdot\tilde{\mathbf{x}} + i\mathbf{P}\cdot\mathbf{X}] \exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma-1)\,\hat{\mathbf{V}}\hat{\mathbf{V}}\cdot\mathbf{x} \qquad p^0_{a,b} = \sqrt{\mathbf{p}^2 + m^2_{a,b}} \qquad k_{a,b} \equiv -\tau_{b,a}(U+W)$$

Single particle wave-packet

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}}\, u(p)\, \phi(p)\, e^{-ip.x}$$

$$u(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \chi.$$

Correct wavefunction for moving bound state

Not product of independent boosts!

$$\varphi_{E\mathbf{P}}(\mathbf{x}_a \quad \mathbf{x}_b\,,\, X^0)_{SM}$$

$$= \frac{E+\mathscr{M}}{2\mathscr{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a{}^0+m_a}{2p_a{}^0}\,\frac{p_b{}^0+m_b}{2p_b{}^0}\right)^{1/2}$$

$$\times \begin{pmatrix} 1 + \dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathscr{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathscr{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{pmatrix} \otimes \begin{pmatrix} 1 - \dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathscr{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathscr{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{pmatrix}$$

$$\times\ \phi_{\mathscr{M}}(\mathbf{p})\,\chi_{SM}\exp[i\mathbf{p}\cdot\tilde{\mathbf{x}}+i\mathbf{P}\cdot\mathbf{X}]\exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma-1)\,\hat{\mathbf{V}}\hat{\mathbf{V}}\cdot\mathbf{x} \qquad p^0_{a,b} = \sqrt{\mathbf{p}^2+m^2_{a,b}} \qquad k_{a,b} \equiv -\tau_{b,a}(U+W)$$

Primack, sjb

Correct reduction of electromagnetic interaction in nonrelativistic limit

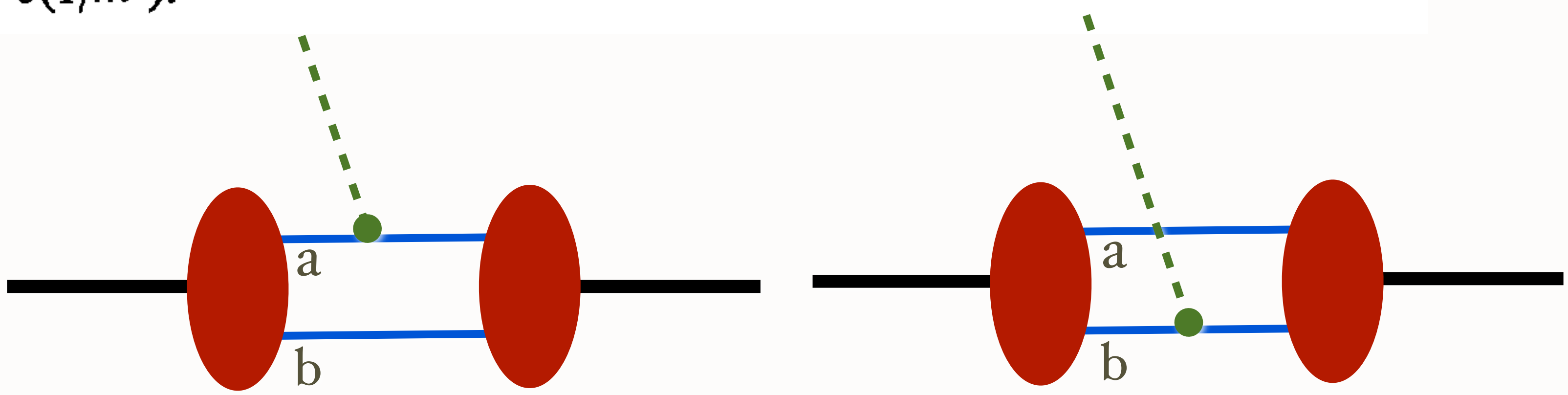
$$H_{\mathrm{NR}}^{\mathrm{em}} = \sum_{s=a,b} \Big[\frac{-\mathbf{p}_s \cdot e_s \mathbf{A}_s}{m_s} + \frac{e_s{}^2 \mathbf{A}_s{}^2}{2m_s} + e_s A_s{}^0 - \mu_s \boldsymbol{\sigma}_s \cdot \mathbf{B}_s$$

$$- \left(2\mu_s - \frac{e_s}{2m_s}\right) \boldsymbol{\sigma}_s \cdot \mathbf{E}_s \times \frac{(\mathbf{p}_s - e_s \mathbf{A}_s)}{2m_s} \Big]$$

Correction to Foldy-Wouthyusen!

$$+ \frac{1}{4M_T} \left(\frac{\boldsymbol{\sigma}_a}{m_a} - \frac{\boldsymbol{\sigma}_b}{m_b} \right) \cdot \left(e_b \mathbf{E}_b \times (\mathbf{p}_a - e_a \mathbf{A}_a) - e_a \mathbf{E}_a \times (\mathbf{p}_b - e_b \mathbf{A}_b) \right)$$

$$+ 0(1/m^3).$$



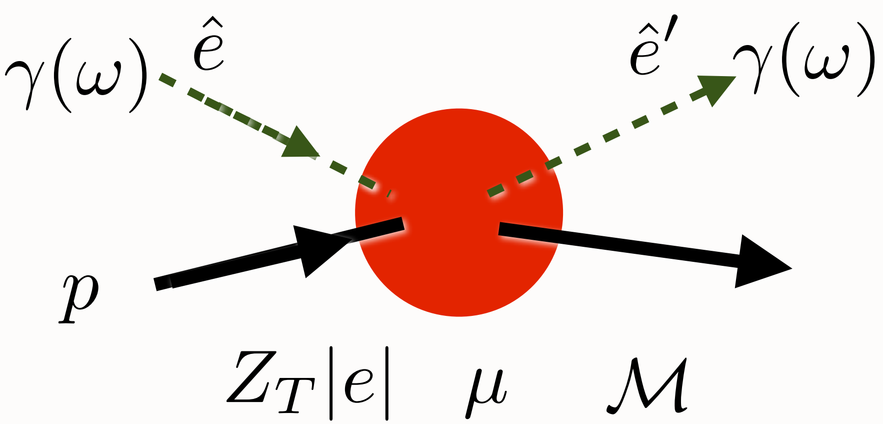
Bound state of two spin-1/2 particles

Low Energy Forward Compton Scattering

Low energy theorem: Spin-1/2 Target

$$S_{fi} = -2\pi i \delta(E_f - E_i)\, M_{fi}$$

$$M_{fi} = \frac{1}{2\omega}(2\pi)^3\,\delta^3(P_f - P_i)\left[\frac{{Z_T}^2 e^2}{\mathcal{M}}\,\hat{\mathbf{e}}'\cdot\hat{\mathbf{e}}\,\delta_{fi} + 2i\omega\left(\mu - \frac{Z_T e}{2\mathcal{M}}\right)^2 \boldsymbol{\sigma}_{fi}\cdot\hat{\mathbf{e}}'\times\hat{\mathbf{e}} + O(\omega^2)\right]$$



Amplitude determined by static properties of target

$$k \cdot p = \omega \mathcal{M}$$

Photon lab energy $\omega \to 0, \theta \to 0$

Drell Hearn Gerasimov Sum Rule

$$\int_{\omega_{\rm th}}^{\infty} \frac{\sigma_P(\omega) - \sigma_A(\omega)}{\omega} d\omega = 8\pi^2 (\mu - \frac{Z_T e}{2\mathcal{M}})^2$$

anomalous magnetic
moment squared

Proof

Optical Theorem from Unitarity

Forward spin-flip amplitude given by LET

Unsubtracted dispersion relation

$$M_{fi} = \frac{1}{2\omega}(2\pi)^3\,\delta^3(P_f - P_i)\left[\frac{Z_T{}^2 e^2}{\mathcal{M}}\hat{\mathbf{e}}'\cdot\hat{\mathbf{e}}\delta_{fi} + 2i\omega\left(\mu - \frac{Z_T e}{2\mathcal{M}}\right)^2 \boldsymbol{\sigma}_{fi}\cdot\hat{\mathbf{e}}'\times\hat{\mathbf{e}} + O(\omega^2)\right]$$

$$M^{\uparrow\rightarrow\downarrow}(\theta = 0)$$

Boost of a Composite System

- Boost is not product of independent boosts of constituents since constituents are already moving

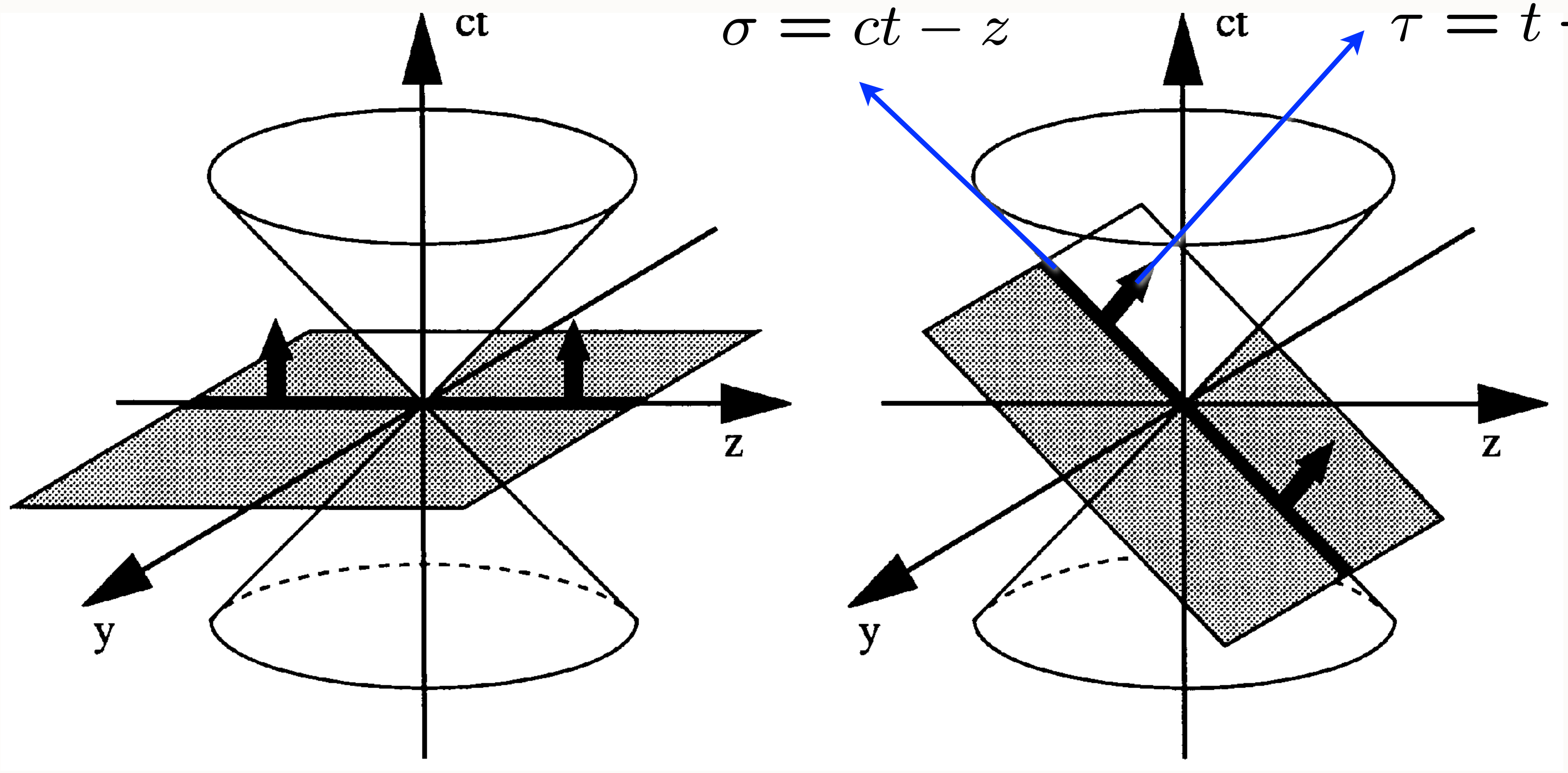
- Only known at weak binding

- Dirac: Boosts are dynamical

- Correct form needed to prove Low Energy Theorem for Compton scattering and Drell-Hearn Gerasimov Sum Rule

Dirac's Amazing Idea: The "Front Form"

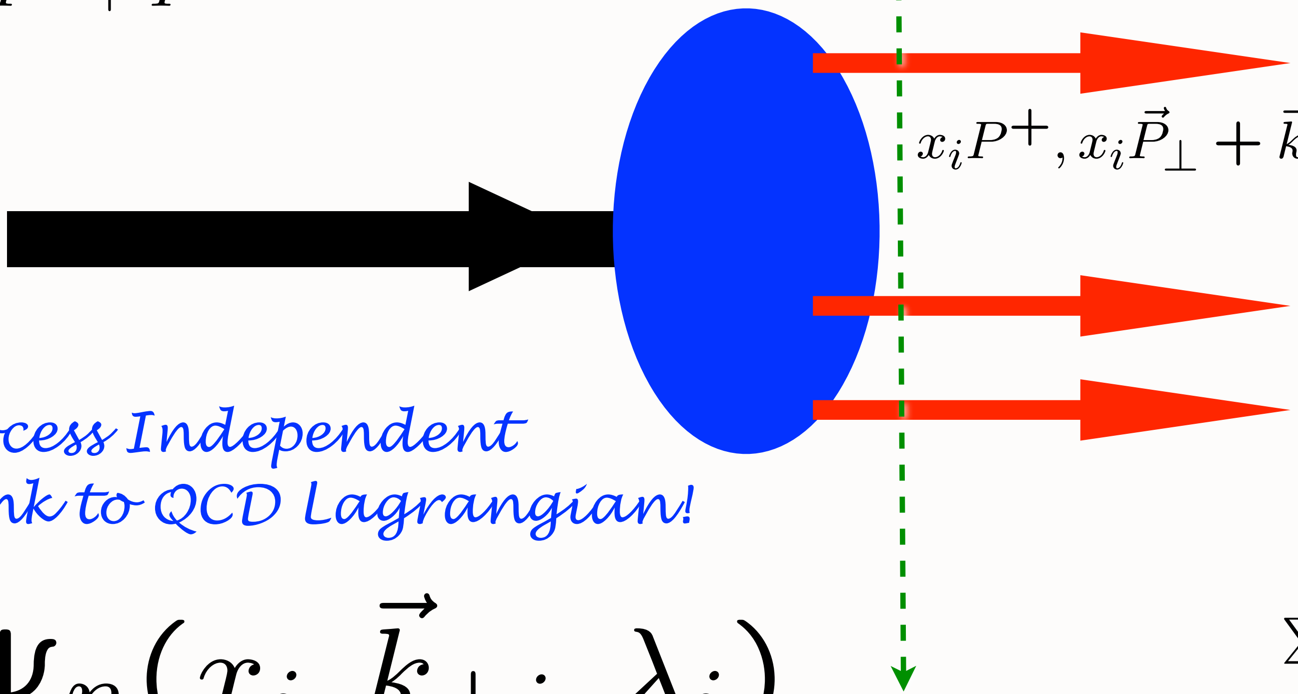
Evolve in light-front time



Instant Form *Front Form*

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Process Independent
Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}^2_\perp$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front QCD

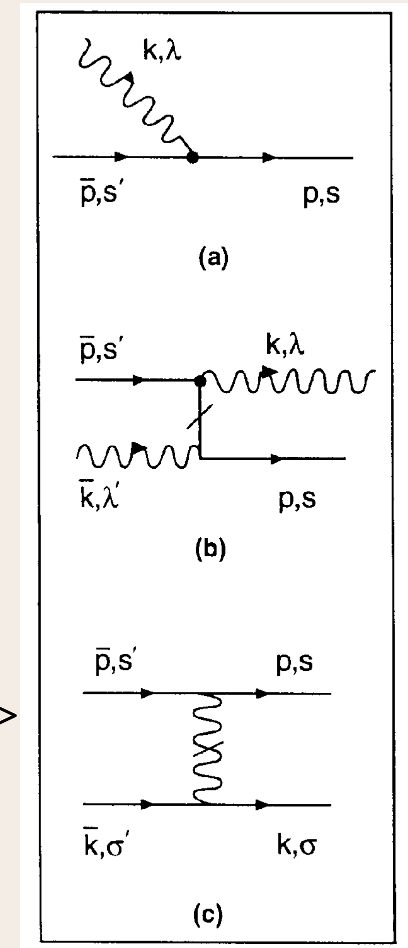
Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \to H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

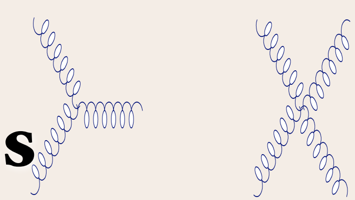


$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass



H_{LF}^{int}

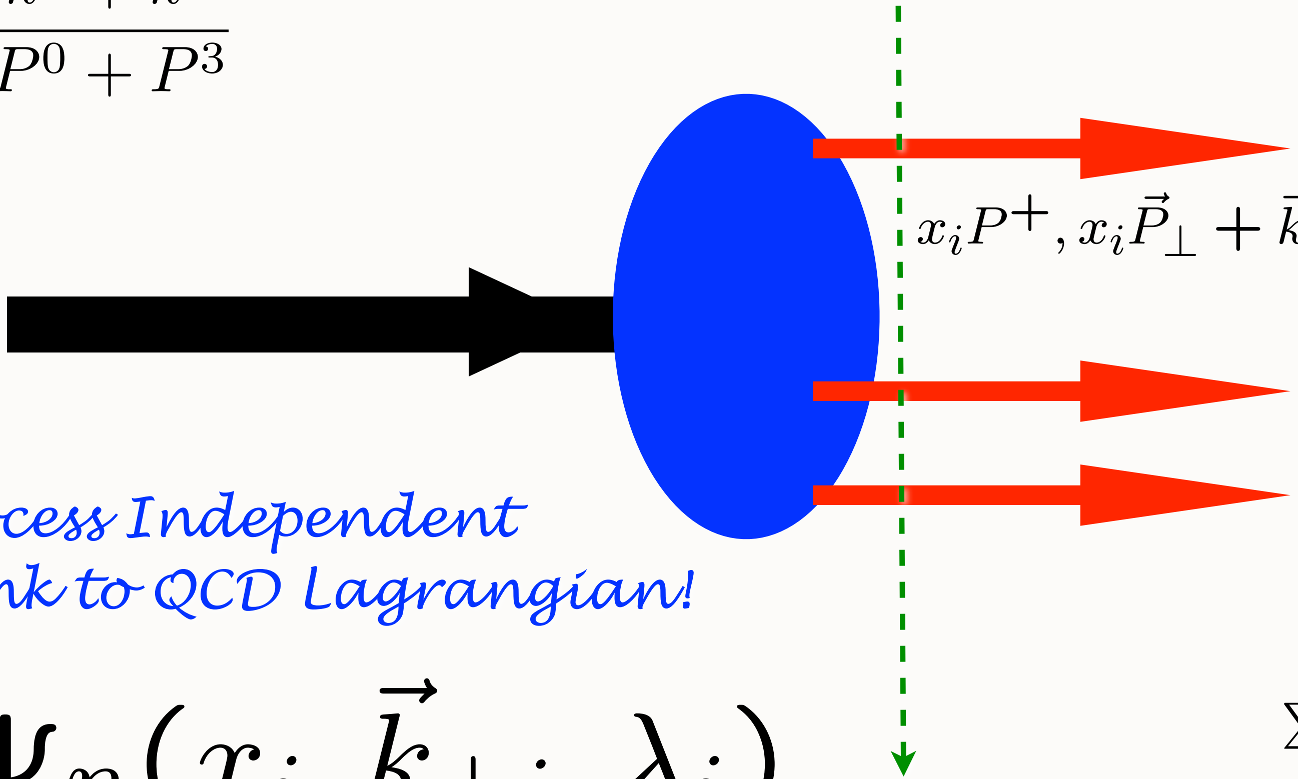
'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing

Montgomery

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Process Independent

Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State!

LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

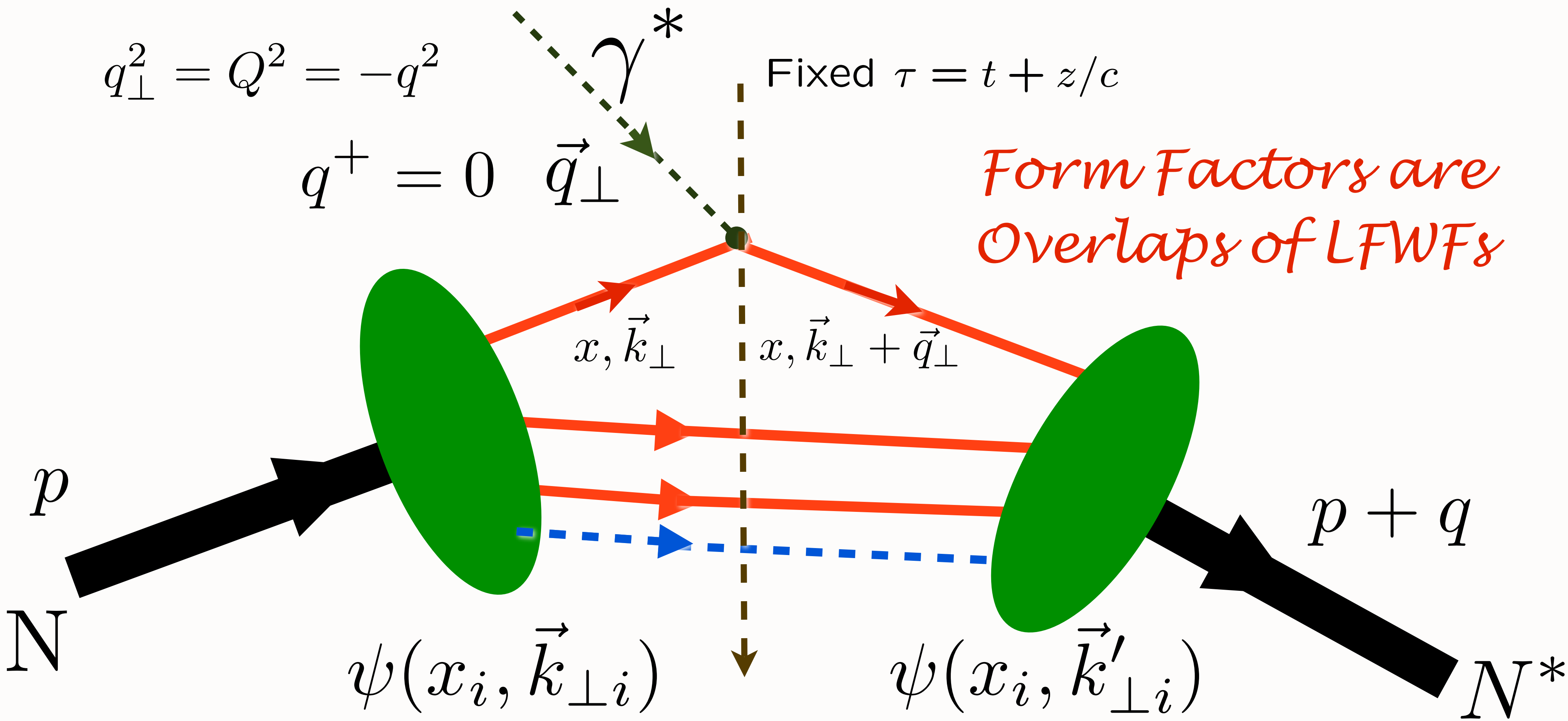
Orbital angular momentum is a property of Light-Front Wavefunctions

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1-x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula
Drell, sjb

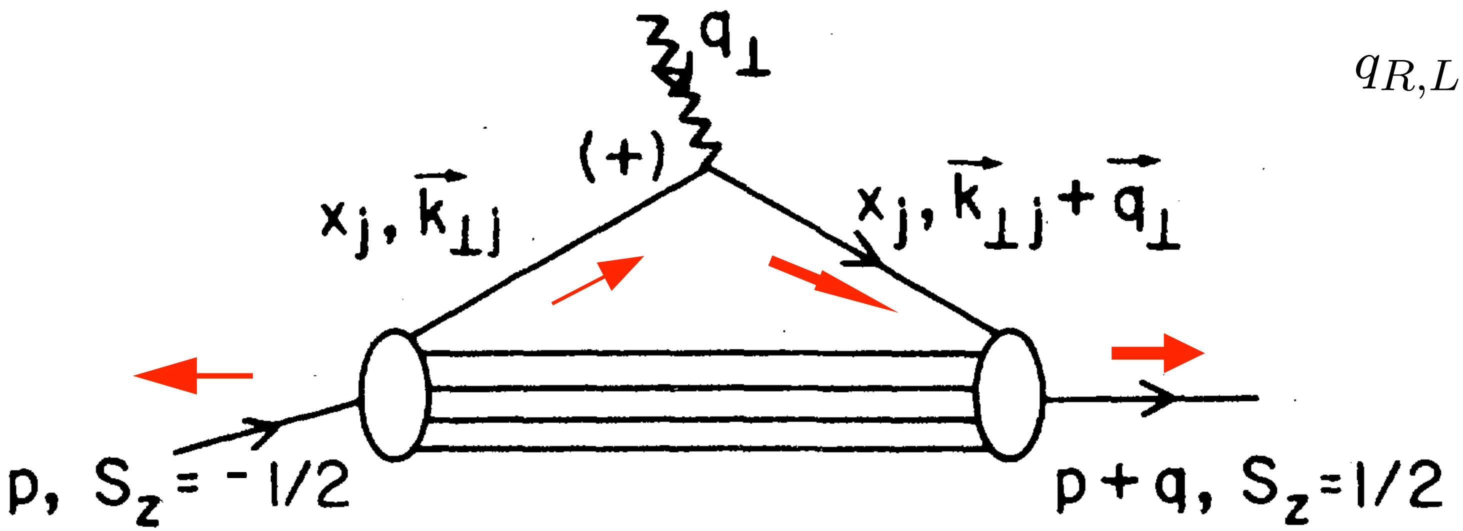
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$

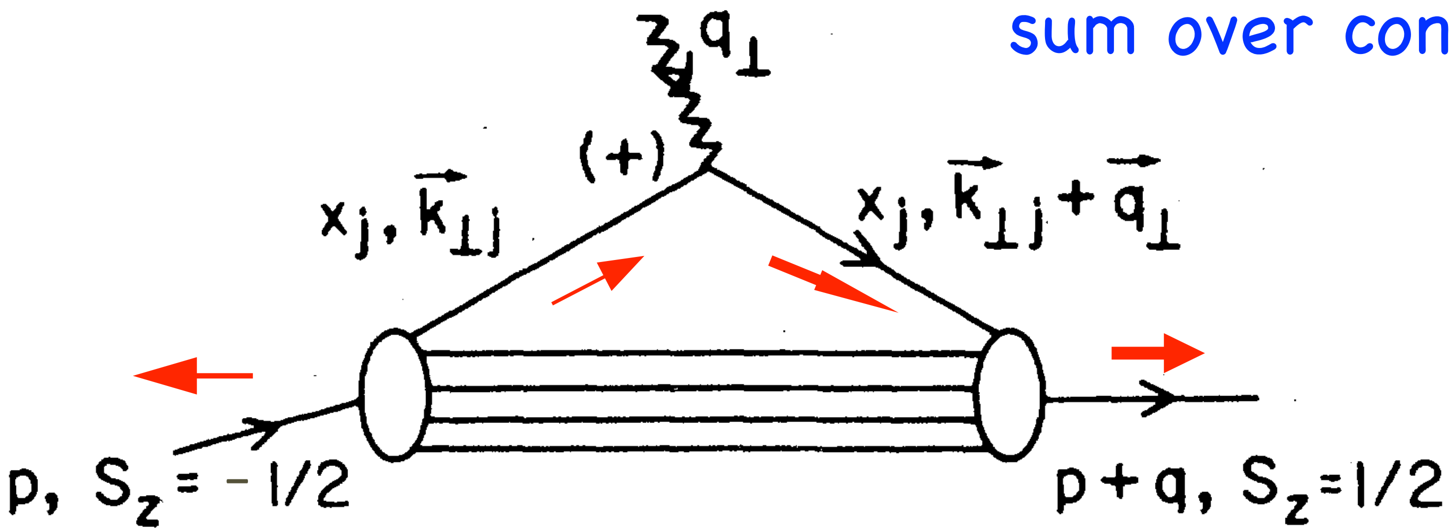
Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents



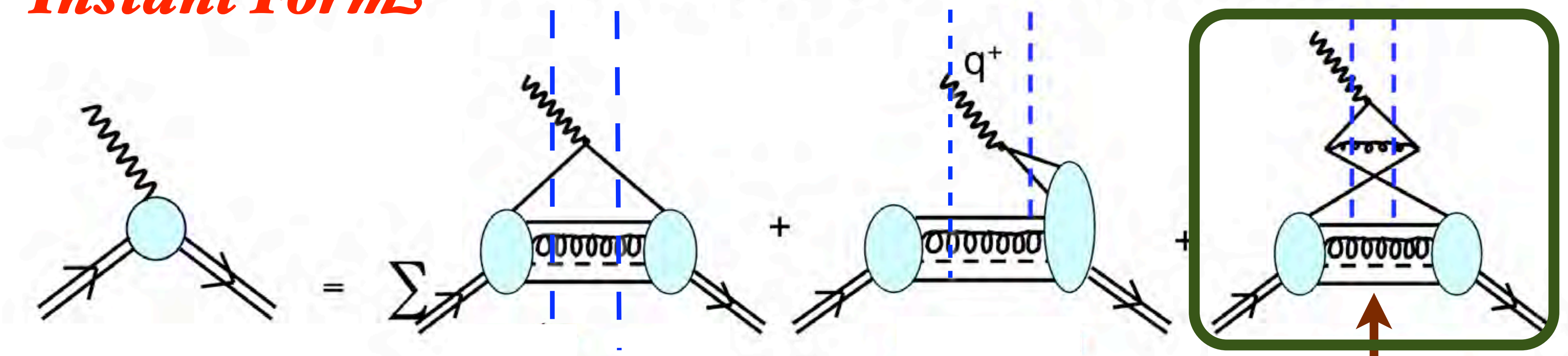
Hwang, Schmidt, sjb; Holstein et al

$$\boxed{B(0) = 0}$$

Each Fock State

Calculation of Form Factors in Equal-Time Theory

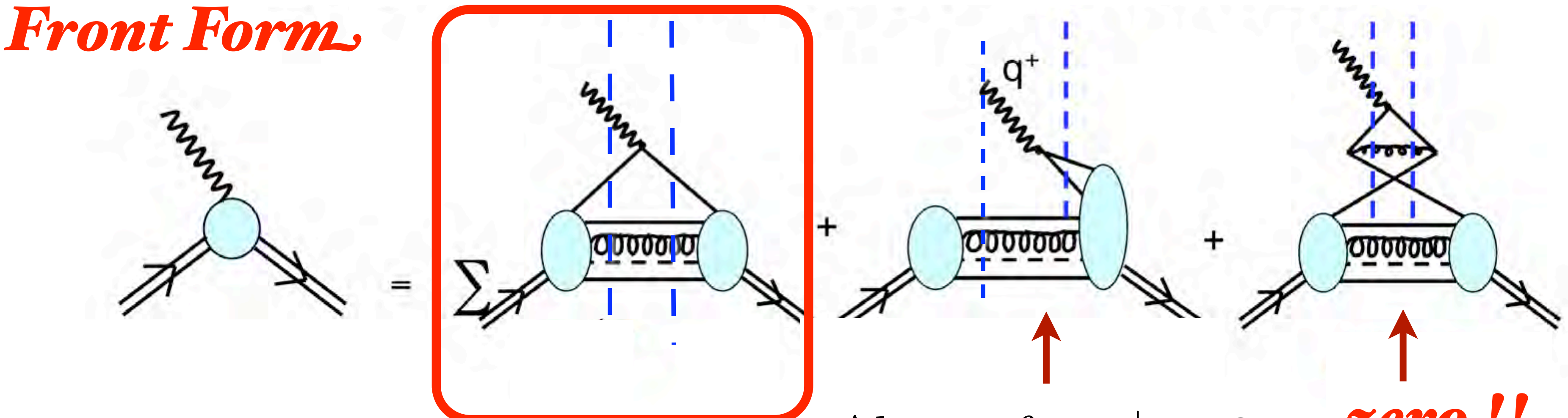
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

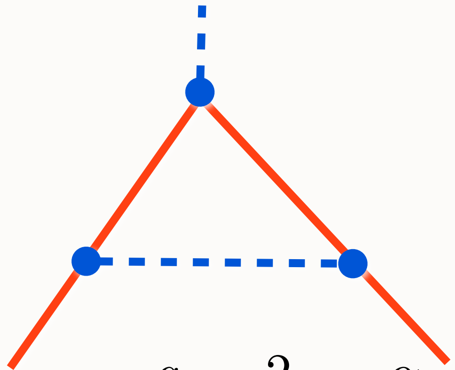
Absent for $q^+ = 0$

zero !!

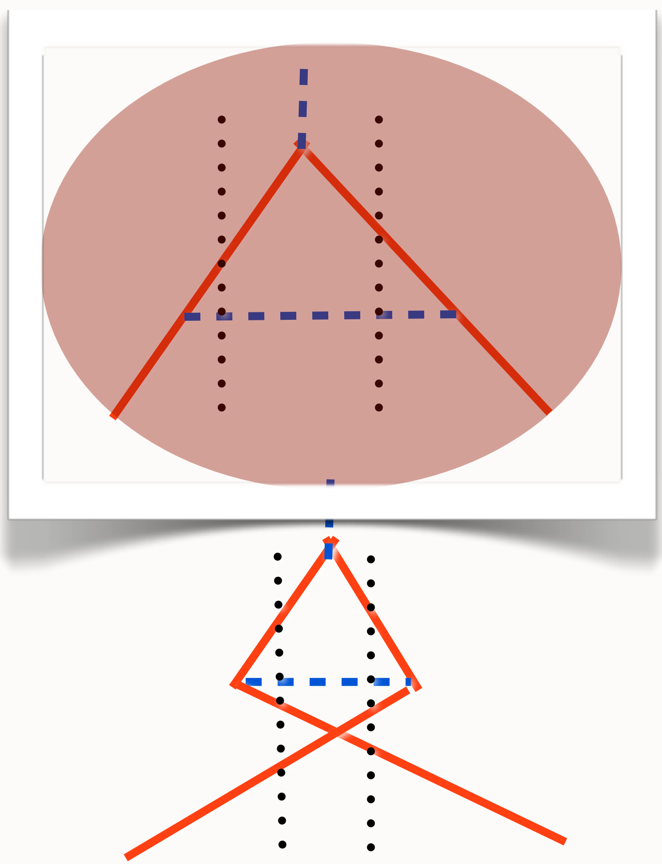
No vacuum graphs

Wick Theorem

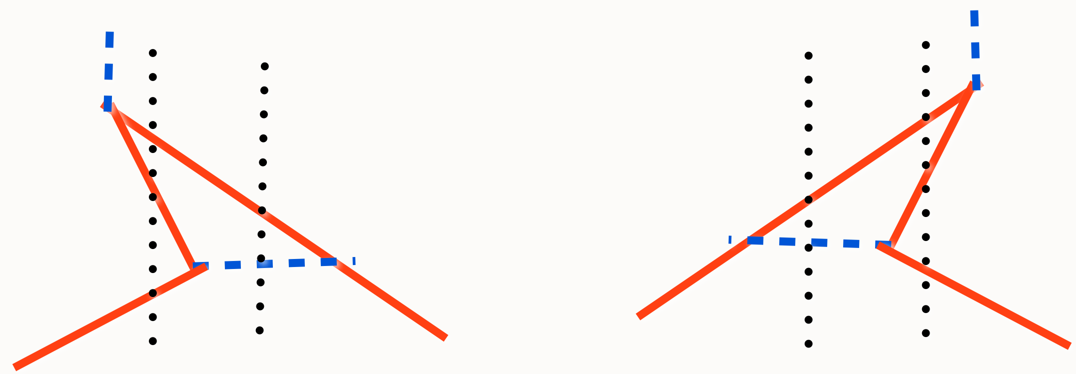
Feynman diagram =
single front-form time-ordered diagram!



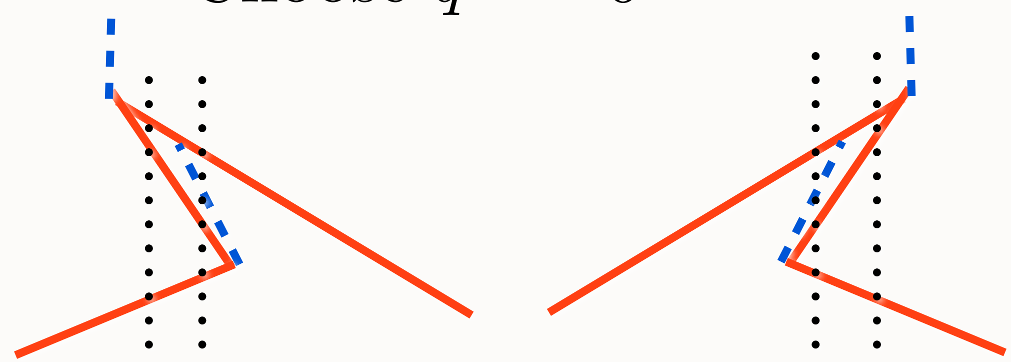
$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$



Also $P \to \infty$ observer frame (Weinberg)

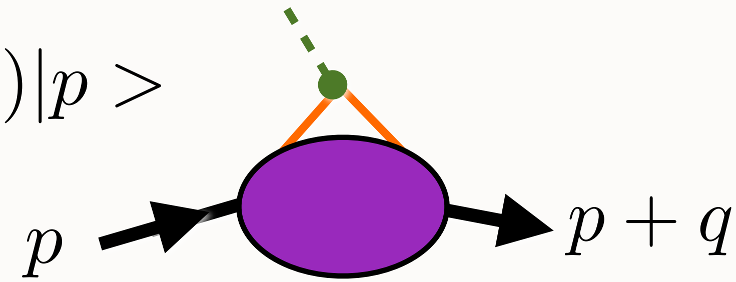


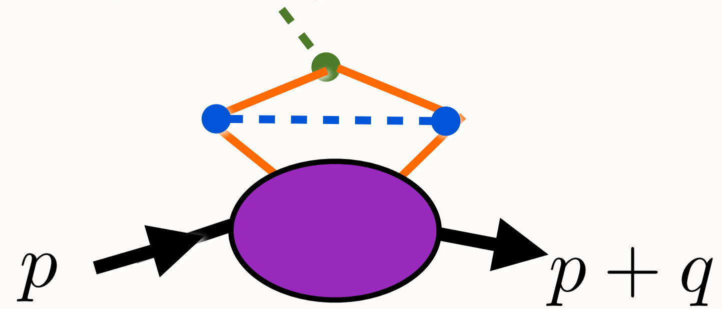
Choose $q^+ = 0$



Calculation of proton form factor in Instant Form

$$< p+q|J^{\mu}(0)|p >$$





- **Need to boost proton wavefunction: p to p+q. Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!! Remain even after normal-ordering**
- **Instant-form WFs insufficient to calculate form factors**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**

Light-Front QED

Heisenberg Matrix Formulation

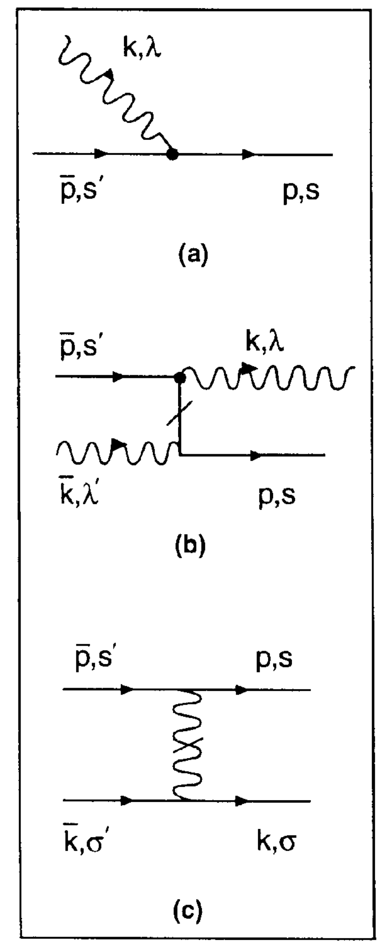
Physical gauge: $A^+ = 0$

$$L^{QED} \rightarrow H^{QED}_{LF}$$

$$H^{QED}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QED}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$



Eigenvalues and Eigensolutions give Positronium Spectrum and Light-Front wavefunctions

Light-Front QCD

Heisenberg Matrix Formulation

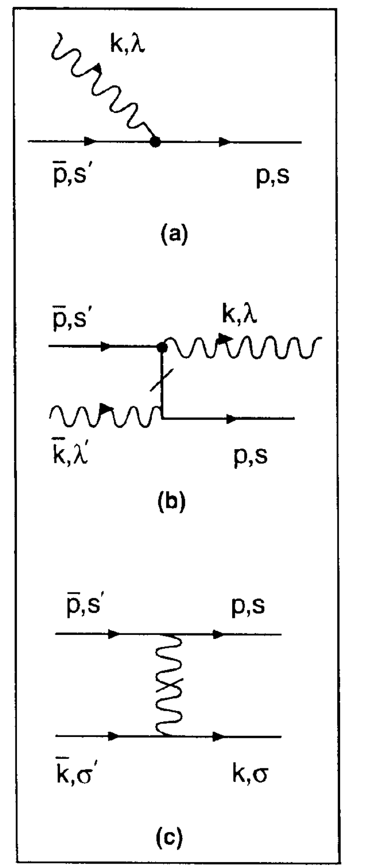
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

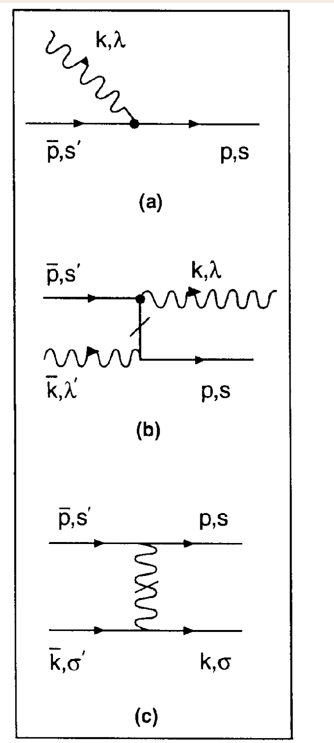
Light-Front QCD

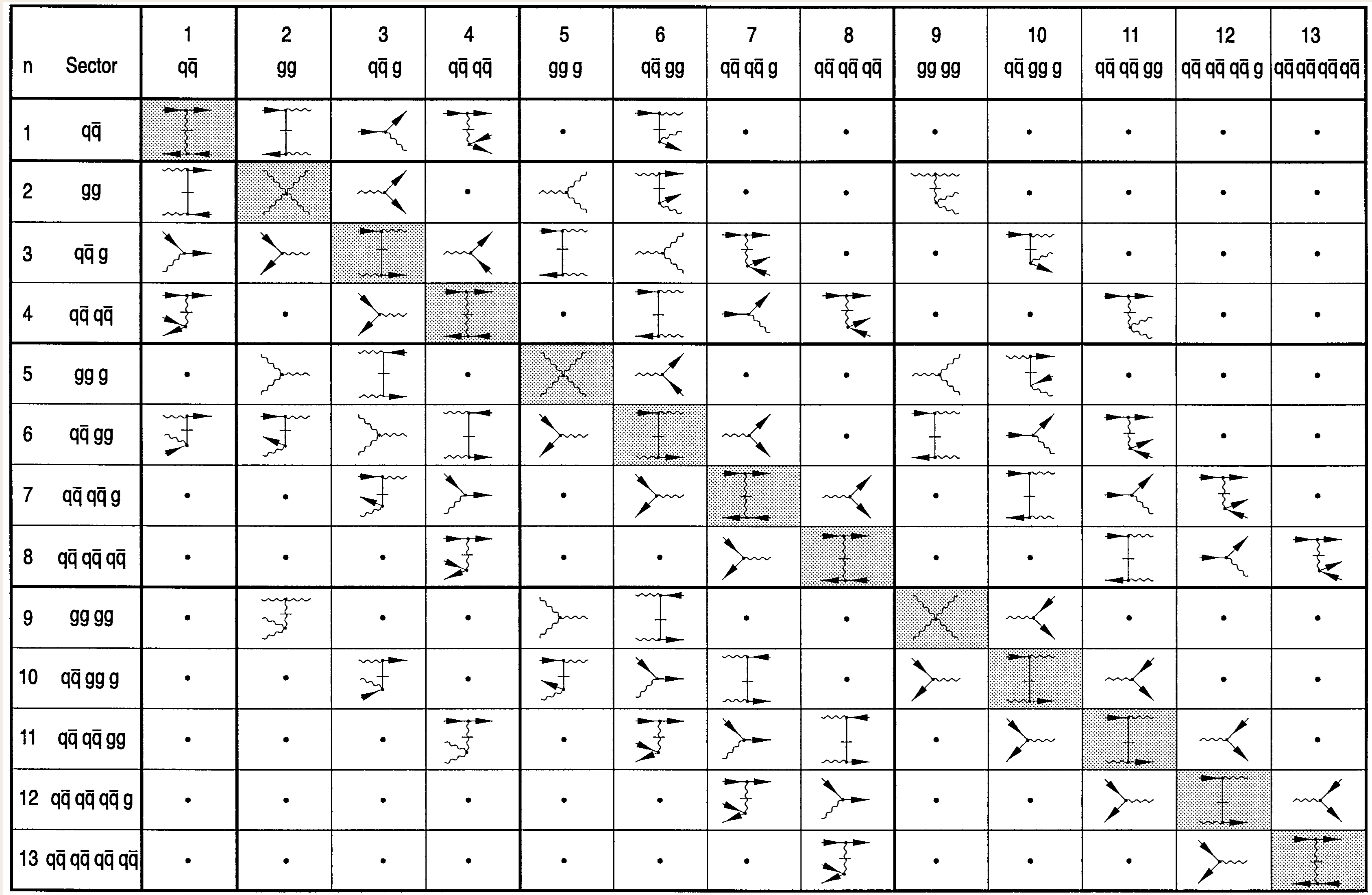
Heisenberg Matrix Formulation

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\,|\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Hans Christian Pauli & sjb

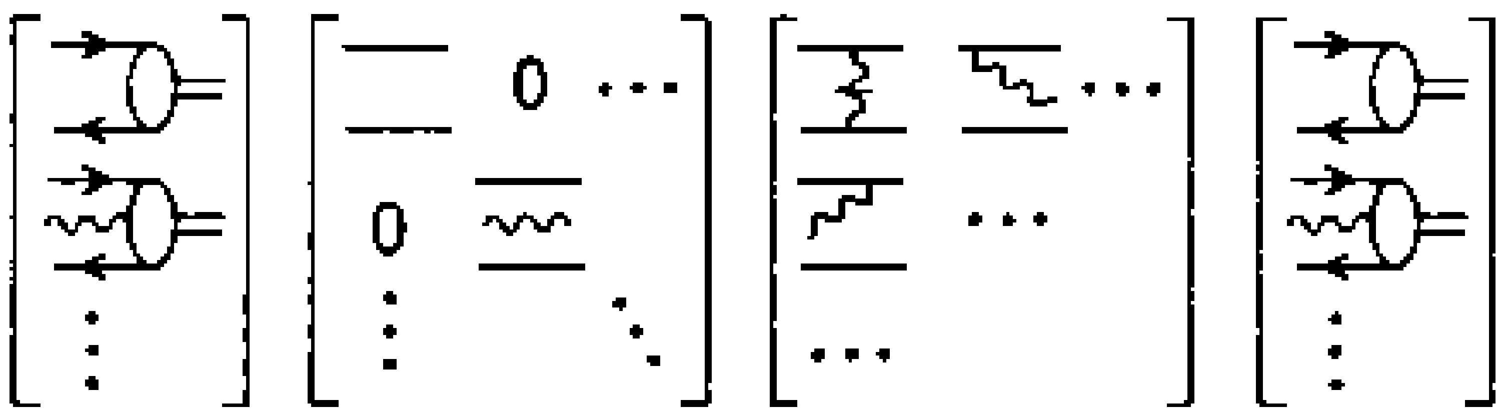
LIGHT-FRONT MATRIX EQUATION

G.P. Lepage, sjb

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$$A^+ = 0$$



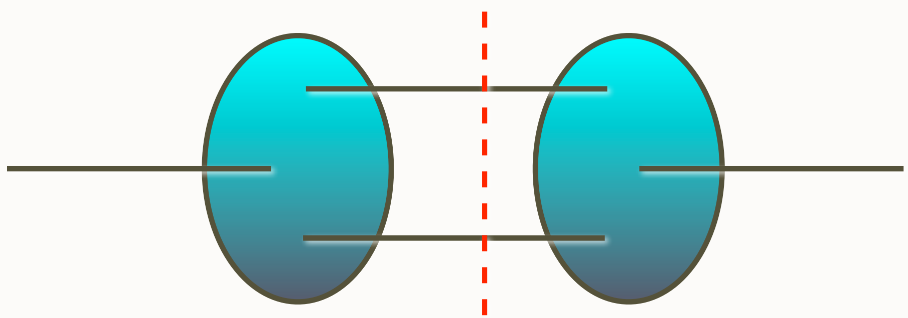
Minkowski space; frame-independent; no fermion doubling; no ghosts

- ***Light-Front Vacuum = Vacuum of Free Hamiltonian!***

Causal, Frame-Independent ***Possible zero modes***

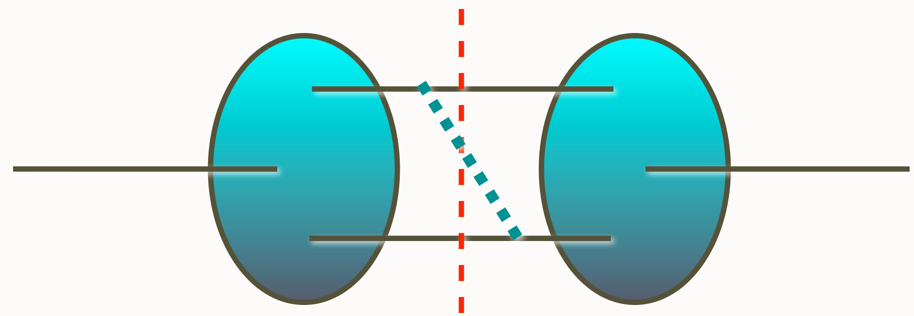
Quantum Mechanics: Uncertainty in p, x, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



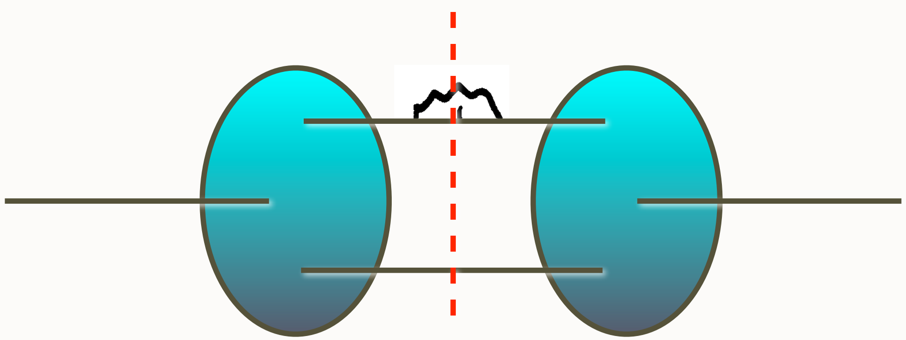
Positronium n=2

e^+e^-



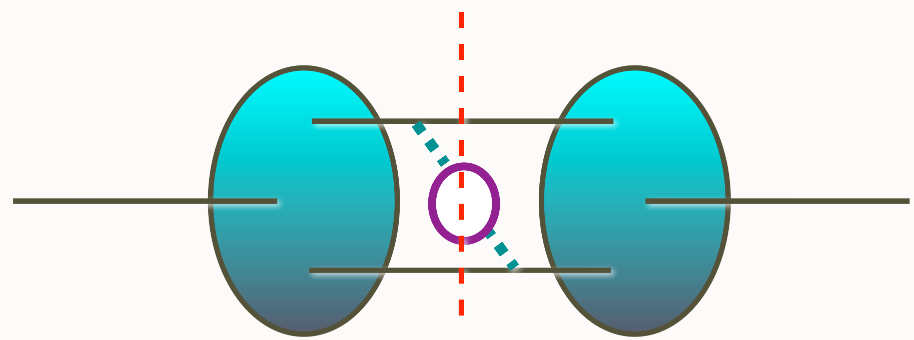
Hyperfine splitting n=3

$e^+e^-\gamma$



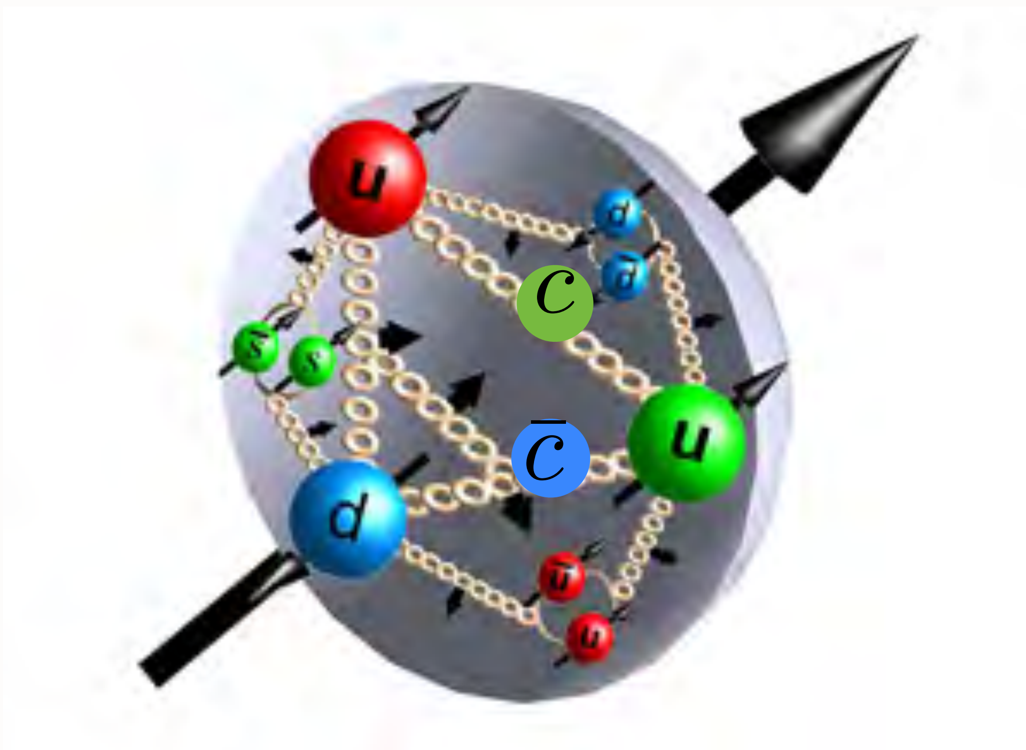
Lamb Shift n=3

$e^+e^-\gamma$

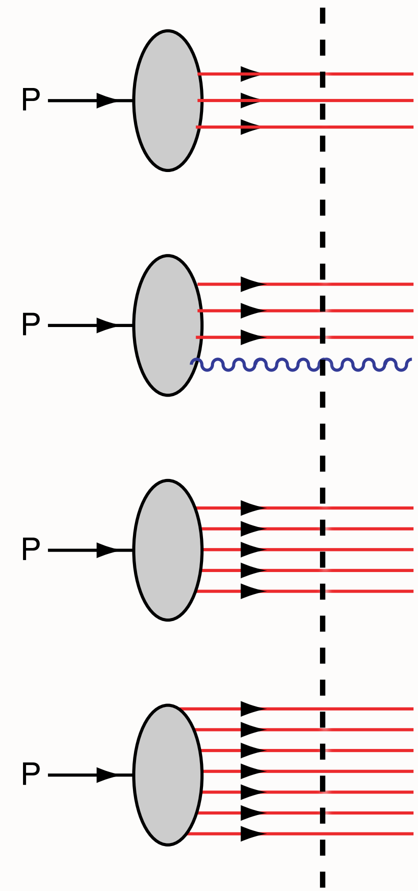


Vacuum Polarization n=4

$e^+e^-e^+e^-$



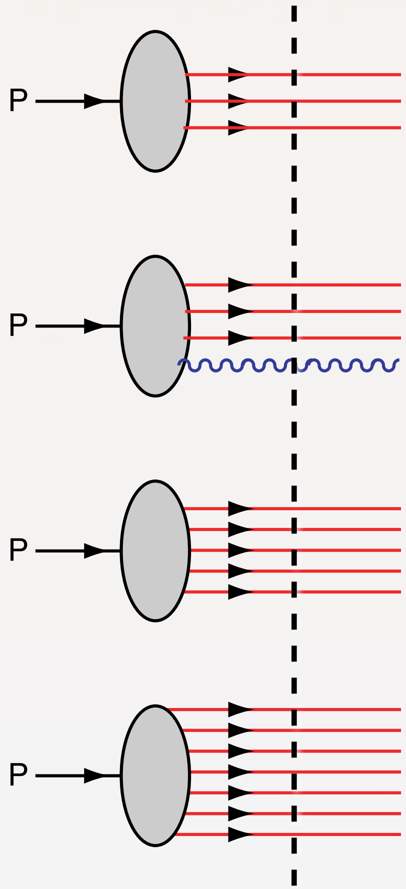
Higher Fock States of the Proton



Fixed LF time

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents



Fixed LF time

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0+k_i^z}{P^0+P^z}$$

are boost invariant.

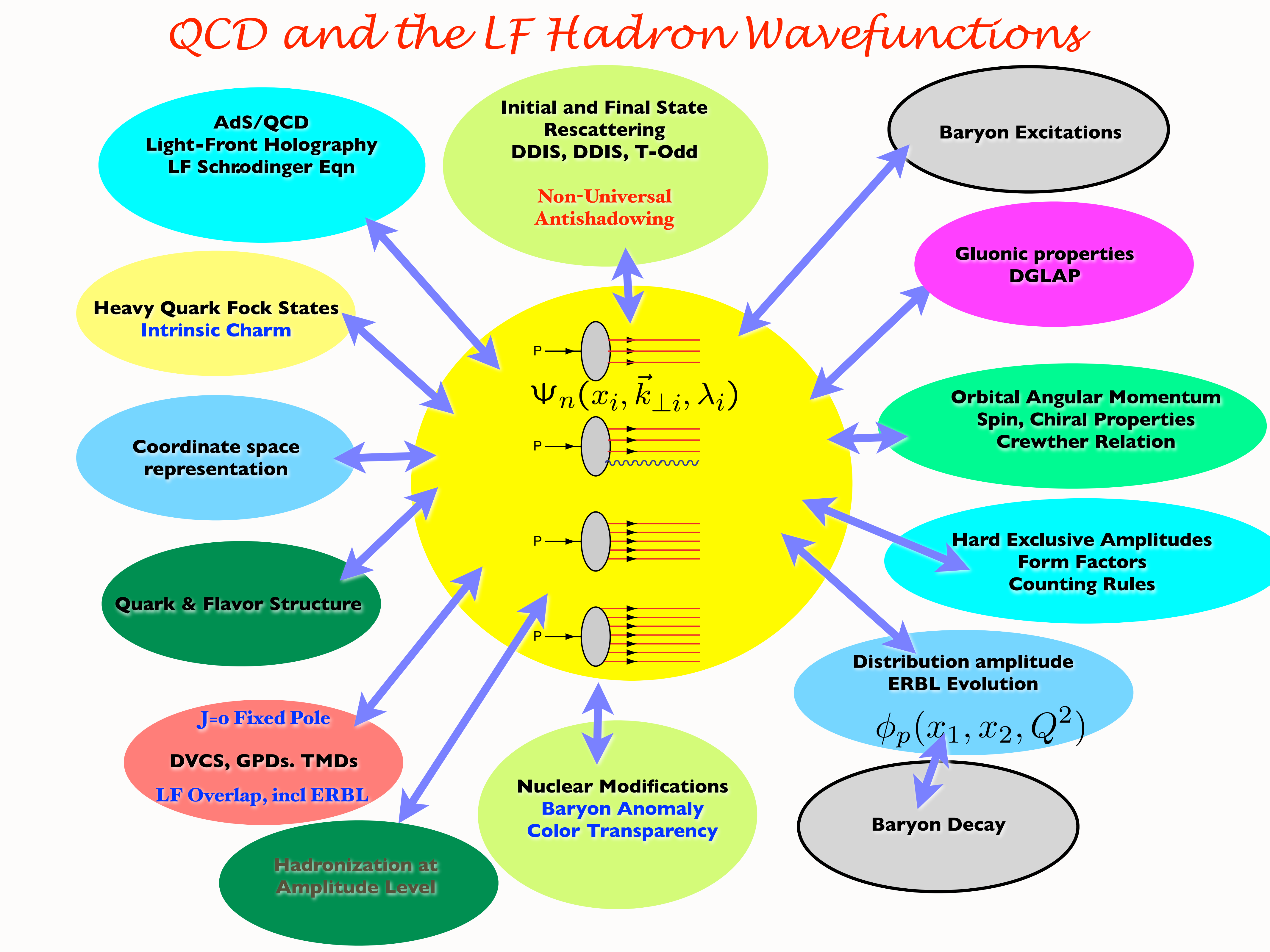
$$\sum_i^n k_i^+ = P^+,\ \sum_i^n x_i = 1,\ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks

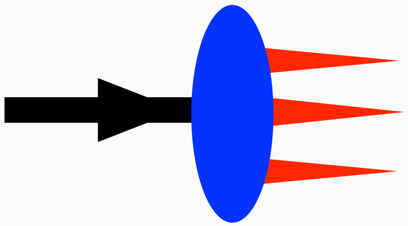
$c(x), b(x)$ at high x

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$


QCD and the LF Hadron Wavefunctions
AdS/QCD
Light-Front Holography
LF Schrodinger Eqn
Initial and Final State
Rescattering
DDIS, DDIS, T-Odd
Non-Universal
Antishadowing
Baryon Excitations
Gluonic properties
DGLAP
Heavy Quark Fock States
Intrinsic Charm
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation
Coordinate space
representation
Hard Exclusive Amplitudes
Form Factors
Counting Rules
Quark & Flavor Structure
Distribution amplitude
ERBL Evolution
$\phi_p(x_1, x_2, Q^2)$
J=0 Fixed Pole
DVCS, GPDs. TMDs
LF Overlap, incl ERBL
Nuclear Modifications
Baryon Anomaly
Color Transparency
Baryon Decay
Hadronization at
Amplitude Level

- LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian

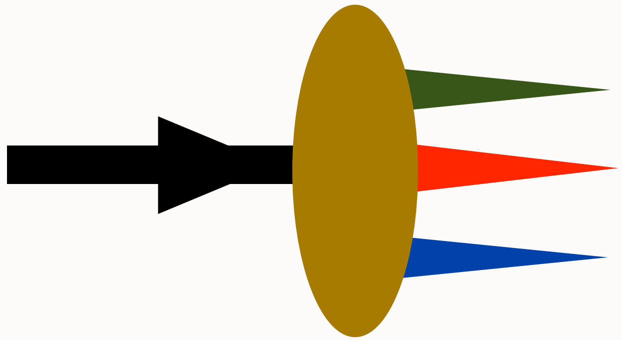
- Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors

- Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs

- Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!

- *Hadron Physics without LFWFs is like Biology without DNA!*

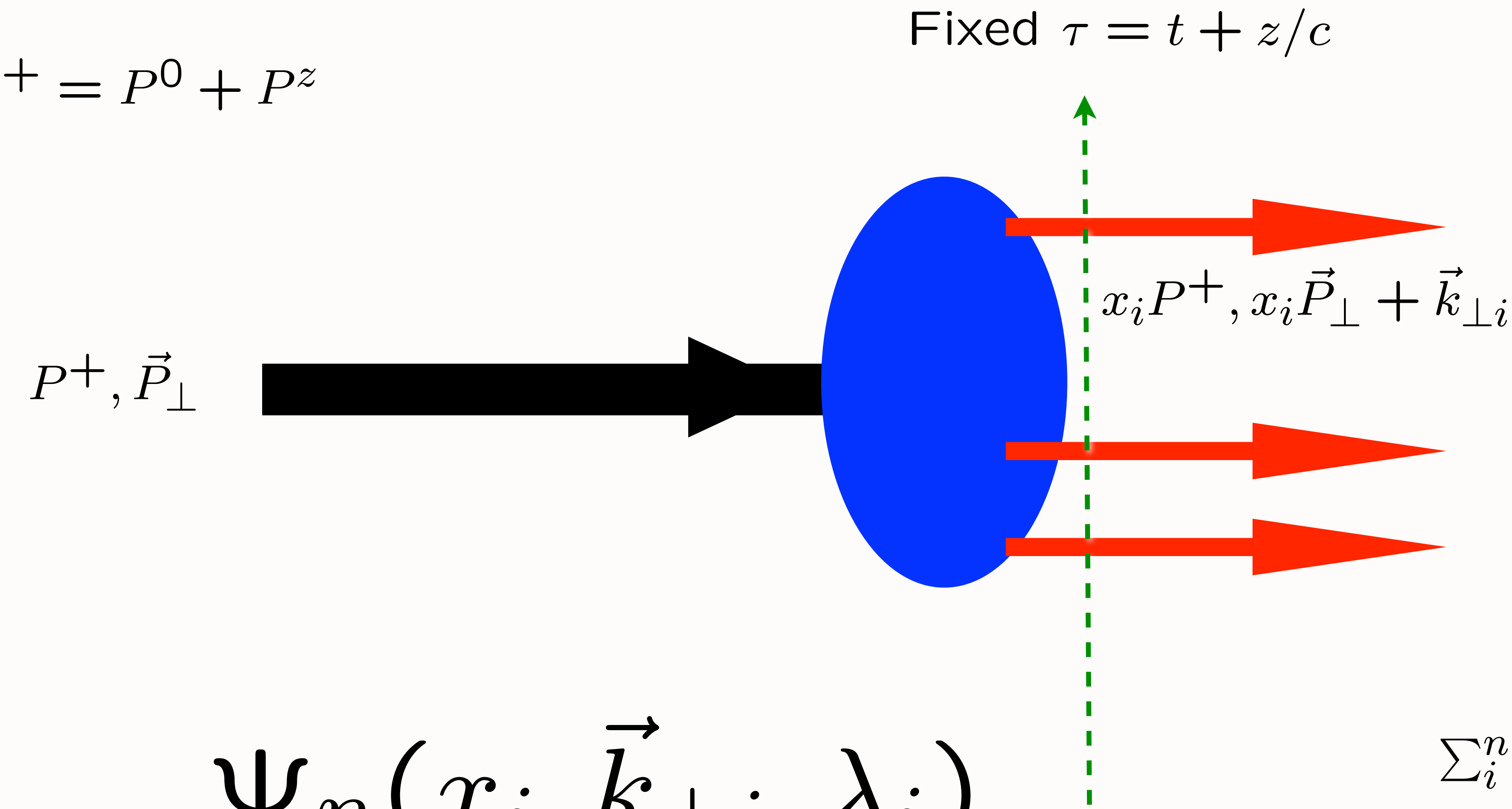
- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

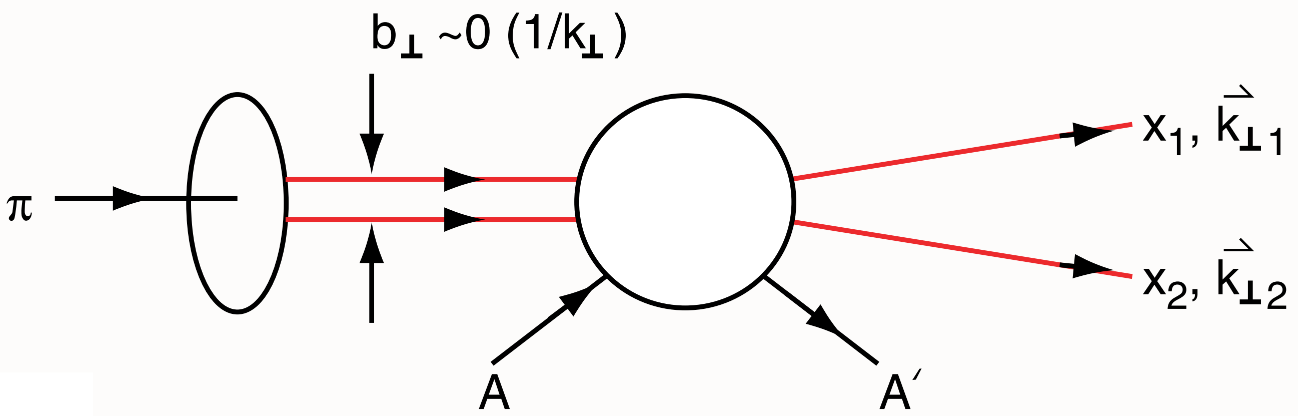
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



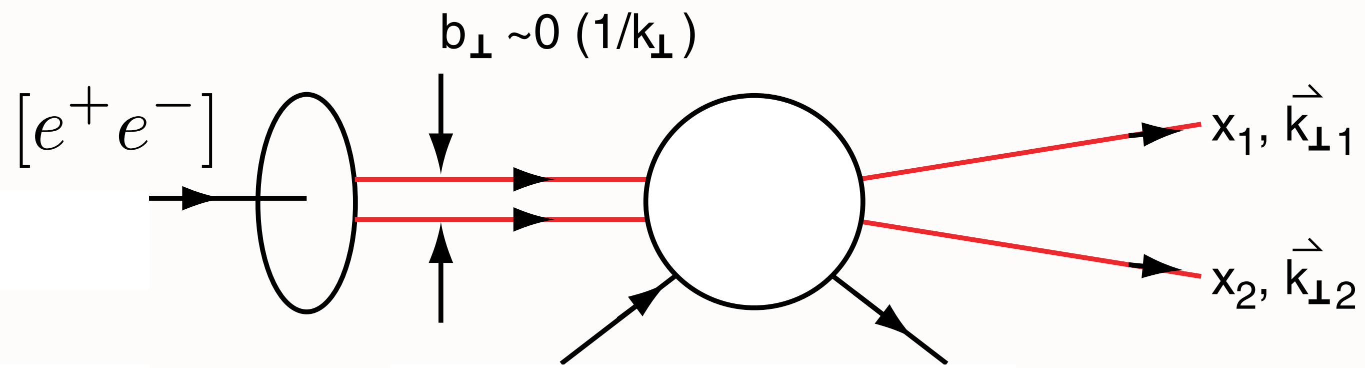
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

Diffractive Dissociation of Atoms

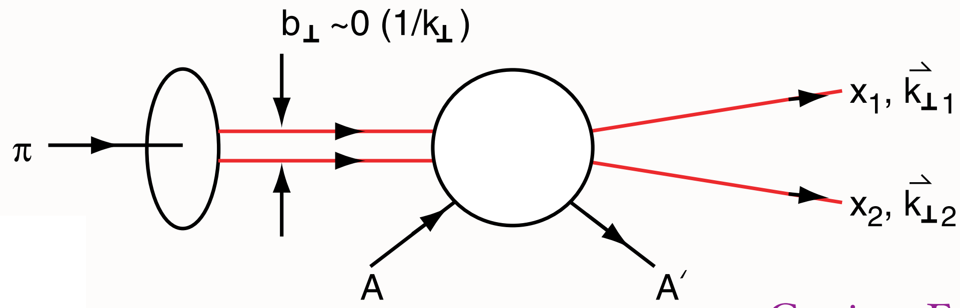


$$M \propto \frac{\partial}{\partial \vec{k}_\perp} \psi_{e^+e^-}(x, \vec{k}_\perp)$$

Measure Light-Front Wavefunction of Positronium
and Other Atoms

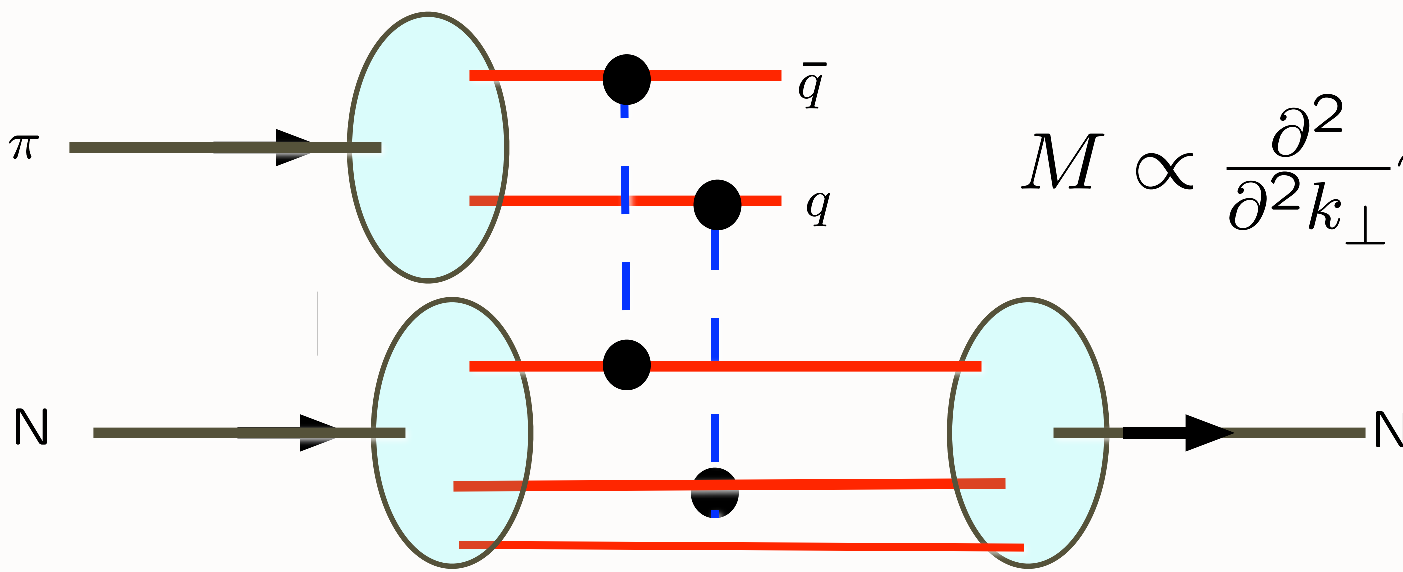
Minimal momentum transfer to Target
Target left Intact!

E791 FNAL Diffractive DiJet

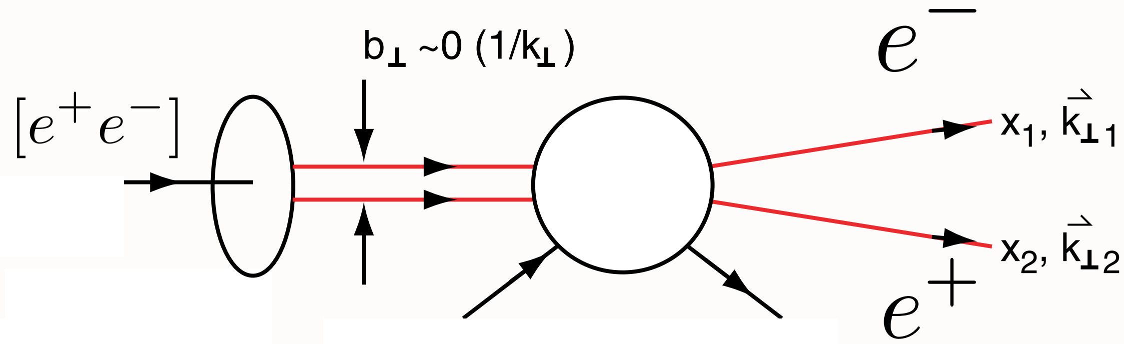


Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

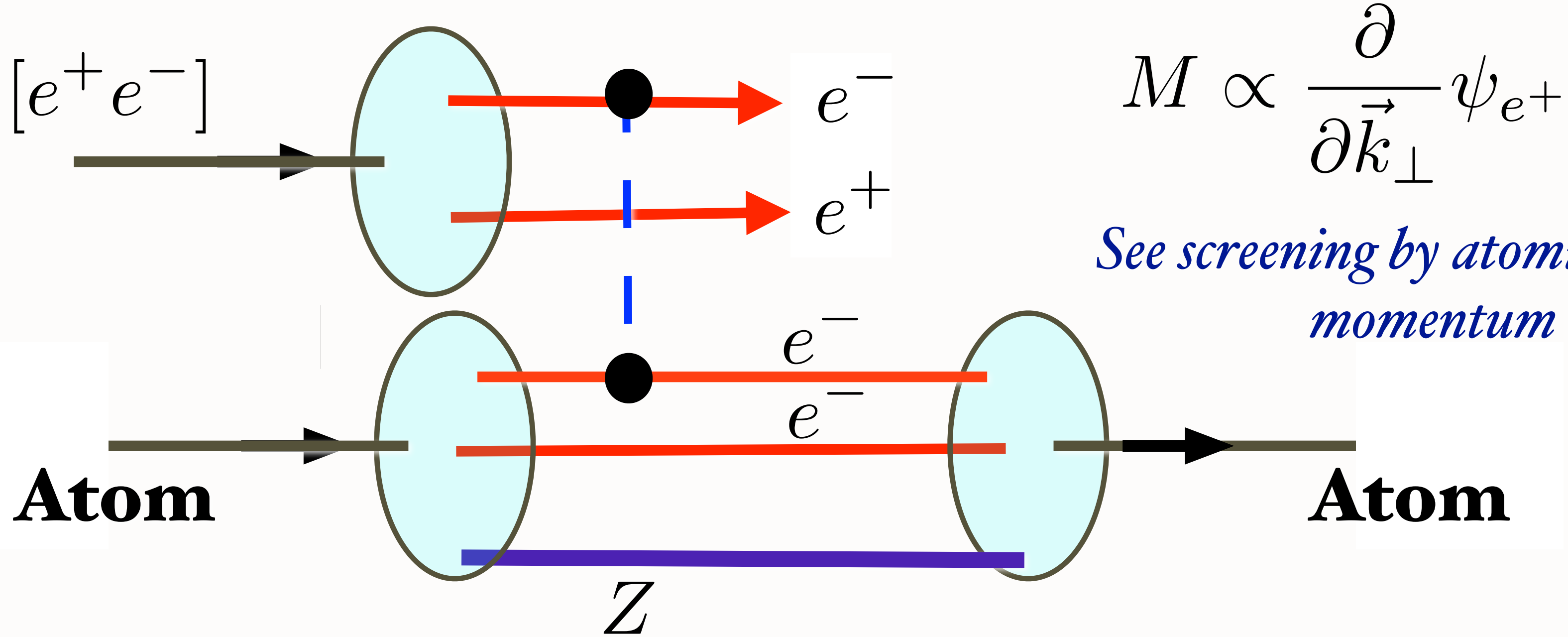
Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$



Coulomb-Photon exchange measures the derivative of the positronium light-front wavefunction



$$M \propto \frac{\partial}{\partial \vec{k}_\perp} \psi_{e^+e^-}(x, \vec{k}_\perp)$$

See screening by atomic electrons at low momentum transfer

Positronium LFWF

$$\left(M^2 - \frac{k_\perp{}^2 + m_1{}^2}{x_1} - \frac{k_\perp{}^2 + m_2{}^2}{x_2}\right)\psi(x_i, k_\perp) = \int_0^1 [dy] \int_0^\infty \frac{d^2 l_\perp}{16\pi^3} \tilde{K}(x_i, k_\perp; y_i, l_\perp; M^2)\psi(y_i, l_\perp)$$

$$\tilde{K} \simeq \frac{-16e^2 m^2}{(k_\perp - l_\perp)^2 + (x-y)^2 m^2}$$

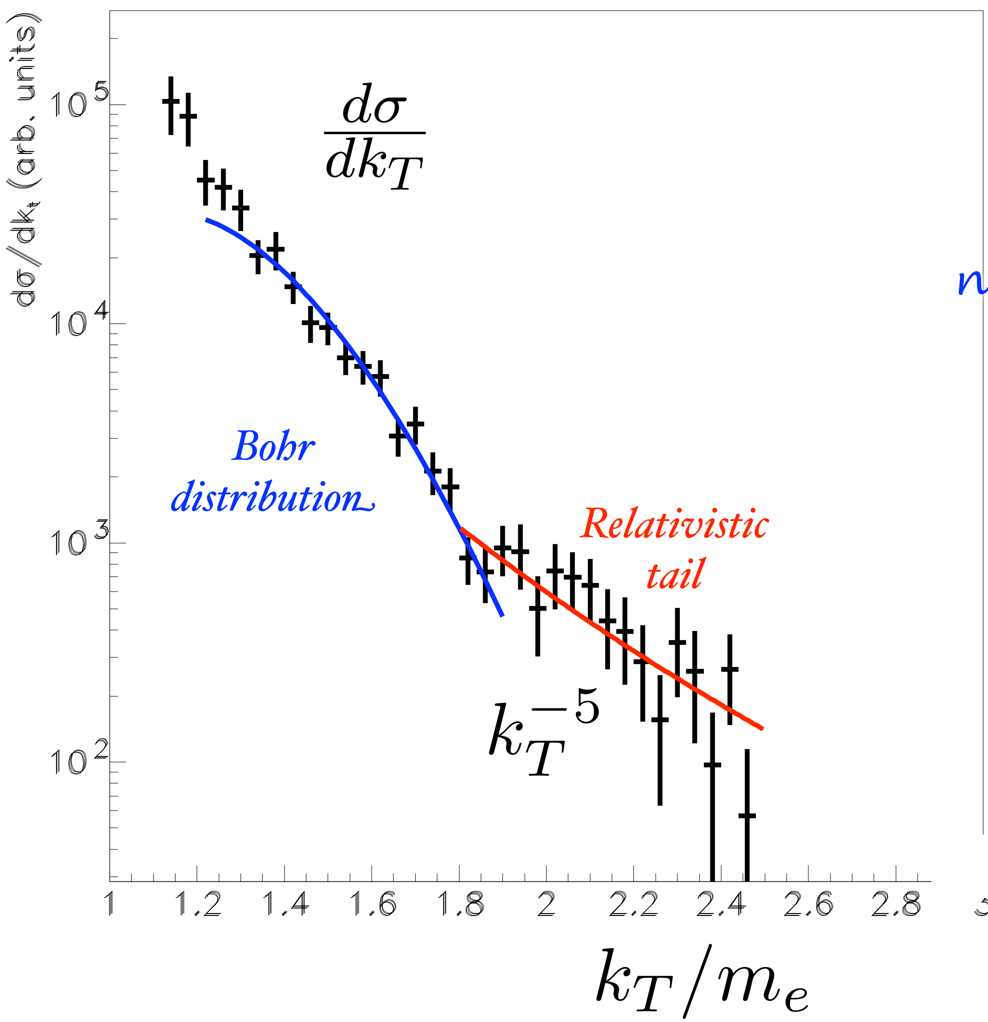
Non-relativistic limit

$$\Psi = \left(\frac{m\beta^3}{\pi}\right)^{1/2} \frac{64\pi\beta x_1 x_2}{[k_\perp{}^2 + (x_1 - x_2)^2 m^2 + \beta^2]^2} \times \begin{cases} \dfrac{u_\uparrow \bar{v}_\downarrow - u_\downarrow \bar{v}_\uparrow}{(2x_1x_2)^{1/2}}, & \text{parapositronium}, \\ \dfrac{u_\uparrow \bar{v}_\uparrow}{(x_1x_2)^{1/2}}, & \text{orthopositronium}, \\ \cdots, \end{cases}$$

$$\psi_{NR}(x, k_\perp) \sim \frac{1}{k_\perp^4} \qquad \beta = \alpha m_e/2$$

At large $k_\perp^2 >> m_e^2$, $\psi(x, k_\perp) \sim \frac{1}{k_\perp^2}$

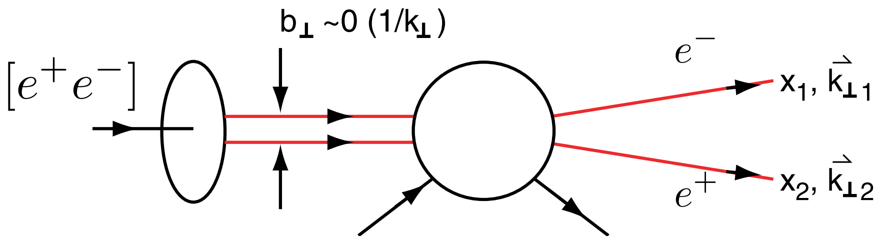
Simulated diffractive transverse momentum distribution for positronium



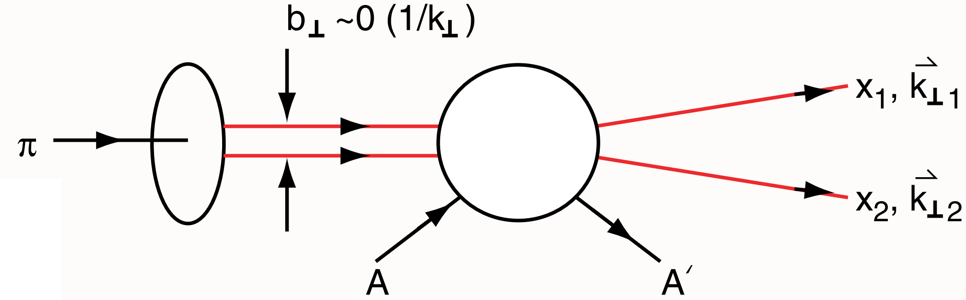
Two Components:

Low momentum dependence from non-relativistic Coulomb interaction

High Transverse momentum dependence from relativistic QED

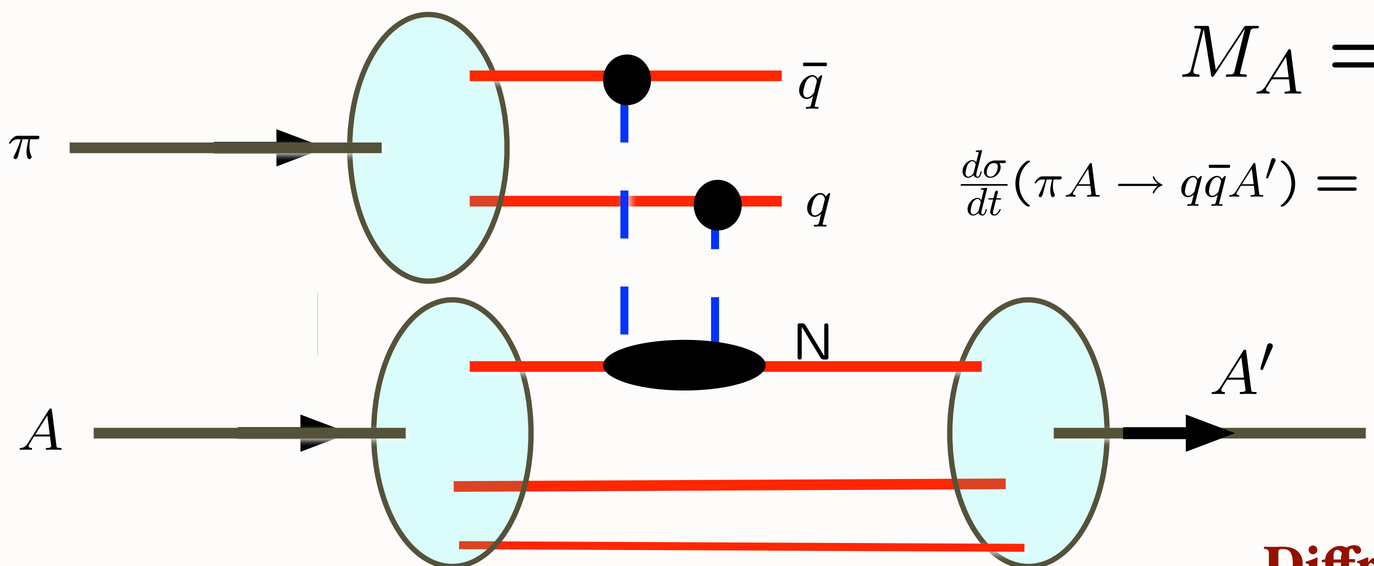


Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller
Strikman

Small color-dipole moment pion not absorbed;
interacts with each nucleon coherently
QCD COLOR Transparency



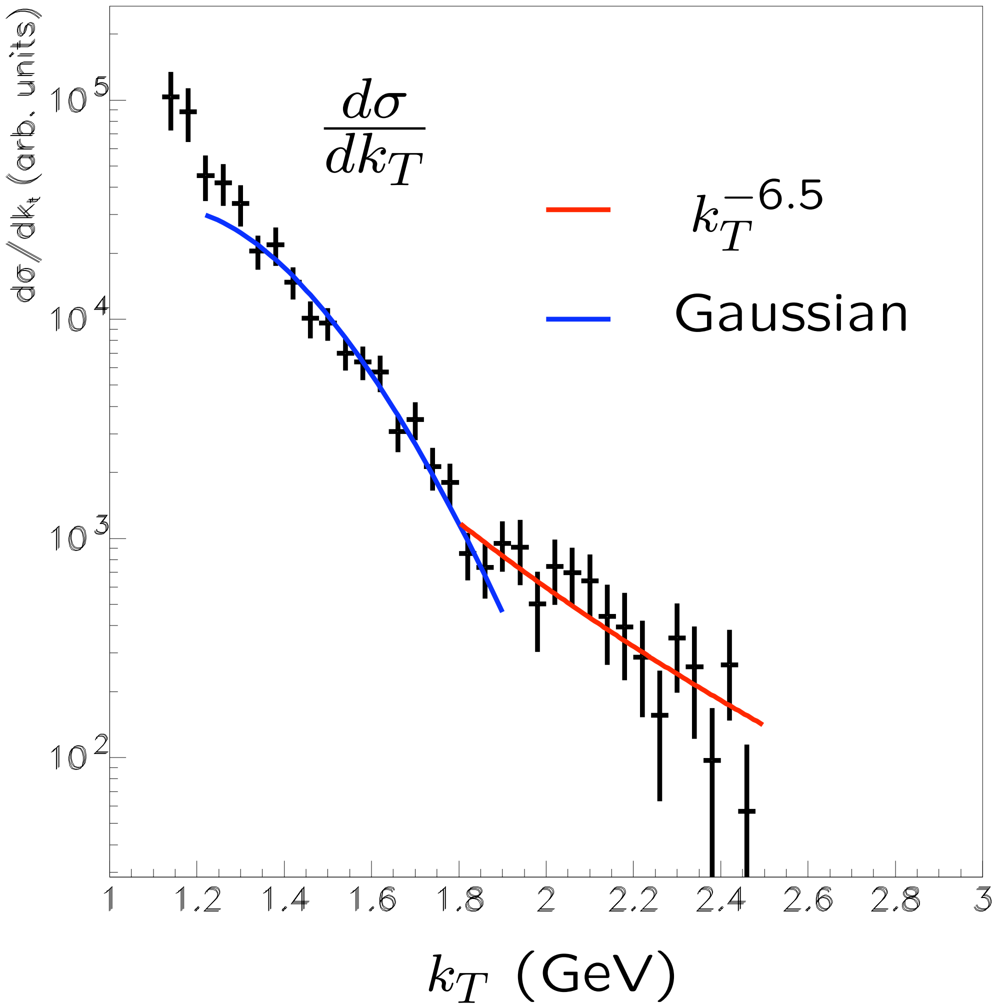
$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F^2_A(t)$$

Target left intact

Diffraction, Rapidity gap

E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with PQCD, ERBL Evolution $k_T^{-6.5}$

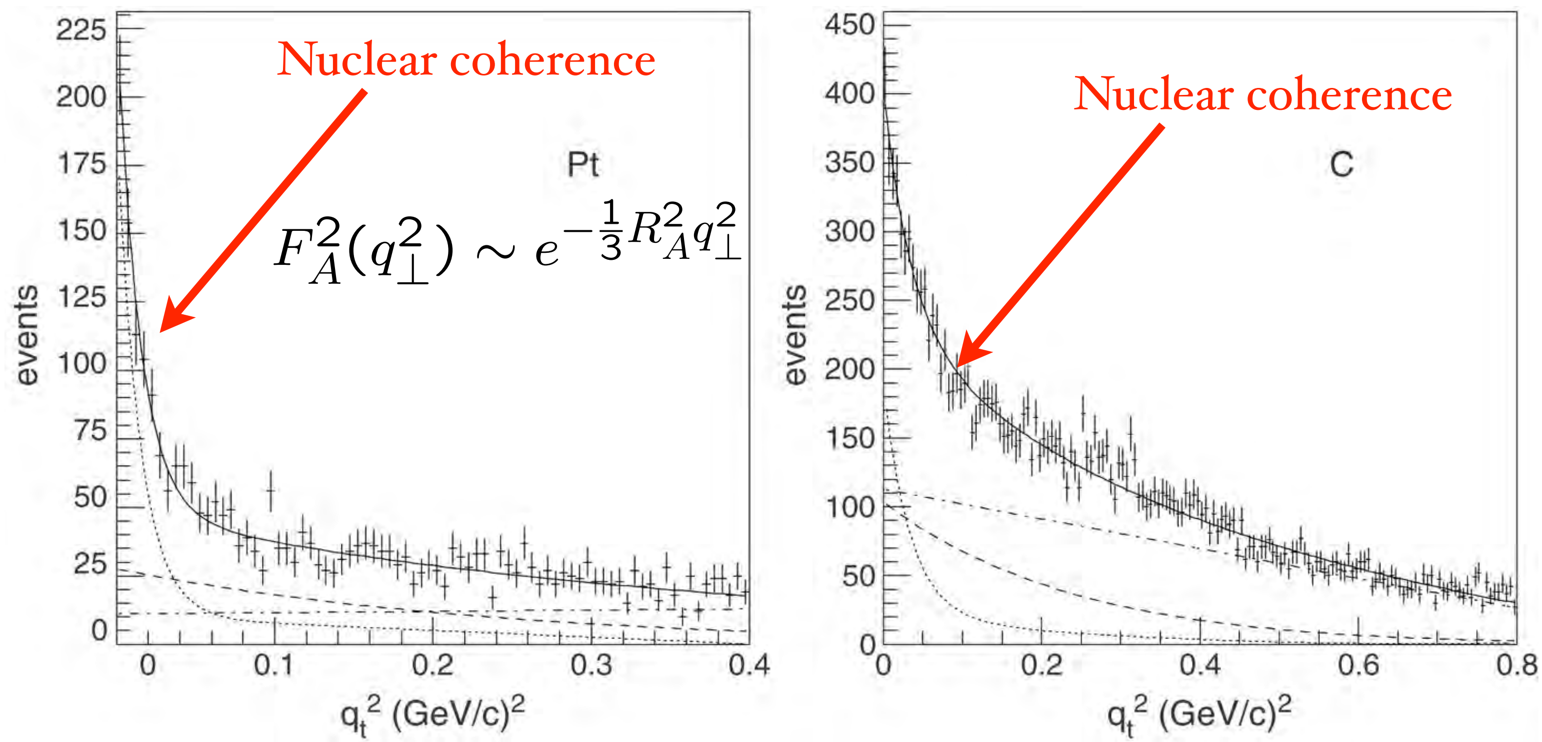
Gaussian component similar to AdS/CFT HO LFWF

- **Fully coherent interactions between pion and nucleons.**
- **Emerging Di-Jets do not interact with nucleus.**

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

$\mathbf{k}_t$ range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

Ashery E791

α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

Factor of 7

Color Transparency

Bertsch, Gunion, Goldhaber, sjb

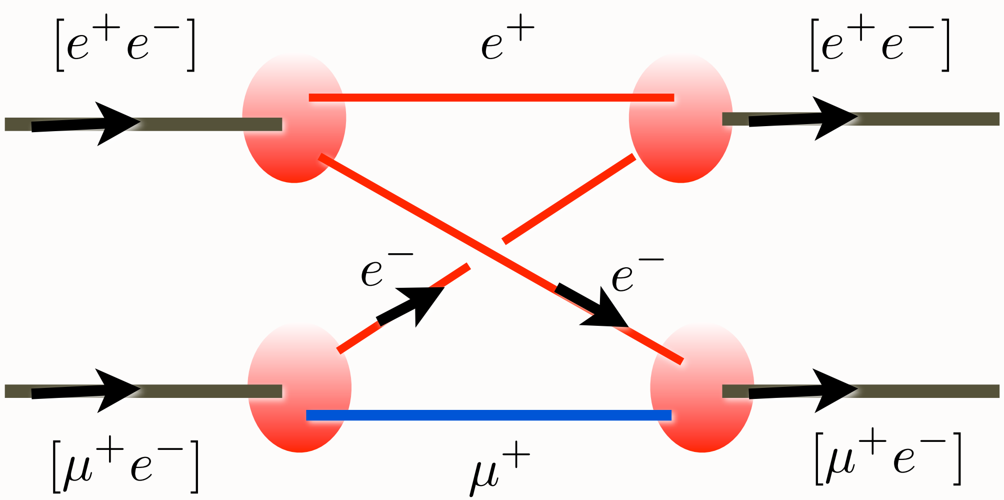
A. H. Mueller, sjb

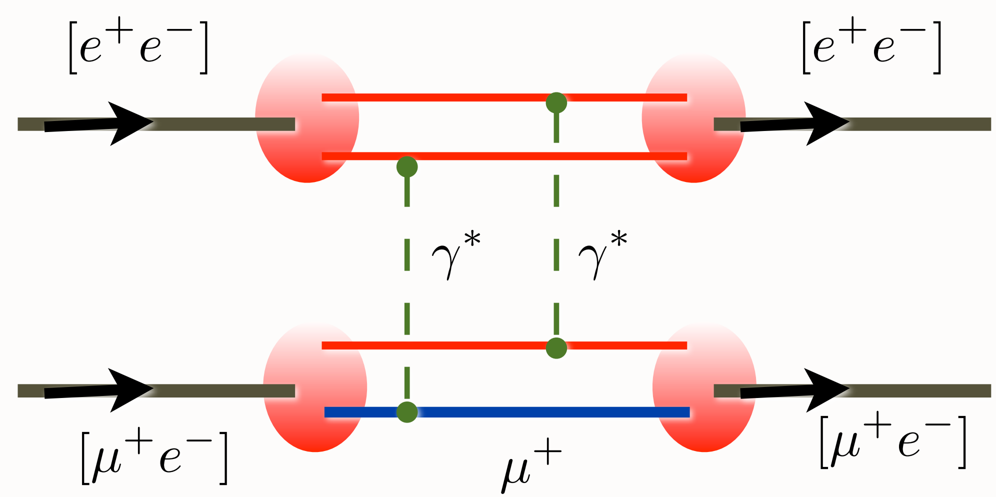
- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Atomic Transparency

- **Fundamental test of gauge theory in atomic physics**
- **Small electric dipole moments interact weakly in target**
- **Complete coherence at high energies -- crystals!**

CIM: Blankenbecler, Gunion, sjb





Constituent Interchange

Spin exchange in atom-atom scattering

Two-Photon Exchange

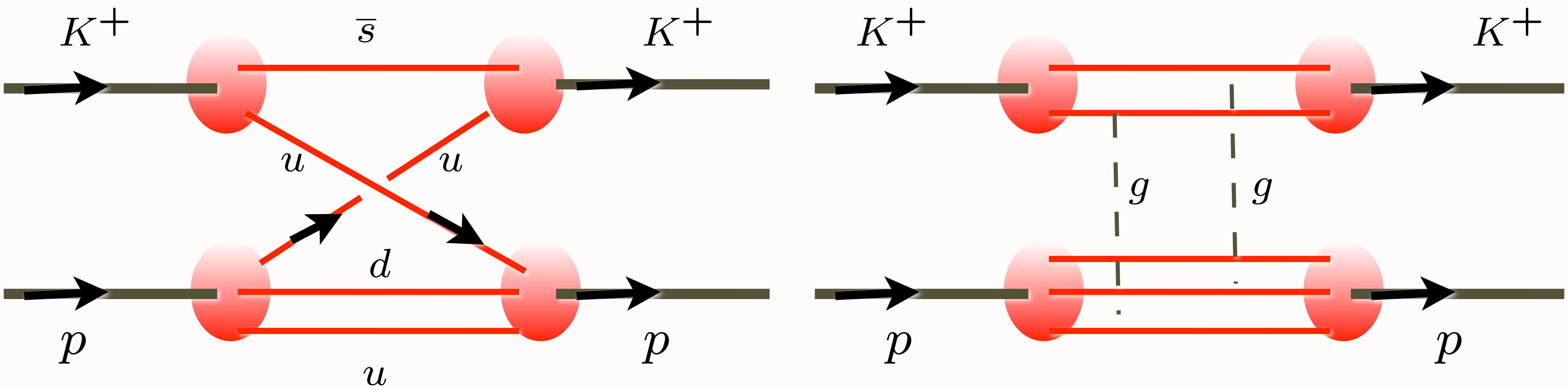
(Van der Waal)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)

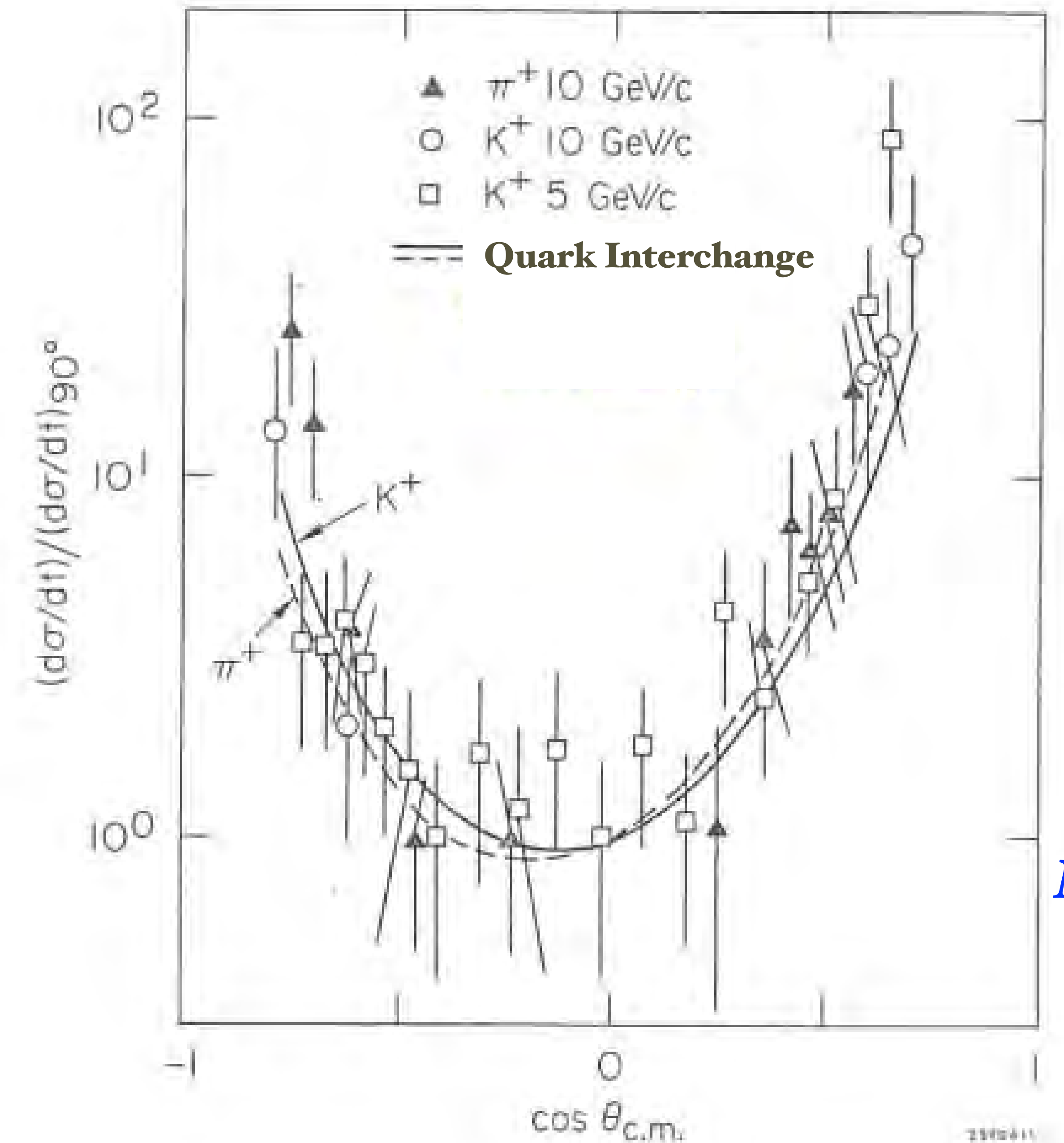
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

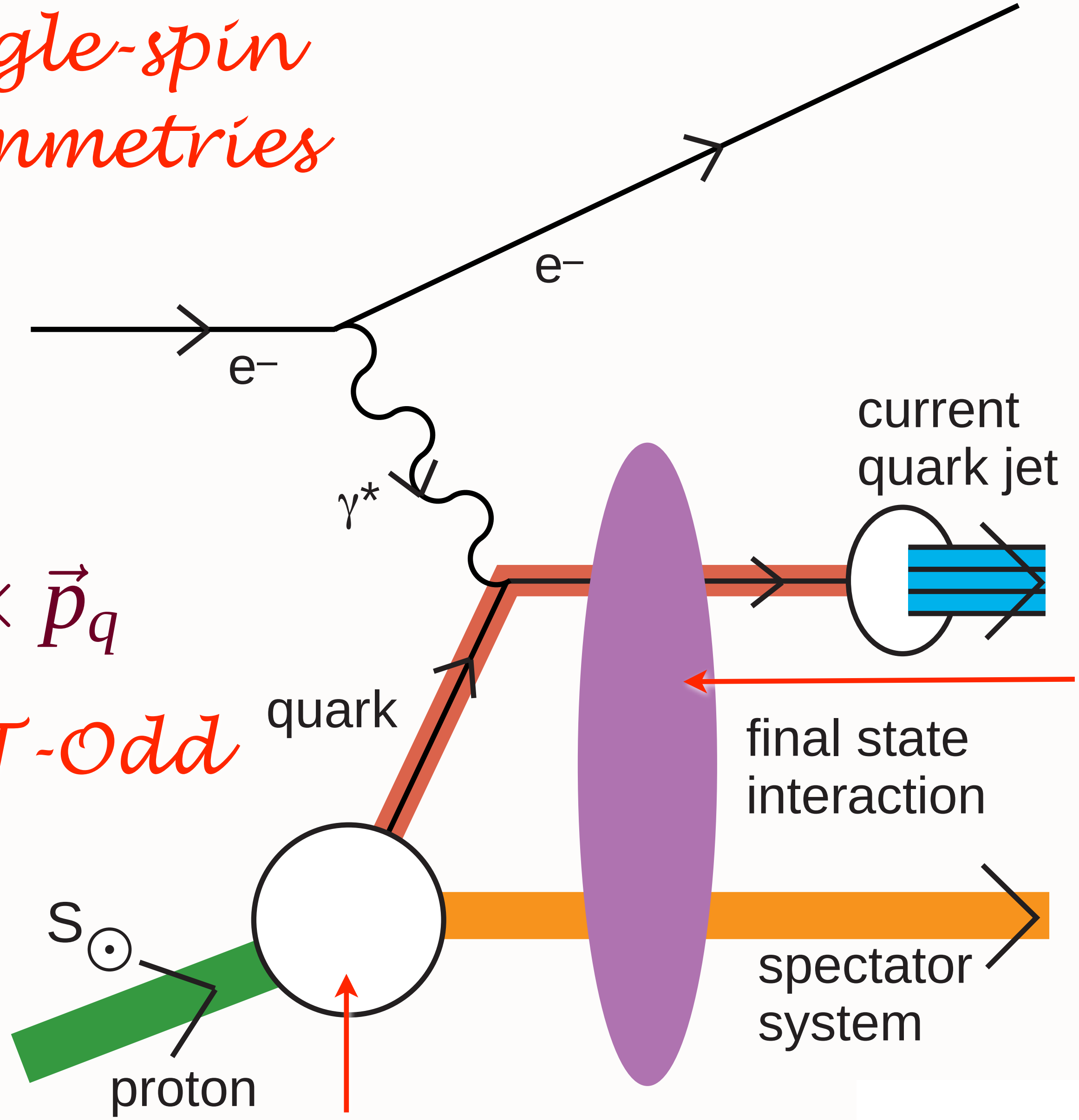
$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Leading-Twist Sivers Effect

Single-spin asymmetries



Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases --Wilson Line

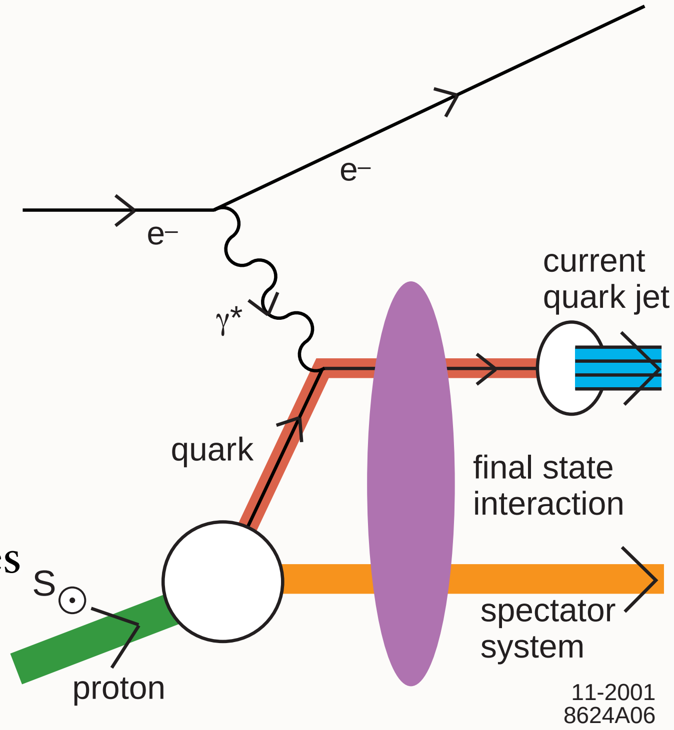
Light-Front Wavefunction S and P- Waves

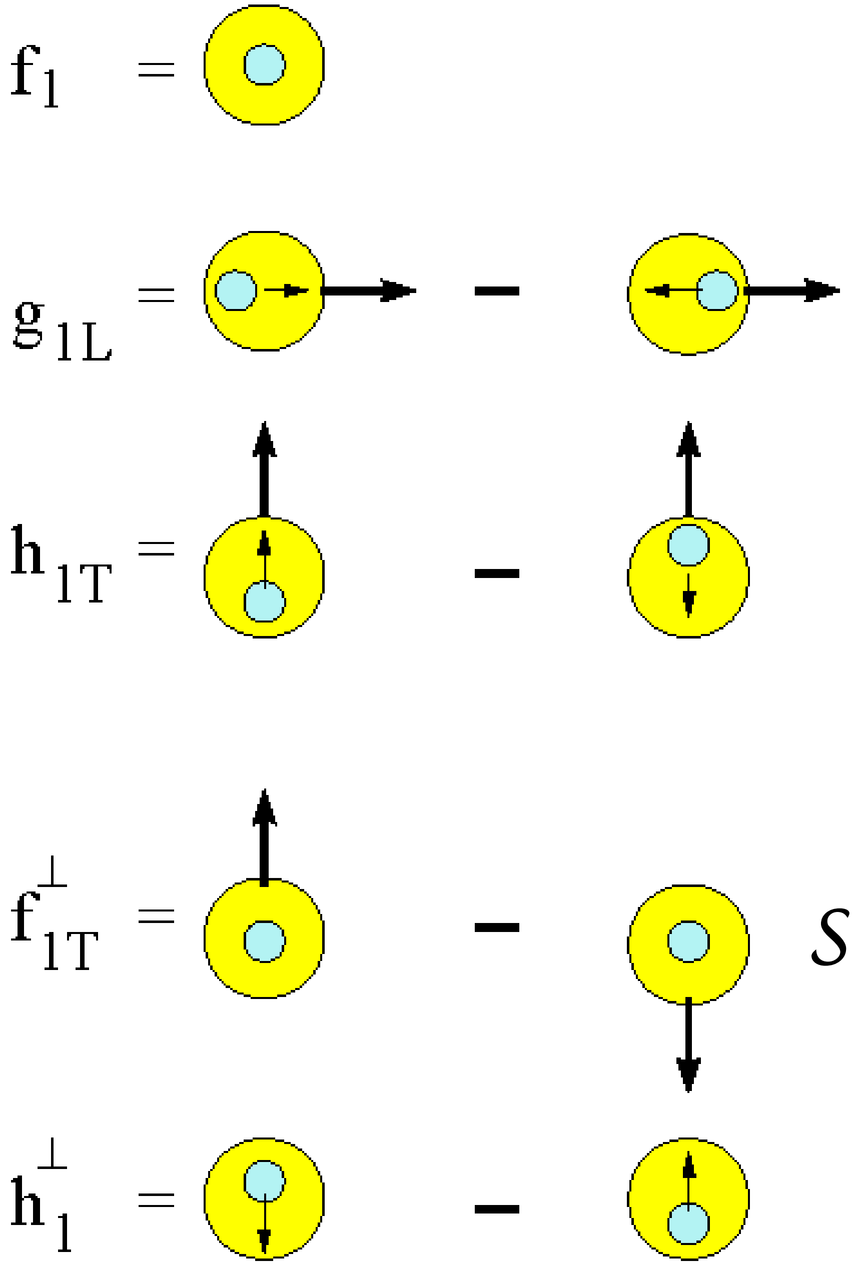
Analog of QED FSIs

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

$$\mathbf{i}\ \vec{S}\cdot\vec{p}_{jet}\times\vec{q}$$





Unpolarized Distribution

Bj Sum Rule

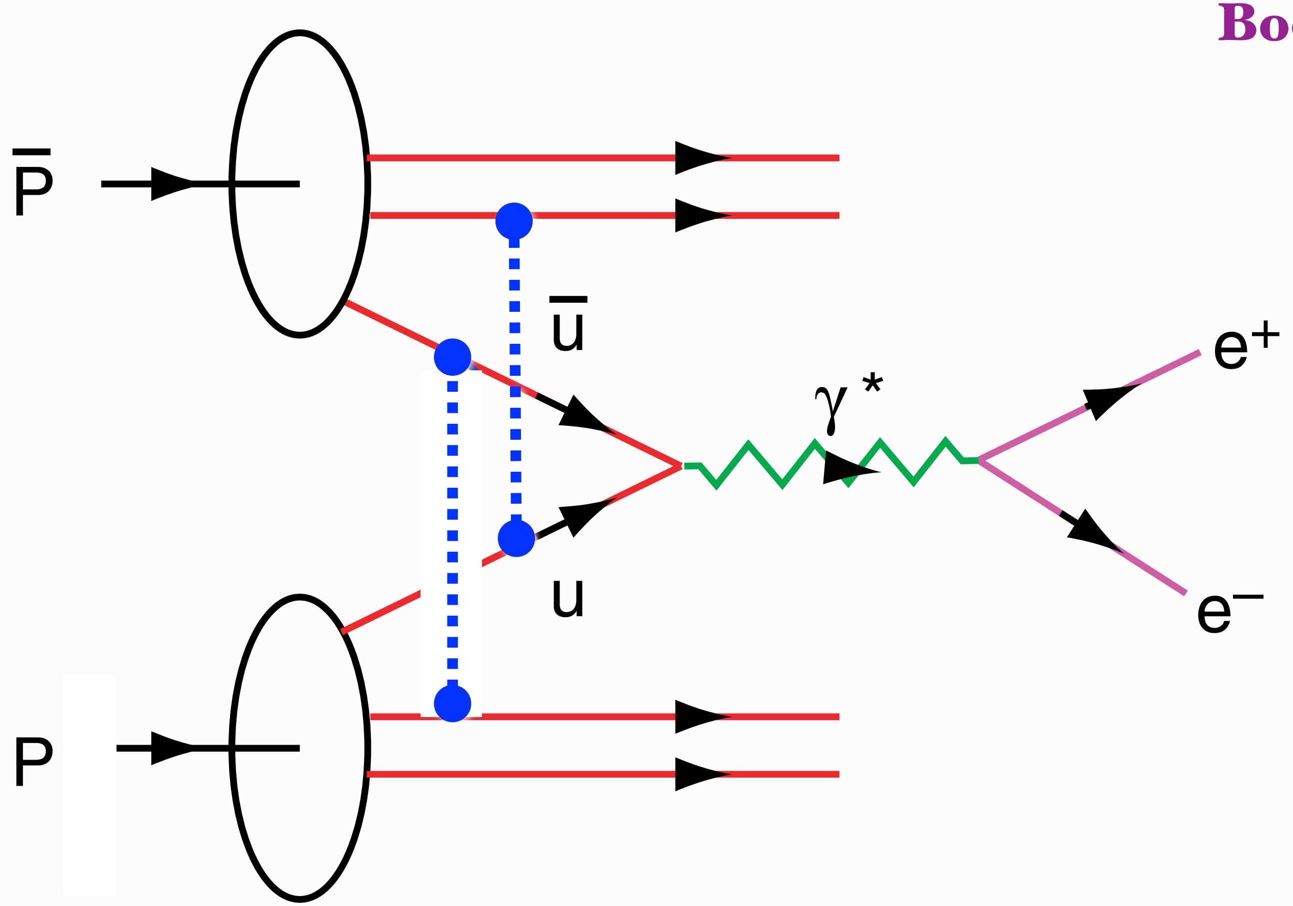
Transversity

Sivers Function

Boer-Mulders Function

T-Odd:
Require ISI or FSI

Boer, Hwang, sjb



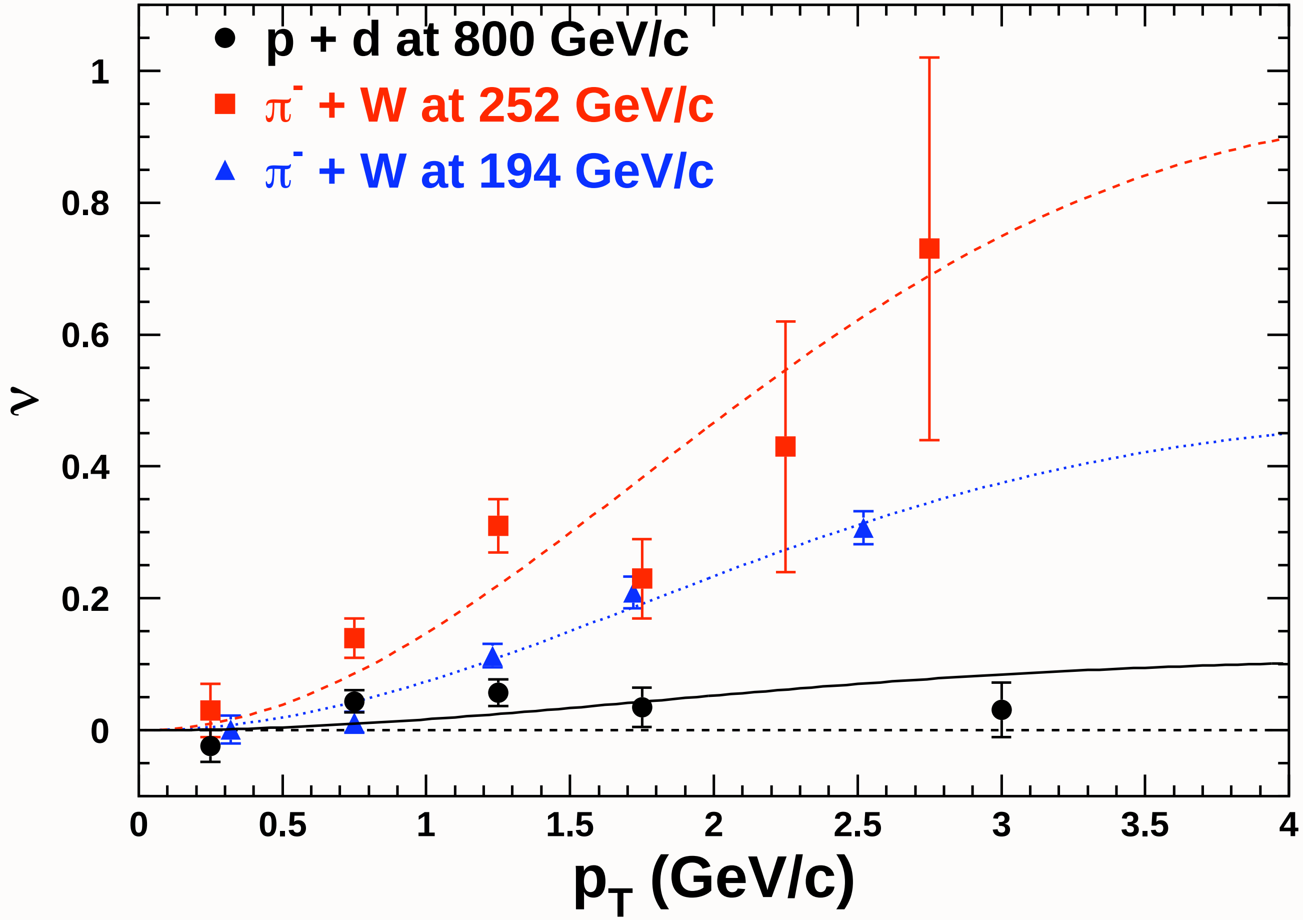
DY $\cos 2\phi$ correlation at leading twist from double ISI

Product of Boer - Mulders Functions

$$h_1^{\perp}(x_1, \boldsymbol{p}_{\perp}^2) \times \bar{h}_1^{\perp}(x_2, \boldsymbol{k}_{\perp}^2)$$

Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)



Huge Effect in

$\pi W \rightarrow \mu^+\mu^- X$

Negligible Effect

$pd \rightarrow \mu^+\mu^- X$

Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

Double Initial-State Interactions generate anomalous $\cos 2\phi$

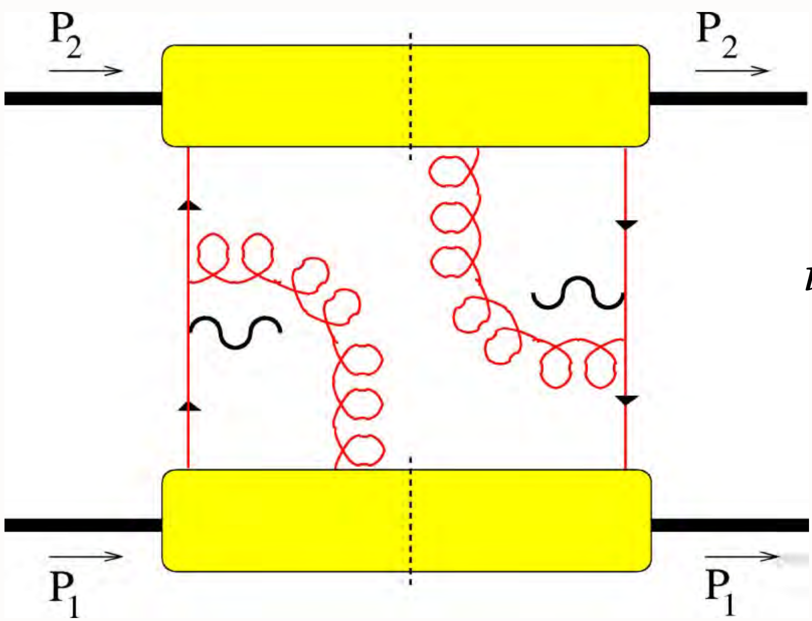
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta\, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

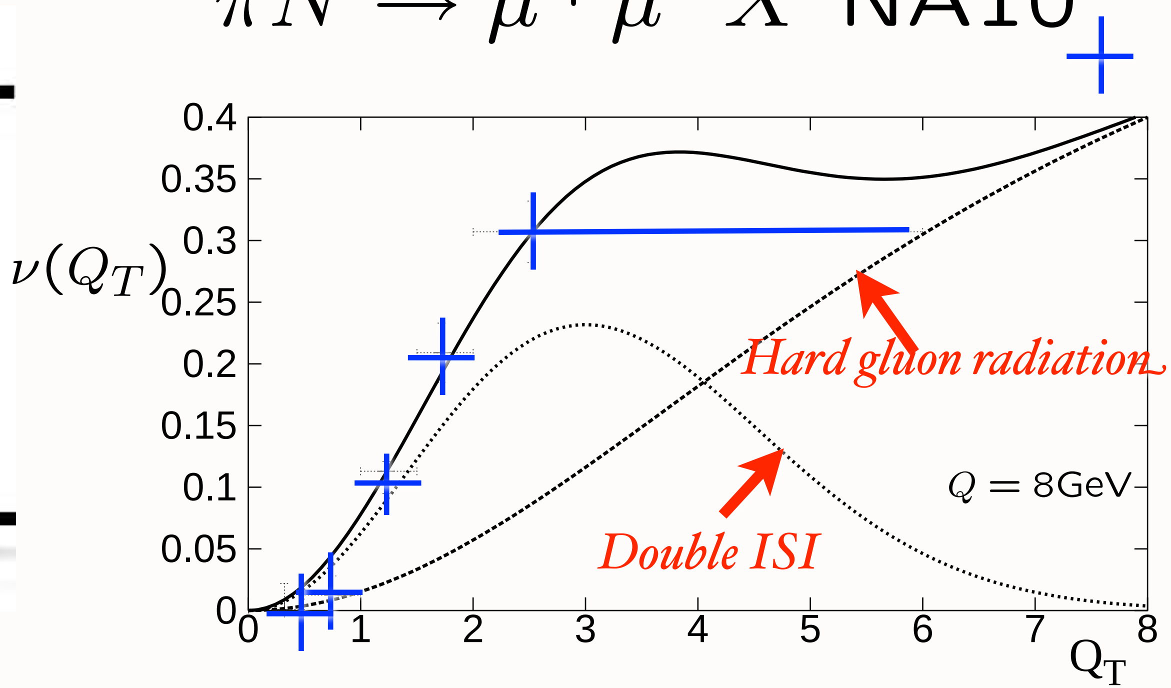
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$



Violates Lam-Tung relation!

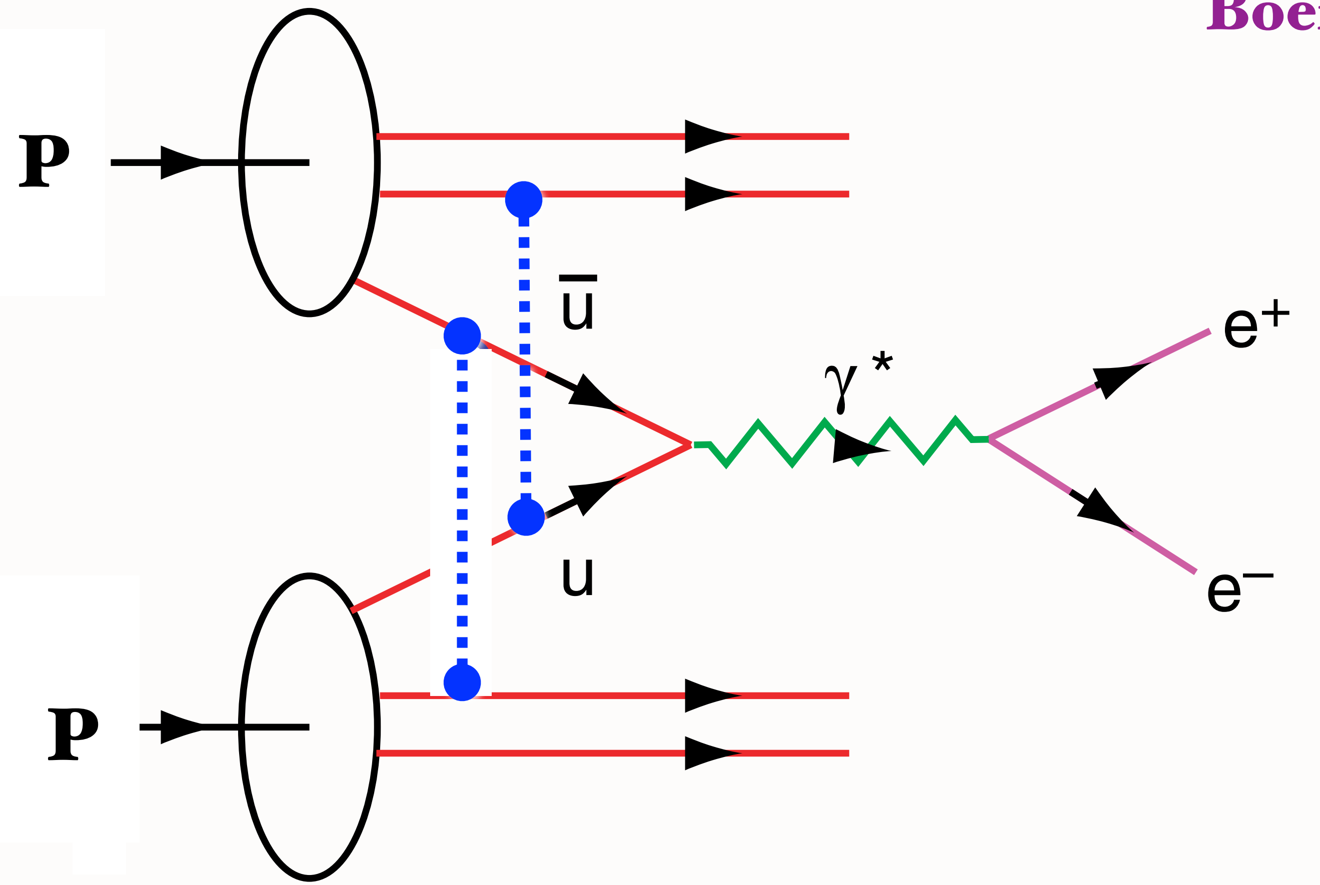
$\pi N \to \mu^+\mu^- X$ NA10



Model: Boer,

LHC Experiment

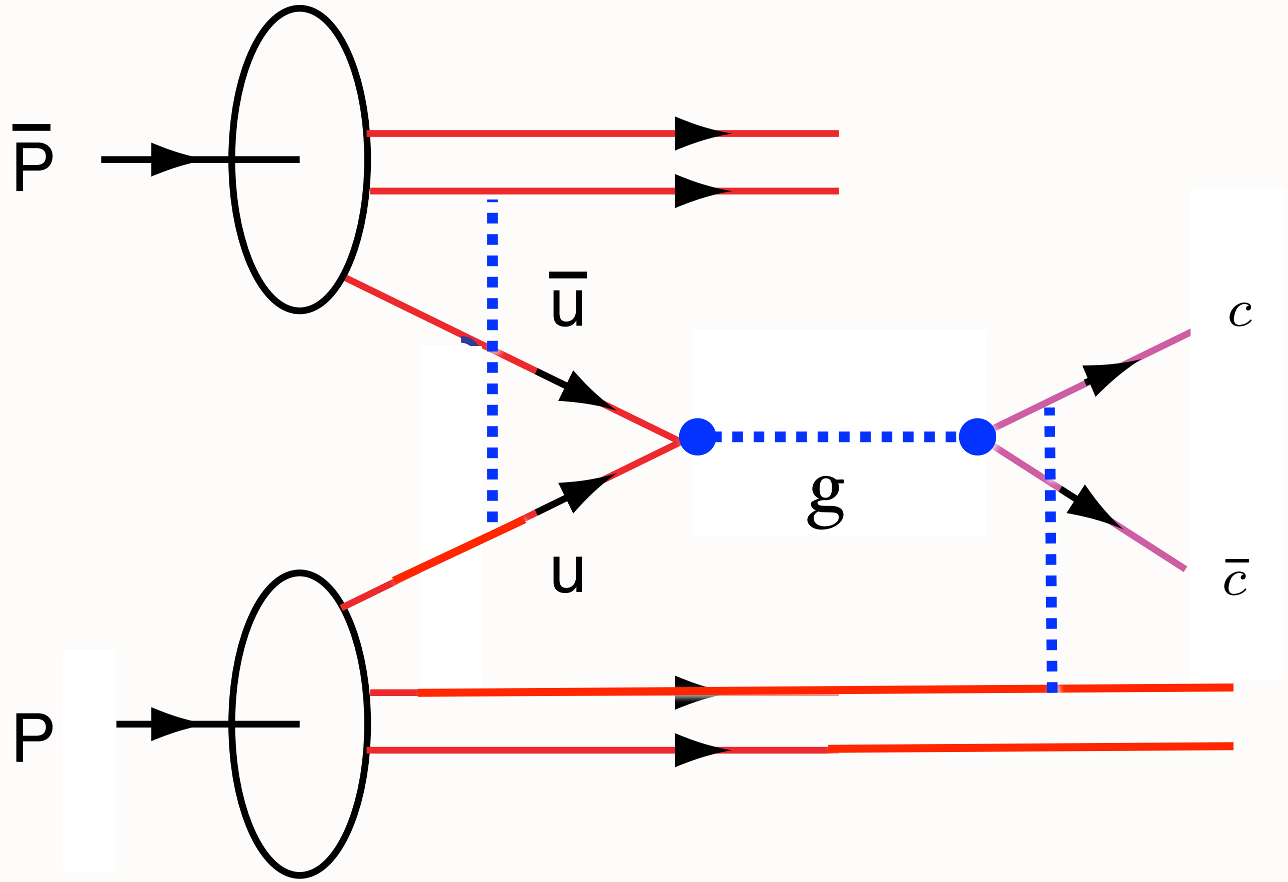
Boer, Hwang, sjb



DY $\cos 2\phi$ **correlation at leading twist from double ISI**

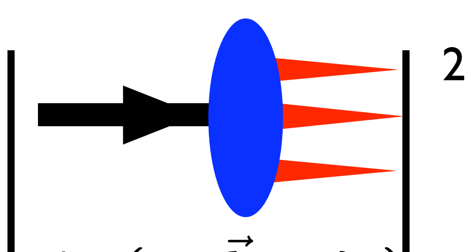
Product of Boer - Mulders Functions

$$h_1^\perp(x_1, \boldsymbol{p}_\perp^2) \times \bar{h}_1^\perp(x_2, \boldsymbol{k}_\perp^2)$$

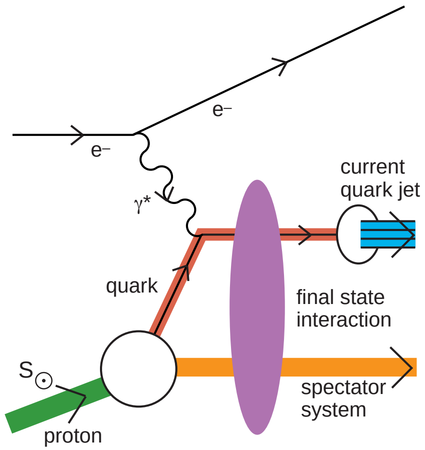


Problem for factorization when both ISI and FSI occur!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

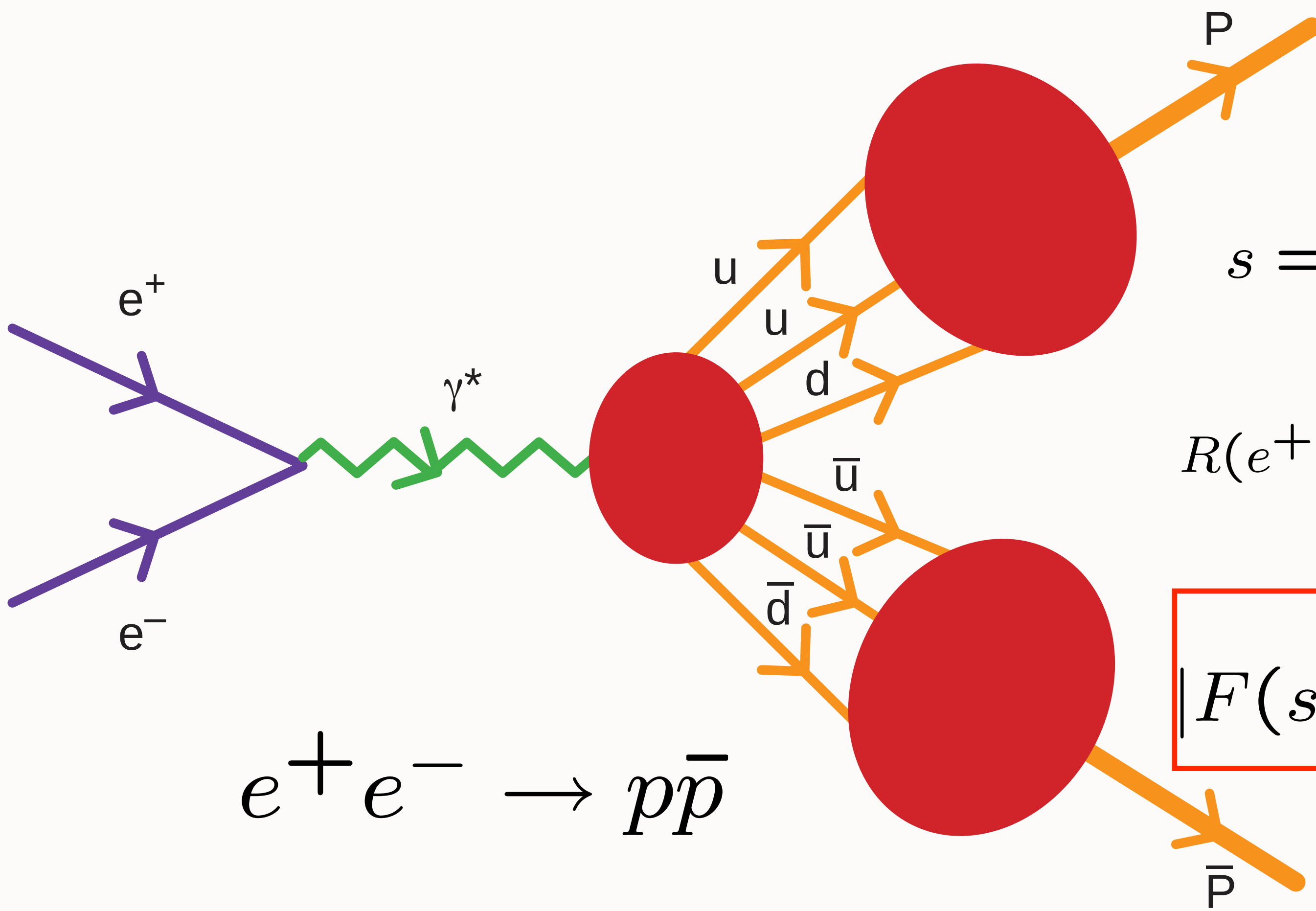


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Exclusive Processes

What if we ask for a specific final state?



$$s = (E_{e^+} + E_{e^-})^2$$

$$R(e^+e^- \to H\bar{H}) \propto |F(s)|^2$$

$$\boxed{|F(s)| \propto [\frac{1}{s}]^{n_q - 1}}$$

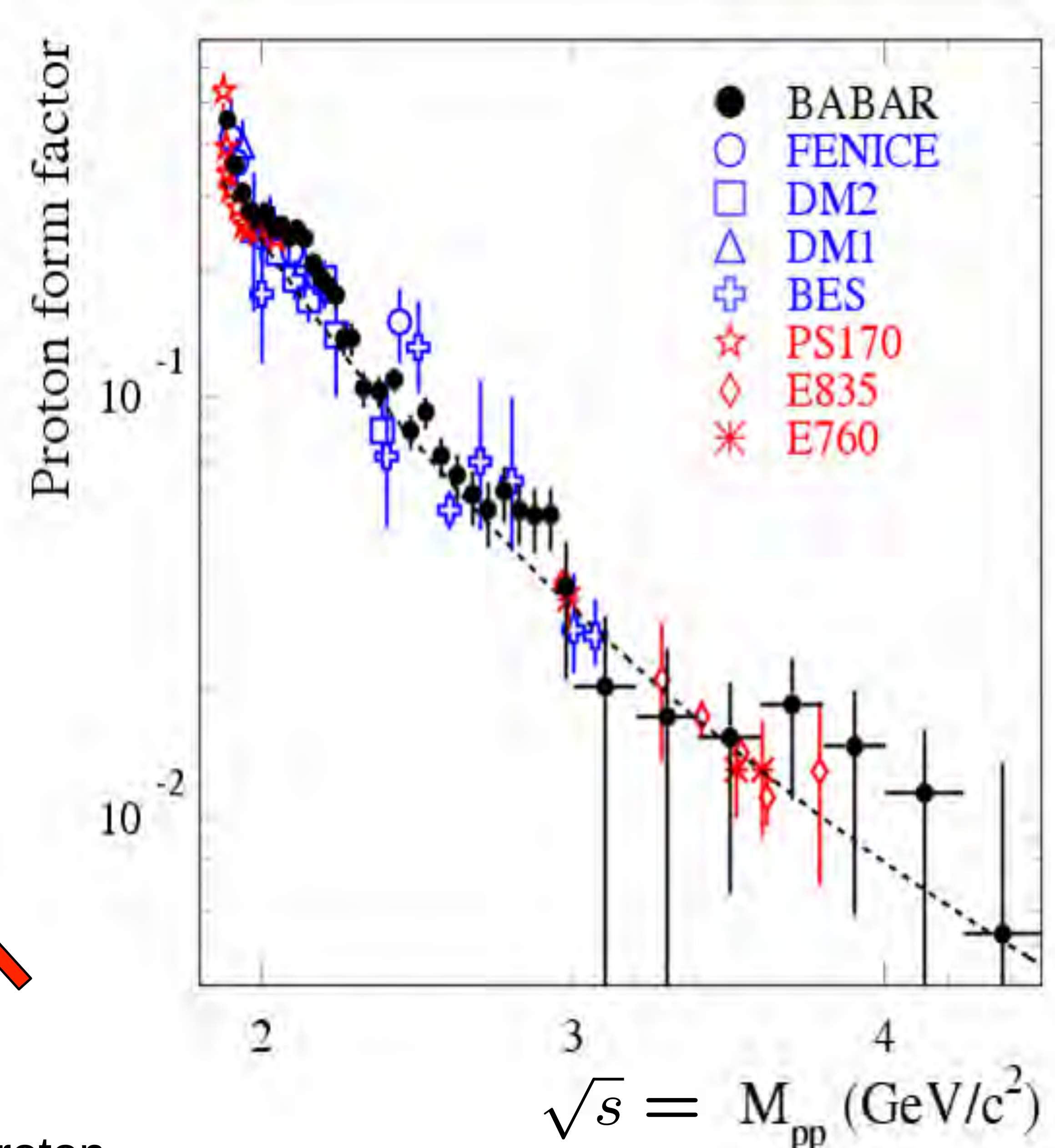
$$e^+e^- \to p\bar{p}$$

Probability decreases with number of constituents!

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2,$$

$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$

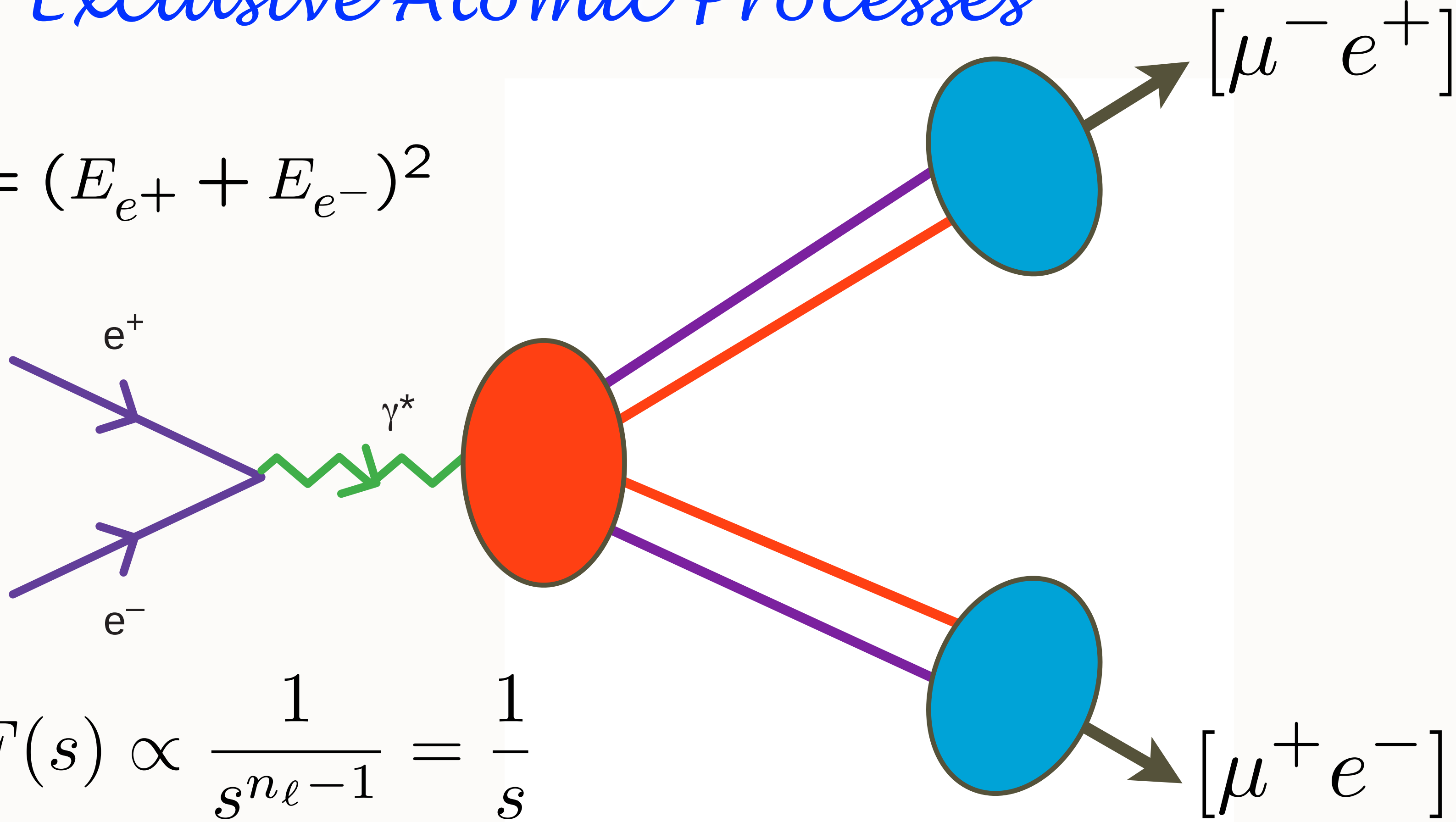
n_q - 1 = 3 - 1 = 2



Quark counting for 3 quarks in proton

Exclusive Atomic Processes

$$s = (E_{e^+} + E_{e^-})^2$$

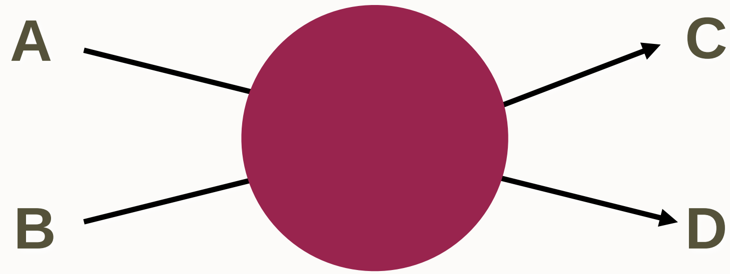


$$F(s) \propto \frac{1}{s^{n_\ell - 1}} = \frac{1}{s}$$

$$R = \frac{\sigma(e^+e^- \to [\mu^+e^-] + [\mu^-e^+])}{\sigma(e^+e^- \to \mu^+\mu^-)} = |F(s)|^2 \propto \frac{1}{s^2}$$

Probability decreases with number of constituents

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

Farrar & sjb;
Matveev, Muradyan, Tavkhelidze

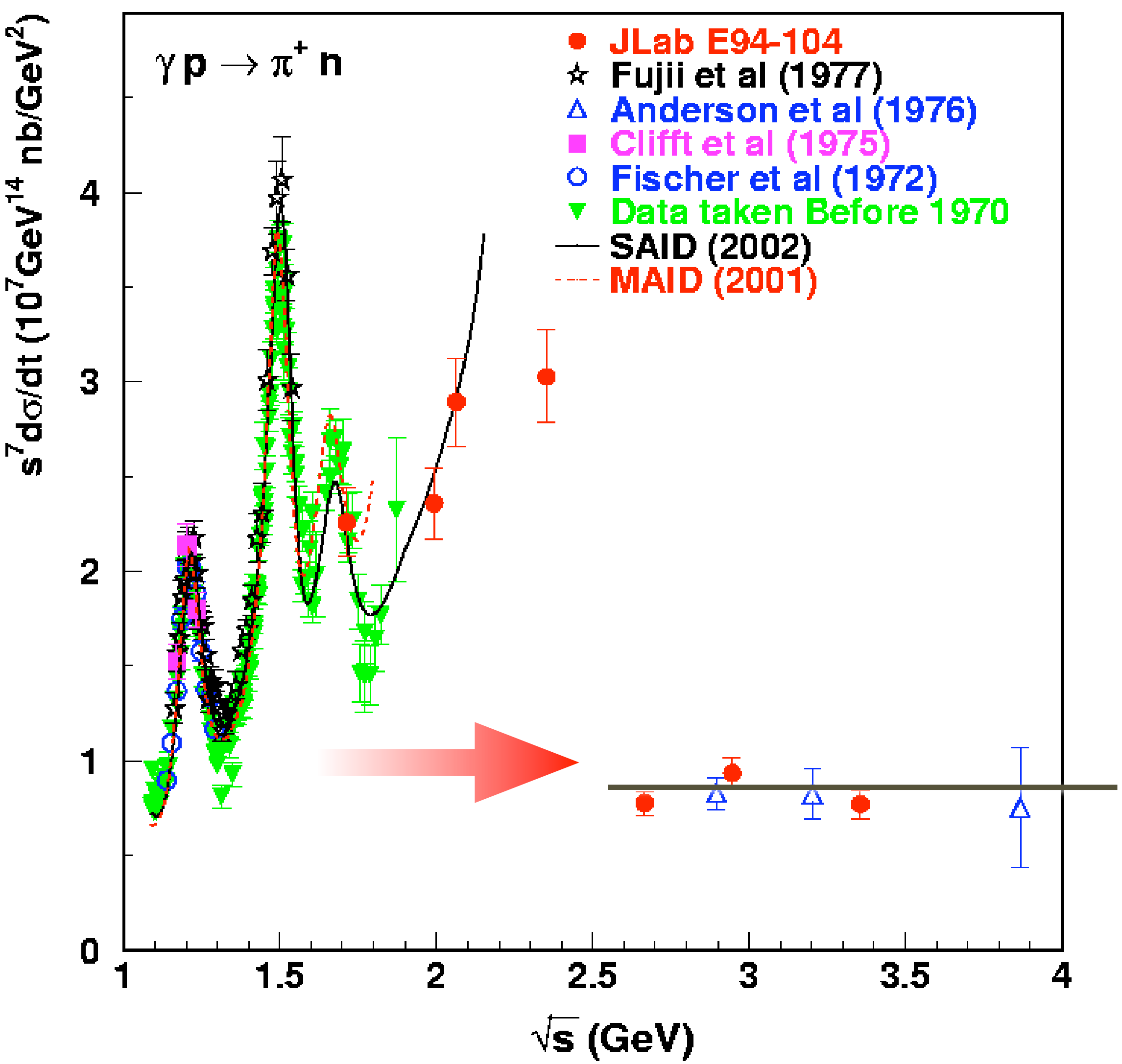
QED predicts leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

$$s, -t >> m_\ell^2$$

Test of Scaling Laws

Constituent counting rules

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719



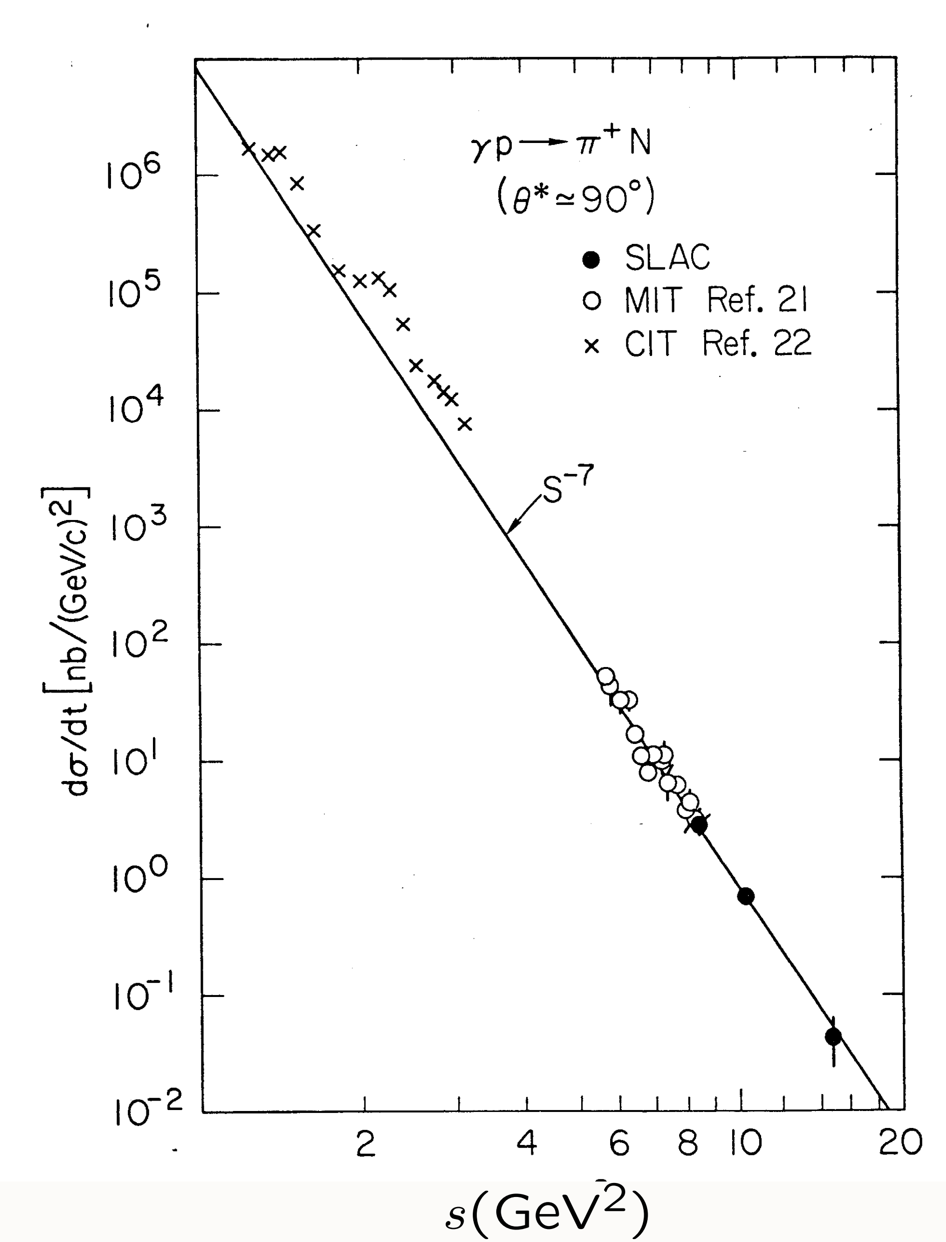
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \rightarrow C+D) = F_{A+B\rightarrow C+D}(\theta_{CM})$$

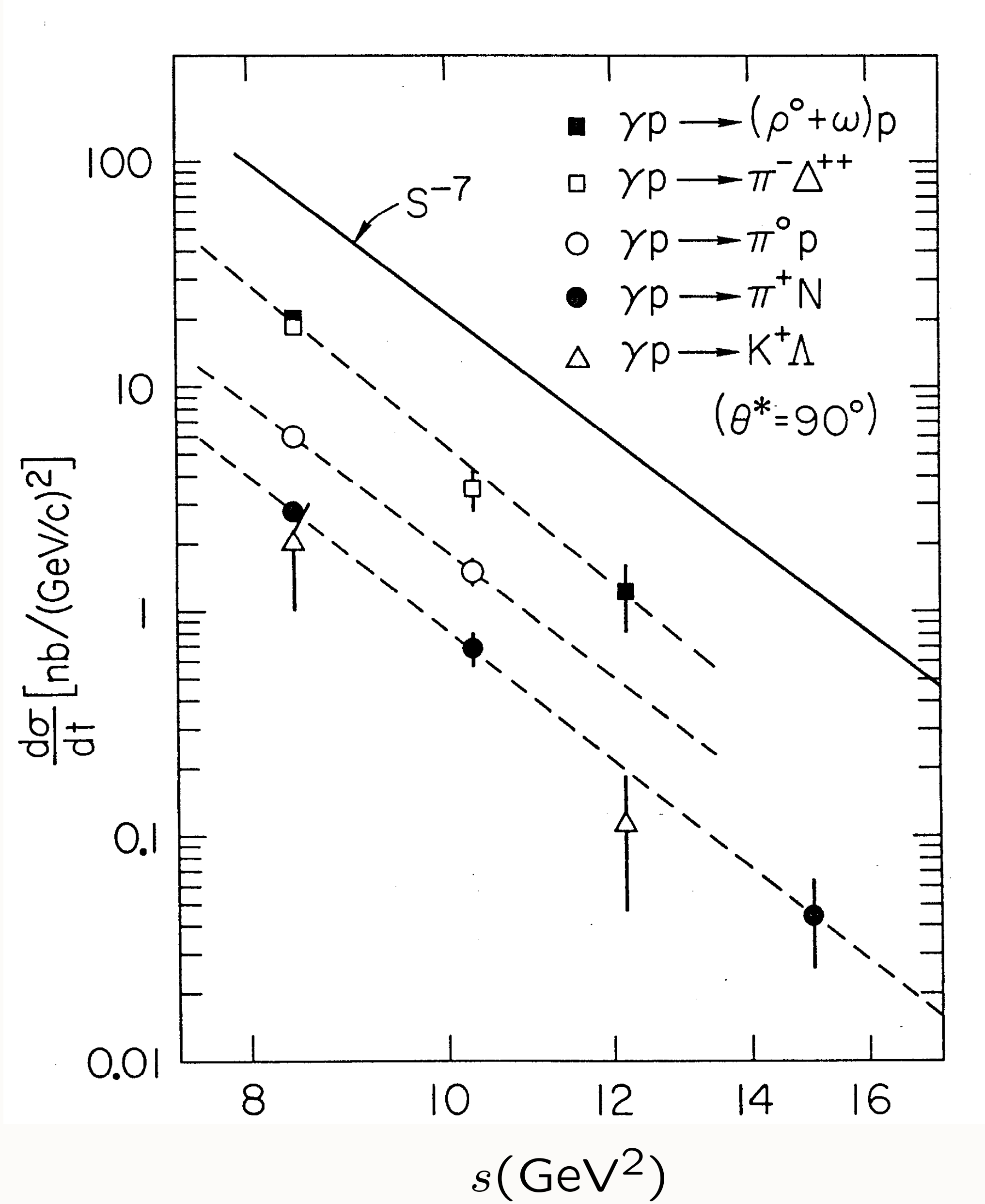
$$s^7\frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

$s^7 d\sigma/dt(\gamma p \rightarrow \pi^+ n) \sim const$
$fixed\ \theta_{CM}$ scaling

Conformal invariance at high momentum transfers!

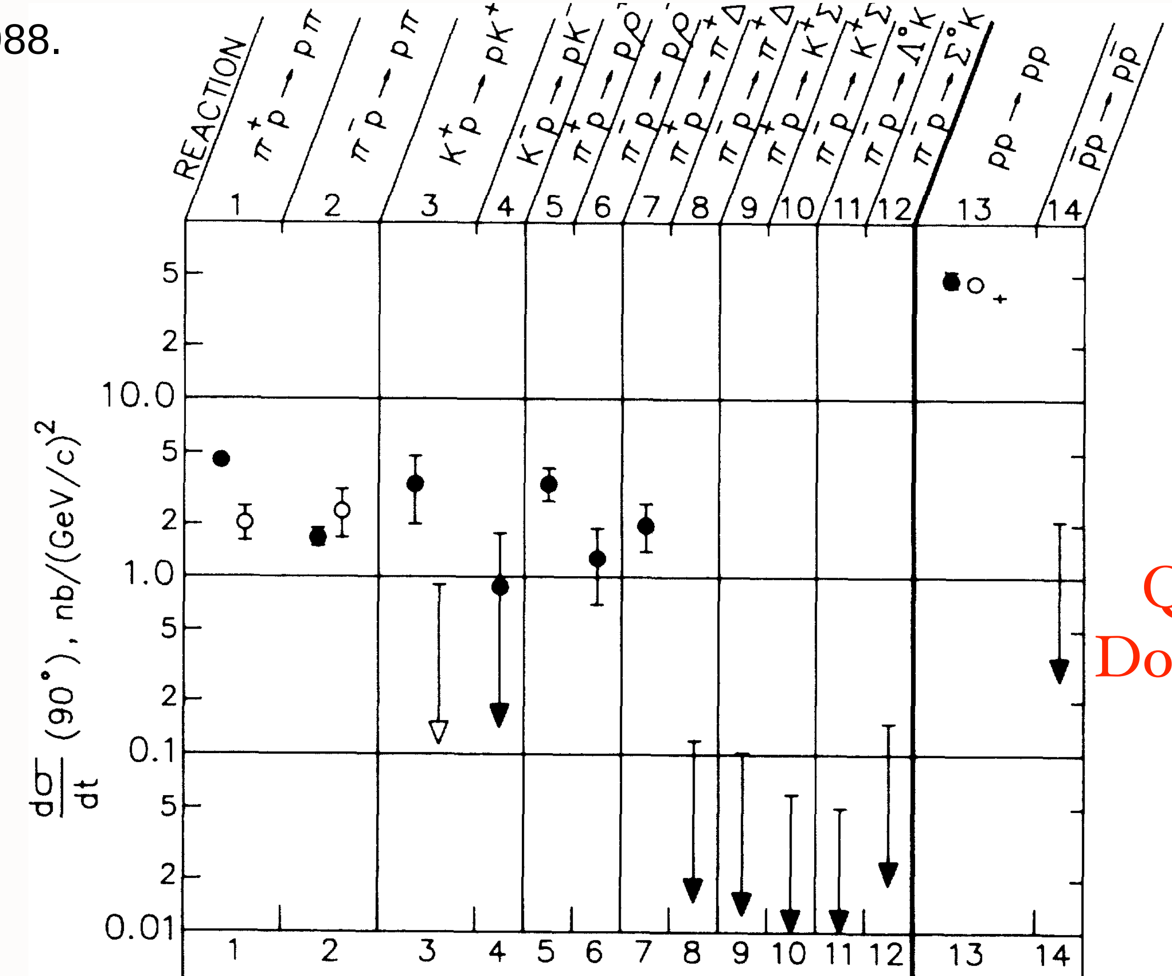




Counting Rules: n=9

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

B.R. Baller *et al.*. 1988.
Published in
Phys.Rev.Lett. 60:1118-1121,1988



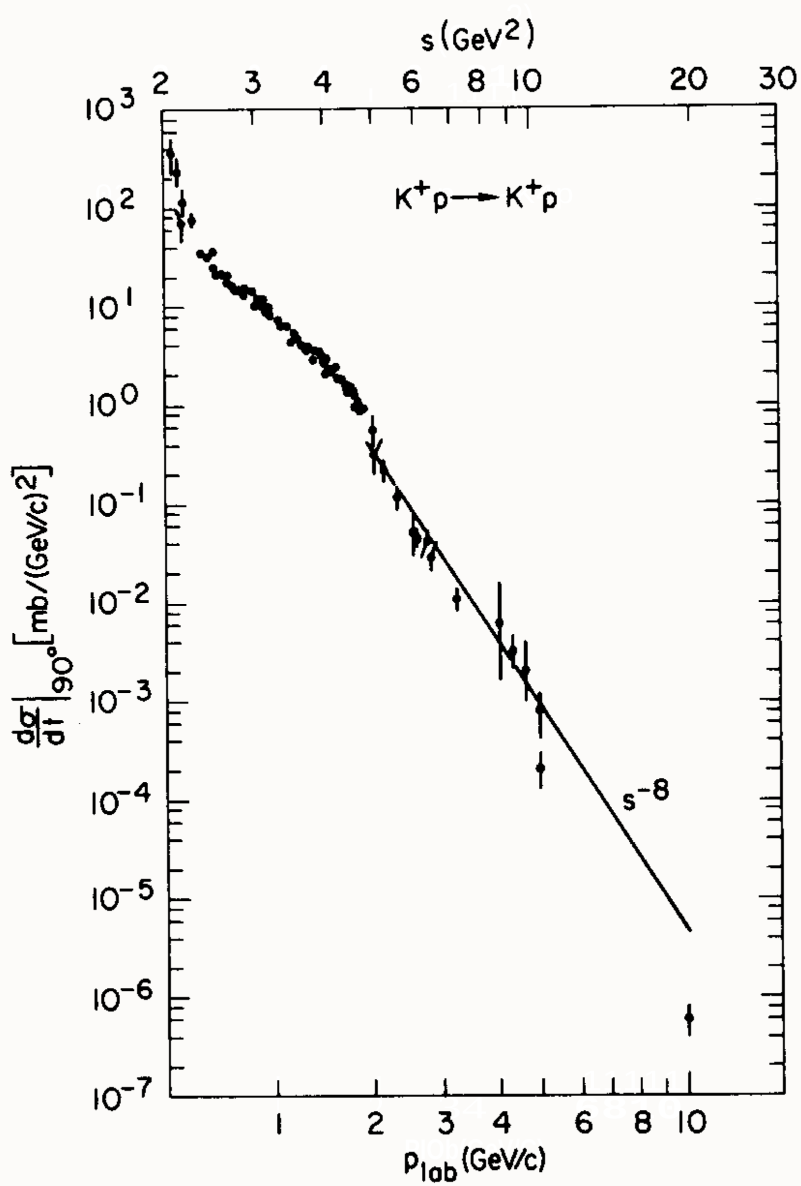
Comparison of Exclusive Reactions at Large *t*

Quark Interchange: Dominant Dynamics at large t, u

Relative Rates Correct

The cross section and upper limits (90% confidence level) measured by this experiment are indicated by the filled circles and arrowheads. Values from this experiment and from previous measurements represent an average over the angular region of $-0.05 < \cos\theta_{c.m.} < 0.10$. The other measurements were obtained from the following references: $\pi^+ p$ and $K^+ p$ elastic, Ref. 5; $\pi^- p \to p\pi^-$, Ref. 6; $pp \to pp$, Ref. 7: Allaby, open circle; Akerlof, cross. Values for the cross sections [(Reaction), cross section in nb/(GeV/c)^2] are as follows: (1), 4.6 ± 0.3; (2), 1.7 ± 0.2; (3), 3.4 ± 1.4; (4), $0.9^{+0.7}_{-0.9}$; (5), 3.4 ± 0.7; (6), 1.3 ± 0.6; (7), 2.0 ± 0.6; (8), < 0.12; (9), < 0.1; (10), < 0.06; (11), < 0.05; (12), < 0.15; (13), 48 ± 5; (14), < 2.1.

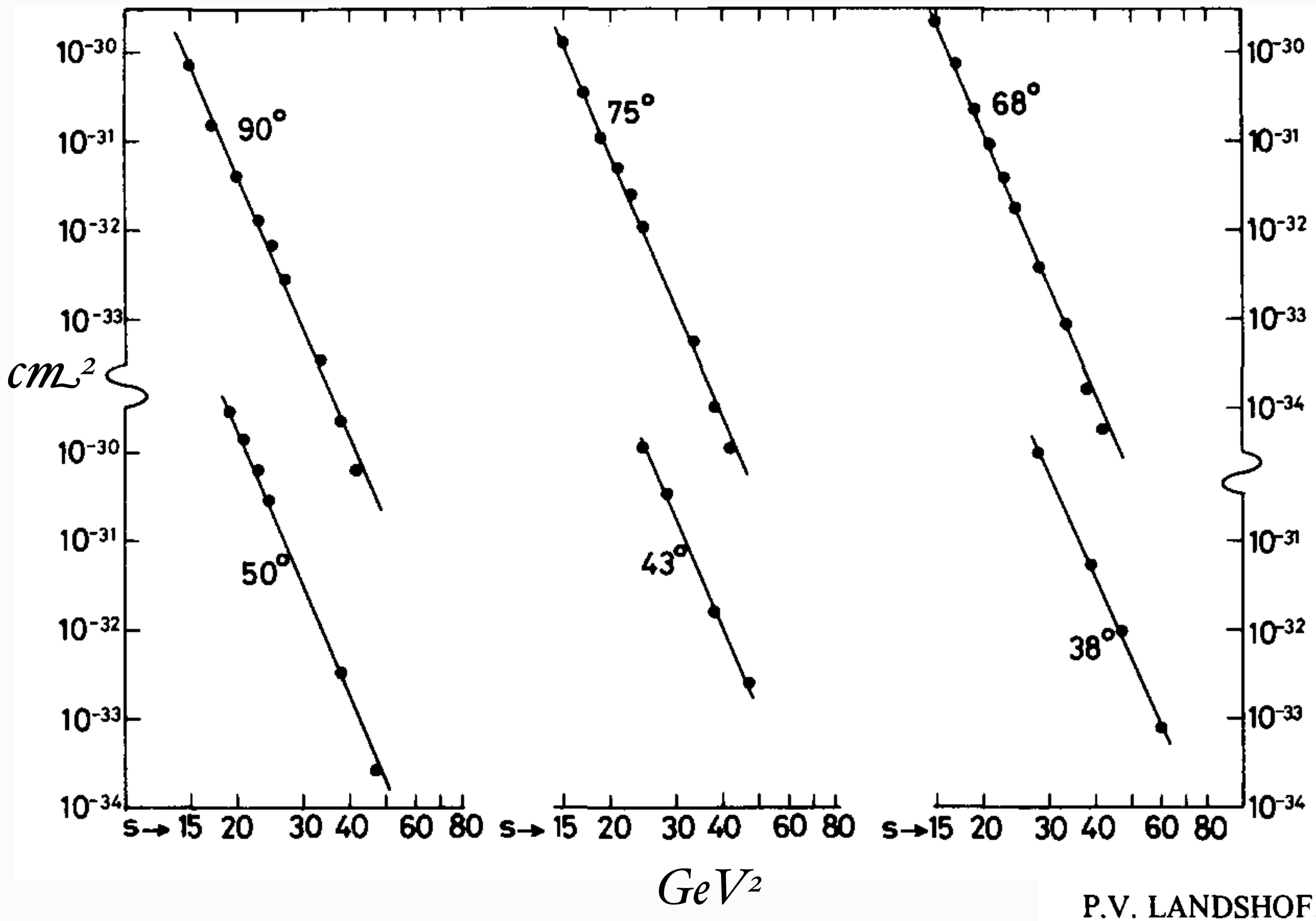
Quark-Counting



$$\frac{d\sigma}{dt}(K^+p \rightarrow K^+p) = \frac{F(\theta_{CM})}{s^8}$$

$$n = 2 \times 3 + 2 \times 2 - 2 = 8$$

Quark-Counting: $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$

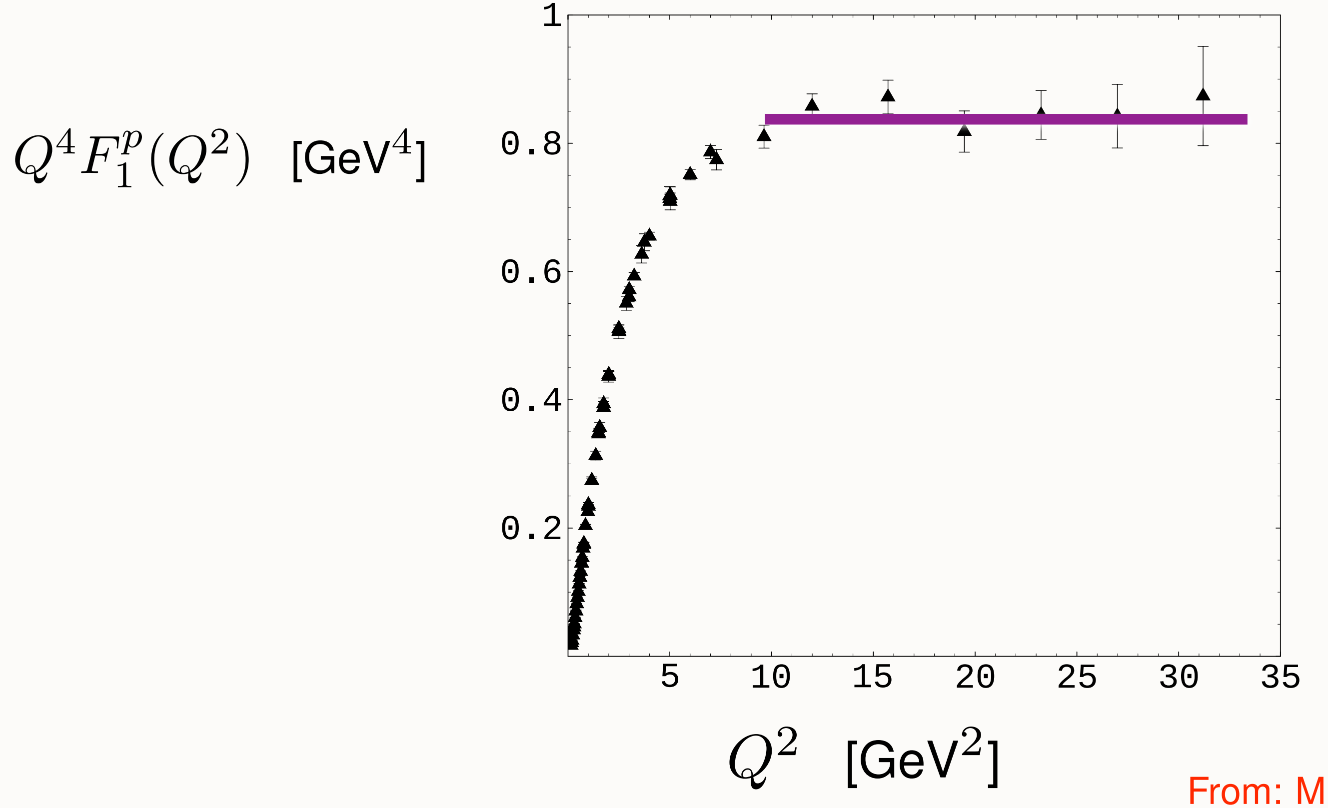


P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions



$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$

measured in electron-proton elastic scattering

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

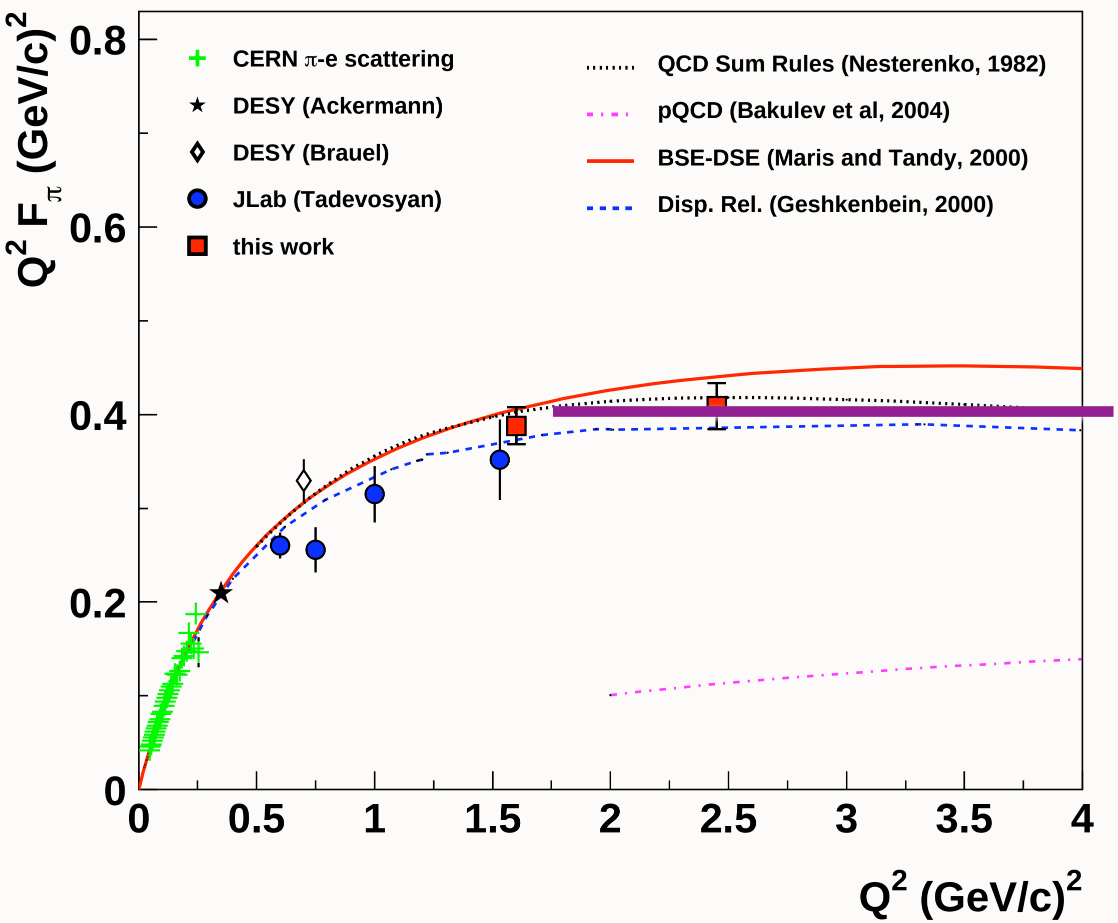
$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

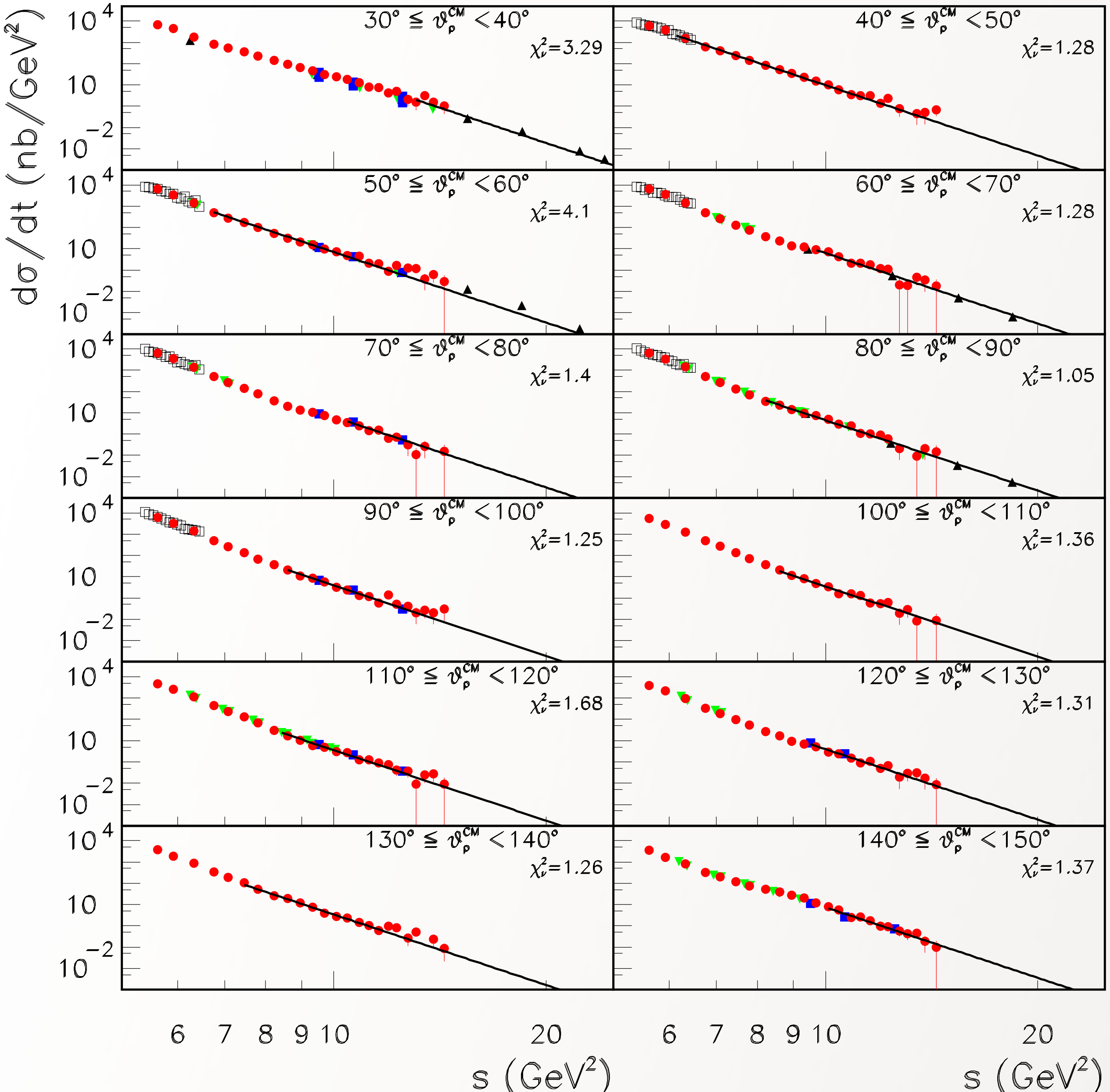
- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const



Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$$n_{tot}-2 = (1+6+3+3)-2 = 11$$

Reflects conformal invariance

- Remarkable Test of Quark Counting Rules

- Deuteron Photo-Disintegration $\gamma d \to np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$$

at high p_T

Primary Evidence for Quarks

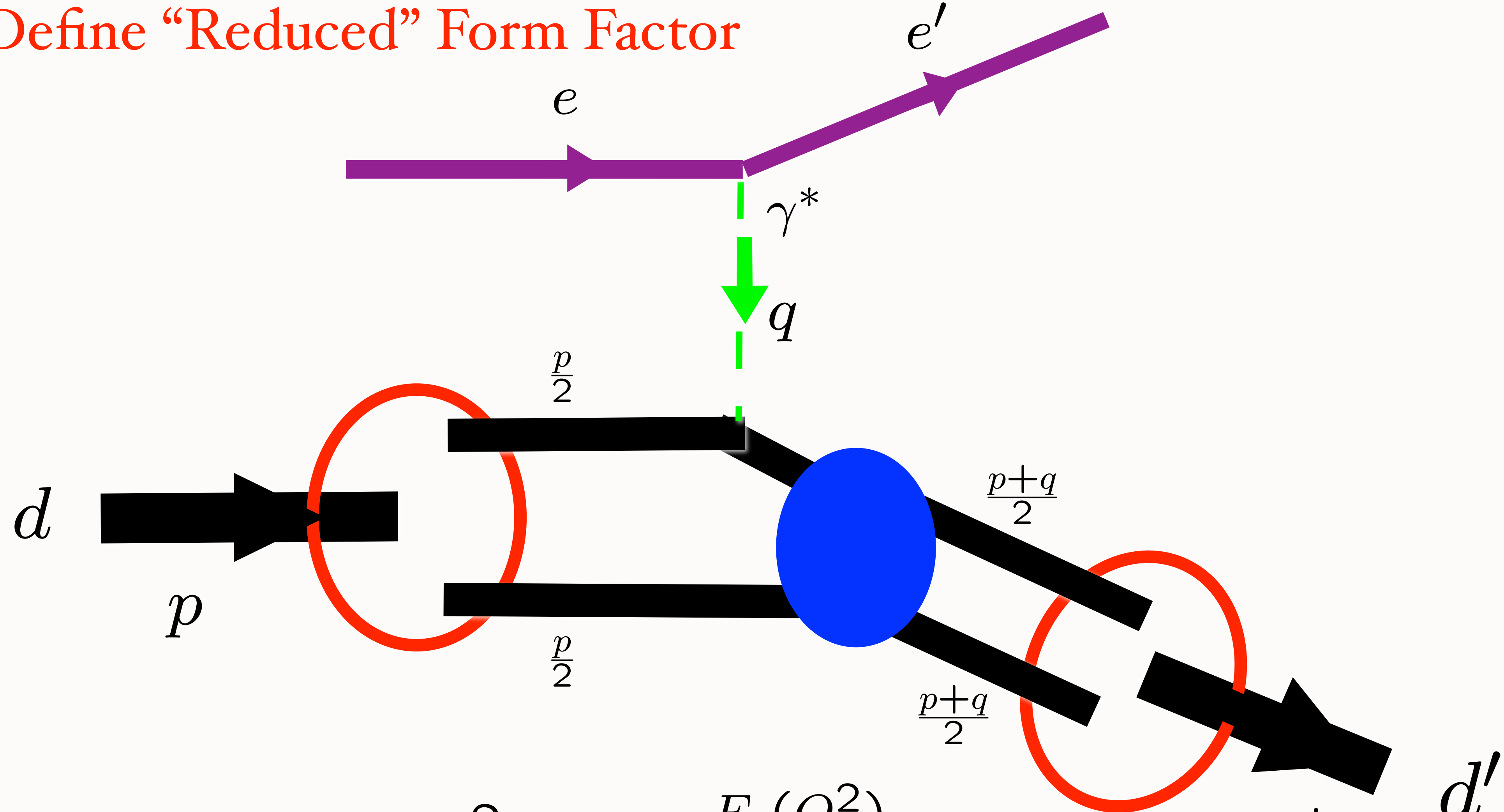
- Electron-Proton Inelastic Scattering: $ep \to e'X$
  Electron scatters on pointlike constituents with fractional charge; final-state jets

- Electron-Positron Annihilation: $e^+e^- \to X$
  Production of pointlike pairs with fractional charges and 3 colors; quark, antiquark, gluon jets

- Exclusive hard scattering reactions: $pp \to pp, \ \gamma p \to \pi^+ n, \ ep \to ep$
  probability that hadron stays intact counts number of its pointlike constituents:

Quark Counting Rules

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2)=\left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d-\gamma_m^d}\left[1+O\left(\alpha_s(Q^2),\frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2)\equiv\frac{F_d(Q^2)}{F_N^{\,2}(Q^2/4)}\,.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2)\sim\frac{\alpha_s(Q^2)}{Q^2}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-(2/5)\,C_F/\beta}$$

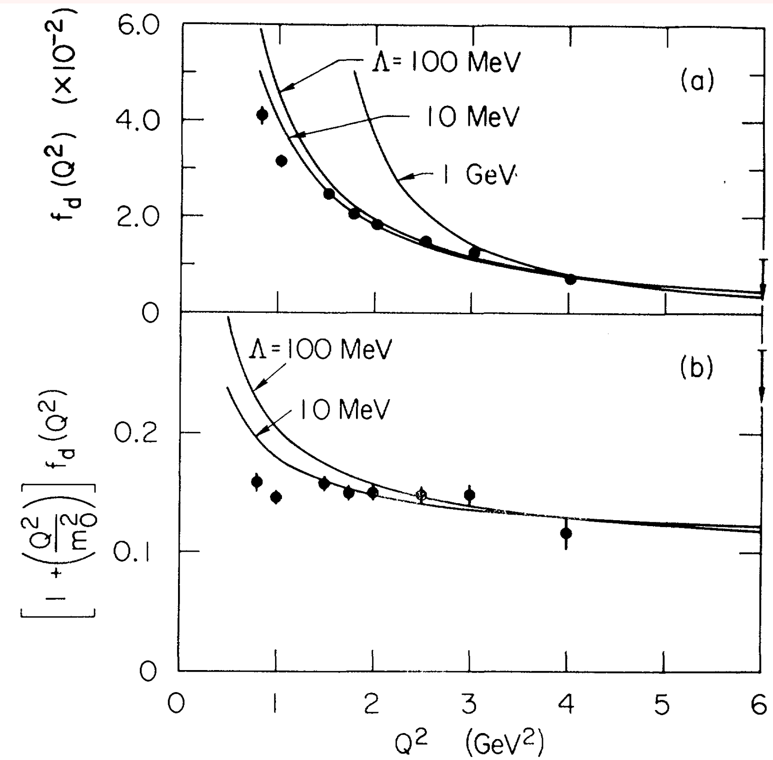
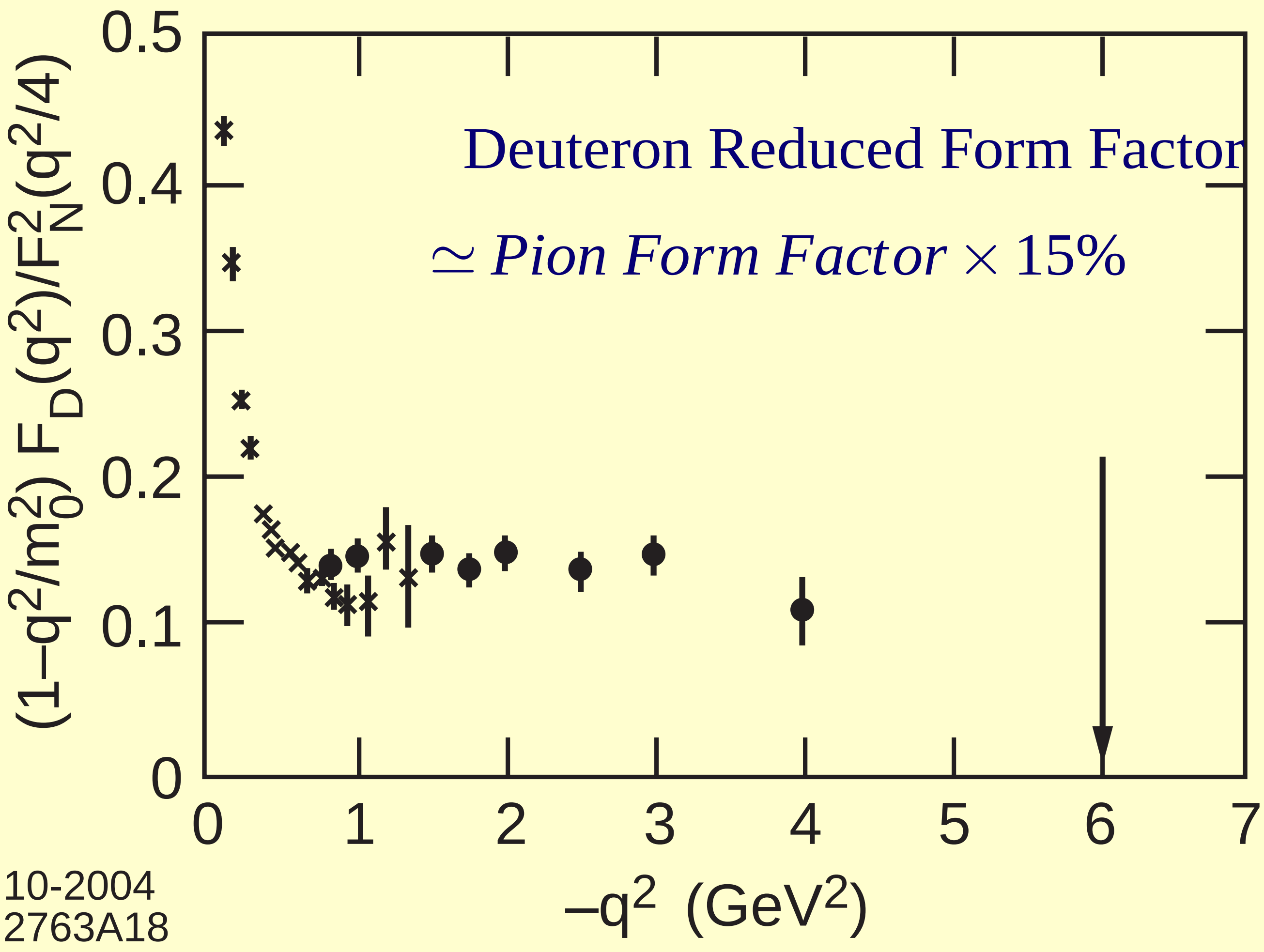


FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2)\propto(1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2)=[1+Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2=4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1+(Q^2/m_0^2)]f_d(Q^2)\propto[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2=0.28\ \mathrm{GeV}^2$ is used (Ref. 8).



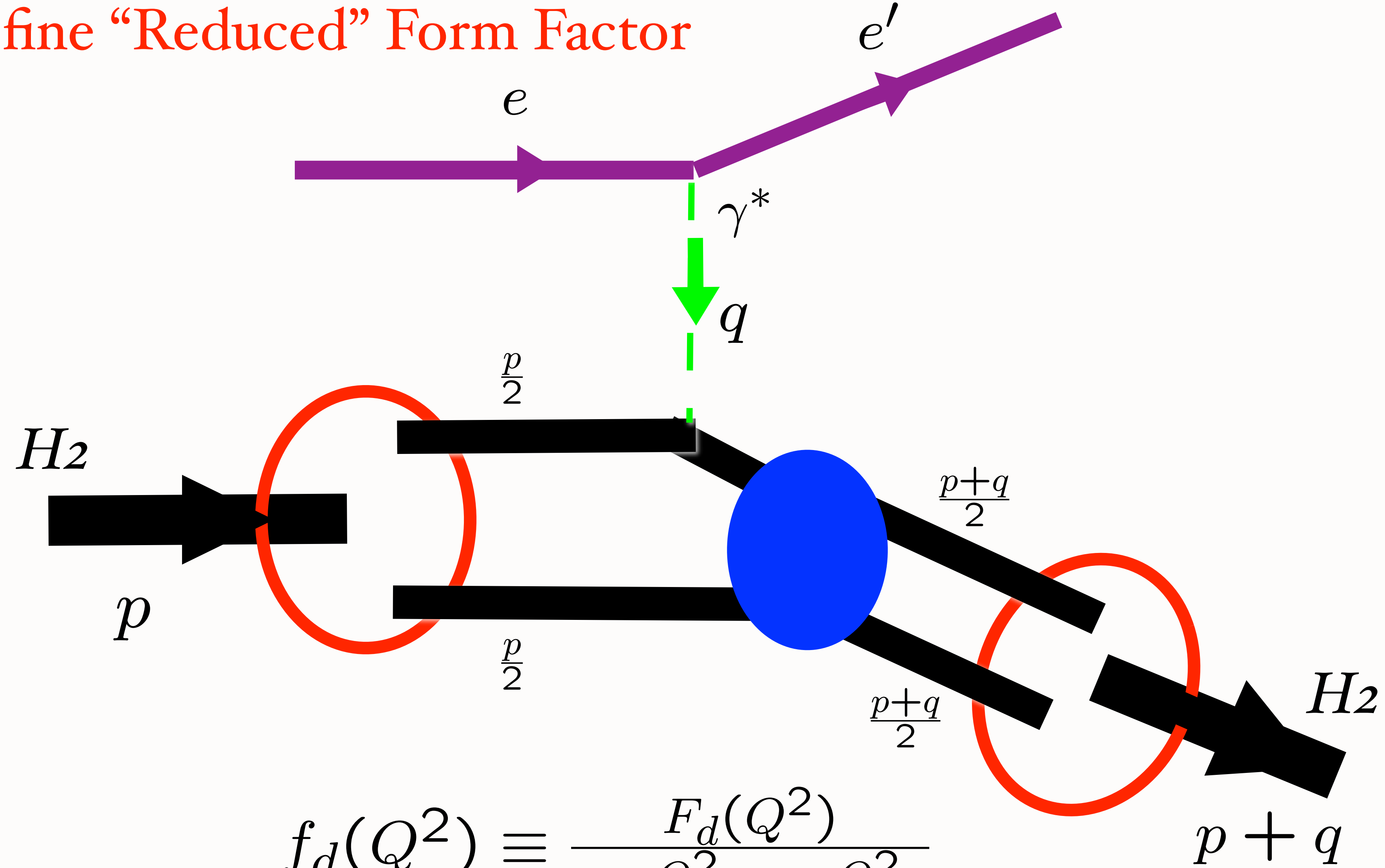
- 15% Hidden Color in the Deuteron

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high Q^2

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-molecule scattering!

Quark Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
- $$F_H(Q) \propto \frac{1}{(Q^2)^{n-1}}$$ n = # elementary constituents

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

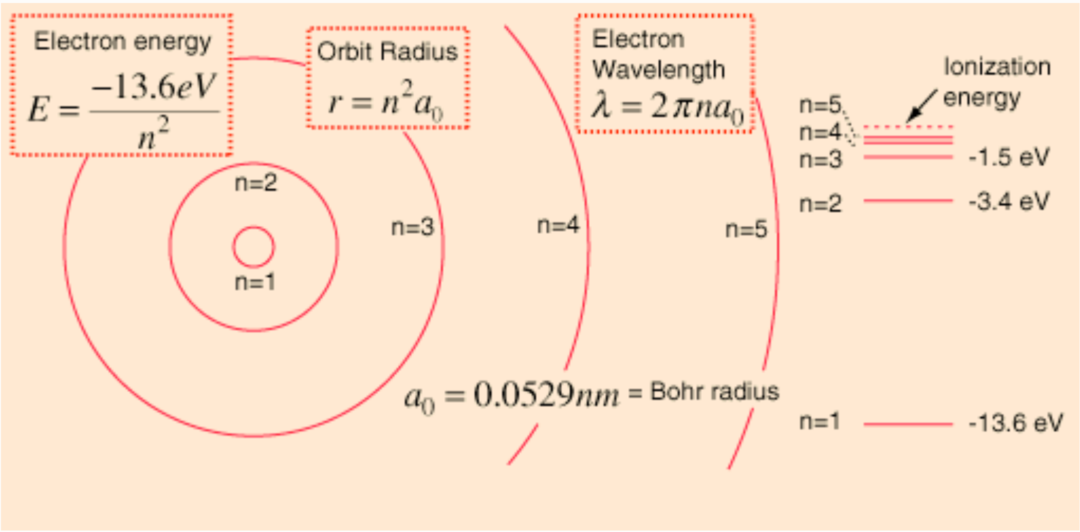
Coulomb potential

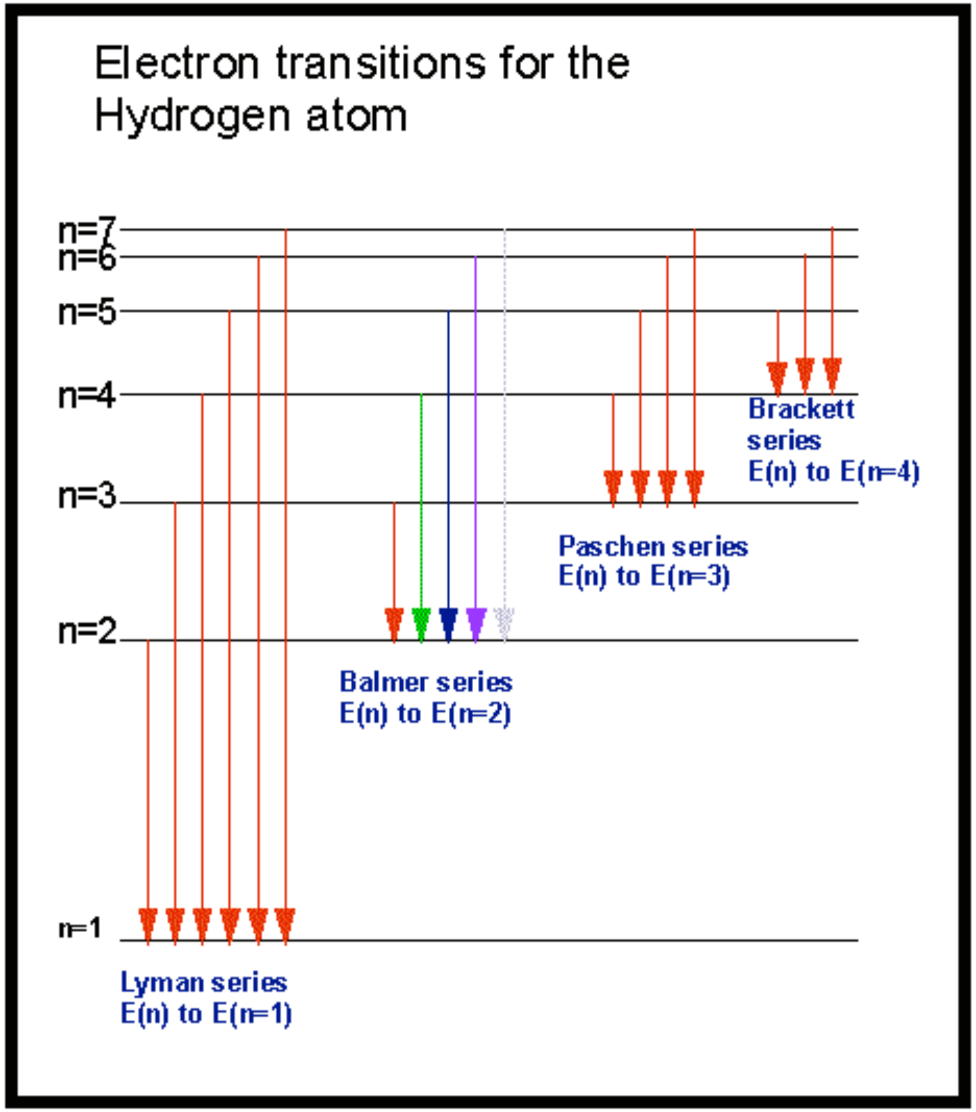
Bohr Spectrum

Semiclassical first approximation to QED

Schrödinger Eq.

Bohr Atom


Electron energy
$E = \frac{-13.6eV}{n^2}$
Orbit Radius
$r = n^2 a_0$
Electron Wavelength
$\lambda = 2\pi n a_0$
Ionization energy
n=5
n=4
n=3
-1.5 eV
n=2
-3.4 eV
n=1
-13.6 eV
n=2
n=1
n=3
n=4
n=5
$a_0 = 0.0529nm$ = Bohr radius


Electron transitions for the Hydrogen atom
n=7
n=6
n=5
n=4
n=3
n=2
n=1
Brackett series
E(n) to E(n=4)
Paschen series
E(n) to E(n=3)
Balmer series
E(n) to E(n=2)
Lyman series
E(n) to E(n=1)

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis ζ, ϕ

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables

$$(\vec{\zeta},\varphi), \; \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta)U(\zeta)\phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

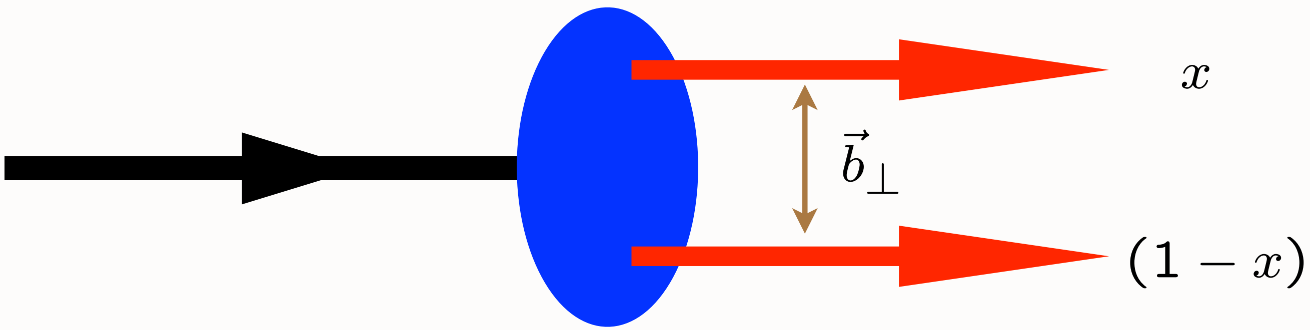
Light-Front Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall
confining potential:

Light-Front Schrödinger Equation

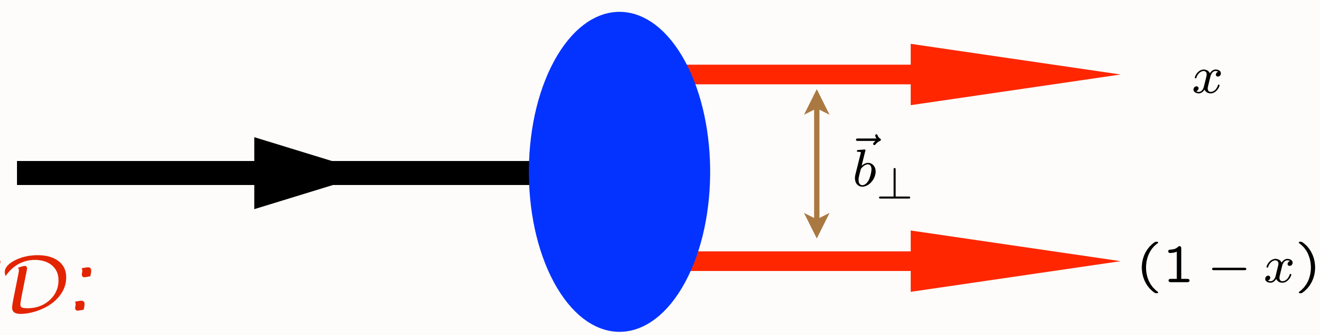
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

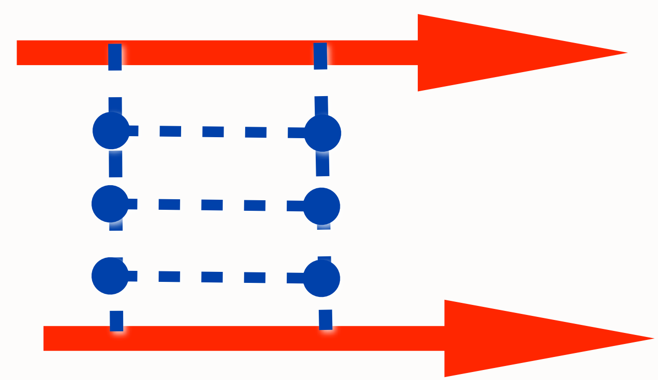
$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \, \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



AdS/QCD:

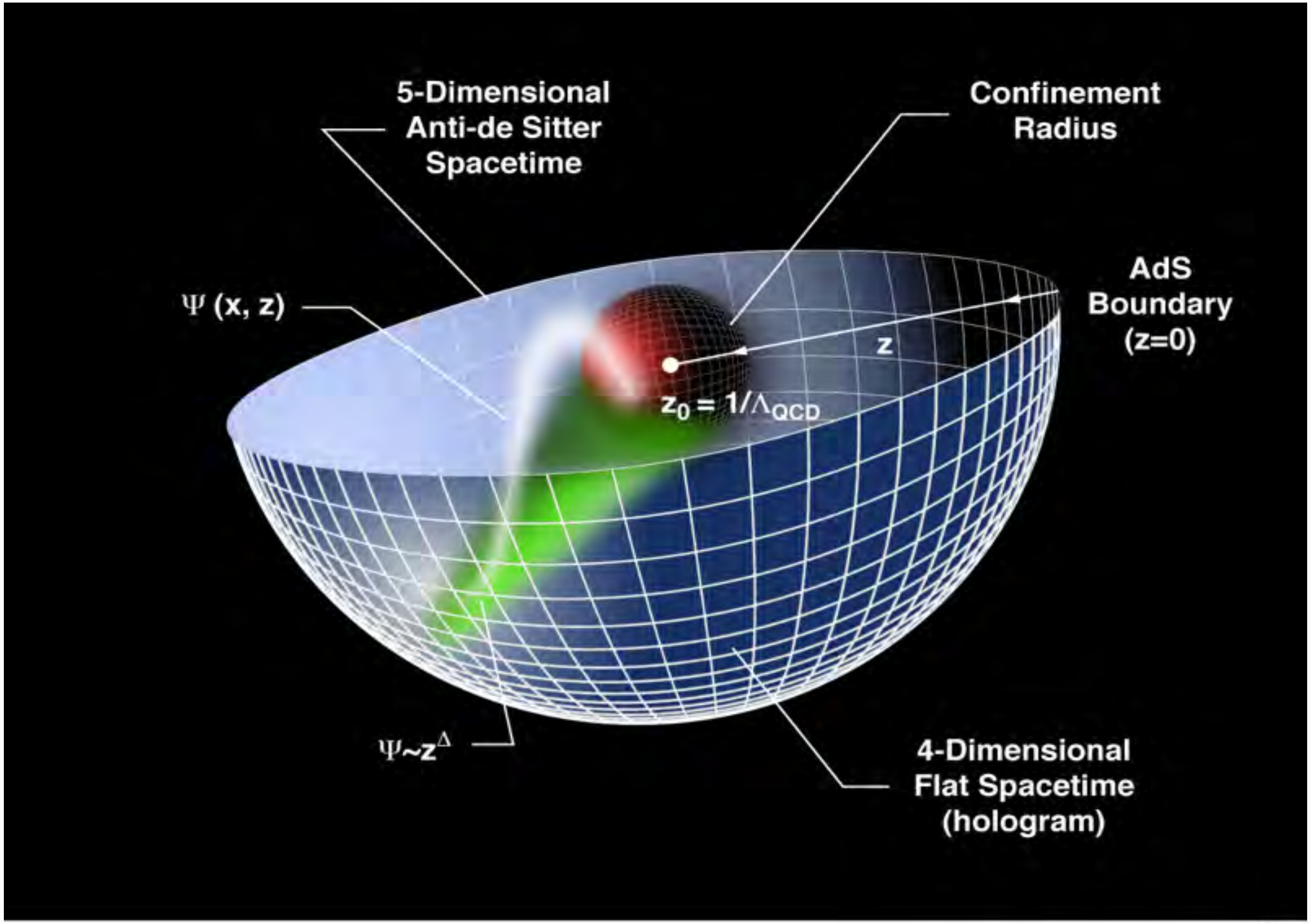
$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$



U is the exact QCD potential

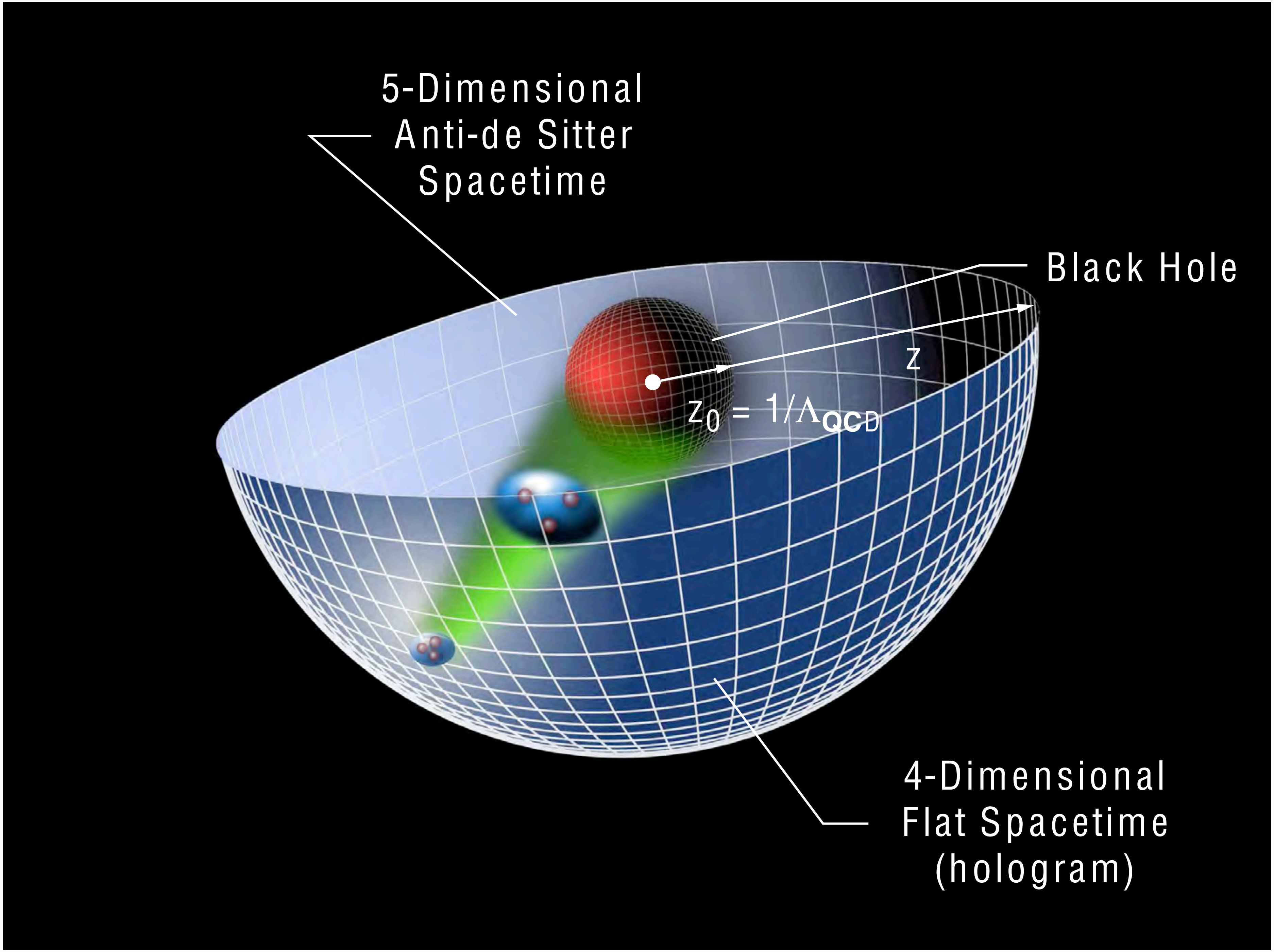
Conjecture: 'H'-diagrams generate U?

Applications of AdS/CFT to QCD

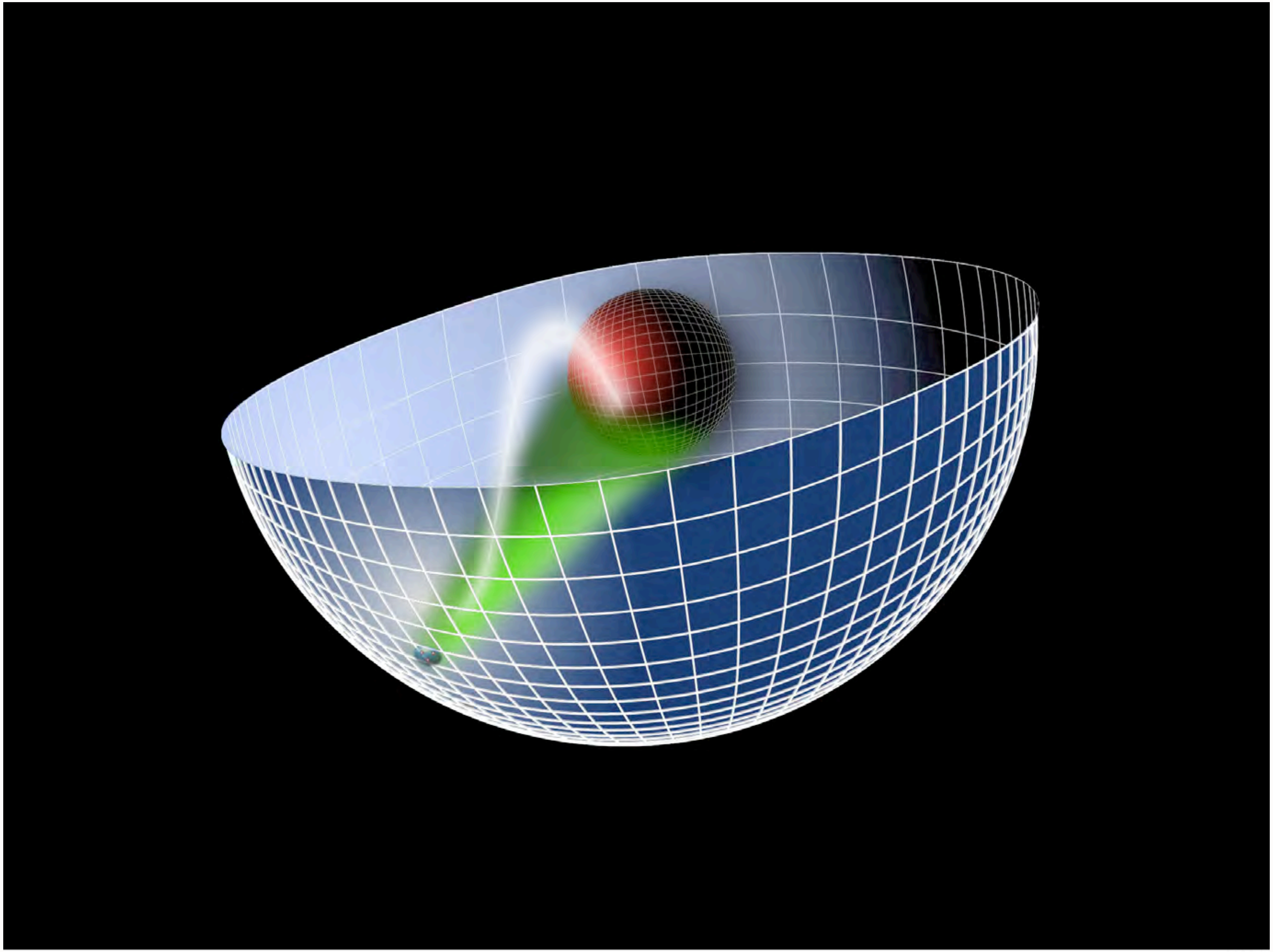


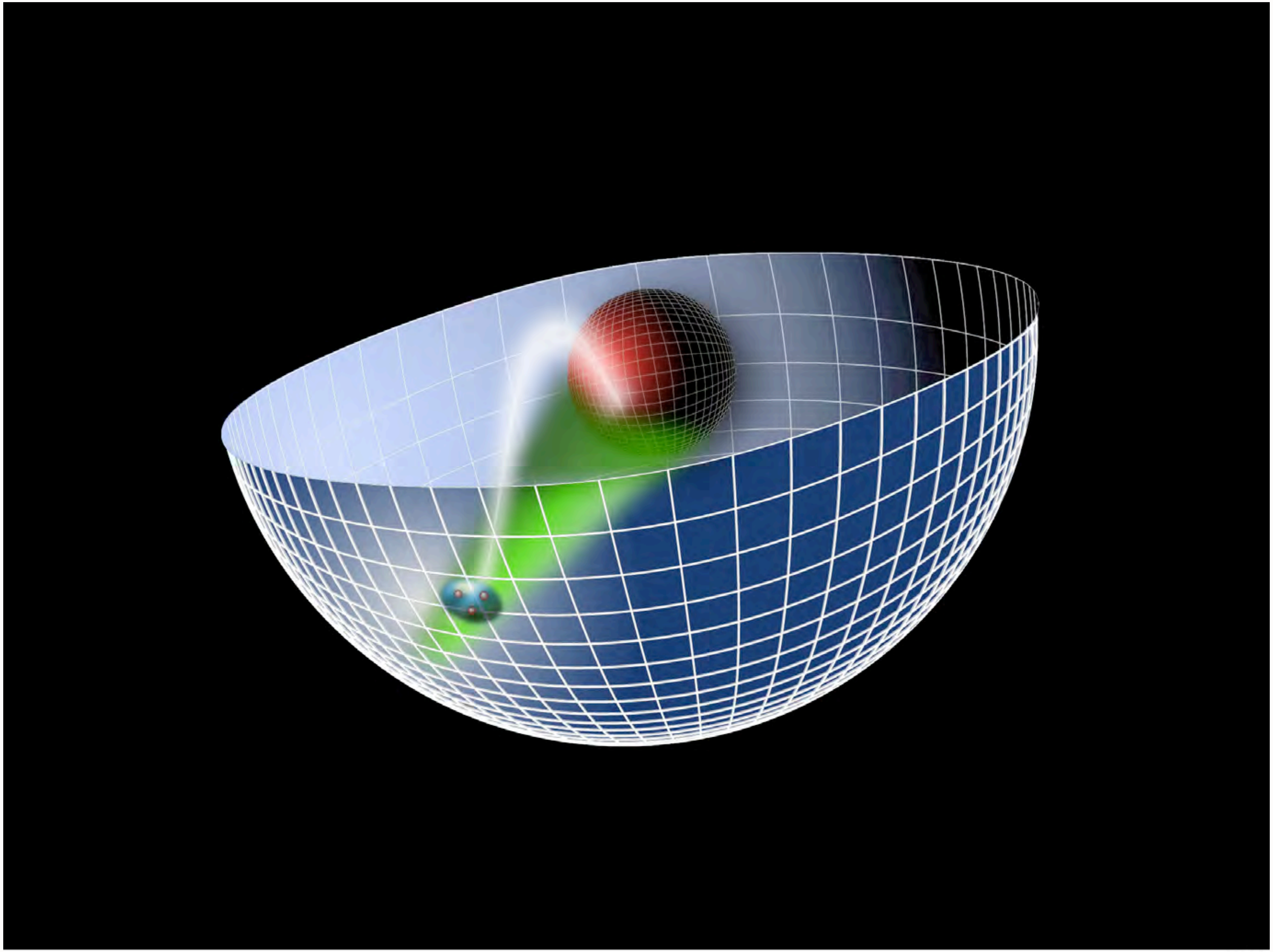
Changes in physical length scale mapped to evolution in the 5th dimension z

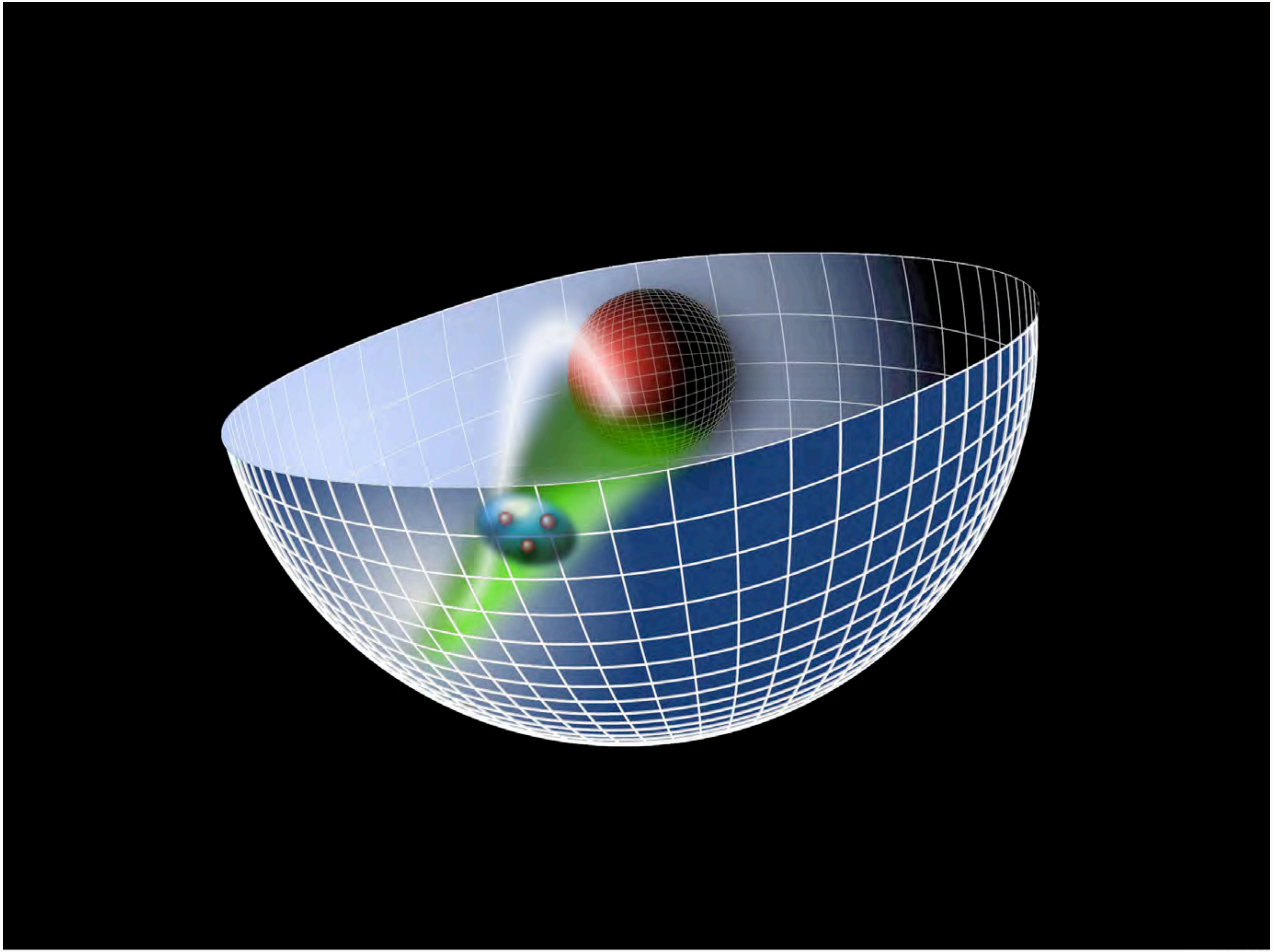
in collaboration with Guy de Teramond

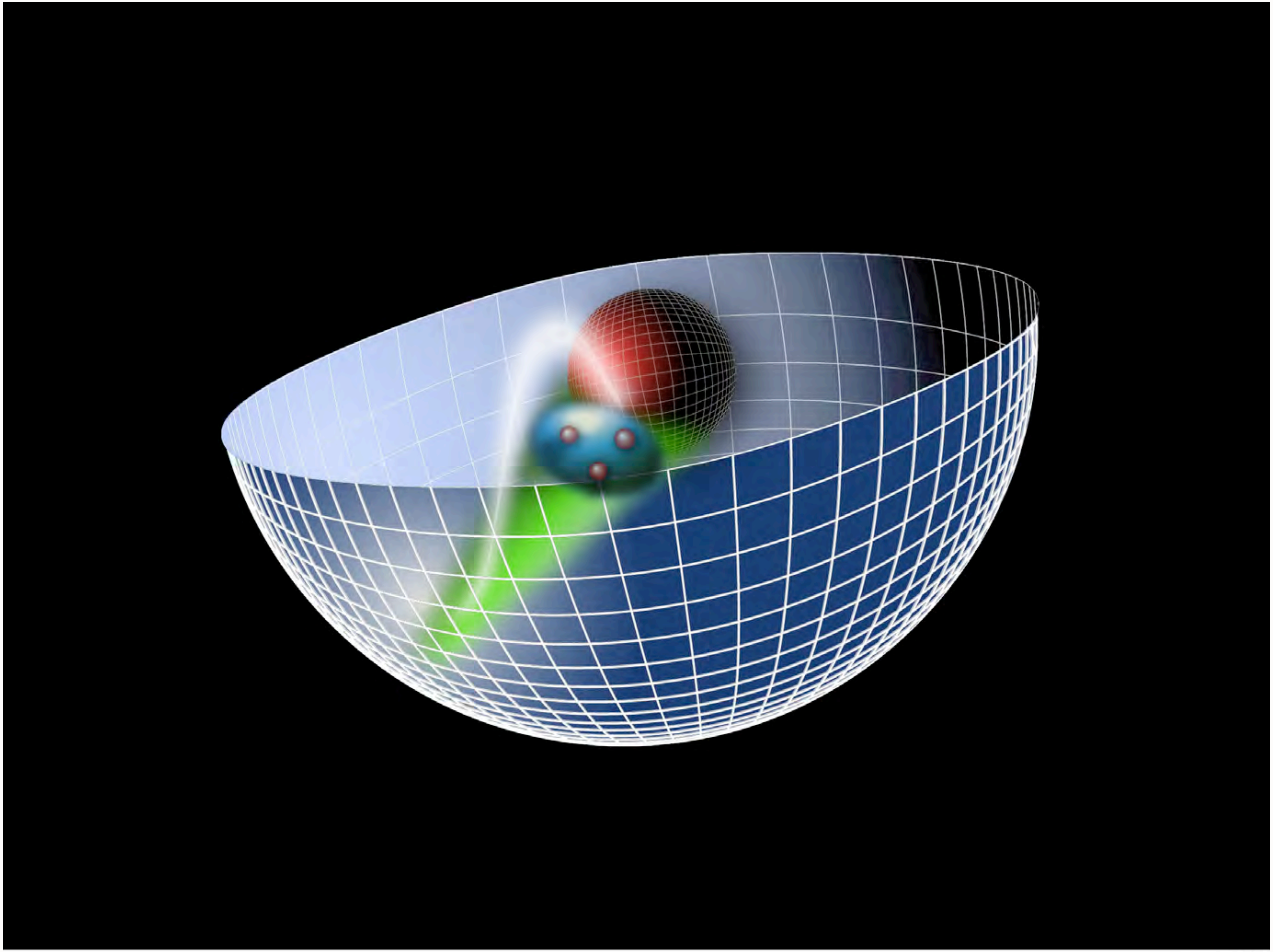


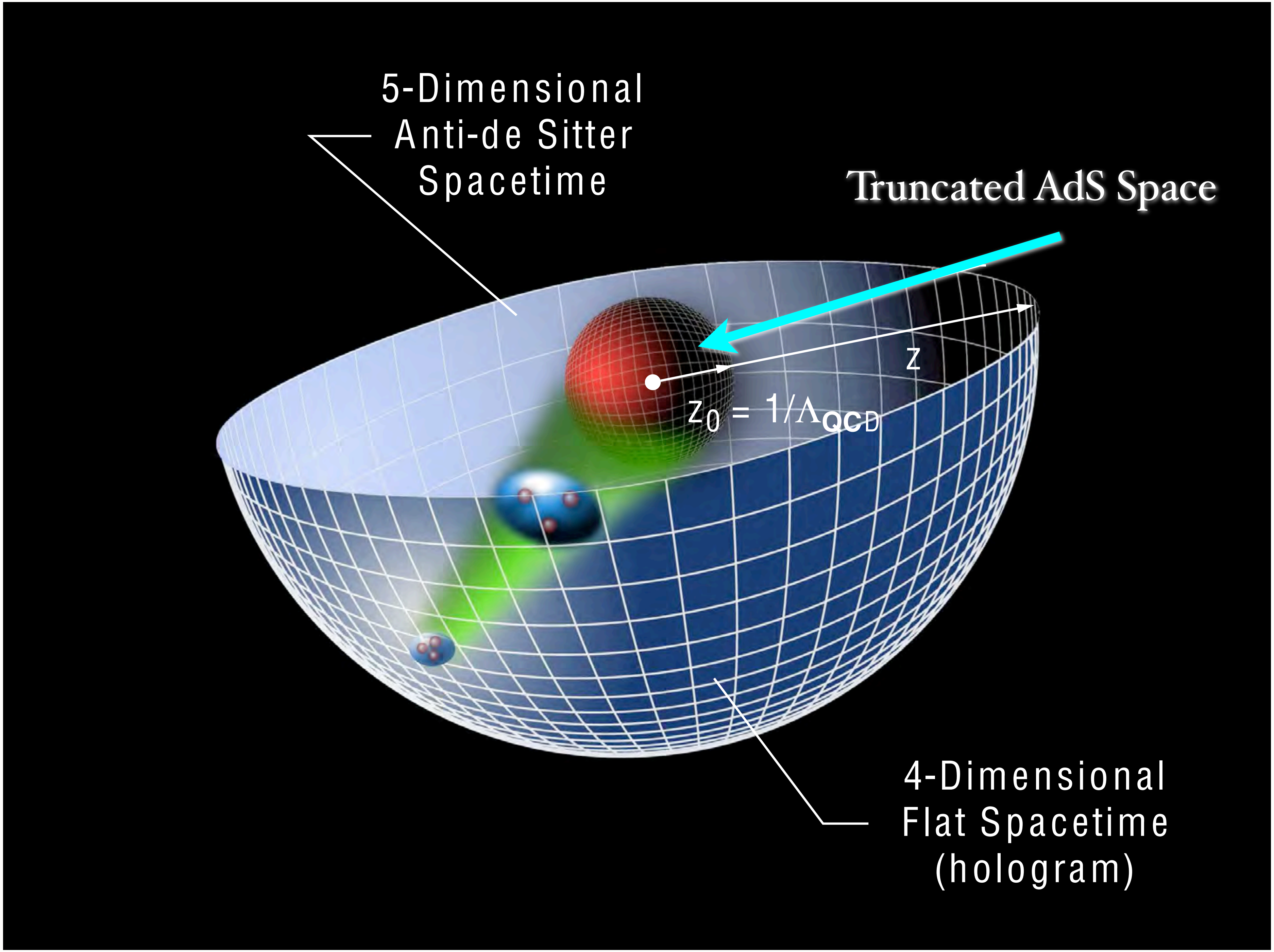
5-Dimensional Anti-de Sitter Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional Flat Spacetime (hologram)









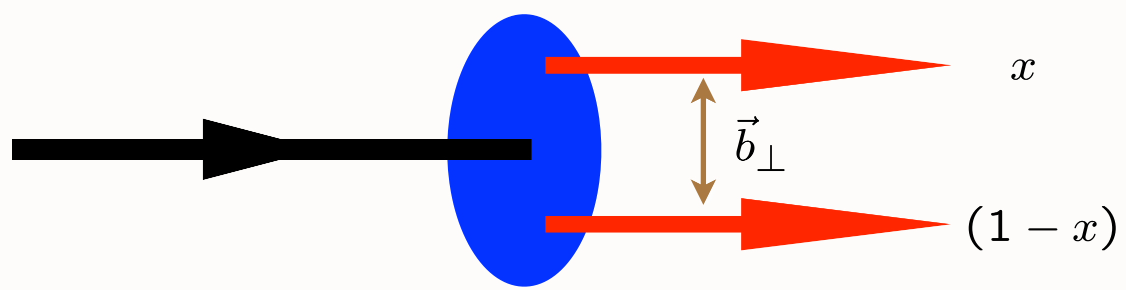

5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

LF(3+1) $\quad\quad$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \quad \longleftrightarrow \quad z$$

$\psi(x, \vec{b}_\perp)$



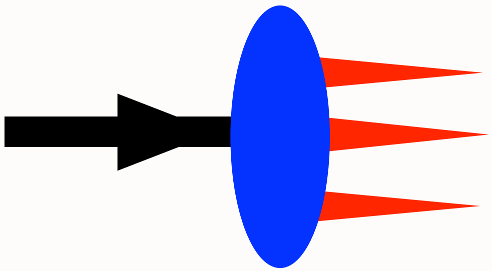
$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

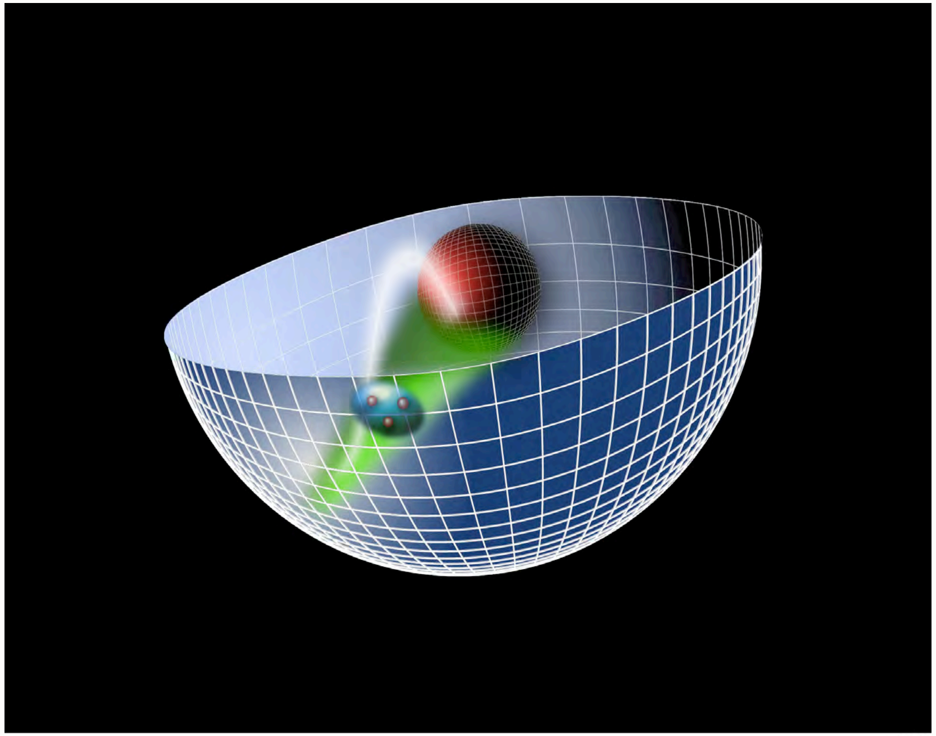
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

- **Analogous to Schrödinger Theory for Atomic Physics**

- *AdS/QCD Light-Front Holography*

- *Hadronic Spectra and Light-Front Wavefunctions*

Light-Front Schrödinger Equation

Light-Front Schrödinger Equation

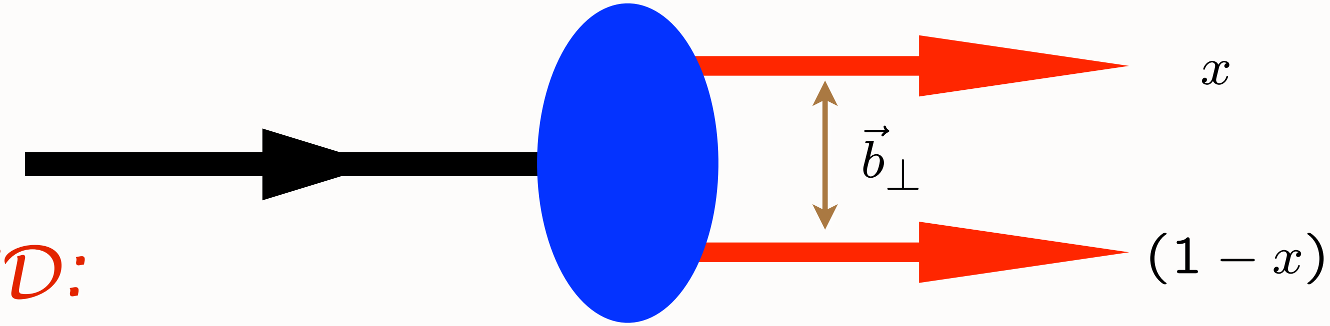
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



AdS/QCD:

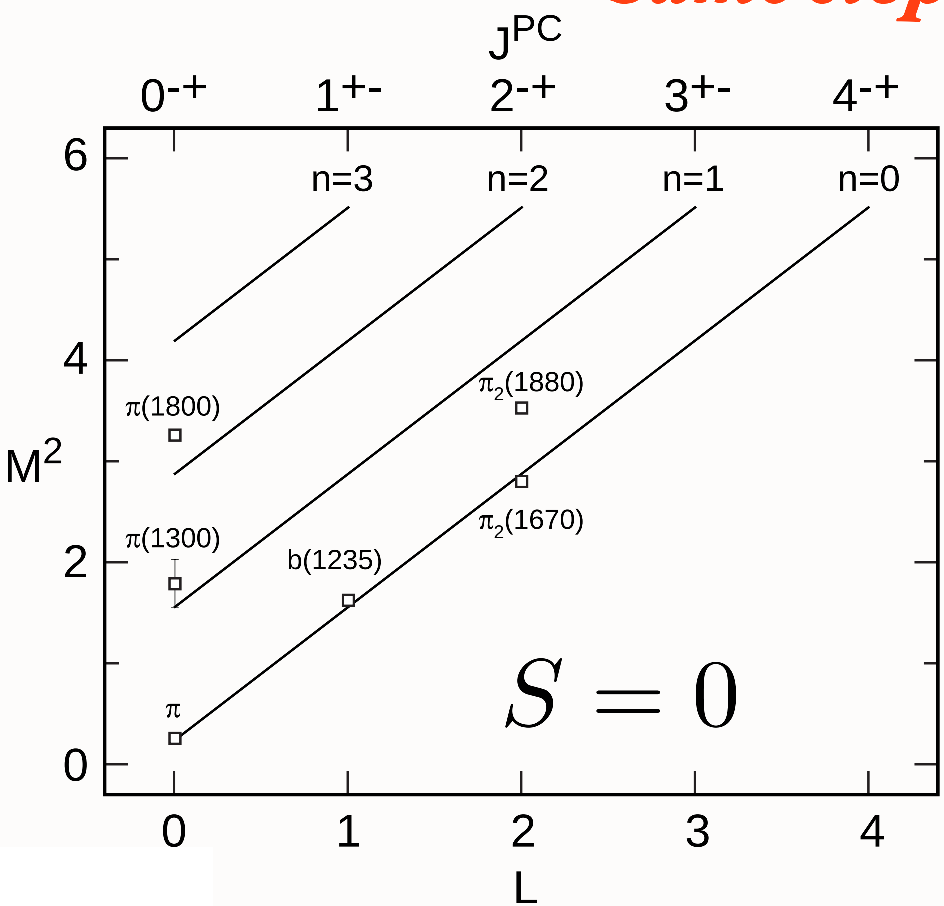
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

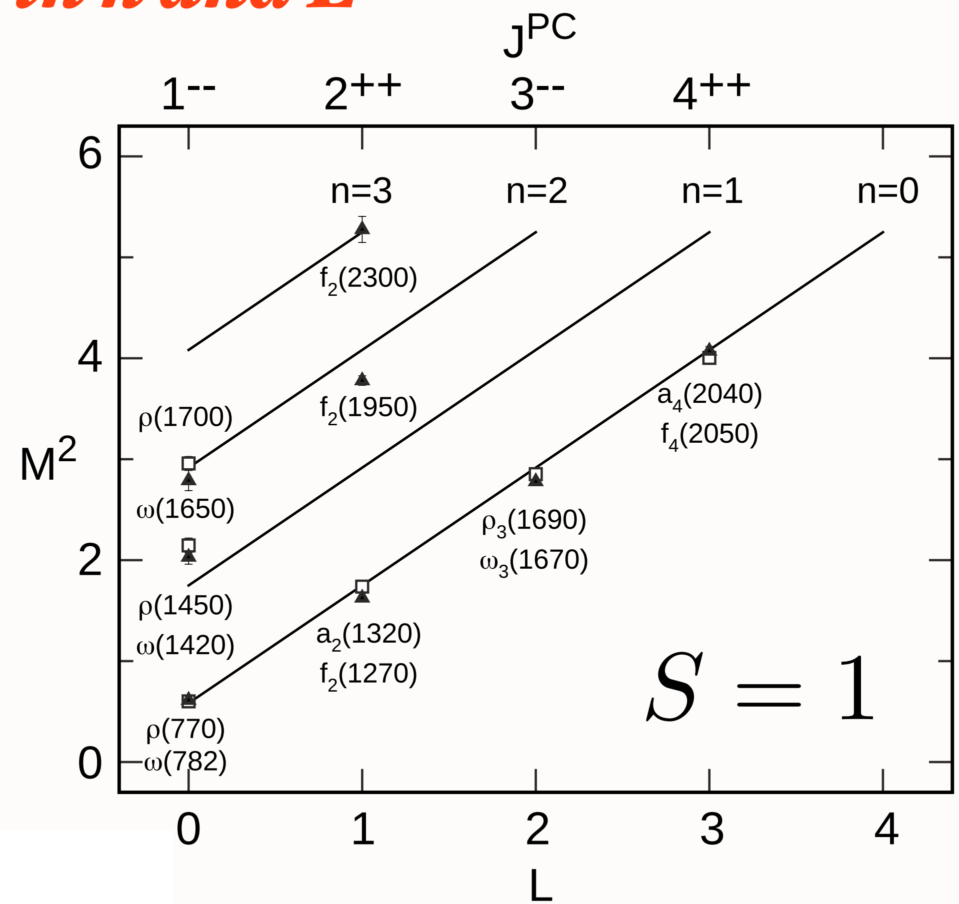
Bosonic Modes and Meson Spectrum

$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \rightarrow 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

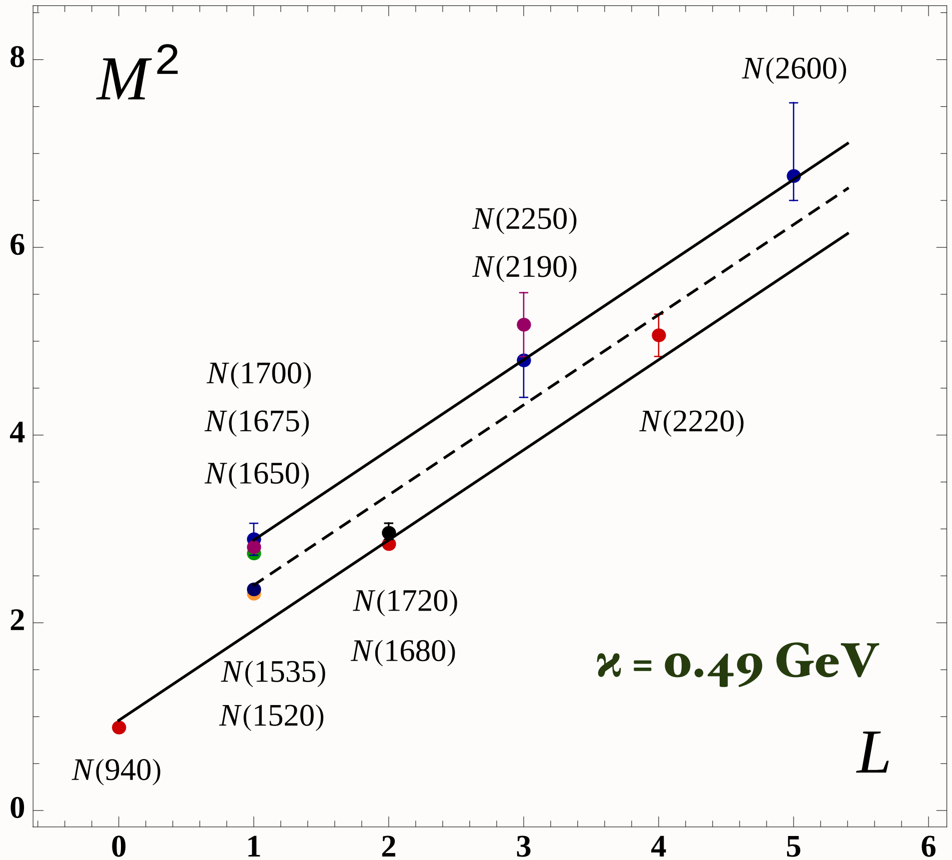
Same slope in n and L

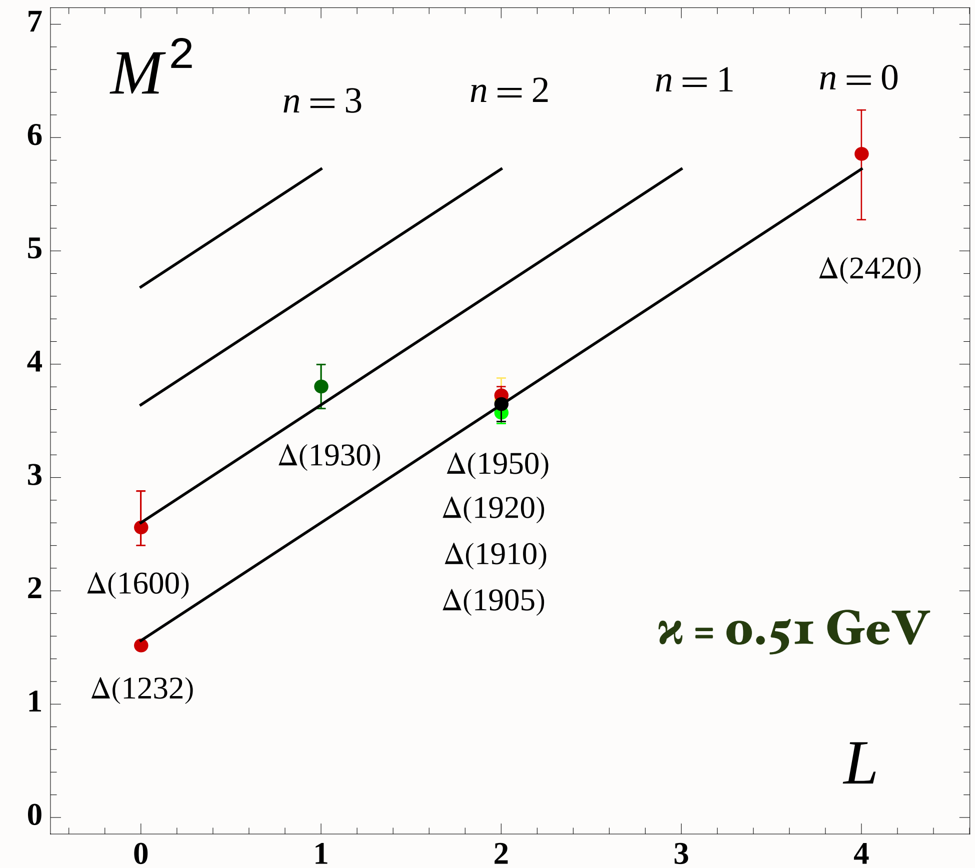




Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

Balmer series of QCD





de Teramond, sjb

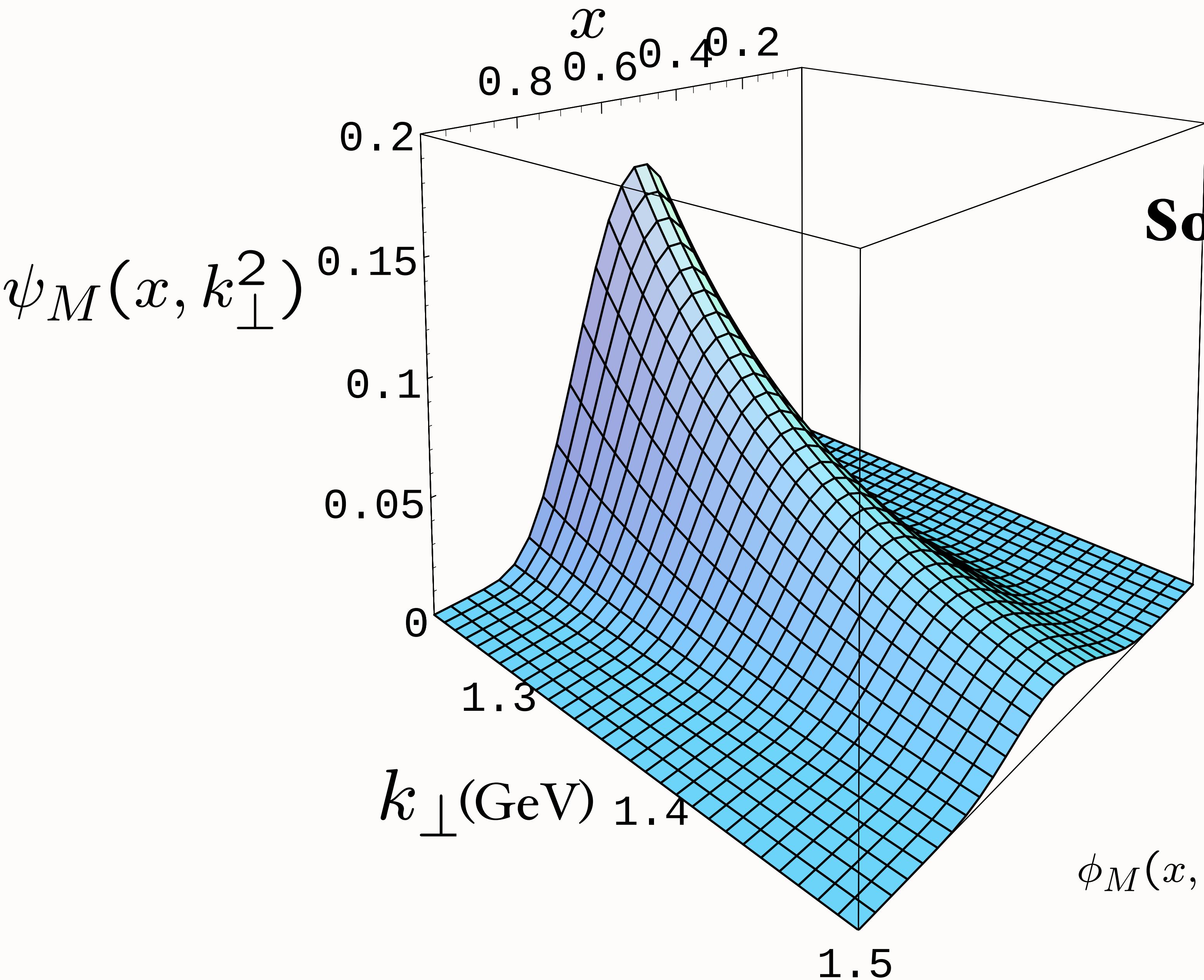
All confirmed resonances from PDG 2012

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ **positive parity**

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ **negative parity**

See also Forkel, Beyer, Federico, Klempt

Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

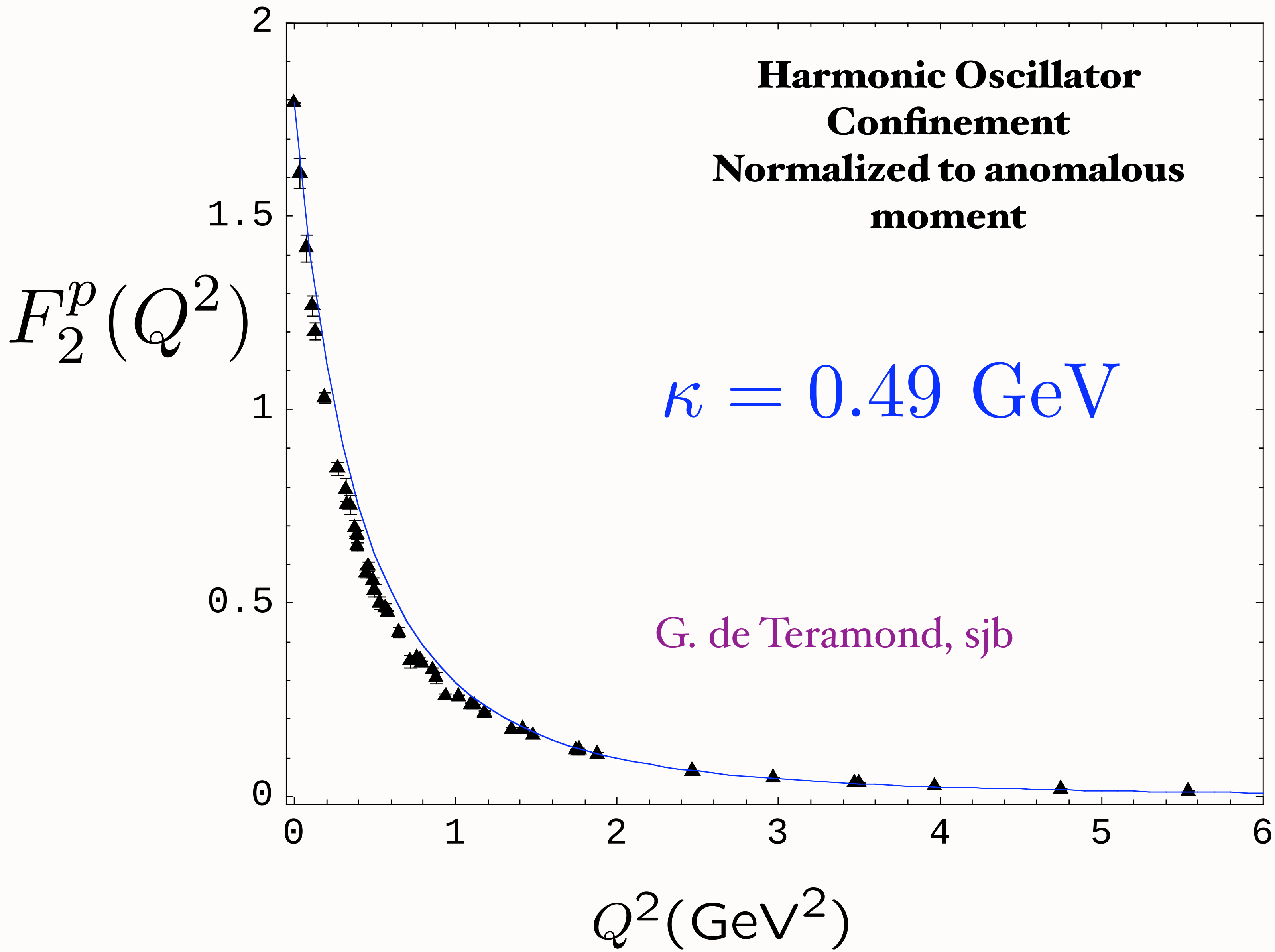
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

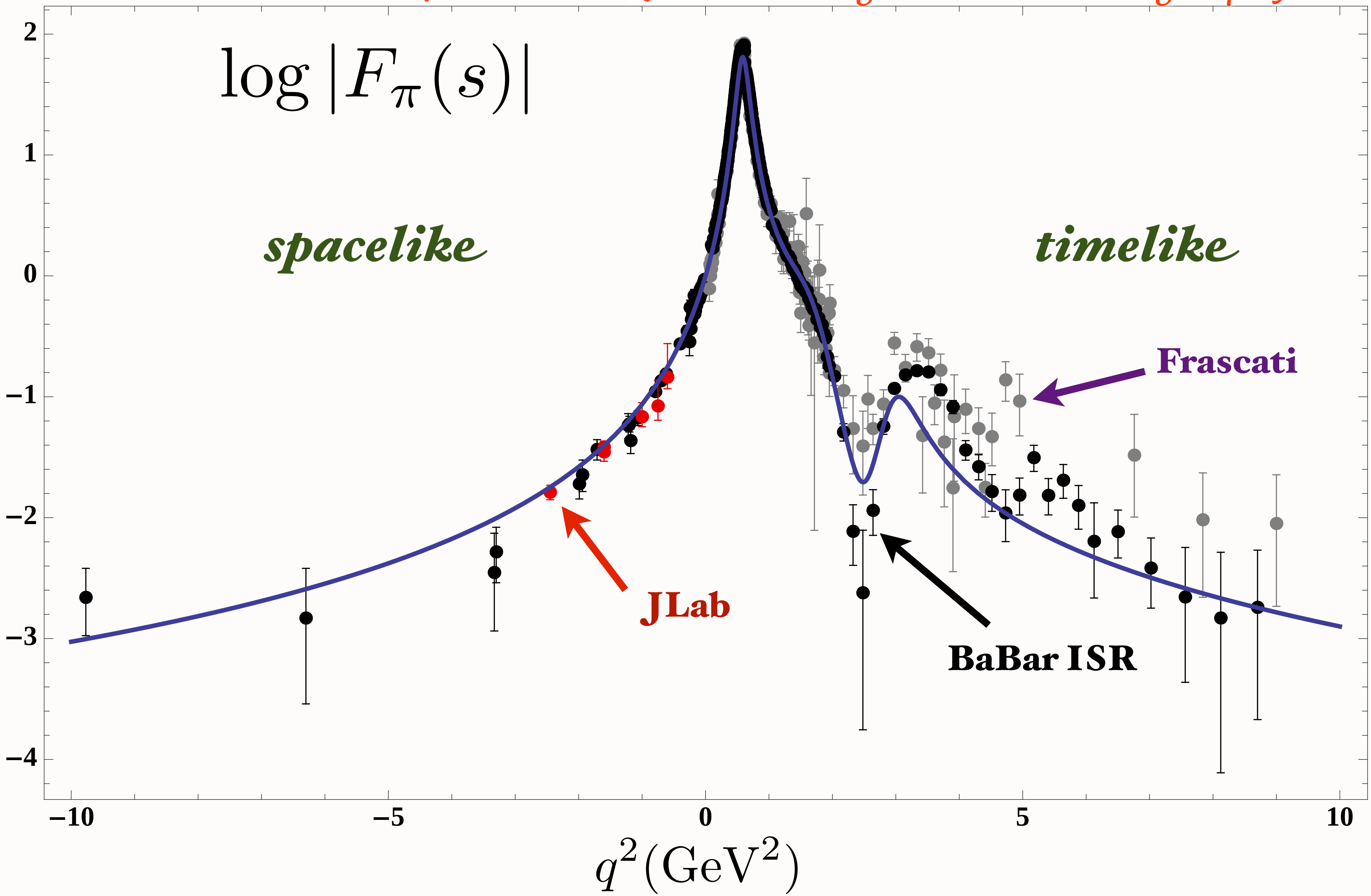
Spacelike Pauli Form Factor

Preliminary

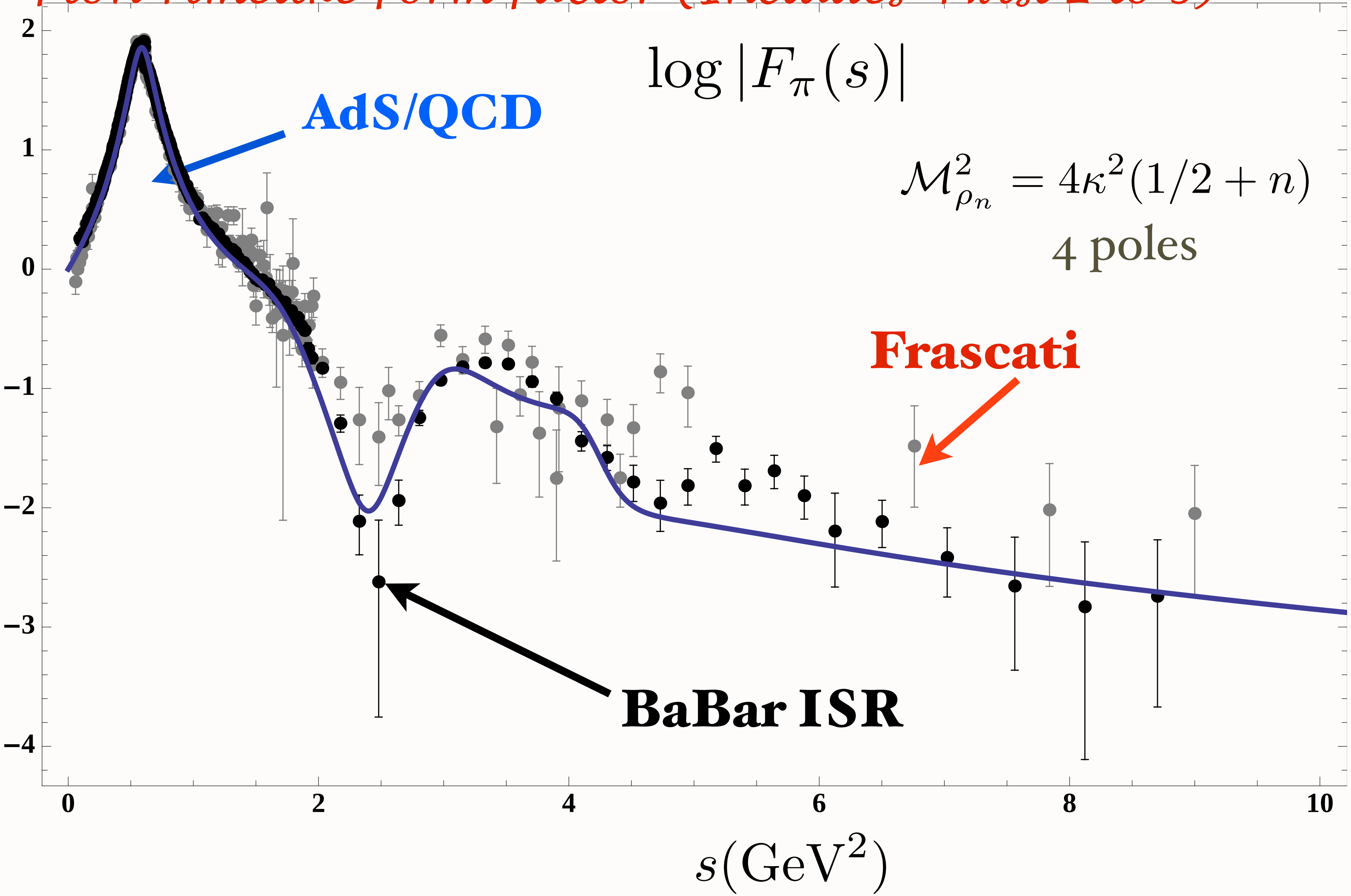
From overlap of $L = 1$ and $L = 0$ LFWFs



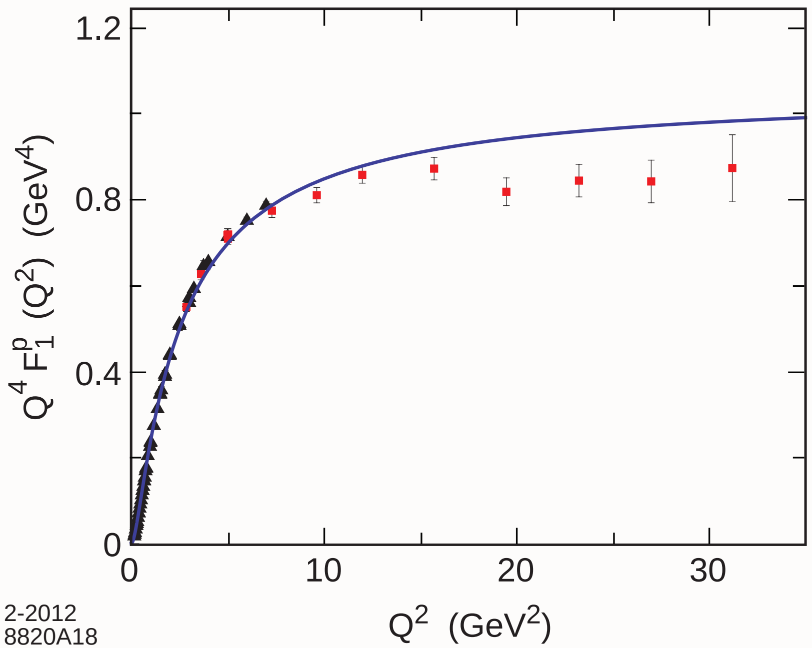
Pion Form Factor from AdS/QCD and Light-Front Holography



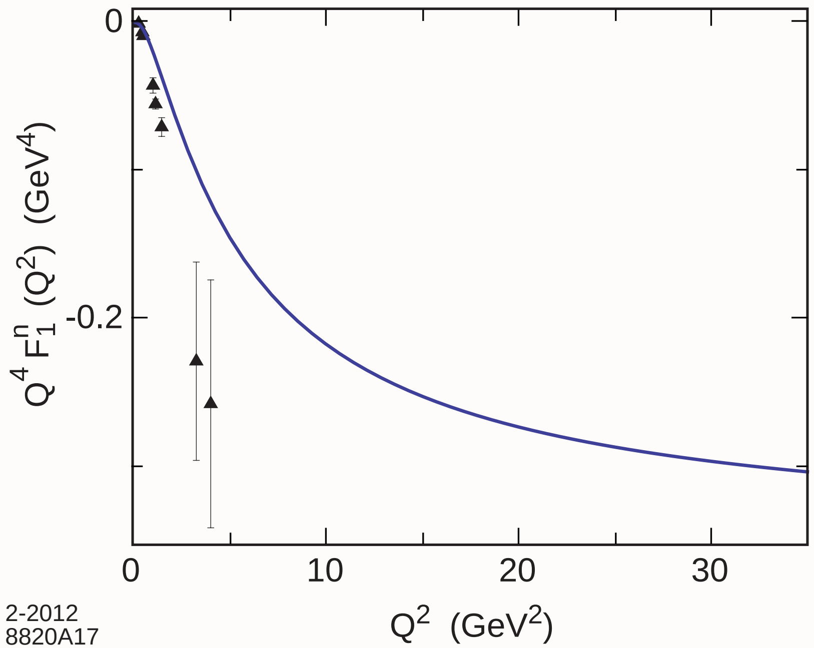
Pion Timelike Form Factor (Includes Twist 2 to 5)



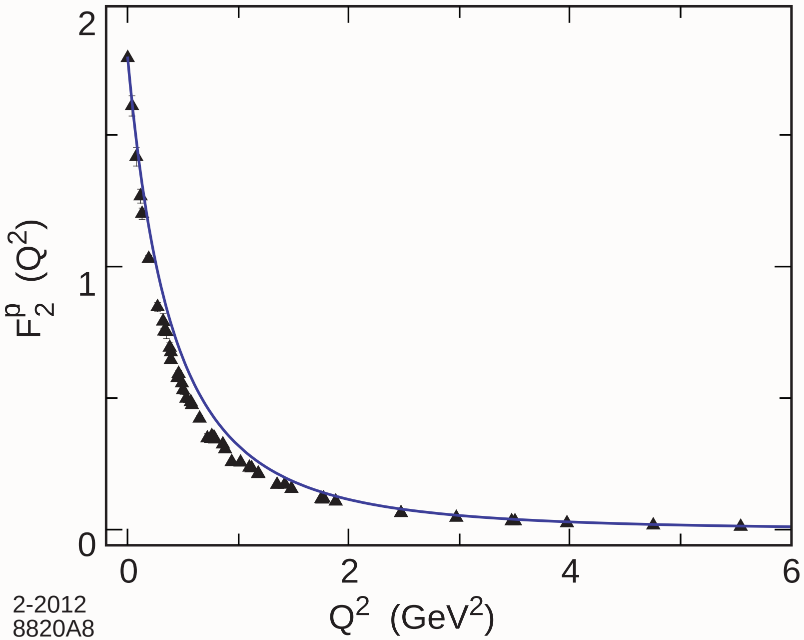
Using $SU(6)$ flavor symmetry and normalization to static quantities



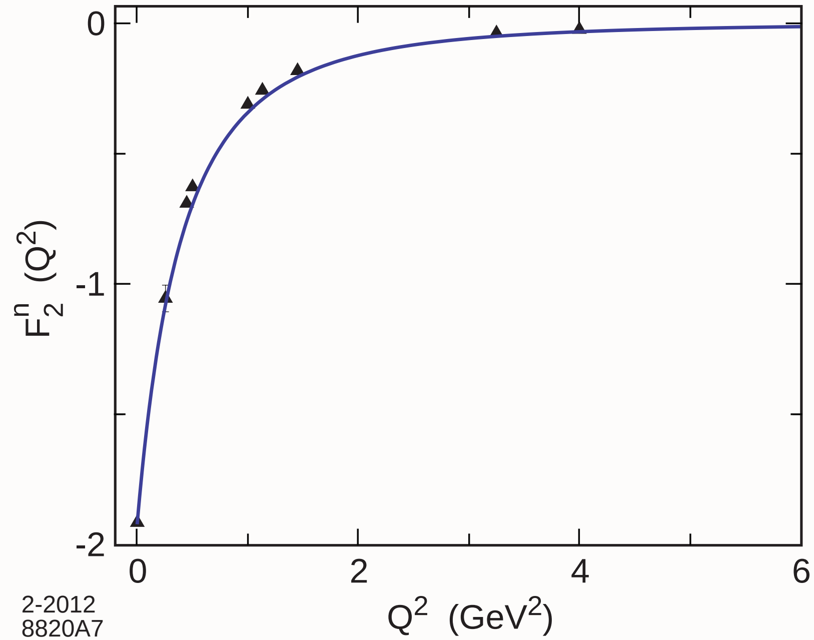
2-2012
8820A18



2-2012
8820A17



2-2012
8820A8



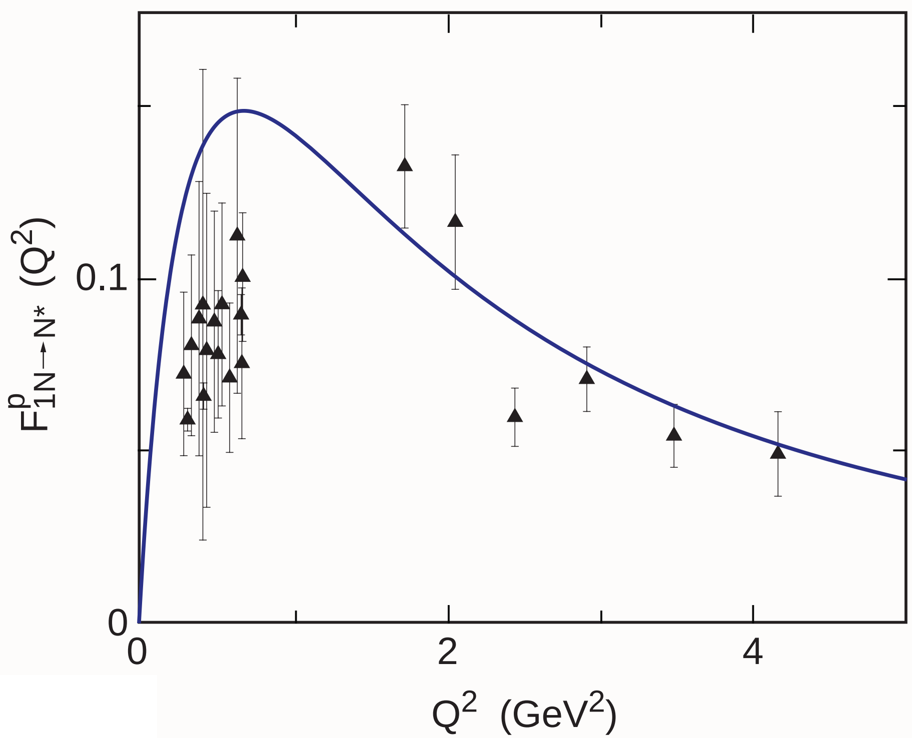
2-2012
8820A7

Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$

AdS\QCD
Light-Front
Holography

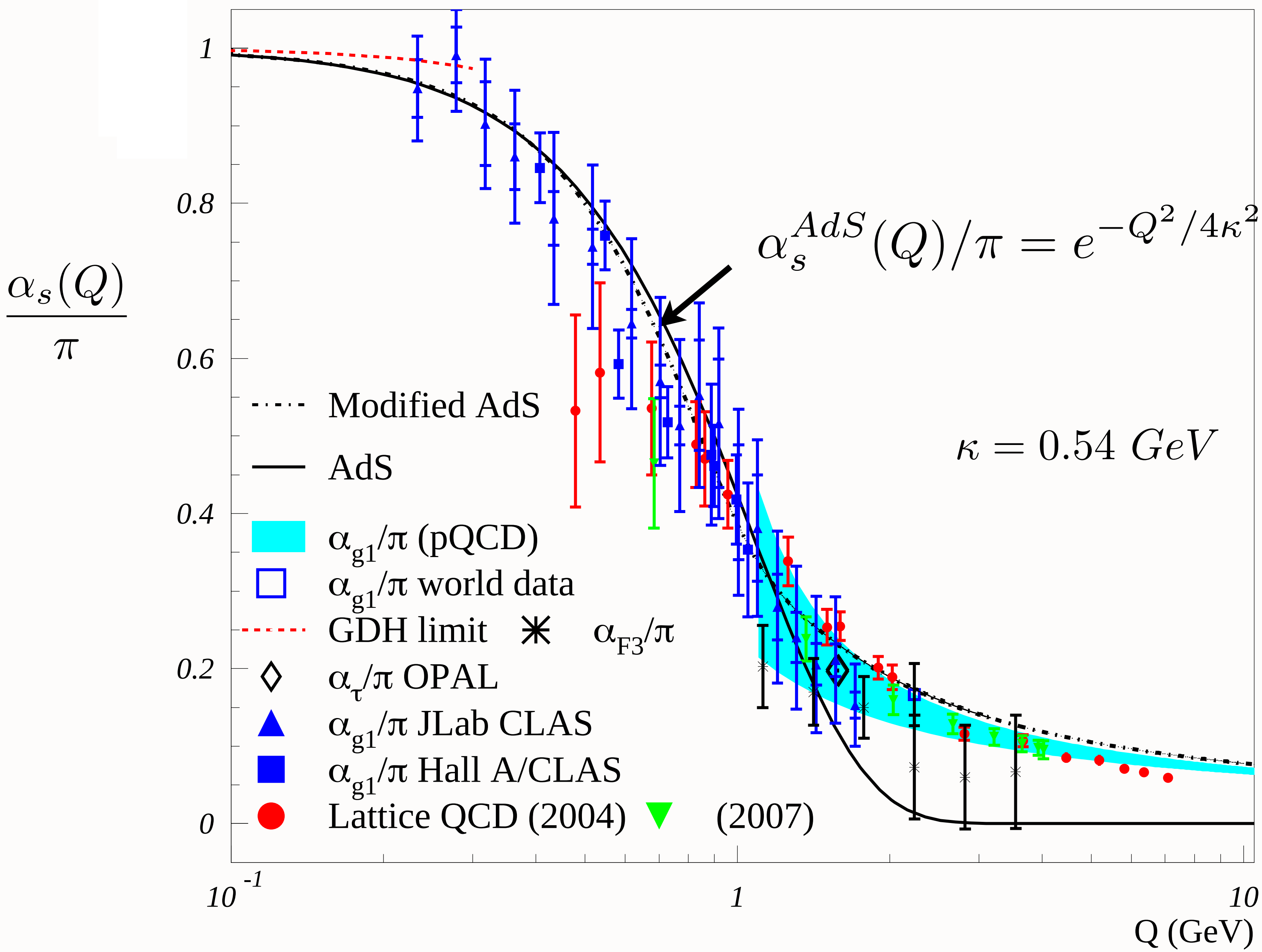
G. de Teramond, sjb



Proton transition form factor to the first radial excited state. Data from JLab

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Deur, de Teramond, sjb

AdS/QCD and Light-Front Holography

- AdS/QCD: Incorporates scale transformations characteristic of QCD with a single scale -- RGE
- Light-Front Holography; unique connection of AdS_5 to Front-Form
- Profound connection between gravity in 5th dimension and physical 3+1 space time at fixed LF time τ
- Gives unique interpretation of z in AdS to physical variable ζ in 3+1 space-time

Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion ($m_q = 0$)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S.**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit required**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

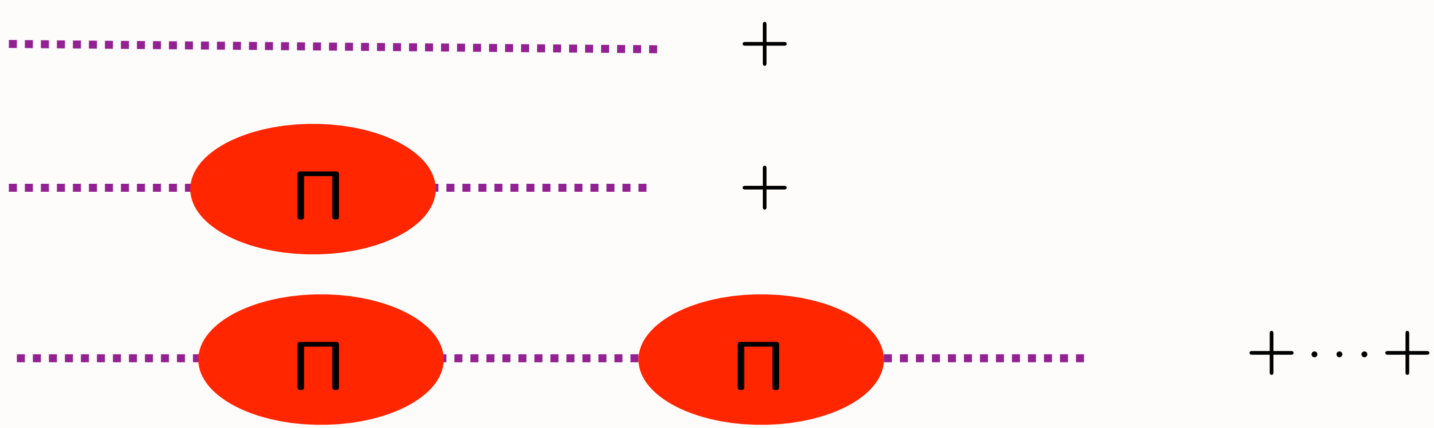
$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton loop corrections to dressed photon propagator

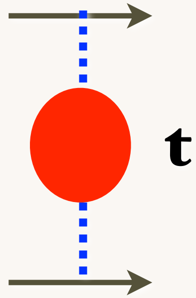


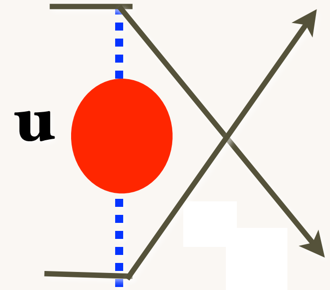
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Initial scale t_o is arbitrary -- Variation gives RGE Equations
Physical renormalization scale t not arbitrary

Electron-Electron Scattering in QED

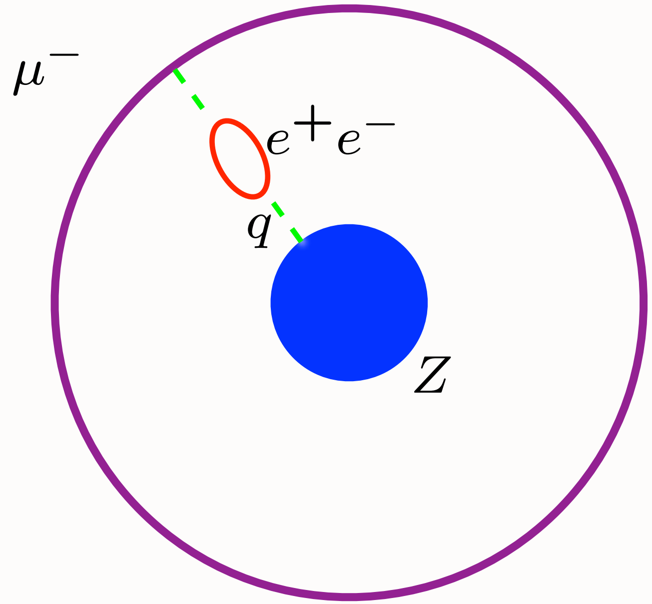
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$





- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**
- **If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- *No renormalization scale ambiguity!*
- *Two separate gauge invariant physical scales.*

Scale Setting in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1 - \Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Need to set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

↓

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

↓

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

$through\ the\ PMC - BLM\ correspondence\ principle$

↓

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\} - terms$

↓

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

Same as QED Scale Setting

Apply to Evolution kernels, hard subprocesses

Eliminates unnecessary systematic uncertainty

Xing-Gang Wu
Leonardo di Giustino, SJB

Principle of Maximum Conformality

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

Higher Twist from Hadron Dynamics

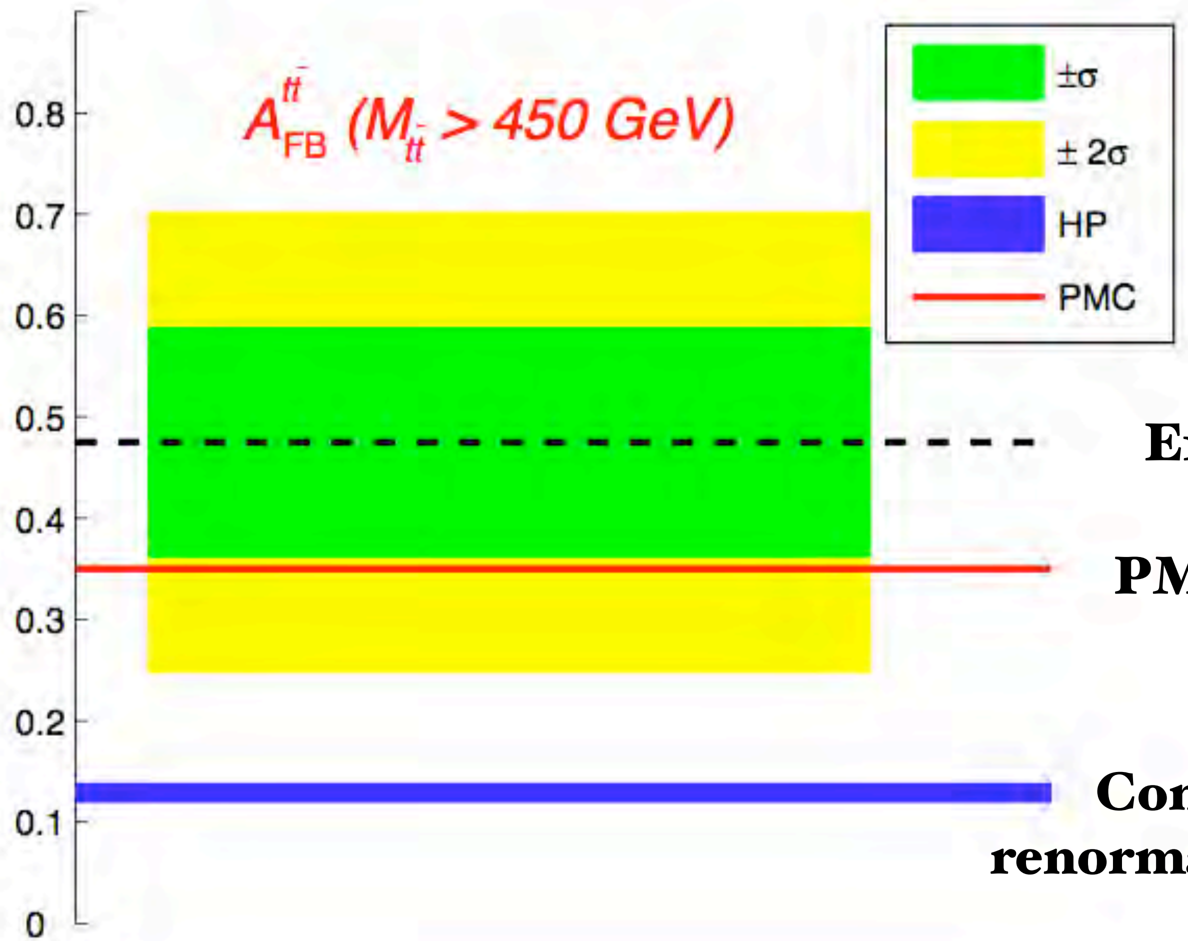
Intrinsic Heavy Quarks

Light by Light Loops

BLM/PMC: Absorb β-terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

PMC Prediction

Conventional: guess for renormalization scale and range

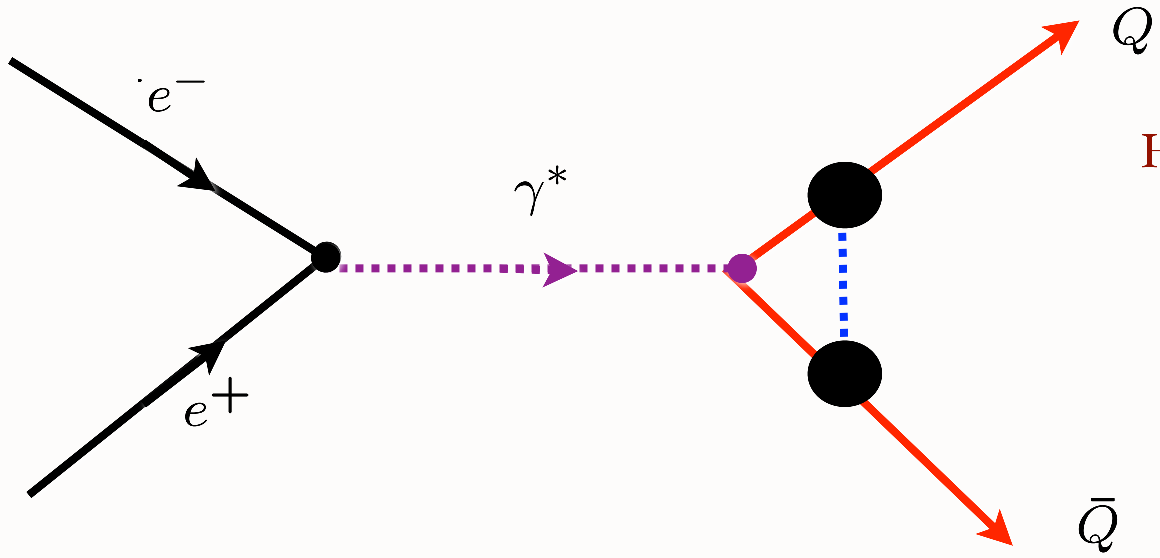
$t\bar{t}$ asymmetry predicted by pQCD NNLO within $1\ \sigma$ of CDF/D0 measurements using PMC/BLM scale setting

Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb Phys.Rev.D28:228,1983

- **"Principle of Maximum Conformality"** **Di Giustino, Wu, sjb**
- **All terms associated with nonzero beta function summed into running coupling**
- **Standard procedure in QED**
- **Resulting series identical to conformal series**
- **Renormalon n! growth of PQCD coefficients from beta function eliminated!**
- *Scheme Independent* **!!!**
- **In general, BLM/PMC scales depend on all invariants**
- **Single Effective PMC scale at NLO**



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Myths concerning scale setting

- **Renormalization scale "unphysical": No optimal physical scale**
- **Can ignore possibility of multiple physical scales**
- **Accuracy of PQCD prediction can be judged by taking arbitrary guess with an arbitrary range**
- **Factorization scale should be taken equal to renormalization scale**

$$\mu_F = \mu_R$$

Guessing the scale: Wrong in QED. Scheme dependent!

$$
\begin{aligned}
\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} &+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right] \\
&+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2 \right. \\
&+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f \\
&+ \left. \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2} \right\}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} &+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right] \\
&+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2 \right. \\
&+ \left. \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648}f^2 \right\}.
\end{aligned}
$$

Eliminate MSbar,
Find Amazing Simplification

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} {e_q}^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

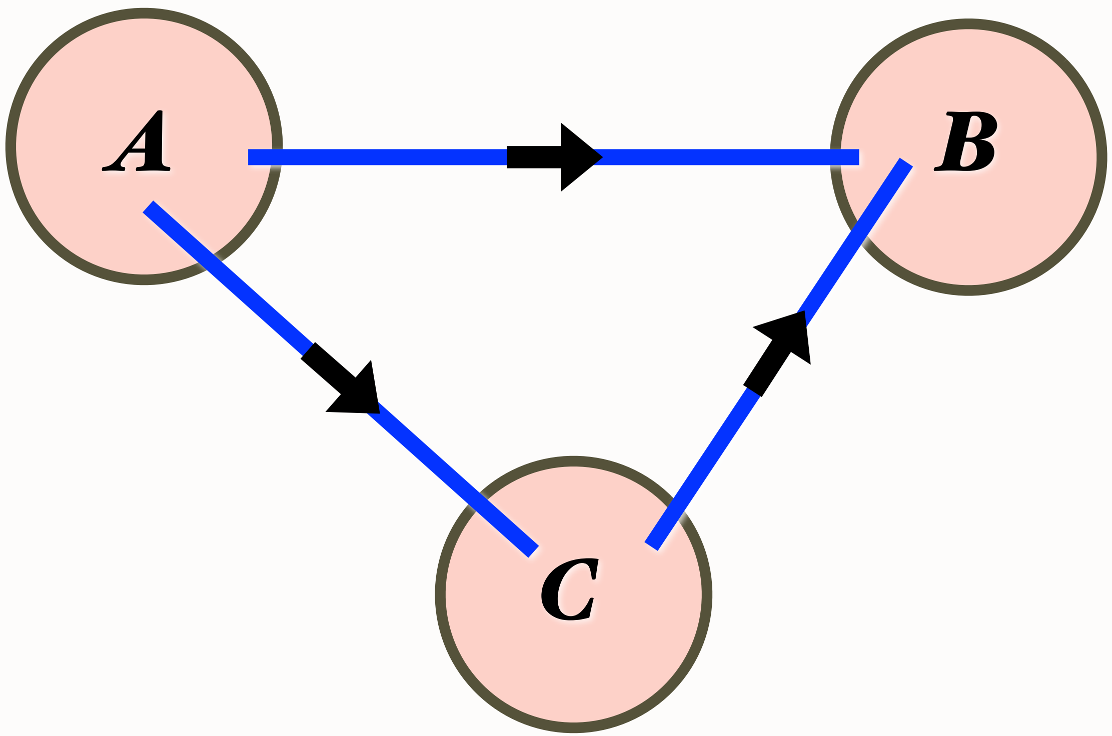
No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)
No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

Transitivity Property of Renormalization Group

Relation of observables must be independent of intermediate scheme



A → C **C → B** ***identical to*** **A → B**

Violated by PMS!

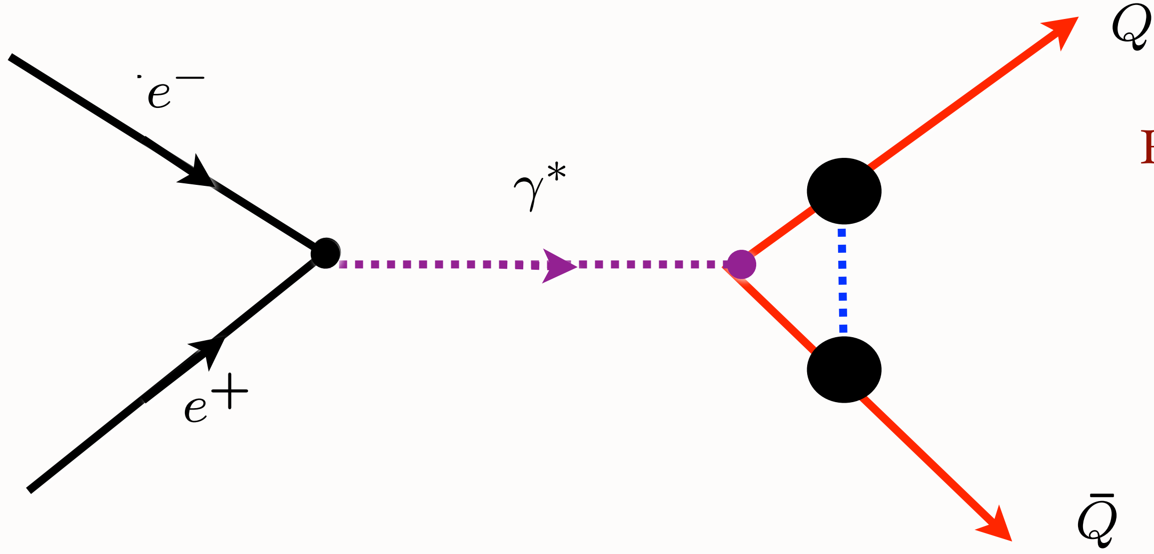
Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(s\beta^2)}{4\beta}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity β

The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, QGP, CGC, ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities. —Mark Twain

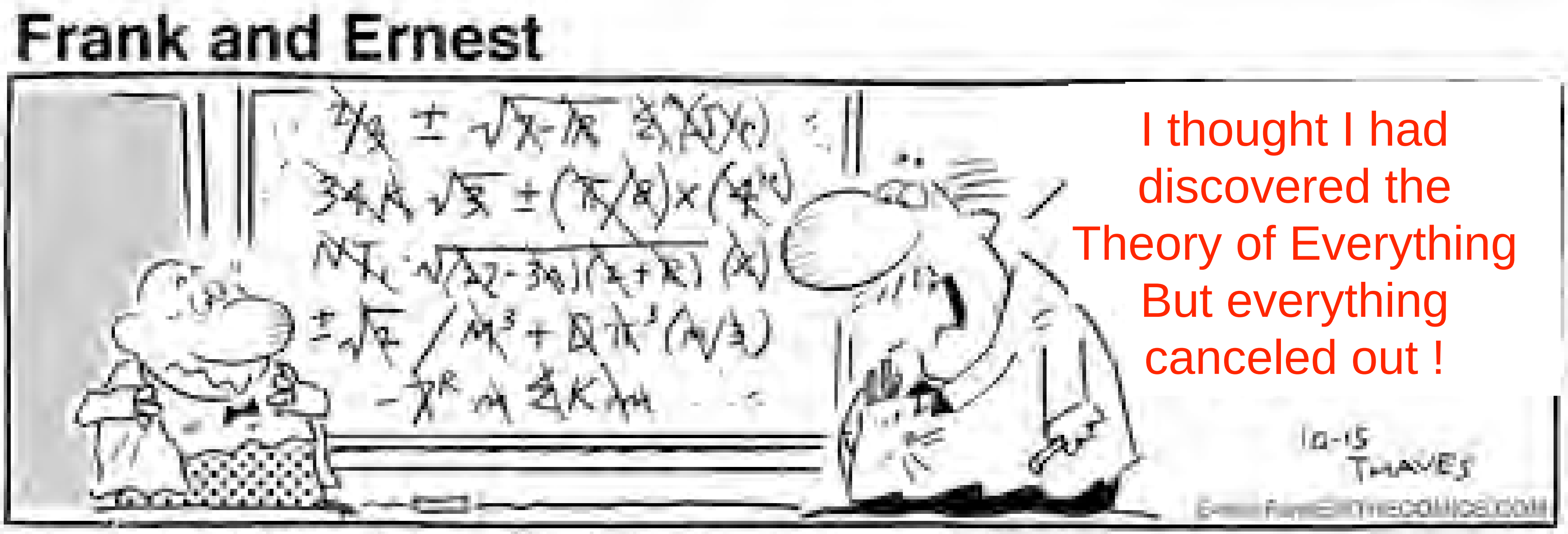
Many Analogs: QED/QCD

- Diffractive Dissociation of Atoms/Hadrons
- Atomic/Color Transparency
- Light-Front Wavefunctions
- Atomic Alchemy/B decay
- Atom Formation/Hadronization
- Spontaneous pair production/ Confinement
- Intrinsic heavy leptons/Intrinsic Charm
- True Muonium/Quarkonium

SCIENCE VOL 265 15 SEPTEMBER 1995

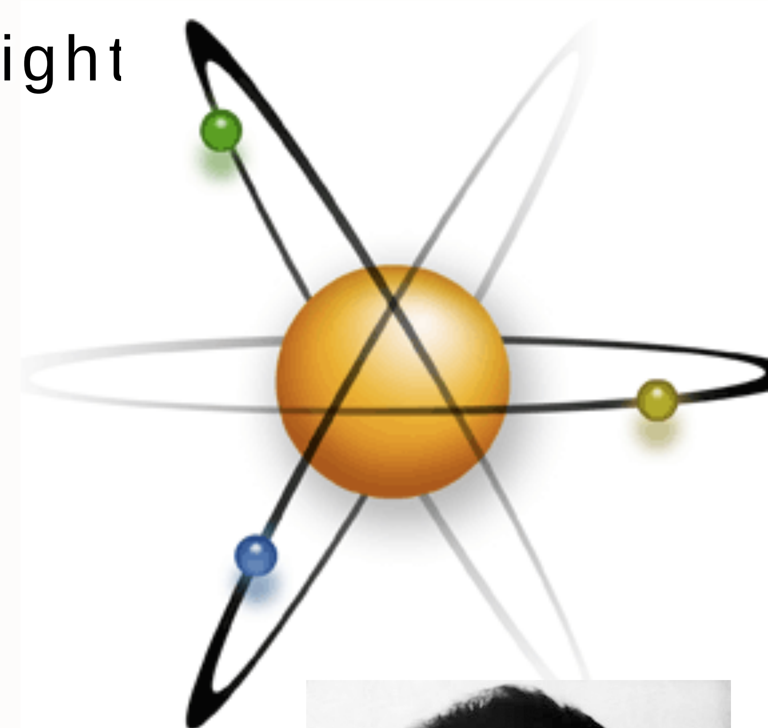
A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.



The remarkable connections between atomic and hadronic physics

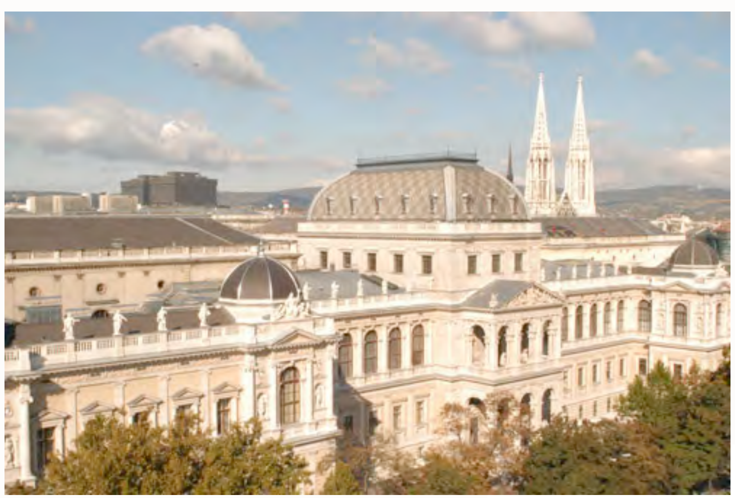
and Exotic Atoms in Flight



Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Stanford University



universität wien