Theoretical Frameworks for Neutrino Masses

Ferruccio Feruglio
Universita' di Padova

Vienna, January 20th 2011

Plan

1. Neutrino oscillations and summary of data
2. How to extend the SM to incorporate neutrino masses
3. Purely Dirac neutrino masses
4. Neutrino masses from D=5 operator
5. The see-saw mechanism
1. Tests of D=5 operator
2. Flavour symmetries

Two-flavour neutrino oscillations (ν_e,ν_μ)

here ν_e are produced with average energy E

$\longleftarrow$ L $\longrightarrow$

+---+

source detector

here we measure

$$P_{ee} \equiv P(\nu_e \to \nu_e)$$

neutrino interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}}_{U} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu \nu_l$$

$$t \approx L$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2} \approx \frac{m_2^2}{2E} - \frac{m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E}$$

$$P_{ee} = \left|\langle \nu_e | \psi(L) \rangle\right|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

no dependence on the phase α more on this later on ....

to see any effect, if Δm^2 is tiny, we need both θ and L large

Three-flavour neutrino oscillations $(\nu_e, \nu_\mu, \nu_\tau)$

survival probability as before, with more terms

$$P_{ff} = P(\nu_f \to \nu_f) = \left|\langle \nu_f | \psi(L) \rangle\right|^2 = 1 - 4\sum_{k<j} |U_{fk}|^2 |U_{fj}|^2 \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right)$$

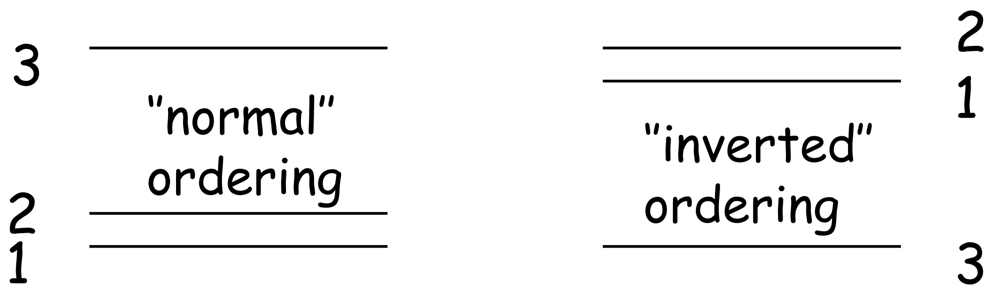
similarly, we can derive the disappearance probabilities $P_{ff'} = P(\nu_f \to \nu_{f'})$

conventions: $[\Delta m^2_{ij} \equiv m^2_i - m^2_j]$

$m_1 < m_2$

$\Delta m^2_{21} < |\Delta m^2_{32}|, |\Delta m^2_{31}|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



[we anticipate that $\Delta m^2_{21} << |\Delta m^2_{32}|, |\Delta m^2_{31}|$]

Mixing matrix U=U$_{PMNS}$ (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi}\nu_i$$

$$(f = e,\mu,\tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23}e^{-i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{-i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12},\ldots$$

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\quad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 6 combinations

$$\Delta m^2_{21}, \Delta m^2_{32}, \quad \vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23} \quad \delta$$

Summary of data

$m_\nu < 2.2\, eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

$\Delta m^2_{atm} \equiv |\Delta m^2_{32}| = (2.38 \pm 0.27) \times 10^{-3}\ \text{eV}^2$

$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.66 \pm 0.35) \times 10^{-5}\ \text{eV}^2$

$[2\sigma \quad \text{errors} \, (95\%\, \text{C.L.})]$

$\sin^2 \vartheta_{13} = 0.016 \pm 0.010$

$\sin^2 \vartheta_{23} = 0.45^{+0.16}_{-0.09} \qquad [2\sigma]$

$\sin^2 \vartheta_{12} = 0.326^{+0.05}_{-0.04} \qquad [2\sigma]$

violation of individual lepton number implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown

$\text{sign}\, [\Delta m^2_{32}] \quad \text{unknown}$

[complete ordering (either normal or inverted hierarchy) not known]

$\delta, \alpha, \beta \quad \text{unknown}$

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

by unitarity

this pattern is called tri-bimaximal completely different from the quark mixing pattern: two angles are large

historically Δm_{21}^2 and $\sin^2\theta_{12}$ were first determined by solving the solar neutrino problem, i.e. the disappearance of about one third of solar electron neutrino flux, for solar neutrinos above few MeV. The desire of detecting solar neutrinos, to confirm the thermodynamics of the sun, was the driving motivation for the whole field for more than 30 years. Electron solar neutrinos oscillate, but the formalism requires the introduction of matter effects, since the electron density in the sun is not negligible. Experiments: SuperKamiokande, SNO

Beyond the Standard Model

a non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model [SM]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

[by definition, right-handed neutrinos $\nu^c = (1,1,0)$ do not exist in the SM]

the requirement of invariance under the gauge group $G=SU(3) \times SU(2) \times U(1)_Y$ forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

$$\Phi \quad \underbrace{\Psi \Psi'}_{\text{same helicity}}$$

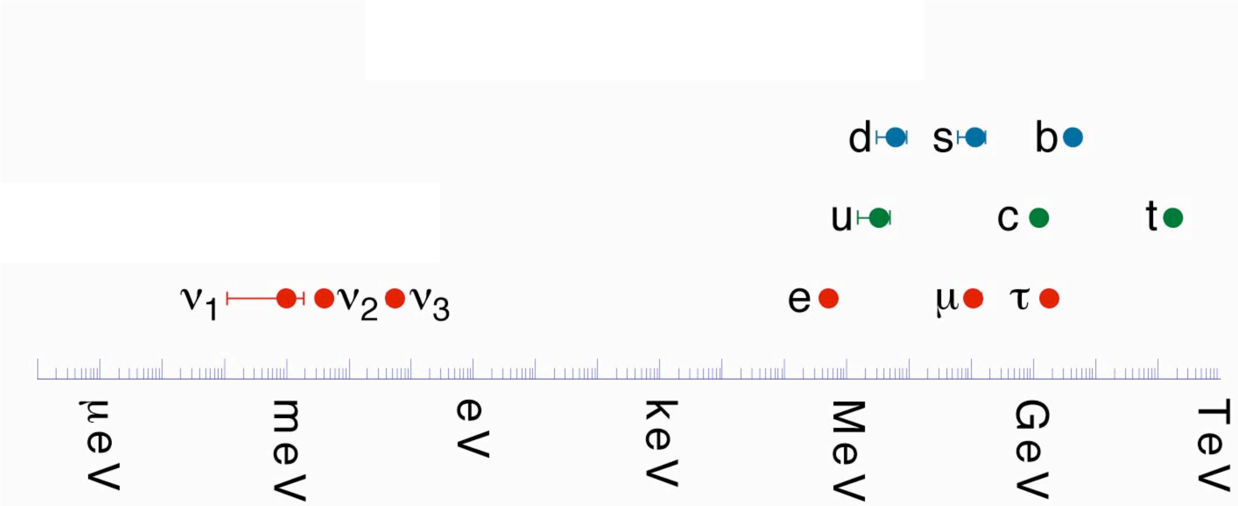
Higgs ⟷ Φ

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$$\lambda \approx 0.22$$

How to modify the SM?

the SM, as a consistent RQFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content — three copies of (q, u^c, d^c, l, e^c)
   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L)

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 2. (and/or 3.) anyway…

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

- add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)
- ask for (global) invariance under B-L (no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = d^c y_d (\Phi^+ q) + u^c y_u (\tilde{\Phi}^+ q) + e^c y_e (\Phi^+ l) + \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

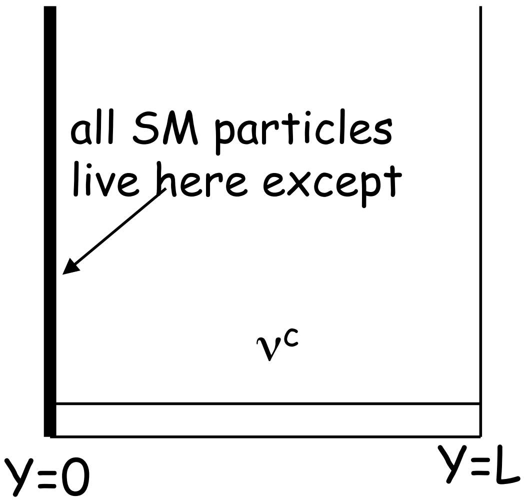
the particle content can be modified in several different ways in order to account for non-vanishing neutrino masses (additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,...). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \leq 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$

$$= \frac{1}{\sqrt{L}} \nu_0^c (\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale) then neutrino Yukawa coupling is suppressed

Second possibility: abandon (2) renormalizability

A disaster?

$$L = L_{d\le 4}^{SM} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,… contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \to \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \to \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\, GeV}{10^{15}\, GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top} !

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$

full singlet under G=SU(3)xSU(2)xU(1)

P. Minkowski, Phys. Lett. 67B, 421 (1977)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2}\nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out" the field ν^c

$$L_{eff}(l) = -\frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + ...$$

terms suppressed by more powers of M^{-1}

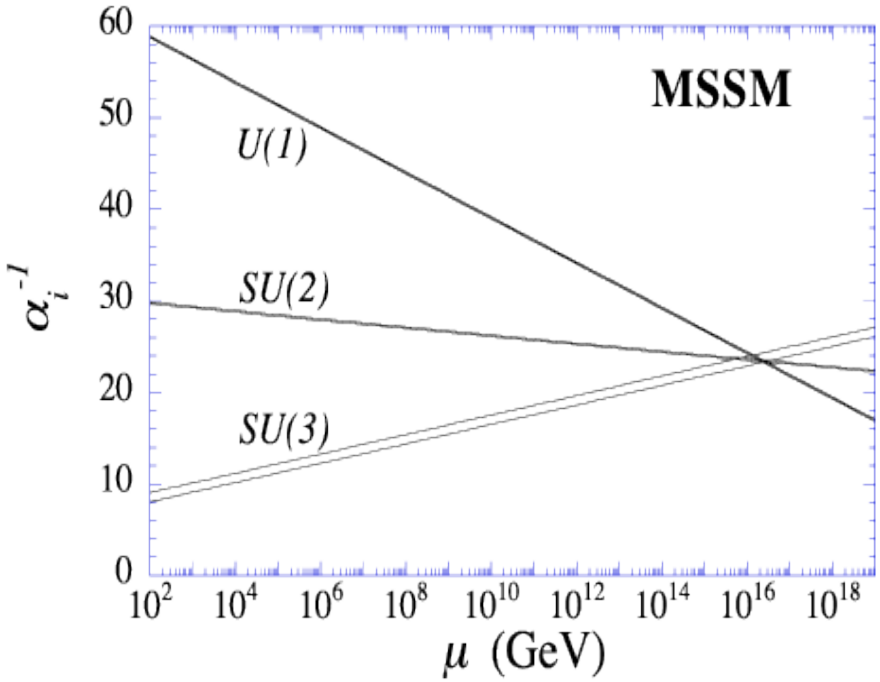
this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),…



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu \right] v^2$$

Example with 2 generations

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

δ<<1
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\overline{B}})}{s} \approx 6 \times 10^{-10}$$

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles and 3 phases

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

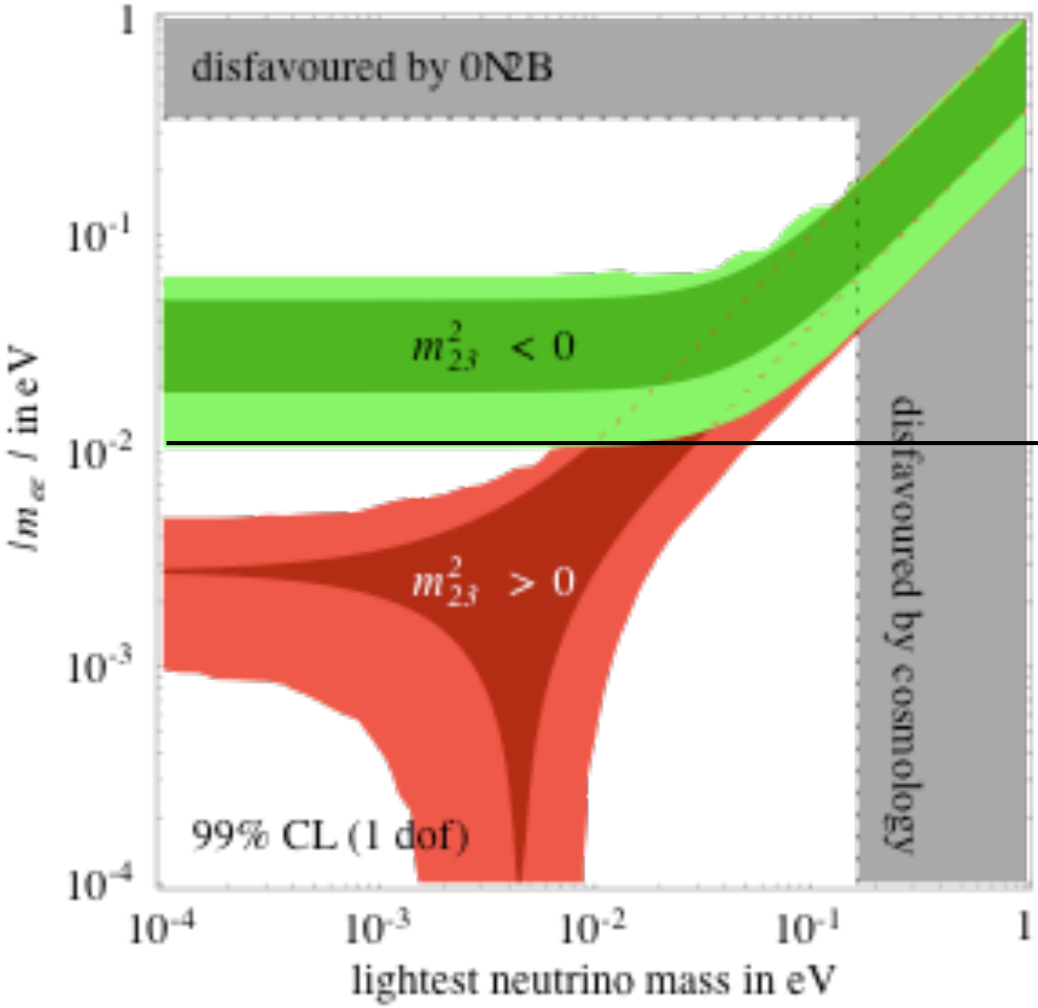
[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is $0\nu\beta\beta$ decay: $(A,Z) \to (A,Z+2) + 2e^-$
this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12}e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13}e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m_{ij}^2, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why ξ_i<<1 consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When ξ_i=0 the theory becomes invariant under a flavour symmetry F

Example: why y_e<<y_{top}? Assume F=$U(1)_F$

F(t)=F(t^c)=F(h)=0 $\qquad y_{top}(h+v)t^c t \qquad$ allowed

F(e^c)=p>0 F(e)=q>0 $\qquad y_e(h+v)e^c e \qquad$ breaks $U(1)_F$ by (p+q) units

if ξ=<φ>/Λ<1 breaks U(1) by one negative unit $\qquad y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$

provides a qualitative picture of the existing hierarchies in the fermion spectrum

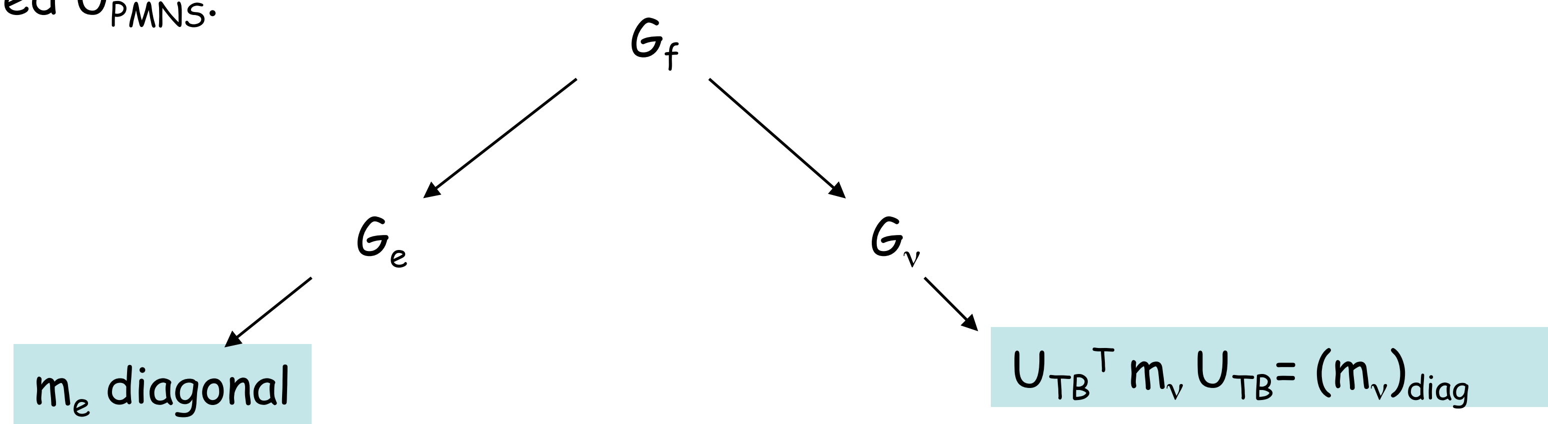
Flavor symmetries II (the lepton mixing puzzle)

why $$U_{PMNS} \approx U_{TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$?

[TB=TriBimaximal]

$$U_{PMNS} = U_e^{+} U_\nu$$

Consider a flavor symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector. m_e is invariant under G_e and m_ν is invariant under G_ν. If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.



The simplest example is based on a small discrete group, $G_f=A_4$. It is the subgroup of SO(3) leaving a regular tetrahedron invariant. The elements of A_4 can all be generated starting from two of them: S and T such that

$$S^2 = T^3 = (ST)^3 = 1$$

S generates a subgroup Z_2 of A_4
T generates a subgroup Z_3 of A_4

simple models have been constructed where $G_e=Z_3$ and $G_\nu=Z_2$ and where the lepton mixing matrix U_{PMNS} is automatically U_{TB}, at the leading order in the SB parameters. Small corrections are induced by higher order terms.

the generic predictions of this approach is that θ_{13} and $(\theta_{23}-\pi/4)$ are very small quantities, of the order of few percent: testable in a not-so-far future.

Conclusion

theory of neutrino masses

it does not exist! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$
gauge invariance

all fermion-gauge boson interactions in terms of 2 parameters: g and g'

?

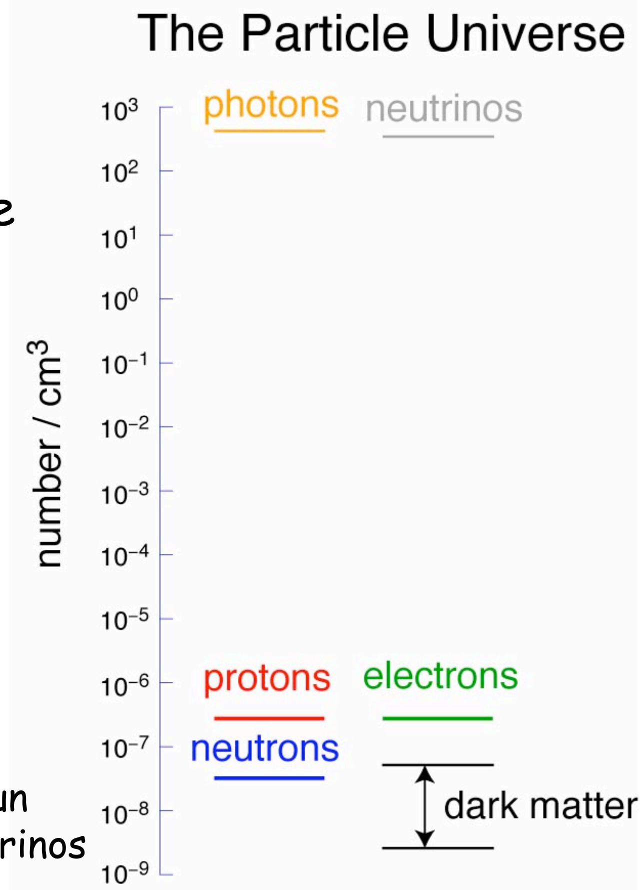
Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

only few ideas and prejudices about neutrino masses and mixing angles

caveat: several prejudices turned out to be wrong in the past!
- $m_\nu \approx 10$ eV because is the cosmologically relevant range
- solution to solar is MSW Small Angle
- atmospheric neutrino problem will go away because it implies a large angle

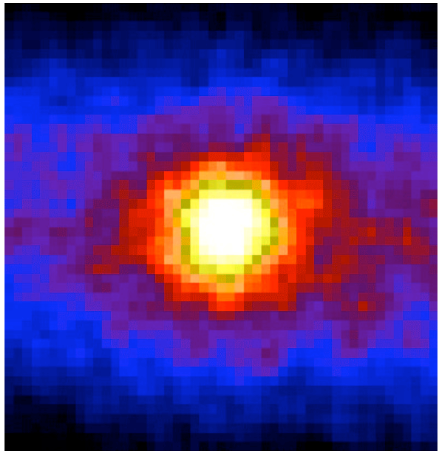
Backup slides

General remarks on neutrinos



the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3

produced by stars: about 3% of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 000 solar neutrinos go through your bodies each second.



this is a picture of the sun reconstructed from neutrinos

electrically neutral and extremely light:
they can carry information about extremely large length scales
e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed 23 years ago

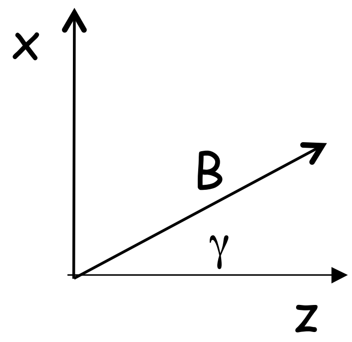
in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass)
the discovery that they are massive (twelve anniversary now!) allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments (more on this later on...)

Neutrino oscillations

from quantum interference, better exemplified in a two-state system

elementary spin 1/2 particle in a constant magnetic field $\vec{B} = (B\sin\gamma, 0, B\cos\gamma)$

$$H = -\vec{\mu}\cdot\vec{B} = \frac{e}{m}\vec{S}\cdot\vec{B} \qquad (g = 2 \qquad \hbar = c = 1)$$



$$H|E_i\rangle = E_i|E_i\rangle \qquad E_{1,2} = \pm\frac{eB}{2m}$$

at t=0 the system has spin +1/2 along the z-axis

$$|\psi(0)\rangle = |u\rangle$$

$$S_z|u\rangle = +\frac{1}{2}|u\rangle \qquad S_z|d\rangle = -\frac{1}{2}|d\rangle$$

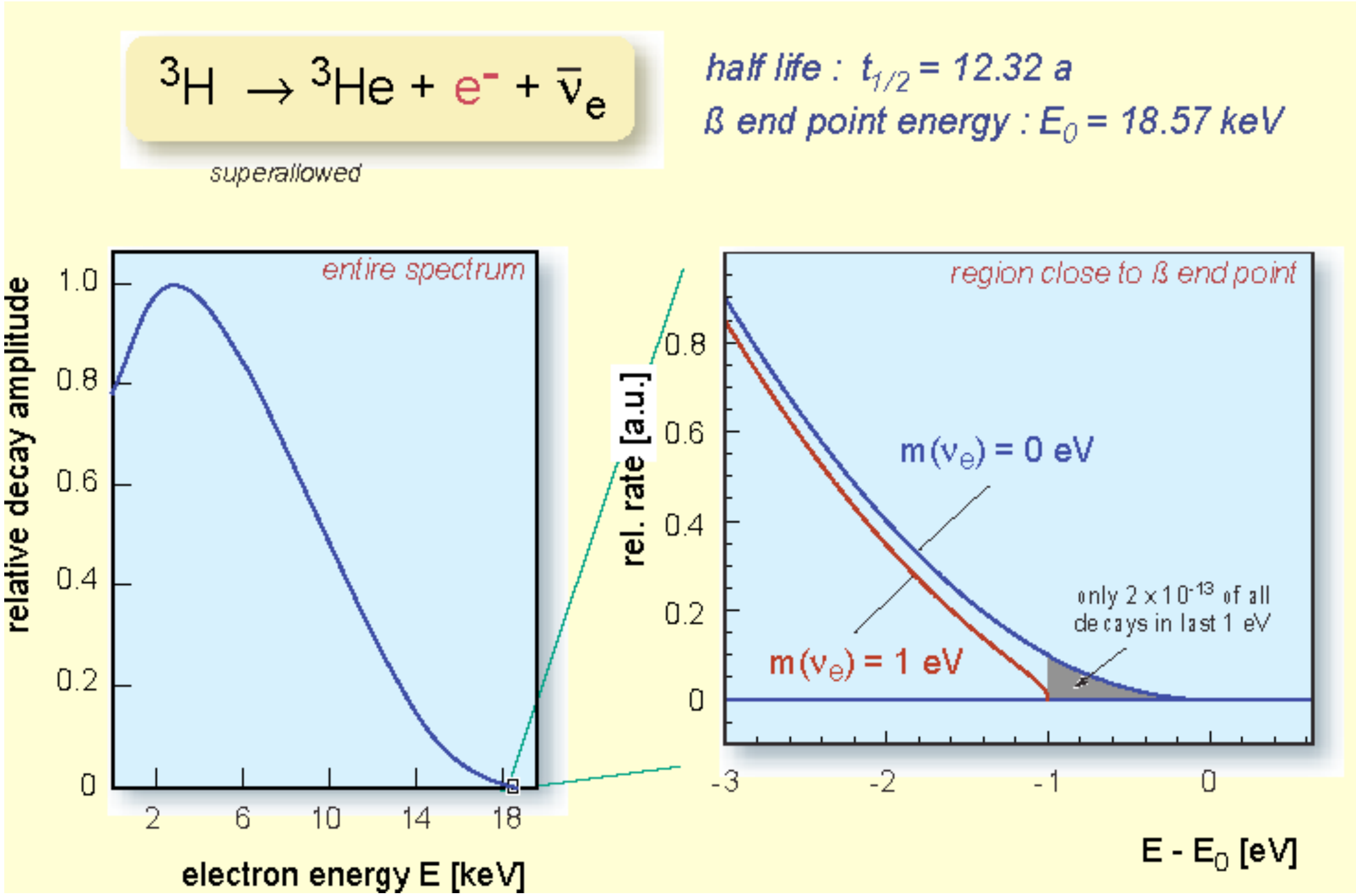
$$|s\rangle = \sum_i U_{si}^*|E_i\rangle \qquad s = u,d$$

$$U = \begin{pmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{pmatrix}$$

$$|\psi(t)\rangle = U_{u1}^* e^{-iE_1t}|E_1\rangle + U_{u2}^* e^{-iE_2t}|E_2\rangle$$

$$P_{uu}(t) = \left|\langle u|\psi(t)\rangle\right|^2 = 1 - \underbrace{4|U_{u1}|^2|U_{u2}|^2}_{\sin^2\gamma}\sin^2\left(\frac{E_1 - E_2}{2}t\right)$$

Upper limit on neutrino mass (laboratory)



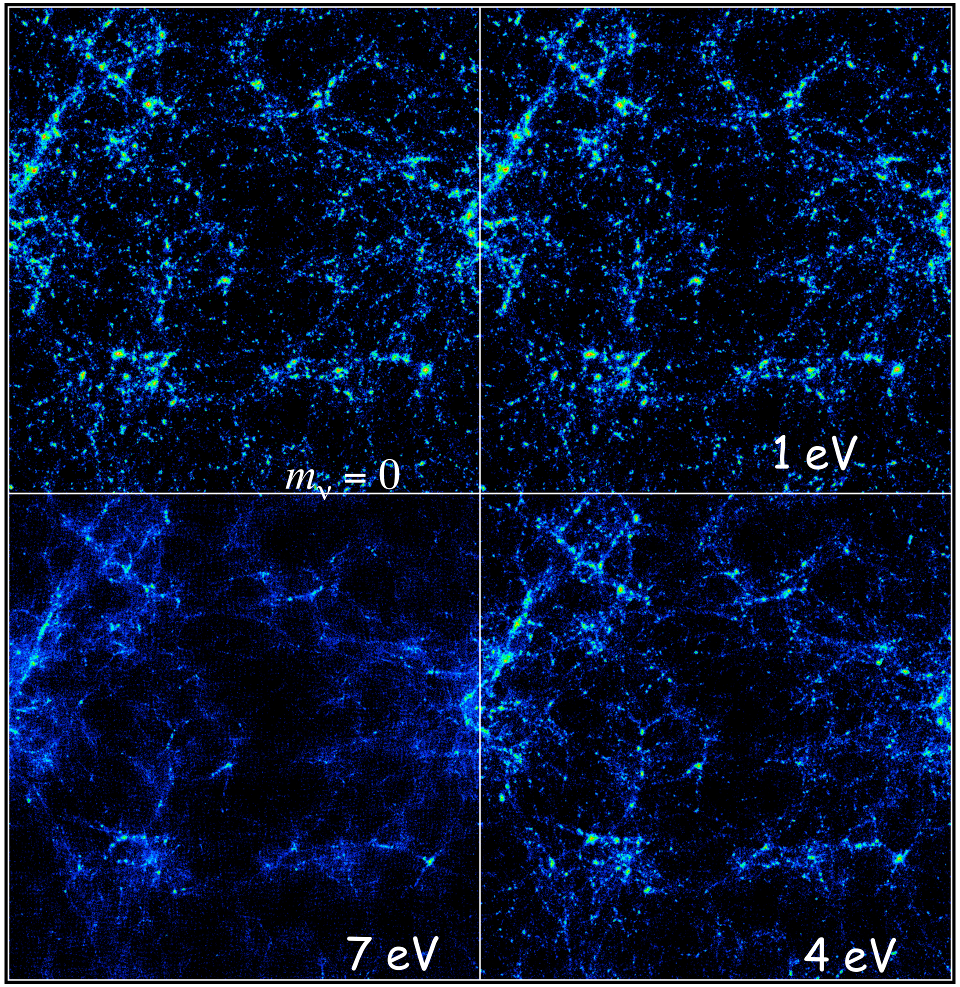
$$m_\nu < 2.2\, eV \quad (95\%\ CL)$$

Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed



$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} P(\vec{k})$$

regimes

$$P_{ee} = |\langle \nu_e | \psi(L) \rangle|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$\frac{\Delta m^2 L}{4E} << 1$		$P_{ee} \approx 1$
$\frac{\Delta m^2 L}{4E} >> 1$	$\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \frac{1}{2}$	$P_{ee} \approx 1 - \frac{\sin^2 2\vartheta}{2}$
$\frac{\Delta m^2 L}{4E} \approx 1$		$P_{ee} = P_{ee}(E)$

by averaging over ν_e energy at the source

useful relation $\frac{\Delta m^2 L}{4E} \approx 1.27 \left(\frac{\Delta m^2}{1\, eV^2}\right)\left(\frac{L}{1\, Km}\right)\left(\frac{E}{1\, GeV}\right)^{-1}$

source	L(km)	E(GeV)	$\Delta m^2 (eV^2)$
ν_e, ν_μ (atmosphere)	10^4 (Earth diameter)	1-10	10^{-4} - 10^{-3}
anti- ν_e (reactor)	1	10^{-3}	10^{-3}
anti- ν_e (reactor)	100	10^{-3}	10^{-5}
ν_e (sun)	10^8	10^{-3} - 10^{-2}	10^{-11} - 10^{-10}

neglecting matter effects

θ_{13} is small

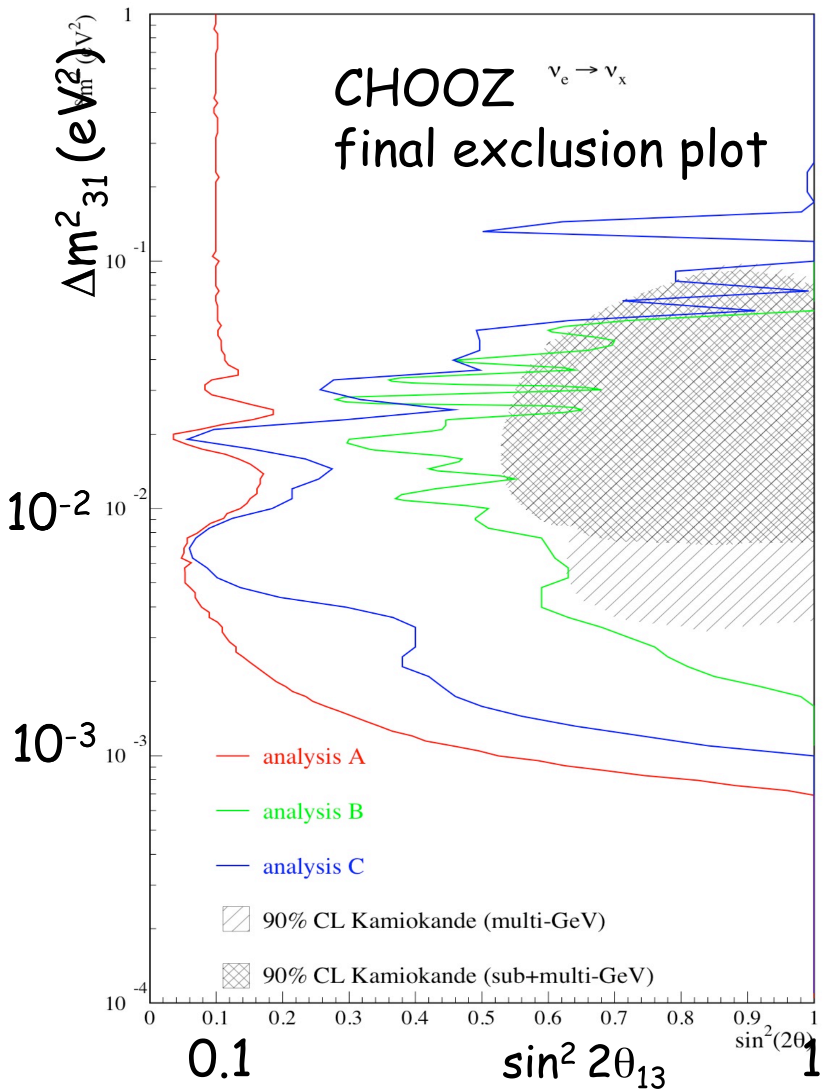
$\Delta m^2_{21} << |\Delta m^2_{32}|, |\Delta m^2_{31}|$ $\longrightarrow$ set $\Delta m^2_{21} = 0$ in general formula for P_{ee}

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

P_{ee} has been measured by the CHOOZ experiment that has not observed any sizeable disappearance. Electron anti-neutrinos are produced by a reactor ($E \approx 3$ MeV, $L \approx 1$ Km) and $P_{ee}^{reactor} \approx 1$ (by CPT the survival probability in vacuum is the same for neutrinos and antineutrinos and matter effects are negligible).

For a sufficiently large Δm_{31}^2 (above 10^{-3} eV^2), such that $P_{ee} = 1 - (\sin^2 2\theta_{13})/2$

$$|U_{e3}|^2 \equiv |\sin^2 \vartheta_{13}|^2 < 0.05 \quad (3\sigma)$$



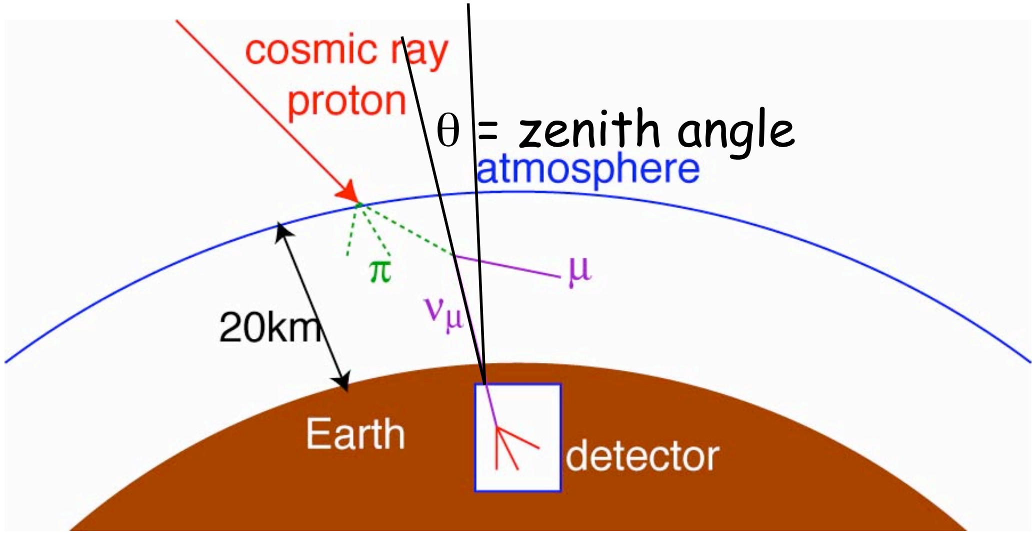
$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & \text{small} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

in what follows, for illustrative purposes, we will work in the approximation

$$U_{e3} = \sin\vartheta_{13} = 0$$

[dependence on CP violating phase δ is lost in this limit]

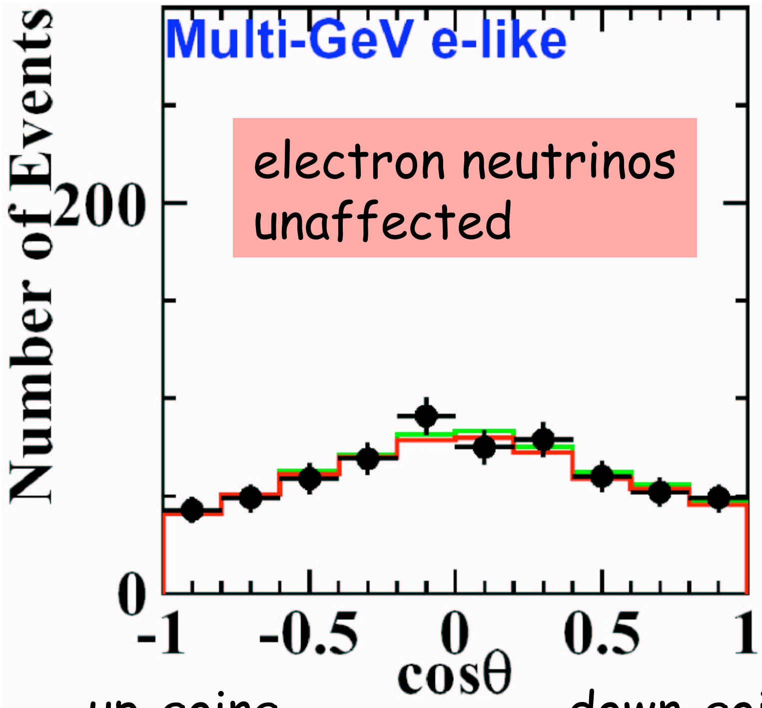
Atmospheric neutrino oscillations

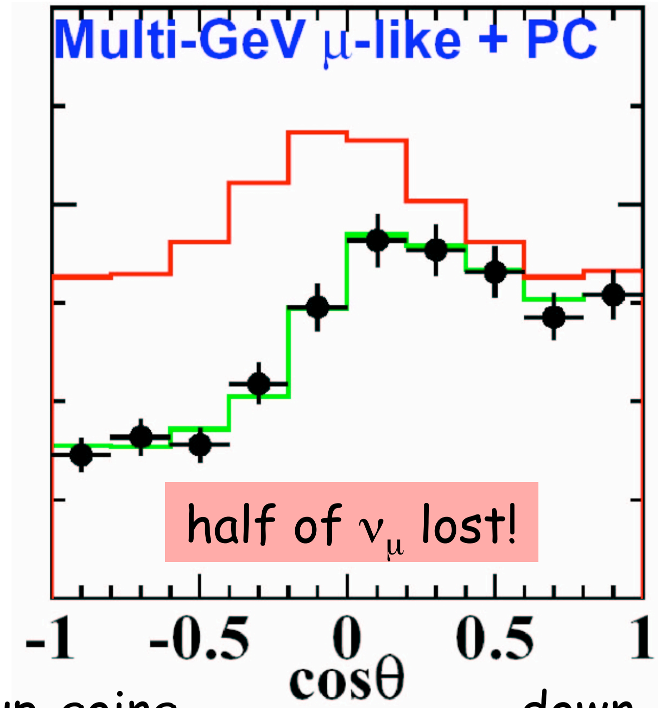


[this year: 10th anniversary]

Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiment:
SuperKamiokande (Japan)





up-going down-going

up-going down-going

electron neutrinos do not oscillate

by working in the approximation $\Delta m^2_{21} = 0$

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \approx 1$$

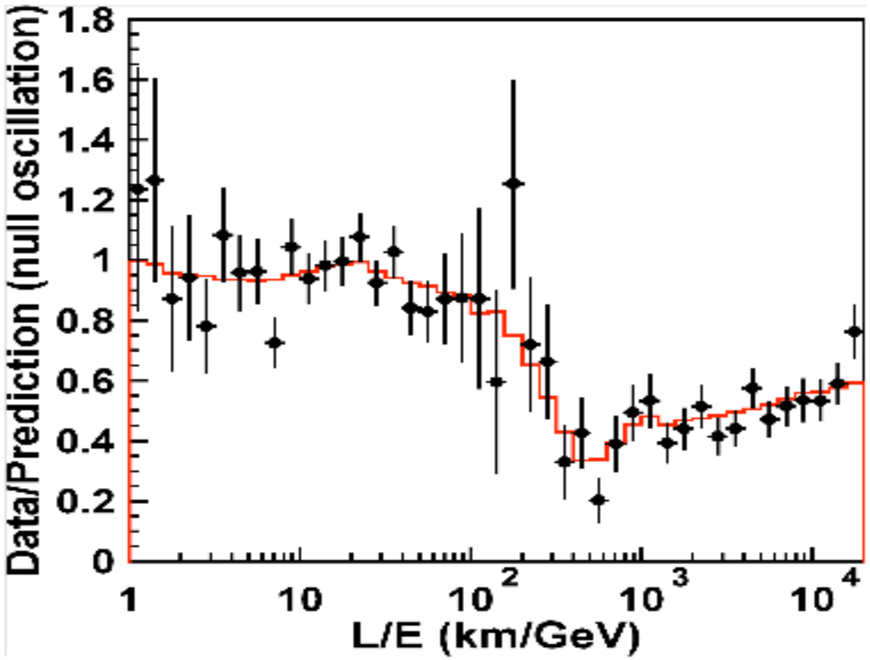
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$|\Delta m^2_{32}| \approx 2\cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

maximal mixing!
not a replica of the quark mixing pattern

this picture is supported by other terrestrial esperiments such as
K2K (Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1 GeV)
and MINOS (USA, from Fermilab to Soudan mine L ≈ 735 Km E ≈ 5 GeV)
that are sensitive to Δm_{32}^2 close to 10^{-3} eV^2,

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

KamLAND experiment exploits the low-energy electron anti-neutrinos ($E \approx 3$ MeV) produced by Japanese and Korean reactors at an average distance of $L \approx 180$ Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

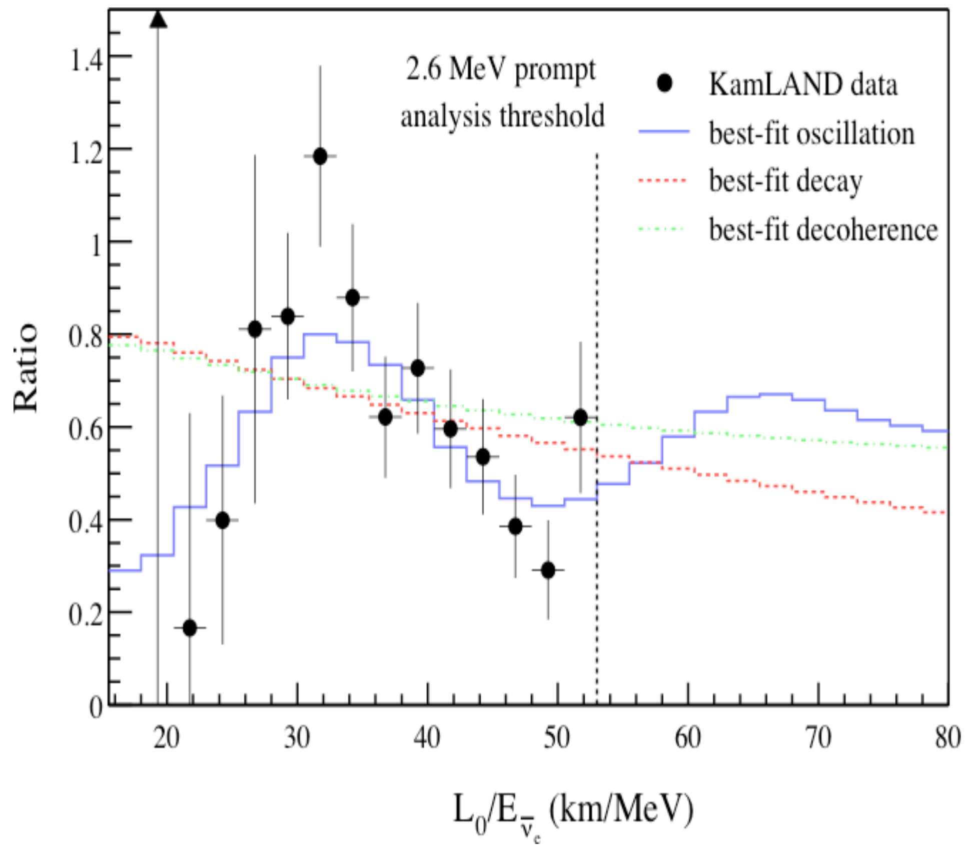
by working in the approximation

$$U_{e3} = \sin\vartheta_{13} = 0$$ we get

$$P_{ee} = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\Delta m_{21}^2 \approx 8 \cdot 10^{-5} \quad eV^2$$

$$\sin^2\vartheta_{12} \approx \frac{1}{3}$$



Tri-Bimaximal Mixing

a good approximation of the data [Harrison, Perkins and Scott; Zhi-Zhong Xing 2002]

$$\sin^2\vartheta_{12}^{TB} = \frac{1}{3} \qquad \sin^2\vartheta_{23}^{TB} = \frac{1}{2} \qquad \sin^2\vartheta_{13}^{TB} = 0$$

quality set by the solar angle

$$\vartheta_{12}^{TB} = 35.3^0 \quad \Longleftrightarrow \quad \begin{array}{l} \vartheta_{12}^{Fogli} = \left(34.8^{+3.0}_{-2.5}\right)^0 \quad [2\sigma] \\ \vartheta_{12}^{Schwetz} = \left(33.5^{+1.4}_{-1.0}\right)^0 \end{array}$$

correct within a couple of degrees, about 0.035 rad, less than ϑ_C^2

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bimaximal mixing

$$\nu_3 = \frac{-\nu_\mu + \nu_\tau}{\sqrt{2}}$$ maximal

$$\nu_2 = \frac{\nu_e + \nu_\mu + \nu_\tau}{\sqrt{3}}$$ trimaximal

What is the best 1st order approximation to lepton mixing?

in the quark sector

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\vartheta_C)$$

[Wolfenstein 1983; Zhi-Zhong Xing 1994,...]

in the lepton sector

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \dots$$

agreement of ϑ_{12} suggests that only tiny corrections [$O(\vartheta_C^2)$] are tolerated. If all corrections are of the same order, then

$\vartheta_{13} \approx O(\vartheta_C^2)$ expected

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \dots$$

can be reconciled with the data through a correction of $O(\vartheta_C)$, for instance a rotation in the 12 sector [from the left side]

$\vartheta_{13} \approx O(\vartheta_C)$ expected

[quark-lepton complementarity ?] [Smirnov; Raidal; Minakata and Smirnov 2004]

$\vartheta_{23} - \pi/4 \approx O(\vartheta_C^2)$

common feature: $\vartheta_{23} \approx \pi/4$ [maximal atm mixing]

... or anarchical U_{PMNS} ? [Hall, Murayama, Weiner 1999]

θ_{23} maximal from some flavour symmetries ?

a no-go theorem
[F. 2004]

$\vartheta_{23} = \pi/4$ can never arise in the limit of an exact realistic symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is exact

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan\vartheta_{23}^0 = \tan\vartheta_{23}^\nu \cos\vartheta_{12}^e + \left(\frac{\tan\vartheta_{13}^\nu}{\cos\vartheta_{23}^\nu}\right)\sin\vartheta_{12}^e$$

undetermined

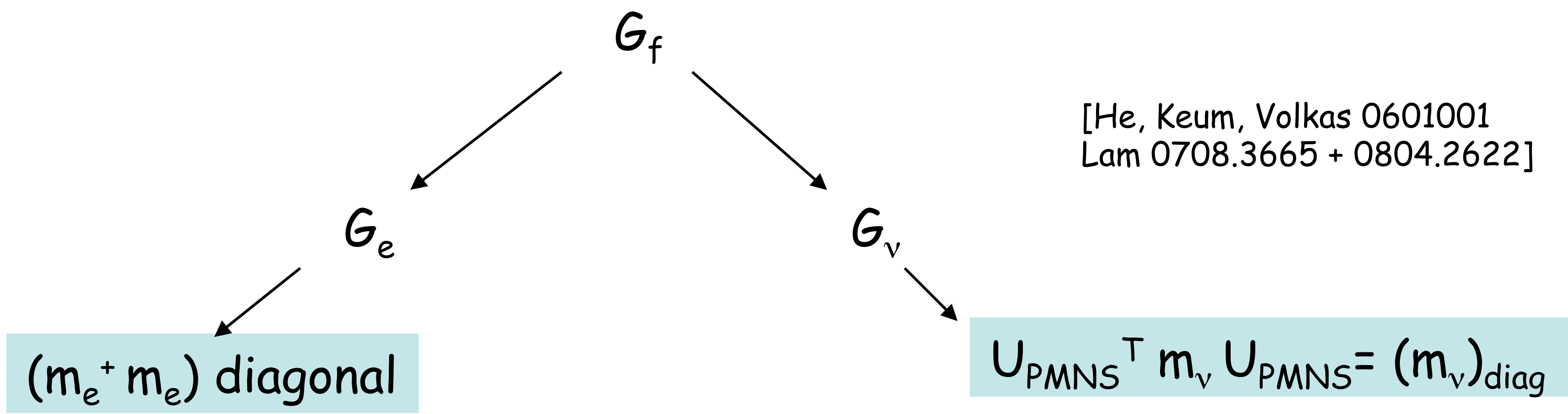
$$\vartheta_{23} = \frac{\pi}{4}$$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

Lepton mixing from symmetry breaking

Consider a flavor symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector. $(m_e^+ m_e)$ is invariant under G_e and m_ν is invariant under G_ν. If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.

For instance we can select G_e in such a way that $(m_e^+ m_e)$ is diagonal and G_ν in such a way that m_ν is responsible for the whole lepton mixing.



[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

TB mixing from symmetry breaking

it is easy to find a symmetry that forces $(m_e^+ m_e)$ to be diagonal;
a "minimal" example (there are many other possibilities) is

$$G_T=\{1,T,T^2\}$$

$$T=\begin{pmatrix}1 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega\end{pmatrix} \qquad \omega=e^{i\frac{2\pi}{3}}$$

[$T^3=1$ and mathematicians call a group with this property Z_3]

$$T^+ (m_e^+ m_e)\, T = (m_e^+ m_e) \quad \Rightarrow \quad (m_e^+ m_e)=\begin{pmatrix}m_e^2 & 0 & 0\\ 0 & m_\mu^2 & 0\\ 0 & 0 & m_\tau^2\end{pmatrix}$$

in such a framework TB mixing should arise entirely from m_ν

$$m_\nu(TB) \equiv \frac{m_3}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix} + \frac{m_2}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1}{6}\begin{pmatrix}4 & -2 & -2\\ -2 & 1 & 1\\ -2 & 1 & 1\end{pmatrix}$$

most general neutrino mass matrix giving rise to TB mixing

easy to construct from the eigenvectors:

$$m_3 \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ -1\\ 1\end{pmatrix} \qquad m_2 \leftrightarrow \frac{1}{\sqrt{3}}\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} \qquad m_1 \leftrightarrow \frac{1}{\sqrt{6}}\begin{pmatrix}2\\ -1\\ -1\end{pmatrix}$$

a "minimal" symmetry guaranteeing such a pattern [C.S. Lam 0804.2622]

$G_S \times G_U$ $G_S=\{1,S\}$ $G_U=\{1,U\}$

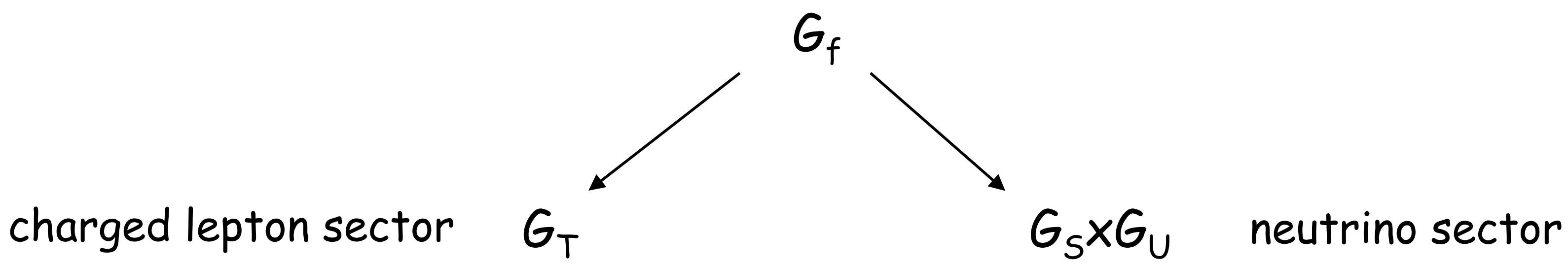
$$S = \frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix} \qquad U = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

[this group corresponds to $Z_2 \times Z_2$ since $S^2=U^2=1$]

$$S^T m_\nu S = m_\nu \qquad U^T m_\nu U = m_\nu \quad \Rightarrow \quad m_\nu = m_\nu(TB)$$

Algorithm to generate TB mixing

- start from a flavour symmetry group G_f containing G_T, G_S, G_U
- arrange appropriate symmetry breaking



if the breaking is spontaneous, induced by $\langle\varphi_T\rangle$,$\langle\varphi_S\rangle$,... there is a vacuum alignment problem

Minimal choice

G_f generated by S and T (U can arise as an accidental symmetry) they satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

these are the defining relations of A_4, group of even permutations of 4 objects, subgroup of SO(3) leaving invariant a regular tetrahedron. S and T generate 12 elements

[Ma and Rajasekaran 2001, Ma 2002, Babu, Ma and Valle 2003, ...]

$$A_4 = \left\{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\right\}$$

there are many many non-minimal possibilities: $G_f = S_4$, $\Delta(27)$, $\Delta(108)$, ...

[Medeiros Varzielas,
King and Ross 2005 and 2006;
Luhn, Nasri and Ramond 2007,
Blum, Hagedorn and Lindner 2007,...]

A_4 has 4 irreducible representations: 1, 1′, 1″ and 3

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

$$\begin{array}{lll} 1 & S=1 & T=1 \\ 1' & S=1 & T=\omega^2 \\ 1'' & S=1 & T=\omega \end{array}$$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Building blocks of a minimal model [AF1, AF2]

	l	e^c	μ^c	τ^c	h_u	h_d	φ_T	φ_S	ξ_i
A_4	3	1	1''	1'	1	1	3	3	1

matter fields (l, e^c, μ^c, τ^c) — Higgses (h_u, h_d) — A_4 breaking sector (φ_T, φ_S, ξ_i)

[change of notation: Higgs doublets are denoted by h_u and h_d]

SU(2)xU(1)xA_4x… invariant Lagrangian:

$$L = \frac{y_e}{\Lambda} e^c h_d (\varphi_T l) + \frac{y_\mu}{\Lambda} \mu^c h_d (\varphi_T l)' + \frac{y_\tau}{\Lambda} \tau^c h_d (\varphi_T l)''$$

$$+ \frac{x_a}{\Lambda^2} h_u h_u \xi (ll) + \frac{x_b}{\Lambda^2} h_u h_u (\varphi_S ll) + V(\xi, \varphi_S, \varphi_T) \ldots$$

[(…) denotes an A_4 singlet,…]

higher dimensional operators in $1/\Lambda$ expansion [Λ = cutoff]

additional symmetry: Z_3, acts as a discrete lepton number; avoids additional invariants

$\varphi_S \leftrightarrow \varphi_T$

$x(ll)$

under appropriate conditions (SUSY,...) a natural minimum of the scalar potential V is

$$\frac{\langle\varphi_T\rangle}{\Lambda} = (u,0,0)$$

$$\frac{\langle\varphi_S\rangle}{\Lambda} = y_b(u,u,u)$$

$$\frac{\langle\xi\rangle}{\Lambda} = y_a u$$

⟺ breaks A_4 down to G_T

⟺ breaks A_4 down to G_S

[y_a and y_b are numbers of order one]

then:

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d u$$

⇒ charged fermion masses

$$m_f = y_f v_d u$$

free parameters as in the SM at this level

$$m_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$a \equiv 2x_a y_a u$$

$$b \equiv 2x_b y_b u$$

2 complex parameters in ν sector (overall phase unphysical)

is also invariant under G_U (accidental symmetry)

TB mixing automatically guaranteed by pattern of symmetry breaking

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

independent from
|a|, |b|, $\Delta \equiv \arg(a)-\arg(b)$!!

ν spectrum

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$

requires a (moderate) tuning

- in this minimal model the mass spectrum is always of normal hierarchy type
the model predicts

$$m_1 \geq 0.017 \text{ eV} \qquad \sum_i m_i \geq 0.09 \text{ eV} \qquad |m_3|^2 = |m_{ee}|^2 + \frac{10}{9}\Delta m^2_{atm}\left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

- in a see-saw realization both normal and inverted hierarchies can be accommodated

Sub-leading corrections

arising from higher dimensional operators, depleted by additional powers of $1/\Lambda$. → they affect m_l , m_ν and they can deform the VEVs.

results

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(u)$$

TB pattern is preserved if corrections are $\leq \vartheta_C^2 \approx 0.04$

generic prediction for ϑ_{13}
$\vartheta_{13}=O(u)$

range of VEVs:

$$m_\tau = y_\tau v_d u$$
$$y_\tau < 4\pi$$

→

$$u > 0.002(0.02)$$
$$\tan\beta = 2.5(30)$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$0.002 \leq u \leq 0.04$$

the range expected for ϑ_{13} is similar additional tests are possible if there is new physics at a scale M close to TeV

$$L_{eff} = i\frac{e}{M^2} l^c h_d \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{M}(\langle\varphi\rangle) l + [4\text{-fermion}] + h.c. + \ldots$$

dominant 4-fermion LFV operators

$$\frac{1}{M^2}\bar{e}^c \bar{\tau}^c \mu^c \mu^c \qquad \frac{1}{M^2}(\bar{l}\,\bar{l}\,l\,l)$$

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$

$$\tau^- \to \mu^+ e^- e^- \qquad \tau^- \to e^+ \mu^- \mu^-$$

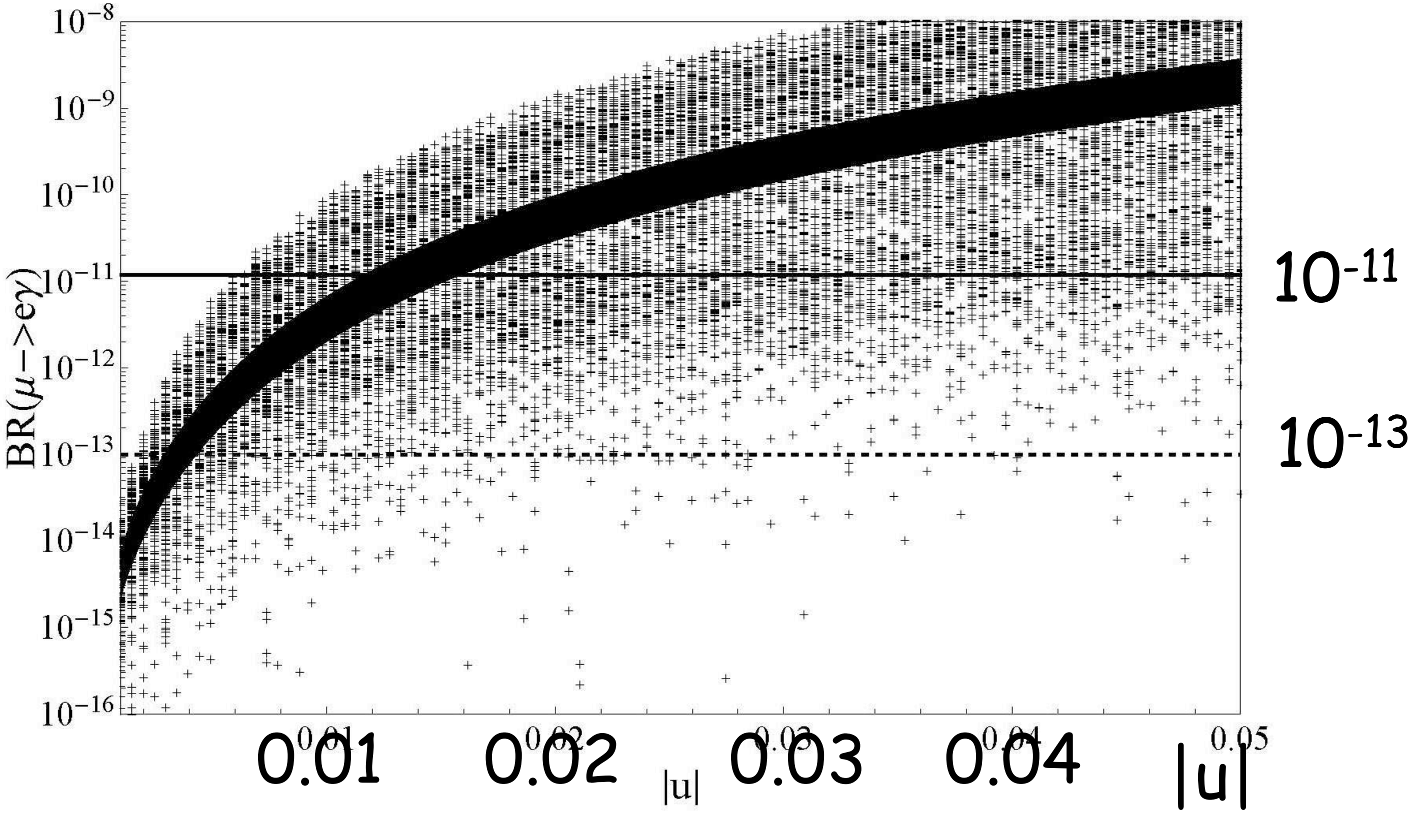
this term contributes to magnetic dipole moments and to LFV transitions such as $\mu \to e\gamma$ $\tau \to \mu\gamma$ $\tau \to e\gamma$ usually discussed in terms of

$$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$$

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from u

$\tau \to \mu\gamma$ $\tau \to e\gamma$ below expected future sensitivity

In a SUSY realization of this model



O(1) coefficient

$$BR(\mu \to e\gamma) = \underbrace{\frac{12\pi^3\alpha_{em}}{G_F^2 m_\mu^4}(\delta a_\mu)^2}_{0.0014\times\left(\frac{\delta a_\mu}{30\times 10^{-10}}\right)^2}[\gamma u]^4$$

[other slides]

many models predicts a **large** but **not necessarily maximal** θ_{23}

an example: abelian flavour symmetry group $U(1)_F$

$$F(l) = (\times,0,0) \qquad [\times \neq 0]$$

$$F(e^c) = (\times,\times,0)$$

$$m_e = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \end{pmatrix} v_d \qquad m_\nu = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \\ \cdot & O(1) & O(1) \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1)$$

maximal only by a fine-tuning!

similarly for all other abelian charge assignements

$$F(l) = (1,-1,-1)$$

$$m_\nu = \begin{pmatrix} \cdot & O(1) & O(1) \\ O(1) & \cdot & \cdot \\ O(1) & \cdot & \cdot \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1) + \text{charged lepton contribution}$$

no help from the see-saw mechanism within abelian symmetries…

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale m_2, m_3, ϑ_{23}

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m} \cos 2\vartheta_{23}$$

[possible only if $\delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log \frac{\Lambda}{Q}$$

gives the scale Q at which $\theta_{23}(Q)$ becomes maximal ⇒ m_2, m_3, ϑ_{23} fine tuned to obtain Q at the e.w. scale

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2 \vartheta_{13} \sin^2 2\vartheta_{23}}{(\sin^2 \vartheta_{23} \cos^2 \vartheta_{13} + \sin^2 \vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

if $\vartheta_{23} = \frac{\pi}{4}$

$$\sin^2 2\vartheta_{12} = \frac{4 \sin^2 \vartheta_{13}}{(1 + \sin^2 \vartheta_{13})^2} < 0.2 \ \ \text{(Chooz)}$$

wrong!

Alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses are already diagonal

$$m_e << m_\mu << m_\tau$$

can be reproduced by U(1) flavour symmetry

$$Q(e^c) = 4 \quad Q(\mu^c) = 2 \quad Q(\tau^c) = 0$$
$$Q(l) = 0$$
$$Q(\vartheta) = -1 \qquad \langle \vartheta \rangle \neq 0$$

compatible with A_4

$$y_e \approx \frac{\langle \vartheta \rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle \vartheta \rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$

[see also Lin hep-ph/08042867 for a realization without an additional U(1)]

Quark masses - grand unification

quarks assigned to the same A_4 representations used for leptons?

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- fermion masses from dim ≥ 5 operators, e.g. $\frac{\tau^c \varphi_T l H_d}{\Lambda}$
  good for leptons, but not for the top quark
- naïve extension to quarks leads diagonal quark mass matrices and to $V_{CKM}=1$
  departure from this approximation is problematic
  [expansion parameter (VEV/Λ) too small]

possible solution within T′, the double covering of A_4

[FHLM1]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1′ 1″ 3 2 2′ 2″

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$(t \quad b)$	t^c	b^c	η	ξ''
T'	2''	2''	2''	1	1	1	2'	1''

[older T′ models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators) at the leading order

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \otimes \\ 0 & \otimes & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243$ $\quad$ 0.2257 ± 0.0021

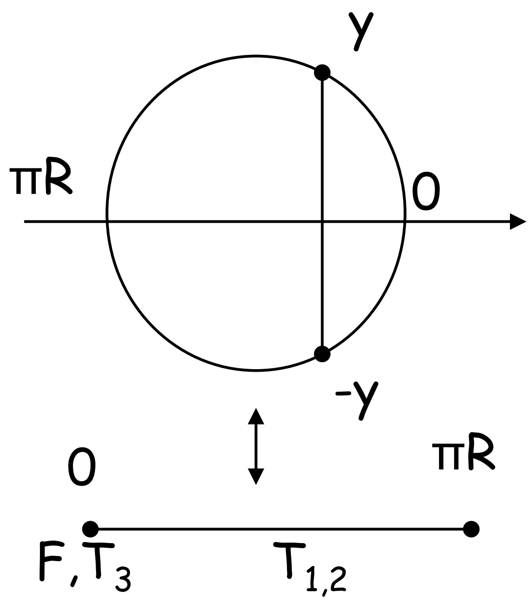
$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

other option: SUSY SU(5) in 5D=$M_4 x(S^1 x Z_2)$ + flavour symmetry $A_4 xU(1)$

[AFH]



DT splitting problem solved
via SU(5) breaking induced by compactification

dim 5 B-violating operators forbidden!
p-decay dominated by gauge boson exchange (dim 6)

unwanted minimal SU(5) mass relation $m_e=m_d^T$ avoided by assigning $T_{1,2}$ to the bulk

the construction is compatible with A_4!

	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$
$SU(5)$	1	$\bar{5}$	10	10	10	5	$\bar{5}$
A_4	3	3	1''	1'	1	1	1'

reshuffling of singlet reps.

unsuppressed top Yukawa coupling $T_3 T_3$

realistic quark mass matrices
by an additional U(1) acting on $T_{1,2}$

neutrino masses from see-saw
compatible with both normal and
inverted hierarchy

TB mixing + small corrections

A_4 as a leftover of Poincare symmetry in D>4

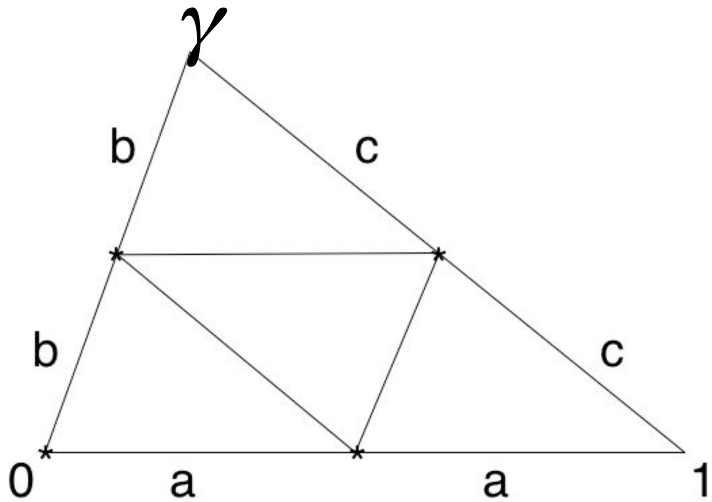
[AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

⟺

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \to z + \frac{1}{2}$$ [translation]

$$T: \quad z \to \gamma^2 z$$ [rotation by 120^0]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)] the four fixed points (z_1,z_2,z_3,z_4) are permuted under the action of S and T

$$S: \qquad (z_1,z_2,z_3,z_4) \to (z_4,z_3,z_2,z_1)$$
$$T: \qquad (z_1,z_2,z_3,z_4) \to (z_2,z_3,z_1,z_4)$$

S and T satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

the compact space is invariant under a remnant of 2-translations x SO(2) isomorphic to the A_4 group

Field Theory

brane fields $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$, $\varphi_4(x)$ transform as 3 + (a singlet) under A_4

The previous model can be reproduced by choosing l, e^c, μ^c, τ^c, $H_{u,d}$ as brane fields and φ_T, φ_S and ξ as bulk fields.

String Theory [heterotic string compactified on orbifolds]

in string theory the discrete flavour symmetry is in general bigger than the isometry of the compact space. [Kobayashi, Nilles, Ploger, Raby, Ratz 2006]

orbifolds are defined by the identification

$$(\vartheta\, x) \approx x + l \qquad \begin{cases} l = n_a e_a & \text{translation in a lattice} \\ \vartheta & \text{twist} \end{cases}$$

group generated by (ϑ, l) is called space group

fixed points: special points x_F satisfying

$$x_F \equiv (\vartheta_F^K\, x_F) + l_F \qquad \text{for some} \quad (\vartheta_F^K, l_F)$$

twisted states living at the fixed point $x_F=(\vartheta_F^K, l_F)$ have couplings satisfying space group selection rules [SGSR]. Non-vanishing couplings allowed for

$$\prod_F (\vartheta_F^K, l_F) \equiv (1,0)$$

G_f is the group generated by the orbifold isometry and the SGSR

Example: S^1/Z_2

1 2

Isometry group = S_2 generated by σ^1 in the basis {|1>,|2>}

SGSR = $Z_2 \times Z_2$ generated by $(\sigma^3,-1)$

[allowed couplings when number n_1 of twisted states at |1> and the number n_2 of twisted states at |2> are even]

G_f = semidirect product of S_2 and $(Z_2 \times Z_2) \equiv D_4$

group leaving invariant a square

relation between A_4 and the modular group [AF2]

modular group PSL(2,Z): linear fractional transformation

complex variable

$$z \to \frac{az+b}{cz+d}$$

$a,b,c,d \in Z$

$ad - bc = 1$

discrete, infinite group generated by two elements

$$\underbrace{z \to -\frac{1}{z}}_{S} \qquad \underbrace{z \to z+1}_{T}$$

obeying

$$S^2 = (ST)^3 = 1$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$A_4 = \frac{PSL(2,Z)}{H}$$

⇒ representations of A_4 are representations of PSL(2,Z)

infinite discrete normal subgroup of PSL(2,Z)

future improvements on atmospheric and reactor angles

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \rightarrow \nu_\mu$ disappearance channel

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

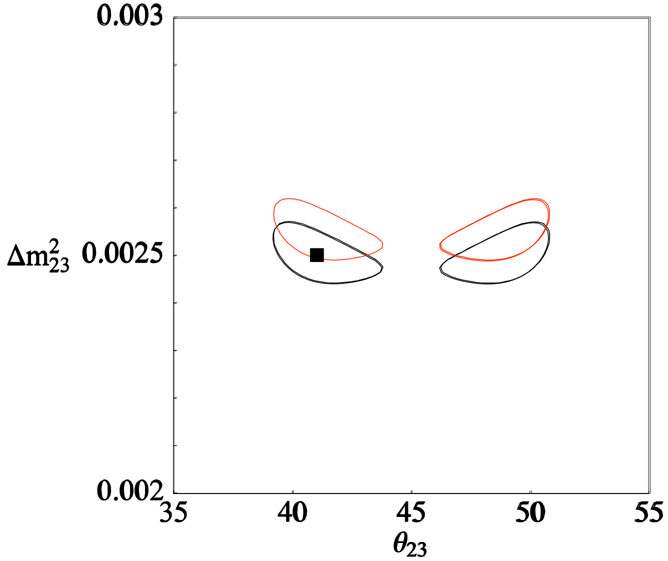
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\text{rad} \leftrightarrow 2.9^0$$

improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]