Collider Physics and Higher Order Corrections

Gudrun Heinrich

Max-Planck Institute for Physics, Munich

Universität Wien, 15.12.2011

Early Colliders

- luminosity low
- detectors poor

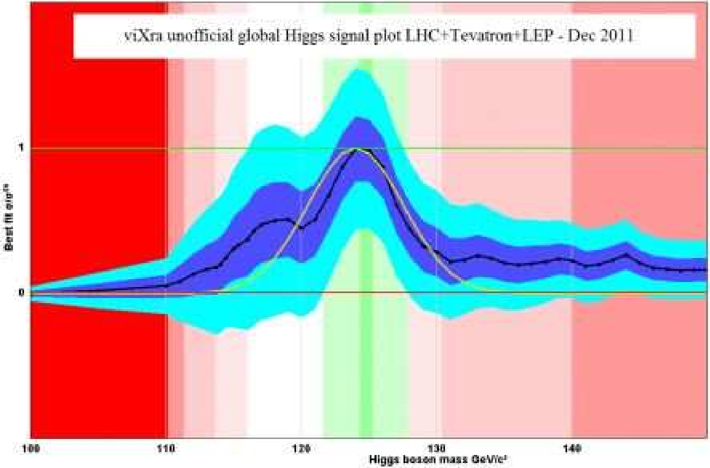
Early Theory

Modern Colliders

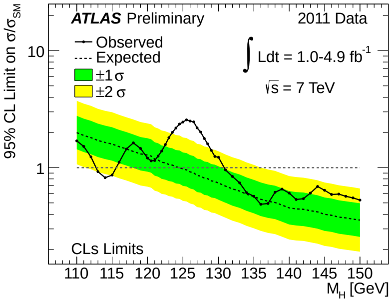
- luminosity higher
- detectors better

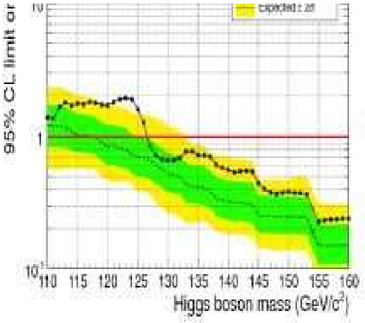
Modern Theory

Modern Phenomenology



Modern Phenomenology





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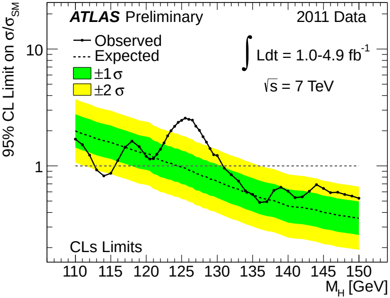
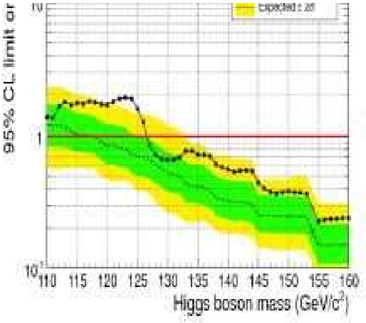
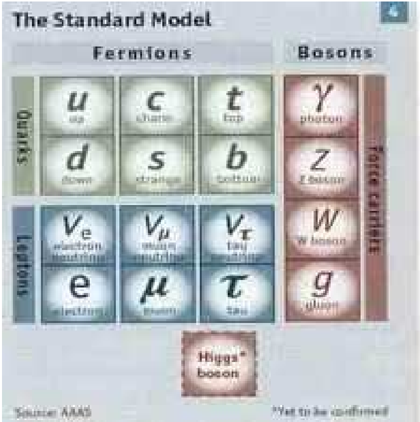




we need precise theory predictions for "Expected" !

The Standard Model



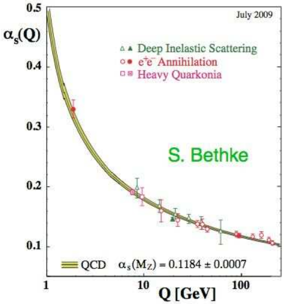
important concepts:

- local gauge theory $SU(2) \times U(1) \times SU(3)_c$
- renormalizability
- perturbation theory

Strong Interactions

basic principles of Quantum Chromo-Dynamics (QCD):

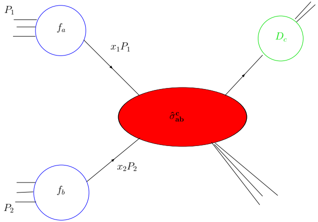
- asymptotic freedom: coupling $\alpha_s(Q^2) \rightarrow 0$ for $Q^2 \rightarrow \infty$



constituents of hadrons (quarks and gluons) can be considered as freely interacting at high energies (i.e. short distances)

- factorisation: systematic separation of long-distance effects (non-perturbative) and short-distance cross sections ("hard scattering")

factorisation



$$\sigma_{pp\to X} \;=\; \sum_{a,b,c} f_a(x_1,\mu_f^2) f_b(x_2,\mu_f^2) \otimes \hat{\sigma}_{ab}\left(p_1,p_2,\frac{Q^2}{\mu_f^2},\frac{Q^2}{\mu_r^2},\alpha_s(\mu_r^2)\right)$$

$$\otimes D_{c\to X}(z,\mu_f^2) \;+\mathcal{O}(\Lambda/Q)$$

f_a, f_b: parton distribution functions (from fits to data)

$\hat{\sigma}_{ab}$: partonic hard scattering cross section

calculable order by order in perturbation theory

$D_{c\to X}(z,\mu_f^2)$: describing the final state e.g. fragmentation function, jet observable, etc.

Perturbative expansion

expansion in strong coupling α_s:

$$\hat{\sigma} = \alpha_s^k(\mu)\,\left[\,\hat{\sigma}^{\mathrm{LO}} + \alpha_s(\mu)\,\hat{\sigma}^{\mathrm{NLO}}(\mu) + \alpha_s^2(\mu)\,\hat{\sigma}^{\mathrm{NNLO}}(\mu) + \ldots\,\right]$$

μ-dependence comes from truncation of perturbative series

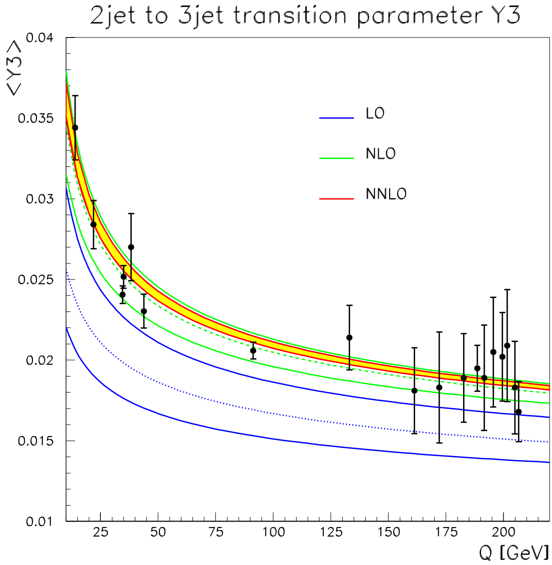
calculation at n-th order:

$$d\hat{\sigma}^{(n)}/d\ln(\mu^2) = \mathcal{O}(\alpha_s^{n+1})$$

truncation of perturbative series at LO

$\Rightarrow$ large renormalisation/factorisation scale dependence

shortcomings of leading order predictions



example:

3-jet observable

in e^+e^- annihilation

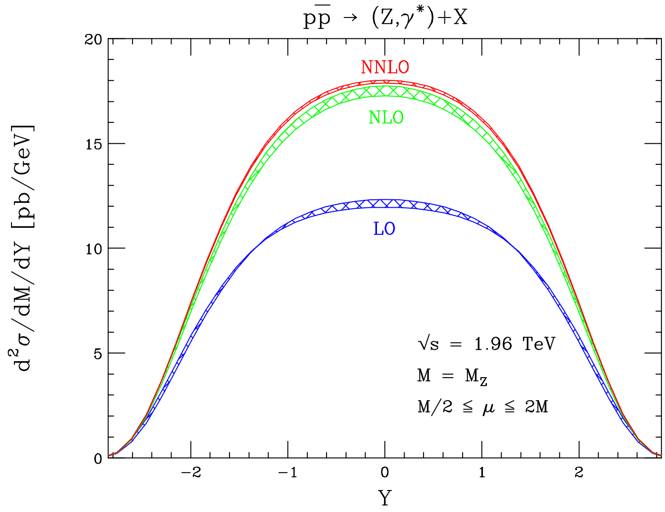
[A. Gehrmann-De Ridder,

T. Gehrmann, N. Glover, GH '09]

uncertainty bands:

$M_Z/2 < \mu < 2\,M_Z$

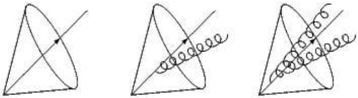
Shortcomings of Leading Order Predictions



[Anastasiou et al. 04]

Shortcomings of Leading Order Predictions

- poor jet modelling



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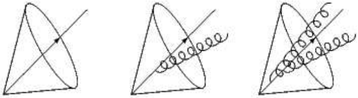

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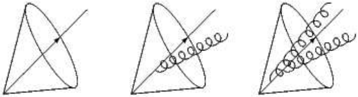

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- Minimal Supersymmetric Standard Model (MSSM):
  would be ruled out already without radiative corrections:
  mass of lightest Higgs boson at LO: $M_h \leq min(M_A, M_Z) \cdot |\cos 2\beta|$

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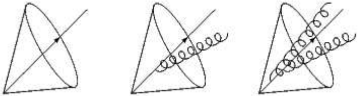

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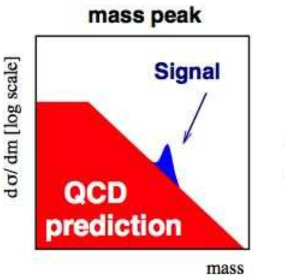
- ...

Identifying New Physics at Hadron Colliders



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- shape: hard
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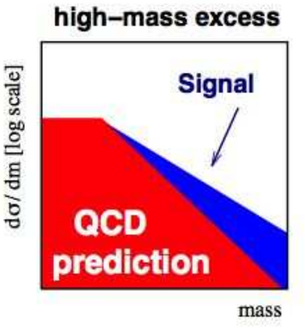
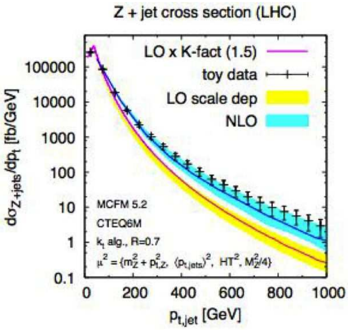
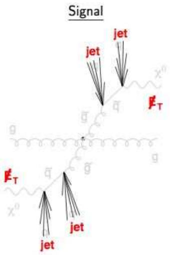


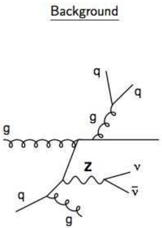


Identifying New Physics at Hadron Colliders

- peak: easy, backgrounds can be measured
- shape: hard
  need signal/background shapes from theory
- rate (e.g. $H \to W^+W^-$): very hard (counting experiment)
  need both shape and normalisation from theory

example heavy SUSY particles





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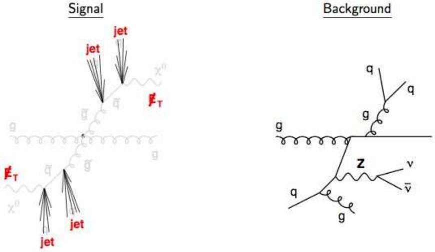
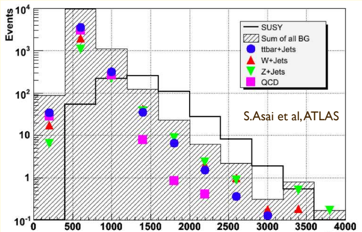


Overall result, after the complete detector simulation, etc....



multi-particle final states

- to establish signals of New Physics
- to measure model parameters

  Leading Order is not sufficient !

- at LHC: typically multi-particle final states

  $\Rightarrow$ calculations of higher orders increasingly difficult

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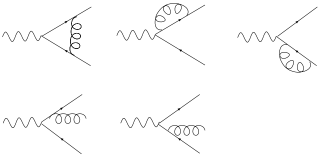
- example for time scale to add one parton:
  $pp \rightarrow 2$ jets at NLO (4-point process): Ellis/Sexton 1986
  $pp \rightarrow 3$ jets at NLO (5-point): Bern et al, Kunszt et al '93-95
  $pp \rightarrow 4$ jets at NLO (6-point): not yet available

ingredients for m-particle observable at NLO



virtual part (one-loop integrals):

$$\mathcal{A}_{NLO}^{V} = A_2/\epsilon^2 + A_1/\epsilon + A_0^{(v)}$$

$$d\sigma^V \sim Re\left(\mathcal{A}_{LO}^{\dagger}\,\mathcal{A}_{NLO}^{V}\right)$$

real radiation part: soft/collinear emission of massless particles

$\Rightarrow$ need subtraction terms

$$\Rightarrow \int_{\mathrm{sing}} d\sigma^S = -A_2/\epsilon^2 - A_1/\epsilon + A_0^{(r)}$$

$$\sigma^{NLO} = \underbrace{\int_{m+1}\left[d\sigma^R - d\sigma^S\right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{cancel poles}} + \underbrace{\int_{\mathrm{s}} d\sigma^S}_{\text{analytically}}\right]_{\epsilon=0}}_{\text{numerically}}$$

NLO calculations

exploit modular structure

Tree Modules		One-Loop Module		IR Modules
$\lvert\mathcal{A}^{LO}\rvert^2$	$\oplus$	$2\,Re(\mathcal{A}^{LO\,\dagger}\mathcal{A}^{NLO,virt})$	$\oplus$	integrated IR subtraction terms
$\lvert\mathcal{A}^{NLO,real}\rvert^2$			$\ominus$	soft/collinear subtraction terms

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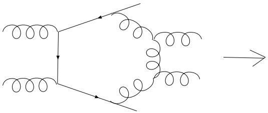
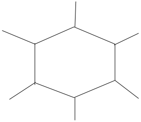
has been bottleneck so far

One-loop methods

basically two categories:

- methods based on Feynman diagrams

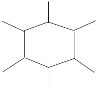




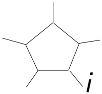
$+$ integrals with less legs

non-trivial tensor structure

scalar 6-point function



$= \sum_{i=1}^{6} b_i$

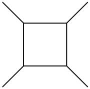


i

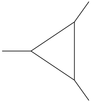
... factorial growth in complexity

reduction to set of basis integrals (4-, 3- and 2-point funcs.)

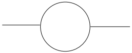
$\mathcal{A} = C_4$



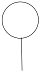
$+ C_3$



$+ C_2$



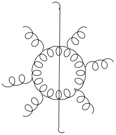
$+ C_1$



$+ \mathcal{R}$

One-loop methods

- "unitarity based":

  $\mathcal{A} = \sum_{\text{cuts}} \int d\, PS$  $+ \quad \mathcal{R}$

  - use analyticity structure to compose loop amplitudes from cuts
  - efficient for coefficients of boxes, triangles, bubbles (use cuts in $D = 4$)
  - cut conditions lead to systems of equations which can be solved numerically [OPP method, Ossola, Papadopoulos, Pittau]
  - obtaining rational terms $\mathcal{R}$ needs $D \neq 4$

Progress monitor

Les Houches NLO wishlist for LHC, Status 2007

process ($V \in \{Z, W, \gamma\}$)	status
1. $pp \to V\,V$ jet	*WW jet:* Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi; Binoth, Guillet, Karg, Kauer, Sanguinetti
2. $pp \to V\,V\,V$	*ZZZ:* Lazopoulos, Melnikov, Petriello
3. $pp \to t\bar{t}\,b\bar{b}$	
4. $pp \to t\bar{t} + 2$ jets	
5. $pp \to V\,V\,b\bar{b}$	
6. $pp \to V\,V + 2$ jets	
7. $pp \to V + 3$ jets	
8. $pp \to b\bar{b}b\bar{b}$	
9. $pp \to 4$ jets	
10. EW corrections to W,Z prod.	

Status Les Houches 2009

$pp \to W\,W$ jet	Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi
	Binoth/Guillet/Karg/Kauer/Sanguinetti
$pp \to Z\,Z$ jet	Binoth/Gleisberg/Karg/Kauer/Sanguinetti; Dittmaier/Kallweit
$pp \to t\bar{t}\,b\bar{b}$	Bredenstein/Denner/Dittmaier/Pozzorini;
	Bevilacqua/Czakon/Papadopoulos/Pittau/Worek
$pp \to t\bar{t} + 2$ jets	Bevilacqua/Czakon/Papadopoulos/Worek
$pp \to Z\,Z\,Z$	Lazopoulos/Melnikov/Petriello; Hankele/Zeppenfeld
$pp \to V\,V\,V$	Binoth/Ossola/Papadopoulos/Pittau; Zeppenfeld et al.
$pp \to V\,V\,b\bar{b}$	
$pp \to W\,\gamma$ jet	Campanario/Englert/Spannowsky/Zeppenfeld
$pp \to V\,V + 2$ jets	VBF: Bozzi/Jäger/Oleari/Zeppenfeld, VBFNLO coll.
$pp \to W + 3$ jets	BlackHat coll.; Ellis/Giele/Kunszt/Melnikov/Zanderighi*
$pp \to Z + 3$ jets	BlackHat collaboration
$qq \to b\bar{b}b\bar{b}$	Binoth/Greiner/Guffanti/Guillet/Reiter/Reuter

• done • partial results * leading colour only

Now

- $pp \to W^+W^-b\bar{b}$ Denner, Dittmaier, Kallweit, Pozzorini '10 ; Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11
- $pp \to W/Z + 4$ jets BlackHat collaboration '10/'11
- $pp \to W/Z/\gamma + 3$ jets BlackHat collaboration '09/'10
- $pp \to t\bar{t}$ +2 jets Bevilacqua, Czakon, Papadopoul., Worek '10
- $pp \to t\bar{t}\,b\bar{b}$ Bredenstein, Denner, Dittmaier, Pozzorini '09; Bevilacqua, Czakon, Papadopoulos, Worek '09
- $pp \to W\gamma\gamma j$ Campanario, Englert, Rauch, Zeppenfeld '11
- $pp \to W^+W^+jj$ Melia, Melnikov, Rontsch, Zanderighi '10
- $pp \to W^+W^-jj$ Melia, Melnikov, Rontsch, Zanderighi '11
- $pp \to 4\,b$ Binoth et al '09; Greiner, Guffanti, Reiter, Reuter '11
- NGluon ($N <\sim 14$) Badger, Biedermann, Uwer '11 (public)
- $e^+e^- \to 5$ jets Frederix, Frixione, Melnikov, Zanderighi '10

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- also: BIG advances in automation

Automated NLO Tools

One-loop

- FeynArts/FormCalc/LoopTools (public) Thomas Hahn et al
- GRACE Fujimoto et al.
- Helac-NLO (public) Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek
- MadLoop/ aMC@NLO Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau
  uses CutTools (public) [Ossola, Papadopoulos, Pittau] and MadFKS
- GoSam (public) Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
  Samurai (public) [Mastrolia, Ossola, Reiter, Tramontano],
  golem95 (public) [Binoth, Cullen, Guillet, GH, Kleinschmidt, Pilon, Reiter, Rodgers]
- dedicated programs also involve high level of automation
  Denner, Dittmaier, Pozzorini et al, VBFNLO coll., Blackhat, Rocket, MCFM, . . .

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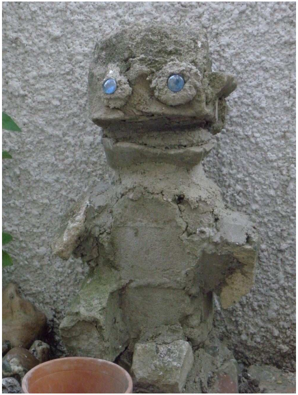
automation of subtraction for IR divergent real radiation

- MadDipole Frederix, Greiner, Gehrmann 08
- TevJet Seymour Tevlin 08
- AutoDipole Hasegawa, Moch, Uwer 08,09
- Helac-Phegas Czakon, Papadopoulos, Worek 09; polarized
- MadFKS Frederix, Frixione, Maltoni, Stelzer 09

Golem-Samurai (GoSam)

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]



Golem-Samurai (GoSam)

General One-Loop Evaluator of Matrix elements &
Scattering Amplitudes from Unitarity based Reduction At Integrand level
[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

- algebraic generation of D-dimensional integrands based on Feynman diagrams
  - QCD, EW, BSM
  - uses QGraf [Nogueira], FeynRules [Duhr et al], Form/Spinney [Vermaseren/Cullen et al], Haggies [Reiter] to generate integrands

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- reduction by D-dimensional extension of cut-based method
  options:
  - OPP-type reduction
    [Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov]
  - traditional tensor reduction (using golem95 library)
  - tensorial reduction at integrand level GH, Ossola, Reiter, Tramontano '10

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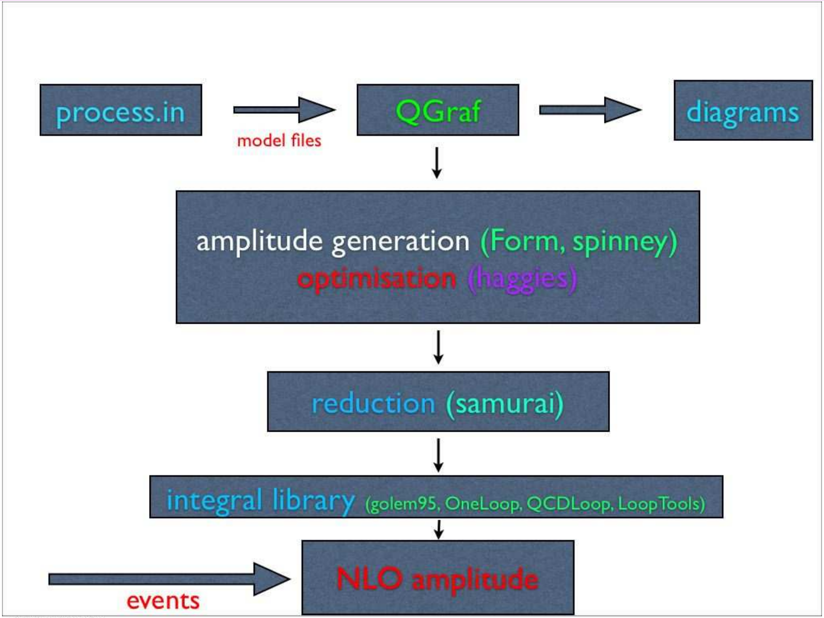
  - QCD, EW, BSM
  - uses QGraf [Nogueira], FeynRules [Duhr et al], Form/Spinney [Vermaseren/Cullen et al], Haggies [Reiter] to generate integrands
- reduction by D-dimensional extension of cut-based method
  options:
  - OPP-type reduction
    [Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov]
  - traditional tensor reduction (using golem95 library)
  - tensorial reduction at integrand level GH, Ossola, Reiter, Tramontano '10
- interface with existing tools for real radiation
  (MadGraph/MadEvent, Sherpa, Powheg, . . .)

Golem-Samurai structure



Monday, July 25, 2011

Golem-Samurai

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
```

models can be added via **FeynRules** (Duhr) or **LanHEP** (Semenov)

```
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

- gosam.py process.in

Golem-Samurai

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- make doc ⇒ documentation and diagram pictures

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- make doc ⇒ documentation and diagram pictures
- make source ⇒ source files

Golem-Samurai

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```

- gosam.py process.in
- make doc ⇒ documentation and diagram pictures
- make source ⇒ source files
- make compile ⇒ fully compiled code

Example $u\,\bar{d} \to W^- W^+\,\bar{s}c \to e^-\bar{\nu}_e\mu^+\nu_\mu\,\bar{s}c$

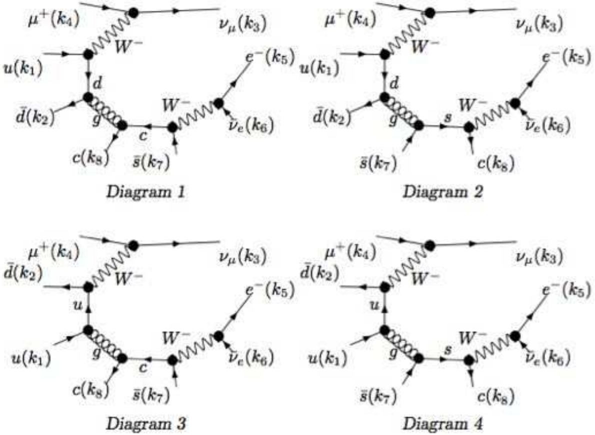


Diagram 1 Diagram 2

Diagram 3 Diagram 4

5 One-Loop Diagrams

General Information

QGraf Setup

Example $u\,\bar{d} \to W^- W^+ \,\bar{s}c \to e^- \bar{\nu}_e \mu^+ \nu_\mu \,\bar{s}c$

NLO sample diagrams

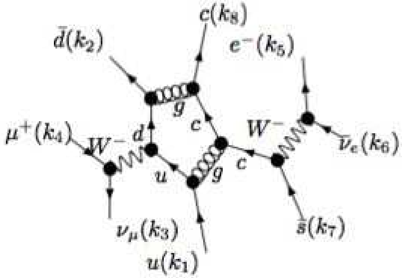


Diagram 8

$S' = S^{\{4\}}_{Q\to -q-(k3-k2+k4)}$, rk = 3

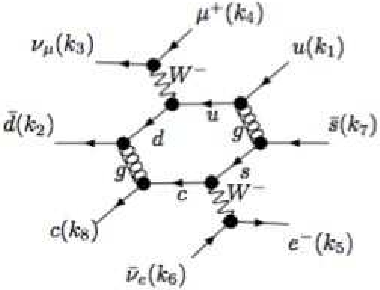
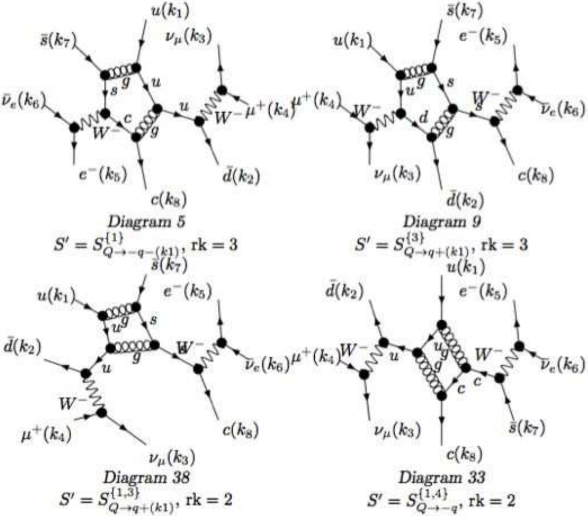


Diagram 1

$S' = S_{Q\to q+(k1)}$, rk = 4

Example $u\,\bar{d} \to W^- W^+\,\bar{s}c \to e^-\bar{\nu}_e\mu^+\nu_\mu\,\bar{s}c$



Example $u\,\bar{d} \to W^- W^+ \,\bar{s}c \to e^- \bar{\nu}_e \mu^+ \nu_\mu \,\bar{s}c$

code generation:

```
Form is processing loop diagram 80 @ Helicity 0
  2.72 sec out of 2.74 sec
Haggies is processing abbreviations for loop diagram 80 @ Helicity 0
Form is processing loop diagram 81 @ Helicity 0
  0.71 sec out of 0.73 sec
Haggies is processing abbreviations for loop diagram 81 @ Helicity 0
Form is processing loop diagram 82 @ Helicity 0
  0.73 sec out of 0.75 sec
Haggies is processing abbreviations for loop diagram 82 @ Helicity 0
Form is processing loop diagram 83 @ Helicity 0
  0.70 sec out of 0.71 sec
Haggies is processing abbreviations for loop diagram 83 @ Helicity 0
Form is processing loop diagram 84 @ Helicity 0
  0.73 sec out of 0.73 sec
Haggies is processing abbreviations for loop diagram 84 @ Helicity 0
Form is processing loop diagram 85 @ Helicity 0
```

Example $u\,\bar{d} \to W^- W^+\,\bar{s}c \to e^-\bar{\nu}_e\mu^+\nu_\mu\,\bar{s}c$

```
==================================================
                 GoSam-1.0
==================================================

#     NLO/LO, finite part:  -15.91575118714612
#     NLO/LO, single pole:    7.587050495888512
#     NLO/LO, double pole:  -5.333333333333234

 CPU time (secs):    1.2997999999999991E-002
```

result compared with

Melia, Melnikov, Rontsch, Zanderighi (MMRZ) 1104.2327 [hep-ph]

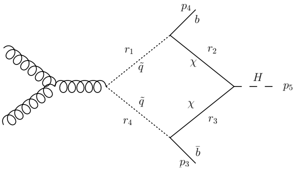
NLO/LO	GoSam	MMRZ
$1/\epsilon^2$	-5.333333333	-5.333333
$1/\epsilon$	7.5870504959	7.587051
finite	-15.915751119	-15.91575

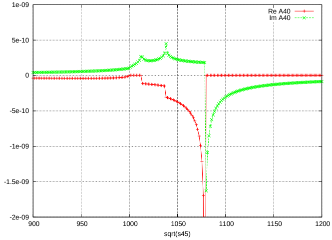
Tested 5- or 6-point processes

- $u\,\bar{d} \to W^+\, s\bar{s} \to e^+\nu_e\, s\bar{s}$
- $u\,\bar{d} \to W^+\, gg \to e^+\nu_e gg$
- $d\,\bar{d} \to Z\, gg \to e^+e^-\, gg$
- $u\,\bar{d} \to W^+\, b\bar{b} \to e^+\nu_e\, b\bar{b}$ also with massive b's
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
- $q\,\bar{q} \to b\bar{b}b\bar{b}$
- $gg \to b\bar{b}b\bar{b}$
- $u\,\bar{d} \to W^+W^+\, s\bar{c} \to e^+\nu_e\mu^+\nu_\mu\, s\bar{c}$
- $u\,\bar{u} \to W^+W^-\, \bar{c}c \to e^-\bar{\nu}_e\mu^+\nu_\mu\, \bar{c}c$
- $u\,\bar{d} \to W^+W^-\, s\bar{c} \to e^-\bar{\nu}_e\mu^+\nu_\mu\, \bar{s}c$
- $u\,\bar{d} \to W^+\, g \to e^+\nu_e\, g$ EW corrections
- plus a large number of $2 \to 2$ processes

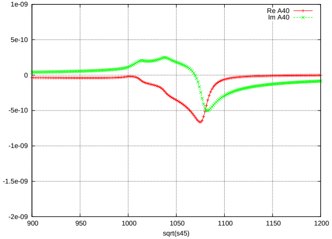
golem95 integral library

Example: production of a heavy neutral MSSM Higgs and a $b\bar{b}$ pair with unstable particles (squarks, neutralinos) in the loop





real masses



complex masses

contained in golem95C library: 1101.5595 [hep-ph]

Binoth, Cullen, Guillet, GH, Kleinschmidt, Pilon, Reiter, Rodgers

http://projects.hepforge.org/~golem/95/

Options

reduction:

- samurai, sampling of groups of diagrams
- samurai, sampling of individual diagrams
- tensorial reconstruction + samurai
- tensor reduction with golem95
- samurai + tensor reduction with golem95 if reconstruction fails
- tensorial reconstruction with PJFry [V.Yundin]

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- golem95C Cullen et al., link to LoopTools [T.Hahn] possible
- QCDLoop Ellis, Zanderighi
- OneLOop A. van Hameren

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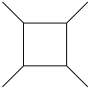
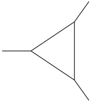
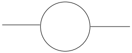
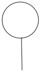
renormalisation/regularisation schemes:

- 't Hooft/Veltman
- DRED (dimensional reduction)
- CDR (conventional dimensional regularisation)
- on-shell (mass counter terms for massive quarks)

Rational Parts

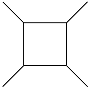
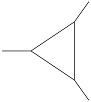
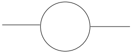
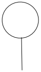
$\mathcal{A} = C_4$  $+ C_3$  $+ C_2$  $+ C_1$  $+ \mathcal{R}$

two categories: $\mathcal{R} = R_1 + R_2$ [Ossola, Papadopoulos, Pittau]

$N(q) = \hat{N}(\hat{q}) + \tilde{N}(q, \mu^2, \epsilon)\ ,\ q^2 = \left(\hat{q}^{(4)}\right)^2 - \tilde{q}^2 = \hat{q}^2 - \mu^2$

$$R_2 = \int \frac{d^D k}{(2\pi)^4} \frac{\tilde{N}(q, \mu^2, \epsilon)}{D_0 \dots D_{n-1}}$$

Rational Parts

$\mathcal{A} = C_4$  $+ C_3$  $+ C_2$  $+ C_1$  $+ \mathcal{R}$

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$$R_2 = \int \frac{d^D k}{(2\pi)^4} \frac{\tilde{N}(q, \mu^2, \epsilon)}{D_0 \ldots D_{n-1}}$$

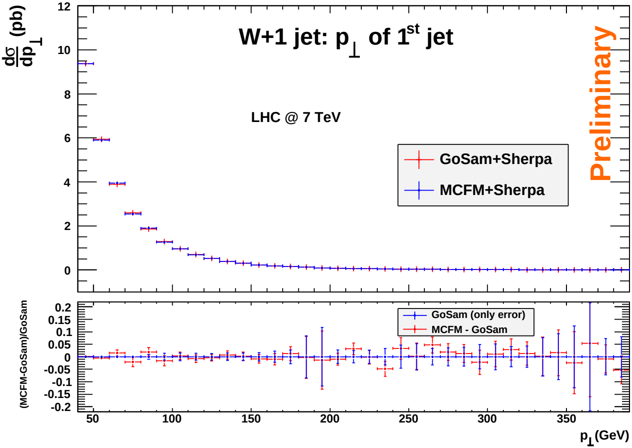
Golem-Samurai offers different options for calculation of R_2

- implicit: μ^2 terms are kept in the numerator and reduced at runtime
- explicit: μ^2 terms are reduced analytically
- only: only the R_2 term is kept in the final result

  (does not require any additional libraries)
- off: all μ^2 terms are set to zero

Interface

- standard interface to real radiation programs (Binoth Les Houches Accord) implemented
- tested with Sherpa and Powheg
- example $pp \to W$+jet [figures by G. Luisoni, J. Archibald]



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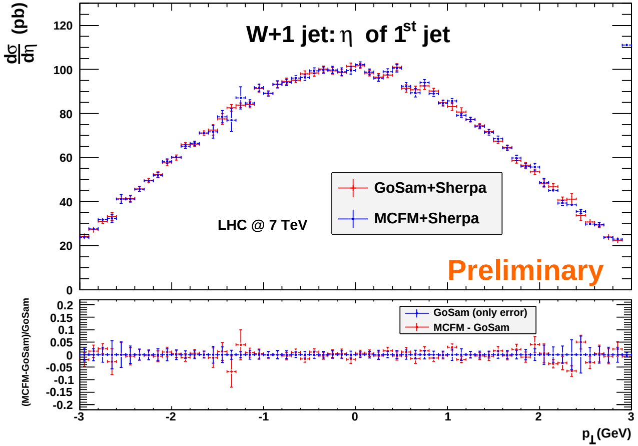


Example MSSM: $pp \to \chi_1^0 \chi_1^0$

NLO SUSY-QCD corrections

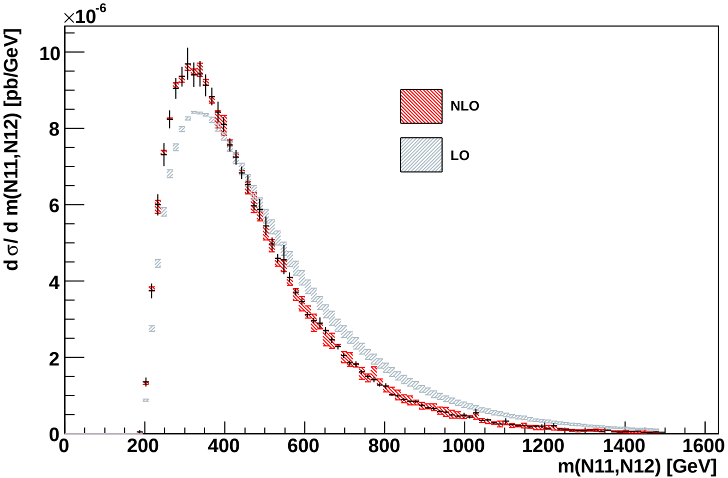
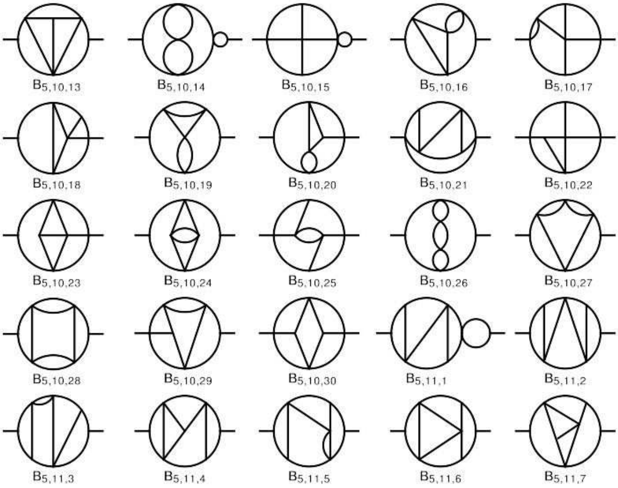


figure by G. Cullen, N. Greiner

Beyond One Loop



NNLO

- full NNLO cross sections:
  - e^+e^-: partonic event generator program EERAD3 for 3-jet observables in e^+e^- annihilation

    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]

    [S. Weinzierl '08/'09]

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    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]
    [S. Weinzierl '08/'09]
  - hadronic collisions:
    - one colour-neutral final state particle (W/Z, Higgs)
      Anastasiou, Dixon, Melnikov, Petriello; Grazzini, Catani, DeFlorian, Cieri, Ferrera
    - $t\bar{t}$, W^+W^-, $\gamma\gamma$, V+jet, dijet under construction
- different methods for double real radiation
  - antenna subtraction Gehrmann-DeRidder, Gehrmann, Glover '05
  - Dipole-like subtraction Grazzini, Catani, DeFlorian; Trocsanyi, Somogyi et al.
  - sector decomposition Binoth, GH '00, Anastasiou, Melnikov, Petriello '03
  - FKS+sector decomposition Czakon '10/'11, Boughezal, Melnikov, Petriello '11

Sector Decomposition

- allows to extract UV and IR singularities from (dimensionally regulated) parameter integrals in an automated way
- produces a Laurent series in ϵ
- coefficients are finite parameter integrals
  $\Rightarrow$ integrate numerically
- can be applied to multi-loop integrals and phase space integrals

Sector Decomposition

public programs:

- sector_decomposition (uses `Ginac`) Bogner, Weinzierl '07
- FIESTA (uses `Mathematica`) A. Smirnov, V.Smirnov,M. Tentyukov '08
- SecDec (uses `Mathematica` and Fortran/C) Jon Carter, GH '10

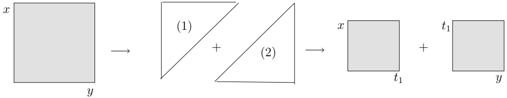
http://projects.hepforge.org/secdec

limitation until recently:
multi-scale integrals limited to Euclidean region
(e.g. no thresholds)

extension of SecDec to general kinematics under construction

method: contour integration in complex plane
S. Borowka, J. Carter, GH

basics of sector decomposition



$$I \;=\; \int_0^1 dx \int_0^1 dy\, x^{-1-\epsilon}\,(x+y)^{-1}\,[\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

subst. (1) $y = x\,z$ (2) $x = y\,z$ to remap to unit cube

$$\begin{aligned} I \;=\; & \int_0^1 dx\, x^{-1-\epsilon} \int_0^1 dz\,(1+z)^{-1} \\ & + \int_0^1 dy\, y^{-1-\epsilon} \int_0^1 dz\, z^{-1-\epsilon}\,(1+z)^{-1} \end{aligned}$$

singularities are disentangled, number of integrals doubled

SecDec

SecDec 1.0 : [J. Carter, GH]

program for the automated numerical evaluation of

- multi-loop integrals

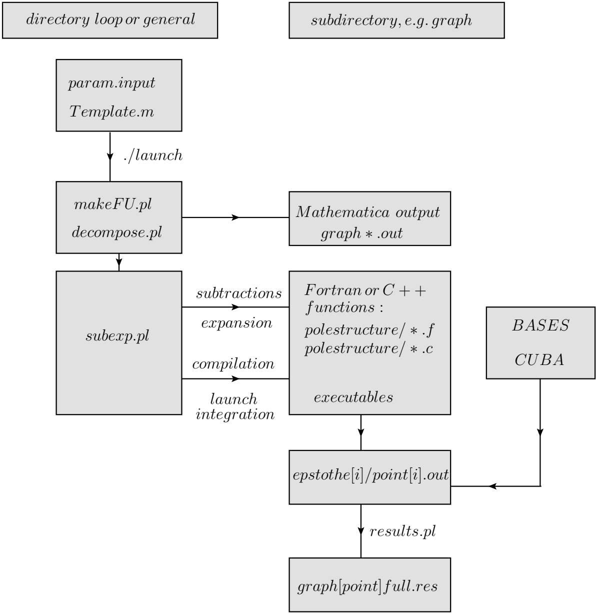
$$G = \frac{(-1)^N}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j\; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\; \frac{\mathcal{U}^{N-(L+1)D/2}}{\mathcal{F}^{N-LD/2}}$$

- general multi-dimensional parameter integrals (e.g. phase space integrals)

$$\int \mathrm{d}\Phi^{(D)}\,|\mathrm{ME}|^2 \sim \int \mathrm{d}s_{13}\,\mathrm{d}s_{23}\; s_{13}^{-1-\epsilon}\,\frac{\mathcal{F}(s_{13},s_{23})}{s_{13}+s_{23}}$$

$$\sim \int_0^1 dx\,dy\, x^{-1-\epsilon}\,\frac{\mathcal{F}(x,y)}{x+y}$$

SecDec


directory loop or general
subdirectory, e.g. graph
param.input
Template.m
./launch
makeFU.pl
decompose.pl
Mathematica output
graph * .out
subexp.pl
subtractions
expansion
compilation
launch
integration
Fortran or C++
functions :
polestructure/ * .f
polestructure/ * .c
executables
BASES
CUBA
epstothe[i]/point[i].out
results.pl
graph[point]full.res

SecDec

user input:

parameter.input:

```
############### input parameters for sector decomposition ########
#
# ############## all lines beginning with # are comments #########
#-----------------------------------------------------------------
# graphname
graph=A91
#-----------------------------------------------------------------
# number of propagators:
propagators=9
#-----------------------------------------------------------------
# number of external legs:
legs=3
#-----------------------------------------------------------------
# number of loops:
loops=3
##########################################################
# parameters for subtractions and epsilon expansion
#########################################################
# epsord: level up to which expansion in eps is desired
# (default is epsord=0: Laurent series is cut after finite part eps^
# series will be calculated from eps^(-maxpole) to eps^epsord
# note that epsord is negative if only some pole coeffs are required
#
epsord=0
#-----------------------------------------------------------------
```

SecDec

user input:

template_graph.m:

```
(* USER INPUT: *)

(* give -list of loop momenta (momlist)
          -list of propagators (proplist):
    masses m_i^2 must be called ms[i],  example k^2-m^2= k^2-ms[1]
    -numerator: list of scalar products of loop momenta contracted with
     external vectors or loop momenta;
     for scalar integrals numerator={1};
    -list of propagator powers (powerlist), default is 1:
     powers of propagators as listed in proplist          *)

(* example is a 3-loop vertex diagram, for definition and analytical result see e.g.
hep-ph/0607185 *)
(* NOTE: A61 below is with power 1+3*eps for propagator 1 *)

momlist={k,r,q};
proplist={k^2,(k+p1+p2)^2,(r-k)^2,(r+p1)^2,(k-q)^2,(q+p1)^2};

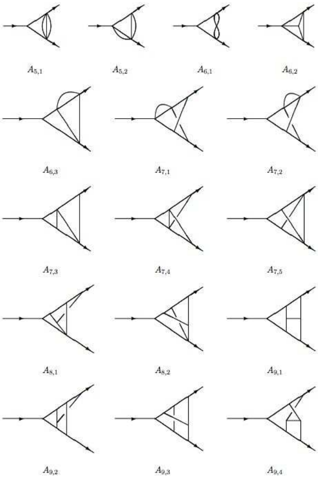
numerator={1};

powerlist={1+3*eps,1,1,1,1,1};

(* optional: give on-shell conditions *)
```

3-loop example

master integrals for 3-loop form factors

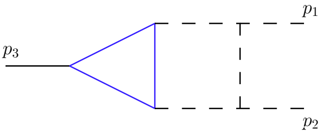


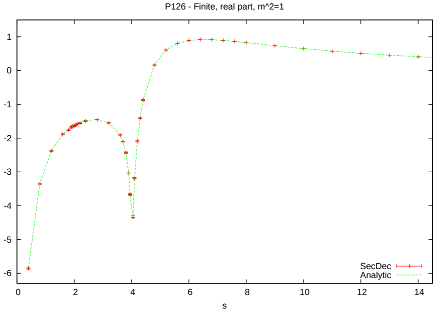
3-loop example

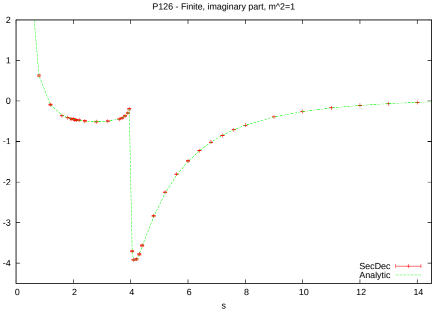
$$A_{9,*} = i\Gamma(1-\epsilon)^{-3}(-q^2 - i\eta)^{-3-3\epsilon}(C_{-6}/\epsilon^6 + C_{-5}/\epsilon^5 + C_{-4}/\epsilon^4 + C_{-3}/\epsilon^3 + C_{-2}/\epsilon^2 + C_{-1}/\epsilon + C_0)$$

$A_{9,4}$	Analytic	SecDec	Longest time(s)	Total time(s)
C_{-6}	0.111111	0.11111	8	17
C_{-5}	0.888889	0.8889	60	154
C_{-4}	-4.65541	-4.652	529	1397
C_{-3}	-33.1607	-33.14	5705	16297
C_{-2}	-42.8359	-42.83	7879	93598
C_{-1}	117.400	117.5	15375	382138
C_0	1948.17	1948	20330	1371382

2-loop example with threshold







S. Borowka, J. Carter, GH

Summary and Outlook

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- Golem/Samurai (GoSam) is a powerful and flexible automated tool for one-loop multi-leg amplitudes
- not limited to Standard Model
- numerically robust due to stability test and rescue systems

Summary and Outlook

- automation is necessary to have NLO predictions as a standard at LHC
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- not limited to Standard Model
- numerically robust due to stability test and rescue systems
- standard interface to real radiation programs (BLHA)

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- useful beyond one loop: program SecDec available at

http://projects.hepforge.org/secdec/

backup slides

Numerical stability

several detection and rescue systems

- local/global $N = N_{\rm rec}$ test: use decomposition of numerator function after coefficients have been determined:

$$
\begin{aligned}
N(\bar{q}) = & \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} D_h + \\
+ & \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} D_h + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} D_h + \\
+ & \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} D_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} D_h
\end{aligned}
$$

  and compare with original numerator for

  - local: comparison only for specific cuts
  - global: compare full numerator function at arbitrary q values

Numerical stability

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Numerical stability

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- power test: check certain combinations of coefficients which should sum to zero if reconstruction was successful
  (e.g. if power of integration momentum is higher than in original function)
- tensorial reconstruction
  rewrite numerator function as a linear combination of tensors

$$\mathcal{N}(q) = \sum_{r=0}^{R} C_{\mu_1\ldots\mu_r} q_{\mu_1} \ldots q_{\mu_r}$$

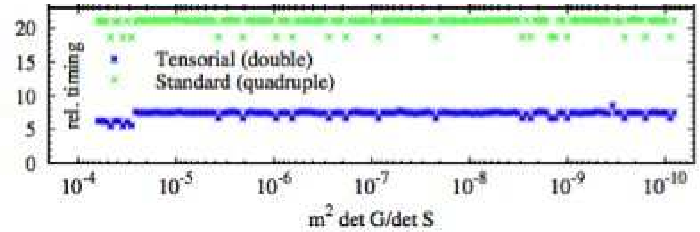
$$C_{\mu_1\ldots\mu_r} q_{\mu_1} \cdots q_{\mu_r} = \sum_{(i_1,i_2,i_3,i_4)\vdash r} \hat{C}^{(r)}_{i_1\,i_2\,i_3\,i_4} \cdot (q_1)^{i_1}(q_2)^{i_2}(q_3)^{i_3}(q_4)^{i_4}$$

determine the coefficients by sampling q

Tensorial reconstruction

advantages:

- tensor basis avoids numerical instabilities due to vanishing Gram determinants (as the latter occur in the reduction to a scalar basis)
- "rescue-system": unstable points will be reprocessed automatically using tensorial decomposition + tensor integrals from golem95



Tensorial reconstruction

further advantage:

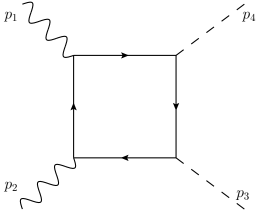
- reconstructed tensor integrand can be used as input for the "standard" reduction (more efficient, as kinematic information is already stored in the tensorial coefficients, disentangles part of integrand depending on the loop momenta from dependence on kinematic invariants)
- $\Rightarrow$ "hybrid method": even for stable phase space points, feeding the reconstructed tensor integrand to the reduction can improve the timings:

# Lines	Time ratio "hybrid"/standard	
N	Rank = 4	Rank = 6
1	1.3	1.6
10	1.1	1.4
100	0.51	0.85
1000	0.30	0.59
10000	0.27	0.55

Numerical stability

Example: massless fermion loop with two light-like and two massive legs

$$
\begin{aligned}
p_{1,2} &= (E, 0, 0, \pm E) \\
p_{3,4} &= (E, 0, \pm Q \sin\theta, \pm Q \cos\theta) \\
E &= \sqrt{M^2 + Q^2}\ ,\ p_{3,4}^2 = M^2
\end{aligned}
$$



$\det G = 32 E^4\, Q^2 \sin^2\theta$

investigate limit $Q^2 \to 0$

Numerical stability