Top-Antitop Threshold - Electroweak corrections

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Outline

I Top-pair production at linear colliders near threshold

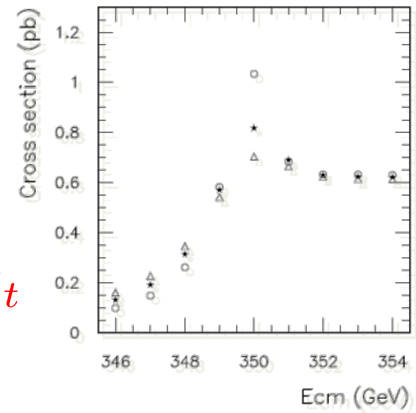
II Non-resonant electroweak NLO contributions

III Phase space matching

IV Results & comparisons

V Conclusions

I. Top-pair production near threshold



Martinez, Miquel '02

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \simeq 350\ \mathrm{GeV}$ will produce lots of $t\bar{t}$ pairs, allowing for a threshold scan of the top cross section

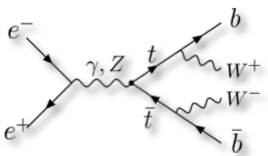
$\hookrightarrow$ Precise determination of the top mass m_t, the width Γ_t and the Yukawa coupling λ_t without the uncertainties/ ambiguities of hadron colliders $\rightarrow$ $\delta m_t^{\mathrm{exp}} \simeq 30$ MeV

$\rightsquigarrow$ m_t is a crucial input for electroweak precision observables!

Requires also precise theoretical prediction
$\Rightarrow$ $\delta\sigma/\sigma \sim 2-3\%$ ($\delta\sigma \sim 5$ fb below threshold)

QCD corrections are known (almost) up to NNLL/NNNLO, but electroweak (NLO) contributions due to top decay were missing!

Note: once EW effects are turned on, the physical final state is $W^+W^-b\bar{b}$



$\Rightarrow$ $\sigma(e^+e^- \rightarrow W^+W^-b\bar{b})$ in the $t\bar{t}$ resonance region and allow for invariant-mass cuts on reconstructed t, $\bar{t}$

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5$ GeV $\gg \Lambda_{\rm QCD}$
$\Rightarrow t\bar{t}$ is perturbative at threshold

Bigi, Dokshitzer, Khoze, Kühn, Zerwas '86

Top quarks move slowly near threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders $\to$ **NRQCD**

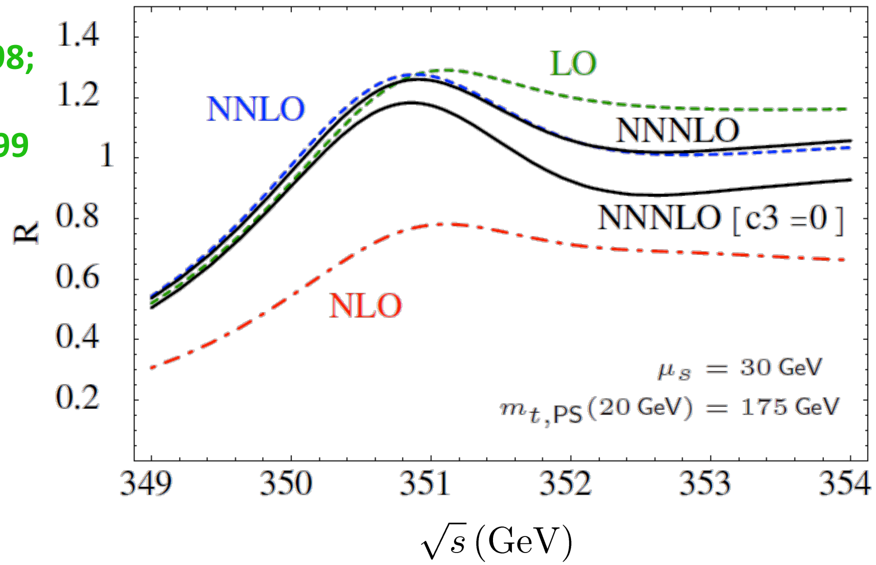
$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\rm LO} + \{\alpha_s, v\}_{\rm NLO} + \{\alpha_s^2, \alpha_s v, v^2\}_{\rm NNLO} + \dots\right)$$

Further RG improvement by summing also $(\alpha_s \ln v)^m$: LL, NLL, ... $\to$ **vNRQCD pNRQCD**

- NNLO QCD corrections
  Hoang, Teubner '98-99; Melnikov, Yelkhovsky '98; Yakovlev ´98; Beneke, Signer, Smirnov '99; Nagano, Ota, Sumino '99; Penin, Pivovarov '98-99

- NNLO & (almost) NNLL
  Hoang, Manohar, Stewart, Teubner '00-01; Hoang ´03; Pineda, Signer '06; Hoang, Stahlhofen '06-11

- (almost) NNNLO
  Beneke, Kiyo, Schuller '05-08 → see figure



Effective field theory (EFT) for pair production of unstable particles near threshold, based on separation of resonant and nonresonant fluctuations

Hoang, Reisser '05 ⟺ Beneke, Chapovsky, Khoze, Signer, Zanderighi '01-04; Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07-08

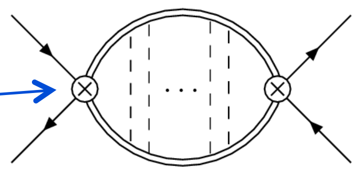
- power counting for finite width effects: $\frac{\Gamma_t}{m_t} \sim \alpha_{\text{EW}} \sim \alpha_s^2 \sim v^2 \ll 1$

- hard modes $\sim m_t$ (including top decay products) are integrated out
  $\rightsquigarrow$ EFT with potential (nearly on-shell) top quarks and ultrasoft gluons

- Extract cross section for $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

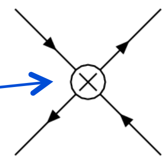
resonant contributions

$t\bar{t}$ production operators $\mathcal{O}_p^{(k)}$

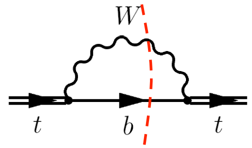


non-resonant contributions

4-electron operators $\mathcal{O}_{4e}^{(k)}$



Electroweak effects at LO Fadin, Khoze '87

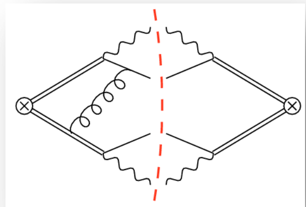


- Replacement rule: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$

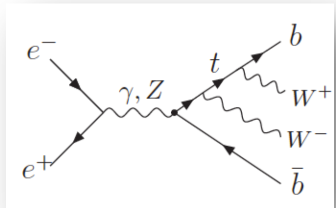
$\Rightarrow$ unstable top propagator $\dfrac{i}{p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$ $\leftarrow$ $\delta\mathcal{L} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}}\, i\,\frac{\Gamma_t}{2}\, \psi_{\mathbf{p}}$

Electroweak effects at NLO

- Exchange of "Coulomb photon": trivially extension of QCD corrections



- Gluon exchange involving the bottom quarks in the final state $\Rightarrow$ these contributions vanish at NLO for the total cross section, Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94
also negligible if loose top invariant-mass cuts are applied; remains true at **NNLO** Hoang, Reisser '05; Beneke, Jantzen, RF '10



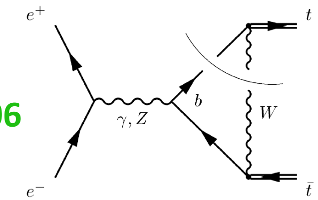
- Non-resonant (hard) corrections to $e^+e^- \to W^+W^-b\bar{b}$ which account for the production of the Wb pairs by highly virtual tops or with only one or no top

$$\hookrightarrow \quad \Delta\sigma_{\text{non-res}} = \frac{1}{s}\sum_k \text{Im}\left[C^{(k)}_{4e}\right] \langle e^+e^-|\mathcal{O}^{(k)}_{4e}|e^+e^-\rangle$$

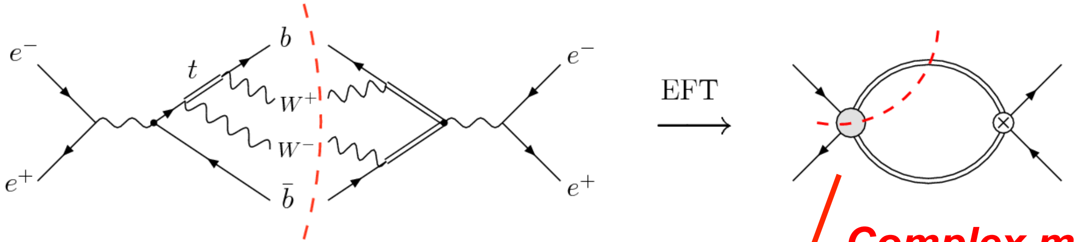
Electroweak (non-trivial) effects at NNLO

- **lifetime dilatation** term $$\delta\mathcal{L} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left(i \frac{\Gamma_t}{2} \frac{\mathbf{p}^2}{2m} \right) \psi_{\mathbf{p}}$$

- **absorptive parts** in the 1-loop matching coeffs. of the production operators (arising from bW cuts) Hoang, Reisser '06

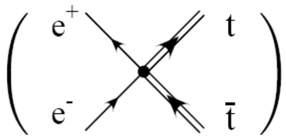


$\Rightarrow$ reproduce interferences between double and single resonant amplitudes



Complex matching conditions

$$J_{\mathbf{p}} = \left[C^{\text{born}}_{\text{LL}} + C^{\text{QCD}}_{\text{NLL}} + C^{\text{QCD}}_{\text{NNLL}} + iC^{bW,\text{abs}}_{\text{NNLL}} + C^{\text{EW}}_{\text{NNLL}} + \ldots \right]$$

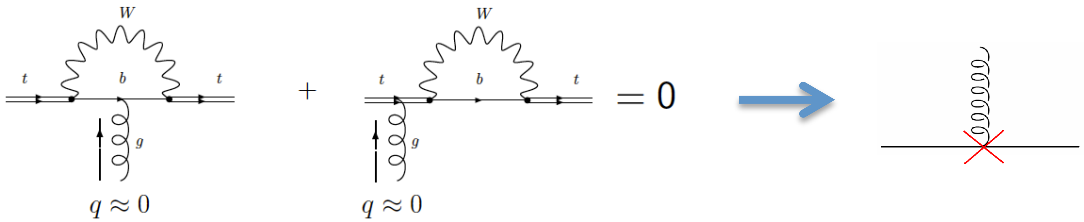


- **real part** of hard one-loop EW corrections

Kuhn, Guth '92; Hoang, Reisser '06

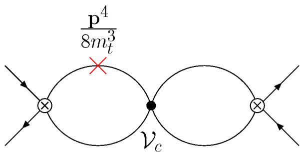
Electroweak effects at NNLO (cont.)

- No EW corrections to the Coulomb potential at NNLO



$\sim g^2 \frac{\mathbf{q}^2}{m_t^2}$ suppressed

- resonant **NNLO** corrections produce "finite-width divergences" (also called "phase space divergences")



$$\Delta\sigma_{\rm tot} \sim m_t^2 \frac{\alpha_s \Gamma_t}{\epsilon}$$

$\hookrightarrow$ anom. dim. of $\mathcal{O}_{4e}^{(k)}$ operators $\quad \delta\mathcal{L} = \sum_k C_{4e}^{(k)}(\mu)\, \mathcal{O}_{4e}^{(k)}$ **Hoang, Reisser '05**

$\hookrightarrow$ $C_{4e}^{(k)}(\mu)$ resums **NLL phase space logs**

- $C_{4e}^{(k)}(m_t)$ determined by the non-resonant contributions. Beyond NLO the exact computation is hard, but dominant terms can be obtained for moderate top invariant mass cuts $\Longrightarrow$ **Phase space matching**

II. Electroweak non-resonant NLO contributions

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Beneke, Jantzen , RF '10

$\Rightarrow$ cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:

  $p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \gg \Sigma(p_t^2) \sim m_t^2 \alpha_{\text{EW}}$

  $\rightarrow \Gamma_t = 0$

- suppressed w.r.t. LO $(\sim v)$ by

  $\alpha_{\text{EW}}/v \sim \alpha_s$

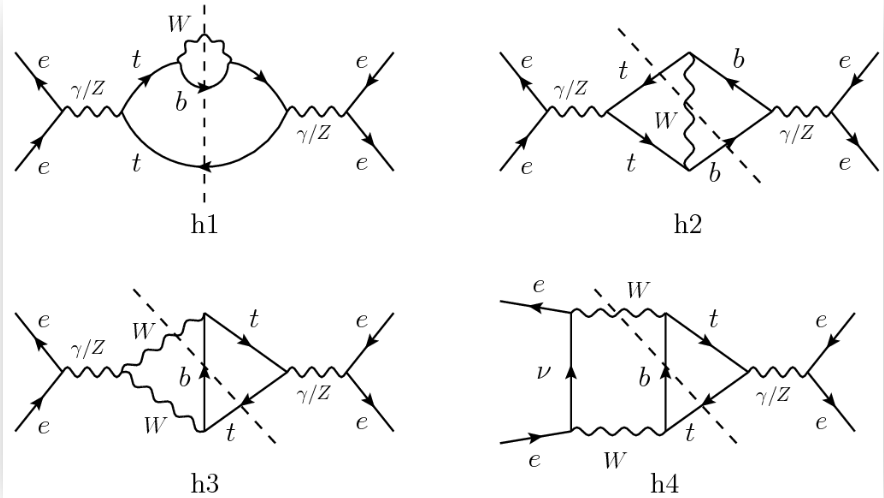
- expansion in

  $\delta = \dfrac{s - 4m_t^2}{4m_t^2}$

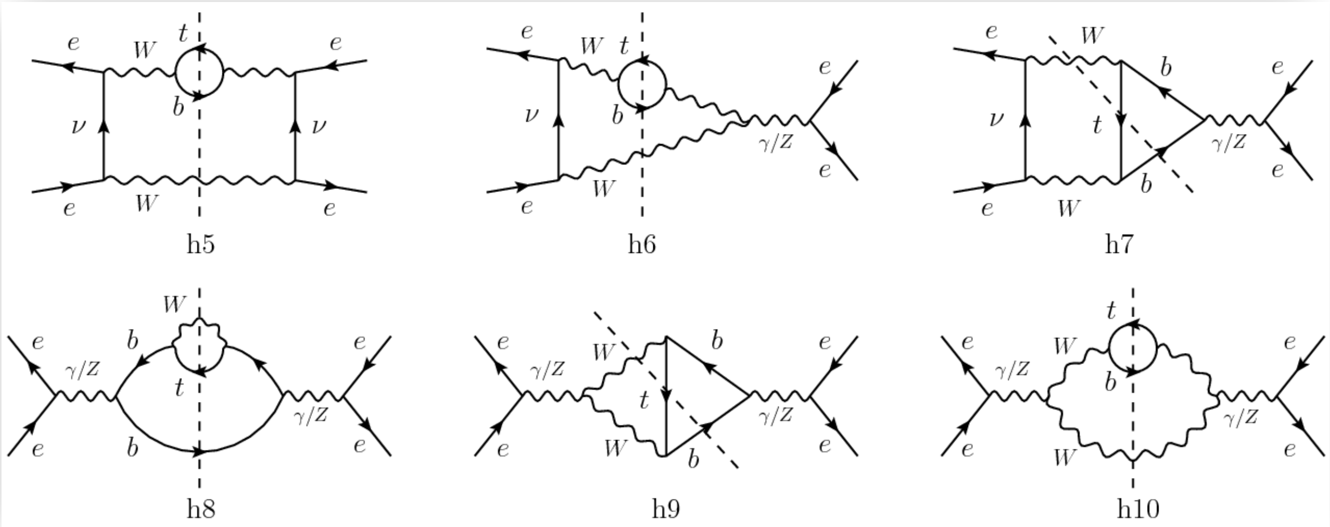
  $\hookrightarrow$ at NLO:

  $s = 4m_t^2$

bW^+ from highly virtual top



bW^+ without intermediate top



Form of non-resonant contributions

In terms of the invariant mass of the bW^+ system, $p_t^2 = (p_b + p_{W^+})^2$, ($p_t \to$ also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section

[Phase-space factor $(m_t^2 - p_t^2)^{1/2-\epsilon}$ in dim. reg. regularizes the end-point singularity for h1]

Applying invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

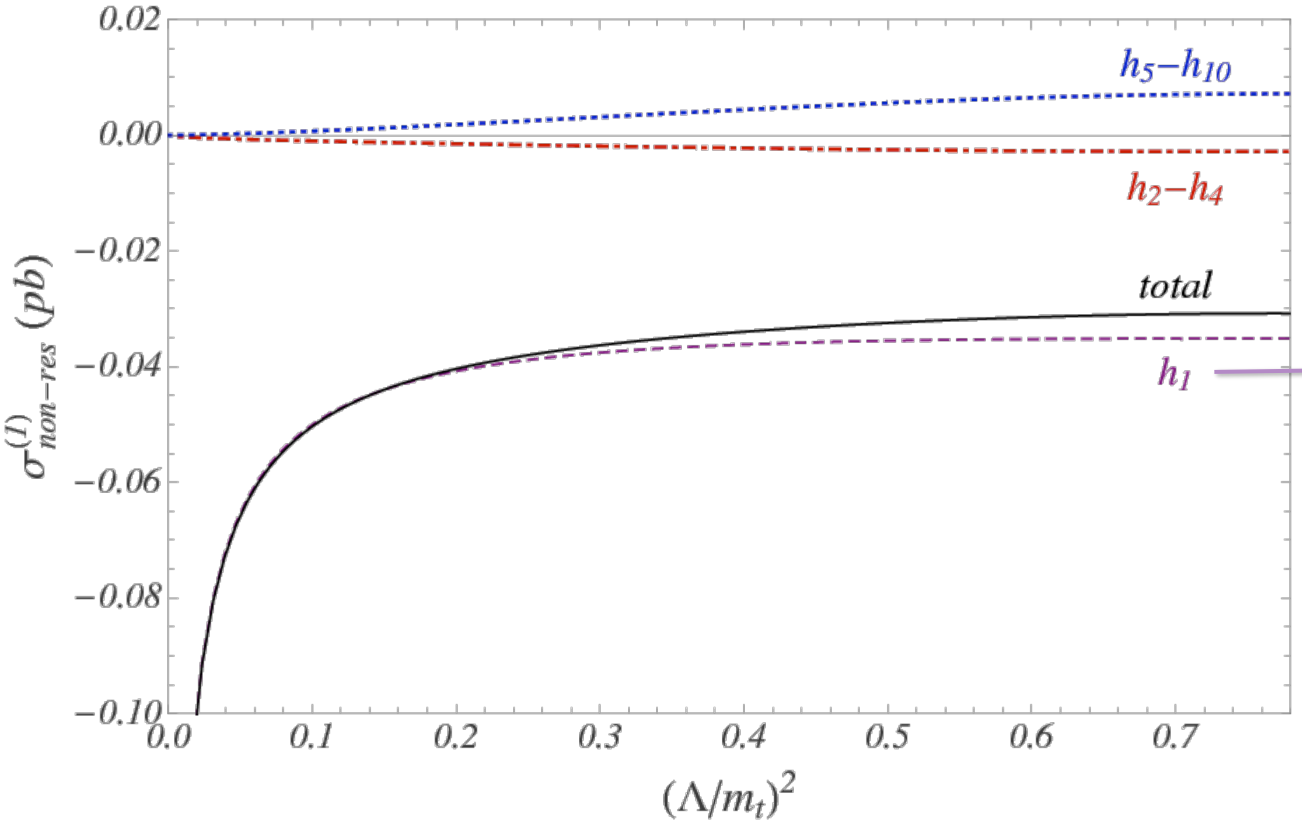
$\hookrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

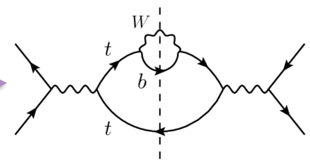
We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (corresponding to $\Delta M_t \gg \Gamma_t$)

$\rightsquigarrow$ cut has no effect in the resonant contributions

[In contrast: for tight cuts with $\Lambda^2 \sim m_t\Gamma_t$ ($\Delta M_t \sim \Gamma_t$), non-resonant contributions vanish and cuts only affect the resonant contributions]

Non-resonant NLO contributions: from numeric integration over p_t^2 (and over one angle for some diagrams), the integrand is an analytic function of p_t^2/m_t^2 and M_W^2/m_t^2; cut-dependence enters through the integration limit





total cross section reached for $(\Lambda/m_t)^2 = 1 - M_W^2/m_t^2$
$\longleftarrow \quad = 0.78$

Parameters: on-shell (pole) masses, $m_t = 172$ GeV, $\Gamma_t = \Gamma_t^{\rm tree} = 1.46550$ GeV, α and $\sin^2\theta_W$ from G_F, M_W, M_Z

III. Phase-space matching

Alternative approach to compute non-resonant contributions

Hoang, Reisser , RF ´10

- Non-resonant contributions obtained for moderate invariant-mass cuts, $m_t\Gamma_t \ll \Lambda^2 \lesssim m_t^2$, as a series:

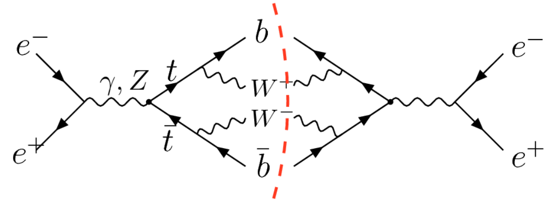
$$\frac{\Gamma_t}{\Lambda}\sum_{n,\ell,k}\left[\left(\frac{m_t\Gamma_t}{\Lambda^2}\right)^n \times \left(\frac{\Lambda^2}{m_t^2}\right)^\ell\right]\times\left(\alpha_s\frac{m_t}{\Lambda}\right)^k \qquad n,\ell,k = 0,1,\ldots$$

- NLO, NNLO and (partial) N^3LO contributions obtained (counting $\Lambda \sim m_t$) ✓
  → NLL, NNLL, N^3LL in the vNRQCD framework

- Assumption: non-resonant background processes are small (✓ at NLO!)

- Beyond NLO, phase space matching approach cannot be applied to larger cuts up to the total cross section ×

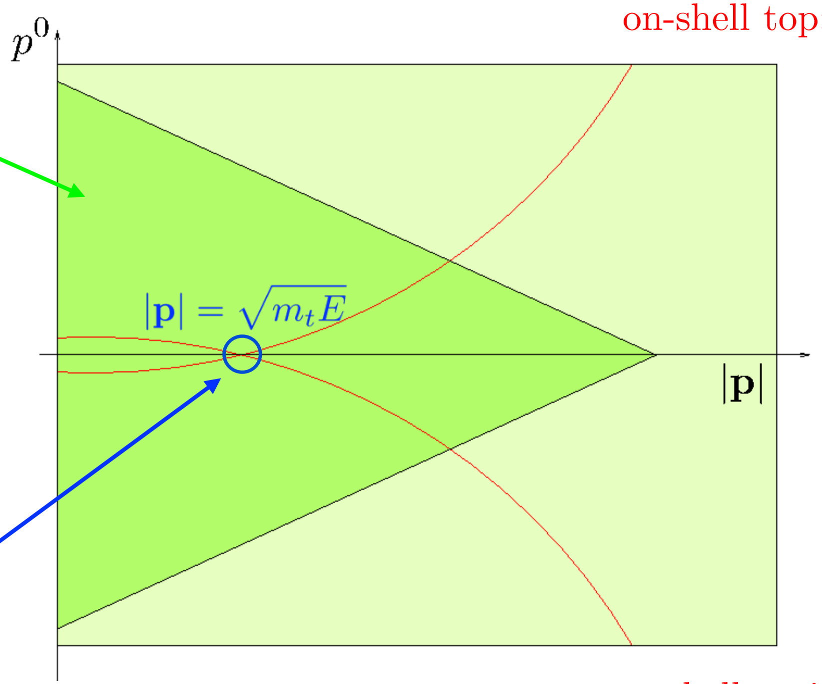
$$\sigma_{t\bar{t}}^{\text{Born}} \sim$$



$$\xrightarrow{\text{LO NR expansion}} \int_{-\infty}^{\infty} dp^0 \int_0^{+\infty} d|\mathbf{p}|\mathbf{p}^2 \frac{\Gamma_t^2}{|\frac{E}{2} + p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2 \, |\frac{E}{2} - p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$

on-shell top: $p^0 = \frac{\mathbf{p}^2}{2m_t} - \frac{E}{2}$

$WWb\bar{b}$ phase space:

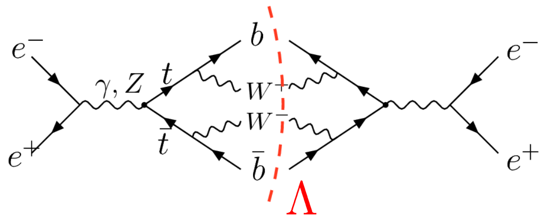


stable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon}$$

$$\longrightarrow (2\pi)\,\delta(p^0 - \frac{\mathbf{p}^2}{2m_t})$$

on-shell antitop: $p^0 = -\frac{\mathbf{p}^2}{2m_t} + \frac{E}{2}$

$\sigma_{t\bar{t}}^{\mathrm{Born}}(\Lambda) \sim$ 

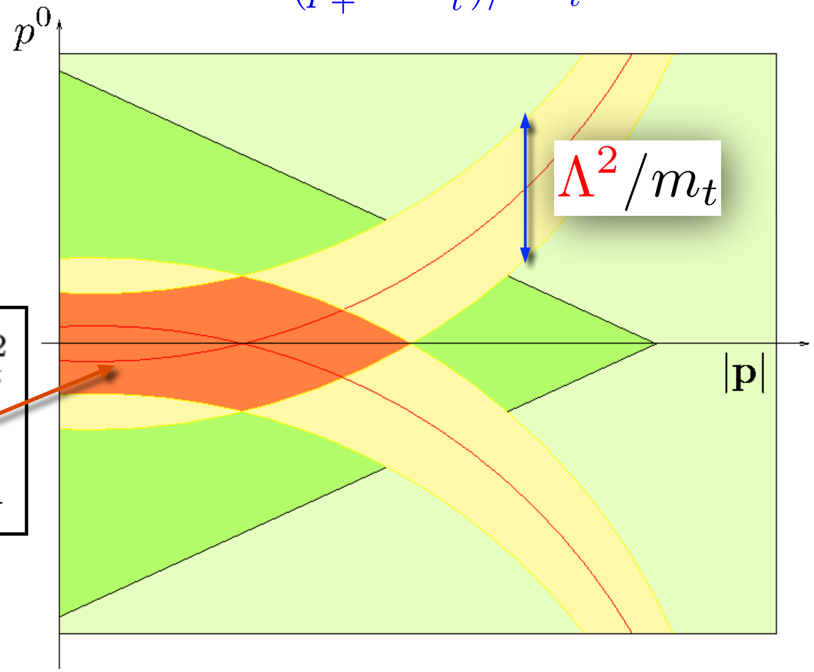
Λ^2 = cut on t and $\bar{t}$ invariant masses

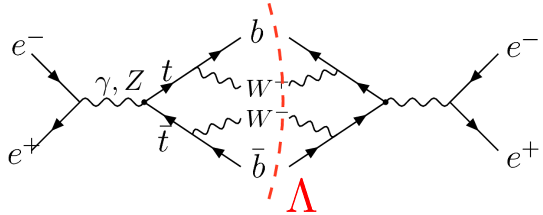
$$\sim \int_{-\Lambda^2/2m_t}^{\Lambda^2/2m_t} dp^0 \int_0^{f(p^0,\Lambda)} d|\mathbf{p}|\mathbf{p}^2 \frac{\Gamma_t^2}{|\underbrace{\frac{E}{2} + p^0 - \frac{\mathbf{p}^2}{2m_t}}_{(p_+^2 - m_t^2)/2m_t} + i\frac{\Gamma_t}{2}|^2 \, |\frac{E}{2} - p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$

$|p_\pm^2 - m_t^2| < \Lambda^2 \ll m_t^2$

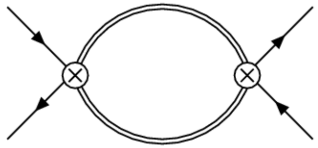
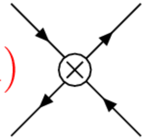
double resonant

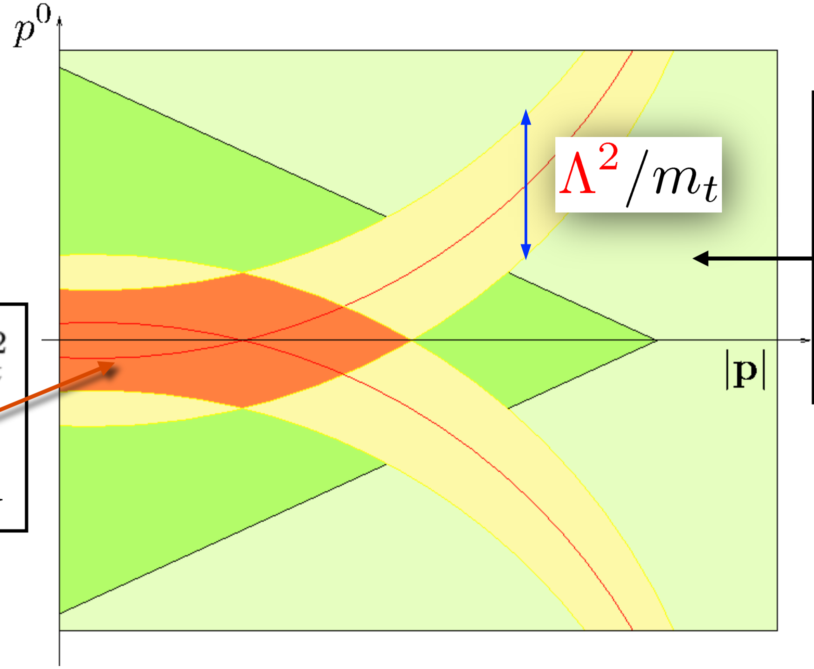
nonrel. expansion valid



$\sigma^{\text{Born}}_{t\bar{t}}(\Lambda) \sim$ 

$+$ expansion for $E, \Gamma_t \ll \Lambda^2/m_t \ll m_t$

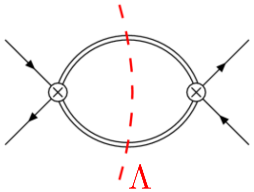
$\xrightarrow{\text{EFT}}$ $2\,\text{Im}\Big[$  $+ \sum_n \tilde{C}^{(n),0}(\Lambda)$  $\Big]$



$|p^2_\pm - m^2_t| < \Lambda^2 \ll m^2_t$
double resonant
nonrel. expansion valid

unphysical region of EFT
single/non-resonant
subtracted in local
expansion by matching
conditions $\tilde{C}^{(n),0}(\Lambda)$

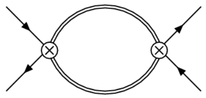
Leading order diagram



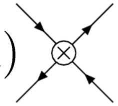
$$\propto \frac{m_t^2}{4\pi}\left(\mathrm{Im}\,(iv) \underbrace{- \frac{2\sqrt{2}}{\pi}\frac{\Gamma_t}{\Lambda} + \frac{4+2\sqrt{2}\sinh(1)}{3\,\pi^2}\frac{m_t\Gamma_t^2}{\Lambda^3}}_{\widetilde{C}^0(\Lambda)} \underbrace{- \frac{2\sqrt{2}}{3\,\pi}\frac{m_t E\,\Gamma_t}{\Lambda^3}}_{\widetilde{C}^{(1),0}(\Lambda)\frac{E}{m_t}} + \dots\right)$$

(NLL) (N^{3}LL)

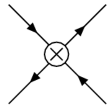
$\mathrm{Im}\,(iv)$ ↓



$\widetilde{C}^0(\Lambda)$



$\widetilde{C}^{(1),0}(\Lambda)\dfrac{E}{m_t}$



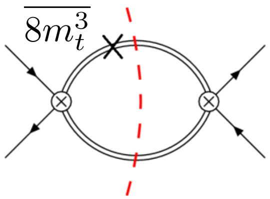
$$\Longrightarrow \quad \frac{\Gamma_t}{\Lambda}\sum_{\ell,k=0,1\dots} \#_{\ell k}\left[\left(\frac{m_t\Gamma_t}{\Lambda^2}\right)^{\ell} \times \left(\frac{m_t E}{\Lambda^2}\right)^{k}\right]$$ series, excellent convergence ✓

Relativistic corrections

$\rightarrow$ introduce powers of $\dfrac{\mathbf{p}^2}{m_t^2} \sim \dfrac{\Lambda^2}{m_t^2}$

example, NNLL kin. insertion

$\dfrac{\mathbf{p}^4}{8m_t^3}$

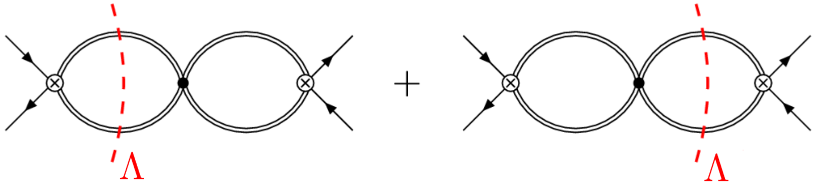


$$\widetilde{C}^{\mathrm{kin},0}(\Lambda) \propto \frac{m_t^2}{4\pi}\frac{9}{8\sqrt{2}\pi}\frac{\Gamma_t\Lambda}{m_t^2} \quad \text{(NLL)}$$

$\rightarrow$ power-counting breaking term

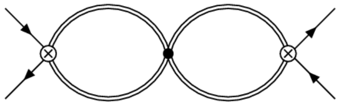
$\Rightarrow$ numerically suppressed for $\Lambda \lesssim 110$ GeV ($\Delta M_t \lesssim 35$ GeV), do not spoil the nonrelativistic expansion

Coulomb-like potentials $\rightarrow$ introduce powers of $\left(\frac{\alpha_s}{v}\right)^n \rightarrow \left(\alpha_s \frac{m_t}{\Lambda}\right)^n$

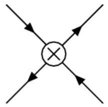


$$\propto \frac{m_t^2}{4\pi}\left[-\alpha_s \,\mathrm{Im}\big[\ln(-i\,v)\big] \quad - 2\alpha_s \frac{m_t\Gamma_t}{\Lambda^2} \quad + \alpha_s \frac{8\sqrt{2}}{3\pi}\frac{m_t^2\Gamma_t}{\Lambda^3}\,\mathrm{Im}\,v + \ldots \right]$$

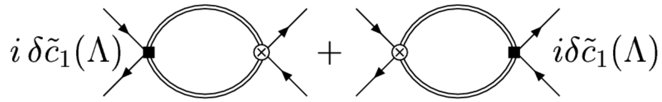
$\mathcal{O}(\alpha_s)$ contribution to LL Coulomb Green function



NNLL

$\widetilde{C}^1(\Lambda)$ 

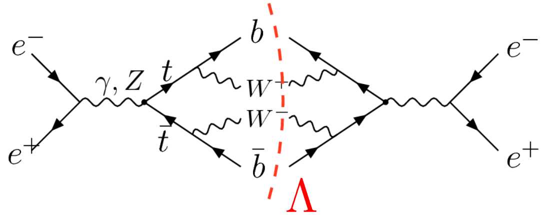
N^{3}LL, nonanalytic in E



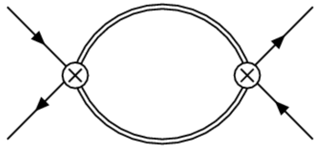
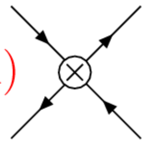
$\rightarrow$ matching for the $t\bar{t}$ currents

$$i\delta\tilde{c}_1(\Lambda) = -iC_F\alpha_s \frac{4\sqrt{2}}{3\pi}\frac{m_t^2\Gamma_t}{\Lambda^3}$$

- α_s-expansion of phase space matching contributions shows good convergence, also for relativistic corrections, for $\Lambda \sim 70 - 110$ GeV ($\Delta M_t \sim 15 - 35$ GeV)
- N^{3}LL $[\mathcal{O}(\alpha_s^2)]$ corrections (not fully known!) needed to meet experimental precision at the future LC

$\sigma^{\mathrm{Born}}_{t\bar t}(\Lambda) \sim$ 

$+$ expansion for $E, \Gamma_t \ll \Lambda^2/m_t \ll m_t$

$\xrightarrow{\mathrm{EFT}}$ $2\,\mathrm{Im}\Big[$  $+ \sum_n \tilde C^{(n),0}(\Lambda)$  $\Big]$

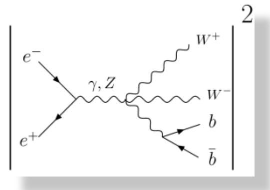
$\sigma_{b\bar b WW}(\Lambda) = \sigma_{\mathrm{NRQCD}}(\Lambda) + \sigma_{\mathrm{rem}}(\Lambda)$

$\sigma_{b\bar b WW}(\Lambda)$: computed in the full relativistic theory

$\sigma_{\mathrm{NRQCD}}(\Lambda)$: $t\bar t$ phase space regions passing the cut on $p^2_{t,\bar t}$, reproduced by NR expansion

$\rightarrow$ use NRQCD rules $+ E, \Gamma_t \ll \Lambda^2/m_t$ to obtain coeffs. $\tilde C^{(n)}(\Lambda)$

$\sigma_{\mathrm{rem}}(\Lambda)$: remainder contributions, for example:



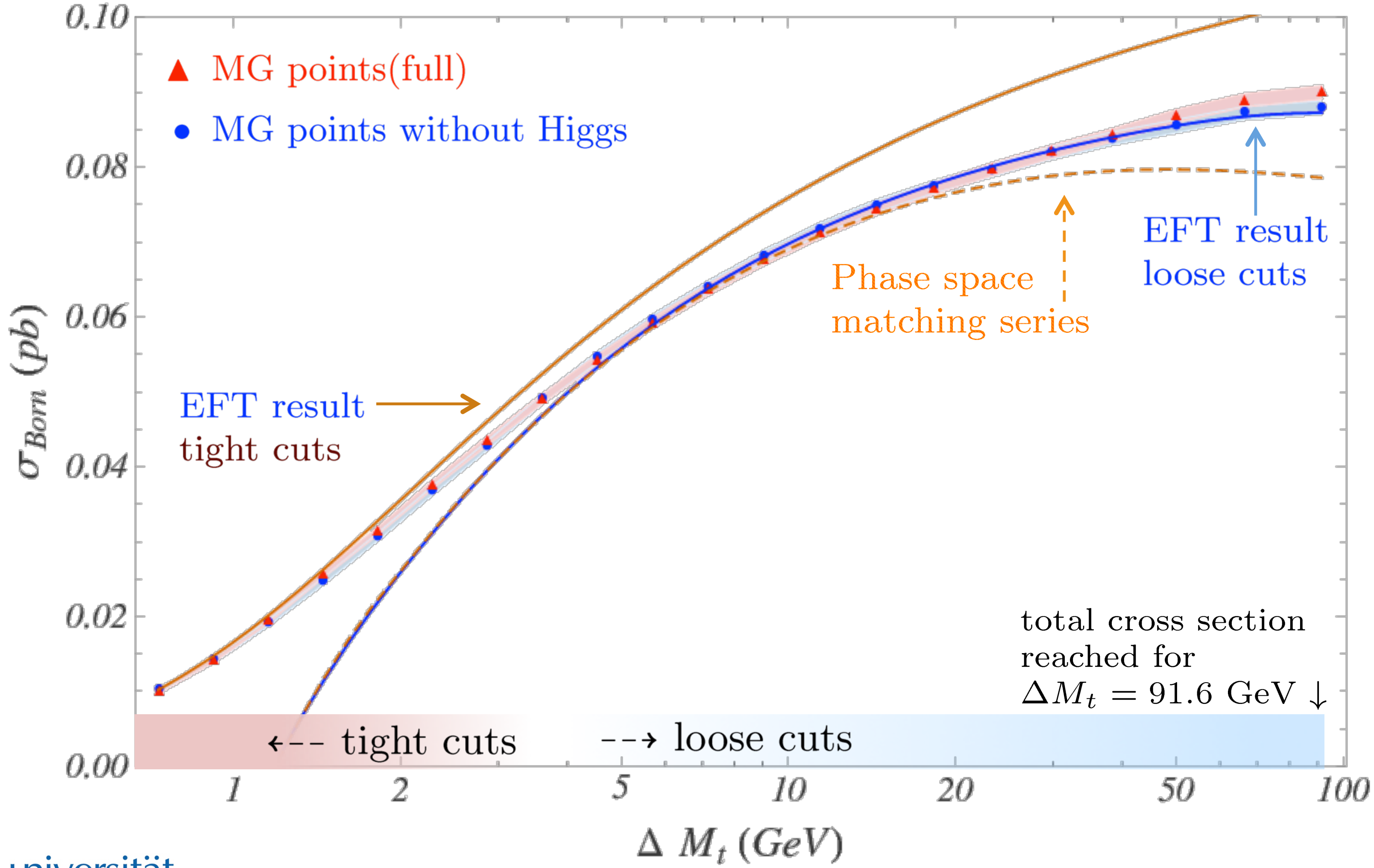
✓ very small for $\alpha_s = 0$

> “matching procedure” inside the EFT itself

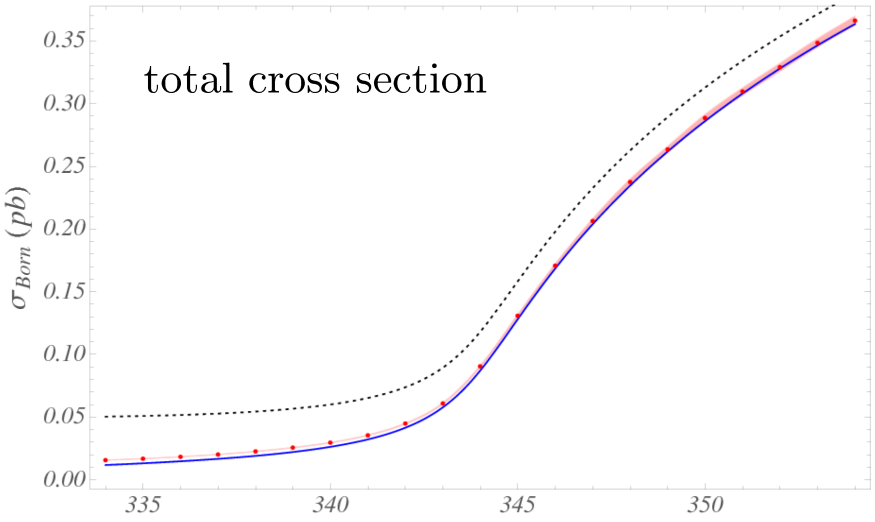
IV. Results & comparisons

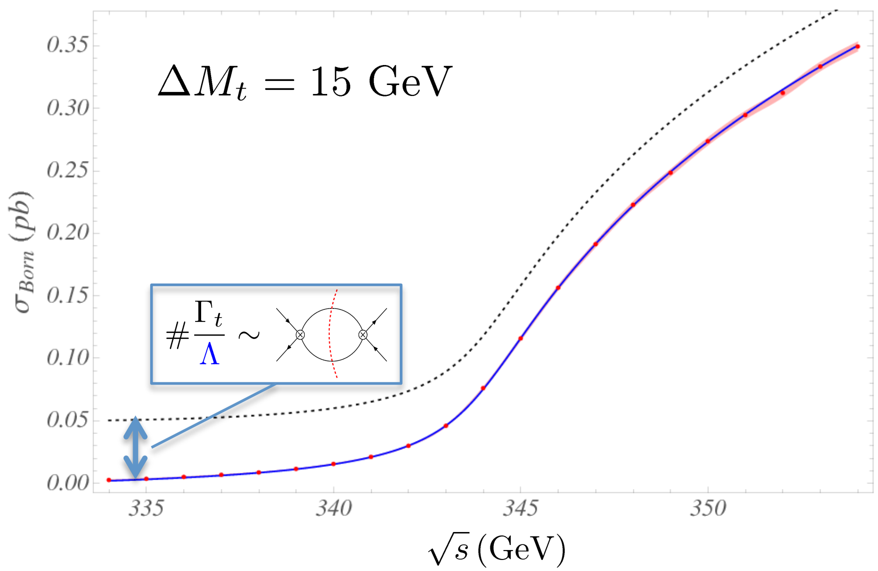
$\hookrightarrow$ generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$ with MadGraph (MG) for $s = 4m_t^2$, and analyzed dependence on the bW invariant-mass cut ΔM_t

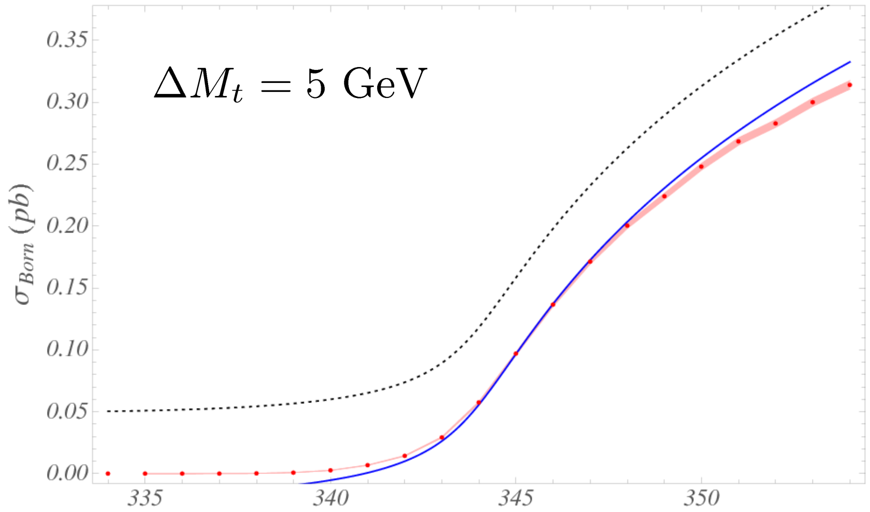
EFT result: resonant LO+NNLO $(\alpha_s = 0)$ + non-resonant NLO



$e^+e^- \to W^+W^-b\bar{b}$ tree-level cross section: energy dependence for different ΔM_t invariant-mass cuts







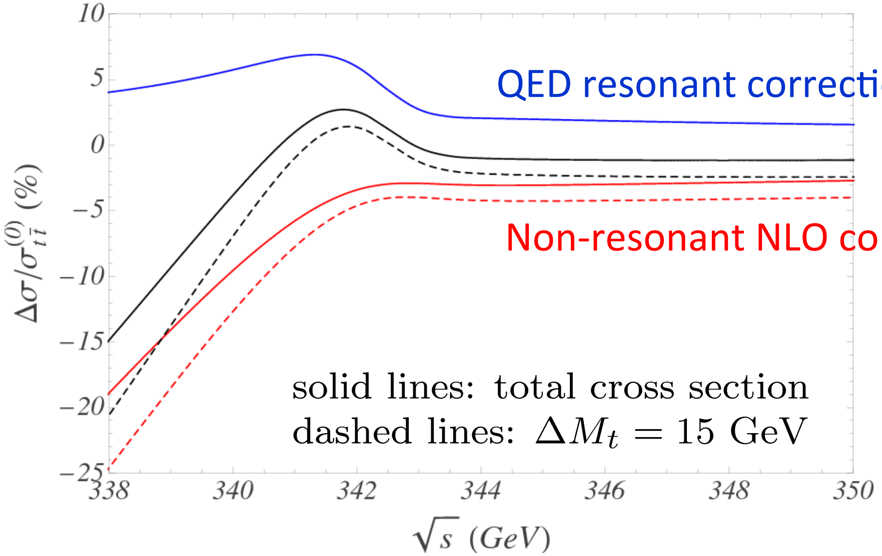
MG (full) points & error band,

EW NNLO tree-level contributions (solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)

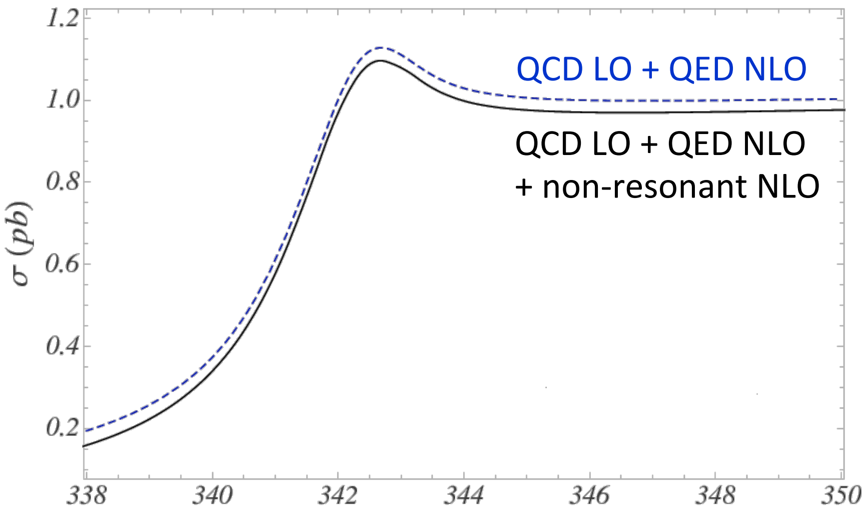
Relative sizes of EW NLO corrections w.r.t. LO (including resummation of Coulomb gluons $\propto (\alpha_s/v)^n$)

$[\alpha_s^{\overline{\mathrm{MS}}}(30\,\mathrm{GeV}) = 0.142]$

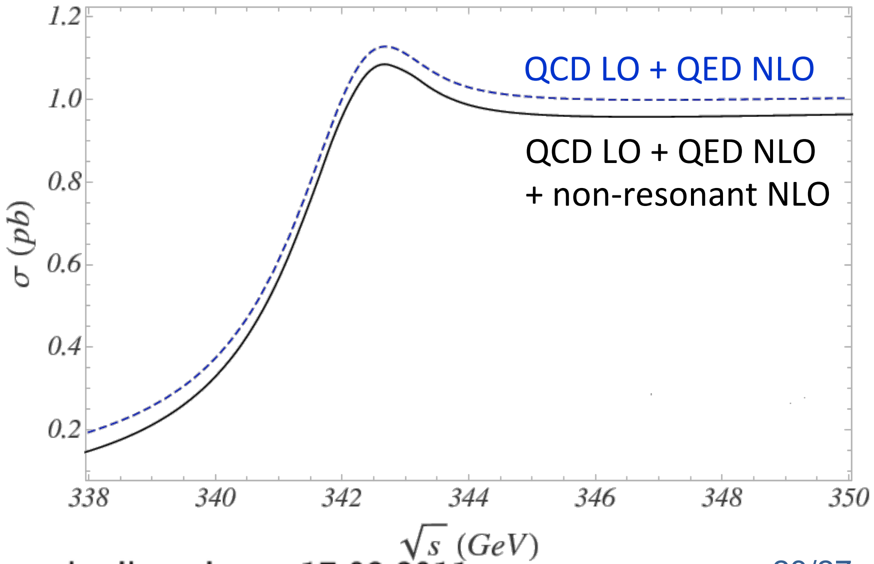


~ -30 fb (-3% above and up to -20% below threshold)

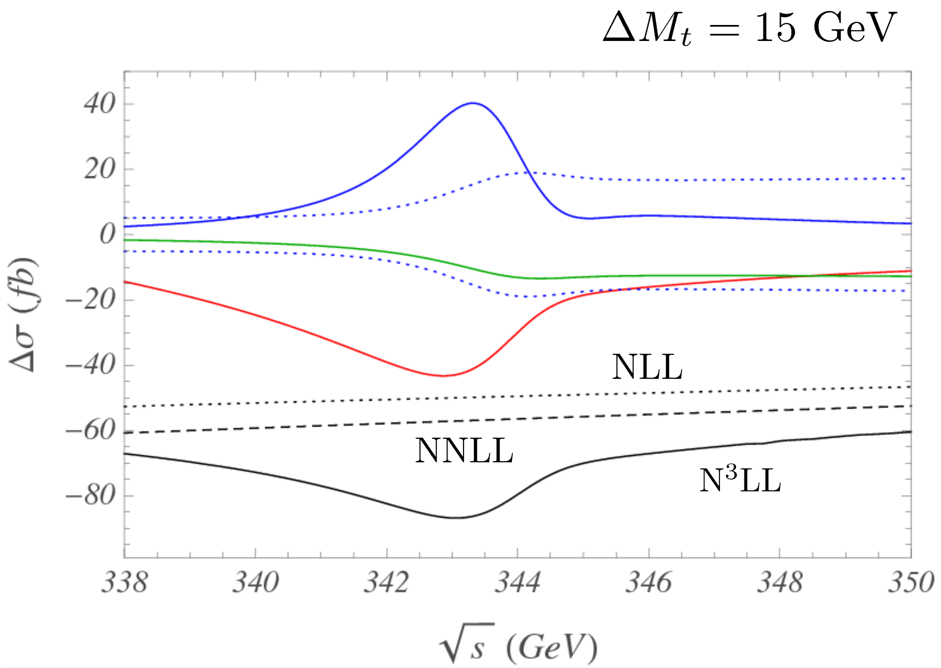
Total cross section

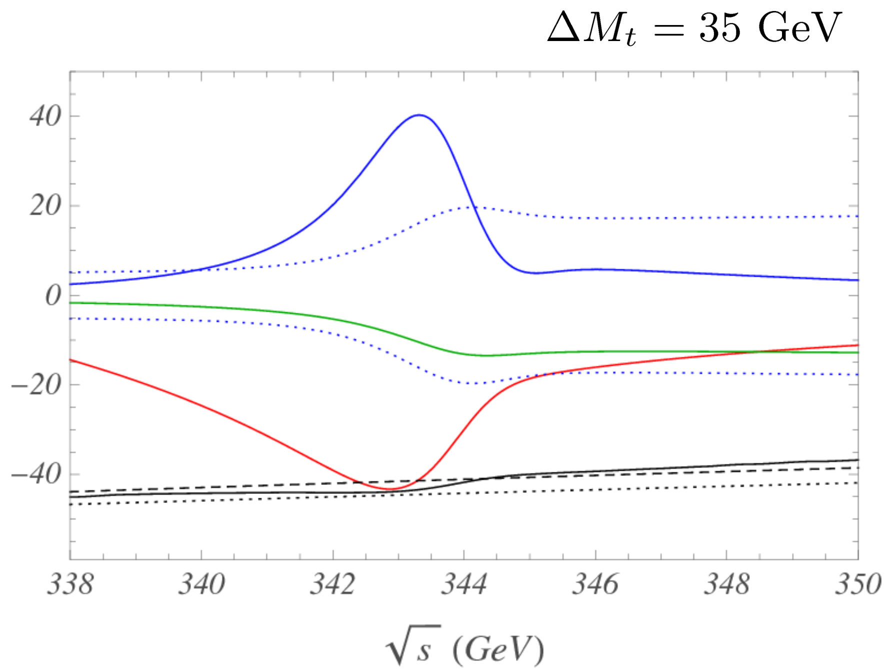


Cross section with $\Delta M_t = 15$ GeV



Sizes of NNLL EW and phase space matching (psm) corrections





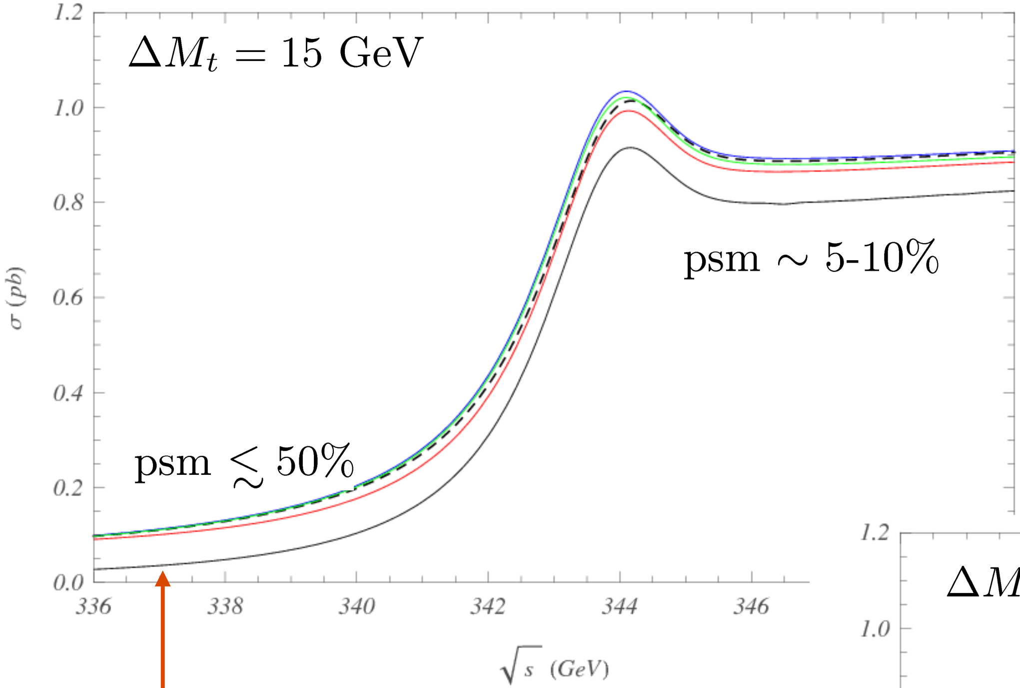
NNLL QED effects

NNLL hard one-loop EW effects

NNLL finite lifetime corrections

Non-resonant corrections
(NLL, NNLL, N^{3}LL phase space matching contributions)

- psm contributions are the largest of the 4 classes of EW effects
- almost constant (small linear $\sqrt{s}$-dependence from γ, Z propagators)
- convergence of the psm procedure particularly good for larger ΔM_t



dashed line: **NNLL pure QCD prediction**

(add step by step)

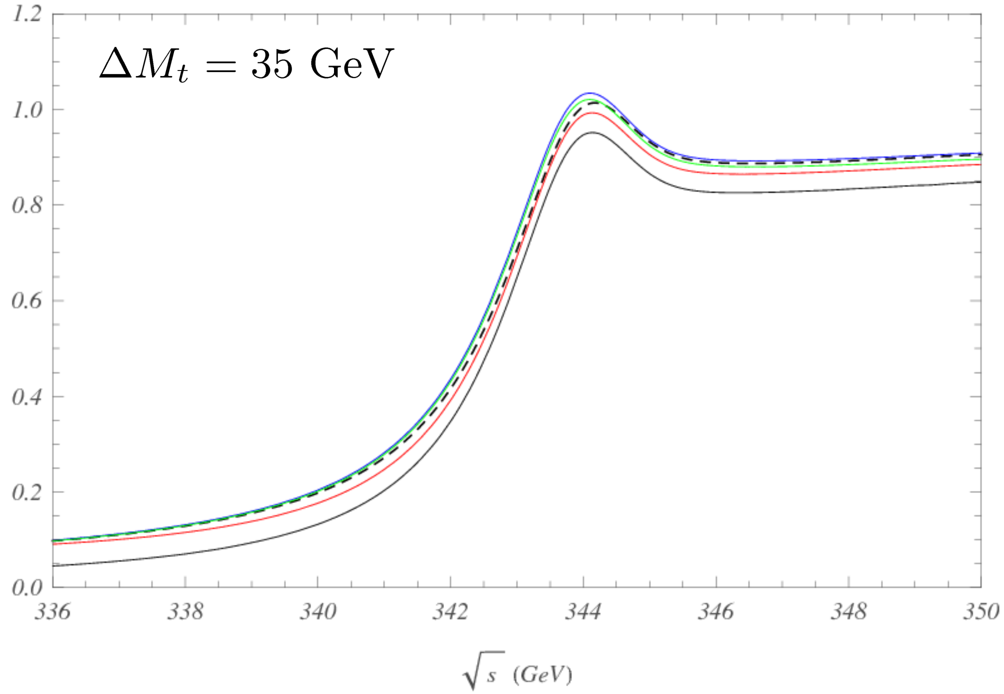
+ NNLL QED effects

+ NNLL hard one-loop EW effects

+ NNLL finite lifetime corrections

+ N^3LL phase space matching contributions

large psm corrections related to the unphysical phase space contributions contained in the pure QCD prediction



V. Conclusions

Precise determinations of top parameters in threshold region

- count number of $t\bar{t}$ events, color singlet state, background non-resonant, physics well understood
- **EFT framework** allows for a separation of resonant and non-resonant fluctuations and to sum up leading contributions
- QCD corrections well under control ⟶ **(almost) NNLL + N^{3}LO**

EW non-resonant corrections to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

- complete NLO non-resonant contributions computed for total cross section and with top invariant-mass cuts
- NLO non-resonant amount ~ -30 fb (-3% above and up to -20% below threshold) for the total cross section, even more with invariant-mass cuts

V. Conclusions

Beyond NLO: Phase space matching approach

- dominant NNLO and NNNLO terms computed in the invariant mass range $\Delta M_t \sim 15 - 35\ \mathrm{GeV}$ show good convergence

  $\Rightarrow$ need to be added to existing QCD results in view of the expected experimental uncertainties at the LC

Outlook

- analysis of **squark pair production** at threshold also possible within scalar NRQCD

  $\Longrightarrow$ full **NLL QCD running** known **Hoang, RF '05**

P-wave production $(e^+e^- \to \tilde{q}\,\bar{\tilde{q}})$: phase-space divergencies more severe

$$G^{L=1}_{\mathrm{coul}} = m^2\left(v^2 + \frac{C_F^2\,\alpha_s^2}{4}\right) G_{\mathrm{coul}} + \ldots \quad \underset{m\,v^2 = E + i\Gamma}{\Longrightarrow} \quad \mathrm{Im}\, G^{L=1}_{\mathrm{coul}} \sim \frac{\alpha_s \Gamma_t}{\epsilon}$$

LO effect!

(work in progress...)

Thank you!