Production of stops in e^+ e^- annihilation close to threshold

Peter Poier
in collabaration with André H. Hoang and Pedro Ruiz-Femenía
University of Vienna

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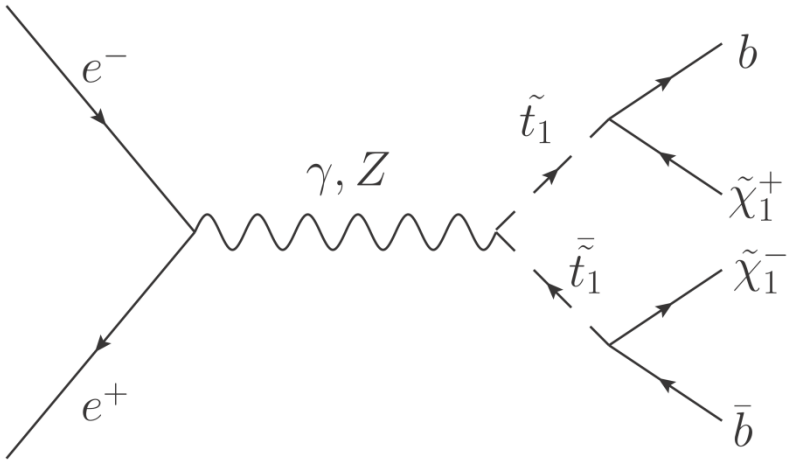

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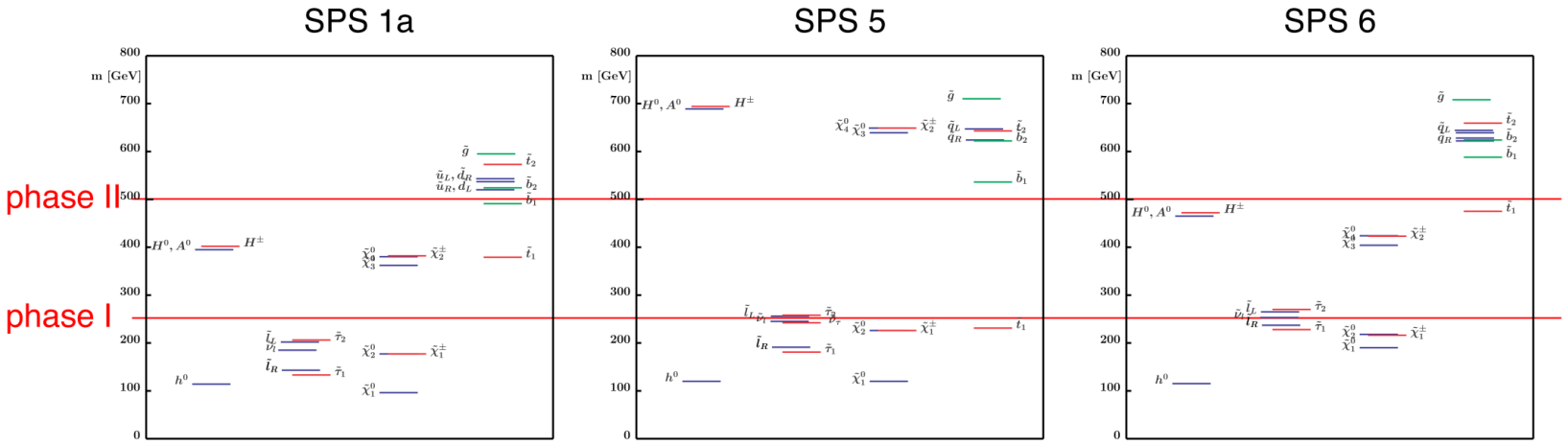
Motivation:

In many SUSY models stop pair production is possible at the ILC:



$m_{\tilde{t}_1} = 396$ GeV

$\tilde{t}_1 \to b\chi_1^+, \ldots$

$\Gamma_{\tilde{t}_1} = 1.92$ GeV

$m_{\tilde{t}_1} = 240$ GeV

$\tilde{t}_1 \to b\chi_1^+, c\chi_1^0$

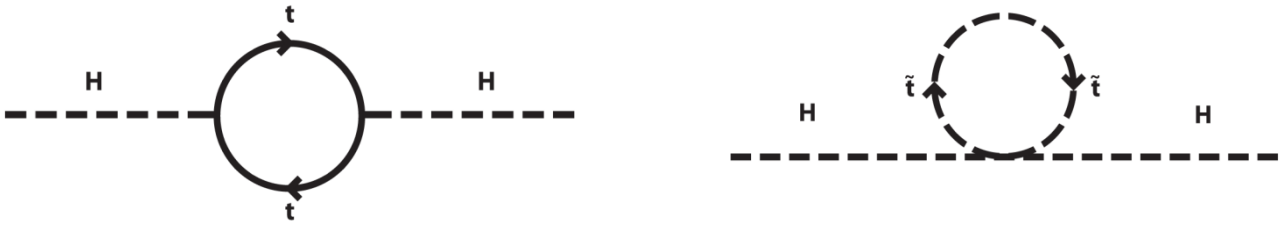
$\Gamma_{\tilde{t}_1} = 0.04$ GeV

$m_{\tilde{t}_1} \simeq 490$ GeV

$\tilde{t}_1 \to b\chi_1^+, \ldots$

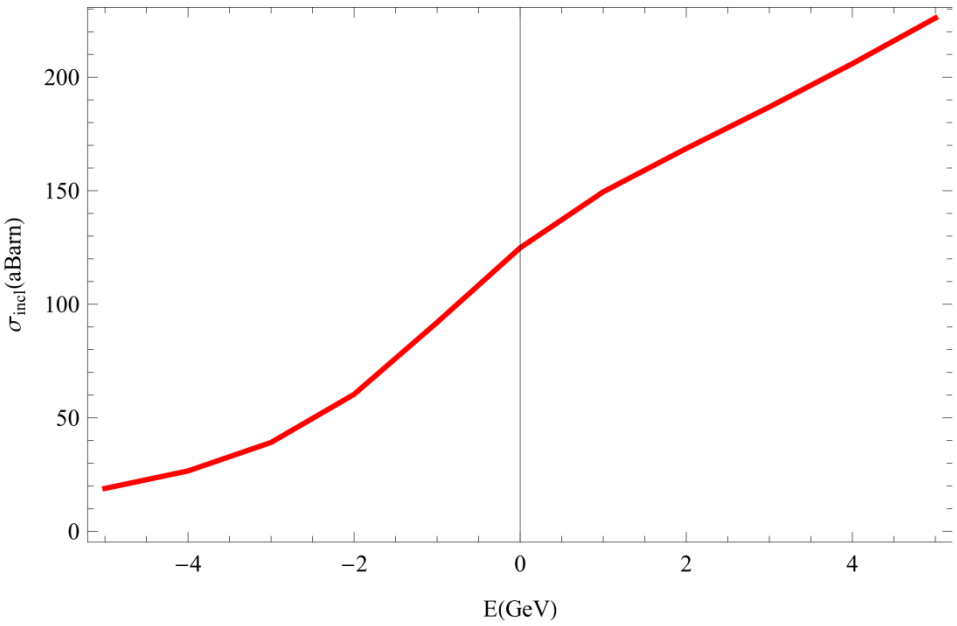
$\Gamma_{\tilde{t}_1} \simeq 3.2$ GeV

Stop mass is supposed to be small



Motivation:

Threshold scan:



determine:

$$m,\ y_{\tilde{t}_1},\ \Gamma_{\tilde{t}_1}$$

Precision aim:

$$\frac{\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1)}{\sigma(e^+e^- \to t\bar{t})} \sim v^2 \sim \frac{1}{100} \Rightarrow \frac{\Delta\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1)}{\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1)} \sim 10\frac{\Delta\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to t\bar{t})} \sim 10-20\%$$

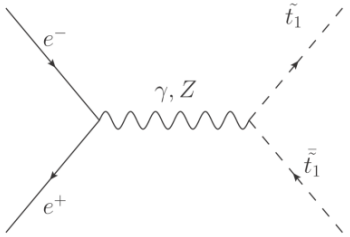
Outline:

- 1-loop calculation
- EFT framework and factorization
- PS matching
- Comparison to Monte Carlo simulation, Results

1-Loop calculation

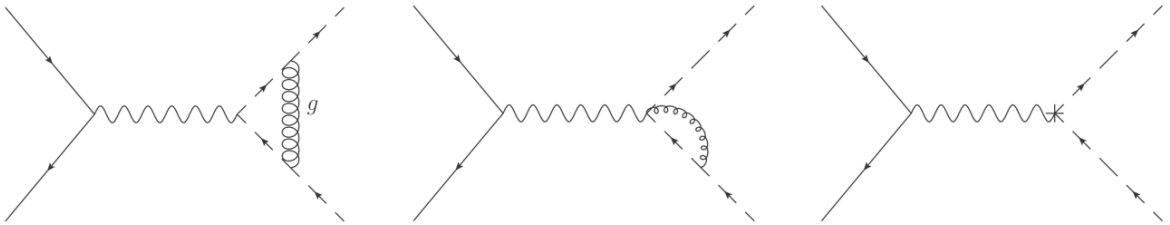
Diagrams contributing to $\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1)$ at 1-loop order (neglecting electroweak corrections):

Tree level diagram:



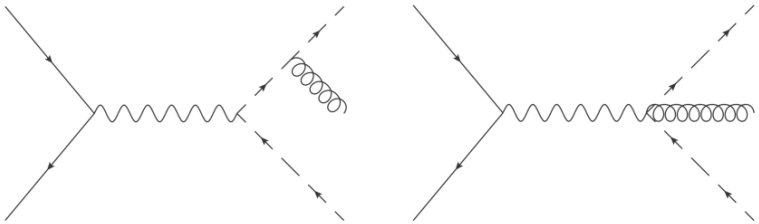
(a)

Virtual corrections:



(b)

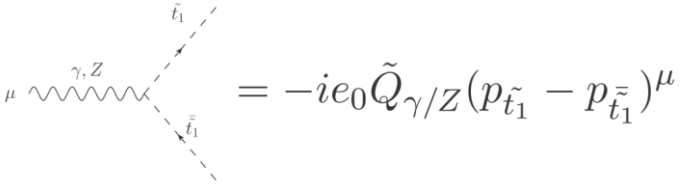
Real radiation:



(c)

Effect of the vertex correction

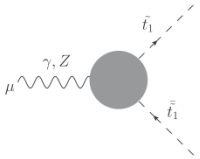
Tree level vertex:



$$= -ie_0\tilde{Q}_{\gamma/Z}(p_{\tilde{t}_1} - p_{\bar{\tilde{t}}_1})^\mu$$

1-loop level vertex:

$$\Gamma^\mu \equiv -ie_0\tilde{Q}_{\gamma/Z}(p_{\tilde{t}_1} - p_{\bar{\tilde{t}}_1})^\mu F(q^2) =$$



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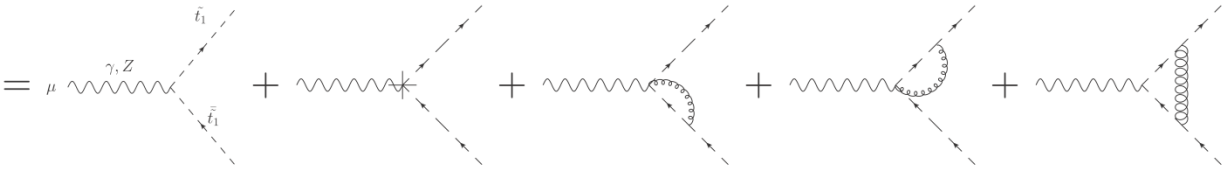


Virtual correction to the cross-section:

$$|F|^2 = |1 + \delta F|^2 = 1 + 2Re(\delta F) + O(\alpha_S^2)$$

$$\sigma\left(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1\right) = \sigma^{(\text{tree})} + \delta\sigma^{(\text{virtual})} = (1 + 2Re\delta F)\sigma^{(\text{tree})}$$

$O(\epsilon)$ term needed!

includes IR divergences

I.R. divergences in $\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1) = (1 + 2Re\delta F)\sigma^{(\text{tree})}$

$$2Re\,\delta F = C_F \frac{\alpha_S}{\pi}\left[\frac{(1+\beta^2)}{\beta}\left(\boxed{-\frac{\log(w)}{2\epsilon_{IR}}} - \frac{1}{2}\log(w)\log\left(\frac{\mu_{IR}^2}{m^2}\right) + Li_2(w)\right.\right.$$

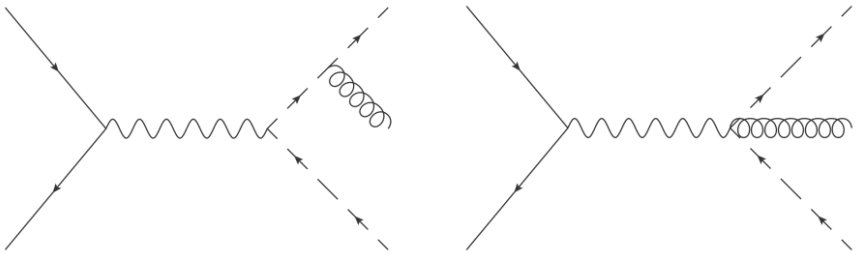
$$\left. -\frac{1}{4}\log^2(w) + \log(1-w)\log(w) - \log(w) + \frac{\pi^2}{3}\right)$$

$$\left.\boxed{-\frac{1}{\epsilon_{IR}}} - \log\left(\frac{\mu_{IR}^2}{m^2}\right) - 2\right]. \qquad \beta = \sqrt{1 - \tfrac{4m}{q^2}} \text{ and } w = \tfrac{1-\beta}{1+\beta}$$

$\sigma\left(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1\right) + \sigma\left(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1 g_{soft}\right)$ is free of I.R. divergences!

virtual **real** (Lee-Nauenberg theorem)

Bremsstrahlung diagrams:



1-Loop calculation result

$$\sigma^{(\text{1-loop})} = \sigma\left(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1\right) + \sigma\left(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1 g\right)$$
$$= \sigma^{(\text{tree})}\left(1 + C_F\frac{\alpha_S}{\pi}f(\beta)\right)$$

$$\beta = \sqrt{1-\frac{4m}{q^2}} \text{ and } w = \frac{1-\beta}{1+\beta}$$

$$f(\beta) = \frac{1+\beta^2}{\beta}\left[\frac{3}{2\beta} + \log(w)\log(1+w) + 2\log(w)\log(1-w) + 4Li_2(w) + 2Li_2(-w)\right]$$
$$- 4\log(1-w) - 2\log(1+w) + \left[3 + \frac{1}{\beta^3}\left(2 - \frac{5}{4}(1+\beta^2)^2\right)\right]\log(w)$$

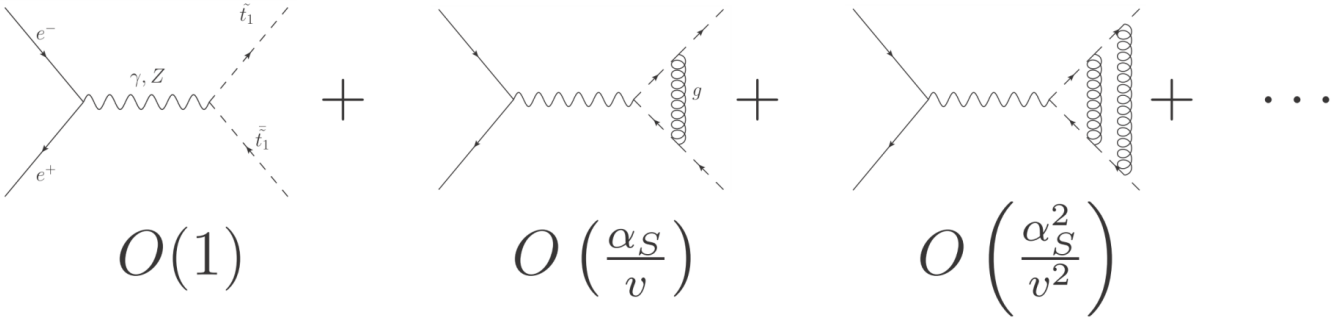
$$= \frac{\pi^2}{2\beta} + O(\beta)$$

$$\sigma^{(\text{1-loop})} = \sigma^{(\text{tree})}\left(1 + \frac{C_F\pi}{2}\frac{\alpha_S}{\beta} + O(\alpha_S\beta^0)\right)$$

no convergence for $\alpha_S \sim \beta$

Motivation for applying vNRQCD

Problem of Coulomb singularities: $\alpha_S \sim v \rightarrow$ NRQCD



Problem of large logarithms:

3 scales $\quad m\ (\text{hard}) \gg \vec{p} \sim mv\ (\text{soft}) \gg E \sim \Gamma \sim mv^2\ (\text{usoft})\ (\gg \Lambda_{\rm QCD})$

$$\ln\left(\frac{m^2}{\mathbf{p}^2}\right),\ \ln\left(\frac{m^2}{E^2}\right),\ \ln\left(\frac{\mathbf{p}^2}{E^2}\right)$$

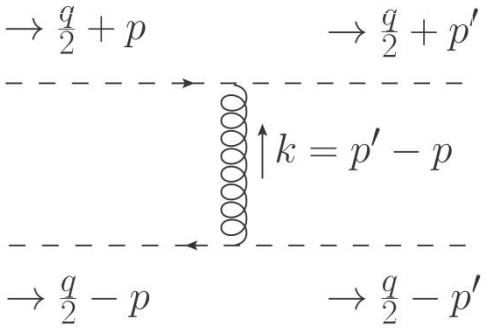
$\rightarrow$ two correlated renormalization scales:
$\mu_S = m\nu,\ \mu_U = m\nu^2,\ \nu \sim v$

'v'NRQCD

$\rightarrow$ RGEs resum $[\alpha_S \ln v]^n,\ \alpha_S\,[\alpha_S \ln v]^n,\ \alpha_S^2\,[\alpha_S \ln v]^n,\ \ldots$ terms

LL **NLL** **NNLL**

Expansion of the Gluon propagator in the potential regime:



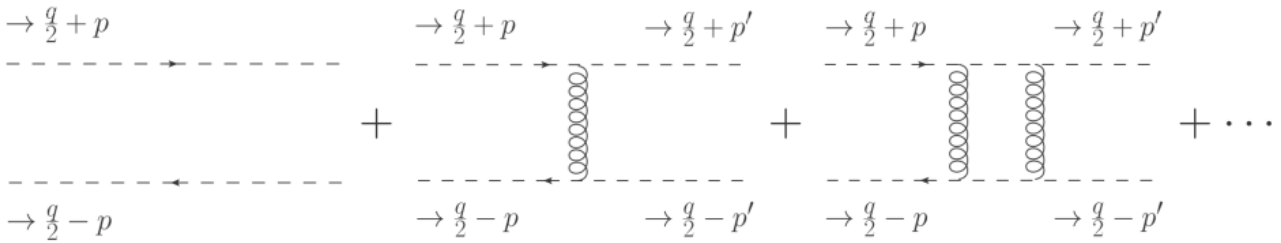
potential: $(k^0, \mathbf{k}) \sim (mv^2, mv)$

$$\frac{-i}{(p-p')^2 + i\epsilon} = \frac{-i}{\underbrace{(p^0 - p'^0)^2}_{\ll (\mathbf{p}-\mathbf{p}')^2} - (\mathbf{p}-\mathbf{p}')^2 + i\epsilon} \approx \frac{i}{(\mathbf{p}-\mathbf{p}')^2 - i\epsilon} \sim \tilde{V}_C(|\mathbf{p}-\mathbf{p}'|)$$

$$V_C(r) \equiv -\frac{a}{r}$$

Expansion for the Gluon propagator in the potential regime gives the Fourier transform of the Coulomb potential

Summing up ladder diagrams:

$\tilde{G}_D(p', p, E) \equiv$ 

Green's function of the Schrödinger equation

$$V_C(r) \equiv -\frac{a}{r}$$

$$\left(-\frac{\nabla^2_{\mathbf{x}'}}{m} + V_C(x') - (E + i\Gamma_{\tilde{t_1}})\right) G_C(\mathbf{x}', \mathbf{x}, E) = \delta^3(\mathbf{x}' - \mathbf{x})$$

$$\tilde{G}_C(\mathbf{p}', \mathbf{p}, E) \equiv \int d^3\mathbf{x}' d^3\mathbf{x} e^{-i\mathbf{p}'\cdot\mathbf{x}'} G_C(\mathbf{x}', \mathbf{x}, E) e^{i\mathbf{p}\cdot\mathbf{x}}$$

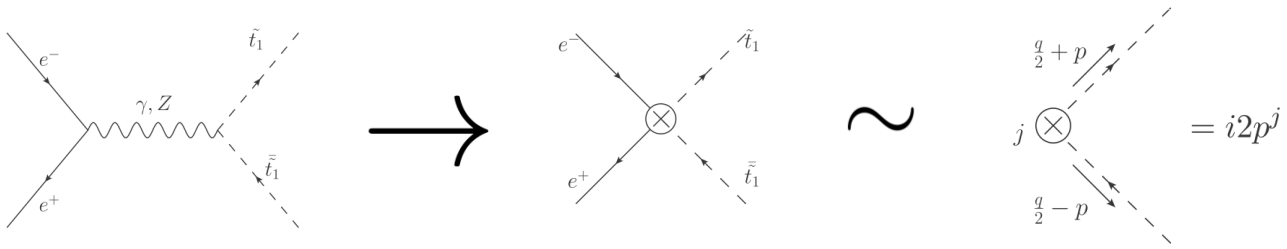
Connection between Green's function and the sum of ladder diagrams

$$\tilde{G}_D(p', p, E) \approx f(p')(-i)\tilde{G}_C(\mathbf{p}', \mathbf{p}, E) f(p)$$

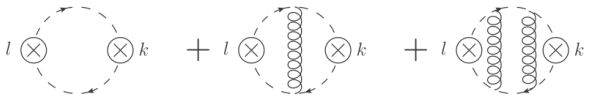
$$f(p) \equiv \frac{i}{\frac{E}{2} + p^0 - \frac{\mathbf{p}^2}{2m} + i\frac{\Gamma_{\tilde{t_1}}}{2}} + \frac{i}{\frac{E}{2} - p^0 - \frac{\mathbf{p}^2}{2m} + i\frac{\Gamma_{\tilde{t_1}}}{2}}$$

Factorization:

$\tilde{t}_1\bar{\tilde{t}}_1$ production vertex



p-wave Green's function



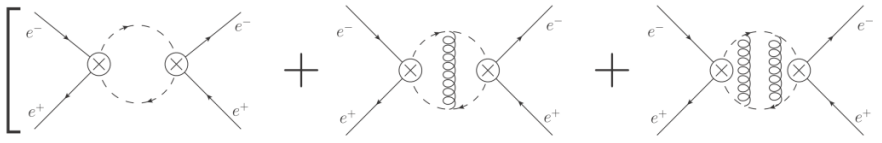
$$\sim \int \frac{d^3p'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3} p'^l \tilde{G}_C(\mathbf{p}', \mathbf{p}, E) p^k \equiv \tilde{G}_C^P \frac{\delta^{lk}}{3}$$

Optical theorem

$$\sigma_{\text{tot}}(k_1, k_2 \to \text{anything}) = \frac{Im(\mathcal{M}_{\text{fwd. scatter}})}{2E_{\text{cm}}p_{\text{cm}}}$$

$\rightarrow$ Factorization formula

$$\sigma_{\text{tot}} \sim \text{Im}\left[\right.$$



$$\left. \right] \sim \text{Im}\left[(C_V^2 + C_A^2)\tilde{G}_C^P\right]$$

PS divergences in $Im\tilde{G}_C^P(E)$

$$\tilde{G}_C^P(E) = \frac{m^4}{4\pi}\left\{ iv^3 - av^2\left[\ln\left(\frac{-iv}{\nu}\right) - 1 + \ln 2 + \gamma_E + \Psi\left(1 - \frac{ia}{2v}\right)\right] + i\frac{va^2}{4} - \frac{a^3}{4}\left[\ln\left(\frac{-iv}{\nu}\right) - \frac{7}{4} + \ln 2 + \gamma_E + \Psi\left(1 - \frac{ia}{2v}\right)\right]\right\} + \frac{m^4}{16\pi}\left(\frac{1}{\epsilon} + \frac{2}{3}\right)\left(v^2 a + \frac{a^3}{8}\right),$$

$$v^2 = \frac{E + i\Gamma}{2m},\ a \sim \alpha_S \sim v$$

We assume a finite width $\Gamma \sim E \sim mv^2$

- $\frac{1}{\epsilon}$ divergence in the imaginary part appears at LO
- Even at LO we need additional matching information to compute σ_{tot}
- Divergence is a PS divergence

Comparison to $e^+e^- \to t\bar{t}$

$$\tilde{G}_C^S(E) \equiv \int \frac{d^3p'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}(\mathbf{p}'\mathbf{p})\tilde{G}_C(\mathbf{p}', \mathbf{p}, E) \quad , Im\tilde{G}_C^S(E) \text{ is finite!}$$

Invariant mass cut-off:

Possible stop decays (Electroweak decay):

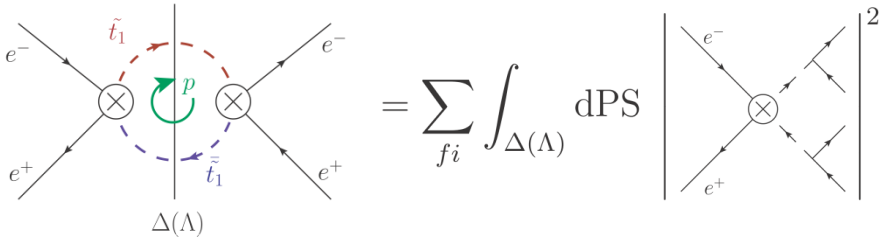
$$\tilde{t}_1 \to b\tilde{\chi}_j^+,\ t\tilde{\chi}_j^0$$

Kinematic Cut:

$$M_{\tilde{t}_1}^2 = p_{\tilde{t}_1}^2 = (p_b + p_{\tilde{\chi}_1^+})^2$$

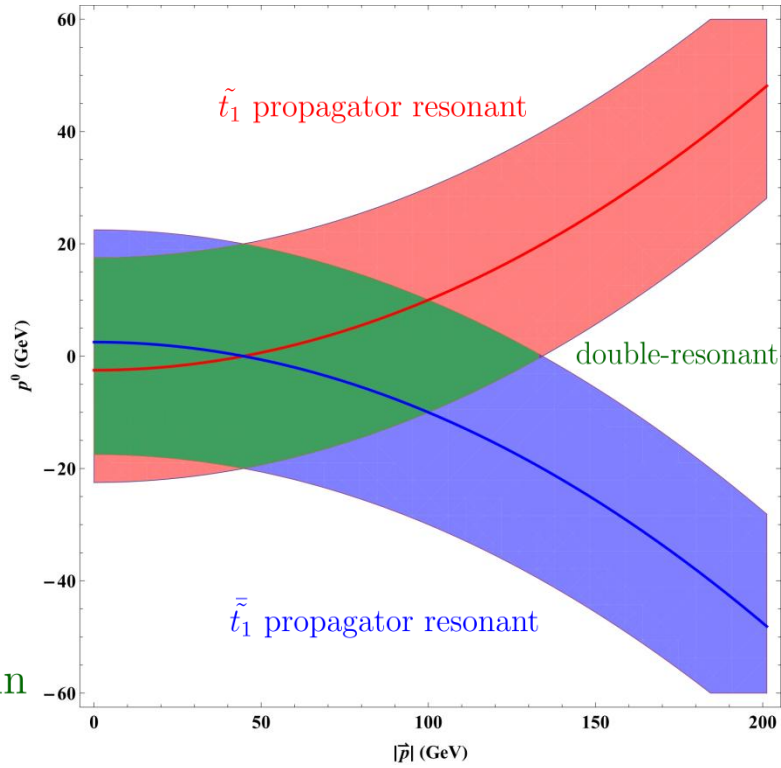
$$\left| m - M_{\tilde{t}_1} \right| \leq \Delta M$$

Meaning of Cutkosky cut with invariant mass cutoff:



$$\Lambda = \sqrt{2m\Delta M}$$

p is restricted to the double-resonant integration domain which is adequately described in vNRQCD



Power Counting:

Scale of ΔM and Λ:

$$\Delta M \sim mv$$

$$\Lambda = \sqrt{2m\Delta M} \sim mv^{1/2}$$

- Λ in between hard (m) and soft (mv) scale
- Physics at $\mathbf{p} \sim \Lambda$ is hard compared to the Physics at the soft (mv) scale
  $\rightarrow$ PS Matching $\rightarrow$ sum up $\ln\left(\frac{\Lambda^2}{\vec{p}^2}\right)$

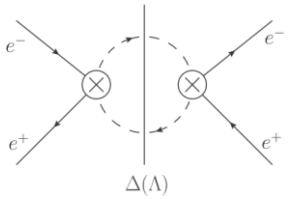
Definition of LL and NLL:

$$\text{LL} = \Big\{ \underbrace{\frac{\Gamma_{\tilde{t}}\Lambda}{m^2}}_{O\left(v^{5/2}\right)}, \underbrace{v^3\left(\frac{\alpha_S}{v}\right)^n, \frac{\alpha_S\Gamma_{\tilde{t}_1}}{m}\left(\alpha_S \ln v\right)^n \ldots}_{O(v^3)} \Big\}$$

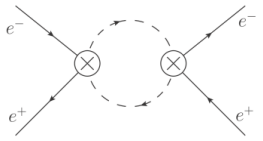
$$\text{NLL} = \Big\{ \underbrace{\frac{\Gamma^2}{m\Lambda}, \frac{v^2\Gamma}{m\Lambda}}_{O\left(v^{7/2}\right)}, \underbrace{\frac{\alpha_S\Gamma^2_{\tilde{t}_1}}{\Lambda^2}, \frac{\alpha_S\Gamma^2_{\tilde{t}_1}}{\Lambda^2}\left(\frac{\alpha_S}{v}\right)^n \ldots}_{O(v^4)} \Big\}$$

PS matching:

Result for PS Integral with kinematic cut at $O(\alpha_S^0)$:



$$= 2Im\left[-i\right.$$

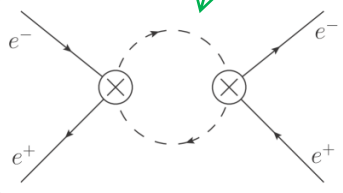


$$\left.\right] + 4N_C m^4 \left[\boxed{\frac{1}{\sqrt{2}\pi^2}\frac{\Gamma_{\tilde{t}_1}\Lambda}{m^2}} - \frac{3}{\sqrt{2}\pi^2}\frac{E\Gamma_{\tilde{t}_1}}{m\Lambda} - \frac{1}{\pi^3}\frac{\Gamma_{\tilde{t}_1}^2}{m\Lambda} + \frac{3\mathrm{arsinh}(1)}{\sqrt{2}\pi^3}\frac{\Gamma_{\tilde{t}_1}^2}{m\Lambda} + O(v^{9/2})\right]$$

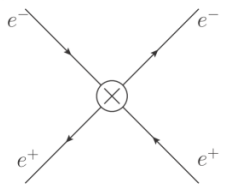
We match the effect of the kinematic cut into local operators $\rightarrow$ sum up $\ln\left(\frac{\Lambda^2}{\vec{p}^2}\right)$

Corresponding expression in EFT with PS matching:

$$2Im\left[-i\left(\right.\right.$$



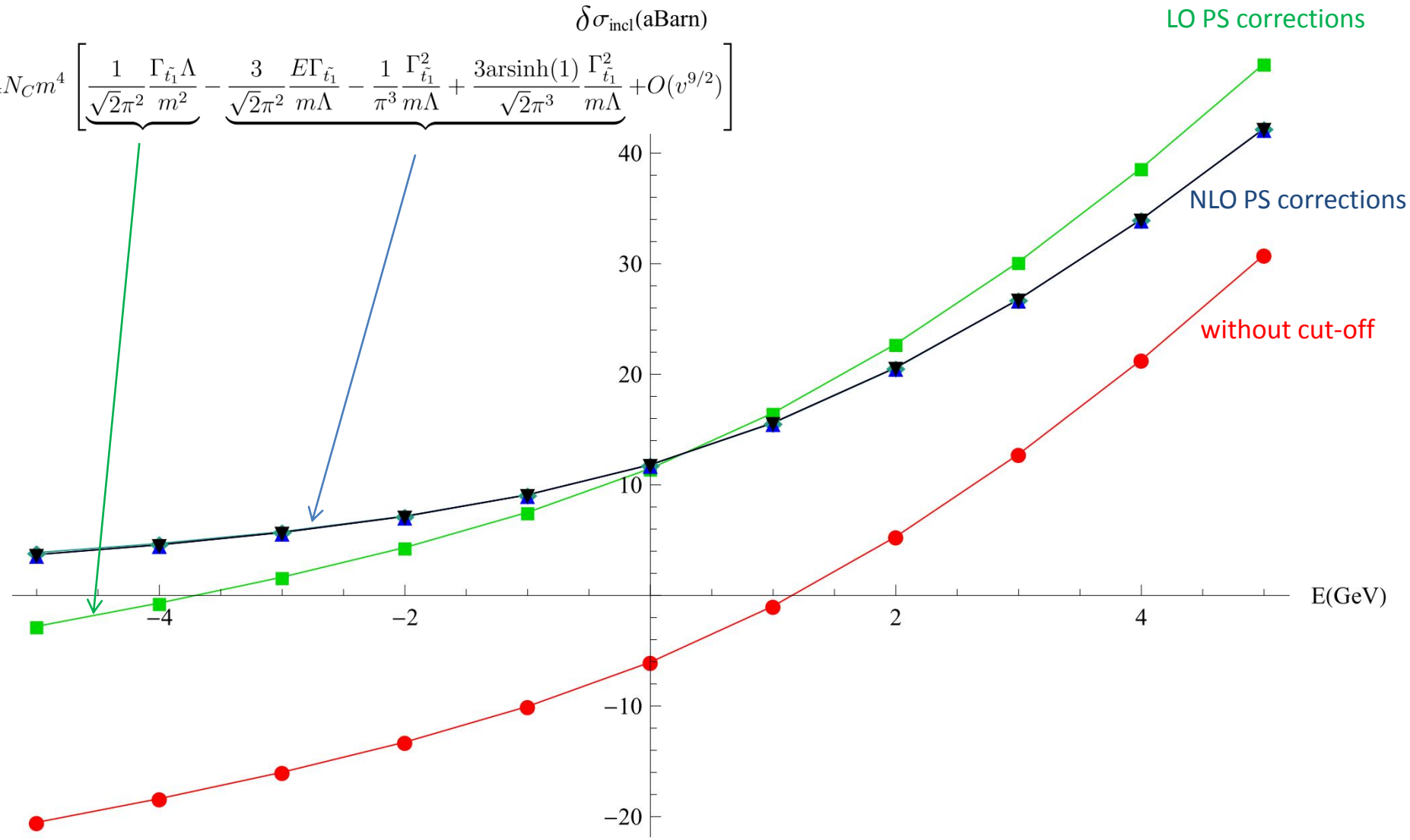
$$+ \sum_{n=0}^{1} \tilde{C}_{V/A,\mathrm{bare}}^{(0,n)}(\Lambda)$$



$$\left.\left.\right)\right]$$

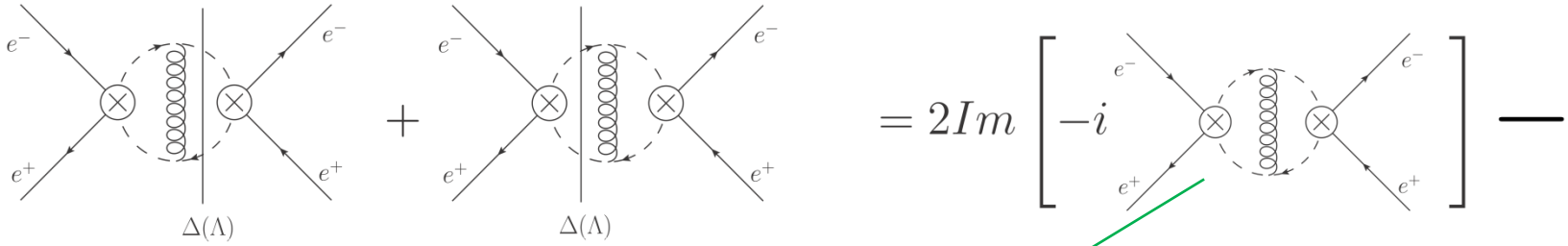
$O(\alpha_S^0)$, $\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma_{\tilde{t}_1} = 1.2\text{GeV}$

$$4N_C m^4 \left[\underbrace{\frac{1}{\sqrt{2}\pi^2}\frac{\Gamma_{\tilde{t}_1}\Lambda}{m^2}} - \underbrace{\frac{3}{\sqrt{2}\pi^2}\frac{E\Gamma_{\tilde{t}_1}}{m\Lambda} - \frac{1}{\pi^3}\frac{\Gamma_{\tilde{t}_1}^2}{m\Lambda} + \frac{3\text{arsinh}(1)}{\sqrt{2}\pi^3}\frac{\Gamma_{\tilde{t}_1}^2}{m\Lambda}} + O(v^{9/2}) \right]$$



PS matching:

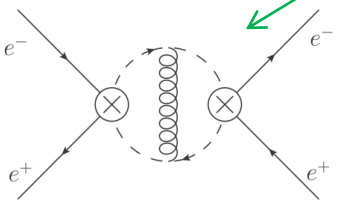
Result for PS Integral with kinematic cut at $O(\alpha_S)$:

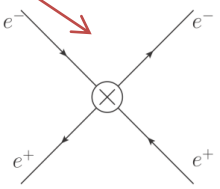


$$\underbrace{\frac{4N_C a m^3 \Gamma_{\tilde{t}_1}}{\pi}\left[\frac{1}{3}+\frac{\ln(2)}{4}+\boxed{\frac{1}{8\epsilon}}-\frac{1}{2}\ln\left(\frac{\Lambda}{m\nu}\right)+\boxed{\frac{Em}{\Lambda^2}}-\frac{\Gamma_{\tilde{t}_1} m}{2\pi\Lambda^2}+O(v^2)\right]}$$

We match the effect of the kinematic cut into local operators $\rightarrow$ sum up $\ln\left(\frac{\Lambda^2}{\vec{p}^2}\right)$

Corresponding expression in EFT with PS matching:

$$2Im\left[-i\left(\right.\right.$$


$$+\sum_{n=0}^{1}\tilde{C}^{(1,n)}_{V/A,\text{bare}}(\Lambda)$$


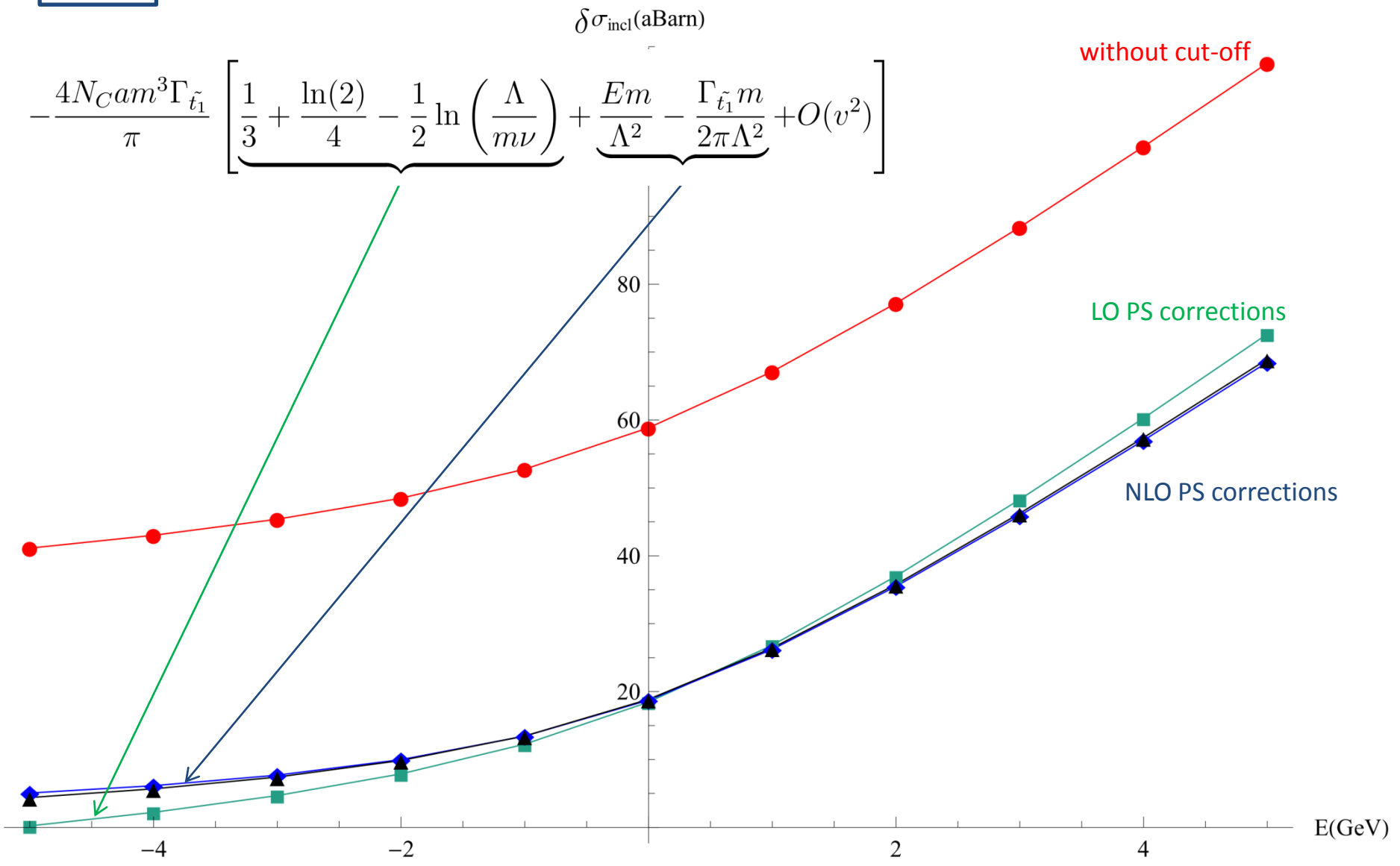
$$\left.\left.\right)\right]$$

RGE running due to the PS divergence:

$$\frac{d\tilde{C}_{V/A}(\nu)}{d\ln(\nu)} = -i\frac{C_{V/A}(\nu)^2 N_C a(\nu) m^3 \Gamma_{\tilde{t}_1}}{3\pi}$$

$O(\alpha_S)$, $\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma_{\tilde{t}_1} = 1.2\text{GeV}$

$$-\frac{4N_C a m^3 \Gamma_{\tilde{t}_1}}{\pi}\left[\underbrace{\frac{1}{3}+\frac{\ln(2)}{4}-\frac{1}{2}\ln\left(\frac{\Lambda}{m\nu}\right)}+\underbrace{\frac{Em}{\Lambda^2}-\frac{\Gamma_{\tilde{t}_1} m}{2\pi\Lambda^2}}+O(v^2)\right]$$

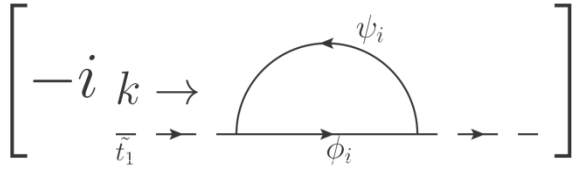


Full theory expression for a cut propagator:

$$\frac{2mi}{(\frac{q}{2}\pm p)^2-m^2+\Pi} \to -2\mathrm{Im}\left(\frac{2m}{(\frac{q}{2}\pm p)^2-m^2+\Pi}\right) =$$
$$\frac{2mi}{(\frac{q}{2}\pm p)^2-m^2+\Pi}\left(\frac{2\mathrm{Im}\Pi}{2m}\right)\left(\frac{2mi}{(\frac{q}{2}\pm p)^2-m^2+\Pi}\right)^*$$

$2\mathrm{Im}\Pi$ corresponds to a sum of PS integrals:

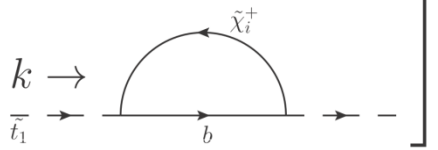
$$2Im\Pi(k^2) = \sum_{i=fi} 2Im\left[-i\right.$$



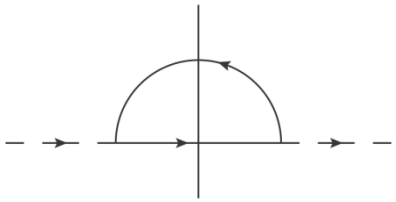
$$\left.\right]$$

We can restrict us to a specific final state:

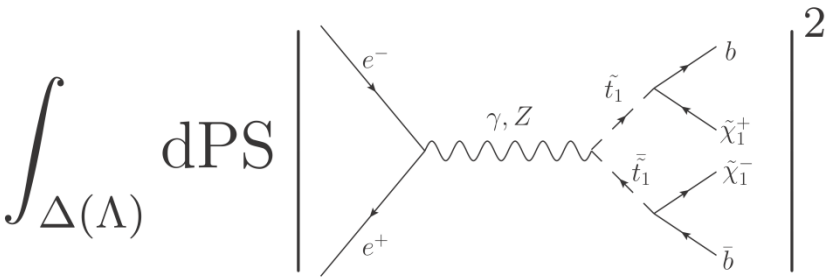
$$2\mathrm{Im}\Pi(k^2) \to 2Im\tilde{\Pi}(k^2) \equiv 2\mathrm{Im}\left[(-i)\right.$$

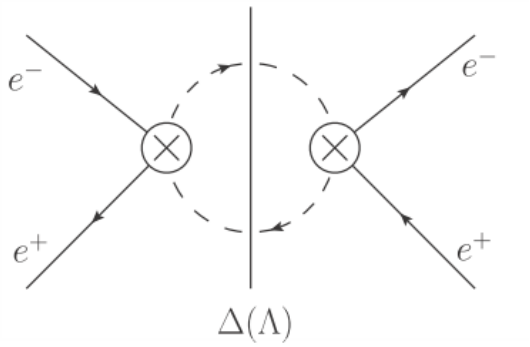


$$\left.\right] =$$

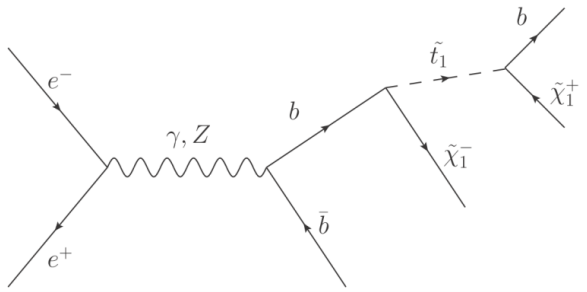


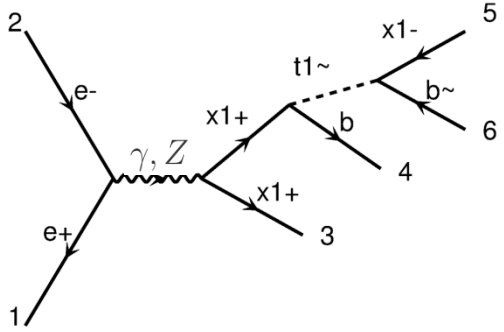
Double-resonant diagrams:



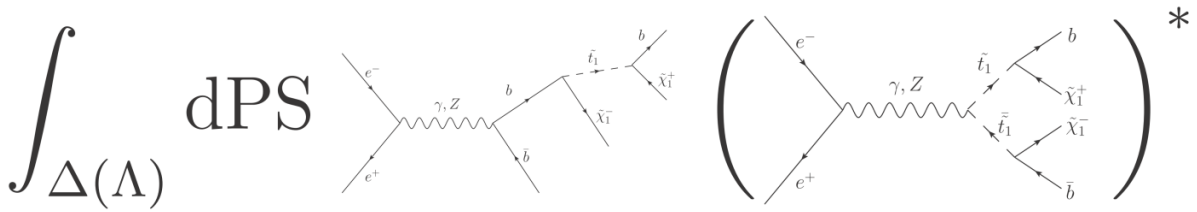


Single-resonant background diagrams:

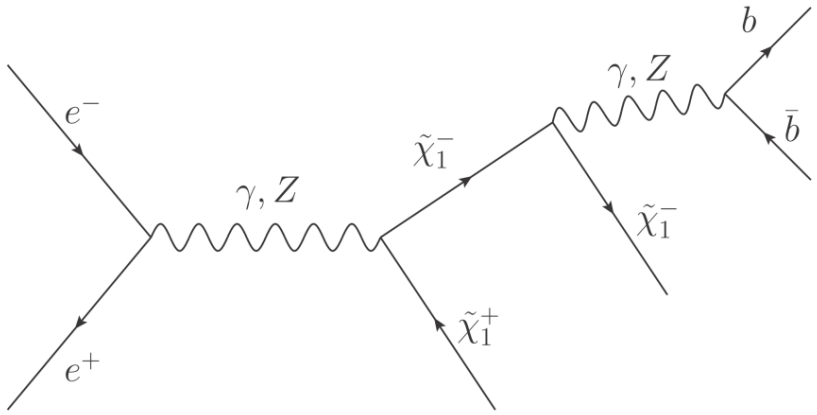


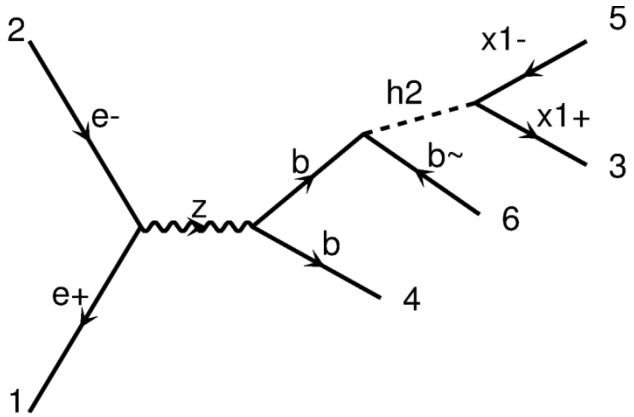


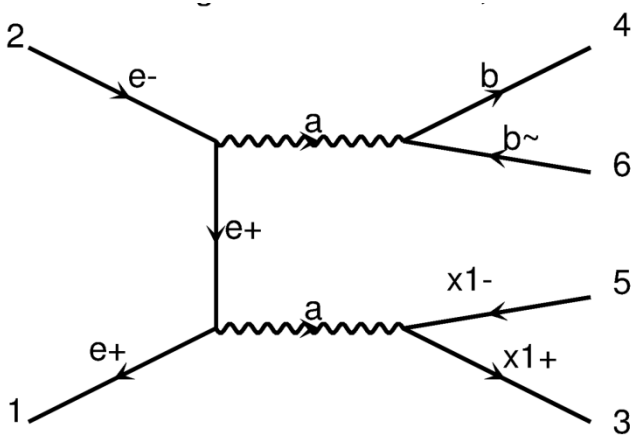
Interference between single and double resonant diagrams:

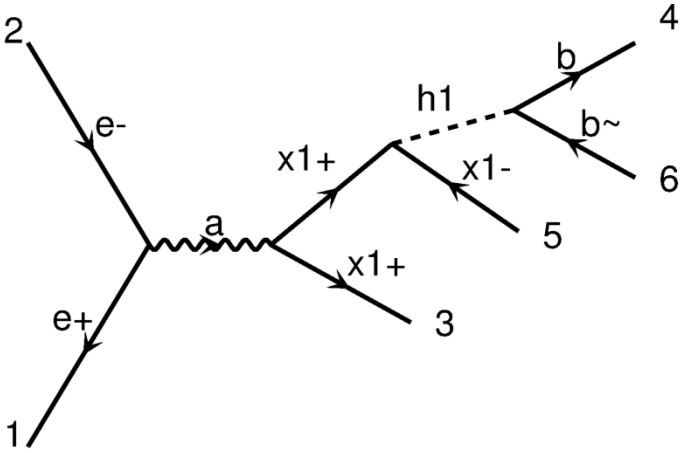


Non-resonant background diagrams:

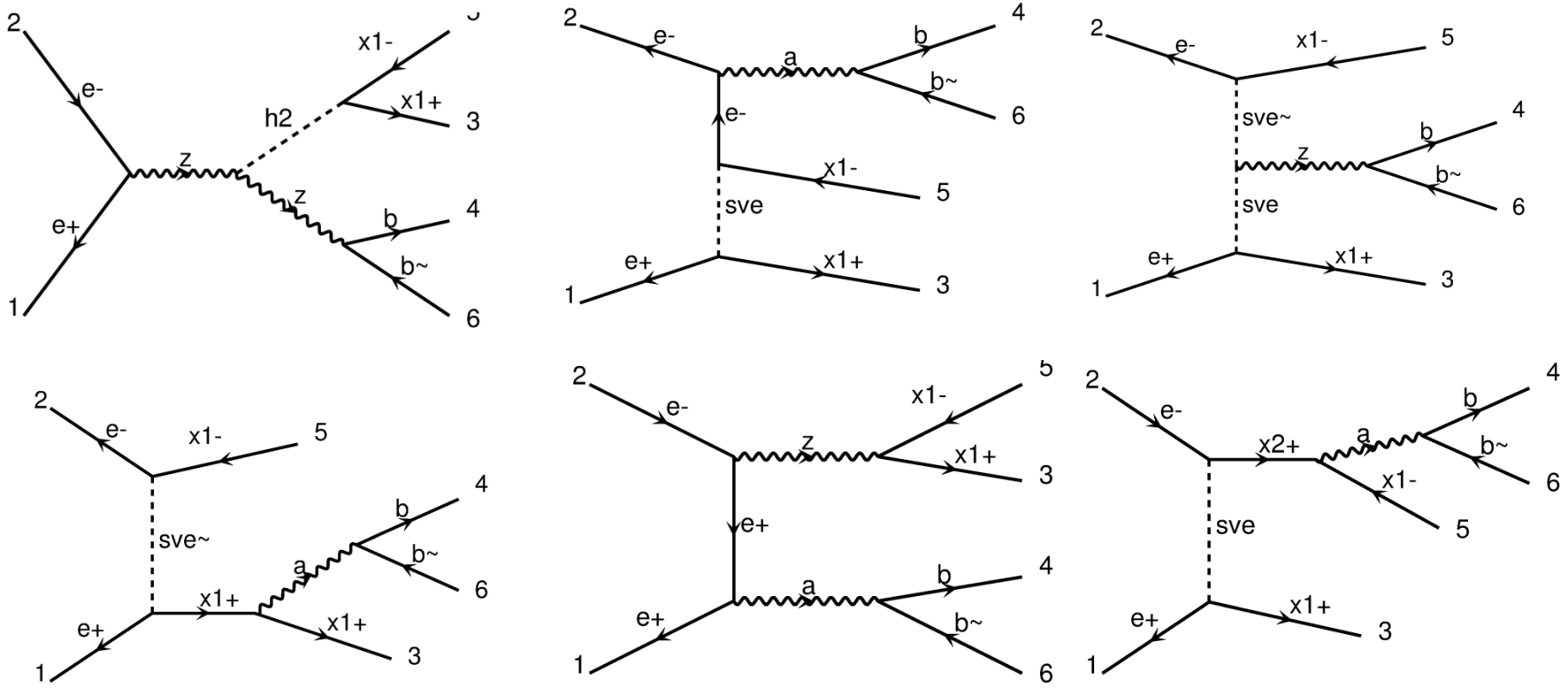








Non-resonant background diagrams:



$\sim$ 100 non-resonant background diagrams

Contributions to $\sigma_{\rm incl}(\Lambda)$

$$\sigma_{\rm incl}(\Lambda) = \sigma_{\rm NRQCD}(\Lambda) + \sigma_{\rm rem}(\Lambda)$$

Validation of our method

We approximate $\sigma_{\rm incl}$ with $\sigma_{\rm NRQCD}$

$\rightarrow$ it has to be checked that we can find a value for ΔM such that

- Double-resonant contributions are described well in vNRQCD
- Expansion of the effect of the kinematic cut in $\frac{Em}{\Lambda}$, $\frac{\Gamma_{\tilde{t}_1} m}{\Lambda}$ converges
- The contribution of the background ($\sigma_{\rm rem}$) is small

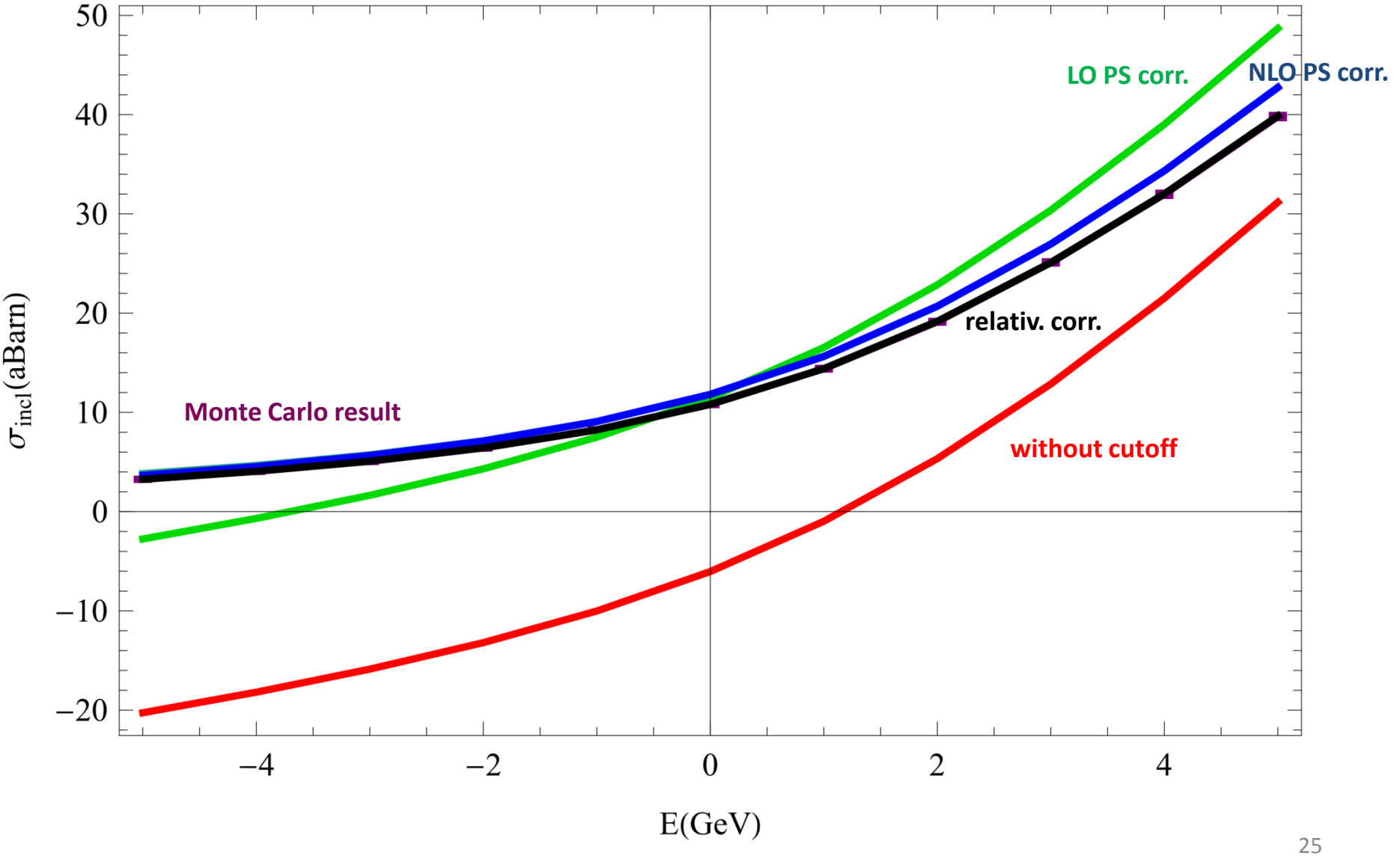
$\rightarrow$ we check this with Monte Carlo simulations in Madgraph for

- $e^+e^- \to \tilde{t}_1 \bar{\tilde{t}}_1 \to b\,\chi_1^+\,\bar{b}\,\chi_1^-$ (only double-resonant diagrams)
- $e^+e^- \to b\,\chi_1^+\,\bar{b}\,\chi_1^-$ (includes background)

Our Monte Carlo simulation is a tree level computation

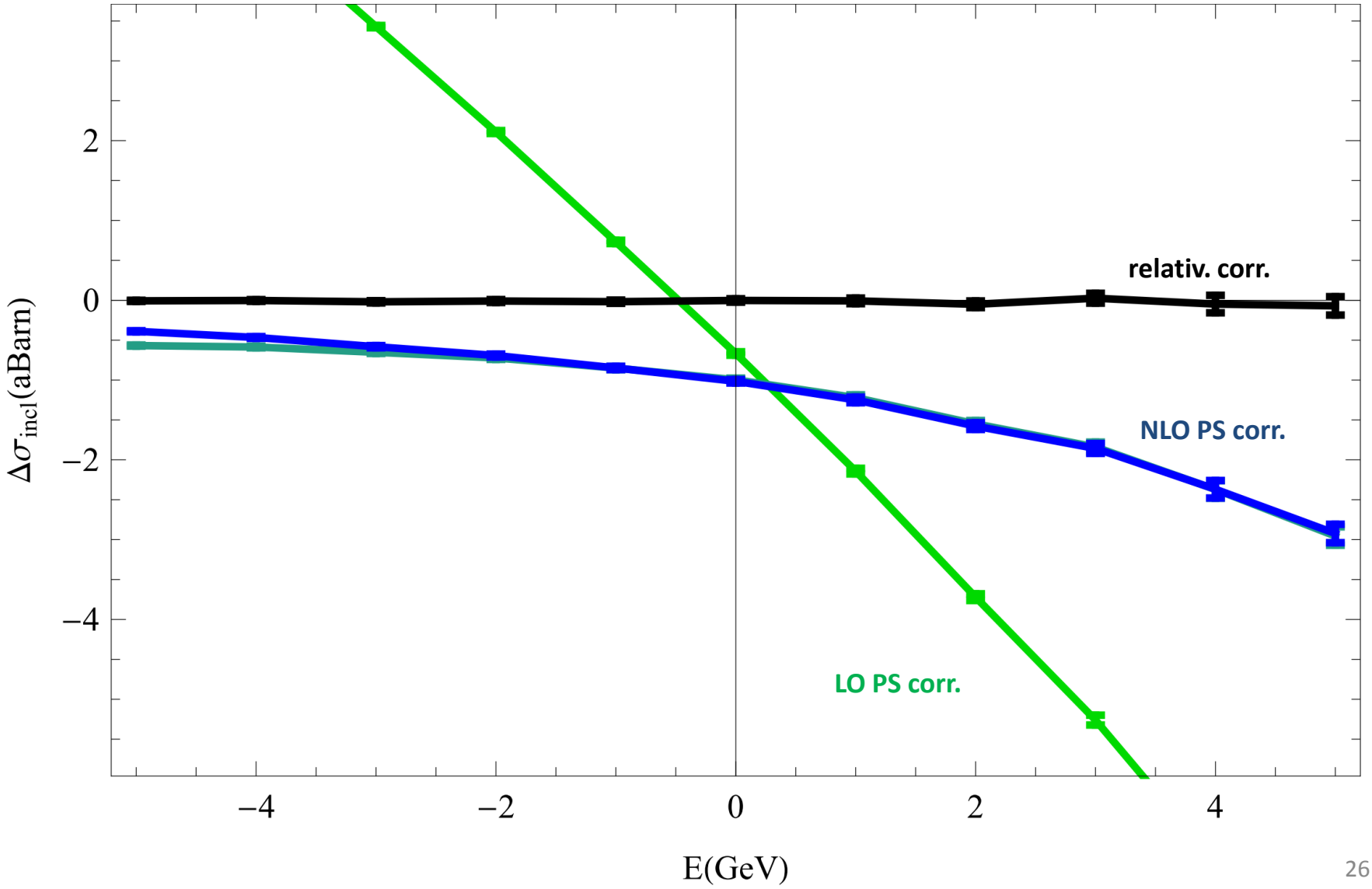
Comparison of our results with a Monte Carlo simulation for $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1 \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$O(\alpha_S^0)$, $\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$



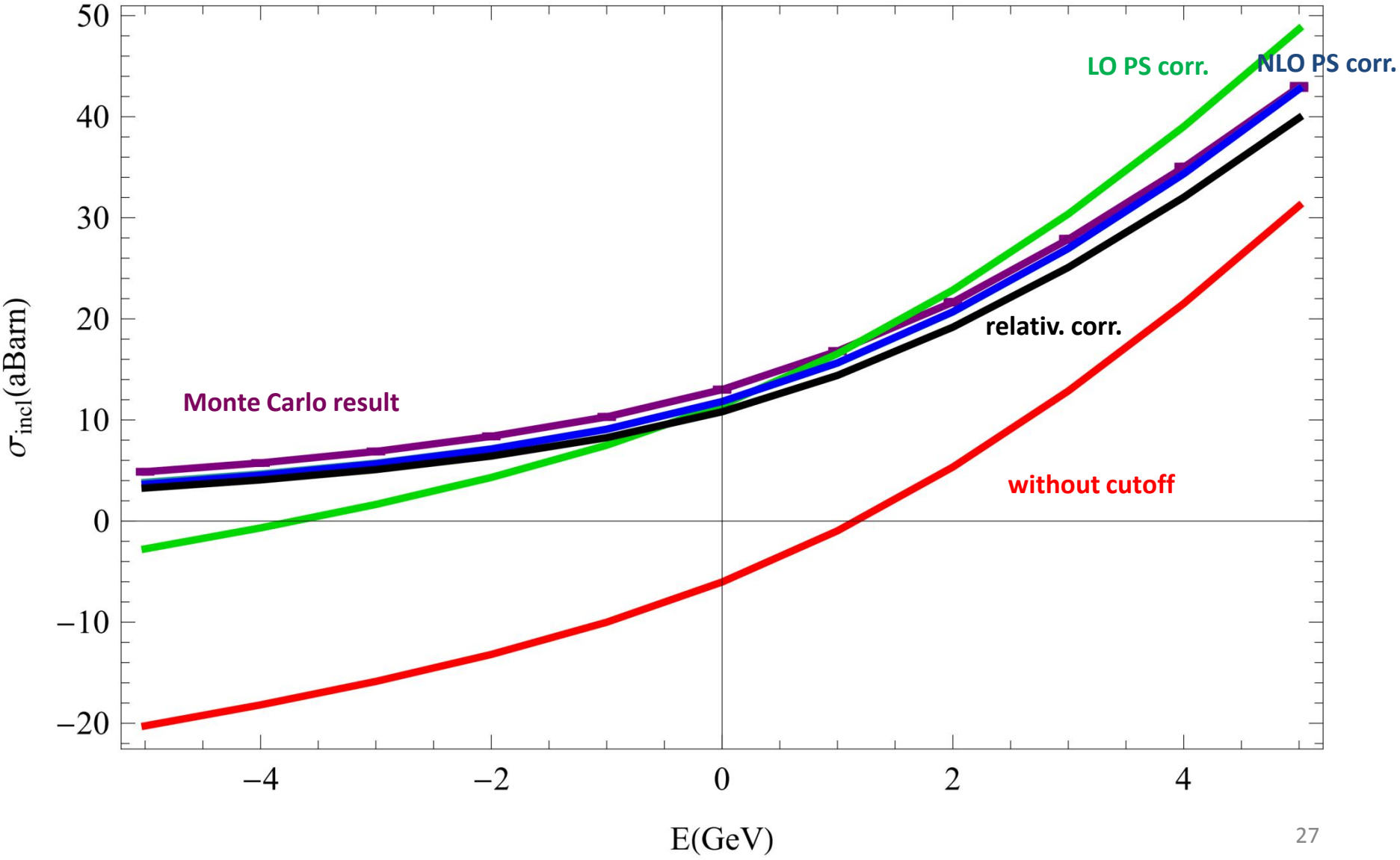
Comparison of our results with a Monte Carlo simulation for $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1 \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$O(\alpha_S^0),\ \Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$



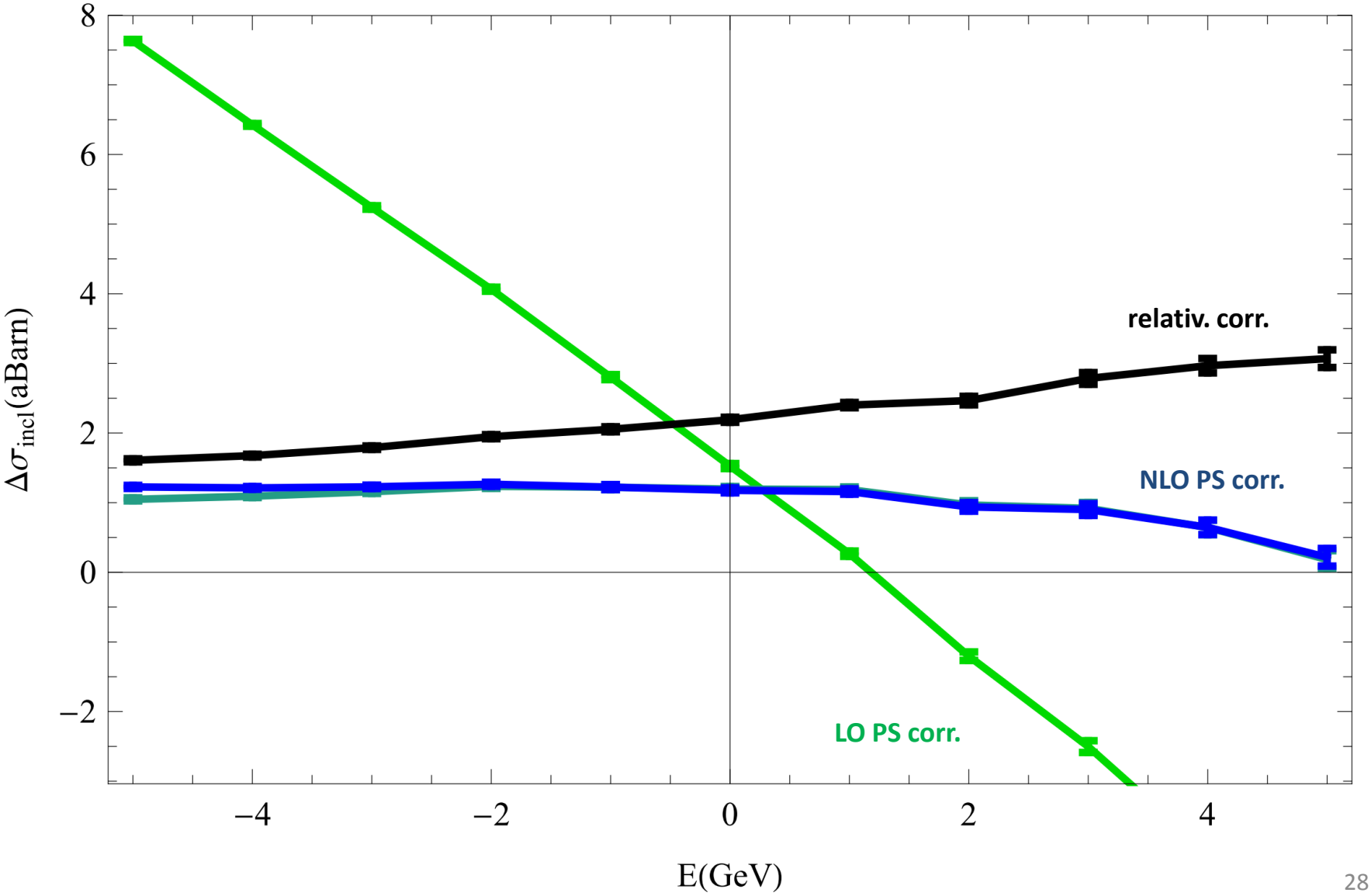
Comparison of our results with a Monte Carlo simulation for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$O(\alpha_S^0)$, $\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$



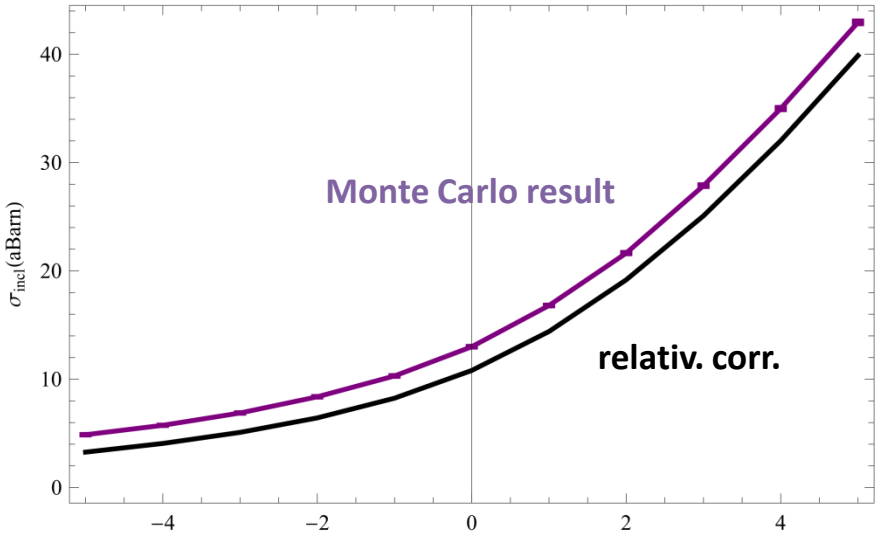
Comparison of our results with a Monte Carlo simulation for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$O(\alpha_S^0),\ \Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$

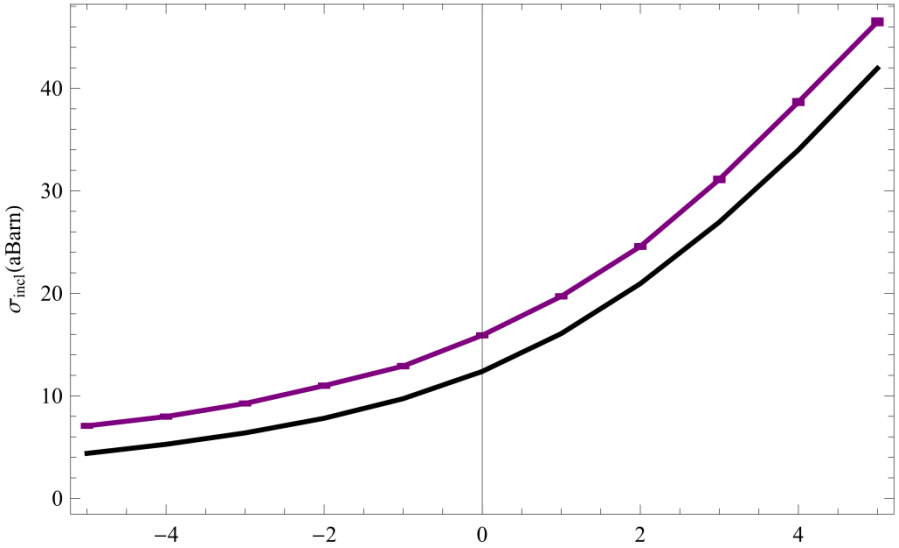


Comparison of our results with a Monte Carlo simulation for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

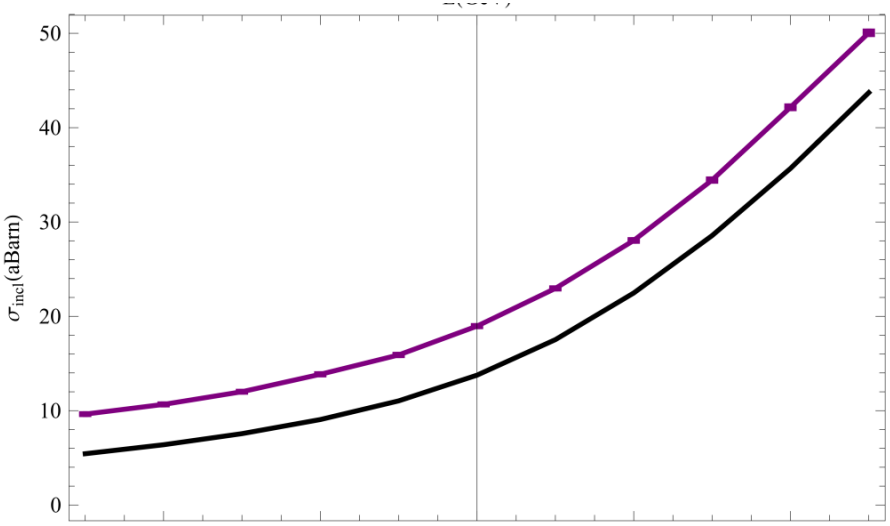
$\Delta M = 21\text{GeV}$



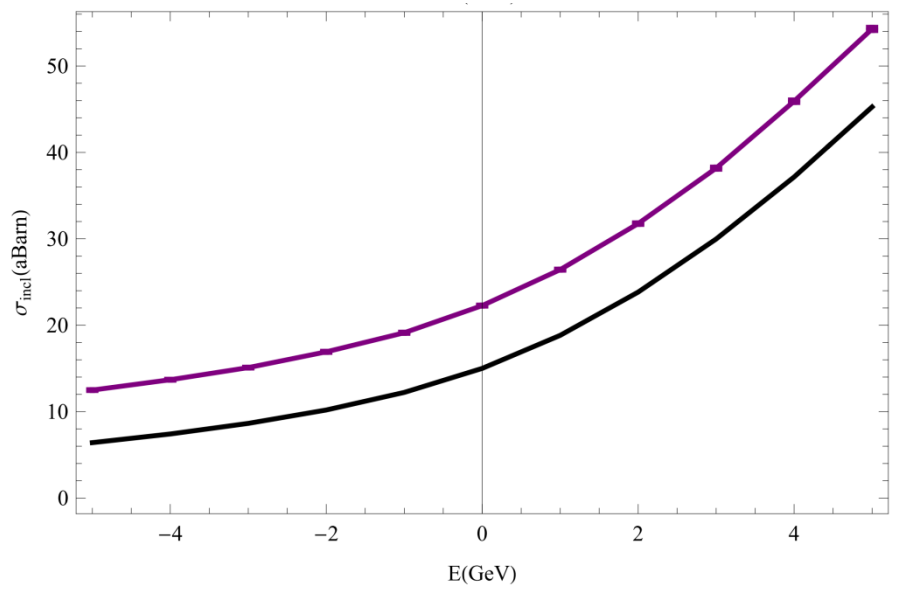
$\Delta M = 26\text{GeV}$



$\Delta M = 31\text{GeV}$



$\Delta M = 36\text{GeV}$

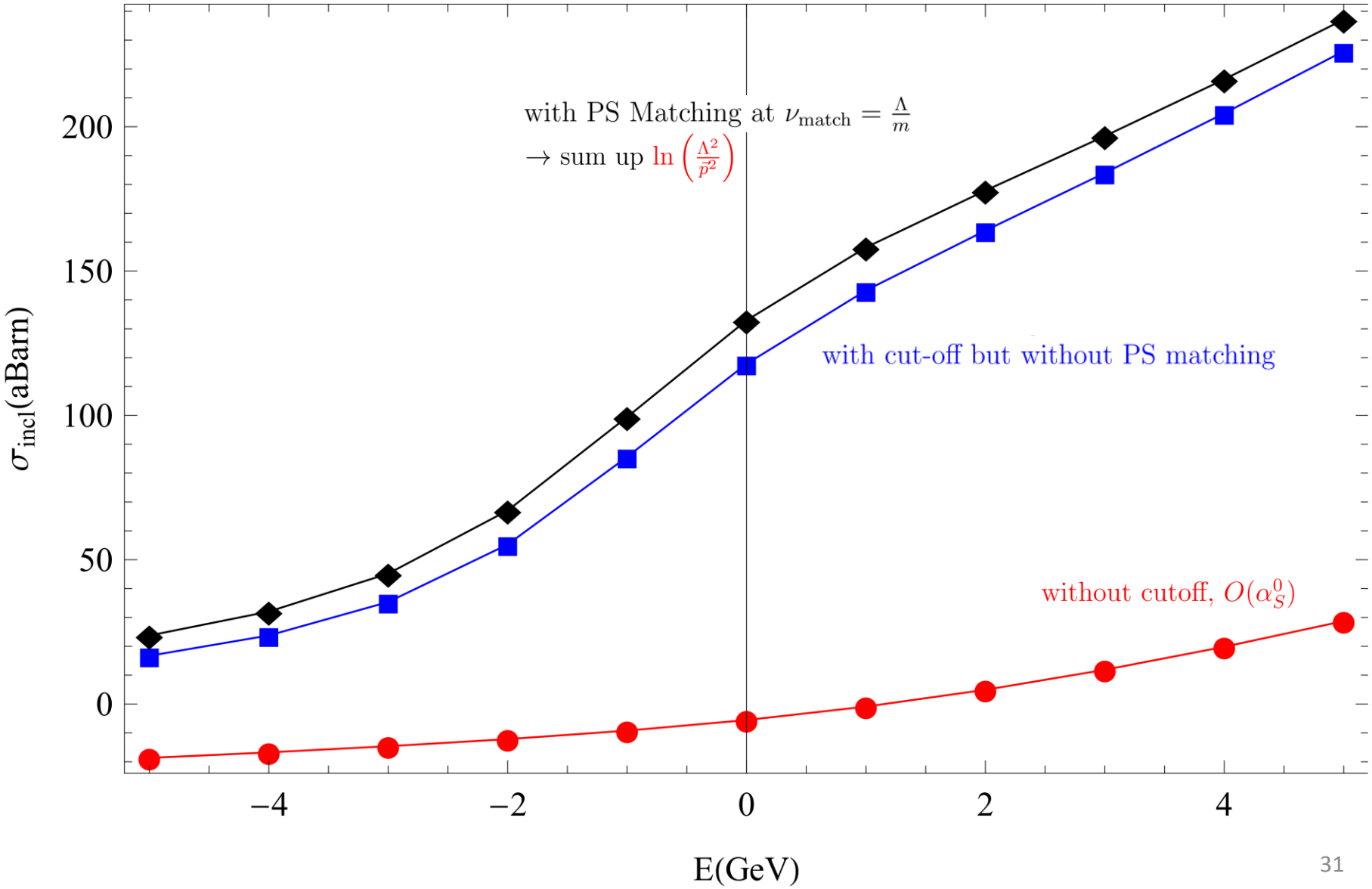


LL result for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

- Match to vNRQCD at $\nu = 1 \to$ sum up $\ln\left(\frac{m^2}{\mathbf{p}^2}\right)$, $\ln\left(\frac{m^2}{E^2}\right)$, $\ln\left(\frac{\mathbf{p}^2}{E^2}\right)$
- PS Matching at $\nu = \frac{\Lambda}{m} \to$ sum up $\ln\left(\frac{\Lambda^2}{\vec{p}^2}\right)$
- Apply EFT for $\nu = v$

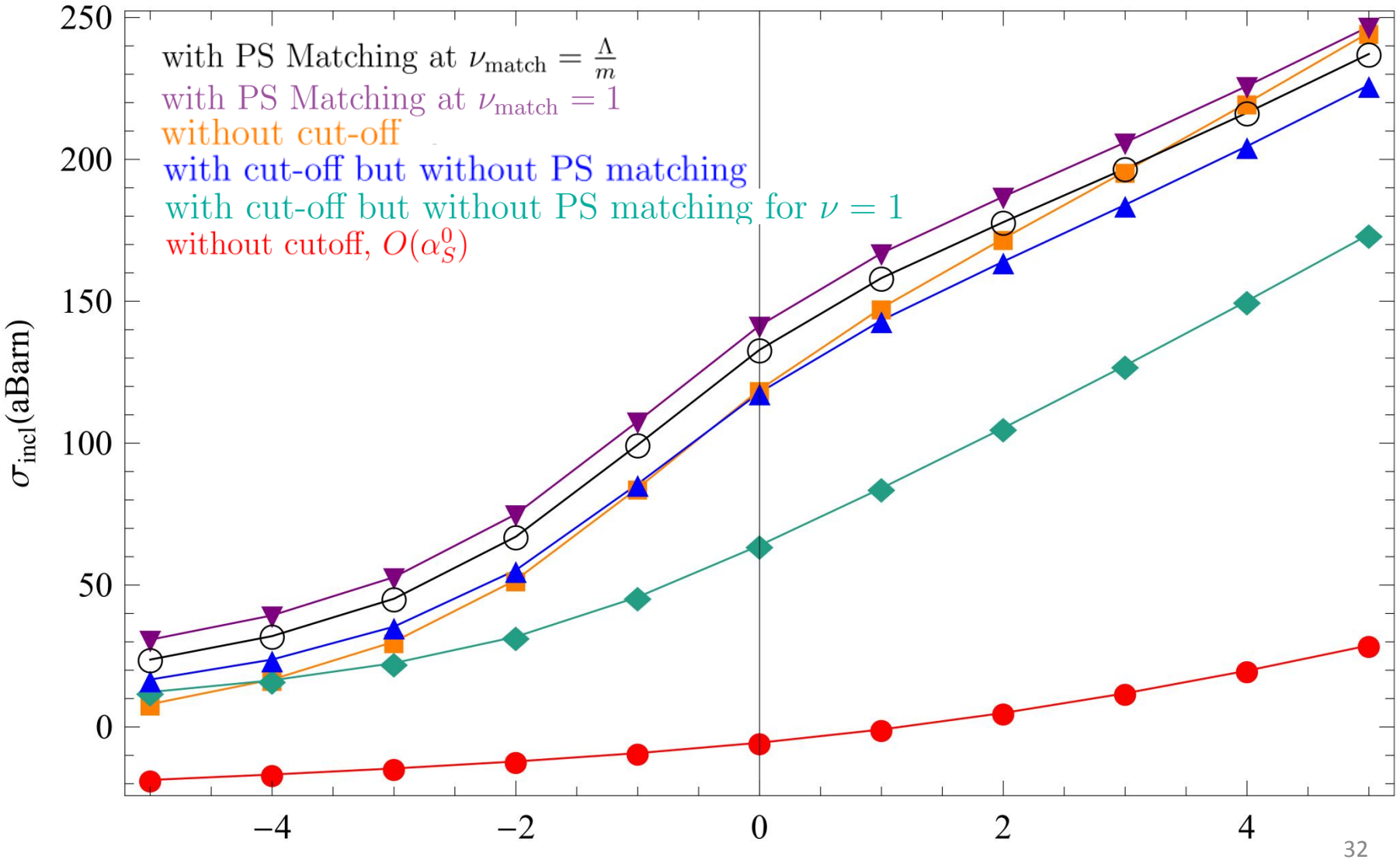
LL result for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$



LL result for $e^+e^- \to b\,\chi_1^+\,\bar{b},\chi_1^-$

$\Delta M = 21\text{GeV},\ m = 400\text{GeV},\ \Gamma\left(\tilde{t}_1 \to b\tilde{\chi}_1^+\right) = 1.2\text{GeV}$



Summary/Outlook:

- Calculation of $\sigma(e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1)$ close to threshold
- For $v \sim \alpha_S$ no convergence in expansion in orders of α_S
- PS divergences appear already at LO
- We avoid PS divergences by introducing a kinematic cut on the final states
- We apply PS-matching to sum up logs of the form $\ln\left(\frac{\Lambda^2}{\vec{p}^2}\right)$
- Compare to $O(\alpha_S^0)$ Monte Carlo simulation for: $e^+e^- \to b\,\chi_1^+\,\bar{b}\,\chi_1^-$
- Corrections to the Coulomb potential as well as effects of single resonant-diagrams are needed for a full NLL result