Confronting τ-decay and $e^{+}+e^{-}\rightarrow$ hadrons with QCD

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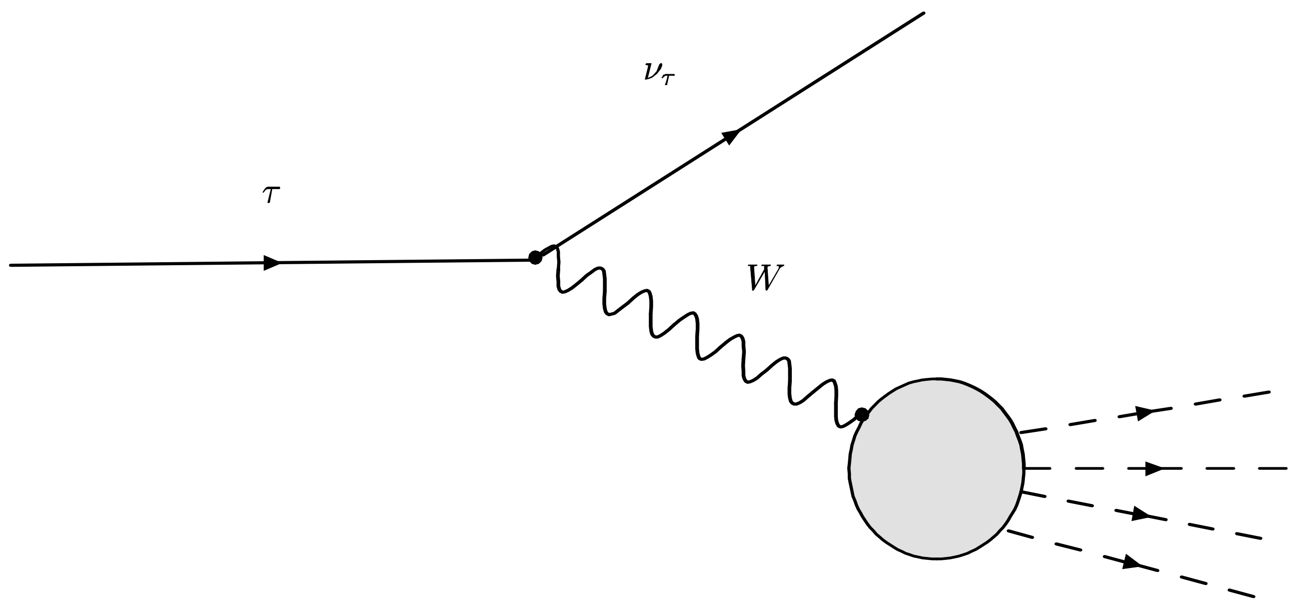
Outline

- τ-decay and hadronic correlators
- Duality in τ-decay
- Chiral sum rules
- Determination of condensates
- Comparison with e^+e^--annihilation
- Conclusions

1 Hadronic spectral functions

The τ-lepton is the only lepton that is heavy enough to decay into hadrons.



The decay matrix element factorizes into a leptonic and a hadronic part,

$$M(\tau^- \to \nu_\tau + hadrons) = \frac{G_F}{\sqrt{2}} |V_{CKM}| l_\mu h^\mu \ ,$$

where

$$l_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)\tau$$

is the leptonic current and

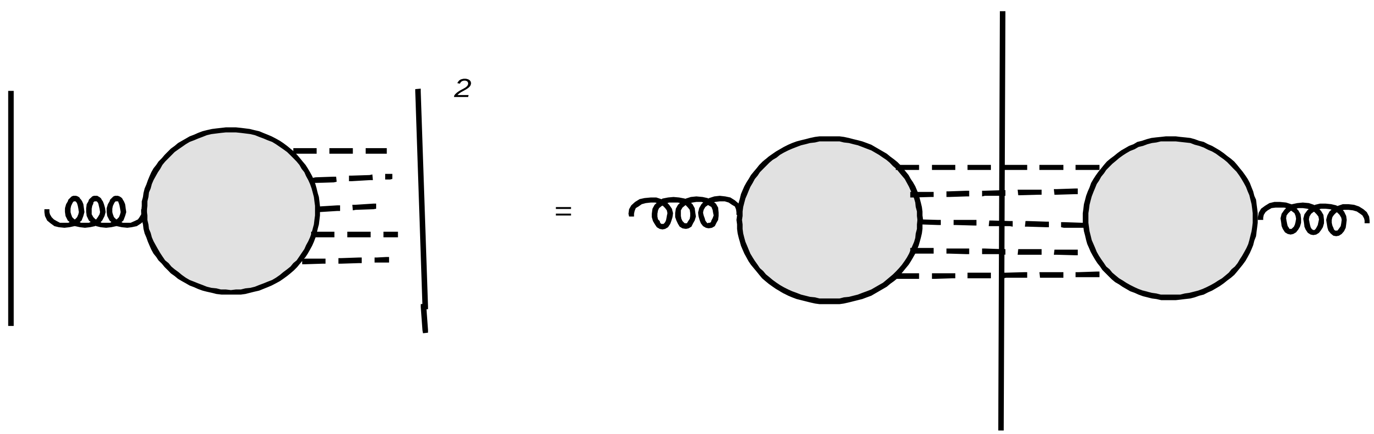
$$h^\mu = \langle hadrons|V^\mu(0) - A^\mu(0)|0\rangle$$

the hadronic one.

Branching ratio for non-strange hadrons

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + hadrons_{s=0})}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} = 3.482 \pm 0.014 \text{ (ALEPH 2005)}$$

Optical theorem:



The total hadronic branching ratio is related via the optical theorem to the correlator

$$\Pi_{\mu\nu}(q^2) = i \int d^4\,x\, e^{iqx} < 0|T(J_\mu(x)\;\; J_\nu^\dagger(0))|0 >$$
$$= (-g_{\mu\nu}\, q^2 + q_\mu q_\nu)\, \Pi_J^{(1+0)}(q^2) - q_\mu q_\nu\, \Pi_J^{(0)}(q^2)$$

$$J_\mu : \quad V_\mu(x) = \bar{u}(x)\gamma_\mu d(x) \;\; or \;\; A_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5 d(x)$$

If only hadrons up to a maximal cCM energy s_0 are counted then

$$R_\tau(s_0) = 24\pi\, |V_{ud}|^2\, S_{EW} \int_0^{s_0} \frac{ds}{s_0}(1 - \frac{s}{s_0})^2$$
$$\left[(1 + 2\frac{s}{s_0})\, \mathrm{Im}\,\Pi_{V+A}^{(0+1)}(s) - 2\frac{s}{s_0}\, \mathrm{Im}\,\Pi_{V+A}^{(0)}(s)\right]$$

$S_{EW} = 1.0194 \pm 0.0040$ (electroweak correction) $|V_{ud}| = 0.9739 \pm 0.0003$

One defines spectral functions $v(s)$ and $a(s)$ by:

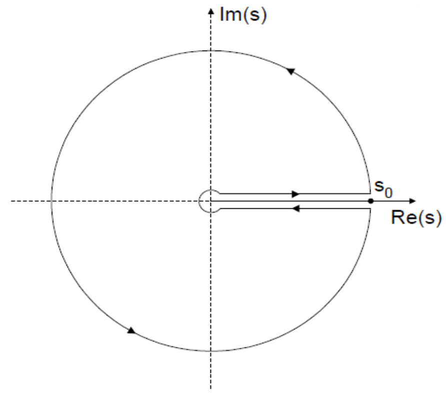
$$(v(s), a(s)) = 4\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{V,A}(s + i\varepsilon),$$

$$a_0(s) = 4\pi \, \mathrm{Im}\, \Pi^{(0)}_{A}(s + i\varepsilon)$$

The individual spectral functions can be separated according to angular momentum parity and flavour, e.g.

$v(s)$	even number of pions
$a(s)$	odd number of pions

Finite energy sum rules



Duality = Cauchy's theorem

$$\frac{1}{\pi}\int_0^R f(s)\,\text{Im}\,\Pi(s)ds = -\frac{1}{2\pi i}\oint_{|s|=R} f(s)\Pi(s)ds$$
$$\simeq -\frac{1}{2\pi i}\oint_{|s|=R} f(s)\Pi_{QCD}(s)ds,$$

where $f(s)$ is an arbitrary holomorphic function, e.g. a polynomial.

Operator Product Expansion (OPE)

$$\Pi(q^2) = \sum_{N=0}^{\infty} \frac{1}{(-q^2)^N}\, C_{2N}(q^2,\mu^2)\ < 0|\mathcal{O}_{2N}(\mu^2)|0 >\ ,$$

with $\mathcal{O}_0$ being the unit operator corresponding to the pure perturbative term.

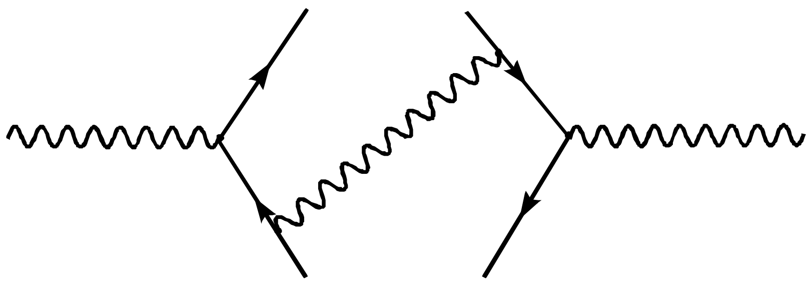
The lowest dimension vacuum expectation values together with commonly used values (at scale $\mu^2 = 1GeV^2$) are:

$$\begin{aligned}
\langle 0|\bar{q}q|0\rangle &= -(225 \pm 25MeV)^3 \qquad \text{(D=3)}\\
\langle 0|\alpha_s G_{\mu\nu}G^{\mu\nu}|0\rangle &= 0.04 \pm .0.02GeV^4 \qquad \text{(D=4)}\\
\langle 0|g_s\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle &= 0.8 \pm 0.2GeV^5 \qquad \text{(D=5)}\\
\left\langle 0|(\bar{q}q)^2|0\right\rangle &= -0.0020\ GeV^6 \qquad \text{(D=6)}
\end{aligned}$$

The chiral correlator

$$\Pi_{V-A} \equiv \Pi_V^{(0+1)} - \Pi_A^{(0+1)} \underset{QCD}{\simeq} \frac{m_u m_d}{q^4} + \ldots.$$

This correlator allows the **direct** study of non-perturbative properties of QCD



$$\Pi_{V-A}(q^2) = \frac{1}{(-q^2)^3} C_6 O_6 + \mathcal{O}(1/Q^8)$$
$$= \frac{32\pi}{9} \frac{\alpha_s < \bar{q}q >^2}{q^6} + \mathcal{O}(1/Q^8)$$
$$= \left\{1 + \frac{\alpha_s(\mu^2)}{4\pi} \left[\frac{247}{12} + \ln\left(\frac{\mu^2}{-q^2}\right)\right]\right\} + \mathcal{O}(1/Q^8) \, .$$

Π_{V-A} serves as a non-perturbative order parameter of spontaneous chiral symmetry breaking.

These condensates enter in other calculations (O_6 enters in the prediction of the CP-violating parameter ε'/ε of K^0-decay).

Chiral Sum Rules

FESR for the chiral correlator

$$\frac{1}{4\pi^2}\int_0^{s_0} ds\, s^N\,(v(s)-a(s))^{data}_{cont} - f_\pi^2(m_\pi^2)^N$$
$$= -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^N\,\Pi^{QCD}(s)$$
$$= (-)^N < C_{2N} O_{2N} >$$

For $N = 2, 3$ the sum rules project the $d = 6, 8$ vacuum condensates, respectively. To first order in α_s, radiative corrections to the vacuum condensates do not induce mixing of condensates of different dimension in a given FESR.

Weinberg sum rules:

In the chiral limit $< C_2 O_2 > = < C_4 O_4 > = 0$.

For $N = 0, 1 \longrightarrow$ we get the first two (Finite Energy) Weinberg sum rules:

$$\frac{1}{4\pi^2} \int_0^{s_0} ds\, (v(s) - a(s)) = f_\pi^2 \quad \text{first WSR}$$

$$\frac{1}{4\pi^2} \int_0^{s_0} ds\, s\, (v(s) - a(s)) = 0 \quad \text{second WSR}$$

Das-Mathur-Okubo sum rule:

The finite remainder of the chiral correlator at zero momentum

$$\bar{\Pi}(0) = \frac{1}{4\pi^2} \int_0^{s_0} \frac{ds}{s} \left(v(s) - a(s)\right)$$

$\bar{\Pi}(0)$ is related to the finite part of the counter term $\bar{L}_{10}$ of the $\mathcal{O}(p^4)$ Lagrangian of chiral perturbation theory

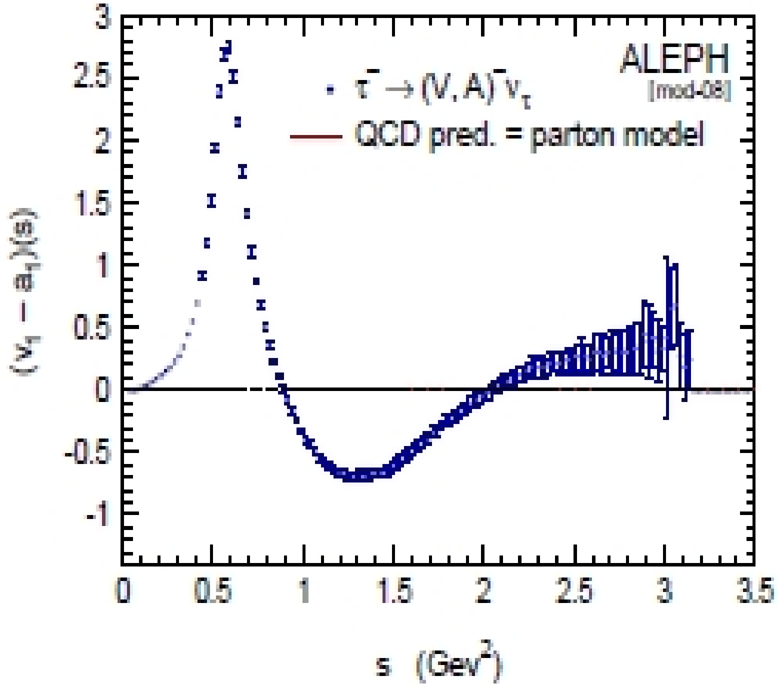
$$\bar{\Pi}(0) = -4\bar{L}_{10}$$

The DGLMY sum rule:

$$\frac{1}{4\pi^2}\int_0^{s_0\to\infty} ds s \ln\frac{s}{\Lambda^2}(v(s)-a(s)) = -\frac{4\pi f_\pi^2}{3\alpha}(m_{\pi^\pm}^2 - m_{\pi^0}^2)$$

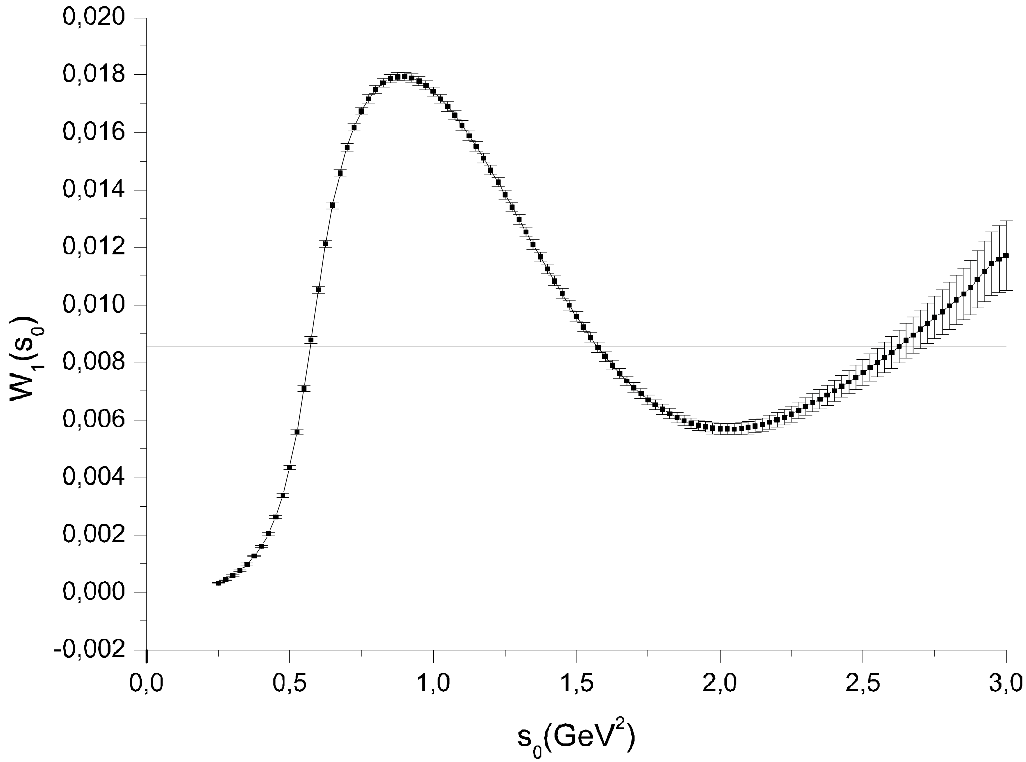
(Das, Guralnik, Low, Mathur and Young 1967). Experimentally $m_{\pi^\pm} - m_{\pi^0} = 4.59 MeV$.

The V-A spectral function data



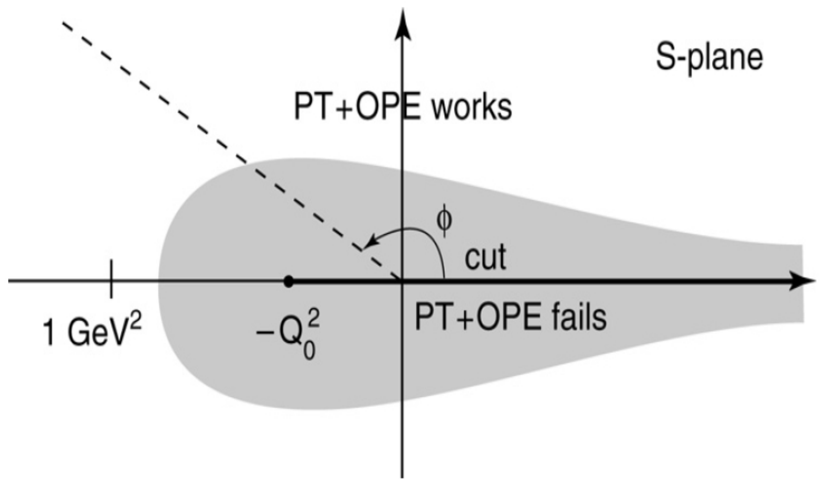
The data clearly show that the asymptotic regime has not been reached, not even at the highest momenta attainable in τ-decay.

Weinberg sum rule:



There is no indication of precocious saturation.

Pinched sum rules

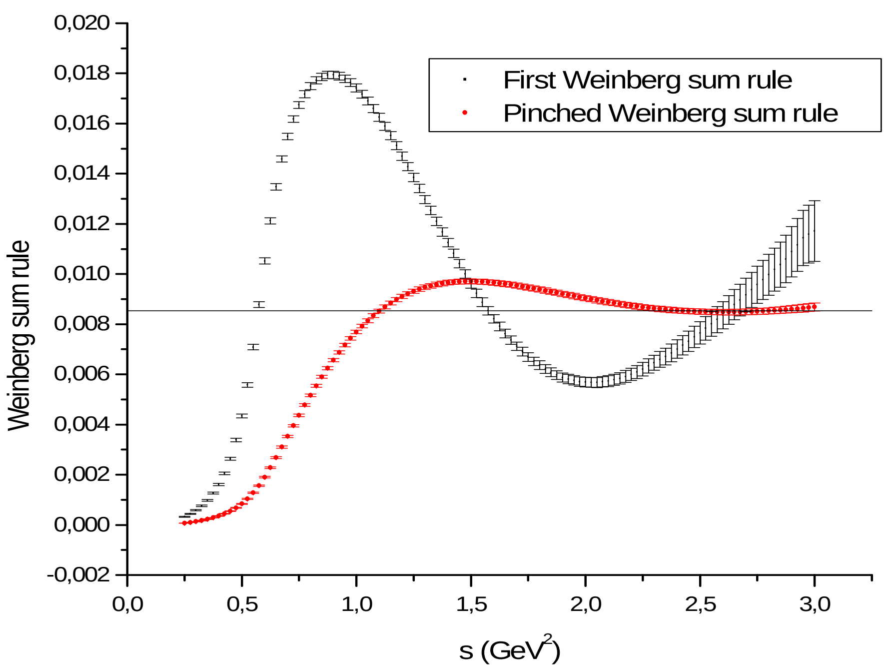


FESR involving factors of $(1-\frac{s}{s_0})$ minimize the contribution near the cut.

Assume: There are no operators of dimension $d=2$ nor $d=4$ (chiral limit)

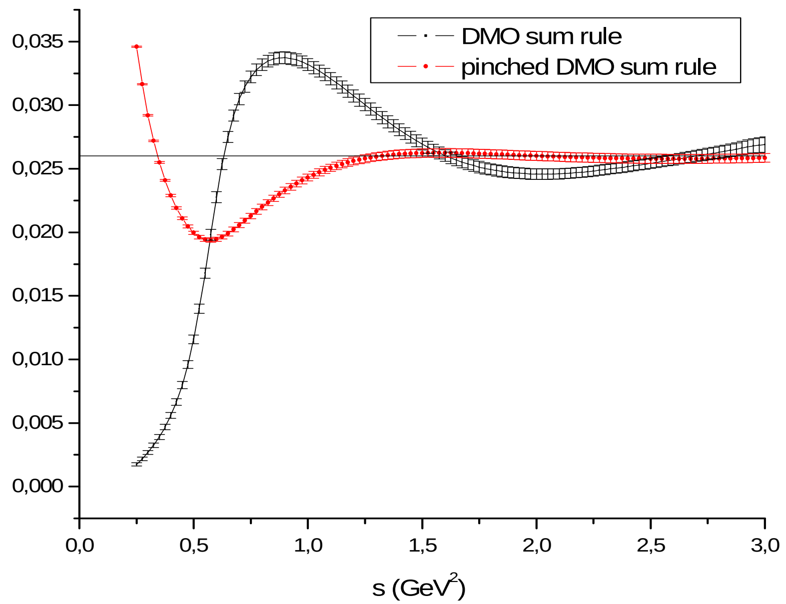
We begin by considering a linear combination of the first two Weinberg sum rules

$$\bar{W}_1(s_0) \equiv \frac{1}{4\pi^2}\int_0^{s_0} ds\, (1-\frac{s}{s_0})\, [v(s)-a(s)] = f_\pi^2.$$



Pinched DMO sum rule

$$\bar{\Pi}(0) = \frac{1}{4\pi^2} \int_0^{s_0} \frac{ds}{s} \, (1 - \frac{s}{s_0}) \, [v(s) - a(s)] + \frac{f_\pi^2}{s_0}$$



Numerically, we find from the DMO sum rule

$$-4\bar{L}_{10} = \bar{\Pi}(0) = 0.02579 \pm 0.00023\ ,$$

(remarkable accuracy for a strong interaction parameter prediction)

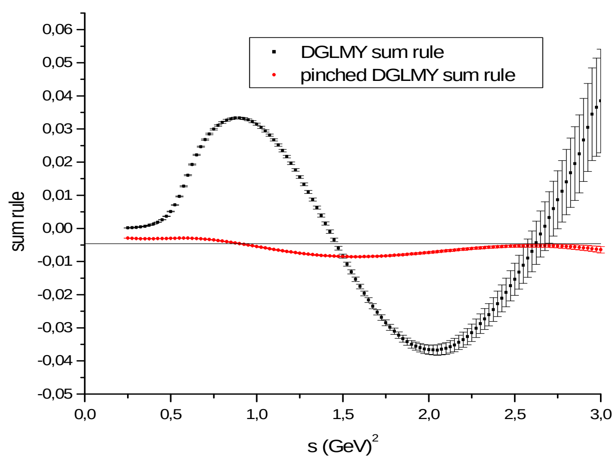
Chiral perturbation theory yields

$$\left[\frac{1}{3} f_\pi^2 < r_\pi^2 > - F_A\right] = -4\bar{L}_{10}\ = 0.026 \pm 0.001\ ,$$

where $< r_\pi^2 >= 0.439 \pm 0.008\ fm^2$, and F_A is the axial-vector coupling measured in radiative pion decay, $F_A = 0.0058 \pm 0.0008$.

Pinched DGLMY sum rule

$$\int_0^{s_0\to\infty} ds \left[s \ln\frac{s}{\Lambda^2} - s_0 \ln\frac{s_0}{\Lambda^2}\right] \rho(s) + s_0 \ln\frac{s_0}{\Lambda^2} f_\pi^2 = -\frac{4\pi f_\pi^2}{3\alpha}(m_{\pi^\pm}^2 - m_{\pi^0}^2)$$



At $s_0 = 2.8 GeV^2$ the sum rule yields $(4.0 \pm 0.8) MeV$ while $\left(m_{\pi^\pm} - m_{\pi^0}\right)_{\text{exp}} = 4.59 MeV$.

Extraction of the condensates

The philosophy:

1. dimension $d = 2$ and $d = 4$ operators are absent in the OPE of the chiral current

2. to require that the polynomial projects out only one operator of the OPE at a time

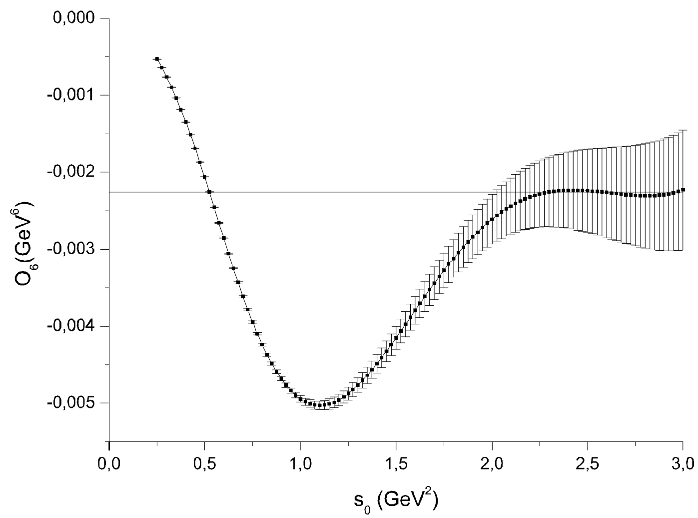
3. require that the polynomial and its first derivative vanish on the integration contour of radius $|s| = s_0$.

In this way one obtains for $N \geq 3$ the sum rules

$$\begin{aligned}
&\mathcal{O}_{2N}(s_0) \\
&= (-1)^{N-1} \frac{1}{4\pi^2} \int_0^{s_0} ds \\
&\times [(N-2)s_0^{N-1} - (N-1)s_0^{N-2}s + s^{N-1}] \\
&\times [v(s) - a(s)] - (-1)^{N-1}(N-2)s_0^{N-1} f_\pi^2
\end{aligned}$$

Pinch factor $(s - s_0)^2$in the polynomial.

Strong stability: *The r.h.s. of the sum rule should be constant for all* s_0 *larger than some some critical value.*



Stability region: $2.3 \leq s_0(GeV^2) \leq 3$:

$$\mathcal{O}_6(2.7\ GeV^2) = -(0.00226 \pm 0.00055)\ GeV^6\ .$$

This value is consistent with the one found from the vacuum saturation approximation $\mathcal{O}_6^{VS} = -0.0020\ GeV^6$ with $< \bar{q}q > (s_0) = -0.019\ GeV^3$, and $\alpha(s_0)/\pi = 0.1$.

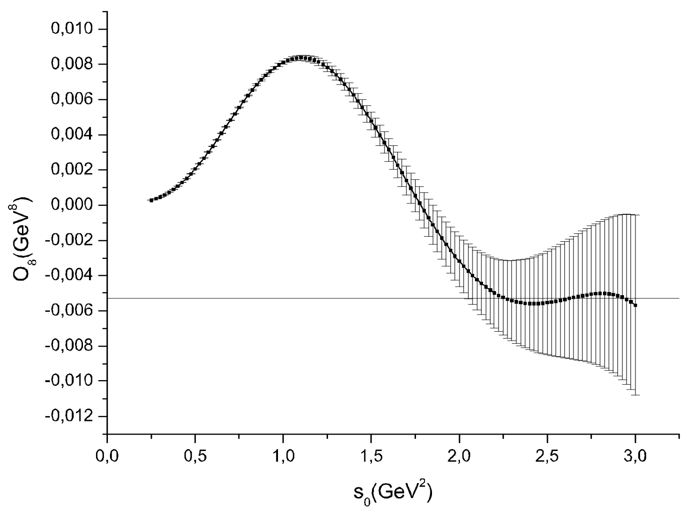
The sum rule for O_8 reads

$$\mathcal{O}_8(s_0) = -\frac{1}{4\pi^2}\int_0^{s_0} ds\, [2s_0^3 - 3s_0^2 s + s^3][v(s) - a(s)] + 2s_0^3 f_\pi^2\ .$$

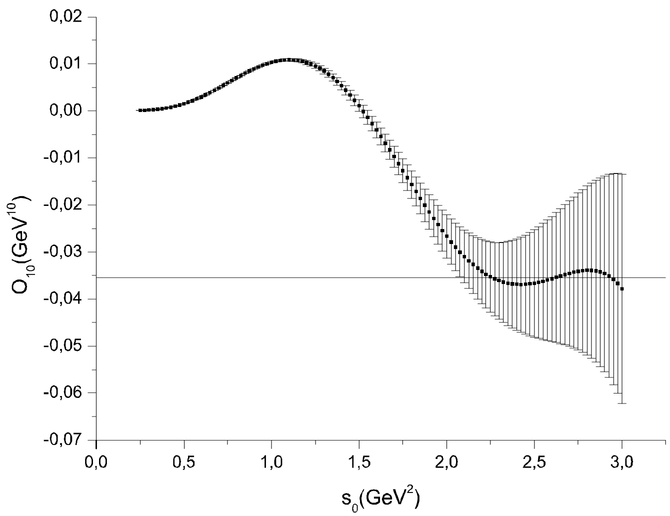
$$(2s_0^3 - 3s_0^2 s + s^3) = (s_0 - s)^2\,(s + 2s_0)).$$

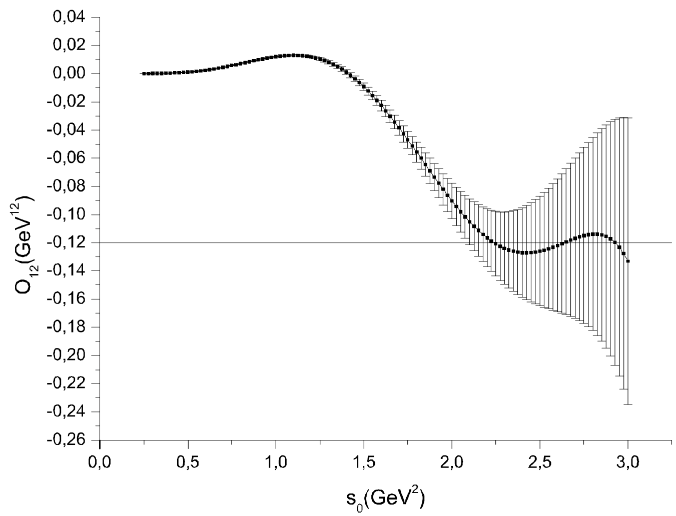
Region of duality in the interval: $2.3 \leq s_0(GeV^2) \leq 3$, which yields

$$\mathcal{O}_8(2.6\ GeV^2) = -(0.0054 \pm 0.0033)\ GeV^8$$



Both the sign and the numerical value of this condensate are controversial.





Strong stability ?

$\mathcal{O}_{10}(2.5\ GeV^2) = (0.036 \pm 0.014) GeV^{10}$, $\mathcal{O}_{12}(2.5\ GeV^2) = -(0.12 \pm 0.05) GeV^{12}$

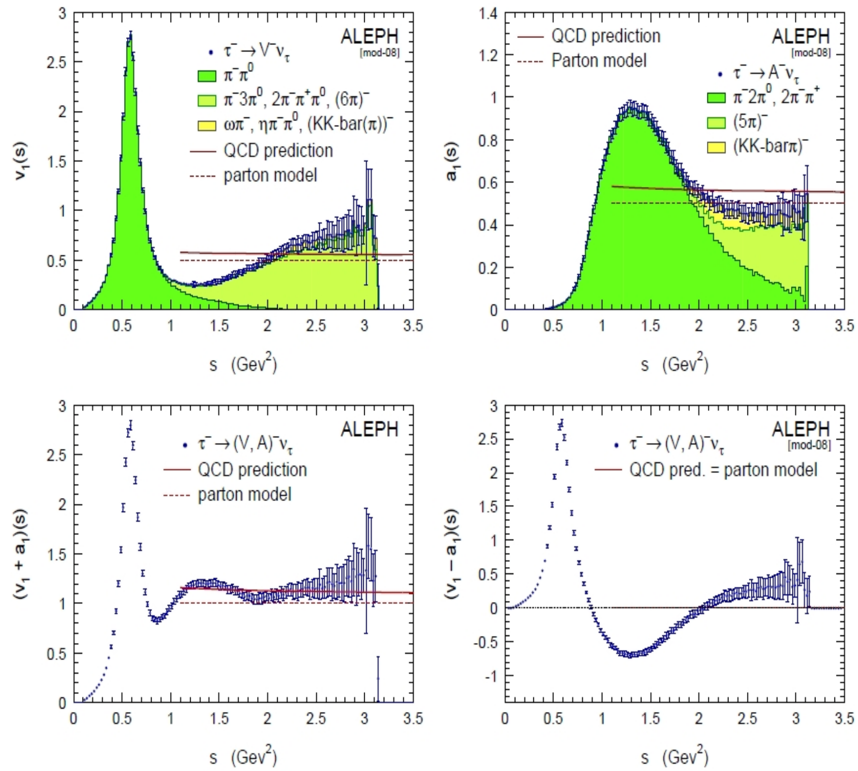
Lessons learnt:

- Perturbative QCD seems to be applicable in the space-like region already at rather small momentum transfers of $O(1 - 2GeV^2)$.

- The same cannot be seen in the time-like region. Duality has in general not been reached even at $q^2 = m_\tau^2$.

- If the weight in the spectral integrals is shifted away from the real axis by pinching, duality is satisfied precociously to a remarkable extend.

2 The V+A spectral function



The $V+A$ spectral function, including the pion pole, roughly exhibits the features expected from global quark-hadron duality: Despite the huge oscillations due to the prominent $\pi, \rho(770), A_1(1450)$, the spectral function qualitatively averages out to the quark contribution from perturbative QCD and approximately reaches the free quark model result for $s \to m_\tau^2$.

- But not quite! V, V+A, V-A spectral functions lie for $s \to m_\tau^2$ a bit above the asymptotic QCD, A below.

- The situation is much better for (pinched) integrated observables.

The total decay rate

We consider the τ-decay rate as a function of the upper limit of integration

$$R_\tau(s_0) = 24\pi\,|V_{ud}|^2\,S_{ew}\int_0^{s_0}\frac{ds}{s_0}(1-\frac{s}{s_0})^2$$
$$\times\left[(1+2\frac{s}{s_0})\,\mathrm{Im}\,\Pi^{(1)}_{V+A}(s)+\mathrm{Im}\,\Pi^{(0)}_{V+A}(s)\right]$$

$(1-\frac{s}{s_0})^2(1+2\frac{s}{s_0})=\frac{1}{s_0^3}\left(2s^3+s_0^3-3s^2s_0\right)$ the gluon condensate does not contribute.

The results of ALEPH:

$$R_\tau(m_\tau^2) = 3.482 \pm 0.014$$

QCD Correlator (V or A, 5 loops)

In the minimal subtraction scheme for one massless flavor:

$$\begin{aligned}
8\pi^2\Pi_V(q^2) = c - L - La - a^2\left[Lk_2 - \frac{1}{2}L^2\beta_0\right] \\
- a^3\left[Lk_3 + \frac{1}{3}L^3\beta_0^2 + L^2\left(-\frac{1}{2}\beta_1 - \beta_0 k_2\right)\right] \\
- a^4\left[Lk_4 - \frac{1}{4}L^4\beta_0^3 + \frac{1}{6}L^3\beta_0\left(5\beta_1 + 6\beta_0 k_2\right)\right. \\
\left. + L^2\left(-\frac{1}{2}\beta_2 - \frac{3}{2}\beta_0 k_3 - \beta_1 k_2\right)\right]
\end{aligned}$$

where

$$a \equiv \frac{\alpha(\mu)}{\pi}; \qquad L \equiv \ln\frac{-q^2}{\mu^2}$$

c is a constant related to external renormalzation,effects of the u, d quark masses are negligible.

n is the number of flavours, $z = 1.2020569$ (z is $\zeta(3)$)

$$\beta_0 = 1/4 * (11 - 2/3n)$$

$$\beta_1 = 1/16 * (102 - 38/3 * n)$$

$$\beta_2 = 1/64 * (2857/2 - 5033/18 * n + 325/54 * n * n)$$

$$\beta_3 = 1/(4^4) * (149753/6 + 3564 * z - (1078361/162 + 6508/27 * z) * n$$
$$+(50065/162 + 6472/81 * z) * n^2 + 1093/729 * n^3)$$
$$= 47.228\,040 \quad \text{for } n = 3.$$

$$k_0 = k_1 = 1, \quad k_2 = 1.63982, \quad k_3 = 6.37101, \quad k_4 = 49.076 \quad \text{(for 3 flavors)}$$

Parton model result:

To lowest order in QCD

$$R_\tau(s_0) = 3\,|V_{ud}|^2\,S_{EW}$$

Simple approach (FOPT):

$$R_\tau(s_0) = 24\pi^2\,|V_{ud}|^2\,S_{EW}\int_0^{s_0}\frac{ds}{s_0}\frac{1}{\pi}\,\mathrm{Im}\,\Pi^{(1)}_{V+A}(s)$$
$$\times\left[(1-3(\frac{s}{s_0})^2+2(\frac{s}{s_0})^3\right]$$

by introducing the QCD moments and using Cauchy's Theorem,

$$M_N(s_0) \equiv 8\pi^2 \int_0^{s_0} \frac{ds}{s_0} \left[\frac{s}{s_0}\right]^N \frac{1}{\pi} \,\mathrm{Im}\, \Pi_{QCD}(s)$$

$$= \int_0^{s_0} \frac{ds}{s_0} \left[\frac{s}{s_0}\right]^N (v(s) + a(s))$$

$$= 8\pi^2 \frac{1}{2i} \oint_{|s|=s_0} \frac{ds}{s_0} \left[\frac{s}{s_0}\right]^N \Pi_{QCD}(s)$$

for **fixed** $\alpha_s(\mu^2)$. Then

$$R_\tau(s_0) = 6\,|V_{ud}|^2\, S_{EW}\,[M_0(s_0) - 3M_2(s_0) + 2M_3(s_0)]$$

The RGE is only applied at the very end for the finite observable $R_\tau(s_0)$ by setting $\mu^2 = s_0$.

Alternative approach (CIPT):

Apply the RGE to the correlator first and then do the contour integration (K.S., M. D.Tran, 1984). The RG is applied more easily to finite observables. Here this is the **Adler function**

$$D(s) \equiv -s\frac{d}{ds}\Pi(s) \ .$$

The Adler function is introduced by partial integration

$$\oint\limits_{|s|=s_0} ds g(s)\Pi(s) = -\oint\limits_{|s|=s_0} \frac{ds}{s}[G(s) - G(s_0)]s\frac{d}{ds}\Pi(s)$$

$$\text{with} \qquad G(s) = \int_0^s ds' g(s') \ .$$

Then

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW}$$
$$\times (-2\pi i) \oint_{|s|=s_0} \frac{ds}{s}[1 - \frac{s}{s_0} + 2(\frac{s}{s_0})^3 - (\frac{s}{s_0})^4] D(s)$$

The Adler function has been calculated to fourth order in QCD perturbation theory,

$$D(s,\mu^2) = \frac{1}{4\pi^2}\sum_{n=0}^{4} K_n(\mu^2) a_s^n(\mu^2) \qquad \left(a = \frac{\alpha_s}{\pi}\right)$$

The RGE is solved by simply replacing μ^2 by $-s$. Then, for $n_f = 3$

$$K_0 = K_1 = 1, \quad K_2 = 1.640,$$
$$K_3 = 6.371, \quad K_4 = 49.076$$

The integral over the circle: substitute on the circle $s = -s_0 e^{i\varphi}$,

$$\frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s}[1 - \frac{s}{s_0} + 2(\frac{s}{s_0})^3 - (\frac{s}{s_0})^4] a_s^n(-s)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \left[1 + 2e^{i\varphi} - 2e^{i3\varphi} - e^{i4\varphi}\right] a_s^n(s_0 e^{i\varphi})$$

The s-dependence of the α_s is obtained by solving the RGE

$$\frac{da_s}{d \ln s} = \beta(a_s) = -a_s^2 \sum_{n=0}^{3} \beta_n a_s^n \qquad \left(a = \frac{\alpha_s}{\pi}\right)$$

numerically with $a_s(-s_0)$ as initial value.

For $n_f = 3$ one finally obtains

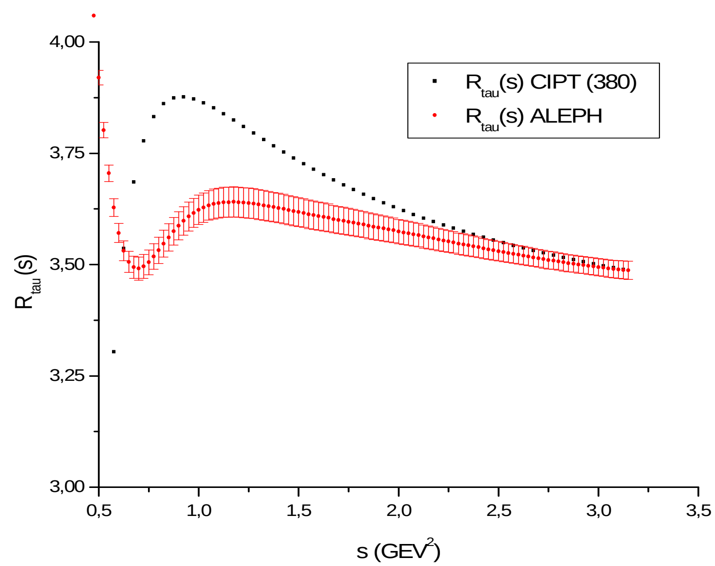
$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW}[1 + 1.364\frac{\alpha_s}{\pi}(s_0) + 2.54\left(\frac{\alpha_s}{\pi}\right)^2 + 9.71\left(\frac{\alpha_s}{\pi}\right)^3 + 64.29\left(\frac{\alpha_s}{\pi}\right)^4 \ldots]$$

Experimentally

$$\alpha_s(m_\tau^2) = 0.344 \pm 0.009$$

The convergence is not great.

Decay rate:

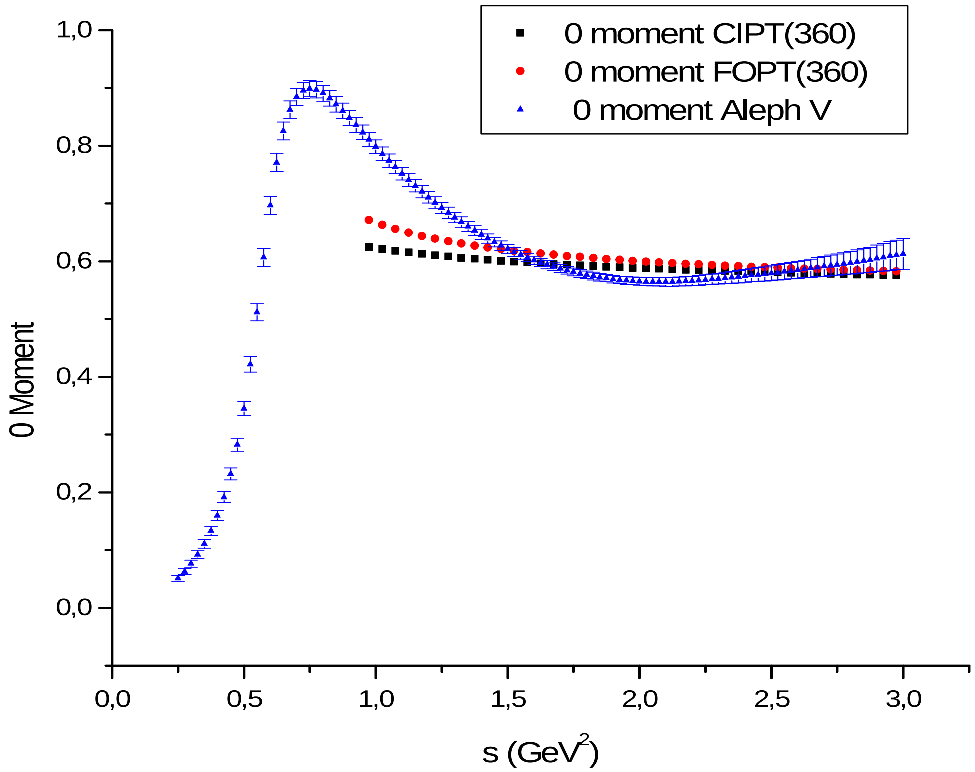


4,00
3,75
3,50
3,25
3,00
R_{tau}(s)
0,5
1,0
1,5
2,0
2,5
3,0
3,5
s (GEV^2)
R_{tau}(s) CIPT (380)
R_{tau}(s) ALEPH

Condensates V+A case

$$\Pi(s)_{np} = \frac{1}{24s^2}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left(1 + \frac{7}{6}\frac{\alpha_s}{\pi}\right) + \frac{1}{2s^2}(m_u + m_d)\langle \bar{q}q \rangle$$
$$+ \frac{32\pi^2}{81s^3}\frac{\alpha_s}{\pi}\langle \bar{q}q \rangle_\mu^2 \left[1 + \left(\frac{29}{24} + \frac{17}{18}\ln\frac{-s}{\mu^2}\right)\right]\frac{\alpha_s}{\pi} + \frac{\langle O_8 \rangle}{s^4}$$

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle < 0.008\ GeV^2$$



0 moment CIPT(360)
0 moment FOPT(360)
0 moment Aleph V
0 Moment
s (GeV^2)
1,0
0,8
0,6
0,4
0,2
0,0
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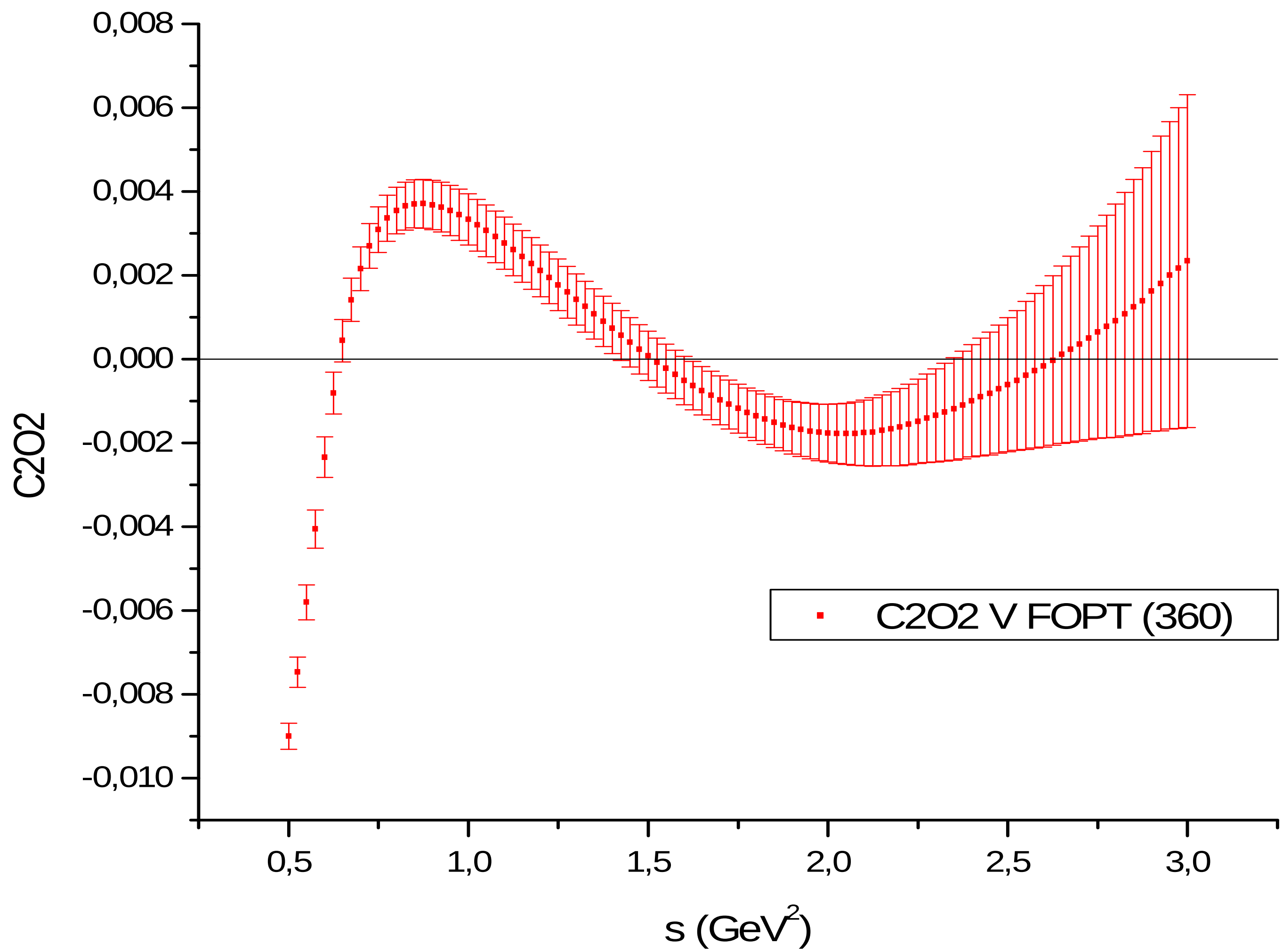
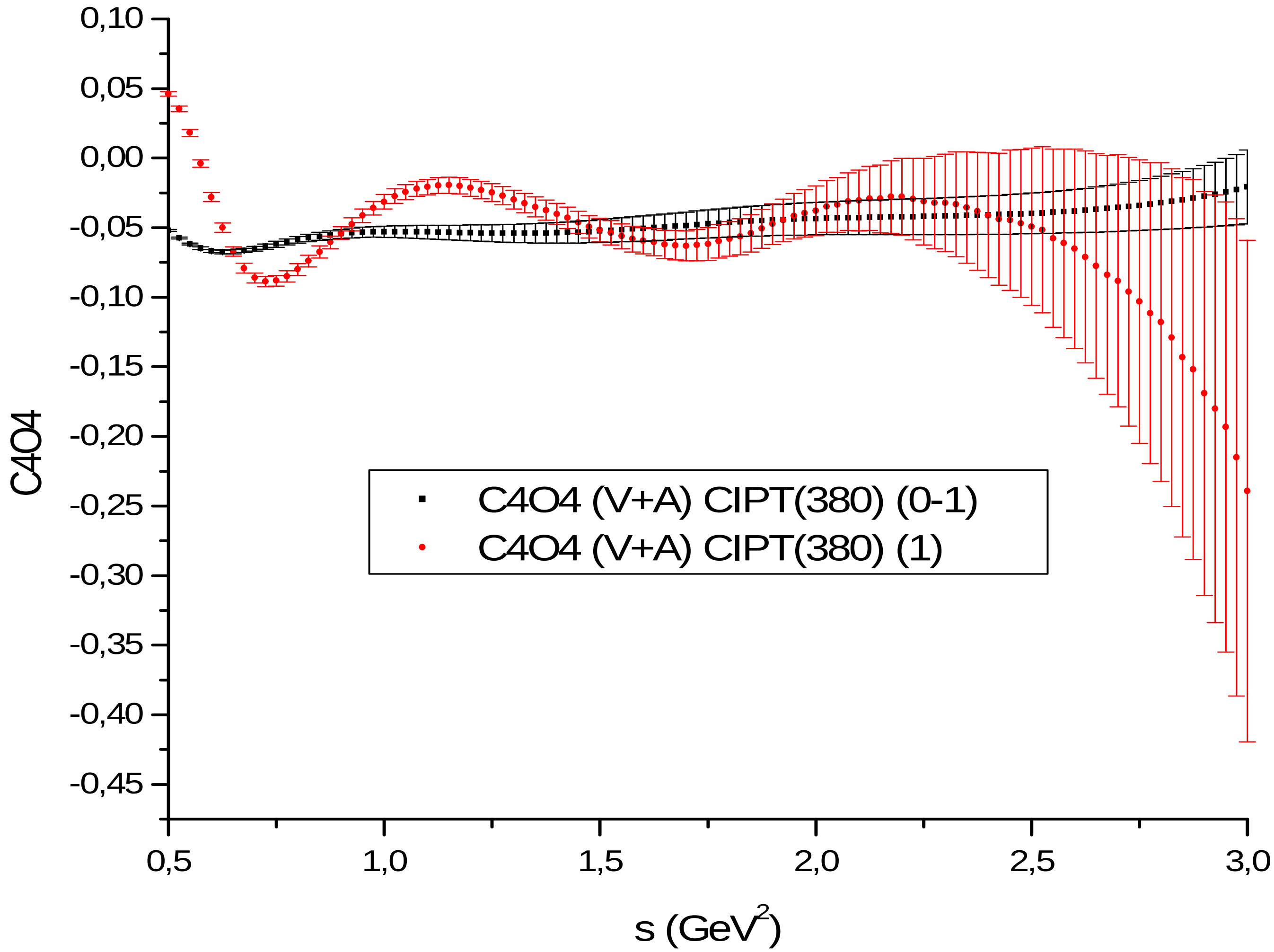
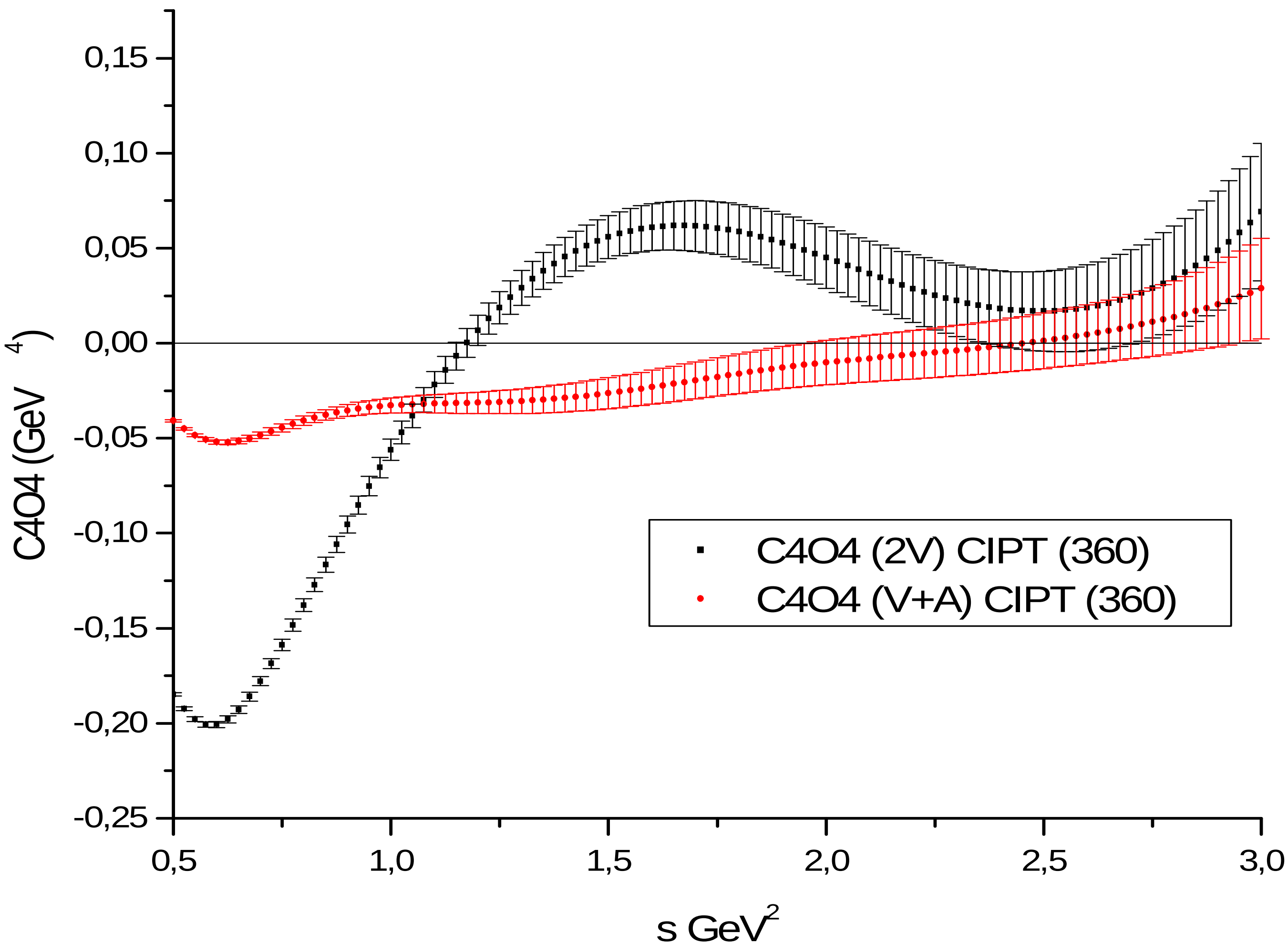
0,5
1,0
1,5
2,0
2,5
3,0


0,008
0,006
0,004
0,002
0,000
-0,002
-0,004
-0,006
-0,008
-0,010
C2O2
0,5
1,0
1,5
2,0
2,5
3,0
s (GeV^2)
C2O2 V FOPT (360)



0,10
0,05
0,00
-0,05
-0,10
-0,15
-0,20
-0,25
-0,30
-0,35
-0,40
-0,45
C4O4
C4O4 (V+A) CIPT(380) (0-1)
C4O4 (V+A) CIPT(380) (1)
0,5
1,0
1,5
2,0
2,5
3,0
s (GeV2)



0,15
0,10
0,05
0,00
-0,05
-0,10
-0,15
-0,20
-0,25
C4O4 (GeV 4)
0,5
1,0
1,5
2,0
2,5
3,0
s GeV2
C4O4 (2V) CIPT (360)
C4O4 (V+A) CIPT (360)

The figure (Kernel s and $1 - s/s_0$) from tau decay which demonstrates three things:

- C2O2 must be small
- pinching works well
- $C4O4$ could still rise to $\approx 0.04 GeV^4$ M (from charmonium sum rules) if the tau mass were bigger (or α_s smaller)

$\rightarrow$Conclusions

- Perturbative and non-perturbartive effects separate nicely in hadronic τ-decay.

- The asymptotic region has not quite been reached in the time-like domain, even at the highest energies accessible in τ-decay.

- In the space-like domain and in the nearby complex plane asymptotic QCD is reached precociously.

- The non-perturbative effects isolated in the V, A and especially the V-A sector are well understood.

- The perturbation series is not very convergent. This is due to the large value of α_s.

- All in all, perturbative QCD and the OPE beautifully describe all aspects of τ-decay

- There is no evidence for duality violations.

e^+e^- annihilation

$$
\begin{aligned}
J^\mu_{\mathsf{em}} &= \sum_f Q_f J^\mu_f = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s \\
&= \underbrace{\frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)}_{I=1} + \underbrace{\frac{1}{6}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d - 2\bar{s}\gamma^\mu s)}_{I=0} \,.
\end{aligned}
$$

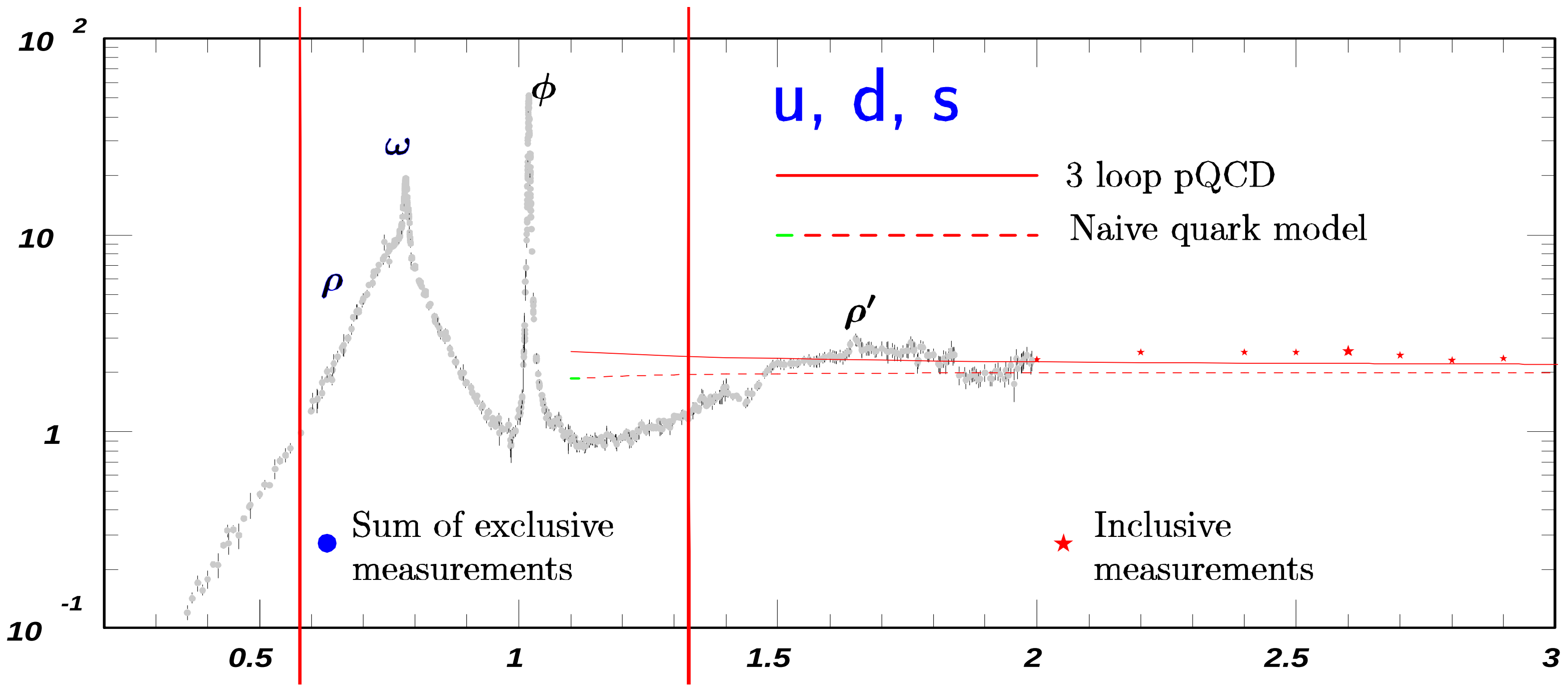
Correlators can be defined for every quark flavor by

$$\Pi_f^{\mu\nu} = i\int d^4x \langle 0|T J_f^{\mu}(x) J_f^{\nu}(0)|0\rangle = i\int d^4x \left\langle 0|\bar{q}_f\gamma_\mu q_f \bar{q}_f \gamma_\mu q_f|\right\rangle$$
$$= \left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right) \Pi_f(q^2) \ ,$$

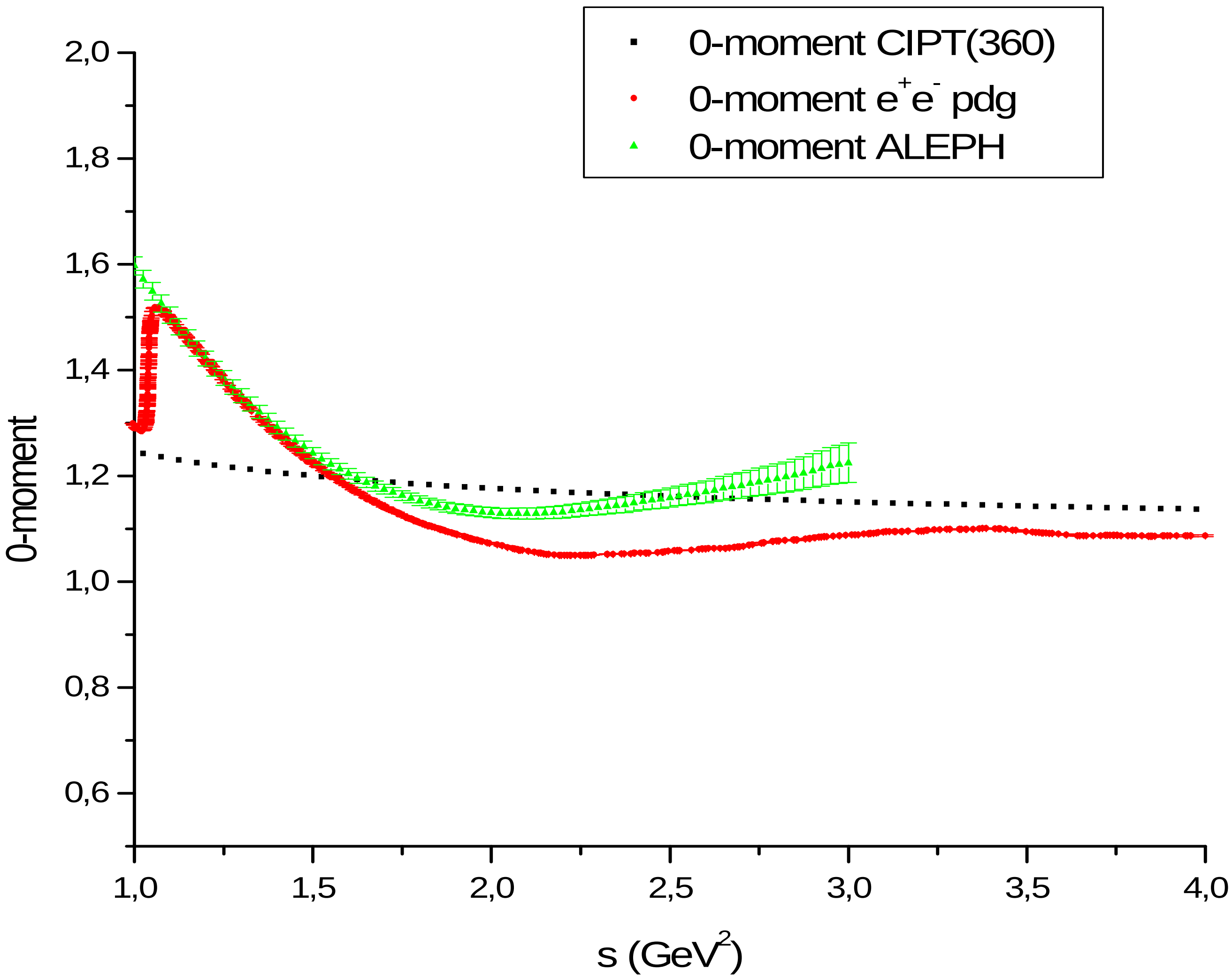
for the electromagnetic current

$$\Pi_{\mathrm{em}}^{\mu\nu} = i\int d^4x \left\langle 0|T J_{\mathrm{em}}^{\mu}(x) J_{\mathrm{em}}^{\nu}(0)|0\right\rangle = \sum_f Q_f^2 \Pi_f^{\mu\nu} \ .$$

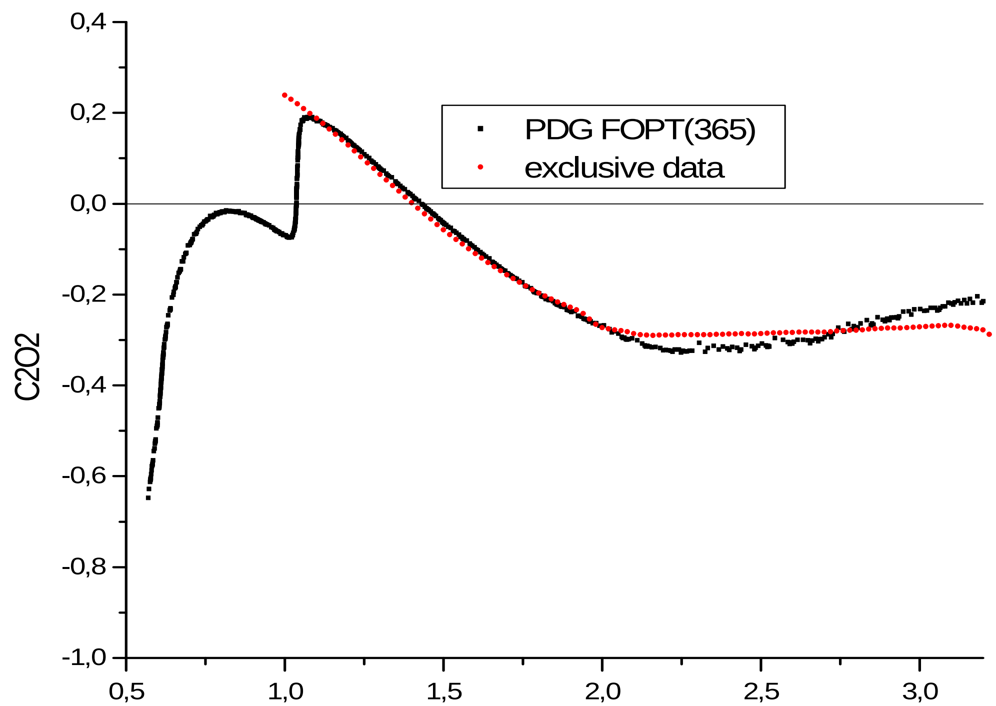
$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$
$$= N_c \sum_i Q_i^2 \left[8\pi^2 \frac{1}{\pi} \mathrm{Im}\,\Pi^i(s)\right]$$



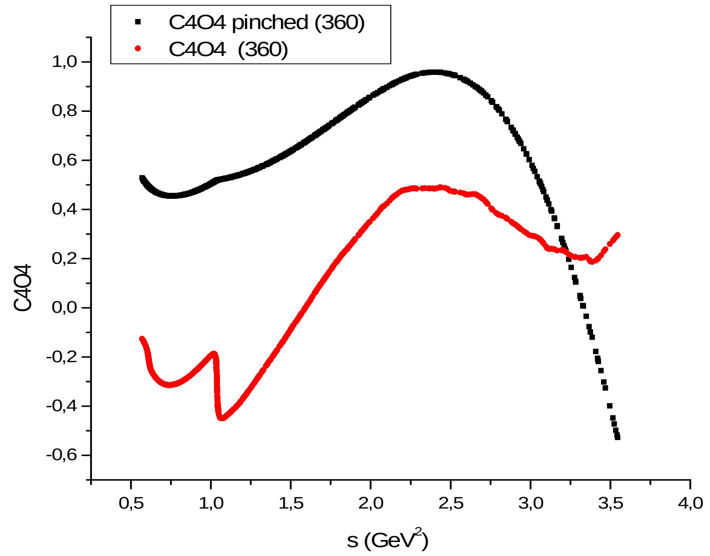
u, d, s
3 loop pQCD
Naive quark model
ρ
ω
φ
ρ′
Sum of exclusive measurements
Inclusive measurements
10^2
10
1
10^-1
0.5
1
1.5
2
2.5
3



0-moment CIPT(360)
0-moment e+e- pdg
0-moment ALEPH
0-moment
s (GeV2)
2,0
1,8
1,6
1,4
1,2
1,0
0,8
0,6
1,0
1,5
2,0
2,5
3,0
3,5
4,0



PDG FOPT(365)
exclusive data
C2O2
0,4
0,2
0,0
-0,2
-0,4
-0,6
-0,8
-1,0
0,5
1,0
1,5
2,0
2,5
3,0



Pinching does not work because $C2O2 \neq 0$.

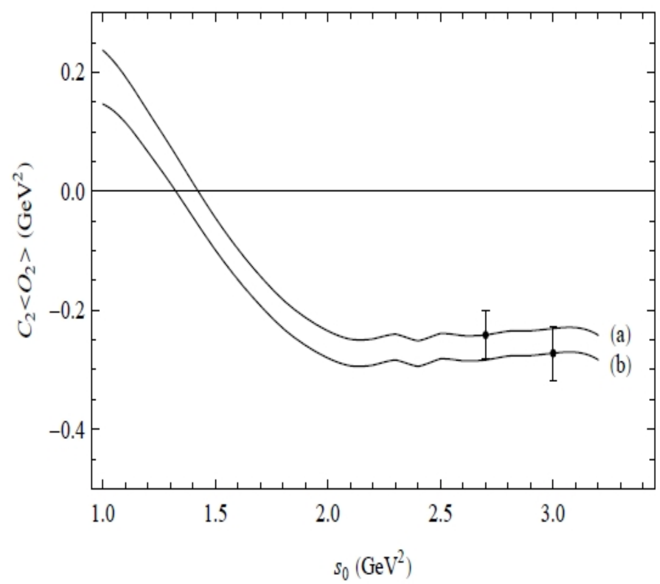


Figure 1: Results for $C_2 \langle \mathcal{O}_2 \rangle$, in units of GeV^2, as a function of s_0. Curve (a) corresponds to $\alpha_s(M_\tau^2) = 0.335$ ($\Lambda = 365$ MeV), and curve (b) to $\alpha_s(M_\tau^2) = 0.353$ ($\Lambda = 397$ MeV). A single representative experimental error is shown for each curve.

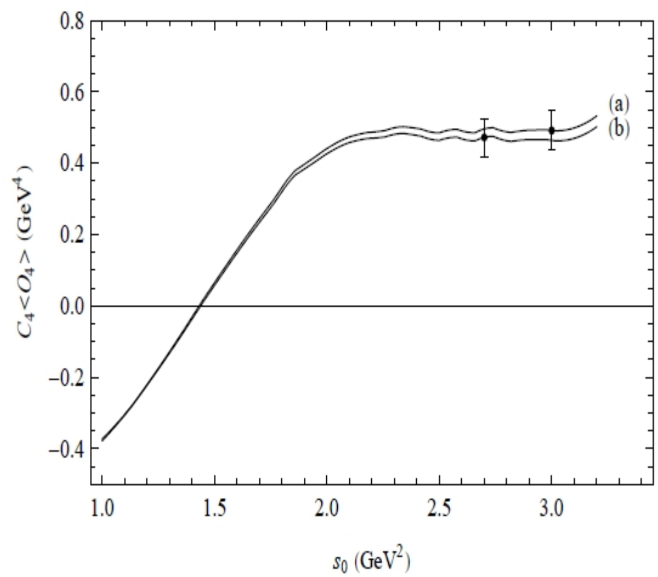


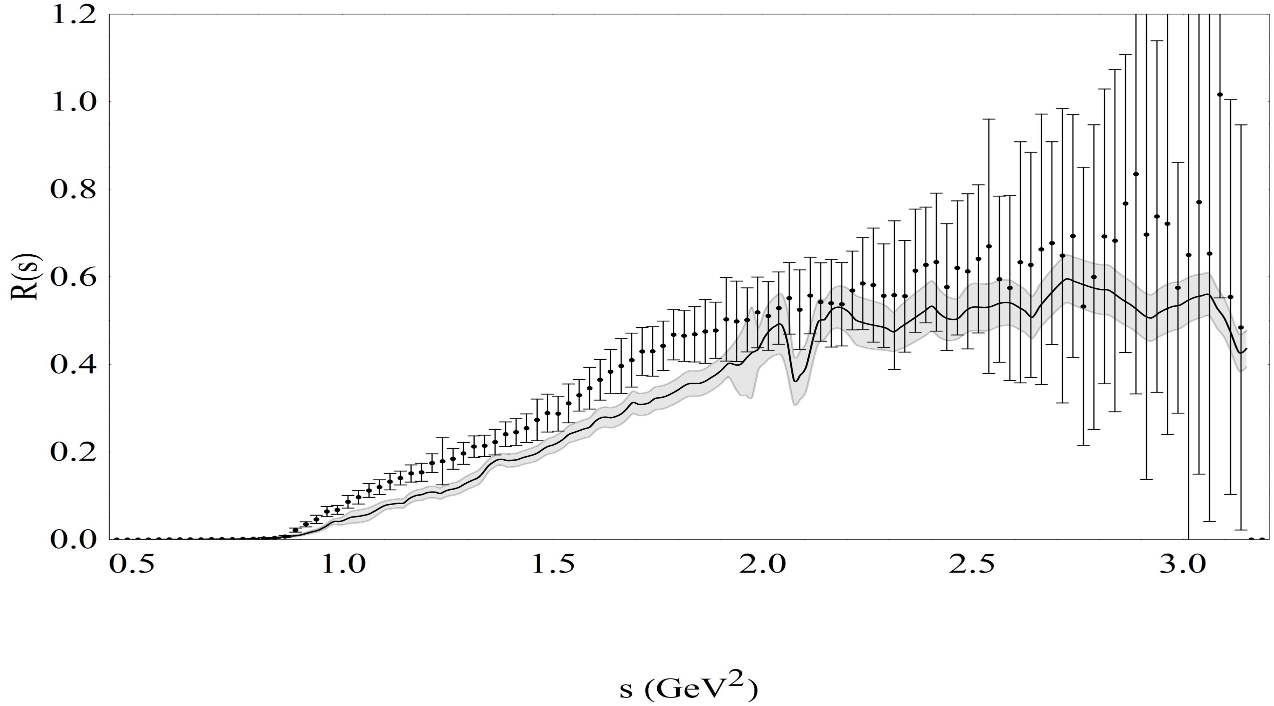
Figure 2: Results for $C_4 \langle \mathcal{O}_4 \rangle$, in units of GeV^4, as a function of s_0. Curve (a) corresponds to $\alpha_s(M_\tau^2) = 0.353$ ($\Lambda = 397$ MeV), and curve (b) to $\alpha_s(M_\tau^2) = 0.335$ ($\Lambda = 365$ MeV). A single representative experimental error is shown for each curve.

There is a problem in the e^+e^- data, but in which energy range?

The low energy 2π region is very precisely measured and agrees with the τ data.

The region above 2 GeV^2 is probably o.k. because otherwise the platau would not be observed

We 'suspect' that the 4π final states are responsible, since these dominate in the energy range betwen 1 and 2 GeV. But we can not exclude that something else is wrong in this energy range



$e^+e^- \to 2\pi^0\pi^+\pi^-$ (shaded) compared with the analogue from τ-deday.