The Top-Antitop Threshold – QCD Contributions

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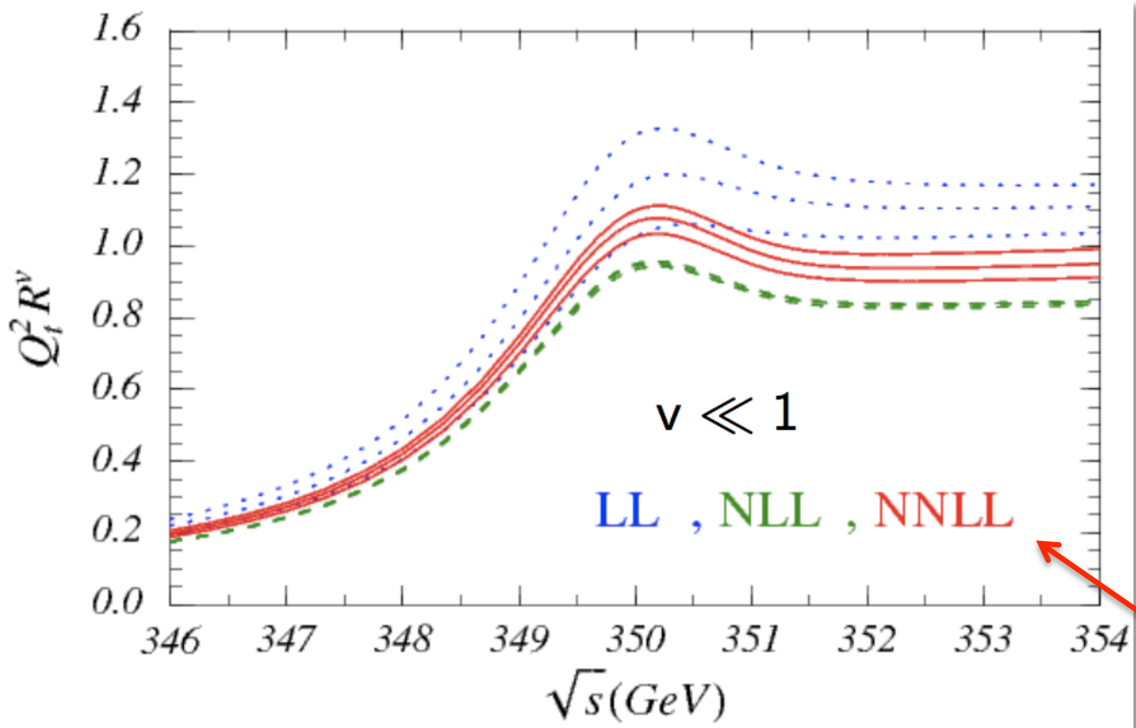
Outline

- The resonance of $\sigma_{\text{tot}}(e^+e^- \longrightarrow t\bar{t})$
  - Measurement
  - Theory
- The effective theory vNRQCD
- Renormalization
  - Currents
  - Potentials
- Recent Results
- Summary/Outlook

The resonance of $\sigma_{tot}(e^+e^- \to t\bar{t})$

Linear Collider: $t\bar{t}$ production at threshold



$\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{QCD}$

- Nonpert. effects suppressed [Fadin, Khoze]
- No sharp resonance peaks

incomplete !!!

Aim: precise determination of

- m_t status: $\delta m_t \sim 100\,\text{MeV}$ ✓
- $y_t,\ \alpha_s,\ \Gamma_t$ status: $\delta\sigma_{tot}^{th}/\sigma_{tot} \sim 6\,\%$

needed: < 3%

The resonance of $\sigma_{tot}(e^+e^- \to t\bar{t})$

EW effects:

- LO: Unstable top $\Rightarrow$ $\Gamma_t \approx 1.5\,\mathrm{GeV} \gg \Lambda_{QCD}$ "IR cutoff"

$$v_{eff} \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \to \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}}\,; \qquad |v_{eff}| \gtrsim 0.1$$

[Fadin, Khoze]

- Higher orders → Pedro's talk

QCD in the resonance region: $v \sim \alpha_s \ll 1$

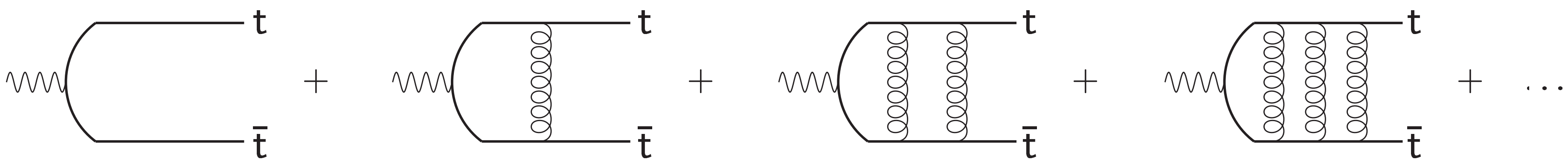
3 scales: $m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{QCD})$

"hard" "soft" "ultrasoft"

Problems:
- "Coulomb singularities"
- Large logs

The resonance of $\sigma_{tot}(e^+e^- \to t\bar{t})$

Problem of Coulomb singularities:



~ 1 $\quad\sim \alpha_s/v$ $\quad\sim (\alpha_s/v)^2$ $\quad\sim (\alpha_s/v)^3$

Production threshold: $\boxed{\alpha_s \sim v \sim 0.1}$ $\Rightarrow$ breakdown of perturbation theory

Solution:

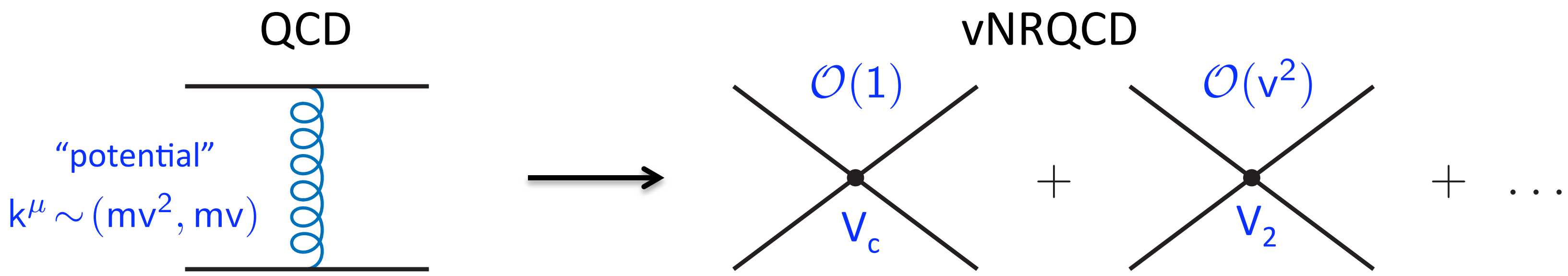
Nonrelativistic effective field theory **vNRQCD**

$\longrightarrow$ summation of $(\alpha_s/v)^n$ terms by means of a Schrödinger Equation !

vNRQCD

<u>Constructing vNRQCD:</u> "Expansion in $v \ll 1$"

→ Integrate out nonresonant degrees of freedom, e.g.:



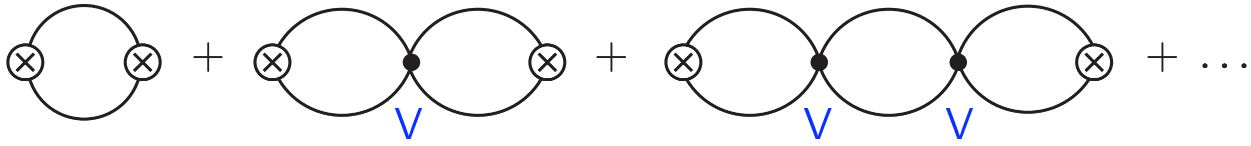
→ $\mathcal{L}_{NR} = \mathcal{L}_{kin} + \Big[\frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots \Big] \psi^\dagger\psi\,\chi^\dagger\chi + \ldots$

→ Separate center-of-mass motion

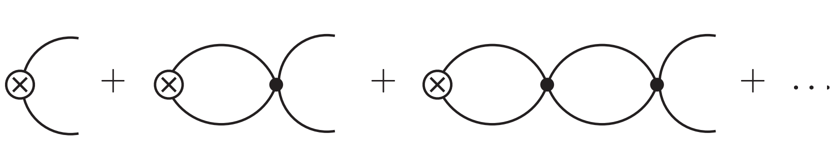
⇒ Schrödinger Equation: $E\,\Psi = \Big[\frac{\mathbf{k}^2}{m} + V + \ldots \Big] \Psi$

vNRQCD

Green function: $\left[-\frac{\nabla_{\vec{r}}^2}{m} + V(r) - E\right] G(\vec{r}, \vec{r}\,', E) = \delta^{(3)}(\vec{r} - \vec{r}\,')$

$G(0,0,E) \sim$ 

Optical theorem:

$\sigma_{\rm tot} \sim \int d\mathrm{PS} \, \Big|$  $\Big|^2 \sim \mathrm{Im}\left[G(0,0,E)\right]$

Unstable top: $G(0,0,E+i\Gamma_t) \sim \sum_n \frac{|\Psi_n(0)|^2}{E_n - E - i\Gamma_t - i\epsilon} + \text{continuum}$

vNRQCD

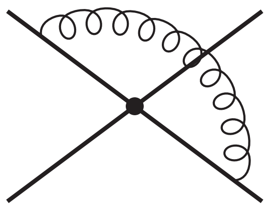
LO Green function: $\left[-\frac{\nabla_{\vec{r}}^2}{m} + V_c(r) - E \right] G^1(\vec{r}, \vec{r}\,', E) \ = \ \delta^{(3)}(\vec{r} - \vec{r}\,')$

→ Analytic solution for $G^1(\vec{r}, \vec{r}\,', E)$ ✓

Higher orders: e.g. $G^2(0,0,E) = -\int d^3\vec{r} \;\; G^1(0,\vec{r},E)\, V^2(\vec{r})\, G^1(\vec{r},0,E)$

↑ $\mathcal{O}(\alpha_s\,, v)$

RG running: e.g.



$\Rightarrow \ V \equiv V(\mu) \ \ \Rightarrow \ \ G(0,0,E) \equiv G(0,0,E,\mu)$

→ $G(0,0,E,\mu)$ known up to NNLL (partly N^{3}LO) ✓

[Hoang, Manohar, Stewart, Teubner; Pineda, Signer] [Beneke, Kiyo, Schuller]

vNRQCD

Problem of large logarithms:

3 scales: $m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2$ $(\sim \Gamma_t \gg \Lambda_{QCD})$

"hard" "soft" "ultrasoft"

→ Logs: $\ln\left(\frac{m^2}{E^2}\right),\ \ln\left(\frac{m^2}{\mathbf{p}^2}\right),\ \ln\left(\frac{\mathbf{p}^2}{E^2}\right)$ e.g. $\alpha_s \ln\left(\frac{m^2}{E^2}\right) \sim -\alpha_s \ln(v^4) \sim 1$

Solution:

Two renormalization scales: $\mu_s = m\nu,\ \mu_u = m\nu^2$ → "v"NRQCD

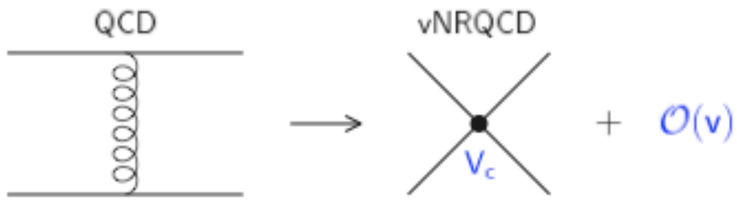
ν "subtraction velocity"

→ RGE's resum $[\alpha_s \ln v]^n,\ \alpha_s[\alpha_s \ln v]^n,\ \alpha_s^2[\alpha_s \ln v]^n$ ... terms

LL NLL NNLL

vNRQCD

- Nonresonant dof's integrated out, e.g.:



- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_q(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in v $\Rightarrow$ consistent power counting in $v \sim \alpha_s$

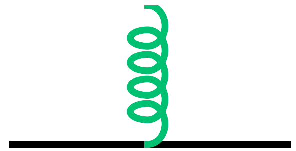
vNRQCD

[Luke, Manohar, Rothstein]

$$\boxed{\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}}$$

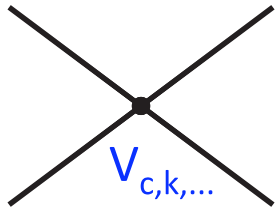
$$D^\mu = \partial^\mu + ig A^\mu(x)$$

$$\mathcal{L}_{\text{usoft}}: \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots\right]\psi_{\mathbf{p}}(x) + \ldots$$

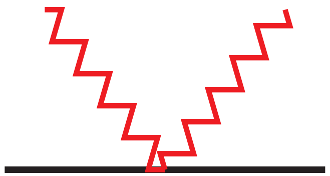


$$\mathcal{L}_{\text{pot}}: \quad -V\ \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$



$\mathcal{L}_{\text{soft}}:$



vNRQCD

[Luke, Manohar, Rothstein]

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$$D^\mu = \partial^\mu + ig A^\mu(x)$$

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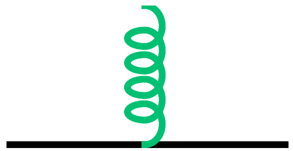
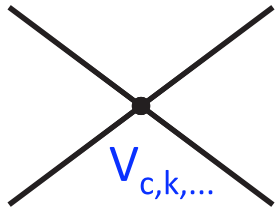
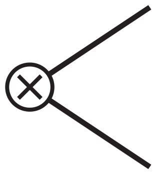


$$\mathcal{L}_{\text{pot}}: \quad - V \; \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$

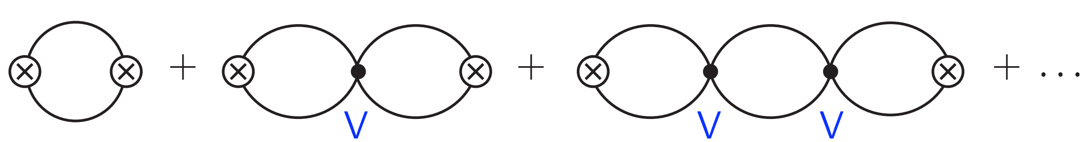


Production/annihilation current ($^3S_1$):



$$\sim \quad c_1(\nu) \cdot \underbrace{\vec{j}_1^{\,\text{eff}}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \ldots \qquad \text{(CMS)}$$

Renormalization

$\sigma_{\text{tot}} \sim \mathrm{Im}\Big[$  $\Big]$

$$\sim \left|c_1(\nu)\right|^2 \cdot \mathrm{Im}\left[-\,i \int d^4x \; e^{i\hat{q}x} \, \langle 0|\, T\, \vec{j}_1^{\,\text{eff}\,*}(x)\, \vec{j}_1^{\,\text{eff}}(0)\, |0\rangle \right] + \ldots$$

$$\sim \left|c_1(\nu)\right|^2 \cdot \mathrm{Im}\left[G(0,0,E,\nu) \right] + \ldots$$

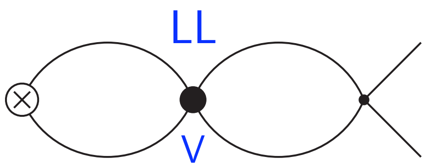
G^{NNLL} known ✓

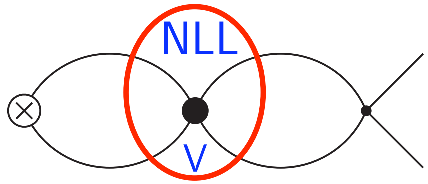
Renormalization

$$\sigma_{\text{tot}} \sim \left| c_1(\nu) \right|^2 \cdot \text{Im}\left[G(0,0,E,\nu) \right] + \dots$$

G^{NNLL} known ✓

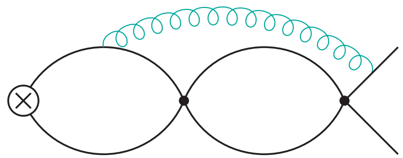
current renormalization





$$\ln\left[\frac{c_1(\nu)}{c_1(1)} \right] = \underbrace{\xi^{\text{LL}}}_{0} + \xi^{\text{NLL}} + \xi^{\text{NNLL}}_{\text{mix}} + \xi^{\text{NNLL}}_{\text{nonmix}}$$

[Hoang]



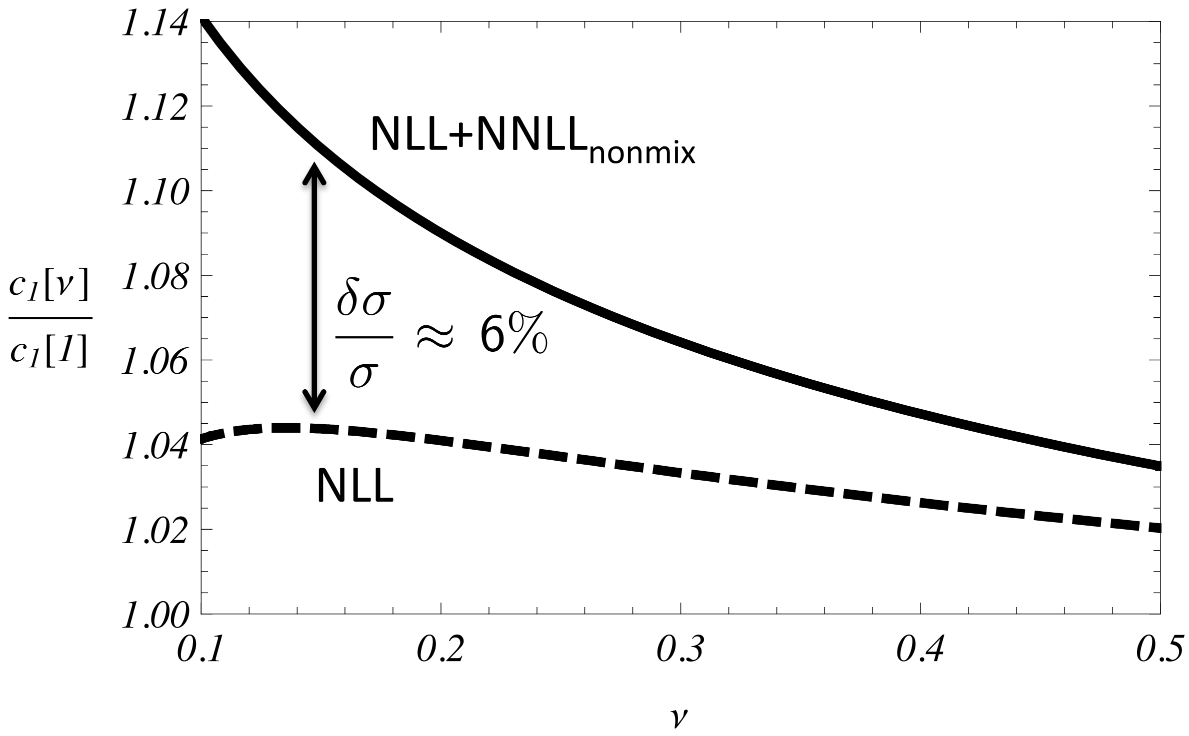
[Luke, Manohar, Rothstein, Pineda, Hoang, Stewart]

missing

Renormalization

$$\sigma_{\text{tot}} \sim |c_1(\nu)|^2 \cdot \text{Im}\left[G(0,0,E,\nu)\right] + \ldots$$

G^{NNLL} known ✓



Missing NNLL_{mix} contribution may reduce th. error of σ_{tot}

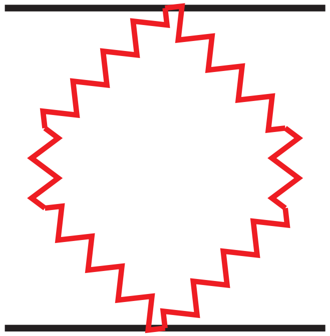
$\Rightarrow$ $\boxed{V^{\text{NLL}}(\nu)}$ needed !

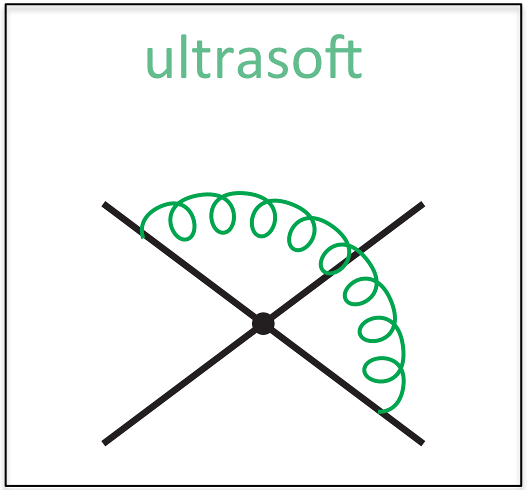
Renormalization

NLL: $$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

NLL running of V:

soft





Renormalization

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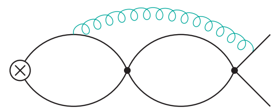
NLL running of V:

- Ultrasoft contributions dominant

$$\boxed{\alpha_s(mv^2) \simeq 0.27 \; > \; \alpha_s(mv) \simeq 0.15} \quad (v \simeq 0.1)$$

- Large ultrasoft contribution to $\xi^{\text{NNLL}}_{\text{nonmix}}$!!! ⟵  [Hoang]
- Soft contributions to V_s known ⟶ tiny [Penin, Pineda, Smirnov, Steinhauser]
- Potentials affected by ultrasoft renormalization:

$$\underbrace{\frac{\mathcal{V}_k \pi^2}{m\mathbf{k}}}_{\mathcal{O}(\alpha_s^2 v)}, \; \underbrace{\frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2}, \; \frac{\mathcal{V}_2}{m^2}}_{\mathcal{O}(\alpha_s v^2)}$$

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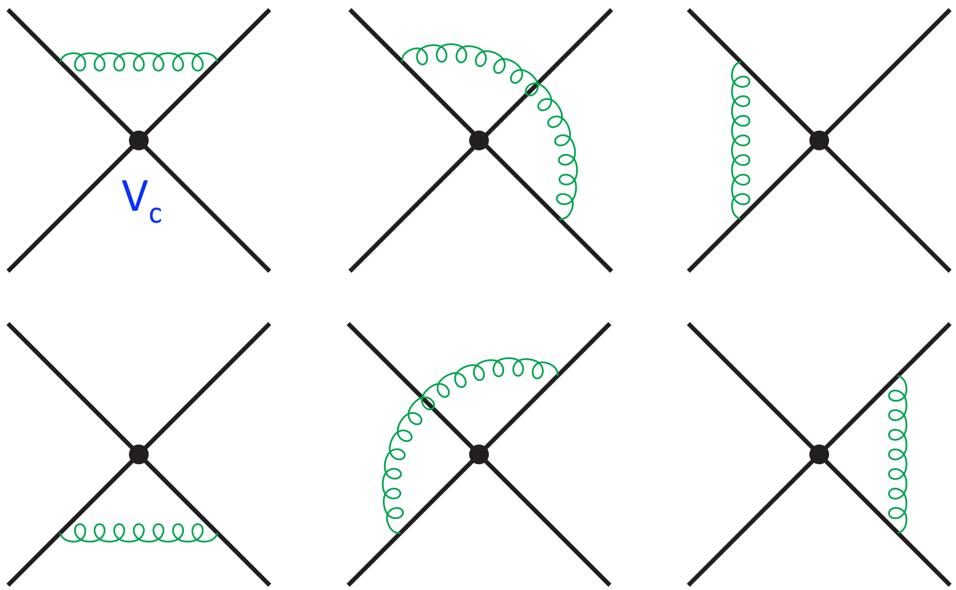
renormalize

<u>One usoft loop:</u>



$$\Rightarrow \quad \boxed{\delta\mathcal{V}_{r,2}^{1\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{LL}}(\nu)}$$

[Manohar, Stewart]

Renormalization

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

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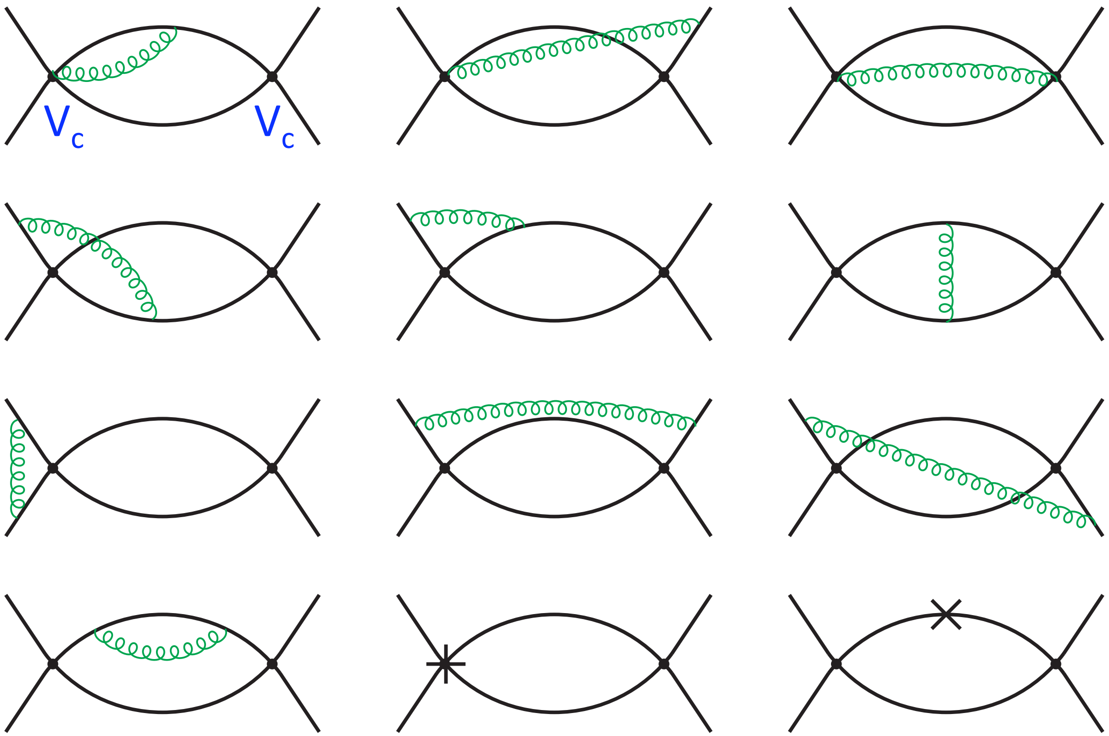

- 2 loops:
  1 x usoft
  1 x potential (finite)

$$\Rightarrow \quad \delta\mathcal{V}_k^{1\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{LL}(\nu)$$

[Manohar, Stewart]

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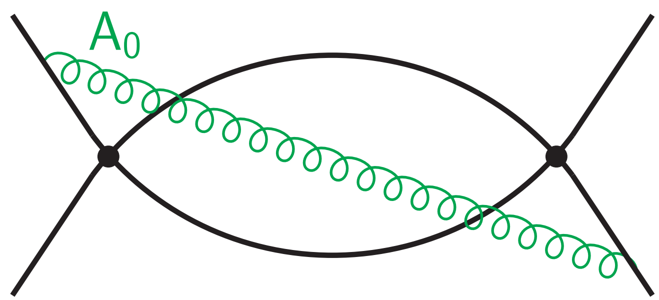
One usoft loop:

Details:

- Feynman/Coulomb gauge
- $\overline{\mathrm{MS}}$, Dim. Reg.
- A_0 and $\mathbf{A}$ couple differently!
- Usoft derivative operator insertions:



$$\mathcal{L}_{\mathrm{usoft}}: \quad \psi_{\mathbf{p}}^{\dagger}\Big[\underbrace{\Big(i\partial_0 - \frac{\mathbf{p}^2}{2m}\Big)}_{\text{HQ propagator}} - gA_0 + \frac{i\mathbf{p}\nabla}{m} + g\frac{\mathbf{p}\mathbf{A}}{m} + \frac{\nabla^2}{2m} + \dots\Big]\psi_{\mathbf{p}}$$

Renormalization

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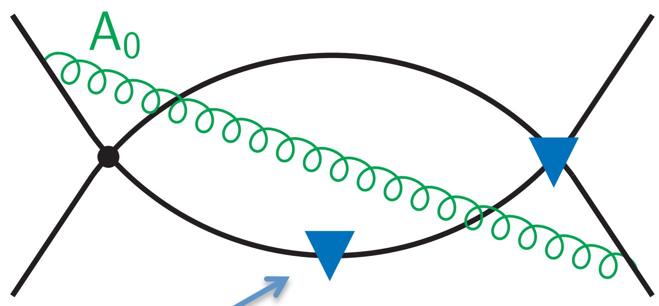

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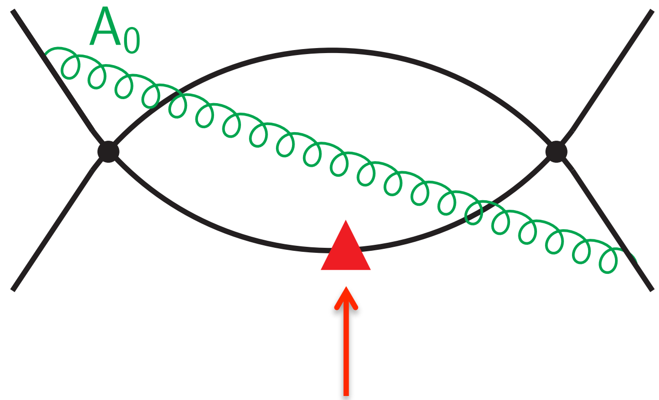

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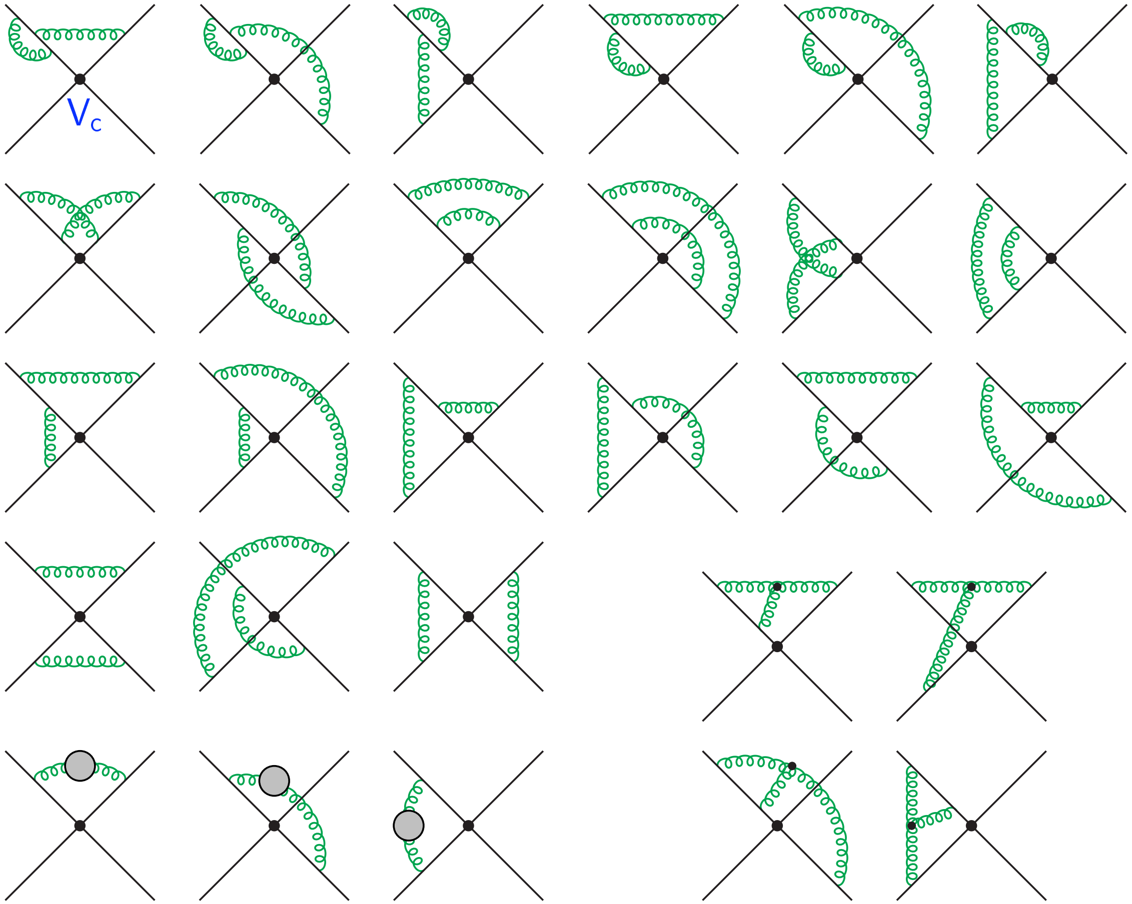
renormalize

Two usoft loops:



- Feynman gauge
- O(10^3) diagrams

$\Rightarrow$ $\delta\mathcal{V}_{r,2}^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{NLL}}(\nu)$

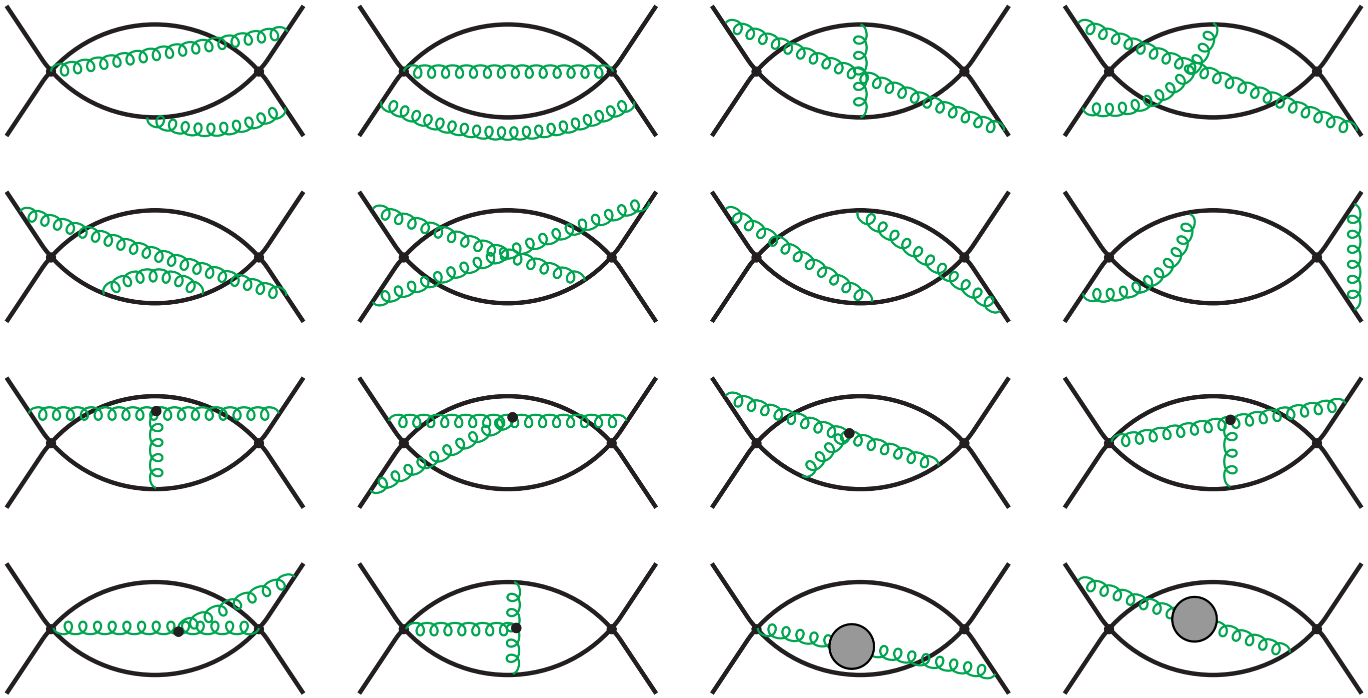
[MS, Hoang]

Renormalization

NLL: $$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

renormalize

Two usoft loops:



- 3 loops:
  2 x usoft
  1 x potential (finite)
- Feynman gauge
- $O(10^4)$ diagrams
- Generation:
  own **Mathematica** code
- Integrals:
  IBP & partial frac.

$$\Rightarrow \quad \delta\mathcal{V}_k^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)$$

[MS, Hoang]

Results

RGE's + matching at hard scale $(\nu = 1)$ give:

LL | NLL

$$\left[\mathcal{V}_2(\nu)\right]^{\rm NLL}_{\rm usoft} = 4\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_2 \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_2 - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_2\right)\right]$$

$$\left[\mathcal{V}_r(\nu)\right]^{\rm NLL}_{\rm usoft} = 8\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_r \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_r - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_r\right)\right]$$

$$\left[\mathcal{V}_k(\nu)\right]^{\rm NLL}_{\rm usoft} = 2\alpha_s^2(m\nu)\left[-\frac{4\pi}{\beta_0}A_k \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_k - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_k\right)\right]$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ B_r \end{bmatrix} = -C_A C_F\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = -C_A C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

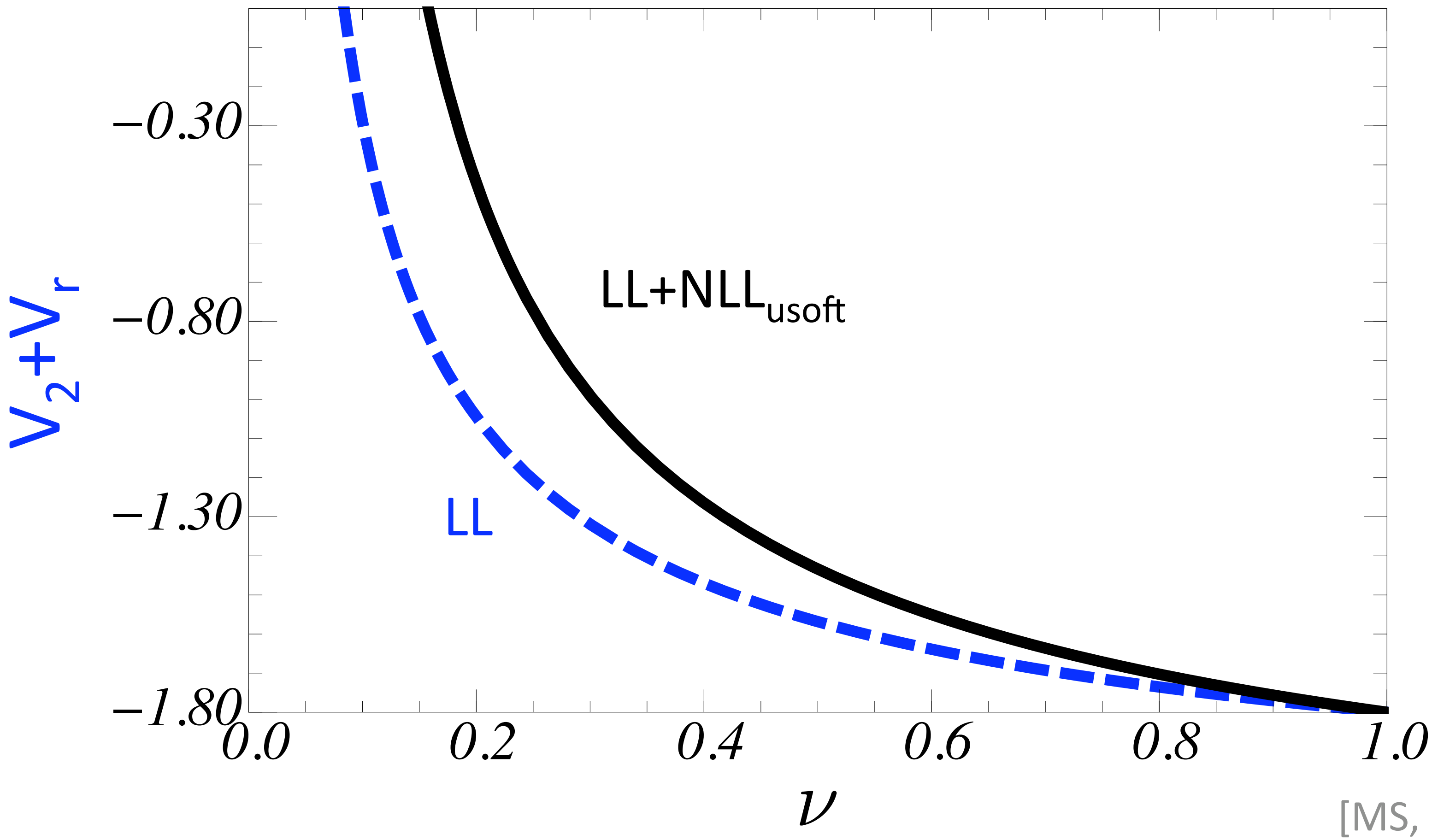
$$A = \frac{1}{3\pi}$$

$$B = \frac{C_A(47+6\pi^2) - 10 n_f T}{108\pi^2}$$

[MS, Hoang]

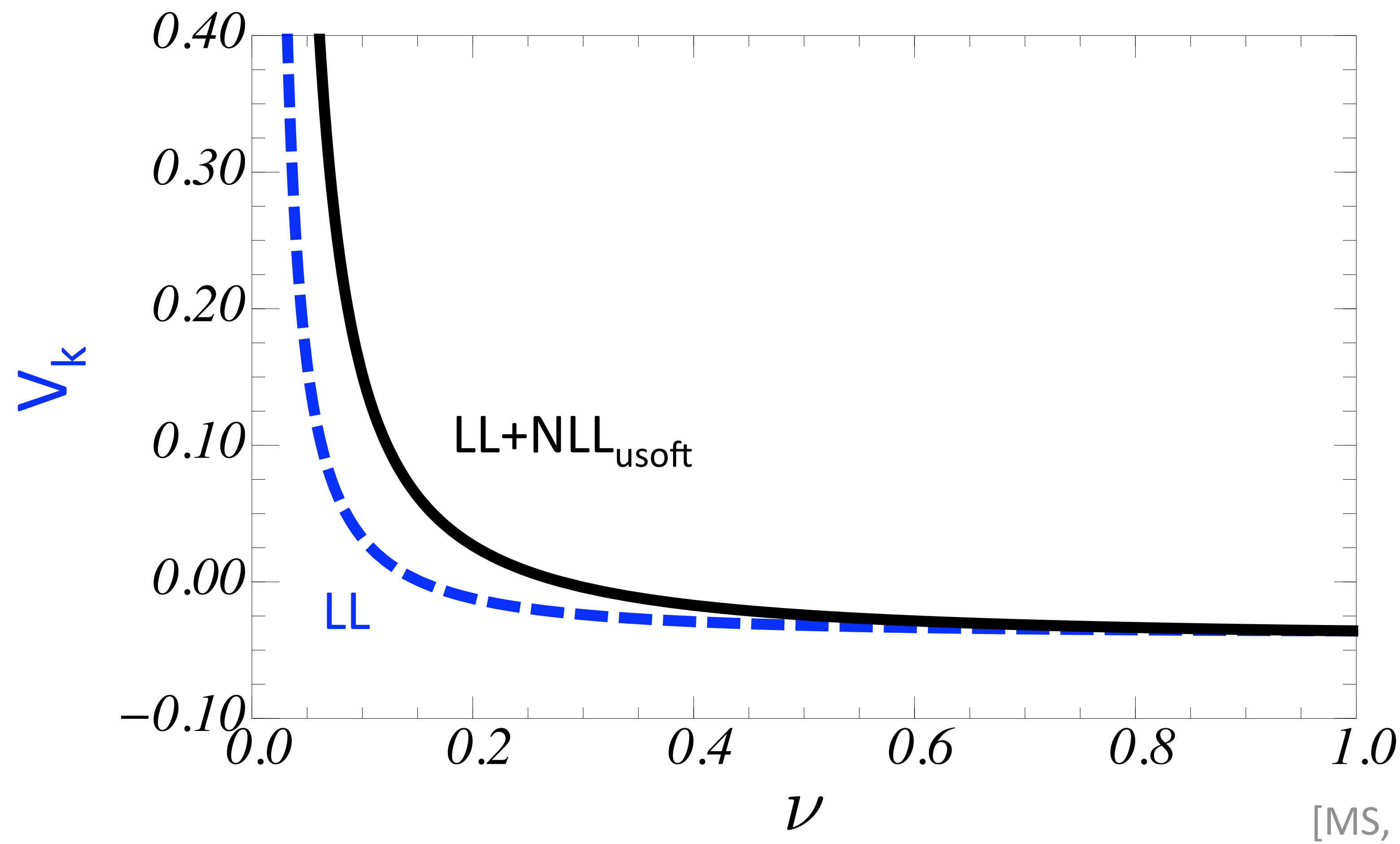
[Pineda]

Results



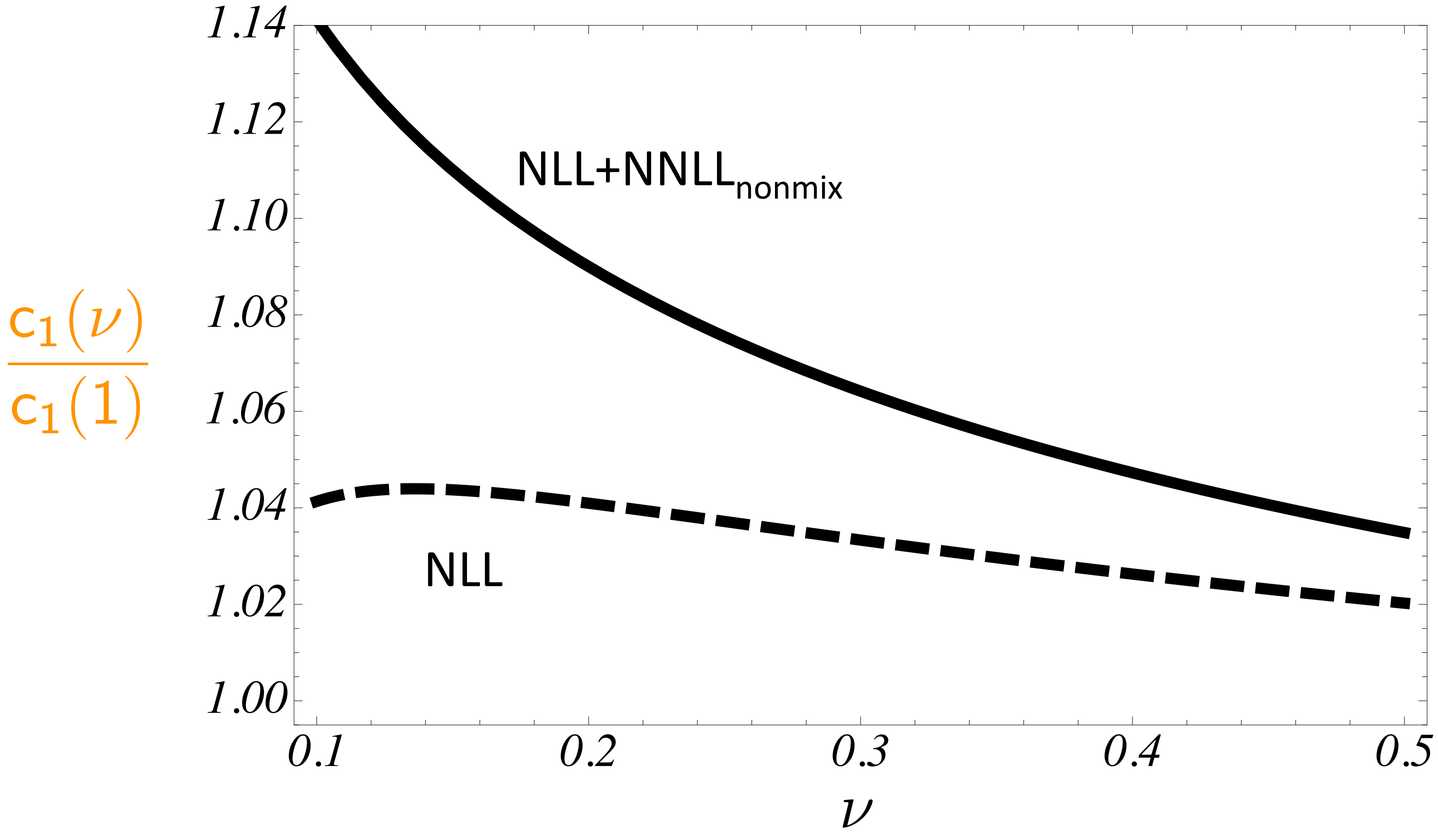
[MS, Hoang]

Results



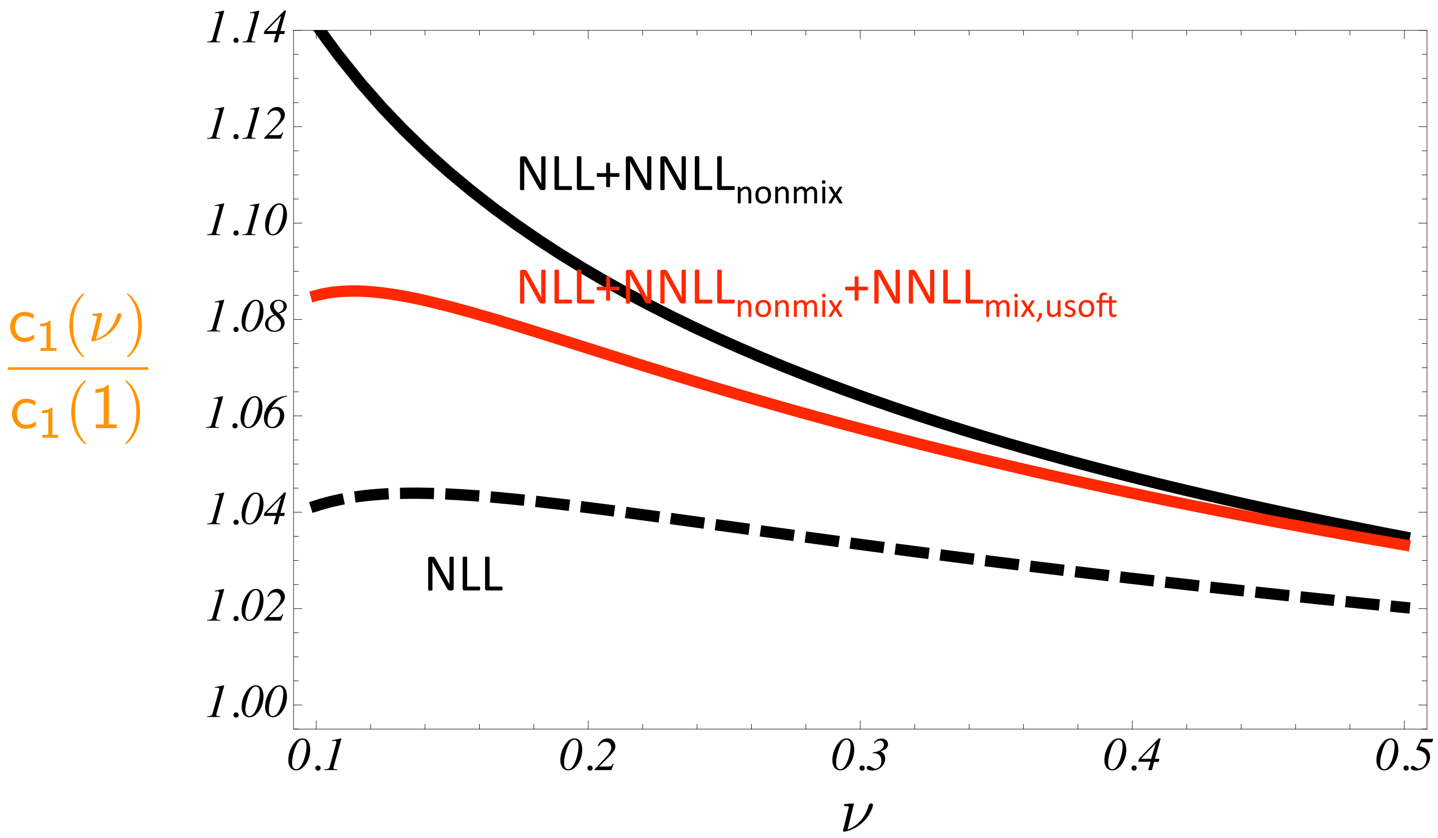
[MS, Hoang]

Results



$\frac{c_1(\nu)}{c_1(1)}$
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
NLL+NNLL$_{\text{nonmix}}$
NLL
0.1
0.2
0.3
0.4
0.5
ν

Results



[MS, Hoang]

- Large usoft NNLL contributions compensate each other ⇒ $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}}$ decreases
- Detailed analysis ⟶ WIP

Summary/Outlook

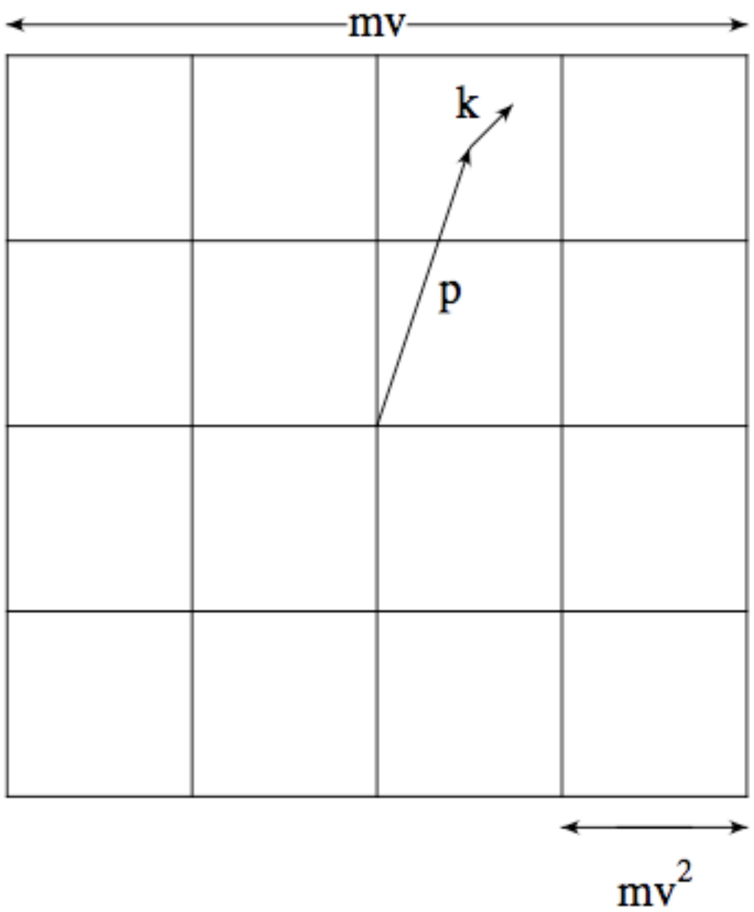
- $y_t,\ \alpha_s,\ \Gamma_t$ from $\sigma_{tot}(e^+e^- \to t\bar{t})$ at threshold
- status: $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}} \approx 6\%$, needed: $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}} \lesssim 3\%$
- $\sigma_{tot} \sim |c_1(\nu)|^2 \cdot \mathrm{Im}\left[G(0,0,E,\nu)\right] + \dots$
- $G(0,0,E,\nu)$ known up to NNLL ✓
- New $\text{NNLL}_{\text{mix,usoft}}$ compensates for large $\text{NNLL}_{\text{nonmix}}$ contribution to $c_1(\nu)$

  → $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}}$ decreases!
- Outlook:
  - Detailed study of $\sigma_{tot}(e^+e^- \to t\bar{t})$ at threshold
  - Determination of bottom mass from nonrel. Υ sum rules

Backup

vNRQCD label formalism:

momentum space:



$$m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2$$

(soft) (usoft)

e.g. nonrel. quark (CMS):

$$p^\mu = (m, 0) + (0, \mathbf{p}) + (k^0, \mathbf{k})$$

label dyn. variable

$$\psi(r) \to \sum_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}}\, \psi_{\mathbf{p}}(x)$$

$\mathbf{x} \sim \frac{1}{mv}$ (small x) $x \sim \frac{1}{mv^2}$ (large x)