Non-relativistic high-energy physics: Precision calculations for top and squark production at the LHC

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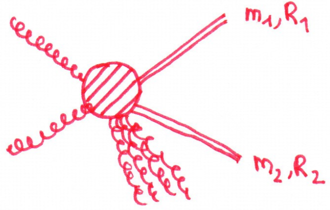
MB, Smirnov, hep-ph/9711391
MB, Kiyo, Schuller, hep-ph/0501289, arXiv:0801.3464 [hep-ph]
MB, Czakon, Falgari, Mitov, Schwinn, 0911.5166 [hep-ph]
MB, Falgari, Schwinn, 0907.1443 [hep-ph], 1007.5414 [hep-ph]; MB, Falgari, Klein, Schwinn, arXiv:1109.1536 [hep-ph]

Heavy particle pair production



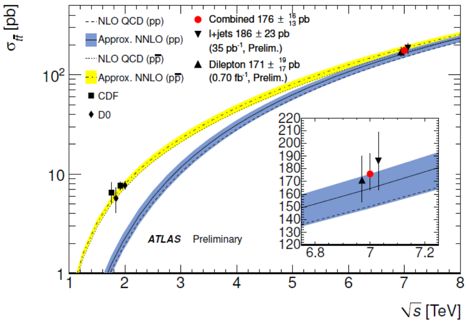
top, $t\bar{t}$
superpartners, squarks and gluinos
etc.

Perturbative

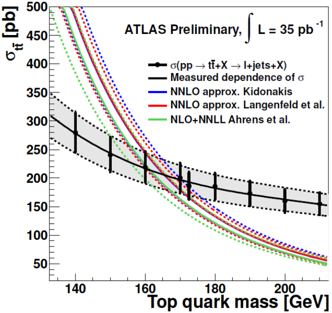
$$\sigma_{\rm had}(\sqrt{s}, m) = \sum_{i,j} \int dx_i dx_j\, f_i(x_i) f_j(x_j)\, \hat{\sigma}_{ij}(\sqrt{\hat{s}}, m)$$

$\hat{s} = x_i x_j s$
$\beta = \sqrt{1 - 4m^2/\hat{s}}$

Top pairs

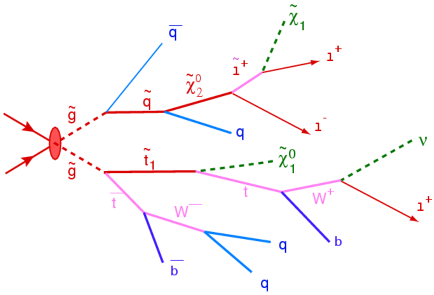


NLO QCD (pp)
Approx. NNLO (pp)
NLO QCD (p$\bar{p}$)
Approx. NNLO (p$\bar{p}$)
CDF
D0
Combined 176 $\pm^{16}_{13}$ pb
l+jets 186 ± 23 pb (35 pb^{-1}, Prelim.)
Dilepton 171 $\pm^{19}_{17}$ pb (0.70 fb^{-1}, Prelim.)
ATLAS Preliminary
$\sigma_{t\bar{t}}$ [pb]
$\sqrt{s}$ [TeV]



ATLAS Preliminary, $\int L = 35$ pb^{-1}
σ(pp → t$\bar{t}$+X → l+jets+X)
Measured dependence of σ
NNLO approx. Kidonakis
NNLO approx. Langenfeld et al.
NLO+NNLL Ahrens et al.
$\sigma_{t\bar{t}}$ [pb]
Top quark mass [GeV]

Sparticle pair production/SUSY parameter determination



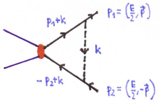
Production of coloured sparticles in pp collisions and cascade decay to the LSP.
Production cross section provides additional important constraint
(Dreiner et al., 1003.2648 [hep-ph])

Fixed-order and breakdown of perturbation theory

NLO is standard (large correction) (Nason et al. 1988 for top, Beenakker et al., 1996 for SUSY)
NNLO for top probably soon.

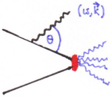
Fixed-order PT not applicable for threshold production (non-relativistic)

- Coulomb force



$$A \sim A_0 \times \frac{g^2}{\beta}$$

- Inhibited (soft) radiation



$$A \sim A_0 \times g^2 \ln^2 \beta$$

Perturbation theory breaks down due to the emergence of small scales $M\beta, M\beta^2 \ll M, \sqrt{\hat{s}}$.
Sum the series of enhanced quantum fluctuations to all orders:

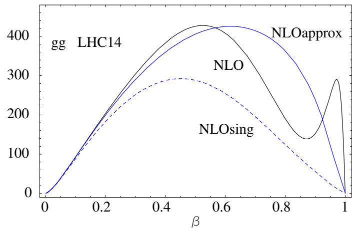
$$\sigma = \sigma_0 \left[1 + g^2 \left\{ \frac{1}{\beta}, \ln^{2,1} \beta \right\} + g^4 \left\{ \frac{1}{\beta^2}, \frac{\ln^{2,1} \beta}{\beta}, \ln^{4,3,2,1} \beta \right\} + \ldots \right]$$

What's the use of the non-relativistic limit and resummation at LHC?

- Strictly valid for high masses $2\bar{m}_H \longrightarrow s_{\rm had}$ (heavy sparticles).
  But cross sections too small.

- Certainly not for tops at LHC7. Invariant mass distribution peaks at 380 GeV, corresponding to $\beta \approx 0.4$.

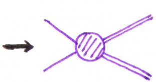
- Assume that threshold expansion provides a good approximation for the integral over all β.



	Tevatron	LHC7	LHC14
$\langle\beta\rangle_{gg,\rm NLO}$	0.41	0.49	0.53
LO	5.25	101.9	563.3
NLO	6.49	149.9	842.7
$\rm NLO_{sing}$	6.76	138.8	751.2
$\rm NLO_{approx}$	7.45	159.0	867.6

Cross section in pb, MSTW2008nnlo PDFs.

What's special about perturbation theory for non-relativistic systems?



$p_1 = \left(\frac{\sqrt{s}}{2}, \vec{p}\right)$

$p_2 = \left(\frac{\sqrt{s}}{2}, -\vec{p}\right)$

$q = (\sqrt{s}, \vec{0})$

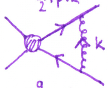
$p = (0, \vec{p})$

$p_i^2 = m^2$

$$\frac{q^2}{4} - m^2 = \vec{p}^{\,2} \ll m^2$$

$$\beta = \sqrt{1 - 4m^2/q^2}\,, \qquad v = \left(\frac{\sqrt{s}-2m}{m}\right)^{1/2}, \qquad 2|\vec{p}|/q$$

all the same (up to $O(\beta^2)$), $\ll 1$



$$\int \frac{d^dk}{(2\pi)^d} \frac{q^2}{[k^2+(q+2p)\cdot k+i\varepsilon][k^2+(-q+2p)\cdot k+i\varepsilon][k^2+i\varepsilon]}$$

(1) (2) (3)

$k^{0\,2} + q^0 k^0 - \vec{k}^{\,2} - 2\vec{p}\cdot\vec{k} + i\varepsilon = 0$

For given $\vec{k}$ calculate k^0 integral by Cauchy integration

Consider $|\vec{k}| \ll m$

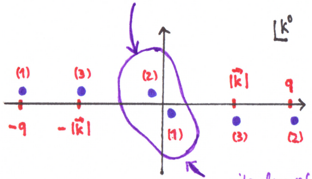
Contribution from (1) in lower plane

$$\frac{(-i)q}{2} \int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}} \frac{1}{\vec{k}^{\,2}(\vec{k}^{\,2}+2\vec{p}\cdot\vec{k})}$$

$$= \text{const} \times q/|\vec{p}| \propto 1/v \gg 1$$

[no other such contribution from other poles, or $\vec{k} \sim m$]

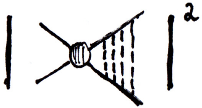
pinch at $k^0 = \pm\frac{\vec{k}^{\,2}+2\vec{p}\cdot\vec{k}}{q} \ll |\vec{k}|$



interplay of two heavy particle propagators

- Perturbation theory breaks down when $v \lesssim \alpha_s$.
- Non-relativistic systems are always non-perturbative – relative to the free theory usually assumed as the starting point in QFT perturbation theory.
- Nevertheless weakly coupled as long as $\alpha_s(mv^2) \ll 1$.

$$\sigma = \int \mathrm{dPS}$$

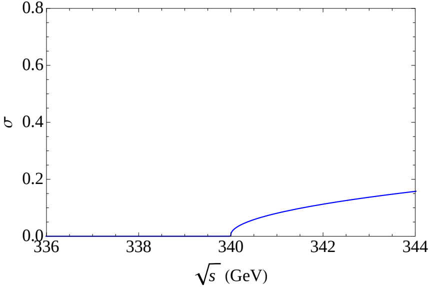


gives ($E = \sqrt{s} - 2m + i\epsilon$)

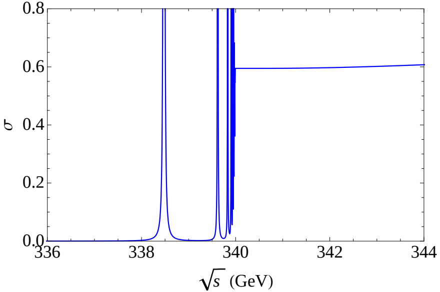
$$\sqrt{E} \to \mathrm{Im}\left[-\sqrt{-E} - \frac{m\alpha_s[-D_R]}{2}\left(\ln\frac{-4mE}{\mu^2} + \frac{1}{2} - \gamma_E - \psi(1-\lambda)\right)\right]$$

with $\lambda = \alpha_s[-D_R]/(2\sqrt{-E/m})$.

Modification of the production threshold ($D_R < 0$)



Coulomb force included



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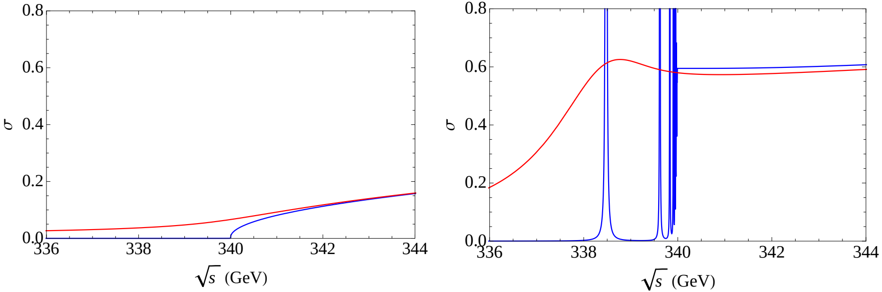


Aim: Systematic approach for precision calculations

Tools

- Diagrammatic threshold expansion
  [MB, Smirnov, 1997]

- Non-relativistic effective field theory
  [Caswell, Lepage, 1986; Lepage et. al, 1992, Bodwin, Braaten, Lepage, 1994; Kinoshita, Nio, 1996; Pineda, Soto, 1997; MB, Signer, Smirnov, 1999]

- Multi-loop technology

- "Non-relativistic" perturbation theory – perturbation theory with a non-trivial unperturbated Lagrangian

Basic methods developed in the late 1990s.
Now:

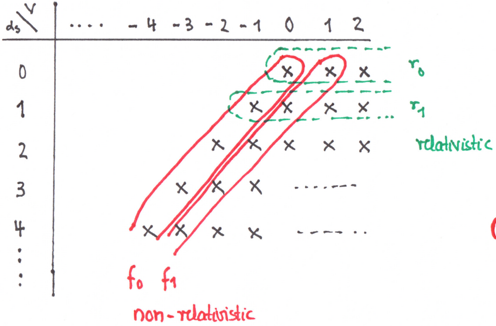
- more precision
- more applications (hadronic, SUSY, DM)
- non-relativistic + soft gluon combined
  [MB, Falgari, Schwinn, 2009ff]

Methods:
Threshold Expansion

Technical statement of the problem

$$= \sum_n r_n(v)\, \alpha_s^n = \sum_n f_n\left(\alpha_s/v\right) \alpha_s^n$$

relativistic PT, $v \sim 1$ | non-relativistic PT, $\alpha_s/v \sim 1$



For non-relativistic systems:

- No need for computation of $r_n(v)$! Only need first few terms of expansion in v
- But need all orders in α_s!

↪ Want a method to compute expansion of $r_n(v)$ directly
– i.e. without computing the full expression

Threshold expansion

$$I = \int \prod_i \frac{d^d k_i}{(2\pi)^d} f(k_i; q; p)$$

(q: large, p: small)

- Taylor expansion of <u>integrand</u> f in p assumes $k_i \sim q$ — hard region
- Wrong if k_i^0 contours are trapped between poles for small $|\vec{k}|$ $\longrightarrow$ additional regions

potential	$k^0 \sim mv^2$, $\vec{k} \sim mv$	heavy particle poles
soft	$k^0 \sim \vec{k} \sim mv$	massless poles
ultrasoft	$k^0 \sim \vec{k} \sim mv^2$	massless poles

since $\vec{p} \sim mv$
$q^0 - 2m \sim mv^2$

Imagine dividing $d^d k$ into regions by cut-offs
In each region expand in small quantities, including loop momenta
In dim. reg. ignore cut-offs and integrate over all $d^d k$ in every region

$$I = \sum_{\text{regions}} I_{\text{region}}$$

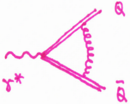
↑ Each term is homogeneous in v, i.e. contributes to a single order in the v-expansion

Expansion rules in every region

$$\frac{1}{\left(k+\frac{q}{2}+p\right)^2-m^2} = \frac{1}{k_0^2-\vec{k}^2+q^0k^0-2\vec{p}\cdot\vec{k}} = \begin{cases} \dfrac{1}{k^2+q\cdot k}+\ldots & \text{hard} \quad \text{off-shell} \\[2ex] \dfrac{1}{2m}\,\dfrac{1}{k^0}+\ldots & \text{soft} \quad \text{static } [\,\delta^{(3)}(\vec{x})\,] \\[2ex] \dfrac{1}{2m}\,\dfrac{1}{k^0-\frac{\vec{k}^2+2\vec{p}\cdot\vec{k}}{2m}}+\ldots & \text{potential} \quad \text{non-relativistic} \end{cases}$$

heavy-particle propagator

Example



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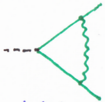
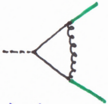


potential
on-shell $Q\bar{Q}$ propagation

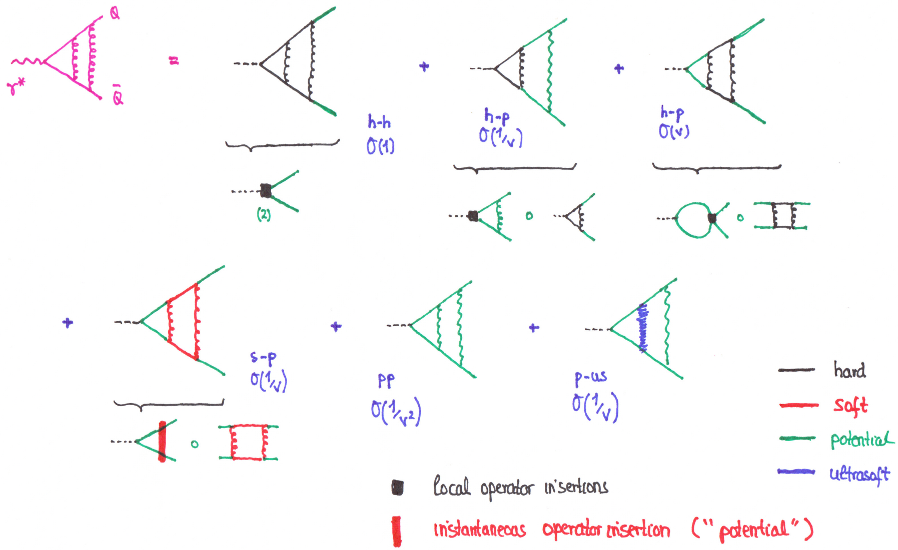
+



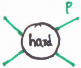
hard
polynomial in $\vec{p}$ ⇒ local vertex

$$= \frac{\alpha_s C_F}{4\pi}\left\{ \frac{\pi m}{\sqrt{-4\vec{p}^{\,2}}}\left(-\frac{1}{2}\right)\left(\frac{1}{\epsilon}-\ln\frac{-4\vec{p}^{\,2}}{\mu^2}\right) - 8 + \mathcal{O}\left(\frac{|\vec{p}|}{m}\right)\right\}$$

At two loops all regions contribute



Recursive structure with effective vertizes suggests effective Lagrangian formulation



s, p, us

Expand in p^0, p^i

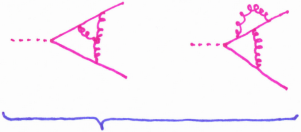
→ local

Expand only in $p^0 \sim mv^2$, not $\vec{p} \sim mv \sim \vec{k}$

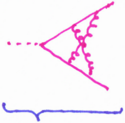
→ non-local in space, local in time

Only the p-p-....-p region contributes to the leading $(\alpha_s/v)^n$ and needs to be summed.

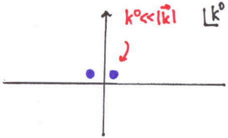
Only ladder diagrams can have an all-p region



don't have two pairs of heavy particle propagators

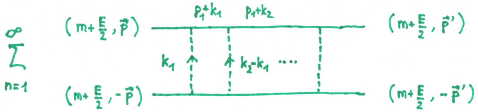


poles not pinched



Diagrammatic resummation

$$H^{(R)}(\mathbf{p},\mathbf{p}';E) \;=$$



$$= \sum_{n=1}^{\infty} (-D_R)^{n+1} \int \left[\prod_{i=1}^{n} \frac{d^d k_i}{(2\pi)^d}\right] \frac{(ig_s)^2 i}{\mathbf{k}_1^2} \,\frac{(ig_s)^2 i}{(\mathbf{k}_2-\mathbf{k}_1)^2} \cdots \frac{(ig_s)^2 i}{(\mathbf{k}_{n+1}-\mathbf{k}_n)^2}$$

$$\cdot \prod_{i=1}^{n} \frac{i}{\frac{E}{2}+k_i^0-\frac{(\mathbf{p}+\mathbf{k}_i)^2}{2m}+i\epsilon} \, \frac{-i}{\frac{E}{2}-k_i^0-\frac{(\mathbf{p}+\mathbf{k}_i)^2}{2m}+i\epsilon}$$

$$G^{(R)}(\mathbf{p},\mathbf{p}';E) = -\frac{(2\pi)^{d-1}\delta^{(d-1)}(\mathbf{p}'-\mathbf{p})}{E+i\epsilon-\frac{\mathbf{p}^2}{m}} + \frac{1}{E+i\epsilon-\frac{\mathbf{p}^2}{m}}\, iH(\mathbf{p},\mathbf{p}';E)\, \frac{1}{E+i\epsilon-\frac{\mathbf{p}'^{\,2}}{m}}$$

$$\left(\frac{\mathbf{p}^2}{m}-E\right) G^{(R)}(\mathbf{p},\mathbf{p}';E) + \mu^{2\epsilon}\int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}}\,\frac{4\pi D_R\alpha_s}{\mathbf{k}^2}\, G^{(R)}(\mathbf{p}-\mathbf{k},\mathbf{p}';E) = (2\pi)^{d-1}\,\delta^{(d-1)}(\mathbf{p}-\mathbf{p}')$$

$$\left(-\frac{\nabla_{(r)}^2}{m}+\frac{D_R\alpha_s}{r}-E\right) G^{(R)}(\mathbf{r},\mathbf{r}';E) = \delta^{(d-1)}(\mathbf{r}-\mathbf{r}')$$

Coulomb Green function [Schwinger, 1964; Voloshin, 1984]

$$G(\mathbf{p},\mathbf{p}';E) \;=\; -\frac{(2\pi)^3\delta^{(3)}(\mathbf{p}'-\mathbf{p})}{E-\frac{\mathbf{p}^2}{m}} + \frac{1}{E-\frac{\mathbf{p}^2}{m}}\,\frac{g_s^2(-D_R)}{(\mathbf{p}-\mathbf{p}')^2}\,\frac{1}{E-\frac{\mathbf{p}'^{\,2}}{m}}$$

$$+\,\frac{1}{E-\frac{\mathbf{p}^2}{m}}\int_0^1 dt\,\frac{g_s^2(-D_R)\,\lambda\,t^{-\lambda}}{(\mathbf{p}-\mathbf{p}')^2\,t-\frac{m}{4E}(E-\frac{\mathbf{p}^2}{m})(E-\frac{\mathbf{p}'^{\,2}}{m})(1-t)^2}\,\frac{1}{E-\frac{\mathbf{p}'^{\,2}}{m}}$$

$$G(\mathbf{r},\mathbf{r}';E) = \sum_{l=0}^{\infty}(2l+1)\,P_l\left(\frac{\mathbf{r}\cdot\mathbf{r}'}{rr'}\right)G^{(l)}(r,r';E)$$

$$G^{(l)}(r,r';E) = \frac{mp}{2\pi}\,(2pr)^l(2pr')^l\,e^{-p(r+r')}\sum_{s=0}^{\infty}\frac{s!\,L_s^{(2l+1)}(2pr)L_s^{(2l+1)}(2pr')}{(s+2l+1)!(s+l+1-\lambda)}$$

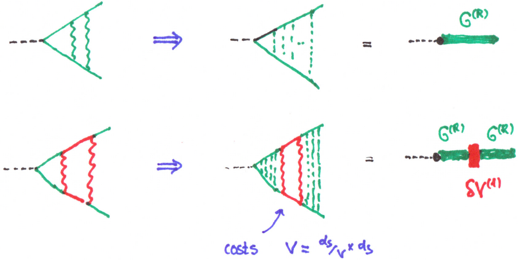
$[p=\sqrt{-mE}]$

Explicit expression known only in $d=4$. Not even $\mathcal{O}(\epsilon)$ terms.
BUT: Every term in the expansion in α_s can be computed in d dimensions.
Enough since $1/\epsilon$ poles disappear beyond a certain order in α_s.

Non-relativistic perturbation theory

In every term replace $G_0 =$  by $G^{(R)} = \sum_{n=0}$ 

Example



contributes to $f_0(\alpha_s/v)$

contributes to $f_1(\alpha_s/v) \cdot \alpha_s$

In this project for e^+e^- up to f_3 []

for hadronic production f_1 + logarithmic terms $\ln v$ in f_2

Methods:
Non-Relativistic Effective Theory

$\mathcal{L}_{QCD}$ ← or any other "full theory"

$m_Q > \Lambda_{QCD}$ ↓

Integrate out hard modes $k^0 \sim \vec{k} \sim m$

i.e. hard subgraphs → effective, point-like vertices ∎

local – defined with cut-off, in dim.reg only through threshold expansion

$\mathcal{L}_{NRQCD}$

non-relativistic fields $\phi \sim \int \frac{d^3\vec{p}}{(2\pi)^3 2p^0}\, u(p,s)\, a(p,s)$

$m_Q v > \Lambda_{QCD}$ ↓

Integrate out soft modes $k^0 \sim \vec{k} \sim mv$

i.e. soft subgraphs → effective instantaneous, non-local vertices ("potentials") ∎

$\mathcal{L}_{PNRQCD}$

- non-local in $\vec{r}$
- contains potential heavy quark field and ultrasoft gluons (light quarks)
- simple v-counting
  starting point for resummation

Integrating out the hard region: non-relativistic effective theory [Caswell, Lepage, 1986; Lepage et. al, 1992, Bodwin, Braaten, Lepage, 1994; Kinoshita, Nio, 1996]

$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger \left(iD^0 + \frac{\mathbf{D}^2}{2m}\right)\psi + \frac{1}{8m^3}\,\psi^\dagger \mathbf{D}^4\psi - \frac{d_1\,g_s}{2m}\,\psi^\dagger \sigma\cdot\mathbf{B}\psi$$
$$+ \frac{d_2\,g_s}{8m^2}\,\psi^\dagger\,(\mathbf{D}\cdot\mathbf{E}-\mathbf{E}\cdot\mathbf{D})\,\psi + \frac{d_3\,ig_s}{8m^2}\,\psi^\dagger\sigma\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})\,\psi$$
$$+ \sum_c \frac{d_{4c}\,g_s^2}{8m^2}\psi^\dagger\,[\kappa_c]\,\psi\,\sum_f \bar{q}_f\,\left[\kappa_c'\right]\,q_f + \text{ antiquark terms } \psi\to\chi$$
$$+ \sum_c \frac{d_{5c}\,g_s^2}{8m^2}\psi^\dagger\,[\kappa_c]\,\psi\,\chi^\dagger\,[\kappa_c]\,\chi + \mathcal{L}_{\rm light}$$

The $\gamma^* Q\bar{Q}$ coupling:

$$\bar{Q}\gamma^i Q = c_1\,\psi^\dagger\sigma^i\chi - \frac{c_2}{6m^2}\,\psi^\dagger\sigma^i(i\,\mathbf{D})^2\chi + \ldots$$

Hadronic $Q\bar{Q}$ production:

$$\mathcal{A}(ij\to\bar{Q}Q)_{\rm hard} \quad\to\quad C_{\{a\alpha\}}\,\phi_{c;\alpha_1 a_1}\phi_{\bar{c};\alpha_2 a_2}\psi^\dagger_{\alpha_3 a3}\chi_{\alpha_4 a_4} + \ldots$$

Integrating out the soft region : PNRQCD

[Pineda, Soto, 1997 ; MB, Signer, Smirnov, 1999 ; Brambilla et al, 1999]

$$\mathcal{L}_{PNRQCD} = \underbrace{\psi^\dagger\Big(\underset{v^{3/2}}{}\, i\partial^0 + \underset{v^2}{}\, gA^0(t,\vec{0}) + \frac{\vec{\partial}^{\,2}}{2m}\Big)\underset{v^{3/2}}{\psi}}_{v^5} + \text{anti-quark term} + \underbrace{\int \underset{1/v^3}{d^3\vec{r}}\, \underset{v^3}{[\psi\psi^\dagger]_{(x+\vec{r})}} \underset{\alpha_s v}{\left(\frac{\alpha_s D_R}{r}\right)} \underset{v^3}{[\chi^\dagger\chi]_{(x)}}}_{\alpha_s v^4 \sim v^5}$$

$$+ \frac{1}{8m^3}\psi^\dagger\vec{\partial}^{\,4}\psi + \text{anti-quark term} + \int d^3\vec{r}\, [\psi\psi^\dagger]_{(x+\vec{r})}\, \delta V\, [\chi^\dagger\chi]_{(x)} - g_s\psi^\dagger\vec{x}\cdot\vec{E}(t,\vec{0})\,\psi + \text{anti-quark term}$$

$$+ \;.....$$

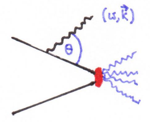
multipole expansion

- Leading Coulomb potential part of $\mathcal{L}^{(0)}_{PNRQCD}$ $\longrightarrow$ Coulomb propagator
  $\left[\psi \sim v^{3/2} \text{ from } \psi\psi^\dagger \propto \int d^4p\, \frac{1}{p^0 - \vec{p}^{\,2}/2m} \sim v^2 v^3 \cdot 1/v^2\right]$
- Multipole expansion of ultrasoft fields $A(t,\vec{x}) = A(t,\vec{0}) + \vec{x}\cdot\vec{\partial}A(t,\vec{0}) + \ldots$ ($1/v \cdot v^2$) corresponds to expansion of heavy particle propagator

$\Rightarrow \vec{x}\cdot\vec{E}$

Hadronic production

- Coloured initial state $\longrightarrow$ threshold enhancement from soft-gluon radiation

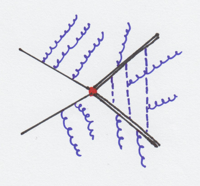


$$A \approx A_0 \times g^2 \int_{M\lambda}^{M\beta^2} \frac{d\omega}{\omega} \int_{\lambda}^{\beta^2} \frac{d\theta}{\theta} \sim A_0 \times g^2 \ln^2 \beta$$

Soft-gluon resummations in Mellin moment space (Sterman 1987; Catani, Trentadue, 1989)
or soft-collinear effective theory to all orders

- For pair production: Factorization of soft, collinear and Coulomb gluons is non-trivial [but well-known for soft-collinear], since soft gluons attach to and between Coulomb ladders and collinear radiation.

Kinematics: $[k_-]_c \sim [k^0]_p \sim k_s^\mu$
Also Coulomb exchange carries colour structure $T_R^A \otimes T_{R'}^A$.



Soft-collinear-potential decoupling

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(i\partial^0 + g_s A_s^0(x_0, \vec{0}\,) + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^{\,\dagger} \left(i\partial^0 + g_s A_s^0(x_0, \vec{0}\,) + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r}\, \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r}\,) \left(\frac{\alpha_s}{r} \right) \left[\psi'^{\,\dagger} \mathbf{T}^{(R')a} \psi' \right](0)$$

Field redefinition:

$$\psi(x) = S_w^{(R)}(x_0)\psi^{(0)}(x) \qquad \text{with} \qquad p^\mu = m w^\mu$$
$$S_w^{(R)}(x) = \mathrm{P}\exp\left[-i g_s \int_0^\infty dt\, w \cdot A_s^c(x+wt)\mathbf{T}^{(R)c} \right]$$
$$S_w^{(R)\dagger} iD_s^0 S_w^{(R)} = i\partial^0$$

Soft-collinear-potential decoupling

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger S_w^{(R)\dagger} \mathbf{T}^{(R)a} S_w^{(R)} \psi \right] (\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger S_w^{(R')\dagger} \mathbf{T}^{(R')a} S_w^{(R')} \psi' \right] (0)$$

In any representation $S_w^{(R)\dagger} \mathbf{T}^{(R)a} S_w^{(R)} = S_{w,ab}^{(\text{ad})} \mathbf{T}^{(R)b}$, hence

$$S_w^{(R)\dagger} \mathbf{T}^{(R)a} S_w^{(R)} \otimes S_w^{(R')\dagger} \mathbf{T}^{(R')a} S_w^{(R')} = \underbrace{S_{w,ab}^{(\text{ad})} S_{w,ac}^{(\text{ad})}}_{\delta^{bc}} \mathbf{T}^{(R)b} \otimes \mathbf{T}^{(R')c} = \mathbf{T}^{(R)a} \otimes \mathbf{T}^{(R')a}$$

Soft interactions disappear from the Lagrangian and survive only in the hard production vertex operator

$$\mathcal{O}_{\{a;\alpha\}}^{(\ell)}(\mu) = \left[\phi_{c;a_1} \phi_{\bar{c};a_2} \psi_{a_3}^\dagger \psi_{a_4}'^\dagger \right] (\mu)$$

Factorisation and resummation

$$\hat{\sigma}(\beta,\mu) = \sum_i H_i(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu).$$

- $H_i(M,\mu)$ – Short-distance production of heavy particle pair
  Sums $(g^2 \ln^2 \beta)^n$ by renormalization group evolution in μ from M to $M\beta^2$.

- $W_i^{R_\alpha}(\omega,\mu)$ – Soft function for the production of a *single* particle in irrep R_α

$$\hat{W}^{R_\alpha}_{\{a\alpha,b\beta\}}(z,\mu) \equiv \langle 0|\overline{\mathrm{T}}[S^{R_\alpha}_{v,\beta\kappa} S^\dagger_{\bar{n},jb_2} S^\dagger_{n,ib_1}](z) \mathrm{T}[S_{n,a_1 i} S_{\bar{n},a_2 j} S^{R_\alpha\,\dagger}_{v,\kappa\alpha}](0)|0\rangle$$

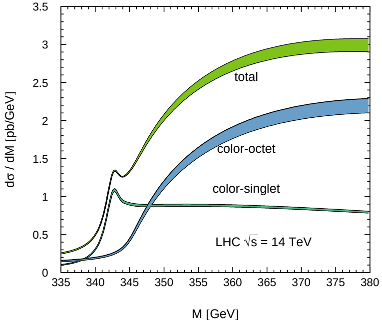
$$\frac{d}{d\ln\mu}\hat{W}_i^{R_\alpha}(L) = \left((\Gamma^r_{\mathrm{cusp}} + \Gamma^{r'}_{\mathrm{cusp}})L - 2\gamma^{R_\alpha}_{W,i}\right)\hat{W}_i^{R_\alpha}(L)$$

  NNLL soft-gluon resummation needs the 3-loop cusp anomalous dimension and 2-loop soft anomalous dimension

- J_{R_α} – Coulomb Green function = related to a correlation function of non-relativistic fields in PNRQCD.
  Sums Coulomb-exchange $(g^2/\beta)^n$ to all orders for HH' in irrep R_α.

Results (top, squarks)

Invariant $t\bar{t}$ mass distributions near threshold (so far NLL)



(Kiyo, Kühn, Moch, Steinhauser, Uwer; 2008)

Also for squark/gluino pair production [Kauth et al., 2010; Hagiwara, Yokoya, 2009]

Total $t\bar{t}$ cross section (NNLL) [MB, Falgari, Klein, Schwinn, 2011]

Table 9: Total cross sections in pb at the Tevatron for $m_t = 165\ldots180$ GeV. The errors denote the scale variation (scale variation+resummation ambiguities for NNLL_2), the NNLO constant variation (for $\mathrm{NNLO}_{\mathrm{app}}$ and NNLL_2) and the PDF+α_s error.

m_t [GeV]	NLO	$\mathrm{NNLO}_{\mathrm{app}}$	NNLL_2
171	$7.18^{+0.39+0.54}_{-0.81-0.49}$	$7.58^{+0.27+0.11+0.75}_{-0.36-0.11-0.57}$	$7.76^{+0.31+0.11+0.77}_{-0.49-0.11-0.59}$
172	$6.96^{+0.38+0.53}_{-0.78-0.47}$	$7.35^{+0.26+0.10+0.72}_{-0.35-0.10-0.55}$	$7.52^{+0.30+0.10+0.75}_{-0.48-0.10-0.57}$
173	$6.74^{+0.37+0.51}_{-0.76-0.46}$	$7.12^{+0.25+0.10+0.70}_{-0.34-0.10-0.53}$	$7.29^{+0.29+0.10+0.72}_{-0.46-0.10-0.55}$
174	$6.54^{+0.36+0.50}_{-0.74-0.44}$	$6.91^{+0.24+0.09+0.67}_{-0.32-0.09-0.51}$	$7.07^{+0.28+0.09+0.70}_{-0.45-0.09-0.53}$

D0 : $7.56^{+0.63}_{-0.56}$ pb
CDF: $7.50^{+0.48}_{-0.48}$ pb

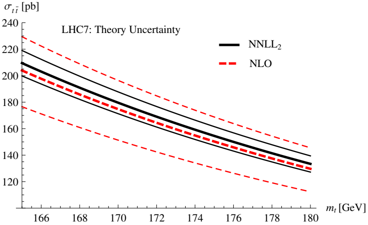
Table 10: Total cross section at the LHC ($\sqrt{s} = 7$ TeV) for $m_t = 165\ldots180$. The errors denote the theory error, the NNLO constant variation (for $\mathrm{NNLO}_{\mathrm{approx}}$ and NNLL_2) and the PDF+α_s error.

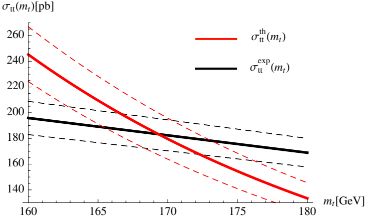
m_t [GeV]	NLO	$\mathrm{NNLO}_{\mathrm{app}}$	NNLL_2
171	$169.5^{+21.0+14.9}_{-22.7-14.0}$	$172.6^{+12.3+5.0+16.2}_{-11.8-5.0-15.6}$	$174.2^{+6.1+5.0+16.5}_{-6.3-5.0-15.8}$
172	$164.4^{+20.3+14.5}_{-22.0-13.6}$	$167.5^{+11.9+4.9+15.8}_{-11.4-4.9-15.1}$	$169.0^{+6.0+4.9+16.0}_{-6.1-4.9-15.3}$
173	$159.6^{+19.7+14.0}_{-21.4-13.3}$	$162.5^{+11.5+4.7+15.4}_{-11.0-4.7-14.7}$	$164.0^{+5.8+4.7+15.6}_{-6.0-4.7-14.9}$
174	$154.8^{+19.1+13.6}_{-20.7-12.9}$	$157.7^{+11.1+4.5+14.9}_{-10.7-4.5-14.3}$	$159.2^{+5.6+4.5+15.1}_{-5.8-4.5-14.4}$

ATLAS: 179.0 ± 11.8 pb ($0.7\mathrm{fb}^{-1}$)
CMS: 165.8 ± 2.2(stat.) $\pm$ 10.6(syst.) $\pm$ 7.8(lumi.) pb (0.8-$1.1\mathrm{fb}^{-1}$)

At LHC $\approx 3\%$ beyond NLO, reduction of theoretical uncertainty from 13% to 5%

Total $t\bar{t}$ cross section (NNLL)





From total cross section (ATLAS data)

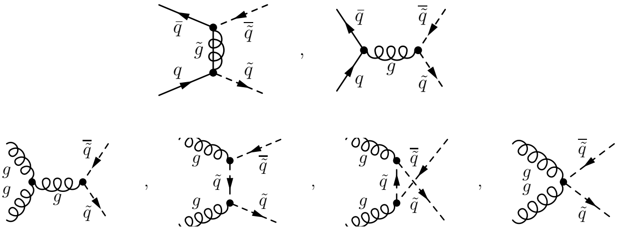
$$m_t = (169.8^{+4.9}_{-4.7})\,\text{GeV}$$

Squark-antisquark production

Born production processes

$$q_i(k_1)\bar{q}_j(k_2) \to \tilde{q}_{\sigma_1 k}(p_1)\bar{\tilde{q}}_{\sigma_2 l}(p_2)$$
$$g(k_1)g(k_2) \to \tilde{q}_{\sigma_1 i}(p_1)\bar{\tilde{q}}_{\sigma_2 j}(p_2)\,,$$

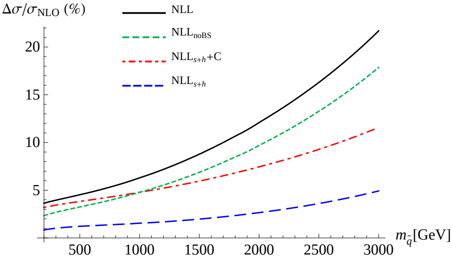
Must be separated into colour-singlet and colour-octet production.



NLL resummation of $pp \to$ squark+antisquark $+ X$ at $\sqrt{s} = 14\,\text{TeV}$

NLL = Tree C and W, 1-loop anomalous dim. + LO Coulomb Green function + matching to NLO fixed order (from Beenakker et al, 1997; Prospino code)

Size of corrections beyond NLO

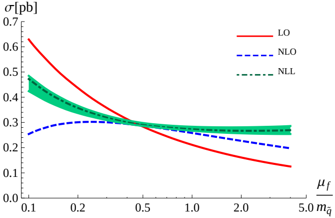


($\sqrt{s} = 14$ TeV; $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

Significant soft-Coulomb interference effect (see also Beenakker et al. 2011)

Resummation is $\approx 10\%$ (mass-dependent) effect at the natural scale.

Scale dependence ($\simeq$ theoretical uncertainty) at LO, NLO, NLL



(pp collisions at $\sqrt{s} = 14$ TeV; $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$; $m_{\tilde{q}} = 1$ TeV [left] and $p\bar{p}$ collisions, $m_{\tilde{q}} = 400$ GeV [right]
Green band: variation of all factorization scales.)

Theoretical uncertainty is reduced.

Conclusion

1) Joint resummation possible due to factorization of soft and Coulomb gluon effects (threshold expansion + SCET $\times$ NRQCD)

2) Top pair cross section at threshold known at $\mathcal{O}(\alpha_s^2)$ at threshold up to the constant term + NNLL to all orders.

   For any coloured heavy particles, given the one-loop hard matching coefficients.

3) Ready for LHC data and discoveries