Top-Antitop Threshold - Electroweak corrections

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Outline

I Top-pair production at linear colliders near threshold

II Non-resonant electroweak NLO contributions

III Phase space matching

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V Conclusions
I. Top-pair production near threshold

**Future linear colliders** (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \approx 350$ GeV will produce lots of $t\bar{t}$ pairs, allowing for a **threshold scan** of the top cross section

$\leftrightarrow$ Precise determination of the top mass $m_t$, the width $\Gamma_t$ and the Yukawa coupling $\lambda_t$ without the uncertainties/ambiguities of hadron colliders → $\delta m_t^{\text{exp}} \approx 30$ MeV

$\sim m_t$ is a crucial input for electroweak precision observables!

**Requires also precise theoretical prediction**

$\Rightarrow$ $\delta\sigma/\sigma \sim 2 - 3\%$ ($\delta\sigma \sim 5$ fb below threshold)

QCD corrections are known (almost) up to NNLL/NNNLO, but electroweak (NLO) contributions due to top decay were missing!

**Note:** once EW effects are turned on, the physical final state is $W^+W^-b\bar{b}$

$\Rightarrow$ $\sigma(e^+e^- \rightarrow W^+W^-b\bar{b})$ in the $t\bar{t}$ resonance region and allow for invariant-mass cuts on reconstructed $t, \bar{t}$
Decay $t \rightarrow bW^+$ with $\Gamma_t \approx 1.5$ GeV $\gg \Lambda_{QCD}$
$\Rightarrow$ $tt$ is perturbative at threshold

Top quarks move slowly near threshold:

$\leftrightarrow$ sum $\left( \frac{\alpha_s}{v} \right)^n$ from “Coulomb gluons” to all orders $\rightarrow$ NRQCD

$$R = \frac{\sigma_{tt}}{\sigma_{\mu+\mu-}} = v \sum_n \left( \frac{\alpha_s}{v} \right)^n \left( \{1\}_{LO} + \{\alpha_s, v\}_{NLO} + \{\alpha_s^2, \alpha_s v, v^2\}_{NNLO} + \ldots \right)$$

Further RG improvement by summing also $(\alpha_s \ln v)^m$: LL, NLL, ... $\rightarrow vNRQCD$

- **NNLO QCD corrections**
  Hoang, Teubner '98-99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99;
  Nagano, Ota, Sumino '99; Penin, Pivovarov '98-99

- **NNLO & (almost) NNLL**
  Hoang, Manohar, Stewart, Teubner '00-01;
  Hoang '03; Pineda, Signer '06;
  Hoang, Stahlhofen '06-11

- **(almost) NNNLO**
  Beneke, Kiyo, Schuller '05-08 $\rightarrow$ see figure
Effective field theory (EFT) for pair production of unstable particles near threshold, based on separation of resonant and nonresonant fluctuations

Hoang, Reisser '05 ↔ Beneke, Chapovsky, Khoze, Signer, Zanderighi '01-04; Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07-08

- power counting for finite width effects:
  \[
  \frac{\Gamma_t}{m_t} \sim \alpha_{\text{EW}} \sim \alpha_s^2 \sim v^2 \ll 1
  \]

- hard modes \( \sim m_t \) (including top decay products) are integrated out
  \( \sim \) EFT with potential (nearly on-shell) top quarks and ultrasoft gluons

- Extract cross section for \( e^+e^- \rightarrow W^+W^-b\bar{b} \) from appropriate cuts of the \( e^+e^- \rightarrow e^+e^- \) forward-scattering amplitude:

  - resonant contributions
  - non-resonant contributions
Electroweak effects at LO

- Replacement rule: \( E = \sqrt{s} - 2m_t \rightarrow E + i\Gamma_t \)

\( \Rightarrow \) unstable top propagator

\[
\delta \mathcal{L} = \sum_p \psi_p^\dagger \frac{i}{2} \frac{\Gamma_t}{p^0 - \frac{p^2}{2m} + i\Gamma_t/2} \psi_p
\]

Electroweak effects at NLO

- Exchange of “Coulomb photon”: trivially extension of QCD corrections

- Gluon exchange involving the bottom quarks in the final state \( \Rightarrow \) these contributions vanish at NLO for the total cross section, \( \text{Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94} \)

also negligible if loose top invariant-mass cuts are applied; remains true at NNLO \( \text{Hoang, Reisser '05; Beneke, Jantzen, RF '10} \)

- Non-resonant (hard) corrections to \( e^+ e^- \rightarrow W^+ W^- b \bar{b} \)

which account for the production of the \( Wb \) pairs by highly virtual tops or with only one or no top

\[
\Delta \sigma_{\text{non-res}} = \frac{1}{s} \sum_k \text{Im} \left[ C_{4e}^{(k)} \right] \langle e^+ e^- | \mathcal{O}_{4e}^{(k)} | e^+ e^- \rangle
\]
Electroweak (non-trivial) effects at NNLO

- **lifetime dilatation** term
  \[ \delta \mathcal{L} = \sum_p \psi_p^\dagger \left( i \frac{\Gamma_t}{2} \frac{p^2}{2m} \right) \psi_p \]

- **absorptive parts** in the 1-loop matching coeffs. of the production operators (arising from $bW$ cuts) Hoang, Reisser '06

\[ \Rightarrow \text{reproduce interferences between double and single resonant amplitudes} \]

\[ J_p = \left[ C_{LL}^{\text{born}} + C_{NLL}^{\text{QCD}} + C_{NNLL}^{\text{QCD}} + i C_{NNLL}^{bW,\text{abs}} + C_{\text{EW NNLL}} + \ldots \right] \left( \begin{array}{c} e^+ \\ e^- \\ \bar{t} \end{array} \right) \]

- **real part** of hard one-loop EW corrections Kuhn, Guth '92; Hoang, Reisser '06
Electroweak effects at NNLO (cont.)

- No EW corrections to the Coulomb potential at NNLO

\[ \sum_{\text{res}} \delta \sigma \sim \frac{q^2}{m_t^2} \text{suppressed} \]

- Resonant NNLO corrections produce “finite-width divergences” (also called “phase space divergences”)

\[ \Delta \sigma_{\text{tot}} \sim m_t^2 \frac{\alpha_s \Gamma_t}{\epsilon} \]

\[ \xrightarrow{\text{anom. dim. of } \mathcal{O}_{4e}^{(k)} \text{ operators}} \]

\[ \xrightarrow{\mathcal{C}_{4e}^{(k)}(\mu) \text{ resums NLL phase space logs}} \]

- \( \mathcal{C}_{4e}^{(k)}(m_t) \) determined by the non-resonant contributions. Beyond NLO the exact computation is hard, but dominant terms can be obtained for moderate top invariant mass cuts \( \Rightarrow \text{Phase space matching} \)
II. Electroweak non-resonant NLO contributions
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Beneke, Jantzen, RF '10

⇒ cuts through \( bW^+\bar{t} \) (see diagrams) and \( \bar{b}W^-t \) (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:
  \[ p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \Rightarrow \sum(p_t^2) \sim m_t^2 \alpha_{\text{EW}} \]
  \( \Rightarrow \Gamma_t = 0 \)

- suppressed w.r.t. LO (\( \sim v \)) by
  \[ \alpha_{\text{EW}}/v \sim \alpha_s \]

- expansion in
  \[ \delta = \frac{s - 4m_t^2}{4m_t^2} \]
  \( \Rightarrow \) at NLO:
  \[ s = 4m_t^2 \]
II. Electroweak non-resonant NLO contributions

Form of non-resonant contributions

In terms of the invariant mass of the $bW^+$ system, $p_t^2 = (p_b + p_{W+})^2$, ($p_t \rightarrow$ also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 (m_t^2 - p_t^2)^{1/2-\epsilon} H_i \left( \frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2} \right)$$

with $\Delta^2 = M_W^2$ for the total cross section

[Phase-space factor $(m_t^2 - p_t^2)^{1/2-\epsilon}$ in dim. reg. regularizes the end-point singularity for h1]

Applying invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

$\leftrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (corresponding to $\Delta M_t \gg \Gamma_t$)

$\sim$ cut has no effect in the resonant contributions

[In contrast: for tight cuts with $\Lambda^2 \sim m_t\Gamma_t$ ($\Delta M_t \sim \Gamma_t$), non-resonant contributions vanish and cuts only affect the resonant contributions]
Non-resonant NLO contributions: from numeric integration over $p_t^2$ (and over one angle for some diagrams), the integrand is an analytic function of $p_t^2/m_t^2$ and $M_W^2/m_t^2$; cut-dependence enters through the integration limit.

Parameters: on-shell (pole) masses, $m_t = 172$ GeV, $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550$ GeV, $\alpha$ and $\sin^2\theta_W$ from $G_F$, $M_W$, $M_Z$.
III. Phase-space matching
Alternative approach to compute non-resonant contributions

- Non-resonant contributions obtained for moderate invariant-mass cuts, $m_t \Gamma_t \ll \Lambda^2 \lesssim m_t^2$, as a series:

$$\frac{\Gamma_t}{\Lambda} \sum_{n,\ell,k} \left[ \left( \frac{m_t \Gamma_t}{\Lambda^2} \right)^n \times \left( \frac{\Lambda^2}{m_t^2} \right)^\ell \right] \times \left( \alpha_s \frac{m_t}{\Lambda} \right)^k \quad n, \ell, k = 0, 1, \ldots$$

- NLO, NNLO and (partial) N$^3$LO contributions obtained (counting $\Lambda \sim m_t$) → NLL, NNLL, N$^3$LL in the vNRQCD framework

- Assumption: non-resonant background processes are small (✓ at NLO!)

- Beyond NLO, phase space matching approach cannot be applied to larger cuts up to the total cross section ×
**III. Phase space matching**

\[
\sigma_{t\bar{t}} \sim \int_{-\infty}^{\infty} dp^0 \int_{0}^{+\infty} d|p| p^2 \frac{\Gamma_t^2}{|E/2 + p^0 - \frac{p^2}{2m_t} + i\frac{\Gamma_t}{2}|^2 |E/2 - p^0 - \frac{p^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}
\]

**WWb\bar{b} phase space:**

**stable tops**

\[
\frac{i}{p^0 - \frac{p^2}{2m_t} + i\epsilon} \rightarrow (2\pi) \delta(p^0 - \frac{p^2}{2m_t})
\]

on-shell top: \( p^0 = \frac{p^2}{2m_t} - \frac{E}{2} \)

on-shell antitop: \( p^0 = -\frac{p^2}{2m_t} + \frac{E}{2} \)


\[ \sigma_{\text{Born}}^{t\bar{t}} (\Lambda) \sim \int_{-\Lambda^2/2m_t}^{\Lambda^2/2m_t} dp^0 \int_0^{f(p^0, \Lambda)} dp^2 \int_0^{\Gamma_{t}^2} \left( \frac{E}{2} + p^0 - \frac{p^2}{2m_t} \right) + i \frac{\Gamma_{t}}{2} \left| \frac{E}{2} - p^0 - \frac{p^2}{2m_t} + i \frac{\Gamma_{t}}{2} \right|^2 \]

\[ (p_{\pm}^2 - m_t^2) / 2m_t \]

\[ \Lambda^2 = \text{cut on } t \text{ and } \bar{t} \]

\[ \text{invariant masses} \]

\[ |p_{\pm}^2 - m_t^2| < \Lambda^2 \ll m_t^2 \]

\[ \text{double resonant} \]

\[ \text{nonrel. expansion valid} \]
CONCEPTS OF PHASE SPACE MATCHING

III. Phase space matching

\[ \sigma_{tt}^{\text{Born}}(\Lambda) \sim \]

\[ + \text{ expansion for } E, \Gamma_t \ll \Lambda^2/m_t \ll m_t \]

\[ \xrightarrow{\text{EFT}} 2 \text{ Im} \left[ \right] \]

\[ + \sum_n \tilde{C}^{(n),0}(\Lambda) \]

unphysical region of EFT single/non-resonant subtracted in local expansion by matching conditions \( \tilde{C}^{(n),0}(\Lambda) \)

\[ |p_+^2 - m_t^2| < \Lambda^2 \ll m_t^2 \]

double resonant nonrel. expansion valid
III. Phase space matching

Leading order diagram

\[
\alpha_s = 0
\]

\[
\Gamma_t \sum_{\ell,k=0,1\ldots} \#\ell_k \left[ \left( \frac{m_t \Gamma_t}{\Lambda^2} \right)^\ell \times \left( \frac{m_t E}{\Lambda^2} \right)^k \right] \text{ series, excellent convergence}
\]

Relativistic corrections → introduce powers of \( \frac{P^2}{m_t^2} \sim \frac{\Lambda^2}{m_t^2} \)

example, NNLL kin. insertion

\[
\bar{C}^{\text{kin},0}(\Lambda) \propto \frac{m_t^2}{4\pi} \frac{9}{8\sqrt{2}\pi} \frac{\Gamma_t \Lambda}{m_t^2} \quad \text{(NLL)}
\]

→ power-counting breaking term

⇒ numerically suppressed for \( \Lambda \lesssim 110 \text{ GeV} \) \( \Delta M_t \lesssim 35 \text{ GeV} \), do not spoil the nonrelativistic expansion
Coulomb-like potentials $\rightarrow$ introduce powers of \( \left( \frac{\alpha_s}{\nu} \right)^n \rightarrow \left( \alpha_s \frac{m_t}{\Lambda} \right)^n \)

\[
\propto \frac{m_t^2}{4\pi} \left[ -\alpha_s \text{Im} \left[ \ln(-i\nu) \right] - 2\alpha_s \frac{m_t \Gamma_t}{\Lambda^2} + \alpha_s \frac{8\sqrt{2}}{3\pi} \frac{m_t^2 \Gamma_t}{\Lambda^3} \text{Im} \nu + \ldots \right]
\]

- $O(\alpha_s)$ contribution to LL
- Coulomb Green function $\tilde{C}^1(\Lambda)$
- NNLL
- $N^3LL$, nonanalytic in $E$

$\rightarrow$ matching for the $t\bar{t}$ currents

\[
i\delta\bar{c}_1(\Lambda) = -iC_F\alpha_s \frac{4\sqrt{2}}{3\pi} \frac{m_t^2 \Gamma_t}{\Lambda^3}
\]

- $\alpha_s$-expansion of phase space matching contributions shows good convergence, also for relativistic corrections, for $\Lambda \sim 70 - 110$ GeV ($\Delta M_t \sim 15 - 35$ GeV)
- $N^3LL$ [$O(\alpha_s^2)$] corrections (not fully known!) needed to meet experimental precision at the future LC
CONCEPTS OF PHASE SPACE MATCHING

III. Phase space matching

\[ \sigma_{\text{Born}}(\Lambda) \sim + \text{expansion for } E, \Gamma_t \ll \Lambda^2/m_t \ll m_t \]

EFT \[ 2 \text{ Im} \left[ + \sum_n \tilde{C}^{(n),0}(\Lambda) \right] \]

\[ \sigma_{b\bar{b}WW}(\Lambda) = \sigma_{\text{NRQCD}}(\Lambda) + \sigma_{\text{rem}}(\Lambda) \]

computed in the full relativistic theory

\( t\bar{t} \) phase space regions passing the cut on \( p_{t,\bar{t}}^2 \),

reproduced by NR expansion

\[ \to \text{ use NRQCD rules } + E, \Gamma_t \ll \Lambda^2/m_t \]

to obtain coeffs. \( \tilde{C}^{(n)}(\Lambda) \)

“matching procedure” inside the EFT itself

\[ \checkmark \text{ very small for } \alpha_s = 0 \]
IV. Results & comparisons
generated \(10^4\) events for \(e^+e^- \rightarrow W^+W^-b\bar{b}\) with MadGraph (MG) for \(s = 4m_t^2\), and analyzed dependence on the \(bW\) invariant-mass cut \(\Delta M_t\).

**EFT result:** resonant LO+NNLO \((\alpha_s = 0)\) + non-resonant NLO

[Graph showing comparison between MG points with and without Higgs, and EFT results with tight and loose cuts.]
$e^+ e^- \rightarrow W^+ W^- b \bar{b}$ tree-level cross section: energy dependence for different $\Delta M_t$ invariant-mass cuts

- Total cross section
- $\Delta M_t = 5$ GeV
- $\Delta M_t = 15$ GeV

MG (full) points & error band,

EW NNLO tree-level contributions (solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)
Relative sizes of EW NLO corrections w.r.t. LO (including resummation of Coulomb gluons $\propto (\alpha_s / v)^n$)

$$[\alpha_s^{\overline{MS}}(30 \text{ GeV}) = 0.142]$$

**QED resonant correction ("Coulomb photons")**

**Combined EW NLO corrections**

**Non-resonant NLO correction** $\sim -30 \text{ fb} (-3\% \text{ above and up to } -20\% \text{ below threshold})$

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**Total cross section**

**Cross section with $\Delta M_t = 15 \text{ GeV}$**

Pedro Ruiz-Femenía · Teilchenphysikseminar · 17.03.2011
NON-QCD CORRECTIONS BEYOND NLO

IV. Results & comparisons

Sizes of NNLL EW and phase space matching (psm) corrections

$\Delta M_t = 15 \text{ GeV}$

$\Delta M_t = 35 \text{ GeV}$

**NNLL QED effects**

**NNLL hard one-loop EW effects**

**NNLL finite lifetime corrections**

**Non-resonant corrections**

(NLL, NNLL, $N^3$LL phase space matching contributions)

- psm contributions are the largest of the 4 classes of EW effects
- almost constant (small linear $\sqrt{s}$-dependence from $\gamma, Z$ propagators)
- convergence of the psm procedure particularly good for larger $\Delta M_t$
TOTAL INCLUSIVE TOP-PAIR PRODUCTION CROSS SECTION

IV. Results & comparisons

\[ \Delta M_t = 15 \text{ GeV} \]

\[ \Delta M_t = 35 \text{ GeV} \]

dashed line: NNLL pure QCD prediction
(add step by step)
+ NNLL QED effects
+ NNLL hard one-loop EW effects
+ NNLL finite lifetime corrections
+ N^3LL phase space matching contributions

large psm corrections related to the unphysical phase space contributions contained in the pure QCD prediction
V. Conclusions

Precise determinations of top parameters in threshold region

- count number of $t\bar{t}$ events, color singlet state, background non-resonant, physics well understood

- **EFT framework** allows for a separation of resonant and non-resonant fluctuations and to sum up leading contributions

- QCD corrections well under control \(\rightarrow\) (almost) NNLL + $N^3LO$

**EW non-resonant corrections to** \(e^+e^- \rightarrow W^+W^-b\bar{b}\) **in the** $t\bar{t}$ **resonance region**

- complete NLO non-resonant contributions computed for total cross section and with top invariant-mass cuts

- NLO non-resonant amount \(~-30\) fb \((-3\%\) above and up to \(-20\%\) below threshold\) for the total cross section, even more with invariant-mass cuts
V. Conclusions

Beyond NLO: Phase space matching approach

- dominant NNLO and NNNLO terms computed in the invariant mass range $\Delta M_t \sim 15 - 35$ GeV show good convergence

$\Rightarrow$ need to be added to existing QCD results in view of the expected experimental uncertainties at the LC

Outlook

- analysis of squark pair production at threshold also possible within scalar NRQCD

$\Rightarrow$ full NLL QCD running known Hoang, RF '05

P-wave production ($e^+e^- \rightarrow \tilde{q}\tilde{q}$): phase-space divergencies more severe

$$G^{L=1}_{\text{coul}} = m^2 \left( v^2 + \frac{G_F^2 \alpha_s^2}{4} \right) G_{\text{coul}} + \ldots$$

$$m v^2 = E + i \Gamma$$

$$\text{Im} \; G^{L=1}_{\text{coul}} \sim \frac{\alpha_s \Gamma t}{\epsilon}$$

LO effect!

(work in progress...)

Thank you!