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Neutrino mass models and sizable θ_{13}

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IP3

Outline

- experimental milestones
- tri-bimaximal lepton mixing (until recently)
- family symmetries like A_4 and S_4
- direct vs. indirect family symmetry models
- strategies to implement sizable reactor angle θ_{13} (post T2K)
  - tri-bimaximal mixing plus corrections
  - non-standard vacuum configurations
  - new family symmetries

A brief history of neutrino mixing

- atmospheric neutrinos
  - ν_μ / $\overline{\nu}_\mu$ disappear – Super-Kamiokande (1998)
- accelerator neutrinos
  - ν_μ disappear – K2K (2002), MINOS (2006)
  - ν_μ converted to ν_τ – OPERA (2010)
  - ν_μ converted to ν_e – T2K (2011), MINOS (2011)
- solar neutrinos
  - ν_e disappear – Chlorine (1998), Gallium (1999 - 2009), Super-Kamiokande (2002), Borexino (2008)
  - ν_e converted to $(\nu_\mu + \nu_\tau)$ – SNO (2002)
- reactor neutrinos
  - $\overline{\nu}_e$ disappear – Double Chooz (2011), Daya Bay (2012)
  - $\overline{\nu}_e$ disappear – KamLAND (2002)

2011/2012 indications of a non-zero θ_{13}

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$ disfavoured at $\sim 2.5\sigma$
- $5° \lesssim \theta_{13} \lesssim 18°$ at 90% C.L.

MINOS [arXiv:1108.0015]

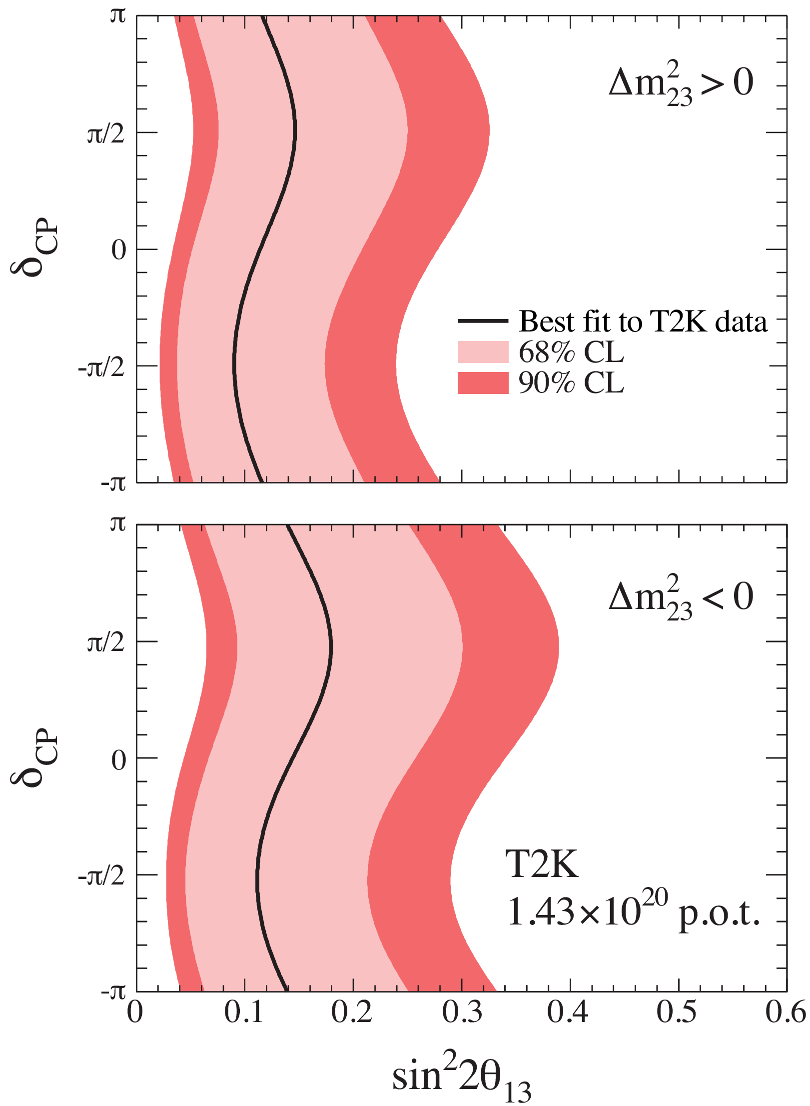
- $\theta_{13} \neq 0$ disfavoured at $\sim 1.6\sigma$
- $\theta_{13} \lesssim 13°$ at 90% C.L.

Double Chooz [arXiv:1112.6353]

- $\theta_{13} \neq 0$ disfavoured at $\sim 2\sigma$
- $4° \lesssim \theta_{13} \lesssim 12°$ at 90% C.L.

Daya Bay [arXiv:1203.1669]

- $\theta_{13} \neq 0$ disfavoured at $\sim 5.2\sigma$
- $7° \lesssim \theta_{13} \lesssim 10°$ at 90% C.L.



Other unresolved puzzles

- sterile neutrinos?

	yes	no
$\overline{\nu}_\mu \to \overline{\nu}_e$	LSND (2001) MiniBooNE (2010)	KARMEN (2002)
$\nu_\mu \to \nu_e$		MiniBooNE (2007)

- superluminal neutrinos?

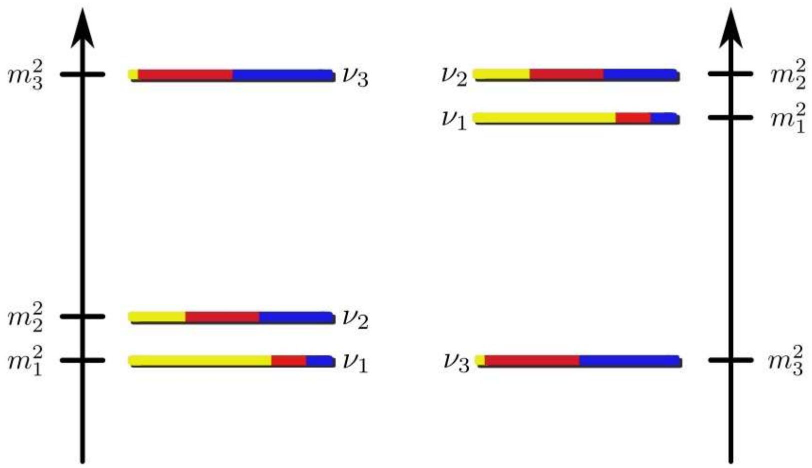
yes	no
OPERA (2011)	SN1987a

- future experiments will have to settle these questions
- not discussed in this talk

Three neutrino flavour mixing

(in diagonal charged lepton basis)

$$\begin{array}{ccc} \text{flavour} & \text{PMNS mixing} & \text{mass} \\ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = & \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} & \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \end{array}$$



$$U_{\text{PMNS}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor + Dirac}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{\frac{\alpha_3}{2}} \end{pmatrix}}_{\text{Majorana}}$$

Tri-bimaximal lepton mixing vs. global neutrino fits

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



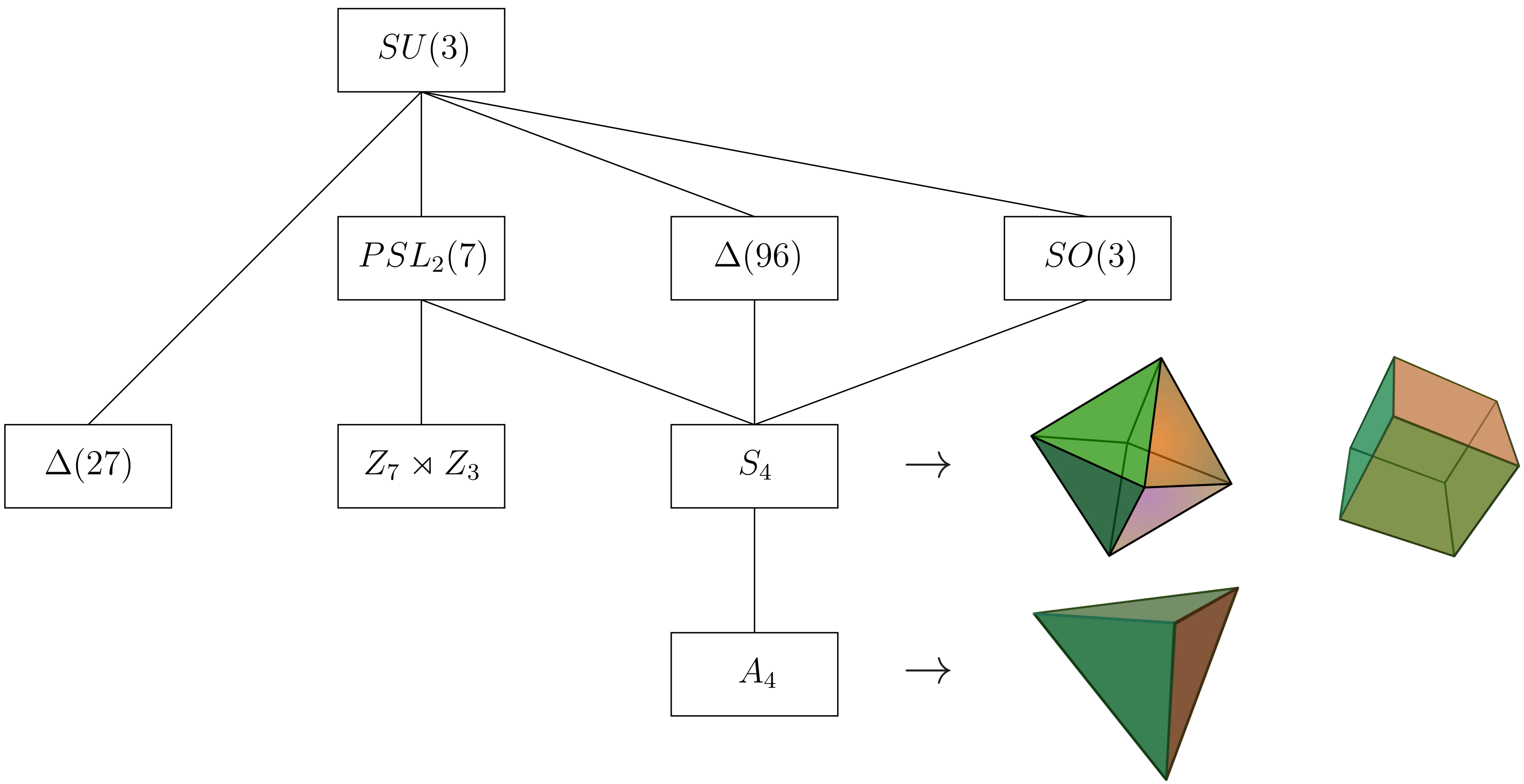
$\Rightarrow$

PMNS-angles	tri-bimax.	1σ exp.	1σ exp.
$\sin^2\theta_{12}$:	$\frac{1}{3}$	$0.297 - 0.329$	$0.296 - 0.329$
$\sin^2\theta_{23}$:	$\frac{1}{2}$	$0.45 - 0.58$	$0.39 - 0.50$
$\sin^2\theta_{13}$:	0	$0.008 - 0.020$	$0.018 - 0.032$
		Schwetz et al. (2011)	Fogli et al. (2011)

- TB mixing fits rather well $\to$ motivation for family symmetry, e.g. A_4, S_4
- how to accommodate sizable $\theta_{13} \sim 5^\circ - 10^\circ$?

Non-Abelian family symmetries

- unify three families in multiplets of family symmetry
- group should have two- or <u>three-dimensional</u> representations

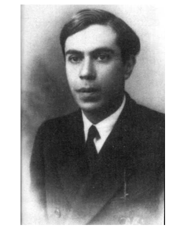


The Klein symmetry of neutrino mixing

mass basis

$$M_\ell^{\text{diag}} = M_\ell \qquad M_\nu^{\text{diag}} = U_{\text{PMNS}}^\dagger M_\nu U_{\text{PMNS}}^*$$

$$M_\ell^{\text{diag}} = \tilde{h}^T M_\ell^{\text{diag}} \tilde{h}^* \qquad M_\nu^{\text{diag}} = \tilde{k}^T M_\nu^{\text{diag}} \tilde{k}$$



$$\tilde{h} = \text{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}}) \qquad \tilde{k} = \text{diag}\,(\pm 1, \pm 1, \pm 1)$$

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flavour basis

$$M_\ell = h^T M_\ell h^* \qquad M_\nu = k^T M_\nu k$$

$$h = \tilde{h} \qquad k = U_{\text{PMNS}}^* \tilde{k}\, U_{\text{PMNS}}^T$$

→ Klein symmetry $\mathcal{K} = \{1, k_1, k_2, k_3\}$ of M_ν depends on U_{PMNS}

for TB mixing:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

Direct vs. indirect models

- direct models
  - Klein symmetry $\mathcal{K} \;\subset\;$ family symmetry $\mathcal{G}$
  - generate Yukawa couplings dynamically through flavon fields ϕ

    neutrino sector $\mathcal{K}$ symmetric: $\quad \mathcal{L}_\nu(\phi_\mathcal{K}) \qquad k_i\langle\phi_\mathcal{K}\rangle = \langle\phi_\mathcal{K}\rangle$

    charged lepton sector diagonal: $\quad \mathcal{L}_\ell(\phi_\mathcal{H}) \qquad h\,\langle\phi_\mathcal{H}\rangle = \langle\phi_\mathcal{H}\rangle$
  - for TB mixing (k_1, k_2, h) generate S_4
- indirect models
  - Klein symmetry $\mathcal{K}$ not necessarily $\;\subset\;$ family symmetry $\mathcal{G}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations
  - for TB mixing

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\; H\,H$$

Typical model setup

M_{Pl}

M_{GUT}

M_{seesaw}

family symmetry broken

$\rightarrow$ M_R / Yukawas generated

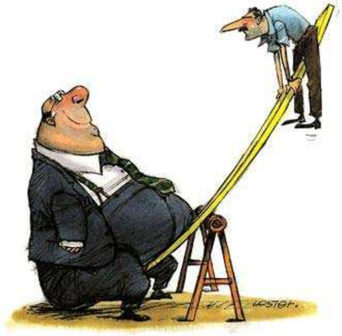
M_{EW}

electroweak symmetry broken

$\rightarrow$ light fermion masses generated

ingredients:

Majorana ν_L



seesaw

Implementing sizable θ_{13}

direct models

(I) TB plus corrections

(a) charged lepton sector

(b) RG running

(c) higher order operators

(d) extra TB breaking flavon field

(III) other family symmetries with non-standard $\mathcal{K}$

indirect models

(I) TB plus corrections

(a) charged lepton sector

(b) RG running

(II) non-standard flavon VEV configurations

(I a) Charged lepton corrections

Lepton mixing with TB neutrino mixing

- charged lepton mass matrix may not be exactly diagonal (GUTs)
- $U_{\rm PMNS} = V_{\ell_L} V^\dagger_{\nu_L}$
- $V^\dagger_{\ell_L} = \underbrace{U^{\ell_L}_{23} U^{\ell_L}_{13} U^{\ell_L}_{12}}_{\text{small mixing}}$ from M_ℓ and $V^\dagger_{\nu_L} = \underbrace{U^{\nu_L}_{23} U^{\nu_L}_{13} U^{\nu_L}_{12}}_{U_{TB}}$ from M_ν

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}^*_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}^*_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}^*_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} s_{12}\, e^{i\delta_{12}} &\approx \frac{1}{\sqrt{3}} \left(e^{i\delta^\nu_{12}} - \theta^\ell_{12}\, e^{i\delta^\ell_{12}} + \theta^\ell_{13}\, e^{i(\delta^\ell_{13} - \delta^\nu_{23})} \right) \\ s_{23}\, e^{i\delta_{23}} &\approx \frac{1}{\sqrt{2}} \left(e^{i\delta^\nu_{23}} - \theta^\ell_{23}\, e^{i\delta^\ell_{23}} \right) \\ s_{13}\, e^{i\delta_{13}} &\approx \frac{1}{\sqrt{2}} \left(-\theta^\ell_{12}\, e^{i(\delta^\ell_{12} + \delta^\nu_{23})} - \theta^\ell_{13}\, e^{i\delta^\ell_{13}} \right) \end{aligned}$$

$$\boxed{\begin{aligned} c_{ij} &= \cos\theta_{ij} \\ \hat{s}_{ij} &= \sin\theta_{ij}\, e^{-i\delta_{ij}} \end{aligned}}$$

Charged lepton mixing in $SU(5)$

up sector $\mathbf{10\ 10\ 5_H}$

$$M_u \sim \begin{pmatrix} \lambda^8 & * & * \\ * & \lambda^4 & * \\ * & * & 1 \end{pmatrix} v_u$$

- symmetric by definition
- mainly diagonal (e.g. $M^u_{12} \lesssim \lambda^6$)
- Cabibbo mixing from down sector

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down sector $\mathbf{\overline{5}\ 10\ \overline{5}_H}$ & $\mathbf{\overline{5}\ 10\ \overline{45}_H}$

$$M_d \sim \begin{pmatrix} 0 & \lambda^3 & * \\ \lambda^3 & \lambda^2 & * \\ * & * & 1 \end{pmatrix} v_d \qquad M_\ell \sim \begin{pmatrix} 0 & \lambda^3 & * \\ \lambda^3 & 3\,\lambda^2 & * \\ * & * & 1 \end{pmatrix} v_d$$

- Georgi-Jarlskog relations $m_b \sim m_\tau$, $m_s \sim \frac{1}{3} m_\mu$, $m_d \sim 3 m_e$
- Gatto-Sartori-Tonin relation $M^d_{12} = M^d_{21} \ \rightarrow \ \theta^d_{12} \sim \sqrt{\frac{m_d}{m_s}} \sim \lambda$
- charged lepton mixing $\theta^\ell_{12} \sim \frac{1}{3}\lambda \sim 4^\circ$

Does it give $\theta_{13} \sim 5^\circ - 10^\circ$?

$$
\begin{aligned}
s_{12}\, e^{i\delta_{12}} &\approx \frac{1}{\sqrt{3}} \left(e^{i\delta^\nu_{12}} - \theta^\ell_{12}\, e^{i\delta^\ell_{12}} + \theta^\ell_{13}\, e^{i(\delta^\ell_{13} - \delta^\nu_{23})} \right) \\
s_{23}\, e^{i\delta_{23}} &\approx \frac{1}{\sqrt{2}} \left(e^{i\delta^\nu_{23}} - \theta^\ell_{23}\, e^{i\delta^\ell_{23}} \right) \\
s_{13}\, e^{i\delta_{13}} &\approx \frac{1}{\sqrt{2}} \left(-\theta^\ell_{12}\, e^{i(\delta^\ell_{12} + \delta^\nu_{23})} - \theta^\ell_{13}\, e^{i\delta^\ell_{13}} \right)
\end{aligned}
$$

- θ^ℓ_{12} alone $\rightarrow$ $\boxed{\theta_{13} \sim \frac{\lambda/3}{\sqrt{2}} \sim 3^\circ}$ too small
- give up Georgi-Jarlskog Antusch, Maurer (2011); Marzocca et al. (2011)
- make use of θ^ℓ_{13} (requires non-symmetric M_d)
- sizable θ^ℓ_{12} and/or θ^ℓ_{13} might lead to a disfavoured solar angle θ_{12} ! ! !
- special phases are essential to avoid this

(I b) RG corrections

(too small – at least with a normal neutrino mass hierarchy)

(I c) Higher order corrections

(too small – without extending the model ...)

(I d) Adding a new TB breaking flavon

(direct models)

A direct S_4 model

matter	L	τ^c	μ^c	e^c	N^c	H_u	H_d
S_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Z_3^ν	1	2	2	2	2	0	0
Z_3^ℓ	0	2	1	0	0	0	0

King, Luhn (2011)

$$\langle \varphi_\ell \rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \qquad \langle \eta_\mu \rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix} \qquad \langle \eta_e \rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

flavons	φ_ℓ	η_μ	η_e	φ_ν	η_ν	ξ_ν	ζ_ν
S_4	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3^ν	0	0	0	2	2	2	0
Z_3^ℓ	1	1	2	0	0	0	0

$$\langle \varphi_\nu \rangle = v_\nu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \eta_\nu \rangle = w_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \xi_\nu \rangle = u_\nu \qquad \langle \zeta_\nu \rangle = z_\nu$$

Charged lepton sector

$$\boxed{W_\ell \;\sim\; \left[\tfrac{1}{M}(L\varphi_\ell)_{1'}\,\tau^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_\mu\,\mu^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_e\,e^c\right]H_d}$$

- Z_3^ℓ controls pairing of flavons with right-handed charged fermions
- different S_4 contractions of $(L\varphi_\ell)$ pick out different L_i components

$$\begin{aligned}
(L\varphi_\ell)_{1'} &= L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \quad \rightarrow \quad L_3 \\
(L\varphi_\ell)_2 &= \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \rightarrow \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}
\end{aligned}$$

- mass matrix diagonal by construction
- m_τ heavier than m_μ and m_e
- hierarchy between m_μ and m_e due to hierarchy of VEVs w_μ and w_e
- just a toy model of charged lepton sector (with GUTs off-diagonals)

Neutrino sector

$$\boxed{W_\nu \;\sim\; LN^cH_u + (\varphi_\nu + \eta_\nu + \xi_\nu)N^cN^c + \frac{1}{M}\zeta_\nu\eta_\nu N^cN^c}$$

- trivial Dirac neutrino Yukawa ✓
- neutrino mixing governed by heavy right-handed neutrinos
- S_4 multiplication rule ($N^c \sim \mathbf{3}$)

$$\mathbf{3} \otimes \mathbf{3} = (\mathbf{3}' + \mathbf{2} + \mathbf{1})_s + \mathbf{3}_a$$

- three TB conserving flavons φ_ν η_ν ξ_ν
- ζ_ν flavon is neutral except for S_4 ($\zeta_\nu \sim \mathbf{1}'$)

$$\mathbf{1}' \otimes (\mathbf{3} \otimes \mathbf{3}) = (\mathbf{3} + \mathbf{2} + \mathbf{1}')_\mathbf{s} + \mathbf{3}'_\mathbf{a}$$

- only one extra term involving ζ_ν
- this breaks TB structure (at higher order) ...

Breaking of the TB Klein symmetry $\mathcal{K}$

Dirac term LN^cH_u respects $\mathcal{K} \subset S_4$

Majorana terms $\left(\varphi_\nu + \eta_\nu + \xi_\nu + \frac{1}{M}\zeta_\nu\eta_\nu\right) N^cN^c$ respect k_1 but break k_2

S_4 irrep	k_1	k_2	alignment
$\mathbf{3'}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle\varphi_\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle\eta_\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle\xi_\nu\rangle \propto 1$
$\mathbf{1'}$	1	-1	$\langle\zeta_\nu\rangle \propto 1$

Resulting mixing

$$M_R = \frac{M_1+M_3}{6}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{2M_2+M_3-M_1}{6}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \frac{M_1+M_2-M_3}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+\ \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \leftarrow \quad \text{small TB breaking term}$$

$$\Longrightarrow \quad \boxed{U_{\text{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}}$$

$$\text{Re}\ \alpha \approx -\sqrt{3}\cdot\left[\text{Re}\left(\frac{\Delta}{M_1 - M_3}\right) + \text{Im}\left(\frac{\Delta}{M_1 - M_3}\right)\frac{\text{Im}\left(\frac{M_1+M_3}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}\right]$$

$$\text{Im}\ \alpha \approx \sqrt{3}\cdot\frac{\text{Im}\left(\frac{\Delta}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}$$

Trimaximal neutrino mixing

- second column of $U_{\text{PMNS}} \;\propto\; (1,1,1)^T$
- one could have guessed this special structure

  (i) $\;(1,1,1)^T$ is an eigenvector of M_R

  (ii) $\;k_1$ generator of TB Klein symmetry $\mathcal{K}$ unbroken
- such a TB breaking affects θ_{13} and θ_{23} – but not θ_{12}
- get correlations between deviation parameters r, a, s

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\, r \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a) \qquad \sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)$$

$$r\cos\delta \;\approx\; -\frac{2}{\sqrt{3}}\,\mathrm{Re}\;\alpha \qquad a \;\approx\; \frac{1}{\sqrt{3}}\,\mathrm{Re}\;\alpha \qquad \delta \;\approx\; \pi + \arg\;\alpha$$

$\rightarrow$ testable sum rules $\boxed{a \approx -\frac{1}{2}\, r\cos\delta \qquad s \;\approx\; 0}$

Alignment mechanism

- SUSY unbroken at scale of family symmetry breaking
- F-terms of driving fields need to vanish

$$\begin{aligned}
W_{\text{flavon}} = \; & \varphi_\nu^0 (g_1 \varphi_\nu \varphi_\nu + g_2 \varphi_\nu \xi_\nu + g_3 \varphi_\nu \eta_\nu) \\
& + \tilde{\varphi}_\nu^0 (g_4 \varphi_\nu \eta_\nu) \\
& + \xi^0 (g_5 \varphi_\nu \varphi_\nu + g_6 \xi_\nu \xi_\nu + g_7 \eta_\nu \eta_\nu) \\
& + D_\nu^0 (g_8 \zeta_\nu \zeta_\nu + g_9 M_0^2) \\
& + \varphi_\ell^0 (h_1 \varphi_\ell \varphi_\ell + h_2 \varphi_\ell \eta_\mu) \\
& + \xi_\ell^0 (h_3 \varphi_\ell \varphi_\ell + h_4 \eta_\mu \eta_\mu) \\
& + \eta_\ell^0 (\tilde{M}_0 \eta_e + h_5 \zeta_\nu \eta_e + h_6 \eta_\mu \eta_\mu + h_7 \varphi_\ell \varphi_\ell) \\
& + D_\ell^0 (h_8 \eta_\mu \eta_e + h_9 M_0^2)
\end{aligned}$$

- yields previously assumed flavon alignments
- get hierarchy $m_e \ll m_\mu$ from $\tilde{M}_0 \gg M_0$

(II) Non-standard flavon alignments

(indirect models)

TB from constrained sequential dominance

- diagonal charged leptons
- diagonal right-handed neutrinos $M_R = \text{diag}\,(M_A, M_B, M_C)$
- Dirac neutrino Yukawa matrix $Y_\nu = (A, B, C)$
- type I seesaw

$$M_\nu = \left(\frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C} \right) v_u^2$$

- sequential dominance assumes hierarchy $\frac{AA^T}{M_A} \gg \frac{BB^T}{M_B} \gg \frac{CC^T}{M_C}$
- implies normal neutrino mass hierarchy $m_3^\nu \gg m_2^\nu \gg m_1^\nu \approx 0$
- consider only dominant (A) and subdominant (B) contributions

- constrained sequential dominance assumes specific A and B
- family symmetry models $\rightarrow$ identify A and B with flavon VEVs

Standard and non-standard alignments

TB structure: dominant flavon — subdominant flavon

$$\langle\phi_A\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \langle\phi_B\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$M_\nu = a\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

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non-TB structure: dominant flavon — subdominant flavon

$$\langle\phi_A\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \langle\phi_B\rangle \propto \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$M_\nu = a\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

what mixing does this induce?

Resulting mixing

- hierarchy $a \gg b \quad \rightarrow \quad$ define expansion parameter $\frac{b}{a} = \epsilon\, e^{i\delta}$

$$\boxed{\frac{m_2^\nu}{m_3^\nu} \approx \frac{3}{2}\,\epsilon} \qquad \theta_{13} = \frac{1}{\sqrt{2}}\,\epsilon \qquad \theta_{23} = \frac{\pi}{4} + \epsilon \cos\delta \qquad \theta_{12} = \arcsin\frac{1}{\sqrt{3}}$$

- correlation between neutrino mass ratio and mixing angles
- normal mass hierarchy with $m_1^\nu = 0 \;\rightarrow\;$ know $\epsilon \approx \frac{2}{15}$

$$\boxed{\text{get a prediction for reactor angle} \quad \theta_{13} \sim 5^\circ - 6^\circ}$$

- sum rules in terms of deviation parameters r, a, s

$$\boxed{a \approx r\cos\delta \qquad s \approx 0} \qquad \leftarrow \quad \text{different from previous ones}$$

- no change in solar angle θ_{12} (from TB value)
- first column of $U_{TB} \;\propto\; (-2, 1, 1)^T$ is an eigenvector of M_ν

Ingredients of an indirect A_4 model

matter	L	τ^c	μ^c	e^c	N_3^c	N_2^c	H_u	H_d
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Antusch, King, Luhn, Spinrath (2011)

$$W_{\mathrm{Yuk}} \sim \frac{1}{M}\left[(L\phi_3^\ell)\tau^c + (L\phi_2^\ell)\mu^c + (L\phi_1^\ell)e^c\right]H_d + \frac{1}{M}\left[(L\phi_3^\nu)N_3^c + (L\phi_{120}^\nu)N_2^c\right]H_u$$

$$\langle\phi_3^\ell\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \langle\phi_2^\ell\rangle \propto \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \langle\phi_1^\ell\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\langle\phi_3^\nu\rangle \propto \begin{pmatrix}0\\1\\-1\end{pmatrix} \qquad \langle\phi_2^\nu\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_1^\nu\rangle \propto \begin{pmatrix}-2\\1\\1\end{pmatrix} \qquad \langle\phi_{120}^\nu\rangle \propto \begin{pmatrix}1\\2\\0\end{pmatrix}$$

Z_N symmetries to control pairing of flavons with τ^c, μ^c, e^c, N_3^c, N_2^c

Obtaining the non-standard alignment ϕ^ν_{120}

- construct standard alignments ϕ^ℓ_i and ϕ^ν_i $(i = 1, 2, 3)$
- add A_4 singlet driving fields D_k which produce orthogonality relations

$$D_1(\phi^\nu_{120} \cdot \phi^\ell_3) \quad \rightarrow \quad \langle \phi^\nu_{120} \rangle \; \propto \; \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$D_2(\phi^\nu_{120} \cdot \phi^\nu_1) \quad \rightarrow \quad \langle \phi^\nu_{120} \rangle \; \propto \; \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

- similarly one could get alignment $(1, 0, 2)^T$
- driving fields charged under Z_N shaping symmetries

(III) Finite groups with new Klein symmetries

(direct models)

Family symmetry $\Delta(96) \supset S_4$

charged leptons $h = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{4\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \end{pmatrix}$

neutrinos $k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ $k_2 = -\frac{1}{3}\begin{pmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \end{pmatrix}$

$\rightarrow$ generators h and k_1 are common with S_4

$\rightarrow$ trimaximal mixing due to k_1 symmetry

PMNS-angles	Adelhart Toorop et al. (2011)	1σ exp.	1σ exp.
$\sin^2\theta_{12}$:	0.349	$0.297 - 0.329$	$0.296 - 0.329$
$\sin^2\theta_{23}$:	0.349 0.651	$0.45 - 0.58$	$0.39 - 0.50$
$\sin^2\theta_{13}$:	0.045	$0.008 - 0.020$	$0.018 - 0.032$

Family symmetry $\Delta(384) \supset \Delta(96) \supset S_4$

PMNS-angles	Adelhart Toorop et al. (2011)	1σ exp.	1σ exp.
$\sin^2\theta_{12}$:	0.337	$0.297 - 0.329$	$0.296 - 0.329$
$\sin^2\theta_{23}$:	0.424 0.576	$0.45 - 0.58$	$0.39 - 0.50$
$\sin^2\theta_{13}$:	0.011	$0.008 - 0.020$	$0.018 - 0.032$

$\rightarrow$ again trimaximal mixing

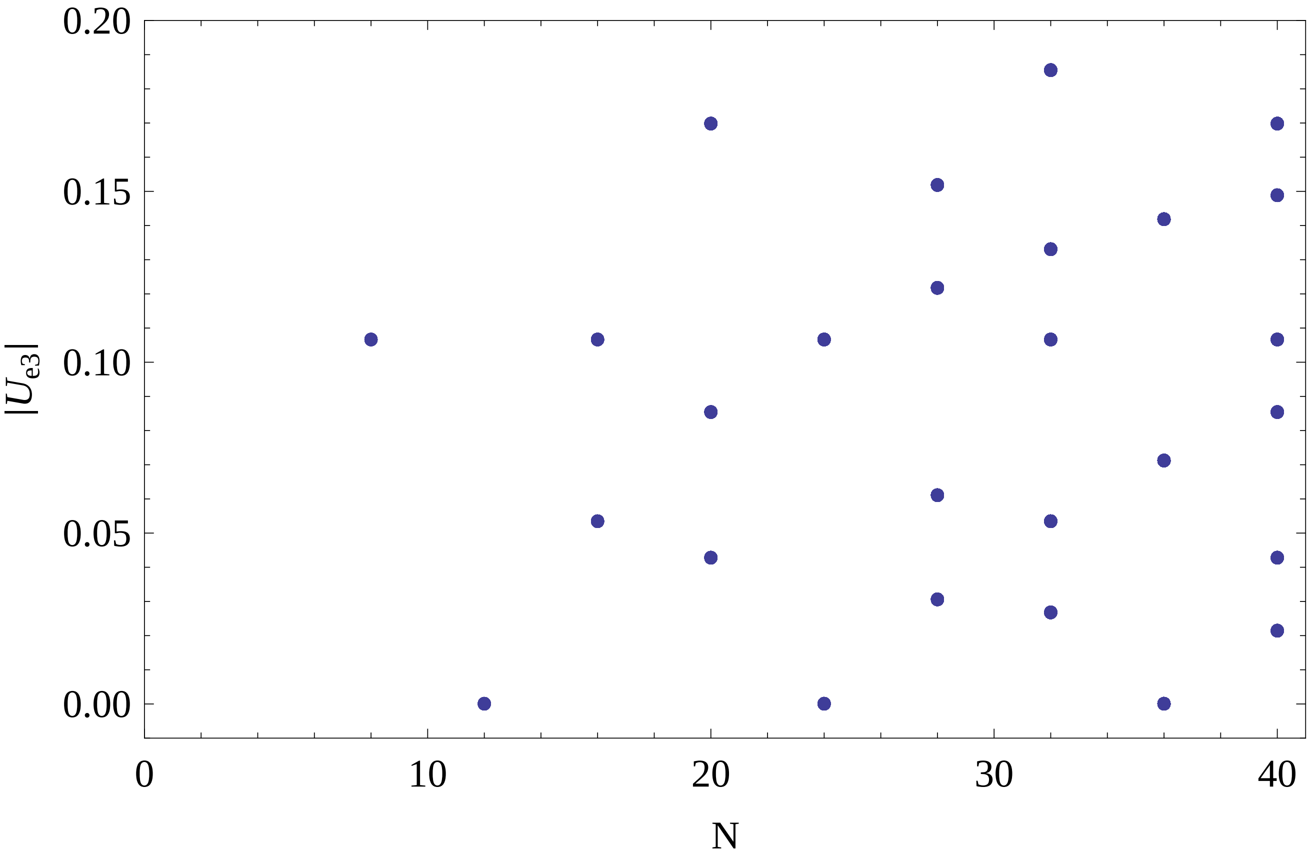
$\rightarrow$ fits experimental values relatively well

Family symmetry $(Z_N \times Z_N \times Z_N) \rtimes Z_3$

N not multiple of 3 $\rightarrow$ $\Delta(3N^2) \times Z_N$

N multiple of 3 $\rightarrow$ $\Sigma(3N^3)$

Ishimori, Kobayashi (2012)



Conclusion

- experimental evidence for sizable θ_{13}
- review role of family symmetries
- possible ways to implement non-zero θ_{13}
  - corrections to TB mixing – not without new ingredients ...
  - non-standard flavon alignments – $(1, 2, 0)^T$ ...
  - new family symmetries – $\Delta(384)$ ...
- TB may go, yet family symmetries remain

Thank you