Non-relativistic high-energy physics: Precision calculations for top and squark production at the LHC

M. Beneke (RWTH Aachen)
Vienna, 26 January 2012

MB, Smirnov, hep-ph/9711391
MB, Czakon, Falgari, Mitov, Schwinn, 0911.5166 [hep-ph]
Heavy particle pair production

\[
\sigma_{\text{had}}(\sqrt{s}, m) = \sum_{i,j} \int dx_i dx_j f_i(x_i)f_j(x_j) \hat{\sigma}_{ij}(\sqrt{\hat{s}}, m)
\]

\[
\hat{s} = x_i x_j s
\]

\[
\beta = \sqrt{1 - 4m^2/\hat{s}}
\]

top, $\bar{t}t$
superpartners, squarks and gluinos etc.

Perturbative
Top pairs

![Graph showing production cross section of top quark pairs as a function of \( \sqrt{s} \) in TeV, with ATLAS Preliminary data and theoretical predictions.]
Production of coloured sparticles in pp collisions and cascade decay to the LSP. Production cross section provides additional important constraint (Dreiner et al., 1003.2648 [hep-ph])
Fixed-order and breakdown of perturbation theory

NLO is standard (large correction) (Nason et al. 1988 for top, Beenakker et al., 1996 for SUSY) NNLO for top probably soon.

Fixed-order PT not applicable for threshold production (non-relativistic)

- Coulomb force

\[ A \sim A_0 \times \frac{g^2}{\beta} \]

- Inhibited (soft) radiation

\[ A \sim A_0 \times g^2 \ln^2 \beta \]

Perturbation theory breaks down due to the emergence of small scales \( M\beta, M\beta^2 \ll M, \sqrt{s} \).

Sum the series of enhanced quantum fluctuations to all orders:

\[
\sigma = \sigma_0 \left[ 1 + g^2 \left\{ \frac{1}{\beta}, \ln^2, \frac{1}{\beta} \right\} + g^4 \left\{ \frac{1}{\beta^2}, \frac{\ln^2, 1}{\beta^2}, \ln^4, 3, 2, 1, \frac{1}{\beta} \right\} + \ldots \right]
\]
What’s the use of the non-relativistic limit and resummation at LHC?

- Strictly valid for high masses $2\tilde{m}_H \rightarrow s_{\text{had}}$ (heavy sparticles).
  But cross sections too small.

- Certainly not for tops at LHC7. Invariant mass distribution peaks at 380 GeV, corresponding to $\beta \approx 0.4$.

- Assume that threshold expansion provides a good approximation for the integral over all $\beta$.

<table>
<thead>
<tr>
<th></th>
<th>Tevatron</th>
<th>LHC7</th>
<th>LHC14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \beta \rangle_{gg, \text{NLO}}$</td>
<td>0.41</td>
<td>0.49</td>
<td>0.53</td>
</tr>
<tr>
<td>LO</td>
<td>5.25</td>
<td>101.9</td>
<td>563.3</td>
</tr>
<tr>
<td>NLO</td>
<td>6.49</td>
<td>149.9</td>
<td>842.7</td>
</tr>
<tr>
<td>NLO$_{\text{sing}}$</td>
<td>6.76</td>
<td>138.8</td>
<td>751.2</td>
</tr>
<tr>
<td>NLO$_{\text{approx}}$</td>
<td>7.45</td>
<td>159.0</td>
<td>867.6</td>
</tr>
</tbody>
</table>

Cross section in pb, MSTW2008nnlo PDFs.
What’s special about perturbation theory for non-relativistic systems?
\[ q = (\sqrt{s}, \beta) \]

\[ p = (0, \beta) \]

\[ p_1 = \left( \frac{\sqrt{s}}{2}, \beta \right) \]

\[ p_2 = \left( \frac{\sqrt{s}}{2}, -\beta \right) \]

\[ p_i^2 = m^2 \]

\[ \frac{q^2}{4} - m^2 = \bar{p}^2 \ll m^2 \]

\[ \beta = \sqrt{1 - \frac{4m^2}{q^2}}, \quad v = \left( \frac{\sqrt{s} - 2m}{m} \right)^{1/2}, \quad 2\bar{p}/q \]

all the same (up to \( O(\beta^3) \)), \( \ll 1 \)

\[ \mathcal{I} \]

\[ \text{For given } \bar{p} \text{ calculate } k^0 \text{ integral by Cauchy integration} \]

\[ \text{Consider } |\bar{k}| \ll m \]

\[ \int \frac{d^d k}{(2\pi)^d} \frac{q^2}{\left[ k^2 + (q+2p) \cdot k + i\varepsilon \right] \left[ k^2 + (-q+2p) \cdot k + i\varepsilon \right] \left[ k^2 + i\varepsilon \right]} \]

\[ k^2 + q^2 k_0 - k_2 \pm 2\bar{p} \cdot k + i\varepsilon = 0 \]

\[ \text{Contribution from (1) in lower plane} \]

\[ \left( -\frac{i}{2} \right) q \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{k^2 \left( k^2 + 2\bar{p} \cdot k \right)} \]

\[ \left( -\frac{i}{2} \right) q \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{k^2 \left( k^2 + 2\bar{p} \cdot k \right)} \]

\[ \left( -\frac{i}{2} \right) q \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{k^2 \left( k^2 + 2\bar{p} \cdot k \right)} \]

\[ \text{const} \cdot \frac{1}{|\bar{p}|^2} \alpha \left( \frac{1}{|\bar{v}|} \right) \ll 1 \]

\[ \text{[no other such contribution from other poles, or } |\bar{k}| \sim m] \]

\[ \text{interplay of two heavy particle propagators} \]
Perturbation theory breaks down when \( v \lesssim \alpha_s \).

Non-relativistic systems are always non-perturbative – relative to the free theory usually assumed as the starting point in QFT perturbation theory.

Nevertheless weakly coupled as long as \( \alpha_s (m v^2) \ll 1 \).

\[
\sigma = \int d\mathcal{P}S
\]

gives \( E = \sqrt{s} - 2m + i\epsilon \)

\[
\sqrt{E} \to \text{Im} \left[ -\sqrt{-E} - \frac{m \alpha_s [-D_R]}{2} \left( \ln \frac{-4mE}{\mu^2} + \frac{1}{2} - \gamma_E - \psi(1 - \lambda) \right) \right]
\]

with \( \lambda = \alpha_s [-D_R] / (2\sqrt{-E/m}) \).
Modification of the production threshold ($D_R < 0$)

Coulomb force included
Modification of the production threshold ($D_R < 0$)

Aim: Systematic approach for precision calculations
Tools

- Diagrammatic threshold expansion
  [MB, Smirnov, 1997]

- Non-relativistic effective field theory
  [Caswell, Lepage, 1986; Lepage et. al, 1992; Bodwin, Braaten, Lepage, 1994; Kinoshita, Nio, 1996; Pineda, Soto, 1997; MB, Signer, Smirnov, 1999]

- Multi-loop technology

- “Non-relativistic” perturbation theory – perturbation theory with a non-trivial unperturbated Lagrangian

Basic methods developed in the late 1990s.
Now:

- more precision
- more applications (hadronic, SUSY, DM)
- non-relativistic + soft gluon combined
  [MB, Falgari, Schwinn, 2009ff]
Methods:
Threshold Expansion
Technical statement of the problem

\[ \sum_n r_n(v) \alpha_s^n = \sum_n f_n(\frac{\alpha_s}{v}) \alpha_s^n \]

- Relativistic PT, \( v \approx 1 \)
- Non-relativistic PT, \( \frac{\alpha_s}{v} \approx 1 \)

For non-relativistic systems:
- No need for computation of \( r_n(v) \)!
- Only need first few terms of expansion in \( \alpha_s \).
- But need all orders in \( \alpha_s \)!

Want a method to compute expansion of \( r_n(v) \) directly
- i.e. without computing the full expression.
Threshold expansion

\[ I = \int \prod_i \frac{d^4 k_i}{(2\pi)^d} f(k_i; q, p) \]

- Taylor expansion of integrand \( f \) w.r.t. \( p \) assumes \( k_i \sim q \) hard region
- Wrong if \( k_i \) contours are trapped between poles for small \( |k_i| \) → additional regions

potential \( k_0, m^2 \)

\( \not\kern-1mu k \sim m v \)

soft \( k_0, \not\kern-1mu k \sim m v \)

ultrasoft \( k_0, \not\kern-1mu k \sim m^2 \)

since \( p \sim m v \)

\( q^0 \sim 2m \sim m^2 \)

Imagine dividing \( d^4 k \) into regions by cut-offs
In each region expand in small quantities, including loop momenta
In dim. reg. ignore cut-offs and integrate over all \( d^4 k \) in every region

\[ I = \sum \text{regions} \]

Each term is homogeneous w.r.t. \( m \), i.e. contributes to a single order in the \( \alpha_s \) expansion
Expansion rules in every region

\[ \frac{1}{(k + q + p)^2 - m^2} = \frac{1}{k_0^2 - k - q_0 k^0 - 2p \cdot k} = \begin{cases} \frac{1}{k^2 + q \cdot k} & \text{hard} \\ \frac{1}{2m} \frac{1}{k_0} & \text{soft} \\ \frac{1}{2m} \frac{1}{k_0 - k + 2p \cdot k} & \text{potential} \end{cases} \]

heavy-particle propagator

Example

\[ \begin{array}{c}
\begin{aligned}
\text{potential} & \text{on-shell } q \bar{q} \text{ propagation} \\
\text{hard} & \text{polynomial in } p \Rightarrow \text{ local vertex}
\end{aligned}
\end{array} \]

\[ = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{\pi m}{\sqrt{-4p^2}} \left( -\frac{1}{2} \left( \frac{1}{\varepsilon} - \ln \frac{-p^2}{m^2} \right) \right) - 8 + O\left( \frac{1}{p^2} \right) \right\} \]
At two loops all regions contribute

\[ \text{local operator insertions} \]

\[ \text{instantaneous operator insertion ("potential")} \]
Recursive structure with effective vertices suggests effective Lagrangian formulation

\[ P \]
\[ S, p, u, s \]
\[ \text{Expand in } P^0 P^\pm \]
\[ \to \text{local} \]

\[ P \]
\[ \text{Expand only in } P^0 m v^2, \to \]
\[ \text{not } \bar{P} = m v \sim k \]
\[ \to \text{non-local in space local in time} \]

Only the \( P-P-\ldots-P \) region contributes to the leading \((\frac{4\pi}{\alpha})^n\) and needs to be summed. Only \textit{Eckler diagrams} can have an all-\( P \) region.

don't have two pairs of heavy particle propagators

poles not \textit{finished}
Diagrammatic resummation

\[ H^{(R)}(p, p'; E) = \sum_{n=1}^{\infty} (-D_R)^{n+1} \int \left[ \prod_{i=1}^{n} \frac{d^d k_i}{(2\pi)^d} \right] \frac{(ig_s)^2 i}{k_1^2} \frac{(ig_s)^2 i}{(k_2 - k_1)^2} \cdots \frac{(ig_s)^2 i}{(k_{n+1} - k_n)^2} \]

\[ \cdot \prod_{i=1}^{n} \frac{E + k_i^0 - \frac{(p+k_i)^2}{2m} + i\epsilon}{E + k_i^0 - \frac{(p+k_i)^2}{2m} + i\epsilon} \]

\[ G^{(R)}(p, p'; E) = -\frac{(2\pi)^{d-1} \delta^{(d-1)}(p' - p)}{E + i\epsilon - \frac{p^2}{m}} + \frac{1}{E + i\epsilon - \frac{p'^2}{m}} iH(p, p'; E) \frac{1}{E + i\epsilon - \frac{p'^2}{m}} \]

\[ \left( \frac{p^2}{m} - E \right) G^{(R)}(p, p'; E) + \mu^{2\epsilon} \int \frac{d^{d-1} k}{(2\pi)^{d-1}} \frac{4\pi D_R \alpha_s}{k^2} G^{(R)}(p - k, p'; E) = (2\pi)^{d-1} \delta^{(d-1)}(p - p') \]

\[ \left( -\frac{\nabla^2}{m} + \frac{D_R \alpha_s}{m} - E \right) G^{(R)}(r, r'; E) = \delta^{(d-1)}(r - r') \]
Coulomb Green function [Schwinger, 1964; Voloshin, 1984]

\[ G(p, p'; E) = -\frac{(2\pi)^3 \delta^{(3)}(p' - p)}{E - \frac{p^2}{m}} + \frac{1}{E - \frac{p^2}{m}} \frac{g_s^2 (-D_R)}{(p - p')^2} \frac{1}{E - \frac{p'^2}{m}} \]

\[ + \frac{1}{E - \frac{p^2}{m}} \int_0^1 dt \frac{g_s^2 (-D_R) \lambda t^{-\lambda}}{(p - p')^2 t - \frac{m}{4E} (E - \frac{p^2}{m})(E - \frac{p'^2}{m})(1 - t)^2} \frac{1}{E - \frac{p'^2}{m}} \]

\[ G(r, r'; E) = \sum_{l=0}^{\infty} (2l + 1) P_l \left( \frac{r \cdot r'}{rr'} \right) G^{(l)}(r, r'; E) \]

\[ G^{(l)}(r, r'; E) = \frac{mp}{2\pi} (2pr)^l (2pr')^l e^{-p(r + r')} \sum_{s=0}^{\infty} \frac{s!}{(s + 2l + 1)! (s + l + 1 - \lambda)} L_s^{(2l+1)}(2pr) L_s^{(2l+1)}(2pr') \]

\[ [p = \sqrt{-mE}] \]

Explicit expression known only in \( d = 4 \). Not even \( \mathcal{O}(\epsilon) \) terms.
BUT: Every term in the expansion in \( \alpha_s \) can be computed in \( d \) dimensions.
Enough since \( 1/\epsilon \) poles disappear beyond a certain order in \( \alpha_s \).
Non-relativistic perturbation theory

In every term replace \( G_o = \frac{2p}{2p} \) by \( G^{(R)} = \sum_{n=0}^{\infty} \).

Example

\[
\begin{align*}
\text{\begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture}} & \Rightarrow \begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture} = \begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture} \quad \text{contributes to } f_0(\alpha_s V)
\end{align*}
\]

\[
\begin{align*}
\text{\begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture}} & \Rightarrow \begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture} = \begin{tikzpicture}
\draw[green, thick] (0,0) -- (1,1) -- (2,0) -- cycle;
\end{tikzpicture} \quad \text{contributes to } f_1(\alpha_s V) \cdot \alpha_s
\end{align*}
\]

\( V = \alpha_s V \times \alpha_s \)

In this project for \( e^+ e^- \) up to \( f_3 \) \( [ \quad \quad \quad ] \)

for hadronic production \( f_1 + \text{logarithmic terms ln} V \) in \( f_2 \)
Methods:
Non-Relativistic Effective Theory
or any other "full theory"

Integrate out hard modes $k^0 \sim k \sim m$

i.e. hard subgraphs $\rightarrow$ effective, point-like vertices

local - defined with cut-off, in dim. reg only through threshold expansion

non-relativistic fields

$\phi \sim \sum \frac{3 \bar{q}^2}{(2m^2 p^2) u(p,s) a(p,s)}$

Integrate out soft modes $k^0 \sim k_\sim mv$

i.e. soft subgraphs $\rightarrow$ effective instantaneous, non-local vertices ("potentials")

non-local in $\phi$

contains potential heavy quark field and ultrasoft gluons (light quarks)

simple $n$-counting

starting point for resummation

\[
\mathcal{L}_{\text{NRQCD}} = \psi^\dagger \left( iD^0 + \frac{D^2}{2m} \right) \psi + \frac{1}{8m^3} \psi^\dagger D^4 \psi - \frac{d_1 g_s}{2m} \psi^\dagger \sigma \cdot B \psi \\
+ \frac{d_2 g_s}{8m^2} \psi^\dagger (D \cdot E - E \cdot D) \psi + \frac{d_3 i g_s}{8m^2} \psi^\dagger \sigma \cdot (D \times E - E \times D) \psi \\
+ \sum_c \frac{d_{4c} g_s^2}{8m^2} \psi^\dagger \left[ \kappa_c \right] \psi \sum_f \bar{q}_f \left[ \kappa'_c \right] q_f + \text{antiquark terms } \psi \rightarrow \chi \\
+ \sum_c \frac{d_{5c} g_s^2}{8m^2} \psi^\dagger \left[ \kappa_c \right] \psi \psi^\dagger \left[ \kappa_c \right] \chi + \mathcal{L}_{\text{light}}
\]

The \( \gamma^* Q\bar{Q} \) coupling:

\[
\bar{Q} \gamma^i Q = c_1 \psi^\dagger \sigma^i \chi - \frac{c_2}{6m^2} \psi^\dagger \sigma^i (D)^2 \chi + \ldots
\]

Hadronic \( Q\bar{Q} \) production:

\[
A(ij \rightarrow Q\bar{Q})_{\text{hard}} \rightarrow C_{\{a\alpha\}} \phi_{c;\alpha_1 a_1} \phi_{\bar{c};\alpha_2 a_2} \psi^\dagger_{\alpha_3 a_3} \chi_{\alpha_4 a_4} + \ldots
\]
Integrating out the soft region: PNRQCD

\[
\mathcal{L}_{\text{PNRQCD}} = \Psi^+(i\partial^0 + gA^0(t,\vec{r}) + \frac{\vec{d}^2}{2m})\Psi + \text{anti-quark term} + \int d^3r \left[ \Psi^\dagger(t,\vec{r}) \left( \frac{dS_R}{\tau} \right) \right] \left[ X^+X \right](x)
\]

\[
= V^{3/2} V^2 V^{3/2} V^5 \frac{1}{V^3} V^3 \frac{1}{dS_V} \sim V^5
\]

\[
+ \frac{1}{8m^3} \Psi^+ \bar{\psi} \psi + \text{anti-quark term} + \int d^3r \left[ \Psi^\dagger(t,\vec{r}) \right] \text{SV} \left[ X^+X \right](x) = g_s \Psi^+ \bar{X} \bar{E}(t,\vec{r}) \Psi + \text{anti-quark term}
\]

\[
\text{multipole expansion}
\]

- Leading Coulomb potential part of \(\mathcal{L}_{\text{PNRQCD}}^{(0)} \rightarrow \text{Coulomb propagator}

\[
\left[ \Psi^\dagger(t,\vec{r}) \right] \sim \int d^4p \frac{1}{p^0 - \vec{p}^2_{2m}} \sim \frac{1}{V^2} \frac{1}{V^2}
\]

- Multipole expansion of ultrasoft fields

\[
A(t,\vec{r}) = A(t,\vec{0}) + \frac{\vec{r}}{4V^2} + \frac{\vec{r}}{V} \rightarrow \frac{\vec{r}}{V^2} \rightarrow \vec{r} \cdot \bar{E}
\]
Hadronic production
• Coloured initial state → threshold enhancement from soft-gluon radiation

\[ A \approx A_0 \times g^2 \int_{M\lambda}^{M\beta^2} \frac{d\omega}{\omega} \int_{\lambda}^{\beta^2} \frac{d\theta}{\theta} \sim A_0 \times g^2 \ln^2 \beta \]

Soft-gluon resummations in Mellin moment space (Sterman 1987; Catani, Trentadue, 1989)
or soft-collinear effective theory to all orders

• For pair production: Factorization of soft, collinear and Coulomb gluons is non-trivial [but well-known for soft-collinear], since soft gluons attach to and between Coulomb ladders and collinear radiation.

Kinematics: \([k_-]_c \sim [k^0]_p \sim k^\mu_s\)

Also Coulomb exchange carries colour structure \(T^A_R \otimes T^A_{R'}\).
Soft-collinear-potential decoupling

\[ \mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left( i\partial^0 + gsA_s^0(x_0, \vec{0}) + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left( i\partial^0 + gsA_s^0(x_0, \vec{0}) + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' \]

\[ + \int d^3\vec{r} \left[ \psi^\dagger T^{(R)a}_\psi \psi \right] (\vec{r}) \left( \frac{\alpha_s}{r} \right) \left[ \psi'^\dagger T^{(R')a}_\psi \psi' \right] (0) \]

Field redefinition:

\[ \psi(x) = S^{(R)}_w(x_0) \psi^{(0)}(x) \quad \text{with} \quad p^\mu = mw^\mu \]

\[ S^{(R)}_w(x) = \text{P exp} \left[ -ig_s \int_0^\infty dt \, w \cdot A_s^c(x + wt) T^{(R)c} \right] \]

\[ S^{(R)}_w^\dagger iD_s^0 S^{(R)}_w = i\partial^0 \]
Soft-collinear-potential decoupling

\( \mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left( i\partial^0 + \frac{\bar{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left( i\partial^0 + \frac{\bar{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' \)

\[ + \int d^3 \vec{r} \left[ \psi^\dagger S_w^{(R)\dagger} T^{(R)a} S_w^{(R)} \right] (\vec{r}) \left( \frac{\alpha_s}{r} \right) \left[ \psi'^\dagger S_w^{(R')\dagger} T^{(R')a} S_w^{(R')} \right] \psi' \]

In any representation \( S_w^{(R)\dagger} T^{(R)a} S_w^{(R)} = S_{w,ab}^{(ad)} T^{(R)b} \), hence

\[ S_w^{(R)\dagger} T^{(R)a} S_w^{(R)} \otimes S_w^{(R')\dagger} T^{(R')a} S_w^{(R')} = S_{w,ab}^{(ad)} S_{w,ac}^{(ad)} T^{(R)b} \otimes T^{(R')c} = T^{(R)a} \otimes T^{(R')a} \]

\[ \delta^{bc} \]

Soft interactions disappear from the Lagrangian and survive only in the hard production vertex operator

\[ O^{(\ell)}_{\{a;\alpha\}}(\mu) = \left[ \phi_{c;a_1} \phi_{\bar{c};a_2} \psi_{a_3}^\dagger \psi_{a_4}'^\dagger \right](\mu) \]
Factorisation and resummation

\[
\hat{\sigma}(\beta, \mu) = \sum_i H_i(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2}) W^{R_\alpha}_i(\omega, \mu).
\]

- \(H_i(M, \mu)\) – Short-distance production of heavy particle pair
  Sums \((g^2 \ln^2 \beta)^n\) by renormalization group evolution in \(\mu\) from \(M\) to \(M/\beta^2\).

- \(W^{R_\alpha}_i(\omega, \mu)\) – Soft function for the production of a single particle in irrep \(R_\alpha\)

\[
\frac{d}{d \ln \mu} \hat{W}^{R_\alpha}_i(L) = \left( (\Gamma^r_{\text{cusp}} + \Gamma^r_{\text{cusp}}) L - 2 \gamma_{W,i}^{R_\alpha} \right) \hat{W}^{R_\alpha}_i(L)
\]

NNLL soft-gluon resummation needs the 3-loop cusp anomalous dimension and 2-loop soft anomalous dimension

- \(J_{R_\alpha}\) – Coulomb Green function = related to a correlation function of non-relativistic fields in PNRQCD.
  Sums Coulomb-exchange \((g^2/\beta)^n\) to all orders for \(HH'\) in irrep \(R_\alpha\).
Results (top, squarks)
Invariant $\bar{t}t$ mass distributions near threshold (so far NLL)

(Kiyo, Kühn, Moch, Steinhauser, Uwer; 2008)

Also for squark/gluino pair production [Kauth et al., 2010; Hagiwara, Yokoya, 2009]
Total $t\bar{t}$ cross section (NNLL) [MB, Falgari, Klein, Schwinn, 2011]

Table 9: Total cross sections in pb at the Tevatron for $m_t = 165 \ldots 180$ GeV. The errors denote the scale variation (scale variation+resummation ambiguities for NNLL$_2$), the NNLO constant variation (for NNLO$_{app}$ and NNLL$_2$) and the PDF+$\alpha_s$ error.

<table>
<thead>
<tr>
<th>$m_t$ [GeV]</th>
<th>NLO</th>
<th>NNLO$_{app}$</th>
<th>NNLL$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>$7.18^{+0.30+0.54}_{-0.31-0.49}$</td>
<td>$7.58^{+0.27+0.75}_{-0.36-0.57}$</td>
<td>$7.76^{+0.31+0.77}_{-0.49-0.59}$</td>
</tr>
<tr>
<td>172</td>
<td>$6.96^{+0.38+0.53}_{-0.78-0.47}$</td>
<td>$7.35^{+0.26+0.72}_{-0.35-0.55}$</td>
<td>$7.52^{+0.30+0.75}_{-0.48-0.57}$</td>
</tr>
<tr>
<td>173</td>
<td>$6.74^{+0.37+0.51}_{-0.76-0.46}$</td>
<td>$7.12^{+0.25+0.70}_{-0.34-0.53}$</td>
<td>$7.29^{+0.29+0.72}_{-0.46-0.55}$</td>
</tr>
<tr>
<td>174</td>
<td>$6.54^{+0.36+0.50}_{-0.74-0.44}$</td>
<td>$6.91^{+0.24+0.67}_{-0.62-0.51}$</td>
<td>$7.07^{+0.28+0.70}_{-0.45-0.53}$</td>
</tr>
</tbody>
</table>

D0: $7.56^{+0.63}_{-0.56}$ pb
CDF: $7.50^{+0.48}_{-0.48}$ pb

Table 10: Total cross section at the LHC ($\sqrt{s} = 7$ TeV) for $m_t = 165 \ldots 180$. The errors denote the theory error, the NNLO constant variation (for NNLO$_{approx}$ and NNLL$_2$) and the PDF+$\alpha_s$ error.

<table>
<thead>
<tr>
<th>$m_t$ [GeV]</th>
<th>NLO</th>
<th>NNLO$_{approx}$</th>
<th>NNLL$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>$169.5^{+21.0+14.9}_{-22.7-14.0}$</td>
<td>$172.6^{+12.3+5.0+16.2}_{-11.8-5.0-15.6}$</td>
<td>$174.2^{+6.1+5.0+16.5}_{-6.3-5.0-15.8}$</td>
</tr>
<tr>
<td>172</td>
<td>$164.4^{+20.3+14.5}_{-22.0-13.6}$</td>
<td>$167.5^{+11.9+4.9+15.8}_{-11.4-4.9-15.1}$</td>
<td>$169.0^{+6.0+4.9+16.0}_{-6.1-4.9-15.3}$</td>
</tr>
<tr>
<td>173</td>
<td>$159.6^{+19.7+14.0}_{-21.4-13.3}$</td>
<td>$162.5^{+11.5+4.7+15.4}_{-11.0-4.7-14.7}$</td>
<td>$164.0^{+5.8+4.7+15.6}_{-6.0-4.7-14.9}$</td>
</tr>
<tr>
<td>174</td>
<td>$154.8^{+19.1+13.6}_{-20.7-12.9}$</td>
<td>$157.7^{+11.1+4.5+14.9}_{-10.7-4.5-14.3}$</td>
<td>$159.2^{+5.6+4.5+15.1}_{-5.8-4.5-14.4}$</td>
</tr>
</tbody>
</table>

ATLAS: $179.0 \pm 11.8$ pb ($0.7$fb$^{-1}$)
CMS: $165.8 \pm 2.2$(stat.$) \pm 10.6$(syst.$) \pm 7.8$(lumi.$)$ pb ($0.8-1.1$fb$^{-1}$)

At LHC $\approx 3\%$ beyond NLO, reduction of theoretical uncertainty from 13\% to 5\%
Total $\bar{t}t$ cross section (NNLL)

From total cross section (ATLAS data)

$$m_t = (169.8^{+4.9}_{-4.7}) \text{ GeV}$$
Squark-antisquark production

Born production processes

\[ q_i(k_1)\bar{q}_j(k_2) \rightarrow \tilde{q}_{\sigma_1}(p_1)\tilde{q}_{\sigma_2}(p_2) \]

\[ g(k_1)g(k_2) \rightarrow \tilde{q}_{\sigma_1}(p_1)\tilde{q}_{\sigma_2}(p_2) , \]

Must be separated into colour-singlet and colour-octet production.
NLL resummation of $pp \rightarrow$ squark+antisquark + $X$ at $\sqrt{s} = 14$ TeV

NLL = Tree $C$ and $W$, 1-loop anomalous dim. + LO Coulomb Green function + matching to NLO fixed order (from Beenakker et al, 1997; Prospino code)

Size of corrections beyond NLO

Resummation is $\approx 10\%$ (mass-dependent) effect at the natural scale.
Scale dependence ($\sim$ theoretical uncertainty) at LO, NLO, NLL

(pp collisions at $\sqrt{s} = 14$ TeV; $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$; $m_{\tilde{q}} = 1$ TeV [left] and $p\bar{p}$ collisions, $m_{\tilde{q}} = 400$ GeV [right]

Green band: variation of all factorization scales.)

Theoretical uncertainty is reduced.
Conclusion

1) Joint resummation possible due to factorization of soft and Coulomb gluon effects (threshold expansion + SCET $\times$ NRQCD)

2) Top pair cross section at threshold known at $\mathcal{O}(\alpha_s^2)$ at threshold up to the constant term + NNLL to all orders.

   For any coloured heavy particles, given the one-loop hard matching coefficients.

3) Ready for LHC data and discoveries