Hard pion Chiral Perturbation Theory and charmonium decays

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Outline

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Chiral Perturbation Theory

$$\mathcal{L}_{QCD} = \sum_{q=1}^{n_f} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q(\bar{q}_R q_L + \bar{q}_L q_R)]$$

(n_f =number of flavours)

If $m_q = 0$ then $SU(n_f)_L \times SU(n_f)_R$ (chiral symmetry)$\Rightarrow$ parity doublets in the spectrum.

They do not exist! $\Rightarrow SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_V$

- $n_f = 2 \to 3$ Goldstone bosons
- $n_f = 3 \to 8$ Goldstone bosons

$m_q \neq 0$ (but small) $\Rightarrow$ chiral symmetry is also explicitly broken, Goldstone bosons are not massless

Construction as Effective Field Theory

Degrees of freedom pseudo-Goldstone bosons (lightest mesons in the spectrum)

- $n_f = 2 \to \pi^+, \pi^-, \pi^0$
- $n_f = 3 \to \pi, K, \eta$

$$u = e^{i\frac{\phi}{\sqrt{2}F_0}} \qquad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Expected breakdown scale Resonances (m_ρ)

Lagrangian All operators allowed by QCD symmetries

$$\mathcal{L}_2 = \frac{F_0{}^2}{4}\left(\langle u_\mu u^\mu \rangle + \langle \chi_+ \rangle\right), \quad u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\},$$

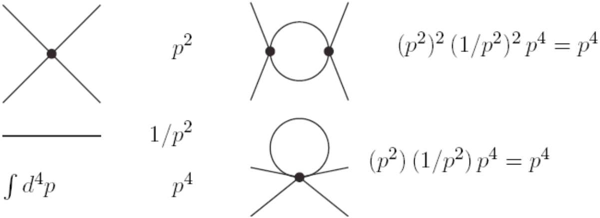
$$\chi_+ = u^\dagger \chi u^\dagger + u\chi^\dagger u, \quad \chi = 2B_0(s + ip)$$

s, p, r_μ, l_μ =external fields, $s = \mathcal{M} + \ldots$ (quark masses)

F_0, B_0 = Low Energy Constants (LECs)

Power counting

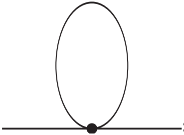
- ChPT at low energies → small momenta (p) and masses
- In $\mathcal{L}_2$ operators with either two derivatives (p^2) or masses (m^2)



- observables depend on $p^2(/(4\pi F_0)^2)$ and $m^2(/(4\pi F_0)^2)$ which are small parameters ($(4\pi F_0)^2 \gg p^2, m^2$)→ use them for perturbative expansion
- so power counting is a dimensional counting!
- $O = O_{p^2} + O_{p^4} + \mathcal{O}(p^6)$

Chiral Logarithms as main prediction of ChPT

Expansion of ChPT is not Taylor expansion: logarithms of masses (and energies) appear



$$\approx \infty + m^2 \log\left(\frac{m^2}{\mu^2}\right) + \cdots$$

m = mass of meson in loop, μ =arbitrary scale

E.g. for the masses

$$M^2 \approx \underbrace{m_0^2}_{\text{LO}} + \underbrace{\frac{m_0^4}{(4\pi F_0)^2} \log\frac{m_0^2}{\mu^2} + \frac{L_i^r}{F_0^2} m_0^4}_{\text{NLO}} + \mathcal{O}(m_0^6)$$

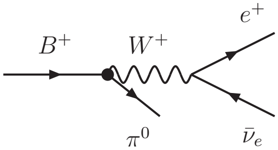
$L_i^r \approx 10^{-2} \Rightarrow$ chiral logarithm $\log\frac{m_0^2}{\mu^2}$ is leading contribution at $\mathcal{O}(p^4)$ (NLO).

The chiral logs encode mass dependence of the observables

Outline

Motivation

- Decays of a heavy meson into light mesons



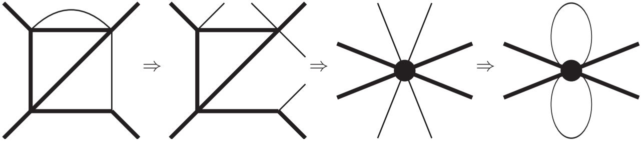
- $m_B \approx 3000$ MeV, $m_D \approx 2000$ MeV, $m_\pi \approx 140$ MeV
- $q^\mu = (p_{B/D} - p_\pi)^\mu$ = momentum transfer to leptons
  $0 \leq q^2 \leq (M - m_\pi)^2 = q^2_{\max}$ with $M = m_B, m_D$
- two different kinematical regimes
  1. $q^2 \approx q^2_{\max} \Rightarrow E_\pi \lesssim 1$ GeV **soft pion** (ChPT ok)
  2. $q^2 \approx 0 \Rightarrow E_\pi > 1$ GeV **hard pion** (ChPT ???)

PROBLEM

- lattice calculates the decay at any q^2 but simulations done with HEAVY pions ($m_\pi > 300$ MeV) $\Rightarrow$ need extrapolation formulas to achieve $m_\pi \sim 140$ MeV

Argument for Hard Pion Chiral Perturbation Theory

Flynn and Sachrajda (2009), Bijnens and Celis (2009), Bijnens and IJ (2010)

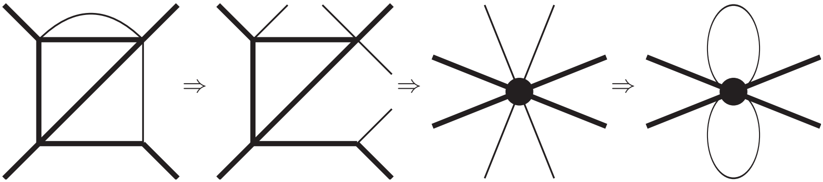


In the Feynman diagrams appear both **hard** and *soft* lines.

The *soft* lines can be separated from the **hard/short-distance** structure of the rest of the diagram.
They are the only responsible of the non analyticities arising for $m_\pi \to 0$ (e.g. the chiral logarithms)

The **hard** part is describable by an effective Lagrangian consistent with all the symmetries and with couplings that depend on hard kinematical quantities.
Assumption: this Lagrangian is sufficiently complete to describe the neighbourhood of the hard process.

Example



1. consider a diagram with *soft (thin)* and **hard (thick)** lines
2. identify the *soft* lines and cut them ⇒ remove the soft singularities
3. the resulting diagram is analytic in the soft part and thus should be describable by a vertex of an effective Lagrangian. The coupling contains information on the hard quantities
4. insert back the loops with the *soft* lines: this last diagram should reproduce the soft singularities of the first one

However only arguments not proof!!!

Sketch of the proofs

- for hard pions $m_\pi^2/(4\pi F_0)^2 \ll 1$, while $p_\pi^2/(4\pi F_0)^2$ is not
- operators with an arbitrary numbers of derivatives on the external π are not negligible
- look at $\langle \pi \,|O|\, B \rangle$ $\quad O$ =operator in $\mathcal{L}$ with more derivatives
- keep only: $O(1)$, $O(m_\pi)$ and $O(m_\pi^2 \log m_\pi^2)$. NO: $O(m_\pi^2)$ without logarithms
- different cases depending on which particle the extra derivative hits
- use partial integration and dimensional analysis
- it must turn out that $\langle \pi \,|O|\, B \rangle$ are all proportional to the lowest order up to terms $O(m_\pi^2)$ (and without logarithms) which are of higher order
- constants of proportionality are absorbed in the couplings of $\mathcal{L}$

What has been done so far

$$O(Q^2, m^2) = E(Q^2, 0) \times \left(1 + \alpha m^2 \log\left(\frac{m^2}{\mu^2}\right) + \mathcal{O}(m^2)\right)$$

α is what we calculate. $E(Q^2, 0)$ depends on the hard quantities $(q^2, m_B, \ldots)$.
Hard pion ChPT applied so far to

- $K \to \pi \ell \nu_\ell$ (two-flavour) Flynn and Sachrajda (2009)
- $K \to \pi\pi$ (two-flavour) Bijnens and Celis (2009)
- results agree with three-flavour standard ChPT
- Bijnens and IJ (2010)-(2011)
  1. $B(D) \to M$ vector transitions with $(M = \pi, K, \eta) \to$ agree with a relativistic formalism,
  2. $B \to D \ell \nu_\ell$
  3. π and K scalar and vector formfactors(two-flavours)
  4. checked including two-loop diagrams for scalar and vector formfactors of the pion$\to$ extension to any order Colangelo et al. (201?)
  5. $\chi_{c0}, \chi_{c2} \to MM$ decays

Beware

- The proof that it all reduces to a single type of lowest order term can be tricky
- E.g. for $SU(3)$ scalar form factor there are two types of LO terms

$$\langle \chi_+ \rangle \quad \text{and} \quad \langle \chi_+ \rangle \langle u^\mu u_\mu \rangle$$

- In $SU(2)$ case these two types are the same
- for vector form factors the second type does not contribute

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$B \to M$ vector transitions: tree level result

(1)

(2)

- HMChPT

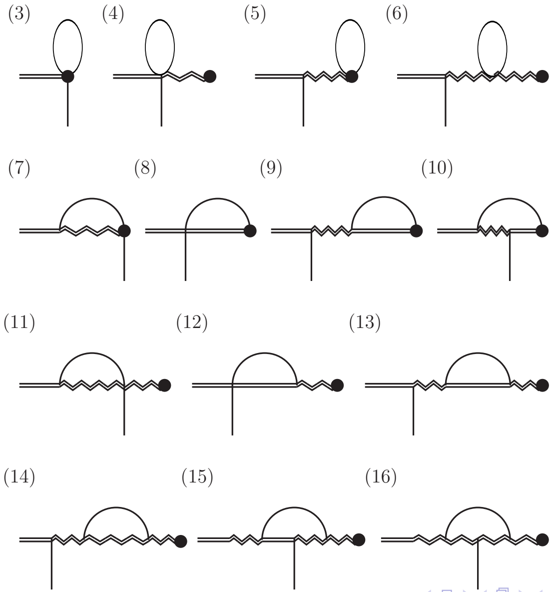
$$f_v^{\text{Tree}}(v\cdot p_\pi) = \frac{\mathcal{C}\sqrt{2}}{F}, \qquad f_p^{\text{Tree}}(v\cdot p_\pi) = \frac{\mathcal{C}\sqrt{2}}{F}\frac{g}{v\cdot p_\pi + \Delta}$$

- relativistic theory

$$f_0^{\text{Tree}}(q^2) = \frac{1}{4}\frac{E_1}{F}, \qquad f_+^{\text{Tree}}(q^2) = \frac{1}{4}\frac{E_3}{F}\frac{m_B}{q^2 - m_B^2}g$$

- near q^2_{max} the propagators become respectively $1/m_\pi$ and $1/(2m_\pi m_B)$ (factor of $2m_B$ due to the different normalizations)
- for $q^2 \approx q^2_{\text{max}}$ see Falk et al. (1993), Becirevic et al.(2003)
- for $q^2 \ll q^2_{\text{max}}$ coupling constants differ at different q^2, can even be complex

One-loop diagrams



The chiral logarithms

- HMCHPT: expand the one-loop calculation Becirevic et al.(2003) for $v \cdot p_\pi \to m_B$, $m_\pi^2 \to 0$
- relativistic theory: calculate the formfactors and then expand the loop integrals for $m_\pi^2 \ll m_B^2, (m_B^2 - q^2)$ (tricky part of the calculations)
- the coefficients of the leading logarithms coincide in the two theories (both at $q^2 \ll q^2_{\max}$ and at $q^2 \approx q^2_{\max}$)

$D \to \pi$ $\qquad q^2 \ll q^2_{\max}$

$$f_{v/p}(v\cdot p_\pi) = f^{\text{Tree}}_{v/p}(v\cdot p_\pi)\left[1 + \left(\frac{3}{4} + \frac{9}{4}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{2} + \frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{12} + \frac{3}{12}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2}\right]$$

$D \to K$ $\qquad q^2 \ll q^2_{\max}$

$$f_{v/p}(v\cdot p_K) = f^{\text{Tree}}_{v/p}(v\cdot p_K)\left[1 + \frac{9}{4}g^2\frac{\overline{A}(m_\pi^2)}{F^2} + \left(1 + \frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{3} + \frac{3}{12}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2}\right]$$

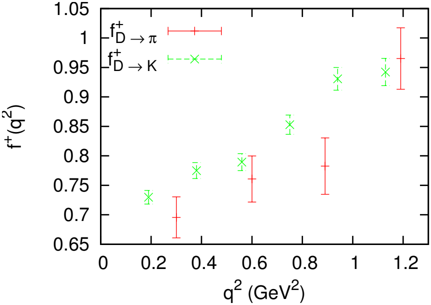
A confirmation

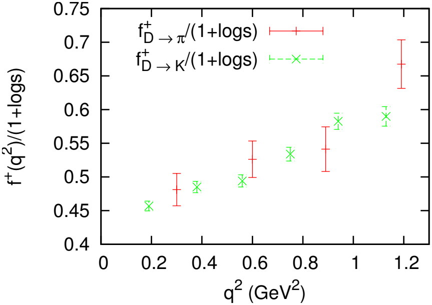
- the chiral logarithms are the same for the scalar formfactor f_0 and for f_+
- same prediction obtained in Large Energy Effective Theory
- Charles et al.(1998) $\rightarrow$ only one form factor for $B \rightarrow M$ transitions
- also confirmed in Soft Collinear Effective Theory Bauer et al. (2001)

Comparison with data

- use $SU(3)$ results and compare $f^+_{D\to\pi}(q^2)$ with $f^+_{D\to K}(q^2)$ CLEO coll. (2009)
- the chiral logs are responsible of the differences between the two decays

$$\frac{f^+_{D\to\pi}}{1+\log_{D\to\pi}} \approx \frac{f^+_{D\to K}}{1+\log_{D\to K}}$$





$\mu^2 = 1\text{Gev}^2 \qquad g^2 = 0.44$

Outline

Charmonium decays to pseudoscalars

- charmonium decays to pseudoscalar mesons again same issue $\rightarrow$ outgoing mesons are hard $\rightarrow$ hard pion ChPT
- charmonium ($c\bar{c}$) singlet under $SU(3)_L \times SU(3)_R$

1. $J/\psi \rightarrow \pi\pi$ vs $J/\psi \rightarrow KK$ difficult to compare different channels:
   - $J/\psi \rightarrow \pi\pi$ violates isospin
   - $J/\psi \rightarrow KK$ U-spin or V-spin
   - similar for χ_{c1}
2. $\chi_c \rightarrow \pi\pi\pi$ vs $\chi_c \rightarrow \pi\pi\pi$
   - tipically more involved (more operators might arise)
   - $\pi\pi\pi$ channel: not all the 3 pions are hard $\Rightarrow$ need to decide where hard pion ChPT is ok
   - $KKK \rightarrow$ Hard Pion ChPT ?
3. $\chi_{c0} \rightarrow MM$ and $\chi_{c2} \rightarrow MM$ are ok!

Lagrangian

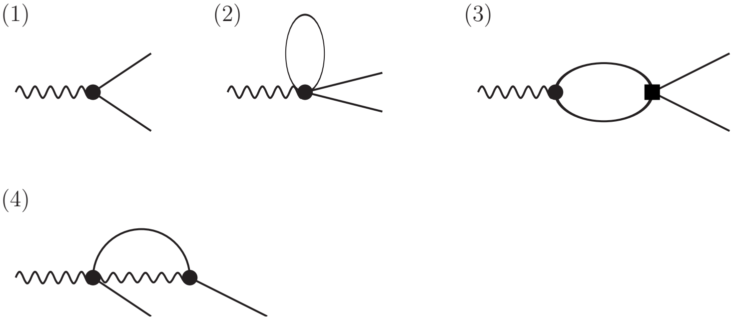
- $\chi_{c0} \rightarrow$ scalar field χ_0
- $\chi_{c2} \rightarrow$ spin-2 field $\chi_2^{\mu\nu}$ (symmetric, traceless etc)
- remember $\chi_c =$ chiral singlet $\rightarrow \mathcal{L}_{\chi_c} \sim \chi_c \times \mathcal{L}_{\text{ChPT}}$

Most general Lagrangian satisfying chiral symmetry

$$\mathcal{L}_{\chi_c} = E_1 F_0^2 \chi_0 \langle u_\mu u^\mu \rangle + E_2 F_0^2 \chi_2^{\mu\nu} \langle u_\mu u_\nu \rangle$$

Extra operators for checks

$$\mathcal{L}_{\chi_c}^E = E_3 F_0^2 \chi_0^2 \langle u_\mu u^\mu \rangle + E_4 F_0^2 \chi_0 \langle \nabla^\mu u_\nu \nabla_\mu u^\nu \rangle$$



- to these add wave-function renormalization
- loop diagrams with derivatives hitting a pseudoscalar propagator are soppressed e.g. (3)
- (4)$\rightarrow$ no chiral logs from diagrams with heavy particle propagators

Results

Tree-level amplitudes

$$
\begin{aligned}
iA(\chi_{c0} \to MM) &= -(m_\chi^2 - 2m_M^2) \times \left(2E_1 + E_4(m_\chi^2 - 2m_M^2)\right) \\
iA(\chi_{c2} \to MM) &= -4E_2 p_1^\mu p_2^\nu T_{\chi\mu\nu}
\end{aligned}
$$

$p =$ momentum of outgoing M

$T_{\chi\mu\nu}$ =polarization tensor of χ_{c2}

- E_4-term gives contribution proportional to LO one (up to order m_M^2)

Results

One-loop amplitudes

$$
\begin{aligned}
iA(\chi_{c0} \to MM) &= -(m_\chi^2 - 2m_M^2) \times \left(2E_1 + E_4(m_\chi^2 - 2m_M^2)\right) \\
iA(\chi_{c2} \to MM) &= -4E_2 p_1^\mu p_2^\nu T_{\chi\mu\nu}
\end{aligned}
$$

$p =$ momentum of outgoing M $\qquad$ $T_{\chi\mu\nu}$ =polarization tensor of χ_{c2}

- at one-loop we found zero corrections!
- the logarithms from diagram (2) exactly cancels with wf renormalization
- result valid for any chiral singlet hence for $T^{\mu\nu}$ (energy-momentum tensor) $\to$ compare with expansion of ChPT calculation Donoghue and Leutwyler (1991): this gives zero as well
- there are no enhanced chiral corrections $\Rightarrow$ no strong conclusions from comparison with data ☹

Comparison with experiments

- nevertheless no chiral corrections should lead to somewhat smaller $SU(3)_V$ breaking effects compared to the "usual" 20% (as in e.g. F_K/F_π)
- at least check how much this statement agrees with data
- compute the amplitudes from the measured BR PDG (2011)
- χ_{c0}

$$G_0 = \sqrt{BR/|\vec{p}_1|}/(p_1.p_2)$$

$p_1 \cdot p_2 = (m_\chi^2 - 2m_M^2)/2 \to$ always in A_0

phase space $= |\vec{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$

- χ_{c2}

$$G_2 = \sqrt{BR/|\vec{p}_1|}/|\vec{p}_1|^2$$

$|\vec{p}_1|^2 \propto \sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} \to$ always in A_2

Comparison with experiments

$$G_0 = \sqrt{BR/|\vec{p}_1|}/(p_1.p_2) \qquad G_2 = \sqrt{BR/|\vec{p}_1|}/|\vec{p}_1|^2$$

	χ_{c0}		χ_{c2}	
Mass Width	3414.75 ± 0.31 MeV 10.4 ± 0.6 MeV		3556.20 ± 0.09 MeV 1.97 ± 0.11 MeV	
Final state	10^3 BR	$10^{10}\, G_0[\mathrm{MeV}^{-5/2}]$	10^3 BR	$10^{10} G_2[\mathrm{MeV}^{-5/2}]$
$\pi\pi$	8.5 ± 0.4	3.15 ± 0.07	2.42 ± 0.13	3.04 ± 0.08
K^+K^-	6.06 ± 0.35	3.45 ± 0.10	1.09 ± 0.08	2.74 ± 0.10
$K_S^0 K_S^0$	3.15 ± 0.18	3.52 ± 0.10	0.58 ± 0.05	2.83 ± 0.12
$\eta\eta$	3.03 ± 0.21	2.48 ± 0.09	0.59 ± 0.05	2.06 ± 0.09
$\eta'\eta'$	2.02 ± 0.22	2.43 ± 0.13	< 0.11	< 1.2

- $\pi\pi$ vs KK about 10% difference for χ_{c0} and χ_{c2}
- $\eta\eta$ borderline
- corrections are indeed reasonably small

Outline

Conclusions and outlook

- extensions of the arguments adopted in Flynn and Sachrajda (2009), Bijnens and Celis (2009)
- still ONLY arguments. Proven for the single processes but they sound very general $\rightarrow$ formalize the approach (with SCET?)
- what discussed here can be found in J. Bijnens, I.J. Nucl Phys B **840** (2010), J. Bijnens, I.J. Nucl Phys B **846** (2011), J. Bijnens, I.J. Eur. Phys. J A **47** (2011)
- extendable for PQChPT (useful on the lattice)