Electromagnetic dipole transitions of heavy quarkonium

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Outline

Why should one study EM transitions?

- EM transitions provide information about the quarkonium spectrum and the wave-functions
- significant contributions to the decay rate (at least for E1)
- reliable experimental data provided in the last and next few years (CLEO, BES, B factories)

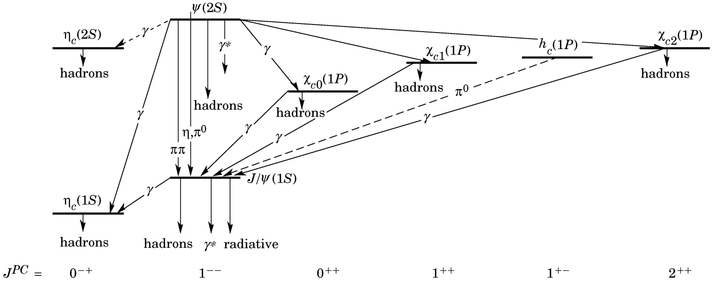

Figure: PDG, J.Phys.G 37 (2010)

What has been done?

Theoretical research on radiative decays

- phenomenological approach: QCD motivated potential models
  `Grotch, Owen, Sebastian, Phys.Rev.D 30 (1984)`
  `Eichten et al., Rev.Mod.Phys.80 (2008)`
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- EFT treatment of radiative decays: pNRQCD
  $\longrightarrow$ M1 transitions
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  $\longrightarrow$ still missing: treatment of E1 transitions

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- lattice QCD (quenched):
  Dudek, Edwards, Richards, Phys.Rev.D 73 (2006)

Basic formalism

EFT for heavy quarkonium
Description of decay processes

Scales in quarkonium

- separation of scales in heavy quarkonium

$$m \gg p \sim mv \gg E \sim mv^2$$

where $v^2 \ll 1$ ($v^2 \approx 0.1$ for $b\bar{b}$, $v^2 \approx 0.3$ for $c\bar{c}$)
$\longrightarrow$ systematic treatment of relativistic corrections in powers of v
$\longrightarrow$ language of effective field theories appropriate

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  $\longrightarrow$ language of effective field theories appropriate
- weakly coupled quarkonia ($p \gg E \gtrsim \Lambda_{QCD}$)
  $\longrightarrow$ perturbative treatment with Coulomb potential at leading order
  (valid for the ground states J/ψ, $\Upsilon(1S)$, η_c, η_b)

$$\begin{aligned} \alpha_s(m) &\sim v^2 \\ \alpha_s(mv) &\sim v \\ \alpha_s(mv^2) &\sim 1 \end{aligned}$$

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- additional scale: photon energy $k_\gamma \sim mv^2$ (different principal quantum number) or $k_\gamma \sim mv^3$, mv^4 (same principal quantum number)

Effective field theories for quarkonium

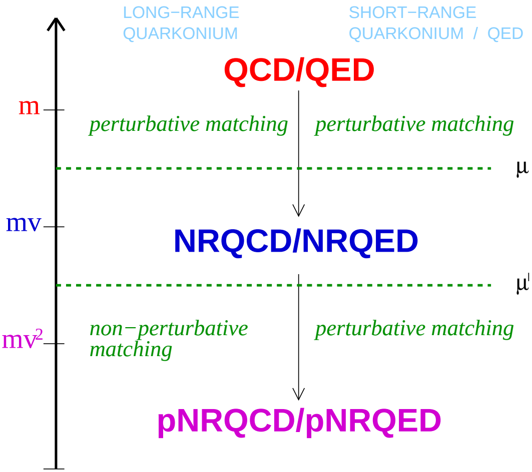

Figure: A.Vairo, arXiv 0902.3346 (2009)

NRQCD

- integrate out energy & momentum modes of order m from QCD
- Lagrangian

$$\begin{aligned}\mathcal{L} &= \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + \dots \right) \psi \\ &+ g\psi^\dagger \left(\frac{c_F}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} + i \frac{c_s}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}] + \dots \right) \psi \\ &+ ee_Q \psi^\dagger \left(\frac{c_F^{em}}{2m} \boldsymbol{\sigma} \cdot \mathbf{B}^{em} + i \frac{c_s^{em}}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}^{em}] + \dots \right) \psi \\ &+ c.c. + \mathcal{L}_{\rm light} + \mathcal{L}_{\rm YM}\end{aligned}$$

coefficients by matching with QCD

pNRQCD (for weak coupling)

- integrate out
  $\longrightarrow$ quarks with energy & momentum $\sim mv$
  $\longrightarrow$ gluons & photons of energy or momentum $\sim mv$
- new degrees of freedom: $Q\bar{Q}$ color singlet and octet fields

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- Lagrangian

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, \mathrm{Tr}\Bigg\{ S^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_S \right) S + O^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_O \right) O + gV_A (O^\dagger \mathbf{r}\cdot\mathbf{E} S + S^\dagger \mathbf{r}\cdot\mathbf{E} O) + gV_B \frac{\{O^\dagger, \mathbf{r}\cdot\mathbf{E}\}}{2} O + \dots \Bigg\} + \mathcal{L}_{\gamma\rm pNRQCD} + \mathcal{L}_{\rm light} + \mathcal{L}_{\rm YM}$$

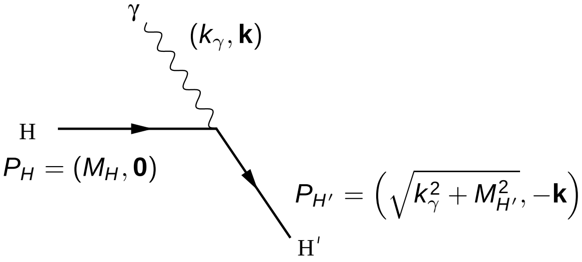
pNRQCD (for weak coupling)

- now: Only relevant degrees of freedom present
- high energy dynamics encoded in Wilson coefficients (obtained by matching with NRQCD at energy mv)
- definite power counting of operators

$$\begin{aligned} r &\sim 1/mv \\ \mathbf{E}, \mathbf{B} &\sim (mv^2)^2 \\ \mathbf{E}^{em}, \mathbf{B}^{em} &\sim k_\gamma^2 \\ \boldsymbol{\nabla} = \partial/\partial\mathbf{R} &\sim mv^2\,, k_\gamma \end{aligned}$$

Radiative decays

- kinematics of single photon transitions $H \to H'\gamma$:



- transition amplitude:

$$\mathcal{A}_{H\to H'\gamma} \sim -\langle H'(\mathbf{P}',\lambda')\gamma(\mathbf{k},\sigma)| \int d^3R\, \mathcal{L}_{\gamma\mathrm{pNRQCD}}\, |H(\mathbf{0},\lambda)\rangle$$

- decay rate:

$$\Gamma_{H\to H'\gamma} = \frac{1}{8\pi^2}\left(1-\frac{k_\gamma}{M_H}\right)\int dk\, k \int d\Omega(\hat{\mathbf{k}})\delta(k-k_\gamma)\frac{1}{N_\lambda}\sum_{\lambda\lambda'\sigma}|\mathcal{A}_{H\to H'\gamma}|^2$$

Quarkonium states

- quarkonium state (leading Fock space component):

$$|H(\mathbf{P},\lambda)\rangle = \int d^3R \int d^3r \, e^{i\mathbf{P}\cdot\mathbf{R}} \mathrm{Tr}\left\{\phi_{H(\lambda)}(\mathbf{r}) \mathrm{S}^\dagger(\mathbf{r},\mathbf{R})|0\rangle\right\}$$

- at leading order:

$$H_S^{(0)}\phi_{H(\lambda)}^{(0)} = \left(-\frac{\boldsymbol{\nabla}_r{}^2}{m} + V_S^{(0)}\right)\phi_{H(\lambda)}^{(0)} = E_{H(\lambda)}^{(0)}\phi_{H(\lambda)}^{(0)}$$

- at higher orders: wave-function corrections due to higher order potentials and singlet-octet transitions

M1 Transitions

Brambilla, Jia, Vairo, Phys.Rev.D 73 (2006)
P.P., master thesis (2011)

General properties

- definition: $|\Delta S| = 1, \Delta L = 0$
- no change in parity, change in C-parity
- allowed transitions: $n = n'$, $k_\gamma \sim mv^4$
  hindered transitions: $n \neq n'$, $k_\gamma \sim mv^2$

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Example

$$1^3S_1 \to 1^1S_0\gamma \quad (J/\psi \to \eta_c\gamma\,,\ \Upsilon(1S) \to \eta_b\gamma)$$
$$1^3P_J \to 1^1P_1\gamma \quad (\chi_c \to h_c\gamma\,,\ \chi_b \to h_b\gamma)$$

Nonrelativistic limit

- leading order operator for M1 transitions

$$\mathcal{L}_{M1} = e e_Q \int d^3r \, \mathrm{Tr} \left\{ \frac{1}{2m} \{ S^\dagger, \boldsymbol{\sigma} \cdot \mathbf{B}^{em} \} S \right\}$$

Nonrelativistic decay rate

$$\Gamma^{(0)}_{n^3S_1 \to n^1S_0 \gamma} = \frac{4}{3} \alpha_{em} e_Q^2 \frac{k_\gamma^3}{m^2} \sim \frac{k_\gamma^3}{m^2}$$

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- comparison with experiment:

Example (J/ψ-decay)

$$\begin{aligned} \Gamma^{(0)}_{J/\psi \to \eta_c \gamma} &= 2.83 \, \mathrm{keV} \\ \Gamma^{\mathrm{exp}}_{J/\psi \to \eta_c \gamma} &= (1.58 \pm 0.37) \, \mathrm{keV} \end{aligned}$$

$\to$ relativistic effects important, now: $\mathcal{O}(v^2)$-corrections

Relevant pNRQCD Lagrangian for decays of order k^3v^2/m^2

$$
\begin{aligned}
\mathcal{L}_{\gamma\mathrm{pNRQCD}} = ee_Q \int d^3r\,\mathrm{Tr}\Big\{ & V^{r\cdot E} S^\dagger \mathbf{r}\cdot\mathbf{E}^{em} S \\
& + \frac{1}{2m}\left(V^{\sigma\cdot B}\{S^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{em}\}S + V_O^{\sigma\cdot B}\{O^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{em}\}O\right) \\
& + \frac{1}{4m}V^{(r\nabla)^2\sigma\cdot B}\{S^\dagger, \boldsymbol{\sigma}\cdot(\mathbf{r}\boldsymbol{\nabla})^2\mathbf{B}^{em}\}S \\
& + \frac{1}{4m^2 r}V^{\sigma\cdot(r\times r\times B)}\{S^\dagger, \boldsymbol{\sigma}\cdot[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{em})]\}S \\
& + \frac{1}{4m^2 r}V^{\sigma\cdot B/r}\{S^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{em}\}S \\
& + i\frac{1}{16m^2}V^{\sigma\cdot(\nabla\times E)}\{S^\dagger, \boldsymbol{\sigma}\}\cdot[\boldsymbol{\nabla}\times, \mathbf{E}^{em}]S \\
& + i\frac{1}{16m^2}V^{\sigma\cdot(\nabla_r\times r\nabla E)}\{S^\dagger, \boldsymbol{\sigma}\}\cdot[\boldsymbol{\nabla}_r\times, (\mathbf{r}\boldsymbol{\nabla})\mathbf{E}^{em}]S \\
& + \frac{1}{4m^3}V^{\nabla_r^2\sigma\cdot B}\{S^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}\}\boldsymbol{\nabla}_r^2 S \Big\}
\end{aligned}
$$

Tree level matching

- project NRQCD Hamiltonian onto the subspace spanned by $\Psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t) \sim \psi_\alpha(\mathbf{x}_1, t)\chi^\dagger_\beta(\mathbf{x}_2, t)$
- decompose $\Psi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2, t)$ into singlet and octet field components
- multipole expand in $r \ll 1/E$

Tree level results

$$\begin{aligned}
V^{r\cdot E} = V^{\nabla_r^2 \sigma\cdot B} &= 1 \\
V^{\sigma\cdot B} = V_O^{\sigma\cdot B} = V^{(r\nabla)^2\sigma\cdot B} &= c_F^{em} \\
V^{\sigma\cdot(\nabla\times E)} = V^{\sigma\cdot(\nabla_r\times r\nabla E)} &= c_S^{em} \\
V^{\sigma\cdot(r\times r\times B)} = V^{\sigma\cdot B/r} &= 0
\end{aligned}$$

Beyond tree level

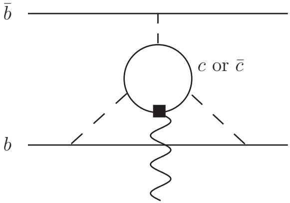
- matching of amplitudes order by order in $1/m$
- required for the perturbative matching at $\mathcal{O}(v^2)$:
  $\rightarrow$ $\mathcal{O}(\alpha_s^2)$ corrections to $V^{\sigma\cdot B}$
  $\rightarrow$ $\mathcal{O}(\alpha_s)$ corrections to $V^{\sigma\cdot B/r}$ and $V^{\sigma\cdot(r\times r\times B)}$
- But: exact relations for all coefficients at $\mathcal{O}(1/m^2)$ can be obtained

Light quark effects

- Loop effects with electromagnetic coupling to u,d and s cancel

$$q_u + q_d + q_s = 0$$

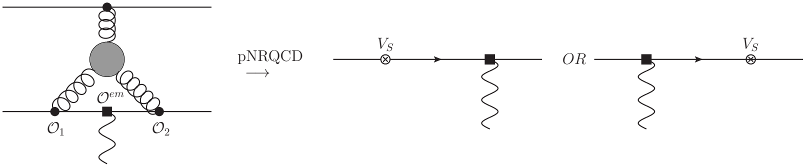
- charm quark effects for bottomonium
  $\longrightarrow$ leading order diagram highly suppressed



$\longrightarrow$ furthermore: decoupling at typical momentum scale
`Brambilla, Sumino, Vairo, Phys.Rev.D 65 (2002)`

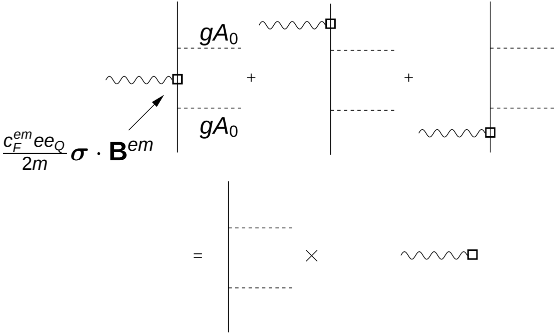
General factorization argument

$[\mathcal{O}^{em}, \mathcal{O}_1] = 0$ OR $[\mathcal{O}^{em}, \mathcal{O}_2] = 0$
$\Rightarrow$ the amplitude factorizes and gives no contribution to the matching of single operators (valid to all orders in α_s and to first order in α_{em})



Magnetic moment of the singlet quarkonium

Example: Exact matching of $V^{\sigma \cdot B}$ possible
Trivial factorization: $[A_0, \mathbf{A}^{em}] = 0$



$\rightarrow$ Tree level result = exact result, no contribution from soft modes

$$V^{\sigma \cdot B} = V^{(r\nabla)^2 \sigma \cdot B} = c_F^{em} \text{ to all orders in } \alpha_s$$

Similarly for other coefficients (except for $V^{\nabla_r^2 \sigma \cdot B}$ at $\mathcal{O}(1/m^3)$)

Effects due to higher-order potentials

Corrections at $\mathcal{O}(v^2)$ due to higher order potentials in $1/m$

- zero-recoil effects (contribution only to hindered transitions)

$$\delta V_S(r) = V_{LS}^{(2)}(r)\mathbf{L}\cdot\mathbf{S} + V_{S^2}^{(2)}(r)\mathbf{S}^2 + V_{S_{12}}^{(2)}(r)\left[3(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1)(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]$$

Effects due to higher-order potentials

Corrections at $\mathcal{O}(v^2)$ due to higher order potentials in $1/m$

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- recoil effect: E1 operator induces effective M1 transition via

$$V_{\mathbf{L}_{CM}\mathbf{S}}(r)(-i\nabla_R \times r)\cdot\mathbf{S}$$

- calculation in QM perturbation theory

Color-octet effects

- higher Fock space components via singlet-octet transitions

$$\mathcal{L} = \int d^3r \, \mathrm{Tr}\left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} S + S^\dagger \mathbf{r} \cdot g\mathbf{E} O \right\}$$

- non-perturbative (chromoelectric field correlators)

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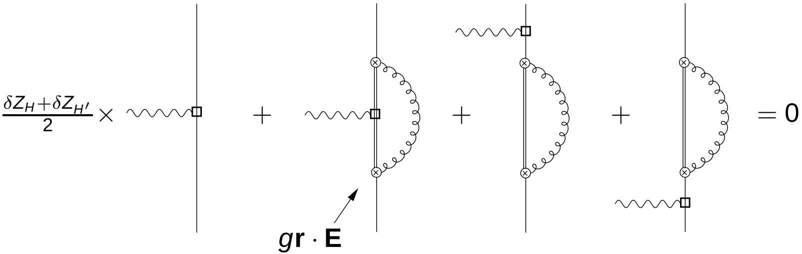
- non-perturbative (chromoelectric field correlators)
- but: factorization



$\rightarrow$ color-octet effects vanish at order v^2

Results

Decay rates for S-wave transitions

- decay rate for allowed S-wave transitions

$$\Gamma_{n^3S_1 \to n^1S_0\gamma} = \frac{4}{3}\,\alpha_{em} e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + \frac{C_F\alpha_s(m)}{\pi} + \frac{5}{3m^2}\left(I_2'' + 2I_1'\right)\right]$$

$$I_N^{(n)}(n0 \to n'0) = \int_0^\infty dr\, r^N R_{n'0}(r)\frac{d^n}{dr^n}R_{n0}(r)$$

- decay rate for hindered S-wave transitions ($n \neq n'$)

$$\Gamma_{n^3S_1 \to n'^1S_0\gamma} = \frac{4}{3}\,\alpha_{em} e_Q^2 \frac{k_\gamma^3}{m^2}\left[-\frac{k_\gamma^2}{24}I_4 + \frac{5}{6m^2}\left(I_2'' + 2I_1'\right) + \frac{2C_F\alpha_s(mv)}{3m^2}\,\frac{R_{n'0}(0)R_{n0}(0)}{E_{n,0}^{(0)} - E_{n',0}^{(0)}}\right]^2$$

Comparison with potential models

Agreement with potential models (Grotch)

Summary: New results

- range of validity: weak-coupling regime
- no scalar interaction term ($V^{\sigma\cdot B/r} = 0$)
- magnetic moment not depending on soft scales
- color-octet effects considered

$J/\psi \to \eta_c\gamma$

- $\alpha_s(m_c) \approx 0.35 \to \Gamma^{\mathrm{pNRQCD}}_{J/\psi\to\eta_c\gamma} = (1.5 \pm 0.7)$ keV
- comparison with lattice calculations $\to \Gamma^{\mathrm{latt.\ mass}}_{J/\psi\to\eta_c\gamma} = (1.61 \pm 0.07)$ keV (Dudek 2006)
- comparison with experiment: $\Gamma^{\mathrm{exp}}_{J/\psi\to\eta_c\gamma} = (1.58 \pm 0.37)$ keV (PDG 2010)

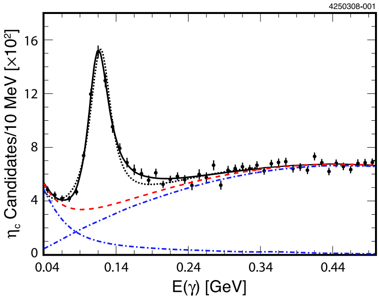


Figure: CLEO Coll., Phys.Rev.Lett.102 (2009)

Predictions: $\Upsilon(1S) \to \eta_b \gamma$

Example: $\Upsilon(1S) \to \eta_b \gamma$

- weakly coupled
- η_b observed in 2008
  $\longrightarrow$ `BABAR Coll., Phys.Rev.Lett.103 (2009)`
  $k_\gamma = (65.9 \pm 5.2)$ MeV
- $\alpha_s(m_b) \approx 0.22 \to \Gamma_{\Upsilon(1S)\to\eta_b\gamma} = (12.5 \pm 3.0^{k_\gamma} \pm 0.4)\ \mathrm{eV}$

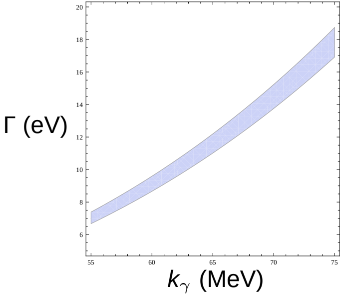


Figure: $\Gamma_{\Upsilon(1S)\to\eta_b\gamma}$ as a function of the photon energy

E1 Transitions

Brambilla, Pietrulewicz, Vairo, preprint: TUM-EFT 25/11

General properties

- definition: $\Delta S = 0, |\Delta L| = 1$
- change in parity, no change in C parity
- for the considered transitions: $k_\gamma \sim mv^2$

Examples

$$1^3P_J \to 1^3S_1\gamma \quad (\chi_c \to J/\psi\gamma\,,\ \chi_b \to \Upsilon(1S)\gamma)$$
$$1^1P_1 \to 1^1S_0\gamma \quad (h_c \to \eta_c\gamma\,,\ h_b \to \eta_b\gamma)$$

Nonrelativistic limit

- leading order operator for E1 transitions

$$\mathcal{L}_{E1} = ee_Q \int d^3r \, \mathrm{Tr} \left\{ S^\dagger \mathbf{r} \cdot \mathbf{E}^{em} S \right\}$$

Nonrelativistic decay rate

$$\Gamma_{n^3P_{J=0,1,2} \to n'^3S_1 \gamma} = \frac{4}{9} \alpha_{em} e_Q^2 k_\gamma^3 I_3^2(n1 \to n'0) \sim \frac{k_\gamma^3}{m^2 v^2}$$

$$I_3(n1 \to n'0) = \int_0^\infty dr \, r^3 R_{n'0}(r) R_{n1}(r)$$

- differences to M1 transitions:
  $\longrightarrow$ leading order amplitude depends on the wave-function
  $\longrightarrow$ enhancement of E1 transitions by factor $1/v^2$
- now: relativistic corrections of $\mathcal{O}(v^2)$

Relevant pNRQCD Lagrangian for decays of order k_γ^3/m^2

$$
\begin{aligned}
\mathcal{L}^{E1}_{\gamma\mathrm{pNRQCD}} = ee_Q \int d^3r\, \mathrm{Tr}\Big\{ & V^{r\cdot E} S^\dagger \mathbf{r}\cdot\mathbf{E}^{em} S + V_O^{r\cdot E} O^\dagger \mathbf{r}\cdot\mathbf{E}^{em} O \\
& + \frac{1}{24} V^{(r\nabla)^2 r\cdot E} S^\dagger \mathbf{r}\cdot(\mathbf{r}\boldsymbol{\nabla})^2 \mathbf{E}^{em} S \\
& + i\frac{1}{4m} V^{\nabla\cdot(r\times B)} S^\dagger \{\boldsymbol{\nabla}\cdot, \mathbf{r}\times\mathbf{B}^{em}\} S \\
& + i\frac{1}{12m} V^{\nabla_r\cdot(r\times(r\nabla)B)} S^\dagger \{\boldsymbol{\nabla}_r\cdot, \mathbf{r}\times(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em}\} S \\
& + \frac{1}{4m} V^{(r\nabla)\sigma\cdot B} [S^\dagger, \boldsymbol{\sigma}]\cdot(\mathbf{r}\boldsymbol{\nabla})\mathbf{B}^{em} S \\
& + \frac{1}{mr} V^{r\cdot E/r} S^\dagger \mathbf{r}\cdot\mathbf{E}^{em} S \\
& - i\frac{1}{4m^2} V^{\sigma\cdot(E\times\nabla_r)} [S^\dagger, \boldsymbol{\sigma}]\cdot(\mathbf{E}^{em}\times\boldsymbol{\nabla}_r) S \Big\}
\end{aligned}
$$

Matching

Tree level results

$$
\begin{aligned}
V_A = V^{r\cdot E} = V_O^{r\cdot E} = V^{(r\nabla)^2 r\cdot E} &= 1 \\
V^{\nabla\cdot(r\times B)} = V^{(r\nabla)\nabla_r\cdot(r\times B)} &= 1 \\
V^{(r\nabla)\sigma\cdot B} &= c_F^{em} \\
V^{\sigma\cdot(E\times\nabla_r)} &= c_S^{em} \\
V^{r\cdot E/r} &= 0
\end{aligned}
$$

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V^{\sigma\cdot(E\times\nabla_r)} &= c_S^{em} \\
V^{r\cdot E/r} &= 0
\end{aligned}
$$

- Required for perturbative matching:
  $\rightarrow$ $\mathcal{O}(\alpha_s^2)$ corrections to $V^{r\cdot E}$
  $\rightarrow$ $\mathcal{O}(\alpha_s)$ corrections to $V^{r\cdot E/r}$
- But: factorization for all relevant operators!
  $\Rightarrow$ tree level results = exact results

Wave-function corrections

- corrections at $\mathcal{O}(v^2)$ due to higher order potentials in $1/m$

$$\delta V_S(r) = \frac{V_r^{(1)}(r)}{m} + \frac{V_{SI}^{(2)}(r)}{m^2} + \frac{V_{SD}^{(2)}(r)}{m^2}$$

$$V_{SI}^{(2)}(r) = V_r^{(2)}(r) + \frac{1}{2}\{V_{p^2}^{(2)}(r), \mathbf{p}^2\} + \frac{V_{L^2}^{(2)}(r)}{r^2}\mathbf{L}^2$$

$$V_{SD}^{(2)}(r) = V_{LS}^{(2)}(r)\mathbf{L}\cdot\mathbf{S} + V_{S^2}^{(2)}(r)\mathbf{S}^2 + V_{S_{12}}^{(2)}(r)\left[3(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1)(\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]$$

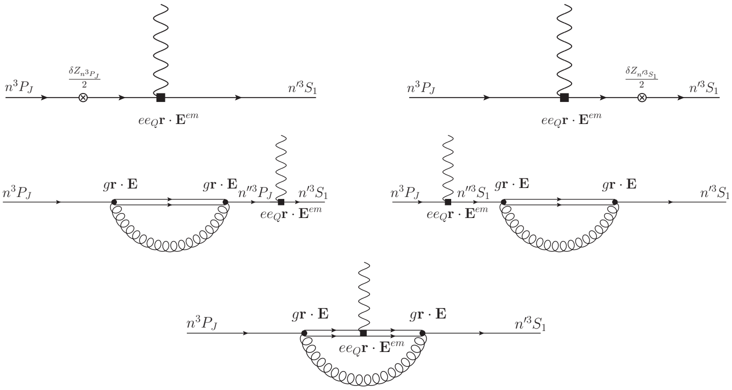
- relativistic kinetic energy correction

$$\delta H_s(r) = -\frac{\mathbf{p}^4}{4m^3}$$

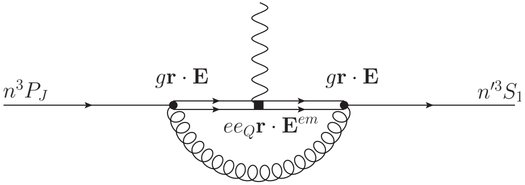
- calculation with QM perturbation theory

Color-octet effects

Example: $n^3P_J \to n'^3S_1\gamma$



Color-octet effects



$$\mathcal{A} = \frac{-2iee_Q k\pi\alpha_s}{3}\int_0^\infty dt\int_0^t dt'\langle 0|\mathbf{E}^a(\mathbf{R},t)\phi(t,0)^{\rm adj}_{ab}\mathbf{E}^b(\mathbf{R},t)|0\rangle$$
$$\times\langle 1^3S_1|\mathbf{r}e^{-i(h_O^{(0)}-E_1^{(0)})(t-t')}\hat{\boldsymbol{\epsilon}}(\sigma)\cdot\mathbf{r}\,e^{-i(h_O^{(0)}-E_2^{(0)})t'}\mathbf{r}|1^3P_J\rangle$$

→ no factorization of amplitudes
→ no cancellation as for M1 transitions
→ evaluation of Wilson loop correlators necessary

however: amplitude vanishes for large N_c for S waves,
strong-coupling description for P states more appropriate

Strong coupling case

- strongly coupled quarkonia ($p \gtrsim \Lambda_{QCD} \gg E$ or $p \gg \Lambda_{QCD} \gg E$) $\longrightarrow$ non-perturbative treatment with confining potential at leading order (valid for excited states χ_c, χ_b,...)
- matching for the relevant operators as before
- non-perturbative potentials taken from lattice simulations/models
- no octet fields
- for $\Lambda_{QCD} \sim mv$ new operators become relevant (for a conservative non-perturbative power counting)
  Ex.: $-\frac{ee_Q g^2}{2m} S^\dagger (\mathbf{r}\cdot\mathbf{B})(\mathbf{r}\times\mathbf{B})\cdot(\mathbf{r}\times\mathbf{B}^{em})S$

Results

Final formula for $n^3P_J \to n'^3S_1\gamma$

$$\Gamma_{E1} = \Gamma_{E1}^{(0)}\left(1 + R - \frac{k_\gamma^2}{60}\frac{I_5}{I_3} - \frac{k_\gamma}{6m} + \frac{k_\gamma(c_F^{em}-1)}{2m}\left[\frac{J(J+1)}{2} - 2\right]\right)$$

$$I_N(n1 \to n'0) = \int_0^\infty dr\, r^N R_{n'0}(r) R_{n1}(r)$$

$R \to$ wave function corrections

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$R \to$ wave function corrections

- comparison with potential models (Grotch):
  equivalence to the given order, but:
  $\to$ range of validity ($E \gtrsim \Lambda_{QCD}$)
  $\to$ systematic inclusion of relativistic corrections (including $V_r^{(1)}$)
  $\to$ color-octet effects included for weak coupling
- analogous formulas for $n^1P_1 \to n'^1S_0\gamma$, $n^3S_1 \to n'^3P_J\gamma$ and $n^1S_0 \to n'^1P_1\gamma$

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- perturbative potentials for short and non-perturbative ones for long distances

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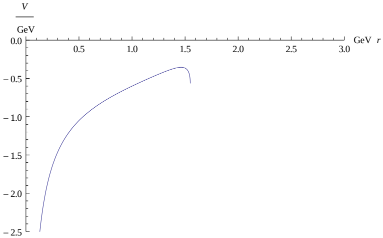
- static potential



$\longrightarrow$ renormalon subtracted potential at NNLL for short distances
`Pineda, J.Phys.G 29 (2003)`

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- perturbative potentials for short and non-perturbative ones for long distances
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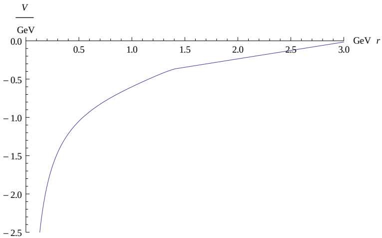

$\longrightarrow$ renormalon subtracted potential at NNLL for short distances
`Pineda, J.Phys.G 29 (2003)`
$\longrightarrow$ linear, confining potential at large distances

- subleading potentials: LL for short distances, expressions from string model for large distances
  `Perez-Nadal, Soto, Phys.Rev.D 79 (2009)`

Numerical results

- Fit to spectrum gives string tension σ and the masses m_c, m_b in our scheme
  $\sigma = (0.21 \pm 0.01)\,\mathrm{GeV}^2$, $m_c = (1.656 \pm 0.005)\,\mathrm{GeV}$,
  $m_b = (4.747 \pm 0.001)\,\mathrm{GeV}$
- Preliminary results for charmonium

process	Γ^{pNRQCD}/keV	$\Gamma^{\mathrm{mod}}_{\mathrm{Grotch}}$/keV	$\Gamma^{\mathrm{lat}}_{\mathrm{Dudek}}$/keV	$\Gamma^{\mathrm{exp}}_{\mathrm{PDG}}$/keV
$\chi_{c0}(1P) \to J/\psi\gamma$	$138 \pm 40 \pm 7$	162-183	232 ± 41	122 ± 11
$\chi_{c1}(1P) \to J/\psi\gamma$	$256 \pm 85 \pm 13$	340-363	487 ± 122	296 ± 22
$\chi_{c2}(1P) \to J/\psi\gamma$	$340 \pm 115 \pm 17$	413-464	-	384 ± 27
$h_c \to \eta_c(1S)\gamma$	$292 \pm 184 \pm 15$	-	601 ± 55	<600

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- Preliminary results for bottomonium

process	Γ^{pNRQCD}/keV	$\Gamma^{\mathrm{mod}}_{\mathrm{Grotch}}$/keV	$\mathcal{B}^{\mathrm{exp}}_{\mathrm{CLEO}} \times \Gamma^{\mathrm{mod}}_{\mathrm{tot,Rosner}}$/keV
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	$24.2 \pm 2.1 \pm 0.5$	25.7-27.0	21.1 ± 4.3
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	$27.0 \pm 2.7 \pm 0.5$	29.8-31.2	32.0 ± 2.5
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	$29.7 \pm 3.1 \pm 0.5$	33.0-34.2	41.1 ± 3.1

Conclusion and Outlook

- Summary:
  EFT treatment for M1 and E1 transitions up to $\mathcal{O}(v^2)$-corrections
  $\longrightarrow$ relevant Lagrangian: exact matching for all operators (for E1)
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Thank you for your attention!

Wave-functions

- S-wave states

$$\phi^{(0)}_{n^1S_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)$$

$$\phi^{(0)}_{n^3S_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n0}(r)\, \boldsymbol{\sigma} \cdot \hat{\mathbf{e}}_{n^3S_1}(\lambda)$$

- P-wave states

$$\phi^{(0)}_{n^1P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\, \hat{\mathbf{e}}_{n^1P_1}(\lambda) \cdot \hat{\mathbf{r}}$$

$$\phi^{(0)}_{n^3P_0}(\mathbf{r}) = \sqrt{\frac{1}{8\pi}} R_{n1}(r) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}$$

$$\phi^{(0)}_{n^3P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{16\pi}} R_{n1}(r)\, \boldsymbol{\sigma} \cdot (\hat{\mathbf{r}} \times \hat{\mathbf{e}}_{n^3P_1}(\lambda))$$

$$\phi^{(0)}_{n^3P_2(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}} R_{n1}(r)\, \boldsymbol{\sigma}^i h^{ij}_{n^3P_2}(\lambda) \hat{\mathbf{r}}^j \,.$$

General non-relativistic formula

$$\Gamma^{(0)}_{n^{2s+1}L_J\to n'^{2s+1}L'_{J'},\gamma} = \frac{4}{3}\,\alpha_{em} e_Q^2 (2J'+1) S^{E1} k_\gamma^3 I_3^2(nl \to n'l')$$

$$S^{E1} = \max(l,l') \left\{ \begin{array}{ccc} J & 1 & J' \\ l' & s & l \end{array} \right\}^2$$

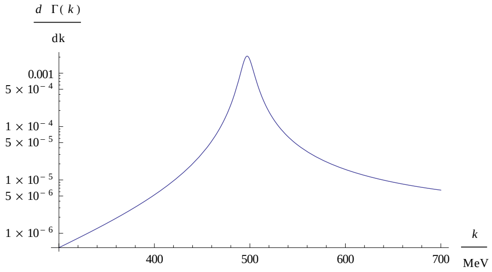
$$I_3(nl \to n'l') = \int_0^\infty dr\, r^3 R_{n'l'}(r) R_{n1}(r)$$

Lineshape of the h_b

Decay $h_b \to \eta_b\gamma \to X\gamma$, resonance in the photon spectrum observable
Lineshape from pNRQCD calculation:

$$\frac{d\Gamma_{h_b}}{dE_\gamma} = \frac{4\alpha_{em}}{81\pi} I_3^2(11 \to 10) E_\gamma^3 \frac{\Gamma_{\eta_b}/2}{(E_\gamma^{\rm peak} - E_\gamma)^2 + \Gamma_{\eta_b}^2/4} \,.$$

with $E_\gamma^{\rm peak} \approx E_{h_b} - E_{\eta_b}$
$\to$ modified Breit-Wigner curve



Numerical results

- $\sigma = 0.22\,\mathrm{GeV}^2$, $m_c = 1.652\,\mathrm{GeV}$, $m_b = 4.747\,\mathrm{GeV}$ in our scheme
- $\Gamma^{\mathrm{pNRQCD}}_{h_b(1P)\to\eta_b(1S)\gamma} = (31.5 \pm 4.4 \pm 0.6)\,\mathrm{keV}$
- Preliminary results for excited bottomonium transitions

process	Γ^{pNRQCD}/keV	$\Gamma^{\mathrm{mod}}_{\mathrm{Grotch}}$/keV	$\mathcal{B}^{\mathrm{exp}}_{\mathrm{CLEO}} \times \Gamma^{\mathrm{mod}}_{\mathrm{tot,Rosner}}$/keV
$\chi_{b0}(2P) \to \Upsilon(1S)\gamma$	$5.5 \pm 1.7 \pm 1.2$	5.3-6.5	7.8 ± 5.2
$\chi_{b1}(2P) \to \Upsilon(1S)\gamma$	$11.1 \pm 1.9 \pm 2.5$	11.0-11.8	4.3 ± 0.7
$\chi_{b2}(2P) \to \Upsilon(1S)\gamma$	$17.6 \pm 2.0 \pm 3.9$	18.2-18.9	10.9 ± 1.5