The role of the transverse gauge links in soft collinear effective theory

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M.G.E., A.I., I.S. arXive:1104.0686[hep-ph] and work in progress

Outline

- SCET and its building blocks
- Gauge invariance for covariant gauges
- Gauge invariance for singular gauges (Light-cone gauge)
- A new Wilson line in SCET: T
- The origin of T-Wilson lines in SCET Lagrangian: gauge conditions for different sectors
- Phenomenology
- Conclusions

SCET, an effective theory of QCD

- SCET (soft collinear effective theory) is an effective theory of QCD
- SCET describes interactions between low energy ,”soft” partonic fields and collinear fields (very energetic in one light-cone direction)
- SCET and QCD have the same infrared structure: matching is possible
- SCET helps in the proof of factorization theorems and identification of relevant scales

SCET: Kinematics

Bauer, Fleming, Pirjol, Stewart, ‘00

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

U-soft

$$n^\mu = (1, \vec{n}), \qquad \bar{n}^\mu = (1, -\vec{n})$$

$$(\vec{n}^2 = 1,\ n^2 = 0,\ \bar{n}^2 = 0)$$

Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$\psi(x) = \sum_n \sum_p e^{-ipx} \psi_{n,p}(x)$$

$$\bar{n}p \sim Q$$

$$p_\perp \sim \lambda Q$$

$$np \sim \lambda^2 Q$$

$$\psi = \left(\frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4} \right) \psi = \xi + \phi$$

Integrated out with EOM

Soft modes $(\lambda, \lambda, \lambda)$
do not interact with
(anti) collinear or u-soft
In covariant gauge

SCET

Bauer, Fleming, Pirjol, Stewart, ‘00

SCET $[\lambda \sim m/Q \ll 1]$		
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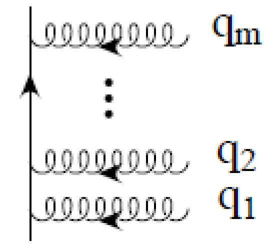


Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$n^\mu = (1, \vec{n}), \qquad \bar{n}^\mu = (1, -\vec{n})$$

$$(\vec{n}^2 = 1,\ n^2 = 0,\ \bar{n}^2 = 0)$$

Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{c,n} = \bar{\xi}_{n,p'} \left\{ i\, n \cdot D + g n \cdot A_{n,q} + \left(\mathcal{P}\!\!\!/_\perp + g A\!\!\!/_{n,q}^\perp \right) \boxed{W \frac{1}{\bar{\mathcal{P}}} W^\dagger} \left(\mathcal{P}\!\!\!/_\perp + g A\!\!\!/_{n,q'}^\perp \right) \right\} \frac{\bar{n}\!\!\!/}{2} \xi_{n,p}$$

$$W_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \bar{n} \cdot A_n(\bar{n}s + x) \right)$$

$$Y_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \bar{n} \cdot A_{us}(\bar{n}s + x) \right)$$

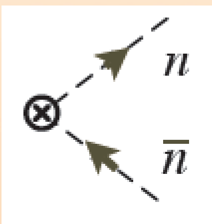
$$\boxed{\begin{aligned} iD_\mu &= i\partial_\mu + gA_{us} \\ inD &= Y_n^\dagger in\partial Y_n \end{aligned}}$$

$$\xi_n^{(0)} = Y_n W_n^\dagger \xi$$

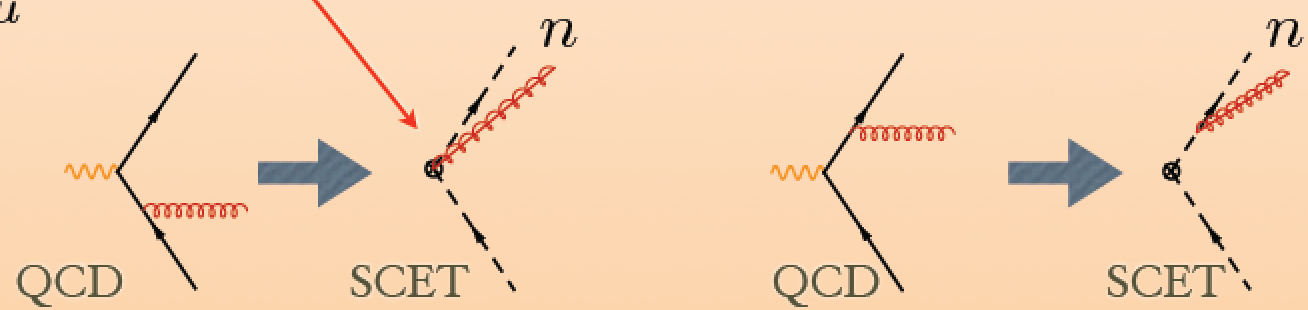
The new fields do not interact anymore with u-soft fields

SCET

Production Current: $Q \gg m$



$$\underbrace{\bar\psi\,\Gamma^\mu\psi}_{\mathcal{J}_i^\mu} \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} = (\bar\xi_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar n} (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$



$$\mathcal{J}_i^\mu(0) = \int d\omega\, d\bar\omega\, C(\omega,\bar\omega,\mu)\, J_i^{(0)\mu}(\omega,\bar\omega,\mu)$$

SCET building blocks

The SCET Lagrangian is formed by gauge invariant building blocks.

Gauge Transformations:

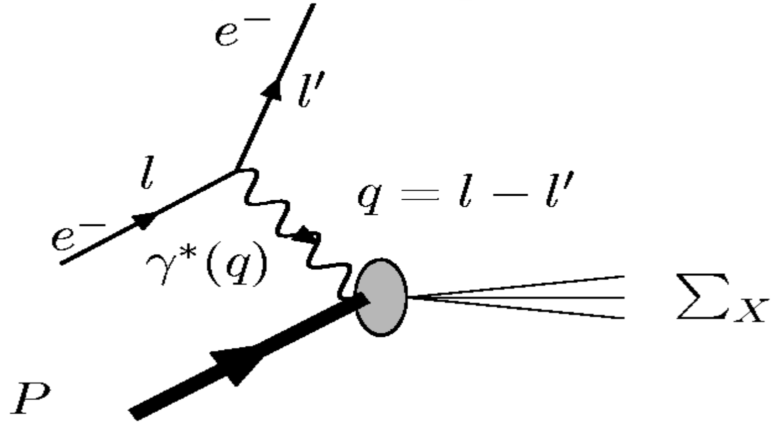
$$\xi \to U\xi$$

$$W_n^+ \to W_n^+ U^+$$

$\longrightarrow$ $W_n^+ \xi$ Is gauge invariant

Factorization Theorem For DIS

$$F_1(x,Q^2) = \sum_f \int_x^1 \frac{dy}{y} C_f\left(\frac{x}{y}, \frac{Q^2}{\mu^2}\right) q_f(y,\mu^2) \ ,$$



•PDF In Full QCD

$$q(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\overline{\psi}(\lambda n)\ \not{n} e^{-ig\int_0^\lambda d\lambda' n\cdot A(\lambda' n)} \psi(0)|P\rangle \ ,$$

•Factorization In SCET $\quad F_1^N(Q^2) = H(Q^2/\mu^2)\phi_N(Q^2/\mu^2)S_N(Q^2/\mu^2)J_N(Q^2/\mu^2) \ ,$

•PDF In SCET: $\phi(x,\mu_I^2) = \left\langle P \left| \bar{\xi}_{\bar{n}} W_{\bar{n}} \delta\left(x - \frac{n\cdot\mathcal{P}_+}{n\cdot p}\right) \frac{\not{n}}{\sqrt{2}} W_{\bar{n}}^\dagger \xi_{\bar{n}} \right| P \right\rangle$

[Neubert et.al, Manohar]

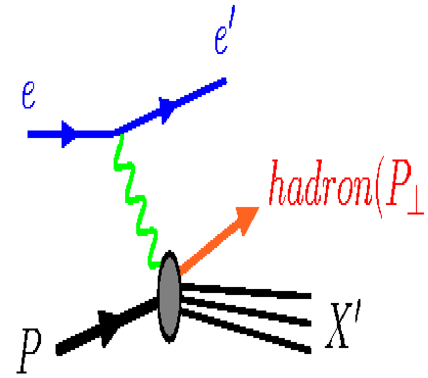
[Stewart et.al]

$\phi(x,\mu_f^2)$ is gauge invariant because each building block is gauge invariant

Factorization Theorem For SIDIS in QCD: Covariant gauge

- In Full QCD And At Low Transverse Momentum:

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \,,$$



Ji, Ma,Yuan '04

- "Naïve" Transverse Momentum Dependent PDF (TMDPDF):

$$q \approx Q/S$$

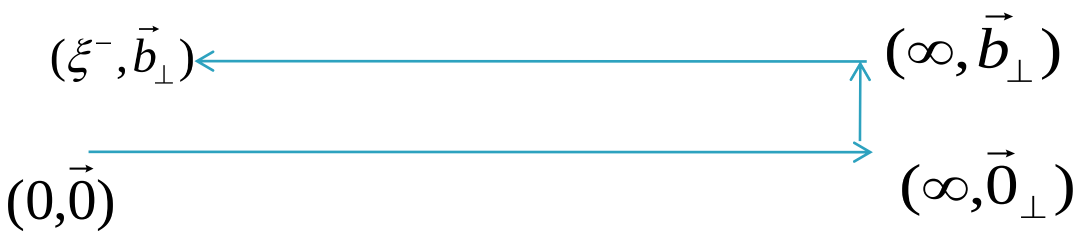
$$\mathcal{Q}(x, k_\perp, \mu, x\zeta) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \left\langle P \left| \overline{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp) \gamma^+ \mathcal{L}_v(\infty; 0) \psi_q(0) \right| P \right\rangle$$

$$\mathcal{L}_v(\infty;\xi) = \exp\left(-ig\int_0^\infty d\lambda v \cdot A(\lambda v + \xi)\right) .$$

Analogous to the W in SCET

This result is true only in "regular" gauges:
Here all fields vanish at infinity

Transverse Gauge Link in QCD



Ji, Ma, Yuan
Ji, Yuan
Belitsky, Ji, Yuan
Cherednikov, Stefanis

- For gauges not vanishing at infinity [Singular Gauges] like the Light-Cone gauge (LC) one needs to introduce an additional Gauge Link which connects $(\infty, \vec{0}_\perp)$ with $(\infty, \vec{b}_\perp)$ to make it Gauge Invariant $A^{\mu\perp}(r_\perp)$
- In LC Gauge This Gauge Link Is Built From The Transverse Component Of The Gluon Field:

$$\tilde{\mathcal{F}}_{i/h}(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp \boldsymbol{\xi}_\perp} \left\langle h | \bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]} [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]} \right.$$
$$\left. \times \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]} [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]} \psi_i(0^-, \boldsymbol{0}_\perp) | h \right\rangle$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_{[\boldsymbol{l}]} \equiv \mathcal{P} \exp \left[ig \int_0^\infty d\tau \, \boldsymbol{l} \cdot \boldsymbol{A}_a t^a (\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau) \right]$$

Gauge Invariant TMDPDF In SCET?

Are TMDPDF fundamental matrix elements in SCET?

Are SCET matrix elements gauge invariant?

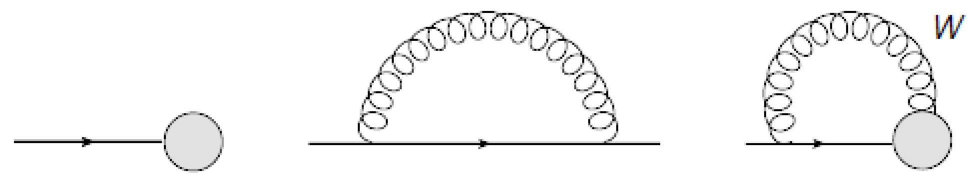
Where are transverse gauge link in SCET?

$$W^{\dagger}\xi \xrightarrow{LC\ gauge} \xi$$

Gauge invariance of SCET building blocks

We calculate $\langle 0 | W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$ at one-loop in Feynman Gauge and in LC gauge

In Feynamn Gauge



$$I_w = \frac{\alpha_s}{4\pi} C_F \, i\not{p} \left[\frac{1}{\epsilon_{\rm UV}} + 1 - \ln \frac{-p^2}{\mu^2} \right],$$

$$W_{n,y} = \mathrm{P} \exp \left(ig \int_{-\infty}^{y} \mathrm{d}s \; \bar{n} \cdot A_n(s\bar{n}) \right)$$

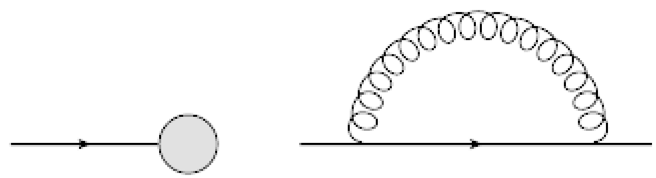
$$I_{\bar{n},Fey} = -2g^2 C_F \mu^{2\varepsilon} i \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ + i0)} \frac{p^+ + k^+}{(p+k)^2 + i0}$$

$$I_{\bar{n}} = -\frac{g^2}{8\pi^2} C_F \gamma^{\mu} \Big[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} + \frac{1}{\epsilon} \ln \frac{-p_1^2}{\mu^2} - \frac{1}{2} \ln^2 \frac{-p_1^2}{\mu^2} + \ln \frac{-p_1^2}{\mu^2} - 2 + \frac{\pi^2}{12} \Big].$$

Gauge invariance of SCET building blocks

We calculate $\left\langle 0 \middle| W_{\bar{n}}^{\dagger} \xi_{\bar{n}} \middle| q \right\rangle$ at one-loop in Feynman Gauge and in LC gauge

In LC Gauge



$$A^{+} = 0 \rightarrow W_{\bar{n}} = W_{\bar{n}}^{\dagger} = 1$$

[Bassetto, Lazzizzera, Soldati]
Canonical quantization imposes ML prescription

$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{\left[k^+\right]} \right)$$

Prescription	$1/[k^+]$
$+i0$	$1/(k^+ + i0)$
$-i0$	$1/(k^+ - i0)$
PV	$1/2(1/(k^+ + i0) + 1/(k^+ - i0))$
ML	$1/(k^+ + i0\mathrm{Sgn}(\mathrm{k}^-))$

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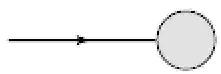
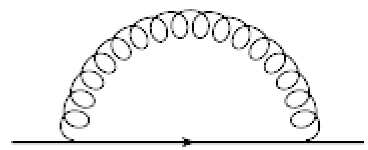
In LC Gauge

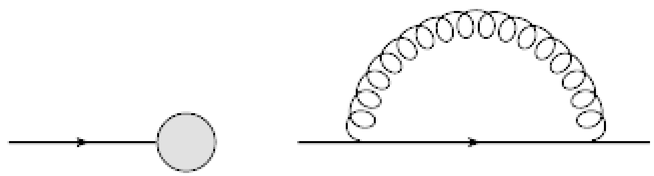
$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{\left[k^+\right]} \right)$$

$$\Sigma_{LC}\left(p\right) = \Sigma_{Fey}\left(p\right) + \Sigma_{Ax}^{(\mathrm{Pres})}\left(p\right) = \left(I_{w,Fey}\left(p\right) + I_{w,Ax}^{(\mathrm{Pres})}\left(p\right)\right)\frac{ip^2}{p^+}\frac{\not{n}}{\sqrt{2}}$$

Gauge invariance of SCET building blocks

We calculate $\left\langle 0 \middle| W_{\bar{n}}^{\dagger} \xi_{\bar{n}} \middle| q \right\rangle$ at one-loop in Feynman Gauge and in LC gauge

In LC Gauge



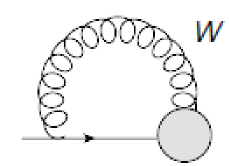
$$\Sigma_{LC}(p) = \Sigma_{Fey}(p) + \Sigma_{Ax}^{(ML)}(p) = \left(I_{w,Fey}(p) + I_{w,Ax}^{(ML)}(p)\right) \frac{ip^2}{p^+} \frac{\not{n}}{\sqrt{2}}$$

$$I_{w,Ax}^{(\mathrm{ML})} = 4ig^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{\left(k^2 + i0\right)\left((p+k)^2 + i0\right)\left[k^+\right]}$$

$$\frac{1}{[k^+]} = \frac{\theta(k^-)}{k^+ + ip^+\eta} + \frac{\theta(-k^-)}{k^+ - ip^+\eta}$$

The gauge invariance is ensured when

$$-\frac{1}{2} I_{w,Ax}^{(ML)} = I_{\bar{n},Fey}$$



The result of this is independent of η and has got only a single pole. Zero-bin subtraction is nul in ML.

Gauge invariance in SCET

The SCET matrix element $\langle 0|W_{\bar{n}}^{\dagger}\xi_{\bar{n}}|q\rangle$ is not gauge invariant . Using LC gauge the result of the one-loop correction depends on the used prescription.

Gauge invariance in SCET

In order to restore gauge invariance we have to introduce a new Wilson line, T, in SCET matrix elements

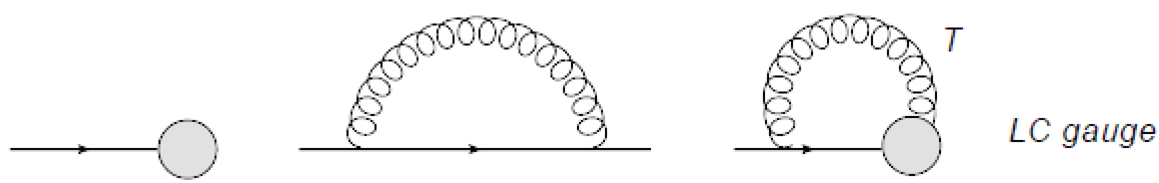
$$T_{\bar{n}}^{\dagger}(x^{+},x_{\perp})=P\exp\left[ig\int_{0}^{\infty}d\tau\,\mathbf{l}_{\perp}\cdot\mathbf{A}_{\perp}(\infty^{-},x^{+};\mathbf{l}_{\perp}\tau+\mathbf{x}_{\perp})\right]$$

And the new gauge invariant matrix element is $\boxed{\langle 0|\,T_{\bar{n}}^{\dagger}W_{\bar{n}}^{\dagger}\xi_{\bar{n}}\,|q\rangle}$

The T-Wilson Line

In covariant gauges $T = T^\dagger = 1$, so we recover the SCET results $\langle 0 | W_{\bar{n}}^\dagger \xi_{\bar{n}} | q \rangle$

In LC gauge

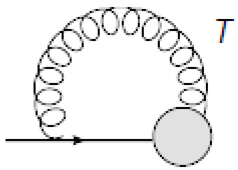


$$\langle 0 | T_{\bar{n}}^\dagger \xi_{\bar{n}} | q \rangle$$

The T-Wilson Line

$$I_{T,Ax}^{(ML)} = 2C_F g^2 \mu^{2\varepsilon} i \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{(k^2 + i0)((p+k)^2 + i0)} \left[\frac{C_\infty^{(ML)}}{k^+ - i0} - \frac{C_\infty^{(ML)}}{k^+ + i0} \right]$$

$$C_\infty^{(ML)} = \theta(-k^-)$$



LC gauge

$$\boxed{I_{\bar{n},Fey} = \frac{-1}{2} I_{w,Ax}^{(\mathrm{Pres})} + I_{T,Ax}^{(\mathrm{Pres})}}$$

All prescription dependence cancels out and gauge invariance is restored no matter what prescription is used

$$\langle 0 | W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle \longrightarrow \langle 0 | T_{\bar{n}}^{\dagger} W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$$

Covariant Gauges — In All Gauges

The T–Wilson Line in other prescriptions

Let us consider the pole part of the interesting integral with $1/[k^{+}]=1/(k^{+}\pm i\eta)$ or the PV prescription. The result is

$$I_{\bar{n}}^{\pm i\eta}=-2\frac{\alpha_s}{4\pi}C_F\left(\frac{\mu^2}{-p^2}\right)^{\varepsilon}\frac{1}{\varepsilon}\int_0^1 dz\,\frac{(1-z)^{1-\varepsilon}z^{-\varepsilon}}{z\mp i\eta}=-\frac{g^2}{4\pi^2}C_F\frac{1}{\varepsilon}\left[1+\ln\mp i\eta\right]+\mathit{finite}$$

And in PV the result does not have any imaginary part. The gauge invariance is restored either with the T with a prescription dependent factor

Prescription	C∞
+i0	0
-i0	1
PV	1/2

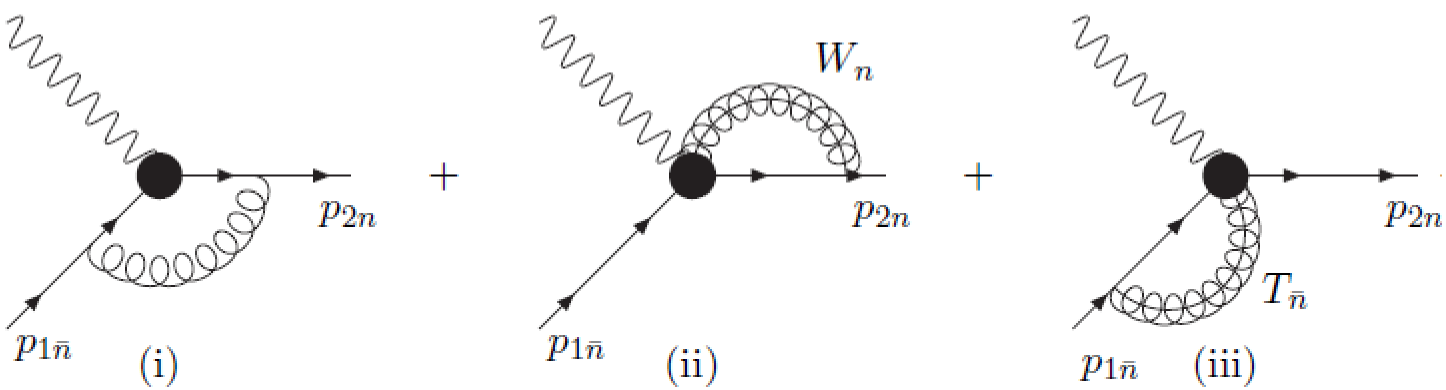
OR with zero-bin Subtraction!!

The values of this constant depend also on the convention for inner/outer moments

Where does the T-Wilson Line come from?

Is there a way to understand the T-Wilson lines from the SCET Lagrangian?

An example, the quark form factor: from QCD to SCET in LCG



The T-Wilson line is born naturally in One loop matching.

Where does the T–Wilson Line come from?

$$(\bar{n}A, nA, A_\perp) \sim Q(1, \lambda^2, \lambda)$$

$$\bar{n}A = 0$$

In the canonical quantization of of the gauge field (Bassetto et al.)

$$A^a_\mu(k) = T^a_\mu(k)\delta(k^2) + \bar{n}_\mu \frac{\delta(\bar{n}k)}{k^2_\perp}\Lambda^a(nk, k_\perp) + \frac{ik_\mu}{k^2_\perp}\delta(\bar{n}k)\boxed{U^a(nk, k_\perp)}$$

$$\bar{n}^\mu T^a_\mu(k) = 0; \quad k^\mu T^a_\mu(k) = 0;$$

We define $A^{(\infty)}(x^+, x_\perp) \overset{def}{=} A(x^+, \infty^-, x_\perp)$

$$\tilde{A}(x^+, x^-, x_\perp) \overset{def}{=} A(x^+, x^-, x_\perp) - A^{(\infty)}(x^+, x_\perp)$$

And we can show

$$i\mathrm{D}^\mu{}_\perp \overset{def}{=} i\mathrm{D}^\mu{}_\perp + gA^{\mu(\infty)}_\perp$$

$$\boxed{i\mathrm{D}^\mu{}_\perp = Ti\mathrm{D}^\mu{}_\perp T^\dagger}$$

$$T^\dagger = P\exp\left[-ig\int_0^\infty d\tau\, l_\perp \cdot A^{(\infty)}_\perp(x^+, x_\perp - l_\perp\tau)\right]$$

The T–Wilson Lines in SCET–I

In SCET–I only collinear and u–soft fields. The first step to obtain the SCET Lagrangian is integrating out energetic part of spinors

$$\mathcal{L} = \bar{\xi}_n \left(inD + i\not{D}_\perp \frac{1}{i\bar{n}D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \xi_n$$

And then applying multipole expansion,

$$x_n \sim 1/Q(1, 1/\lambda^2, 1/\lambda)$$
$$x_{us} \sim 1/Q(1/\lambda^2, 1/\lambda^2, 1/\lambda^2)$$

$$\boxed{\mathcal{L}_{\mathcal{I}} = \bar{\xi}_n \left(in\not{D}_n + gnA_{us}(x^+) + i\not{D}_{n\perp} W_n^T \frac{1}{i\bar{n}\partial} W_n^{T\dagger} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n}$$

Where $W_n^T = T_n W_n$

U–soft field do not give rise to any transverse gauge link!!
There are no transverse u–soft fields and they cannot depend on transverse coordinates!!

The T-Wilson Lines in SCET-II

Now the degrees of freedom are just collinear and soft

$$(\bar{n}A_n, nA_n, A_{n\perp}) \sim Q(1,\eta^2,\eta); \qquad (\bar{n}p_n, np_n, p_{n\perp}) \sim Q(1,\eta^2,\eta);$$

$$(\bar{n}A_s, nA_s, A_{s\perp}) \sim Q(\eta,\eta,\eta); \qquad (\bar{n}p_s, np_s, p_{s\perp}) \sim Q(\eta,\eta,\eta);$$

No interaction is possible for on-shell states

$$\mathcal{L}_{II} = \bar{\xi}_n \left(in\not{D}_n + i\not{D}_{n\perp} W_n \frac{1}{i\bar{n}\partial} W_n^{\dagger} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n$$

Is this true in every gauge?

The T-Wilson Lines in SCET-II

$$(\bar{n}A_n, nA_n, A_{n\perp}) \sim Q(1,\eta^2,\eta); \qquad (\bar{n}p_n, np_n, p_{n\perp}) \sim Q(1,\boxed{\eta^2},\eta);$$

$$(\bar{n}A_s, nA_s = 0, A_{s\perp}) \sim Q(\eta,\varnothing,\eta); \qquad (\bar{n}p_s, np_s, p_{s\perp}) \sim Q(\eta,\boxed{\eta},\eta);$$

The gauge ghost however acts only on some momentum components

$$\prod_i \phi_n^i(x) A_{s\perp}^{\infty}(x^-, x^\perp) \to \prod_i \phi_n^i(x) A_{s\perp}^{\infty}(0, x^\perp)$$

Thus the covariant derivative is $iD^\mu = i\partial^\mu + gA_n^\mu(x) + gA_{s\perp}^{(\infty)\mu}(0^-, x_\perp)$

The decoupling of soft fields requires

$$A_n^{(0)\mu}(x) = T_{sn}(x_\perp) A_n^\mu(x) T_{sn}^\dagger(x_\perp)$$

$$T_{sn} = \bar{P}\exp\left[ig\int_0^\infty d\tau\, l_\perp \cdot A_{s\perp}^{(\infty)}(0^-, x_\perp - l_\perp\tau)\right]$$

The T-Wilson Lines in SCET-II

The new SCET-II Lagrangian is

$$\boxed{\mathcal{L}_{II} = \bar{\xi}_n^{(0)}\left(in D_n^{(0)} + i\not{D}_{n\perp}^{(0)} W_n^{T(0)} \frac{1}{i\bar{n}\partial} W_n^{T(0)\dagger} i\not{D}_{n\perp}^{(0)}\right)\frac{\not{\bar{n}}}{2}\xi_n^{(0)}}$$

$$A_n^{(0)\mu}(x) = T_{sn}(x_\perp) A_n^\mu(x) T_{sn}^\dagger(x_\perp)$$

$$D_n^{(0)\mu} = i\partial^\mu + g A_n^{(0)\mu}$$

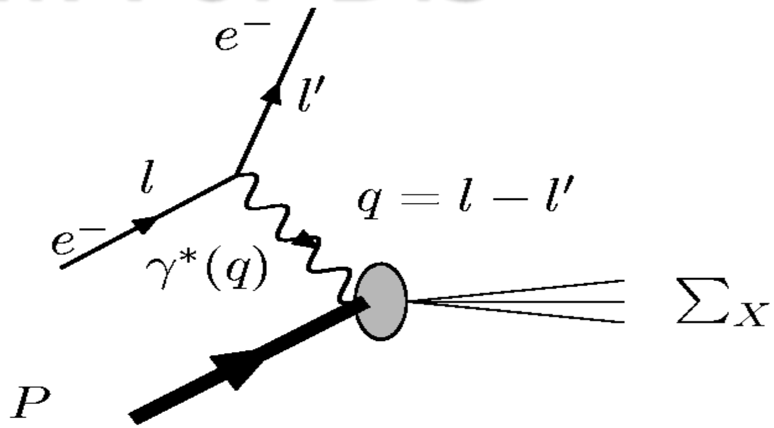
$$\xi_n^{(0)} = T_{sn}(x_\perp)\xi_n(x)$$

$$T_{sn} = \bar{P}\exp\left[ig\int_0^\infty d\tau\, l_\perp \cdot A_{s\perp}^{(\infty)}(0^-, x_\perp - l_\perp\tau)\right]$$

Applications

TMDPDF
Drell-Yan at low Pt [Becher,Neubert]
Higgs production at low Pt [Mantry,Petriello]
Beam functions [Jouttenus,Stewart, Tackmann,Waalewijn]
Heavy Ion physics – Jet Broadening [Ovanesyan,Vitev]
…

Factorization Theorem For DIS



$$F_1(x, Q^2) = \sum_f \int_x^1 \frac{dy}{y} C_f\left(\frac{x}{y}, \frac{Q^2}{\mu^2}\right) q_f(y, \mu^2) \ ,$$

•PDF In Full QCD

$$q(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\overline{\psi}(\lambda n) \not{n} e^{-ig\int_0^\lambda d\lambda' n\cdot A(\lambda' n)} \psi(0)|P\rangle \ ,$$

•Factorization In SCET $F_1^N(Q^2) = H(Q^2/\mu^2)\phi_N(Q^2/\mu^2)S_N(Q^2/\mu^2)J_N(Q^2/\mu^2) \ ,$

[Neubert et.al]

•PDF In SCET: $\phi(x, \mu_I^2) = \left\langle P \left| \bar{\xi}_{\bar{n}} W_{\bar{n}} \delta\left(x - \frac{n\cdot\mathcal{P}_+}{n\cdot p}\right) \frac{\not{n}}{\sqrt{2}} W_{\bar{n}}^\dagger \xi_{\bar{n}} \right| P \right\rangle$

[Stewart et.al]

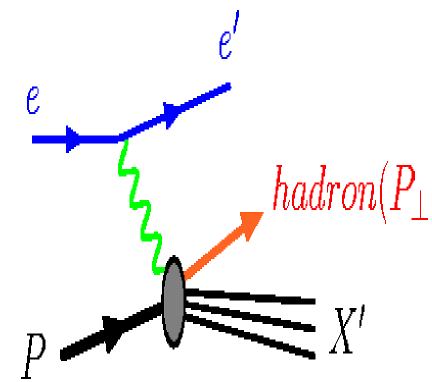
$\phi(x, \mu_f^2)$ is gauge invariant because each building block is gauge invariant

Factorization Theorem For SIDIS in QCD: Covariant gauge

- In Full QCD And At Low Transverse Momentum: Ji, Ma,Yuan '04

$$F(x_B, z_h, P_{h\perp}, Q) = \sum_{q=u,d,s,..} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{l}_\perp \, q(x_B, k_\perp, \mu^2, x_B\zeta, \rho)\, \hat{q}_T(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho)$$

$$\times S(\vec{l}_\perp, \mu^2, \rho)\, H(Q^2, \mu^2, \rho)\, \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{l}_\perp - P_{h\perp})$$



μ Is renorrmalization scale; ρ is a rapidity cut-off

- "Naïve" Transverse Momentum Dependent PDF (TMDPDF):

$$q \approx Q/S$$

$$\mathcal{Q}(x, k_\perp, \mu, x\zeta) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \left\langle P \left| \overline{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp) \gamma^+ \mathcal{L}_v(\infty; 0) \psi_q(0) \right| P \right\rangle$$

$$\mathcal{L}_v(\infty; \xi) = \exp\left(-ig\int_0^\infty d\lambda v \cdot A(\lambda v + \xi)\right)$$

Analogous to the W in SCET

This result is true only in "regular" gauges:
Here all fields vanish at infinity

Transverse Gauge Link in QCD

Ji, Ma, Yuan
Ji, Yuan
Belitsky, Ji, Yuan
Cherednikov, Stefanis

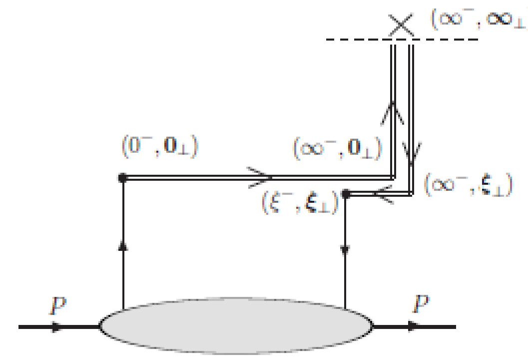
- For gauges not vanishing at infinity [Singular Gauges] like the Light-Cone gauge (LC) one needs to introduce an additional Gauge Link which connects $(\infty, \vec{0}_\perp)$ with $(\infty, \vec{b}_\perp)$ to make it Gauge Invariant $A^{\mu\perp}(r_\perp)$
- In LC Gauge This Gauge Link Is Built From The Transverse Component Of The Gluon Field:

$$\tilde{\mathcal{F}}_{i/h}\left(x, \boldsymbol{k}_\perp\right) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp \boldsymbol{\xi}_\perp} \left\langle h \right| \bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]} [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]}$$

$$\times \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]} [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]} \psi_i(0^-, \boldsymbol{0}_\perp) \left| h \right\rangle$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_{[\boldsymbol{l}]} \equiv \mathcal{P} \exp\left[ig \int_0^\infty d\tau \, \boldsymbol{l} \cdot \boldsymbol{A}_a t^a (\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau) \right]$$

TMDPDF



$$\underline{\chi}_{\bar{n}}(y) \equiv T_{\bar{n}}^{\dagger}(y^+, \mathbf{y}_\perp) W_{\bar{n}}^{\dagger}(y) \xi_{\bar{n}}(y)$$

$$\phi_{q/P} = \langle P_{\bar{n}} | \underline{\bar{\chi}}_{\bar{n}}(y) \delta\left(x - \frac{n\mathcal{P}}{np} \right) \delta^{(2)}(p_\perp - \mathcal{P}_\perp) \frac{\not{n}}{\sqrt{2}} \underline{\chi}_{\bar{n}}(0) | P_{\bar{n}} \rangle$$

We Can Define A Gauge Invariant TMDPDF In SCET (And Factorize SIDIS)

Drell–Yan At Low PT

$$d\sigma = \frac{4\pi\alpha^2}{3q^2 s}\frac{d^4q}{(2\pi)^4}\int d^4x\, e^{-iq\cdot x}\,(-g_{\mu\nu})\,\langle N_1(p)\,N_2(\bar{p})|\,J^{\mu\dagger}(x)\,J^\nu(0)\,|N_1(p)\,N_2(\bar{p})\rangle$$

$$J^\mu \to C_V(-q^2 - i\varepsilon, \mu)\sum_q \left(g_L^q\,\bar{\chi}_{\overline{hc}}\,S_{\bar{n}}^\dagger\,\gamma^\mu\,\frac{1-\gamma_5}{2}\,S_n\,\chi_{hc} + g_R^q\,\bar{\chi}_{\overline{hc}}\,S_{\bar{n}}^\dagger\,\gamma^\mu\,\frac{1+\gamma_5}{2}\,S_n\,\chi_{hc}\right)$$

$$\chi_{\overline{hc}} = W_{\overline{hc}}^\dagger\,\xi_{\overline{hc}}$$

Introduce Gauge Invariant Quark Jet:

$$\begin{aligned} \underline{\chi}_{\bar{n}}(y) &\equiv T_{\bar{n}}^\dagger(y^+, \mathbf{y}_\perp)W_{\bar{n}}^\dagger(y)\xi_{\bar{n}}(y), \\ \phi_{q/P} &= \langle P_{\bar{n}}|\underline{\overline{\chi}}_{\bar{n}}(y)\delta\left(x - \frac{n\mathcal{P}}{np}\right)\delta^{(2)}(p_\perp - \mathcal{P}_\perp)\frac{\not{n}}{\sqrt{2}}\underline{\chi}_{\bar{n}}(0)|P_{\bar{n}}\rangle \end{aligned}$$

The TMDPDF Is Indeed Gauge Invariant.

Collinear Anomaly

$$d\sigma = \frac{4\pi\alpha^2}{3N_c q^2 s}\frac{d^4q}{(2\pi)^4}\int d^4x\, e^{-iq\cdot x}\,|C_V(-q^2,\mu)|^2 \sum_q \frac{|g_L^q|^2+|g_R^q|^2}{2}\,\hat{W}_{\rm DY}(0)$$
$$\times\, \langle N_1(p)|\,\bar{\chi}_{hc}(x_+ + x_\perp)\,\frac{\not{\bar{n}}}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\,\chi_{\overline{hc}}(x_- + x_\perp)\,|N_2(\bar{p})\rangle\,.$$

Notice That The Cross-Section Is Independent Of The Renormalization Scale (RG Invariance).

Also (Up To Three Loop Calculation!) $\mu\frac{dC}{d\mu} = [A\ln\frac{q^2}{\mu^2} + B]C$

$$\hat{W}_{\rm DY}(x) = \frac{1}{N_c}\,\langle 0|\,\mathrm{Tr}\big[\overline{\mathbf{T}}\big(S_n^\dagger(x)\,S_{\bar{n}}(x)\big)\,\mathbf{T}\big(S_{\bar{n}}^\dagger(0)\,S_n(0)\big)\big]\,|0\rangle \qquad \hat{W}_{\rm DY}(0) = 1$$

For Vanishing Soft Function, The Product Of Two TMDPDFs Has to Be Logarithmically Dependent On The Renormalization Scale.
This is Impossible Unless There Is Anomaly.

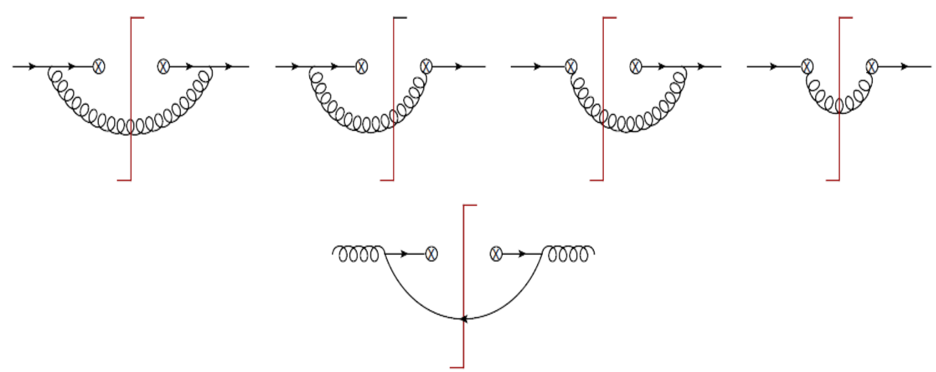
Origin Of Collinear Anomaly

In The Absence Of Soft Interactions Different Collinear Sectors Do Not Interact So There Is No Way To Generate The Q-Dependence

Classically Each Collinear Lagrangian Is Invariant Under Rescaling of Collinear Momentum.

For TMDPDF Quantum Loop Effects Needs Regulation Then Classical Invariance Is Lost However The Q-Dependence Is Obtained

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$



$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p)|\, \bar{\chi}(t\bar{n} + x_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi(0)\, |N(p)\rangle\,,$$

The TMDPDF Is Ill-Defined And We need To Introduce New Set Of NP Matrix Elements

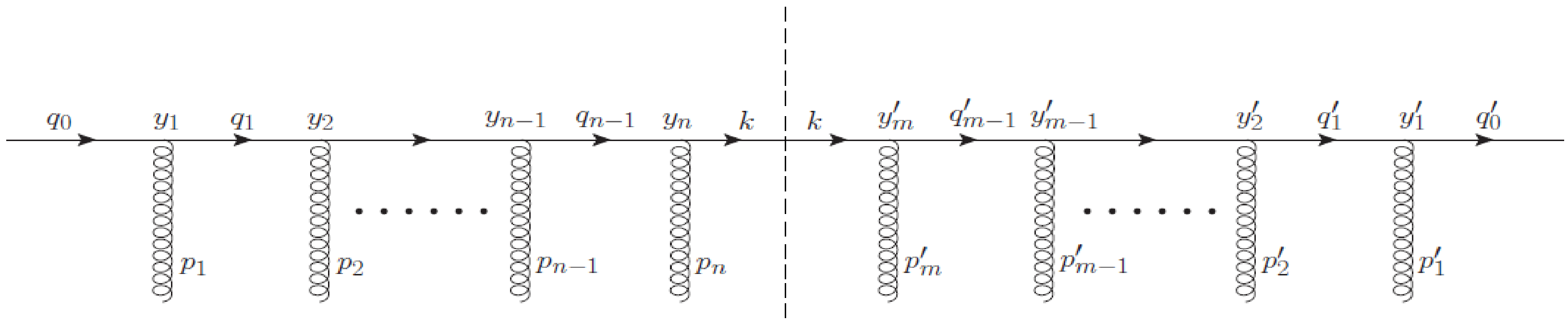
$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu$$

Re-factorization Into The Standard PDF

$$B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, I_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2\, x_T^2)\,.$$

The Analysis Of Becher-Neubert Ignores Two Notions: Transverse Gauge Links And Soft-Gluon Subtraction Needed To Avoid Double Counting!
(Currently Investigated.)

•Application To Heavy-Ion Physics



$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3\, N_c} \int dy^+ dy_\perp dy'_\perp \, e^{-ik_\perp\cdot(y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[y^+, y'_\perp] - 1 \right) \left(W_F[y^+, y_\perp] - 1 \right) \right] \right\rangle$$

D´Eramo, Liu, Rajagopal

$$W_F\,[y^+, y_\perp] \equiv P\left\{ \exp\left[ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp) \right] \right\}$$

In LC Gauge The Above Quantity Is Meaningless. If We Add To It The T-Wilson line Then We Get A Gauge Invariant Physical Entity.

Conclusions

The usual SCET building blocks have to be modified introducing a New Gauge Link, the T-Wilson line.

Using the new formalism we get gauge invariant definitions of non-perturbative matrix elements. In particular the T is compulsory for matrix elements of fields separated in the transverse direction. These matrix elements are relevant in semi-inclusive cross sections or transverse momentum dependent ones.

It is possible that the use of LC gauge helps in the proofs of factorization. The inclusion of T is so fundamental. Work in progress in this direction.

Conclusions

It is definetely possible to understand the origin of T-Wilson lines in a Lagrangian framework for EFT.

Every sector of the SCET can be appropriately written in LCG.

The LCG has peculiar property for loop calculation and can avoid the introduction of new ad-hoc regulators

There is a rich phenomenology to be studied…

so a lot of work in progress!!

THANKS!