Two-Higgs-Doublet Models, Family Replication, Fermion Mass Hierarchies

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Wien, Dec. 2010

Collaborators

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Papers

- EPJ C **48**, 805 (2006)
- EPJ C **49**, 1067 (2007)
- EPJ C **57**, 719 (2008)
- EPJ C **57**, 739 (2008)
- JHEP **05**, 028 (2009)
- JHEP **04**, 027 (2010)
- arXiv 1009.1864 [hep-ph] (2010) (synopsis)

Introduction

Ingredients of gauge theories of particle physics:

- Fermions: leptons and quarks
- Gauge bosons: $\gamma,\ Z,\ W^{\pm}$, Gluons
- Scalars: Higgs bosons (?)

Interactions

- **Gauge interactions**

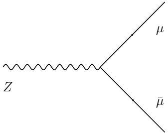
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

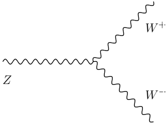
Electroweak symmetry breaking (EWSB) $\downarrow$

$$SU(3)_c \otimes U(1)_{em}$$

$SU(3)_c$: colour group
$SU(2)_L$: weak isospin group
$U(1)_Y$: weak hypercharge group

- Fermion–gauge boson and gauge boson self couplings:
  Very good exp. tests (LEP, HERA etc.)



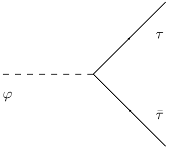


Fermion families

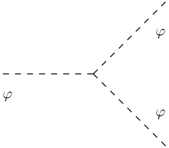
(1)	(2)	(3)	weak isospin t	weak hypercharge y
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	$1/2$	$-1/2$
ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	0	0
e_R	μ_R	τ_R	0	-1
$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b'_L \end{pmatrix}$	$1/2$	$1/6$
u_R	c_R	t_R	0	$2/3$
d_R	s_R	b_R	0	$-1/3$

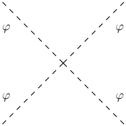
$$\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

- Yukawa interactions: coupling fermions to scalars (?)



- Self interactions of scalars: scalar potential. (??)





- $\Rightarrow$ EWSB; fermion masses; quark mixing, CKM matrix

The Standard Model (SM)

- One complex Higgs-doublet field, hypercharge $y = 1/2$

$$\varphi(x) = \begin{pmatrix} \varphi^+(x) \\ \varphi^0(x) \end{pmatrix}$$

- Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{FB} + \mathcal{L}_{Yuk} + \mathcal{L}_{\varphi}$$

  - Fermion-Boson term

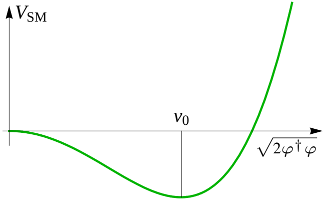
$$\mathcal{L}_{FB} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_R i\gamma^\mu D_\mu \psi_R + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L$$

  - Yukawa term

$$\mathcal{L}_{Yuk} = -\sum_{l=e,\mu,\tau} \bar{l}_R \, c_l \, \varphi^\dagger \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} + h.c. + \text{terms with quarks}$$

Higgs-boson-term

$$\mathcal{L}_\varphi = (D_\mu\varphi)^\dagger(D^\mu\varphi) - V_{SM}(\varphi)$$
$$V_{SM}(\varphi) = -\mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2$$



Stability : $\lambda > 0$

EWSB : $\mu^2 > 0$

- Standard vacuum expectation value (vev):
  $v_0 = \sqrt{\frac{\mu^2}{\lambda}} = 2^{-1/4}G_F^{-1/2} \simeq$ 246 GeV

- In unitary gauge:

$$\langle\varphi(x)\rangle = \begin{pmatrix} 0 \\ \frac{v_0}{\sqrt{2}} \end{pmatrix}, \ \varphi(x) = \langle\varphi(x)\rangle + \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho'(x) \end{pmatrix}$$

- One neutral physical Higgs particle ρ'

$m_{\rho'} > 114$ GeV (direct search at LEP)
$m_{\rho'} = 89^{+35}_{-26}$ GeV (at 68% c.l. from rad. corrections in fits to precision observables. See LEPEWWG 2010)

Fermion masses

$$m_l = c_l \frac{v_0}{\sqrt{2}} \qquad\qquad l = e, \mu, \tau$$

Some questions where the SM gives no good answers

- Why is there family replication?
  Who ordered e, μ, τ?
- Why are there large mass hierarchies?

$$m_e = 0.51 \text{ MeV} \ll m_\mu = 105.6 \text{ MeV} \ll m_\tau = 1777 \text{ MeV}$$

$$\Updownarrow$$

$$c_e \ll c_\mu \ll c_\tau$$

Extending the scalar sector

more Higgs doublets and / or other multiplets.

- Two Higgs doublet models: THDMs
- Minimal Supersymmetric Standard Model: MSSM
  2 Higgs doublets
- Next to Minimal Supersymmetric SM: NMSSM
  2 Higgs doublets $+1$ complex singlet
- General scalar sector investigations:
  W. Bernreuther, O. N., EPJC 9, 319 (1999)

Extended scalar sectors have been studied extensively in the literature. We profited a lot from the work of the Vienna group: Ecker, Grimus, Neufeld.

In our group (M. Maniatis, A. v. Manteuffel, F. Nagel, O. N.) we studied the general THDM.

- We gave precise criteria for
  - stability
  - EWSB
  - explicit and spontaneous CP violation
- We found a class of THDMs having an unusually high degree of CP symmetry in the scalar sector with
  - 4 generalised CP symmetries,
  - EWSB and spontaneous CP violation linked together.

We introduced the **"Principle of maximal CP invariance"** requiring that the full Lagrangian respects these 4 generalised CP symmetries.

Consequences:

- For a single fermion family: $\mathcal{L}_{Yuk} \equiv 0$.
  $\Rightarrow$ For massive fermions we must have **family replication**.
- For two families we can have $\mathcal{L}_{Yuk} \not\equiv 0$. We get - with some additional arguments - one massive plus one massless family. That is, we find theoretical reasons for **mass hierarchies**.

Two-Higgs-Doublet Model

- Higgs fields, hypercharge $y = 1/2$

$$\varphi_1(x) = \begin{pmatrix} \varphi_1^+(x) \\ \varphi_1^0(x) \end{pmatrix}, \quad \varphi_2(x) = \begin{pmatrix} \varphi_2^+(x) \\ \varphi_2^0(x) \end{pmatrix}$$

- Lagrangian:

$$\mathcal{L}_\varphi = \sum_{i=1,2} (D_\mu \varphi_i)^\dagger (D^\mu \varphi_i) - V(\varphi_1, \varphi_2)$$

- Potential

$V(\varphi_1, \varphi_2)$ is required to be **gauge invariant** $\Rightarrow$
it must be built from $\varphi_i^\dagger \varphi_j$,
and **renormalisable** $\Rightarrow$ at most quartic in φ_i

- **Gauge invariant functions**: We write

$$\phi(x) = \begin{pmatrix} \varphi_1^+(x) & \varphi_1^0(x) \\ \varphi_2^+(x) & \varphi_2^0(x) \end{pmatrix}$$

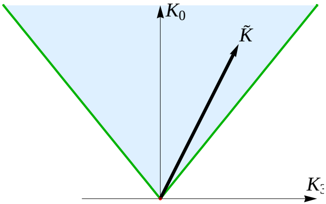
and define gauge invariant quantities:

$$\begin{aligned} \underline{K}(x) &= \phi(x)\phi^\dagger(x) = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix} \\ &= \frac{1}{2}(K_0(x)\mathbb{1}_2 + \mathbf{K}(x)\sigma). \\ \underline{K}(x) &\geq 0 \Rightarrow K_0(x) \geq 0\,,\ K_0^2(x) - \mathbf{K}^2(x) \geq 0 \end{aligned}$$

- The **gauge orbits** of the Higgs fields in the THDM are parametrised by Minkowski-type four-vectors

$$\tilde{K}(x) = \begin{pmatrix} K_0(x) \\ \mathbf{K}(x) \end{pmatrix}$$

lying inside or on the forward light cone in K-space.



- Basis transformations of the Higgs fields

$$\varphi_i'(x) = U_{ij}\,\varphi_j(x)\ ,\ U = (U_{ij}) \in U(2)$$

correspond to **rotations** in K-space.

$$K_0'(x) = K_0(x) \qquad U^\dagger \sigma^a U = R_{ab}(U)\sigma^b$$

$$\mathbf{K}'(x) = R(U)\mathbf{K}(x) \qquad R(U)R^{\mathrm{T}}(U) = \mathbb{1}_3$$

$$\det R(U) = 1$$

- The most general potential V can now be written as

$$V = \xi_0 K_0 + \xi \mathbf{K} + \eta_{00} K_0^2 + 2K_0 \eta \mathbf{K} + \mathbf{K}^{\mathrm{T}} E \mathbf{K}.$$

- 14 Parameters, all real: $\xi_0,\ \eta_{00},$

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}, \quad E = E^{\mathrm{T}} \ (3 \times 3 \text{ Matrix})$$

- By a basis transformation we can diagonalise E:

$$E = \text{diag}\,(\mu_1\ ,\ \mu_2\ ,\ \mu_3).$$

- We have given the precise criteria for V to be stable in a very concise form.
- The global minimum of V determines the vacuum expectation values of the Higgs fields (vevs). We have developed a general algebraic method for determining these vevs. One result is as follows. Let us write

$$\phi_v = \langle\phi(x)\rangle = \begin{pmatrix} \langle\varphi_1^+\rangle & \langle\varphi_1^0\rangle \\ \langle\varphi_2^+\rangle & \langle\varphi_2^0\rangle \end{pmatrix}\ ,\ \underline{K}_v = \phi_v\phi_v^\dagger = \frac{1}{2}(K_{0v}\mathbb{1}_2 + \mathbf{K}_v\sigma)$$

- We have the correct EWSB

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_c \otimes U(1)_{em}$$

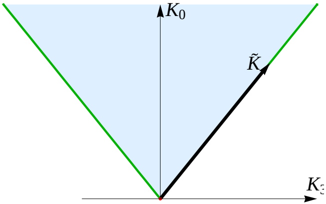
if and only if

$$\tilde{K}_v = \begin{pmatrix} K_{0v} \\ \mathbf{K}_v \end{pmatrix} \quad \text{with } K_{0v} = |\mathbf{K}_v| > 0$$

- That is, we must have $\tilde{K}_v \neq 0$ and $\tilde{K}_v$ must be **on** the forward light cone.

In a suitable basis:

$$\tilde{K}_v = \frac{v_0^2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



CP violation in the Higgs sector

- Standard CP transformation:

$$CP_s : \varphi_i(x) \longrightarrow \varphi_i^*(x') \; , \; x = \begin{pmatrix} x^0 \\ \mathbf{x} \end{pmatrix} \; , \; x' = \begin{pmatrix} x^0 \\ -\mathbf{x} \end{pmatrix}$$

- Generalised CP transformations:

$$\begin{aligned} CP_g \quad : \qquad & \varphi_i(x) \rightarrow U_{ij}\varphi_j^*(x') \; , \; U = (U_{ij}) \in U(2) \\ & K_0(x) \rightarrow K_0(x'), \\ & \mathbf{K}(x) \rightarrow \bar{R}\mathbf{K}(x') \end{aligned}$$

- $\bar{R}$ is an **improper rotation**: $\bar{R}\bar{R}^{\mathrm{T}} = \mathbb{1}_3 \; , \; \det \bar{R} = -1$

- We require that $CP_g \circ CP_g$ gives the identity transformation for the gauge invariant functions $\tilde{K}(x)$.

$$\Rightarrow \bar{R}\bar{R} = \mathbb{1}_3 \Rightarrow \bar{R} = \bar{R}^{\mathrm{T}} \Rightarrow$$

- $\bar{R}$ can be diagonalised by a basis change of the Higgs fields.

$$\bar{R}' = R(U)\bar{R}R^{\mathrm{T}}(U) = \text{diagonal}$$
$$\bar{R}'\bar{R}' = \mathbb{1}_3 \;,\; \det \bar{R}' = -1$$

$$(i) \quad \bar{R}' = -\mathbb{1}_3 = \text{diag}(-1,-1,-1)$$

$$(ii) \quad \bar{R}' = \left\{ \begin{array}{ll} \text{diag}\,(-1,1,1) & =: R_1 \\ \text{diag}\,(1,-1,1) & =: R_2 \\ \text{diag}\,(1,1,-1) & =: R_3 \end{array} \right\} \text{These are equivalent by a basis change}$$

$$\mathcal{L}_\varphi = \sum_{i=1,2} (D_\mu \varphi_i)^\dagger (D^\mu \varphi_i) - V(\varphi_1, \varphi_2),$$
$$V = \xi_0 K_0 + \xi \mathbf{K} + \eta_{00} K_0^2 + 2 K_0 \eta \mathbf{K} + \mathbf{K}^\mathrm{T} E \mathbf{K}.$$

- We have CP_g invariance of $\mathcal{L}_\varphi$ if and only if

$$\bar{R}\xi = \xi \; , \; \bar{R}\eta = \eta \; , \; \bar{R}E\bar{R}^\mathrm{T} = E$$

- A CP_g invariance of $\mathcal{L}_\varphi$ is spontaneously broken if and only if

$$\bar{R}\mathbf{K}_v \neq \mathbf{K}_v$$

Theories with type (i) CP_g invariance

$$CP_g^{(i)} \qquad \varphi_i(x) \to \epsilon_{ij}\varphi_j^*(x') \ , \ \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{K}(x) \to -\mathbf{K}(x') \ , \ \bar{R} = -\mathbb{1}_3$$

- Conditions for the parameters of the potential:

$$\begin{aligned} -\xi &= \xi \ , \ -\eta = \eta \Rightarrow \xi = 0 \ , \ \eta = 0 \\ V &= \xi_0 K_0 + \eta_{00} K_0^2 + \mathbf{K}^{\mathrm{T}} E \mathbf{K}, \\ E &= \text{diag}\left(\mu_1, \mu_2, \mu_3\right) \ , \ \mu_1 \geq \mu_2 \geq \mu_3 \end{aligned}$$

- Stability and EWSB if and only if

$$\eta_{00} > 0 \ , \quad \mu_a + \eta_{00} > 0, \quad \xi_0 < 0, \quad \mu_3 < 0 \qquad a = 1, 2, 3$$

- These models have automatically **three additional type (ii)** CP_g symmetries corresponding to the reflections on the coordinate planes in K space.
  $CP_{g,a}^{(ii)}$, reflection R_a , $a = 1, 2, 3$
- Example:

$$R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} , \; R_3\xi = \xi \text{ for } \xi = 0, \text{ etc.}$$

- EWSB always breaks $CP_g^{(i)}$ and $CP_{g,3}^{(ii)}$ spontaneously.
  Remember:

$$\mathbf{K}_v = \frac{v_0^2}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow -\mathbf{K}_v \neq \mathbf{K}_v \; , \; R_3 \mathbf{K}_v \neq \mathbf{K}_v$$

- vev: $v_0^2 = \frac{-\xi_0}{\eta_{00}+\mu_3}$
- Spectrum of physical Higgs particles:

neutral:

$$
\begin{array}{lrcl}
\rho' : & m^2_{\rho'} & = & 2(-\xi_0) \\
h' : & m^2_{h'} & = & 2v_0^2(\mu_1 - \mu_3) \\
h'' : & m^2_{h''} & = & 2v_0^2(\mu_2 - \mu_3)
\end{array}
$$

charged:

$$
\begin{array}{lrcl}
H^{\pm} : & m^2_{H^\pm} & = & 2v_0^2(-\mu_3)
\end{array}
$$

The principle of maximal CP invariance

- We require as a principle that the complete Lagrangian

$$\mathcal{L} = \mathcal{L}_{FB} + \mathcal{L}_{Yuk} + \mathcal{L}_{\varphi}$$

should respect all four generalised CP symmetries,

$$CP_g^{(i)}\ ,\ CP_{g,a}^{(ii)} \quad a = 1, 2, 3.$$

- This turns out to give no restriction for $\mathcal{L}_{FB}$ but restricts $\mathcal{L}_{Yuk}$ severely.
- First we have to discuss the CP_g transformations of the fermions.

One fermion family

$$CP_g : \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \longrightarrow e^{i\xi_1}\gamma^0 S(C) \begin{pmatrix} \bar{\nu}^{\mathrm{T}}_{eL}(x') \\ \bar{e}^{\mathrm{T}}_L(x') \end{pmatrix}$$
$$e_R(x) \longrightarrow e^{i\xi_2}\gamma^0 S(C)\bar{e}^{\mathrm{T}}_R(x')$$

$$S(C) = i\gamma^2\gamma^0$$

$$\mathcal{L}_{Yuk} = -\bar{e}_R(x) c_{l,i} \varphi_i^\dagger(x) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} + h.c.$$

- Invariance under $CP_g^{(i)}$ requires:

$$\left.\begin{aligned} c^*_{l,1} &= -e^{i(\xi_1-\xi_2)} c_{l,2} \\ c_{l,2} &= e^{-i(\xi_1-\xi_2)} c^*_{l,1} \end{aligned}\right\} \Rightarrow c_{l,1} = c_{l,2} = 0$$

- $\mathcal{L}_{Yuk} \neq 0$ **requires family replication.**

Two fermion families

- We consider the second and third families

$$l_2 \equiv \mu \ , \ \nu_2 \equiv \nu_\mu \ , \ l_3 \equiv \tau \ , \ \nu_3 \equiv \nu_\tau$$

- Most general ansatz:

$$\begin{aligned}
\mathcal{L}_{Yuk} &= -\bar{l}_{\alpha R}(x) C^{(j)}_{\alpha\beta} \varphi^\dagger_j(x) \begin{pmatrix} \nu_{\beta L}(x) \\ l_{\beta L}(x) \end{pmatrix} + h.c. \\
C^{(j)} &= \left(C^{(j)}_{\alpha\beta} \right) \quad \text{arbitrary complex } 2 \times 2 \text{ matrices} \\
j &\in \{1,2\} \ , \ \alpha, \beta \in \{2,3\}
\end{aligned}$$

- Ansatz for CP_g transformations of fermions:

$$CP_g: \quad \begin{pmatrix} \nu_{\alpha L}(x) \\ l_{\alpha L}(x) \end{pmatrix} \longrightarrow U_{L\alpha\beta}\gamma^0 S(C) \begin{pmatrix} \bar{\nu}^{\mathrm{T}}_{\beta L}(x') \\ \bar{l}^{\mathrm{T}}_{\beta L}(x') \end{pmatrix},$$

$$l_{\alpha R}(x) \longrightarrow U_{R\alpha\beta}\gamma^0 S(C)\bar{l}^{\mathrm{T}}_{\beta R}(x'),$$

$$U_L = (U_{L\alpha\beta}) \in U(2) \quad , \quad U_R = (U_{R\alpha\beta}) \in U(2).$$

- We require $CP_g \circ CP_g$ to give back the original fields up to phases,

$$\Rightarrow \quad -U_L U_L^* = \pm \mathbb{1}_2 \; , \; -U_R U_R^* = \pm \mathbb{1}_2$$

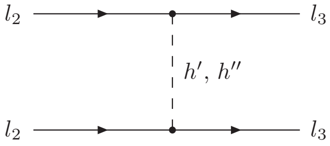
- U_L and U_R are in general different of each other and depend on the CP_g considered.

It is now straightforward but lengthy to work out the forms of $\mathcal{L}_{Yuk}$ which allow implementation of **all four CP_g symmetries**, $CP_g^{(i)}$, $CP_{g,a}^{(ii)}; a = 1, 2, 3$. We find the following results:

- Unequal and non zero masses for μ and τ lead necessarily to large lepton-flavour changing neutral currents, FCNCs.

$$\left.\begin{array}{l} m_\mu \neq 0 \\ m_\tau \neq 0 \\ m_\mu \neq m_\tau \end{array}\right\} \Longrightarrow$$

$$\mu^- + \mu^- \rightarrow \tau^- + \tau^-$$



- We require absence of FCNCs on phenomenological grounds. We have then only the possibilities:
- Equal masses: $m_\mu = m_\tau \neq 0$
- A **large mass hierarchy:** $m_\mu = 0\ ,\ m_\tau \neq 0$
  This leads to a highly symmetric $\mathcal{L}_{Yuk}$

$$\mathcal{L}_{Yuk} = -\frac{m_\tau\sqrt{2}}{v_0} \left\{ \bar{\tau}_R(x)\varphi_1^\dagger(x) \begin{pmatrix} \nu_{\tau L}(x) \\ \tau_L(x) \end{pmatrix} - \bar{\mu}_R(x)\varphi_2^\dagger(x) \begin{pmatrix} \nu_{\mu L}(x) \\ \mu_L(x) \end{pmatrix} + h.c. \right\}$$

- After EWSB:

$$\begin{aligned}\mathcal{L}_{Yuk} =& -m_\tau \left(1 + \frac{\rho'(x)}{v_0}\right) \bar{\tau}(x)\tau(x) \\ &+ \frac{m_\tau}{v_0} h'(x)\bar{\mu}(x)\mu(x) + i\frac{m_\tau}{v_0} h''(x)\bar{\mu}(x)\gamma_5\mu(x) \\ &+ \left\{ \frac{m_\tau}{v_0\sqrt{2}} H^+(x)\bar{\nu}_\mu(x)(1+\gamma_5)\mu(x) + h.c. \right\}\end{aligned}$$

- Couplings of physical Higgs particles:

ρ'	couples exclusively to	τ
$h', h'', H^\pm$	couple exclusively to	μ, ν_μ

- The inclusion of quarks is straightforward

Conclusions

- We have introduced a class of THDMs having four generalised CP symmetries. EWSB and spontaneous CP_g violation are linked.
- We have introduced the principle of maximal CP invariance, requiring the full Langrangian to be invariant under these four CP_g transformations.
- Requiring absence of FCNCs or prescribing the CP_g transformation matrices of the fermions, U_R and U_L, appropriately we get a highly symmetric Yukawa interaction term $\mathcal{L}_{Yuk}$ for two families (2nd and 3rd).
- We add the first family as uncoupled to Higgs fields and call the result the maximally CP symmetric model, MCPM.

- The symmetries leading to the highly symmetric $\mathcal{L}_{Yuk}$ are

$$CP_g: \quad \varphi_i(x) \longrightarrow W_{ij}\varphi_j^*(x')\ ,\ W \in U(2)$$
$$\psi_{\alpha L}(x) \longrightarrow U_{L\alpha\beta}\gamma^0 S(C)\bar{\psi}^{\mathrm{T}}_{\beta L}(x')$$
$$\psi_{\alpha R}(x) \longrightarrow U_{R\alpha\beta}\gamma^0 S(C)\bar{\psi}^{\mathrm{T}}_{\beta R}(x') \qquad \alpha, \beta \in \{2, 3\}$$

CP_g	W	U_R	U_L
$CP_g^{(i)}$	ϵ	ϵ	σ^1
$CP_{g,1}^{(ii)}$	σ^3	$-\sigma^3$	$\mathbb{1}_2$
$CP_{g,2}^{(ii)}$	$\mathbb{1}_2$	$\mathbb{1}_2$	$\mathbb{1}_2$
$CP_{g,3}^{(ii)}$	σ^1	$-\sigma^1$	σ^1

- The third family (τ, t, b) is massive, the second (μ, c, s) and first (e, u, d) families are massless
(at tree level in the symmetry limit).
- Experiment:

$$\frac{m_e}{m_\tau} = 2.9 \times 10^{-4}\,, \quad \left.\frac{m_u}{m_t}\right|_{v_0} = 9.9 \times 10^{-6}\,, \quad \left.\frac{m_d}{m_b}\right|_{v_0} = 1.0 \times 10^{-3}$$

$$\frac{m_\mu}{m_\tau} = 5.9 \times 10^{-2}\,, \quad \left.\frac{m_c}{m_t}\right|_{v_0} = 3.6 \times 10^{-3}\,, \quad \left.\frac{m_s}{m_b}\right|_{v_0} = 1.8 \times 10^{-2}$$

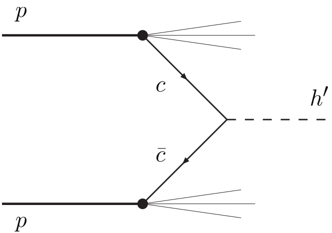
- The CKM matrix $V = \mathbb{1}_3$
  (at tree level in the symmetry limit)
- Experiment:

$$\begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}| \\ |V_{21}| & |V_{22}| & |V_{33}| \\ |V_{31}| & |V_{32}| & |V_{33}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

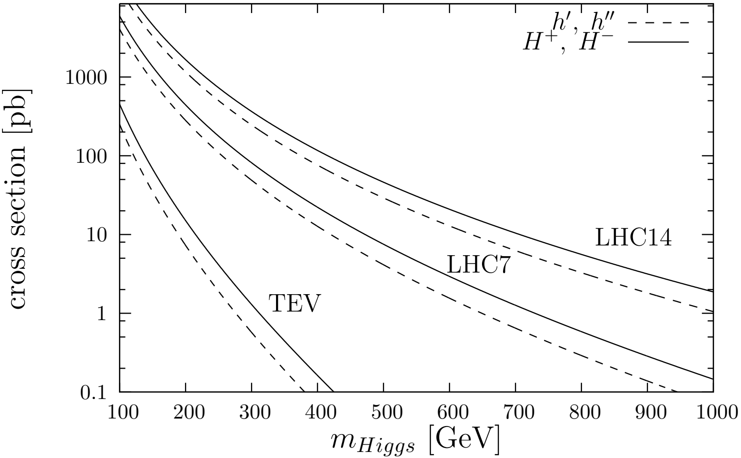
Predictions to be checked at LHC:

- Physical Higgs particles: $\rho', h', h'', H^{\pm}$
- ρ' couples exclusively to (τ, t, b) family.
- $h', h'', H^{\pm}$ couple exclusively to (μ, c, s) family with given strengths.
- The (e, u, d) family is (at tree level in the symmetry limit) uncoupled to Higgs particles.

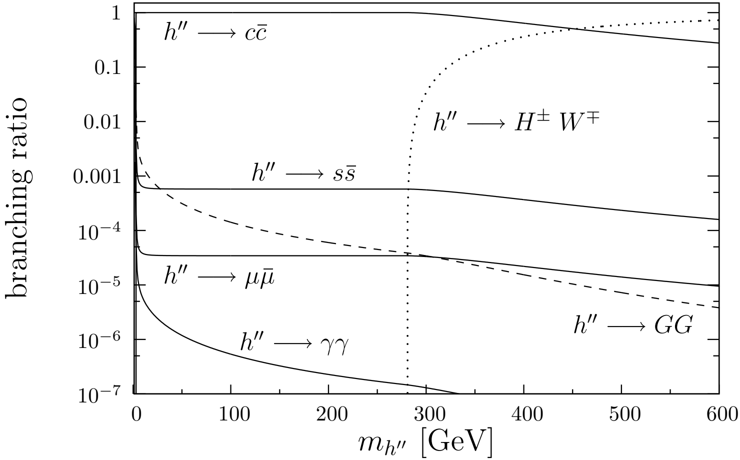
- We predict large cross sections for $h', h'', H^{\pm}$ production via a Drell–Yan type mechanism at the LHC.



- h' decay mostly to $c\bar{c}$ jets.
- $Br(h' \rightarrow \mu\bar{\mu}) = 3 \cdot 10^{-5}$.



- $\int \mathcal{L}dt = 30$ fb^{-1} at LHC14, $m_{h'} = 250$ GeV
  $\Rightarrow$ $30,000,000$ h' produced, 900 $\mu^+\mu^-$ pairs.



- Branching ratios for h'', where $m_{H^\pm} = 200$ GeV is assumed

- **Optimist's view:** The theory has some features which look like a first approximation to what is observed in Nature.
- **Pessimist's view:** We got only a caricature of Nature.
- **Realist's view:** The theory is at least falsifiable.