Isospin violation in leptonic pion and kaon decays

$|V_{us}/V_{ud}|$ from decay widths and lattice data

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March 31, 2011

Isospin violation in $P \to \ell\nu_\ell(\gamma)$

$P = \pi^\pm,\ K^\pm$

Inclusive decay width

$$\Gamma_{P_{\ell 2(\gamma)}} = \frac{G_F^2 |V_P|^2 F_P^2}{4\pi} M_P\, m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 S_{\rm EW} \left(1 + \delta_{\rm EM}^P\right)$$

Fermi constant G_F
CKM matrix elements $V_{\pi^\pm} = V_{ud},\ V_{K^\pm} = V_{us}$
Meson decay constant F_P
Electroweak short-distance enhancement factor $S_{\rm EW}$
Long-distance electromagnetic correction $\delta_{\rm EM}^P$

Long-distance electromagnetic correction

Universal part

$$\delta^P_{\rm EM} = \frac{\alpha}{\pi}\left(F(m_\ell/M_P) + \frac{3}{4}\ln\frac{M_P^2}{M_\rho^2} - C_1^P + \ldots \right)$$

W.J. Marciano, A. Sirlin, 1993

Structure-dependent part

$$\begin{aligned} C_1^\pi &= -\tilde{E}^r(M_\rho) + \frac{Z}{4}\left(3 + 2\ln\frac{M_\pi^2}{M_\rho^2} + \ln\frac{M_K^2}{M_\rho^2}\right) \\ C_1^K &= -\tilde{E}^r(M_\rho) + \frac{Z}{4}\left(3 + 2\ln\frac{M_K^2}{M_\rho^2} + \ln\frac{M_\pi^2}{M_\rho^2}\right) \\ Z &= \frac{M_{\pi^\pm}^2 - M_{\pi^0}^2}{8\pi\alpha F_\pi^2} \simeq 0.8 \end{aligned}$$

M. Knecht, H. N., H. Rupertsberger, P. Talavera, 2000

Ratio of decay widths

$$\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_{K^\pm}^2}{F_{\pi^\pm}^2} \frac{M_{K^\pm}\left(1 - m_\ell^2/M_{K^\pm}^2\right)^2}{M_{\pi^\pm}\left(1 - m_\ell^2/M_{\pi^\pm}^2\right)^2} (1 + \delta_{\rm EM})$$

$$\delta_{\rm EM} = \delta_{\rm EM}^K - \delta_{\rm EM}^\pi = -0.0069(17)$$

$$\frac{|V_{us}|}{|V_{ud}|} \frac{F_{K^\pm}}{F_{\pi^\pm}} = \left(\frac{\Gamma_{K_{\ell 2(\gamma)}} M_{\pi^\pm}}{\Gamma_{\pi_{\ell 2(\gamma)}} M_{K^\pm}}\right)^{1/2} \frac{1 - m_\ell^2/M_{\pi^\pm}^2}{1 - m_\ell^2/M_{K^\pm}^2} (1 - \delta_{\rm EM}/2)$$

Marciano, 2004: use F_K/F_π from lattice $\to |V_{us}/V_{ud}|$

Isospin corrections

V. Cirigliano, H. N., arXiv:1102.0563, 2011

- Lattice determinations of F_K/F_π usually (still) in isospin limit
- Rewrite ratio of decay widths in terms of F_K/F_π:

$$\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{M_{K^\pm}\left(1 - m_\ell^2/M_{K^\pm}^2\right)^2}{M_{\pi^\pm}\left(1 - m_\ell^2/M_{\pi^\pm}^2\right)^2} \left(1 + \delta_{\mathrm{EM}} + \delta_{\mathrm{SU(2)}}\right)$$

$$\delta_{\mathrm{SU(2)}} = \left(\frac{F_{K^\pm} F_\pi}{F_{\pi^\pm} F_K}\right)^2 - 1$$

Isospin limit

$m_u = m_d$, $e = 0$

$M_\pi^2 = M_{\pi^0}^2$, $M_K^2 = \frac{1}{2}\left(M_{K^\pm}^2 + M_{K^0}^2 - M_{\pi^\pm}^2 + M_{\pi^0}^2\right)$

Chiral expansion of $F_{\pi^\pm}$ and $F_{K^\pm}$

J. Gasser, H. Leutwyler, 1985; H. N., H. Rupertsberger, 1996

$$
\begin{aligned}
F_{\pi^\pm} &= F_0\Bigg\{1 + \frac{4}{{F_0}^2}\left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu)M_\pi^2\right] \\
&\quad - \frac{1}{2(4\pi)^2{F_0}^2}\left[2M_\pi^2 \ln\frac{M_\pi^2}{\mu^2} + M_K^2 \ln\frac{M_K^2}{\mu^2}\right]\Bigg\} \\
F_{K^\pm} &= F_0\Bigg\{1 + \frac{4}{{F_0}^2}\left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu)M_K^2\right] \\
&\quad - \frac{1}{8(4\pi)^2{F_0}^2}\left[3M_\pi^2 \ln\frac{M_\pi^2}{\mu^2} + 6M_K^2 \ln\frac{M_K^2}{\mu^2} + 3M_\eta^2 \ln\frac{M_\eta^2}{\mu^2}\right] \\
&\quad - \frac{8\sqrt{3}\,\varepsilon}{3F_0^2}L_5^r(\mu)(M_K^2 - M_\pi^2) \\
&\quad - \frac{\sqrt{3}\,\varepsilon}{4(4\pi)^2F_0^2}\left[M_\pi^2 \ln\frac{M_\pi^2}{\mu^2} - M_\eta^2 \ln\frac{M_\eta^2}{\mu^2} - \frac{2}{3}(M_K^2 - M_\pi^2)\left(\ln\frac{M_K^2}{\mu^2} + 1\right)\right]\Bigg\} \\
M_\eta^2 &= \frac{4}{3}M_K^2 - \frac{1}{3}M_\pi^2, \quad \varepsilon = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}}
\end{aligned}
$$

Strong isospin correction

$$\delta_{\mathrm{SU}(2)} = \sqrt{3}\,\varepsilon \left[-\frac{4}{3}\,(F_K/F_\pi - 1) + \frac{1}{3(4\pi)^2 F_0^2} \left(M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right]$$

$$= -0.0043(5)(11)_{\text{higher orders}}$$

$N_f = 2 + 1$ lattice averages from FLAG (G. Colangelo et al., 2011):

$$\varepsilon = \frac{\sqrt{3}}{4R} = 0.0116(13)\ , \quad F_K/F_\pi = 1.193(6)$$

Update of $|V_{us}/V_{ud}|$

$$\begin{aligned}
\frac{|V_{us}|}{|V_{ud}|}\frac{F_K}{F_\pi} &= \left(\frac{\Gamma_{K_{\ell 2(\gamma)}} M_{\pi^\pm}}{\Gamma_{\pi_{\ell 2(\gamma)}} M_{K^\pm}}\right)^{1/2} \frac{1-m_\ell^2/M_{\pi^\pm}^2}{1-m_\ell^2/M_{K^\pm}^2}\,(1-\delta_{\rm EM}/2-\delta_{\rm SU(2)}/2) \\
&= 0.23922(25)\times\left(\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}}\right)^{1/2} \\
&= 0.2763(5)
\end{aligned}$$

Experimental input:

$\Gamma_{\pi_{\mu 2(\gamma)}} = 38.408(7)\,(\mu{\rm s})^{-1}$ PDG 2010

$\Gamma_{K_{\mu 2(\gamma)}} = 51.25(16)\,(\mu{\rm s})^{-1}$ Antonelli et al. 2010

$$F_K/F_\pi = 1.193(6)\,(\text{lattice}) \rightarrow \frac{|V_{us}|}{|V_{ud}|} = 0.2316(12)$$

Fit of V_{us}, V_{ud}

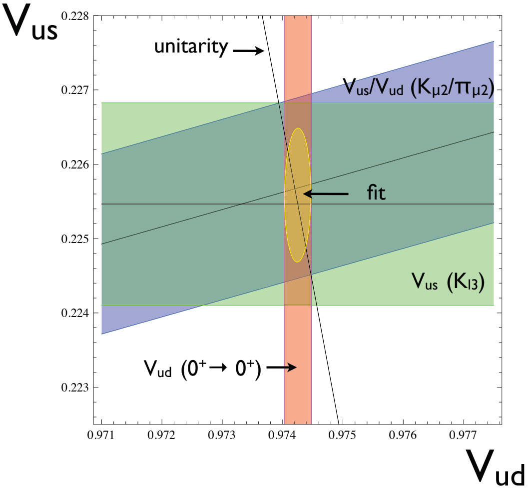
Input:

$|V_{ud}| = 0.97425(22)$ from $0^+ \to 0^+$, J.C. Hardy, I.S. Towner, 2009

$|V_{us}/V_{ud}|$ from $K_{\ell 2}/\pi_{\ell 2}$

$|V_{us}|_{K_{\ell 3}} = 0.2255(5)_{\mathrm{exp}}(12)_{\mathrm{th}}$ from $K_{\ell 3}$ decays and $f_+(0)$ from lattice

$$|V_{ud}| = 0.97425(22)\ , \quad |V_{us}| = 0.2256(9)$$



Vus
0.228
0.227
0.226
0.225
0.224
0.223
unitarity
Vus/Vud (Kμ2/πμ2)
fit
Vus (Kl3)
Vud (0+→ 0+)
0.971
0.972
0.973
0.974
0.975
0.976
0.977
Vud

Conclusions

- EM and strong isospin correction of comparable size:
$$\delta_{\rm EM} = -0.0069(17)\ , \quad \delta_{\rm SU(2)} = -0.0043(12)$$
- Fit result: $|V_{ud}| = 0.97425(22)\ , \quad |V_{us}| = 0.2256(9)$
- Test of CKM unitarity
$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0001(6)$$
- Strong bound on effective scale Λ of dimension-six operators violating quark-lepton universality of charged current weak interactions: $\Lambda > 11\,\mathrm{TeV}\ (90\%\,\mathrm{CL})$