Minimal Flavour Violation and Beyond

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Vienna 2010
Lecture I

1. Grand View
2. TH Framework

Lecture II

3. Minimal Flavour Violation
4. Motivations for BSM and BMFV
5. Models for BMFV (SUSY, LHT, RS, 4G, 2HDM)

Lecture III

6. Patterns of Flavour Violations
7. BSM
8. Outlook
1.

Grand View
The Standard Model

Quarks

\[
\begin{pmatrix}
    u' \\
    d'
\end{pmatrix}_L
\begin{pmatrix}
    c' \\
    s'
\end{pmatrix}_L
\begin{pmatrix}
    t'
\end{pmatrix}_L
\begin{pmatrix}
    u_R \\
    d_R \\
    c_R \\
    s_R \\
    t_R \\
    b_R
\end{pmatrix}
\]

+ 2/3

- 1/3

+ Leptons

Fundamental Forces

Gauge Theory

\[
\text{SU}(3) \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y
\]

QCD

Strong Interactions

\(\text{(Gluons)}\)

Electroweak Interactions

\(\text{(W}\pm, Z^0, \gamma)\)

Mesons

\[
\begin{align*}
    K^0 &= (d\bar{s}) \\
    K^+ &= (u\bar{s}) \\
    K^- &= (\bar{u}s) \\
    \pi^+ &= (u\bar{d}) \\
    \pi^0 &= (\bar{u}u - \bar{d}d) / \sqrt{2} \\
    \pi^- &= (\bar{u}d)
\end{align*}
\]

\[
\begin{align*}
    B^0_d &= (d\bar{b}) \\
    \bar{B}^0_d &= (\bar{d}b) \\
    B^+ &= (u\bar{b}) \\
    B^0_s &= (s\bar{b}) \\
    \bar{B}^0_s &= (\bar{s}b) \\
    B^- &= (\bar{u}b)
\end{align*}
\]

Neutral
Higgs

\(q\bar{q}\)

Bound
States

Vienna2010
1. **Charged Current Interactions only**
   between *left-handed* Quarks

\[ \begin{align*}
W^\pm & \to t_L \\
d_L & \to d_L
\end{align*} \]

\[ \frac{g_2}{2 \sqrt{2}} \gamma^\mu (1 - \gamma_5) \cdot V_{td} \]

2. **Quark Mixing**

\[ \begin{align*}
\begin{pmatrix}
  d' \\
s' \\
b'
\end{pmatrix}
& =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\end{align*} \]

\{ Weak Eigenstates \} \neq \{ Mass Eigenstates \}

3. **GIM Mechanism**

Natural suppression of FCNC

\[ \begin{align*}
\gamma, G, Z^0, H^0 & \to i = 0 \\
j & = 0
\end{align*} \]

\{ Loop Induced Decays, sensitive to \}
\{ short distance flavour dynamics \}
4. **Asymptotic Freedom**

\[ \alpha_{QCD} = \frac{g_{QCD}^2}{4\pi} \]

\[ \alpha_{QCD}(Q) = \frac{4\pi}{\beta_0 \ln \left( \frac{Q^2}{\Lambda_{MS}^2} \right)} \left[ 1 - \frac{\beta_1}{\beta_0} \ln \left( \frac{Q^2}{\Lambda_{MS}^2} \right) + \ldots \right] \]

\[ \Lambda_{MS}^{(5)} = 240 \pm 15 \text{ MeV} \quad \alpha_{MS}^{(5)}(M_Z) = 0.1187 \pm 0.0009 \]

**SD** = Short Distances (Perturbation Theory)

**RG** = Renormalization Group Effects

**LD** = Long Distances (Non-Perturbative Physics)
Kobayashi-Maskawa Picture of CP Violation

CP Violation arises from a single phase $\delta$ in $W^{\pm}$ interactions of Quarks

<table>
<thead>
<tr>
<th>quark pair</th>
<th>matrix element</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ud$</td>
<td>$c_{12}c_{13}$</td>
</tr>
<tr>
<td>$cd$</td>
<td>$-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta}$</td>
</tr>
<tr>
<td>$td$</td>
<td>$s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}$</td>
</tr>
<tr>
<td>$us$</td>
<td>$s_{12}c_{13}$</td>
</tr>
<tr>
<td>$cs$</td>
<td>$c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta}$</td>
</tr>
<tr>
<td>$ts$</td>
<td>$-s_{23}c_{12} - s_{12}s_{23}s_{13}e^{i\delta}$</td>
</tr>
<tr>
<td>$ub$</td>
<td>$s_{13}e^{-i\delta}$</td>
</tr>
<tr>
<td>$cb$</td>
<td>$s_{23}c_{13}$</td>
</tr>
<tr>
<td>$tb$</td>
<td>$c_{23}c_{13}$</td>
</tr>
</tbody>
</table>

Four Parameters: $(\theta_{12} \approx \theta_{\text{cabibbo}})$

$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta$

$c_{ij} \equiv \cos \theta_{ij}; \quad s_{ij} \equiv \sin \theta_{ij}; \quad c_{13} \equiv c_{23} \equiv 1$
**Wolfenstein Parametrization**

### Parameters:

- $\lambda, A, \rho, \eta$

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>$1 - \lambda^2/2$</td>
<td>$\lambda$</td>
<td>$V_{ub}$</td>
</tr>
<tr>
<td>c</td>
<td>$-\lambda$</td>
<td>$1 - \lambda^2/2$</td>
<td>$V_{cb}$</td>
</tr>
<tr>
<td>t</td>
<td>$V_{td}$</td>
<td>$V_{ts}$</td>
<td>1</td>
</tr>
</tbody>
</table>

- $\lambda = 0.22$

- $V_{us} = \lambda + 0(\lambda^7)$
- $V_{cb} = A\lambda^2 + 0(\lambda^8)$
- $V_{ts} = -A\lambda^2 + 0(\lambda^4)$

(A = 0.83 ± 0.02)

\[
V_{ub} \equiv A\lambda^3(\rho - i\eta) \quad V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta})
\]

\[
\bar{\rho} = \rho \left(1 - \frac{\lambda^2}{2}\right) \quad \bar{\eta} = \eta \left(1 - \frac{\lambda^2}{2}\right)
\]

(AJB, Lautenbacher, Ostermaier, 94)

- $R_b \equiv \sqrt{\rho^2 + \eta^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|

- Circle around $(\bar{\rho}, \bar{\eta}) = (0,0)$

- $R_t \equiv \sqrt{(1-\rho)^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|

- Circle around $(\bar{\rho}, \bar{\eta}) = (1,0)$
Unitarity Triangle

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

\( \bar{\eta} \neq 0 \) Signals CP Violation

\[ V_{ub} = |V_{ub}| e^{-i\gamma} \]

\[ V_{td} = |V_{td}| e^{-i\beta} \]

An Important Target of Particle Physics

\[ J_{CP} = \lambda^2 |V_{cb}|^2 \overline{\eta} = 2 \cdot \text{Area of unrescaled UT} \]
Tree Level Decays

$K^+ \rightarrow s \rightarrow u \rightarrow \pi^0$

$V_{us}$

$B^+ \rightarrow b \rightarrow u \rightarrow \rho^{0,\pi^0}$

$V_{ub}$

$B^+ \rightarrow b \rightarrow c \rightarrow D^{0*,D^0}$

$V_{cb}$
Loop Induced FCNC Processes

CP $\varepsilon_K$-Parameter
$\Delta M (K_L-K_S)$

$B^0_d - \overline{B}^0_d$ Mixing

$B^0_s - \overline{B}^0_s$ Mixing

Discovered in 2006
(CDF, DØ)
Loop Induced FCNC Processes

\[
K^+ \to \pi^+ \nu \bar{\nu}, \quad K_L \to \pi^0 \nu \bar{\nu}, \quad K_L \to \mu \bar{\mu}, \quad B_{d,s} \to \mu \bar{\mu}, \quad B \to X_S \nu \bar{\nu}
\]

\[
K_L \to \pi^0 e^+ e^- \quad B \to X_S e^+ e^- \quad X_S \mu \bar{\mu}
\]

\[
B \to X_S \gamma \quad B \to K^* \gamma
\]

\[
B \to X_d \gamma \quad b \to s \text{ gluon}
\]
### CKM Parameters from Tree-Level Decays
(subject to very small NP Pollution)

\[
|V_{us}| = s_{12} = 0.2254 \pm 0.0008 \\
|V_{cb}| = s_{23} = (41.2 \pm 1.1) \cdot 10^{-3} \\
|V_{ub}| = s_{13} = (3.9 \pm 0.4) \cdot 10^{-3} \\
\delta_{\text{CKM}} = \gamma_{\text{UT}} = (75 \pm 15)^{\circ} \\
\beta = (21.1 \pm 0.9)^{\circ}
\]

(-phase of $V_{ub}$)

\[
\left(\sin 2\beta\right)_{\psi K_s} = 0.672 \pm 0.023
\]

(-phase of $V_{td}$)

**Phase of $V_{ts}$:** $\approx - (1.2 \pm 0.1)^{\circ}$

but could be subject to NP pollution
Hierarchical Structure of the CKM Matrix

\[
\begin{pmatrix}
0.97 & s_{12} & s_{13}e^{-i\gamma} \\
-s_{12} & 0.97 & s_{23} \\
s_{12}s_{23} - s_{13}e^{i\gamma} & -s_{23} & 1
\end{pmatrix}
\]

\[s_{13} << s_{23} << s_{12}\]

(4\cdot10^{-3})  (4\cdot10^{-2})  (0.2)

GIM Structure of FCNC’s

Large CP effects in B_d
Small CP effects in B_s
Tiny CP effects in K_L

PMNS: Negligible LFV
(tiny v masses)

\[A_{\text{CP}}(B_d \rightarrow \psi K_s) \approx 0(1)\]

\[A_{\text{CP}}(B_s \rightarrow \psi\phi) \approx 0(10^{-2})\]

\[\varepsilon \approx 0(10^{-3})\]
\[\varepsilon' \approx 0(10^{-6})\]

\[\text{Br}(K_L \rightarrow \pi^0 \nu\bar{\nu}) \approx 0(10^{-11})\]
Unitarity Triangle Fits

(icons of flavour physics)

UT fit

CKM fitter
Impressive Success of the CKM Picture of Flavour Changing Interactions

(Once quark masses determined: only 4 parameters)

1. All leading decays of $K$, $D$, $B_s^0$, $B_d^0$ mesons correctly described

2. Suppressed transitions: $K^0 - \bar{K}^0$, $B_d^0 - \bar{B_d}^0$, $B_s^0 - \bar{B_s}^0$ mixings found at suppressed level

3. CP-violating Data ($K$, $B_d$) correctly described

4. $B \to X_s \gamma$, $B \to X_s l^+ l^-$ OK

$\mathcal{CP}$ in $B_s$?

$(g-2)_\mu$?
Very very highly suppressed transitions in the SM consistent with experiment: (not seen)

<table>
<thead>
<tr>
<th>Standard Model</th>
<th>Exp Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Br}(B_s \rightarrow \mu^+ \mu^-) \approx 3 \cdot 10^{-9}$</td>
<td>$\sim 4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \approx 3 \cdot 10^{-11}$</td>
<td>$\sim 6 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$\text{Br}(K_L \rightarrow \mu e) \approx 10^{-40}$</td>
<td>$\sim 10^{-12}$</td>
</tr>
<tr>
<td>$\text{Br}(\mu \rightarrow e \gamma) \approx 10^{-54}$</td>
<td>$\sim 10^{-11}$</td>
</tr>
<tr>
<td>$d_n \approx 10^{-32} \text{ ecm}$</td>
<td>$\sim 10^{-26} \text{ ecm}$</td>
</tr>
</tbody>
</table>
Impressive Success of the CKM Picture of Flavour Changing Interactions (GIM)

Yet

1. EW-Symmetry Breaking has to be better understood.

2. Hierarchies in Fermion Masses and Mixing Angles have to be understood with the help of some New Physics (NP). This NP could have impact on Low Energies.

3. There is still a lot of room for NP contributions, in particular in rare decays of mesons and leptons, in CP flavour violating transitions and EDM’s.


5. Several tensions between the flavour data and the SM exist.
Superstars of 2010 – 2015
(Flavour Physics)

\[ S_{\psi \phi} \quad (B_s \to \phi \phi) \]

\[ B_s \to \mu^+ \mu^- \quad (B_d \to \mu^+ \mu^-) \]

\[ B^+ \to \tau^+ \nu_\tau \quad (K_L \to \pi^0 \nu \bar{\nu}) \]

\[ K^+ \to \pi^+ \nu \bar{\nu} \quad (B_d \to K^* \mu^+ \mu^-) \]

\[ \gamma \quad \text{from Tree Level Decays} \]

\[ \mu \to e \gamma \]
\[ \tau \to \mu \gamma \]
\[ \tau \to e \gamma \]
\[ \mu \to 3e \]
\[ \tau \to 3 \text{ leptons} \]

\[ \epsilon'/\epsilon \quad \text{(Lattice)} \]

EDM’s
\[ (g-2)_\mu \]
Standard Model Predictions for Superstars

\[ S_{\psi\phi} = 0.035 \pm 0.005 \]
\[ (S_{\psi\phi})_{\text{exp}} = 0.52 \pm 0.20 \]

\[ \text{Br}(B_s \to \mu^+\mu^-) = (3.2 \pm 0.3) \cdot 10^{-9} \]
\[ \text{Br}(B_s \to \mu^+\mu^-)_{\text{exp}} \leq 4.2 \cdot 10^{-8} \]

\[ \text{Br}(B_d \to \mu^+\mu^-) = (1.0 \pm 0.1) \cdot 10^{-10} \]
\[ \text{Br}(B_d \to \mu^+\mu^-)_{\text{exp}} \leq 1.0 \cdot 10^{-8} \]

\[ \text{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (8.4 \pm 0.7) \cdot 10^{-11} \]
\[ \text{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\text{exp}} = (17 \pm 11) \cdot 10^{-11} \]

\[ \gamma = (68 \pm 7)^0 \]
\[ \gamma_{\text{exp}} = (75 \pm 15)^0 \]

\[ \text{Br}(K_L \to \pi^0\nu\bar{\nu}) = (2.8 \pm 0.6) \cdot 10^{-11} \]
\[ \text{Br}(K_L \to \pi^0\nu\bar{\nu})_{\text{exp}} \leq 6 \cdot 10^{-8} \]
2.

Theoretical Framework
The Problem of Strong Interactions

\( B_d^0 - \overline{B_d^0} \) Mixing (SM)

\[ \begin{align*}
B^0 & \longrightarrow W^\pm & \longrightarrow d & \longrightarrow B^0 \\
& \longrightarrow t & \longrightarrow b & \longrightarrow \overline{B^0}
\end{align*} \]

Short Distance

Long Distance

\( B_d^0 - \overline{B_d^0} \) Mixing (MSSM)

\[ \begin{align*}
B^0 & \longrightarrow \chi^\pm & \longrightarrow d & \longrightarrow B^0 \\
& \longrightarrow \tilde{t} & \longrightarrow \tilde{b} & \longrightarrow \overline{B^0}
\end{align*} \]

Short Distance

Long Distance

SD : Perturbative (Asymptotic Freedom)

LD : Non-Perturbative (Confinement)
Effective Field Theory

Full Theory
$$(W^\pm, Z^0, G, \gamma, t, H^0, b, u, d, s, c, l)$$

$\mu \geq M_W$

Effective Theory
$$(G, \gamma, b, u, d, s, c, l)$$

$\mu \equiv 0(m_b)$

"Generalized Fermi Theory" with calculable "couplings" $C_B(\mu), C_2(\mu), ...$
\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i(\mu) Q_i \]

\[ Q_i \leftrightarrow \text{Four Quark Interaction (} s d)_{V-A} (\bar{s} d)_{V-A} \]

\[ C_i(\mu) \leftrightarrow \text{Coupling Constants} \quad C(\mu) = \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\frac{6}{23}} \]

\[ \langle \bar{K}^0 | (s d)_{V-A} (\bar{s} d)_{V-A} | K^0 \rangle = \frac{8\hat{B}_K}{3} F_K^2 m_K^2 \alpha_s(\mu)^{2/9} \]
Master Formula for FCNC Amplitudes

\[ A(\text{FCNC}) = \sum_i B_i \eta_i^{\text{QCD}} V_i^{\text{CKM}} F_i(m_t, \text{NP}) \]

Long Distance
(Non-perturbative Lattice)

Flavour and CPV
in SM and MFV
enters only here

Short Distance
Functions
(Perturbation TH)
(real for MFV)

NP can add
new \( B_i \)'s absent
in SM

(New Operators
with new
Dirac Structures)

Renormalization
Group (QCD)
LD \( \leftrightarrow \) SD

Only Loops
if GIM works

NP can add
new loops and
trees with new
flavour parameters
and CPV phases

\( Q_{LR} \)
(right-handed
currents, S, P)

non-MFV
Possible Dirac Structures in \( K^0 - \bar{K}^0 \) and \( B^0_{d,s} - \bar{B}^0_{d,s} \)

**SM:**
\[
\gamma_\mu \left( 1 - \gamma_5 \right) \otimes \gamma^\mu \left( 1 - \gamma_5 \right)
\]

**Beyond SM:**
\[
\gamma_\mu \left( 1 - \gamma_5 \right) \otimes \gamma^\mu \left( 1 + \gamma_5 \right)
\]
\[
\left( 1 - \gamma_5 \right) \otimes \left( 1 + \gamma_5 \right)
\]
\[
\left( 1 - \gamma_5 \right) \otimes \left( 1 - \gamma_5 \right)
\]
\[
\sigma_{\mu\nu} \left( 1 - \gamma_5 \right) \otimes \sigma^{\mu\nu} \left( 1 - \gamma_5 \right)
\]

MSSM with large \( \tan\beta \)

General Supersymmetric Models

Models with complicated Higgs System

Warped Extra Dimensions

\[ \text{NLO} \left[ \eta^i_{QCD} \right]^{\text{New}} : \quad \text{Ciuchini, Franco, Lubicz, Martinelli, Scimemi, Silvestrini} \]
\[ \text{AJB, Misiak, Urban, Jäger} \]
Basic Diagrams in FCNC Processes

**Penguin Family**

- (GIM broken at one loop)

**Box Diagrams**

**Tree Diagrams**

- (GIM broken at tree level)

New Physics enters here

Similar diagrams in LFV and EDM's

Generated through mixing with New Gauge Bosons

Double Higgs Penguin in SUSY
Most popular BSM Directions

- CMFV (constrained MFV)
- MFV (NMFV) (GMFV)
- LHT (Littlest Higgs with T-parity)
- SUSY (flavour models)
- RS (Randall-Sundrum) (Warped Extra Dimensions)
- 4th G (Hou., Soni., Lenz., Melic) Munich
- Z’ (Langacker…)
- Vector-Like Quarks (Branco…, del Aguila)
- Non-Decoupling

New gauge bosons, fermions, scalars in loops and even trees with often non-CKM interactions.
### 2 x 2 Flavour Matrix of Basic NP Scenarios

(AJB, hep-ph/0101336, Erice)

<table>
<thead>
<tr>
<th></th>
<th>SM Operators</th>
<th></th>
<th>Additional Operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CMFV $(Y_t)$</td>
<td>B</td>
<td>MFV $(Y_t, Y_b)$</td>
</tr>
<tr>
<td>C</td>
<td>beyond CMFV</td>
<td>D</td>
<td>beyond MFV</td>
</tr>
</tbody>
</table>

**A**
- SM, 2 HDM at low $\tan\beta$
- LH without T-parity
- Universal flat ED

**B**
- MSSM with MFV
- 2 HDM at large $\tan\beta$

**C**
- LH with T-parity
- Some $Z'$-models
- 4th generation

**D**
- MSSM with $(\delta_{ij})_{AB} \neq 0$
- RS, Other $Z'$models, LR Models, NMFV
Little Hierarchy Problem

Electroweak Precision Tests

Agreement of the CKM Picture of Flavour and CP Violation with existing Data (FCNC)

$\Lambda_{NP} \approx 1000 \text{ TeV}$ (generic)

$\Lambda_{NP} \approx 5 \text{ TeV}$

Very strong Constraints on Physics beyond SM with scales $O (1 \text{ TeV})$

Necessary to solve the hierarchy problem

$(M_{\text{PLANCK}} >> \Lambda_{EW})$
Message 1: New Physics at TeV-Scale must have a non-Generic Flavour Structure

Message 2: Protection Mechanisms to suppress FCNCs generated by TeV-Scale New Physics required

- MFV
- GIM
- RS-GIM
- T-Parity
- R-Parity
- Alignment
- Degeneracy
- Flavour Symmetries (abelian, non-abelian)
- Custodial Symmetries (continuous, discrete)

Ciuchini et al
Isidori et al
Agashe et al
+50
In our search for a more fundamental theory we need to improve our understanding of Flavour
Towards the Theory of Flavour

- Phenomenology of Quark Flavour Physics
- Construction of New Models
- Phenomenology of Lepton Flavour Physics
- Theory of Quark Masses and Mixing Angles
- Electric Dipole Moments and \((g-2)\mu\)
- Flavour Violation in High Energy Collisions
- Dark Matter and Baryogenesis
- Grand Unification Theories
- New Techniques for Electroweak Symmetry Breaking
- Theory of Lepton Masses and Mixing Angles
- New Directions
- New Concepts
Phenomenology of Quark Flavour Physics

Construction of New Models

Phenomenology of Lepton Flavour Physics

Theory of Quark Masses and Mixing Angles

Electric Dipole Moments and \((g-2)\mu\)

Flavour Violation in High Energy Collisions

Dark Matter and Baryogenesis

Grand Unification Theories

New Techniques for Electroweak Symmetry Breaking

Theory of Lepton Masses and Mixing Angles

New Directions

New Concepts

Flavour Clock
Basic Questions for Flavour Physics

- New Flavour violating CPV phases?
- Flavour Conserving CPV phases?
- Non-MFV Interactions?
- Right-Handed Charged Currents?
- Scalars $H^0, H^\pm$ and related FCNC´s?
- New Fermions? New Gauge Bosons?

How to explain dynamically 22 free Parameters in the Flavour Sector?
3.
Minimal Flavour Violation
General Structure in Models with Constraint Minimal Flavour Violation

Ciuchini, Degrassi, Gambino, Giudice;
AJB, Gambino, Gorbahn, Jäger, Silvestrini;

★ No new Operators (Dirac and Colour Structures) beyond those present in the SM

★ Flavour Changing Transitions governed by CKM. No new complex phases beyond those present in the SM

\[
A(\text{Decay}) = B_i \eta^i_{QCD} V^i_{\text{CKM}} \left[ F^i_{\text{SM}} + F^i_{\text{New}} \right] \]

\( \text{real} \)
Minimal Flavour Violation (MFV)

SM Yukawa Couplings are the only breaking sources of the SU(3)$^5$ flavour symmetry of the low-energy effective theory

D’Ambrosio, Guidice, Isidori, Strumia (02) Chivukula, Georgi (87)

CKM the only source of Flavour Violation but for $Y_t \approx Y_b$ new operators could enter

CMFV

Operator structure of SM remains

AJB, Gambino, Gorbahn, Jäger, Silvestrini (00)
Ali, London

Related Studies:
- Ratz et al (08)
- Smith et al (08)
- Zupan et al (09)
- Kagan et al (09)

Spurion Technology
- Nir et al.
- AGIS
- Feldmann, Mannel

VERY STRONG RELATIONS BETWEEN K and B Physics and generally $\Delta F=2$ and $\Delta F=1$ FCNC Processes

also beyond MFV
The violation of these model independent MFV (CMFV) relations would signal new flavour and CP-violating interactions (and/or new operators)
Relations between $\Delta M_{s,d}$ and $B_{s,d} \rightarrow \mu \bar{\mu}$ in Models with Minimal Flavour Violation

(AJB, hep-ph/0303060)

$$\Delta M_q \sim \hat{B}_q F_{B_q}^2 \left| V_{tq} \right|^2 S(x_t, x_{\text{new}})$$

$$\text{Br}(B_q \rightarrow \mu \bar{\mu}) \sim F_{B_q}^2 \left| V_{tq} \right|^2 Y^2(x_t, x_{\text{new}})$$

Large hadronic uncertainties due to $F_{B_q}^2$

$$F_{B_d} \sqrt{\hat{B}_d} = \left(235 \pm 33 \quad ^{+0}_{-24}\right)\text{MeV} \quad F_{B_d} = (189 \pm 27)\text{MeV}$$

$$F_{B_s} \sqrt{\hat{B}_d} = (276 \pm 38)\text{MeV} \quad F_{B_s} = (230 \pm 30)\text{MeV}$$

$\hat{B}_d = 1.34 \pm 0.12$

$\hat{B}_s = 1.34 \pm 0.12 \quad \frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03$

(No problems with chiral logs and quenching)
Relations between $\Delta M_{s,d}$ and $B_{s,d} \to \mu\bar{\mu}$ in Models with Minimal Flavour Violation

$(AJB, \text{hep-ph/0303060})$

$$\Delta M_q \sim \hat{B}_q F_{B_q}^2 \left| V_{tq} \right|^2 S(x_t, x_{\text{new}})$$

$$\text{Br}(B_q \to \mu\bar{\mu}) \sim F_{B_q}^2 \left| V_{tq} \right|^2 Y^2(x_t, \bar{x}_{\text{new}})$$

Moderate hadronic uncertainties due to $F_{B_q}^2$

$$F_{B_d} \sqrt{\hat{B}_d} = \left( 216 \pm 15 \right) \text{MeV} \quad F_{B_d} = (193 \pm 10) \text{MeV}$$

$$F_{B_s} \sqrt{\hat{B}_d} = (275 \pm 13) \text{MeV} \quad F_{B_s} = (239 \pm 10) \text{MeV}$$

$$\hat{B}_d = 1.26 \pm 0.11$$

$$\hat{B}_s = 1.33 \pm 0.06 \quad \frac{\hat{B}_s}{\hat{B}_d} = 0.95 \pm 0.03$$

(No problems with chiral logs and quenching)
\[ \text{Br}(B_{s,d} \rightarrow \mu \bar{\mu}) \text{ from } \Delta M_{s,d} \]

\[ \text{Br}(B_q \rightarrow \mu \bar{\mu}) = 4.39 \cdot 10^{-10} \frac{\tau(B_q)}{\hat{B}_q} \frac{Y^2(x_t, \bar{x}_{\text{new}})}{S(x_t, x_{\text{new}})} \Delta M_q \]

**SM:**

\[ \text{Br}(B_s \rightarrow \mu \bar{\mu}) = 3.2 \cdot 10^{-9} \left[ \frac{\tau(B_s)}{1.43\text{ps}} \right] \left[ \frac{1.33}{\hat{B}_s} \right] \left[ \frac{\bar{m}_t(m_t)}{164 \text{ GeV}} \right]^{1.6} \left[ \frac{\Delta M_s}{17.8 / \text{ps}} \right] \]

\[ \text{Br}(B_d \rightarrow \mu \bar{\mu}) = 1.0 \cdot 10^{-10} \left[ \frac{\tau(B_d)}{1.52\text{ps}} \right] \left[ \frac{1.26}{\hat{B}_d} \right] \left[ \frac{\bar{m}_t(m_t)}{164 \text{ GeV}} \right]^{1.6} \left[ \frac{\Delta M_d}{0.51 / \text{ps}} \right] \]

(Example)

\[ \Delta M_s = (17.8 \pm 0.1 / \text{ps}) \rightarrow \text{Br}(B_s \rightarrow \mu \bar{\mu}) = (3.2 \pm 0.2) \cdot 10^{-9} \]

\[ \Delta M_d = (0.507 \pm 0.006 / \text{ps}) \rightarrow \text{Br}(B_d \rightarrow \mu \bar{\mu}) = (1.0 \pm 0.1) \cdot 10^{-10} \]

Moreover new Physics Effects can be easier seen.
Testing MFV through $B_{s,d} \to \mu \bar{\mu}$ and $\Delta M_{s,d}$

$$\frac{Br(B_s \to \mu \bar{\mu})}{Br(B_d \to \mu \bar{\mu})} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}$$

(0.95±0.03) Experiment

Valid in MFV models in which only SM operators relevant.

Violation of this relation would indicate the presence of new operators and generally of non-minimal flavour violation.
4. Motivations for BSM and BMFV
Impressive Success of the CKM Picture of Flavour Changing Interactions (GIM)

Yet

1. EW-Symmetry Breaking has to be better understood.

2. Hierarchies in Fermion Masses and Mixing Angles have to be understood with the help of some New Physics (NP). This NP could have impact on Low Energies.

3. There is still a lot of room for NP contributions, in particular in rare decays of mesons and leptons, in $\mathcal{CP}$ flavour violating transitions and EDM´s.


5. Several tensions between the flavour data and the SM exist.
Can SM describe simultaneously $\mathcal{CP}$ in K and $B_d$ Systems?
Can SM describe simultaneously $\mathcal{CP}$ in K and $B_d$ Systems?

\[ R_t^2 \approx \frac{\Delta M_d}{\Delta M_s} \]

\[
\left| \varepsilon^\text{SM}_K \right| \sim \kappa_\varepsilon \hat{B}_K |V_{cb}|^2 \left( \frac{1}{2} |V_{cb}|^2 R_t^2 \sin 2\beta \eta_{tt}^{\text{QCD}} S_0(x_t) + F(\eta_{ct}^{\text{QCD}}, \eta_{cc}^{\text{QCD}}, m_c, \ldots) \right)
\]

BJW (90)  
HN (94)

2009  
2010  
News

\[ \hat{B}_K \equiv 0.72 \pm 0.03 \]

(precise and lower by ~10% vs 2007)

Large N  
\[ \hat{B}_K = 0.70 \]

BBG (87)

\[ \kappa_\varepsilon \equiv 0.94 \pm 0.02 \]

(LD Effects)

RBC-UKQCD  
Aubin et al.  
ETMC

(Nierste; Vysotsky)

2009  
2010  
News

\[ \eta_{cc}^{\text{QCD}}, \eta_{ct}^{\text{QCD}} \]

NNLO QCD calculation

(BG)

\[ |\varepsilon^\text{SM}_K| = (1.92 \pm 0.22) \times 10^{-3} \]

Brod + Gorbahn (10)

\[ |\varepsilon^\text{exp}| = (2.229 \pm 0.012) \times 10^{-3} \]

(BaBar  
Belle)

using \( (\sin 2\beta)_{\psi_K} = 0.672 \pm 0.023 \)  

(NA48, KLOE, KTeV)
Possible Solutions to $\varepsilon_K$ - Anomaly

$$|\varepsilon_K|_{SM}^{SM} \sim \kappa \hat{B}_K |V_{cb}|^2 \left( \frac{1}{2} |V_{cb}|^2 R_t^2 \sin 2\beta \eta_{tt}^{QCD} S_0(x_t) + F(\eta_{ct}^{QCD}, \eta_{cc}^{QCD}, m_c, \ldots) \right)$$

1. Add New Physics to $\varepsilon_K$
   CMFV $S_0(x_t) \rightarrow S_0(x_t) + \Delta S_0^{NP}$ or simply $\Delta \varepsilon_K$ (Non-MFV)

2. Increase $\sin 2\beta \cong 0.67 \Rightarrow 0.85$
   $S_{\psi K_s} = \sin(2\beta + 2\varphi_{NP})$
   (Utfit; BBGT; Ball, Fleischer; Branco et al)

3. Increase $R_t$
   $\gamma = \delta_{CKM} \cong 67^\circ \Rightarrow 82^\circ$

4. Increase $|V_{cb}| \cong (41.2 \cdot 10^{-3}) \Rightarrow (43.5 \cdot 10^{-3})$

AJB Guadagnoli

Lunghi Soni

Super-B

LHC

Super-B

Vienna2010 49
Diego Guadagnoli
### Models investigated by TUM-Teams (Last decade)

<table>
<thead>
<tr>
<th>SM</th>
<th>MFV</th>
<th>MSSM+MFV</th>
<th>Z´-Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>General MSSM</td>
<td>Universal Extra Dimensions</td>
<td>RS with custodial protection</td>
<td>Right-Handed Currents</td>
</tr>
<tr>
<td>Littlest Higgs</td>
<td>Littlest Higgs with T-Parity</td>
<td>SUSY+Flavour Abelian Symmetry (Agashe+Carone)</td>
<td>2 Higgs Doublet Models</td>
</tr>
<tr>
<td>SUSY with SU(3) Flavour (Ross et al) (RVV2)</td>
<td>SUSY with SU(2) Flavour (LH-currents)</td>
<td>Flavour Blind MSSM</td>
<td>4G</td>
</tr>
</tbody>
</table>
4 Generations by 4 Physics Generations

AJB

T. Feldmann S. Recksiegel

B. Duling C. Promberger

T. Heidsieck
5.
Models for BMFV
Most popular BSM Directions

- CMFV (constrained MFV)
- MFV (NMFV) (GMFV)
- LHT (Littlest Higgs with T-parity)
- SUSY (flavour models)
- Z’ (Langacker…)
- RS (Randall-Sundrum) (Warped Extra Dimensions)
- 4th G (Hou.., Soni.., Lenz.., Melic) Munich
- Vector-Like Quarks (Branco…, del Aguila)
- Non-Decoupling

New gauge bosons, fermions, scalars in loops and even trees with often non-CKM interactions.
**4G Model**

The CKM4 matrix: New: $s_{14}$, $s_{24}$, $s_{34}$, $\delta_{14}$, $\delta_{24}$, $m_t$, $m_b$, 300-600 GeV

<table>
<thead>
<tr>
<th>$c_{12}c_{13}c_{14}$</th>
<th>$c_{13}c_{14}c_{12}$</th>
<th>$c_{14}s_{13}e^{-i\delta_{13}}$</th>
<th>$s_{14}e^{-i\delta_{14}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}}$</td>
<td>$c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}}$</td>
<td>$c_{13}c_{24}s_{23}$</td>
<td>$c_{14}s_{24}e^{-i\delta_{24}}$</td>
</tr>
<tr>
<td>$-c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14} - \delta_{24})}$</td>
<td>$-c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14} - \delta_{24})}$</td>
<td>$-s_{13}s_{14}s_{24}e^{i(\delta_{13} + \delta_{24} - \delta_{14})}$</td>
<td></td>
</tr>
<tr>
<td>$-c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23}$</td>
<td>$-c_{12}c_{23}c_{24}s_{34} - c_{23}s_{12}s_{13}s_{23}e^{i\delta_{13}}$</td>
<td>$c_{13}c_{23}c_{34}$</td>
<td>$c_{14}c_{24}s_{34}$</td>
</tr>
<tr>
<td>$-c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}}$</td>
<td>$-c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}}$</td>
<td>$-c_{13}c_{23}s_{24}s_{34}e^{i\delta_{24}}$</td>
<td>$-c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14} - \delta_{13})}$</td>
</tr>
<tr>
<td>$+c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}}$</td>
<td>$-c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}}$</td>
<td>$+s_{12}s_{13}s_{24}s_{34}e^{i(\delta_{13} + \delta_{24})}$</td>
<td></td>
</tr>
<tr>
<td>$+c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13} + \delta_{24})}$</td>
<td>$+s_{12}s_{13}s_{24}s_{34}e^{i(\delta_{13} + \delta_{24})}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vienna2010
Extensive New Interest in 4G

Very many papers: Hou; Hung; Chanowitz; Novikov et al. Kribs et al. + ....

FCNC’s :

Hou, Nagashima, Soddu
Soni, Alok, Giri, Mohanta, Nandi
Herrera, Benovides, Ponce
Bobrowski, Lenz, Riedl, Rohrwild
Eilam, Melic, Trampetic
AJB, Duling, Feldmann, Heidsieck, Promberger, Recksiegel
Lacker, Menzel
New Interest in Higgs-mediated FCNC's

Guidice, Lebedev (08); Agashe, Contino (09), Azatov, Toharia, Zhu (09), AJB, Gori, Duling (09); Duling (09)

Recent: Botella, Branco, Rebelo (09); Joshipura, Kodrani (08, 10), Pich, Tuzon (09), Gupta, Wells (10)

May – June 2010

Dobrescu, Fox, Martin (1005.4238)
(28 May) AJB, Carlucci, Gori, Isidori (1005.5310) \textbf{Neutral Higgs}
(29 May) Aranda, Montano, Ramirez-Zavaleta, Toscano, Tututi (1005.5452)
(31 May) Braeuninger, Ibarra, Simonetto
(2 June) Ligeti, Papucci, Perez, Zupan
(2 June) Jung, Pich, Tuzón \textbf{Charged Higgs}
Problems of the most general 2HDM

- $H_1', H_2$ two Higgs doublets with hypercharges $Y_1 = 1/2$ and $Y_2 = -1/2$

- Most general Yukawa interaction Hamiltonian

$$\mathcal{H}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

where $X_i$ are generic $3 \times 3$ matrices in flavor space

In general too large NP contributions to flavor changing neutral processes, since quark mass matrices and Yukawa couplings are not alligned

FCNCs at the tree level

How to protect the model from too large FCNCs?
Protection mechanisms

\[ \mathcal{H}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.} \]

Largest group which commutes with the SM gauge Lagrangian:

\[ \mathcal{G}_q = \text{SU}(3)_q^3 \otimes \text{U}(1)_B \otimes \text{U}(1)_Y \otimes \text{U}(1)_{\text{PQ}} \]

D'Ambrosio et al., '02

- **Minimal Flavor Violation hypothesis:**
  - SU(3)\(^3\) symmetry broken only by two spurions
  - \(Y_D \sim 3_Q \times 3_D\), \(Y_U \sim 3_Q \times 3_U\)

- **Tree level implication**
  - \(X_{d1} \propto X_{d2}\), \(X_{u1} \propto X_{u2}\)

- Including radiative corrections, one gets
  - \(X_{d1} = Y_d\) (definition)
  - \(X_{d2} = \epsilon_0 Y_d + \epsilon_1 Y_d^\dagger Y_d Y_d + \epsilon_2 Y_u^\dagger Y_u Y_d + \ldots\)
  - \(X_{u1} = \epsilon'_0 Y_u + \epsilon'_1 Y_u^\dagger Y_u Y_u + \epsilon'_2 Y_d^\dagger Y_d Y_u + \ldots\)
  - \(X_{u2} = Y_u\) (definition)

- **Natural Flavor Conservation hypothesis:**
  - only one Higgs field can couple to a given quark species

See also Yukawa alignment, Pich, Tuzon, '09

Glashow, Weinberg, '77
Paschos, '77

- **Natural Flavor Conservation hypothesis:**
  - only one Higgs field can couple to a given quark species

- **Tree level implication**
  - \(X_{d2} = X_{u1} = 0\)

- The hypothesis is enforced by the \(U(1)_{\text{PQ}}\) symmetry

  (also the \(Z_2\) symmetry can do the job)

- **Tree level implication**
  - \(X_{d2} = X_{u1} = 0\) (\(D_R\) and \(H_1\) with opposite PQ charges)

- The symmetry \(U(1)_{\text{PQ}}\) is usually explicitly broken (otherwise appearance of a Goldstone boson)

\[
\begin{align*}
X_{d2} &= \epsilon_d \Delta_d, \quad X_{d1} = Y_d \\
X_{u1} &= \epsilon_u \Delta_u, \quad X_{u2} = Y_u
\end{align*}
\]

FCNCs

S. Gori

Higgs-mediated FCNCs
Constraints on the two hypothesis

Constraint from the K meson mixing system: $\varepsilon_K$

Buras, Carlucci, S.G., Isidori, '10

Natural flavor conservation

$$|\varepsilon_d| \times |\text{Im}[(\Delta_d)_{21}^* (\Delta_d)_{12}]|^{1/2} \lesssim 3 \times 10^{-7} \times \frac{c_\beta M_H}{100 \text{ GeV}}$$

A very high level of fine tuning is required!

found imposing $|\varepsilon_{NP}^K| < 0.2 |\varepsilon_K^{exp}|$

A loop suppression $\varepsilon_d \sim 10^{-2}$ is not sufficient

Minimal flavor violation

$$|a_0| \lesssim 8 \times \frac{M_H}{100 \text{ GeV}} \frac{1}{t_\beta}$$

The constraint is satisfied very naturally, even for relatively light Higgs bosons!

found imposing $|\varepsilon_{NP}^K| < 0.05 |\varepsilon_K^{exp}|$

where

$$a_0 = \frac{\epsilon_2 t_\beta (1 + r_V)^2}{y_t^2 [1 + \epsilon_0 t_\beta]^2}$$

$$r_V \equiv \frac{(\epsilon_2 + \epsilon_3) t_\beta}{1 + (\epsilon_0 + \epsilon_1 - \epsilon_2 - \epsilon_3) t_\beta}$$

S. Gori

Higgs-mediated FCNCs
Few Messages on Higgs-mediated FCNC's

\[ \Delta M_s \]

\[ B_{s,d} \rightarrow \mu^+\mu^- \]

<table>
<thead>
<tr>
<th>SUSY</th>
<th>2HDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\tan\beta)^4)</td>
<td>((\tan\beta)^2) \cdot 1/M_H^2</td>
</tr>
<tr>
<td>((\tan\beta)^6)</td>
<td>((\tan\beta)^4) \cdot 1/M_H^4</td>
</tr>
</tbody>
</table>

MFV more powerful than Natural Flavour Conservation (BCGI)

General 2HDM with MFV and flavour blind phases (A JB, Carlucci, Gori, Isidori)

Aligned 2HDM (Pich, Tuzón) + flavour blind phases

Flavour-Blind phases can be included in MFV

Mercoli, Smith (09)
Kagan, Perez, Volansky, Zupan (09) (could help to generate large CP-phase in B_s-mixing)

Glashow, Weinberg 1977
Little Higgs Models
**Problematic quadratic divergences in $m_H^2$**

<table>
<thead>
<tr>
<th></th>
<th>SUSY</th>
<th>Little Higgs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic divergences canceled by:</td>
<td>(different statistics) super-partners</td>
<td>(same statistics) heavy partners</td>
</tr>
<tr>
<td>Coupling relationships due to:</td>
<td>boson-fermion symmetry</td>
<td>global symmetry</td>
</tr>
</tbody>
</table>
The most economical in matter content: Littlest Higgs (LH)

Valid up to $(4\pi f) \equiv \Lambda$


$$f \approx O(1\text{TeV})$$

Global: SU(5) $\rightarrow$ SO(5)

Local: $[SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2 \rightarrow SU(2)_L \otimes U(1)_Y$

$(g_1) \otimes (g'_1) \otimes (g_2) \otimes (g'_2)$


Theory symmetric under $[SU(2) \otimes U(1)]_1 \leftrightarrow [SU(2) \otimes U(1)]_2$

$g_1 = g_2 \quad g'_1 = g'_2$
Littlest Higgs Models without and with T-Parity

New particles: (with O(f) masses)

**LH**
- Gauge Bosons: $W^\pm_H, Z^0_H, A^0_H$
- Fermions: $T$
- Scalars: $\Phi^\pm, \ldots$

**LHT**
- **T-even Sector**
  - SM Particles + $T_+$
  - Gauge Bosons: $W^\pm_H, Z^0_H, A^0_H$
  - Fermions: $T_-$, Mirror Fermions
  - Scalars: $\Phi^\pm, \ldots$
- **T-odd Sector**
The World of Mirror Fermions

[I. Low, hep-ph/0409025]

Required to cut-off large 4-fermion operators constrained by LEP

Vectorial couplings under $SU(2)_L$

New Flavour Interactions involving SM fermions, Mirror Fermions and $W^\pm_H, Z^0_H, A^0_H$

$$V^\dagger_{Hu} V_{Hd} = V_{CKM}$$

[I. Low, hep-ph/0409025]

[J. Hubisz, S.J. Lee, G. Paz]

Similarly for Leptons

$$m_{u_H} = m_{d_H} \quad i=1,2,3$$
to first order in $v/f$

$$(V_{Hu})_{ij}$$ for:

$$d_{ij} \rightarrow W^\pm_H \sim (V_{Hd})_{ij} \gamma_\mu (1-\gamma_5)$$

$$d_{ij} \rightarrow Z^0_H, A^0_H \sim (V_{Hd})_{ij} \gamma_\mu (1-\gamma_5)$$

$$u_{ij} \rightarrow Z^0_H, A^0_H$$

$$(V_{Hu})_{ij}$$
**LHT goes beyond Minimal Flavour Violation (MFV)**

(without introducing new operators and non-perturbative uncertainties)

```
visible effects in flavour physics are possible```

\[ V_{\text{Hd}} = \begin{pmatrix}
-\frac{s_{12} \cos \theta_{13}}{c_{12}} e^{i \delta_{12}} & \frac{c_{12} c_{13}}{s_{12}} e^{-i \delta_{12}} & \frac{s_{12} c_{13}}{s_{23}} e^{-i \delta_{23}} & \frac{s_{13} c_{23}}{c_{13}} e^{-i \delta_{23}} \\
-\frac{c_{12} s_{13}}{c_{23}} e^{i \delta_{12}} & -\frac{c_{13}}{s_{23}} e^{-i \delta_{12}} & -\frac{s_{13} c_{23}}{c_{13}} e^{-i \delta_{23}} & \frac{s_{23}}{c_{13}} e^{-i \delta_{23}} \\
-\frac{s_{12} c_{13}}{c_{23}} e^{i \delta_{12}} & -\frac{c_{12}}{c_{23}} e^{-i \delta_{12}} & -\frac{s_{13} c_{23}}{c_{13}} e^{-i \delta_{23}} & \frac{s_{23}}{c_{13}} e^{-i \delta_{23}} \\
\end{pmatrix} \]

\( V_{\text{Hu}} V_{\text{Hd}} = V_{\text{CKM}} \)

[Low], [Hubisz, Lee, Paz]

**\( V_{\text{Hd}} \)** parameterization similar to CKM, but with 2 additional phases

(the phases of SM quarks are no more free to be rotated)

[Blanke, AJB, Poschenrieder, Recksiegel, Tarantino, Uhlig, Weiler]

[Similar new interactions and mixing matrices appear in the lepton sector]
General Structure of the Amplitudes

**LH (CMFV Model)**

\[ A(\text{Decay}) = \sum_i B_i^{SM} \eta_{QCD}^i V_{CKM}^i F_i(m_t, m_T, m_{W_H}, ...) \]

- Non-perturbative factors
- Real functions

**LHT**

\[ A(\text{Decay}) = \sum_i B_i^{SM} \eta_{QCD}^i \left[ V_{CKM}^i F_i(m_t, m_T) + V_{Hd}^i G_i(m_H^u, m_H^d, W^\pm, Z^0, A_H^0) \right] \]

- Real functions

**T-even contribution:** CMFV

**T-odd contribution:** New CP and Flavour violating Interactions but only SM operators
Tree-level heavy gauge boson contributions and the triplet $\Phi$ vev make EW precision tests highly constraining:

- $f \geq 2-3$ TeV

- The little hierarchy problem is back
- Only small effects in Flavour Physics

These unwanted contributions are eliminated by a discrete symmetry:

**T-parity**
- SM particles are T-even,
- new particles are T-odd
  (similarly to R-parity in SUSY)

- The little hierarchy problem is solved
- Large effects are possible in Flavour Physics

- Larger $f$ allowed by EW tests

**LH** (without T-parity) vs **LHT** (with T-parity)

- $f \geq 500$ GeV
### General Structure of New Physics Contributions

**SM:**

\[
\lambda_t^{(K)} = V_{ts}^* V_{td} \quad \lambda_t^{(d)} = V_{tb}^* V_{td} \quad \lambda_t^{(s)} = V_{tb}^* V_{ts}
\]

**Amplitudes:**

- \(\lambda_t^{(i)} X_{SM}(m_t)\) with visible in the final state
- \(\lambda_t^{(i)} Y_{SM}(m_t)\) with \(\mu^+ \mu^-\) in the final state

\(i = K, B_d, B_s\)

**LHT:**

- \(X_i = X_{SM}(m_t) + \overline{X}_{even} + \frac{1}{\lambda_t^{(i)}} \xi_i \overline{X}_{odd} \equiv |X_i| e^{i\theta_i^X}\)
- Real part
- Complex part

- \(Y_i = Y_{SM}(m_t) + \overline{Y}_{even} + \frac{1}{\lambda_t^{(i)}} \xi_i \overline{Y}_{odd} \equiv |Y_i| e^{i\theta_i^Y}\)
- Breakdown of Universality
- \(m_X \to m_Y\) (mirror fermions)
Natural Expectations

\[ X_i = X_{SM}(m_t) + \overline{X}_{even} + \frac{1}{\lambda_t^{(i)}} \xi_i \overline{X}_{odd} \equiv |X_i|e^{i\theta_X} \]

(similarly for \( Y_i \))

\[
\frac{1}{\lambda_t^{(K)}} \approx 2 \cdot 10^3 \\
\frac{1}{\lambda_t^{(d)}} \approx 100 \\
\frac{1}{\lambda_t^{(s)}} \approx 25
\]

\[
\{ \text{Natural size of NP contributions} \} : \quad K >> B_d > B_s
\]

But can be reversed for special structures of \( V_{Hd} \)
Randall-Sundrum Framework (Express Summary)
5D spacetime with warped metric:

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu - dy^2 \quad 0 \leq y \leq L \]

- Fermions and gauge bosons live in the bulk
- Higgs localised on IR brane
- Energy scales suppressed by warp factor \( e^{-ky} \)
  natural solution to the gauge hierarchy problem.
- Kaluza-Klein (KK) excitations of both SM fermions and
gauge bosons live close to the IR brane.

(Chang, Okada et al. Grossman, Neubert Gherghetta, Pomarol)
Fermion Localisation and Yukawa Couplings

SM fermion (zero mode) shape function depends strongly on bulk mass parameter characteristic for a given fermion:

\[ f^{(0)}(y, c) \propto e^{\left(\frac{1}{2} - c\right)y} \]

UV brane \quad IR brane

\[ \begin{aligned}
  c > \frac{1}{2} & \quad : \text{localisation near UV brane} \\
  c < \frac{1}{2} & \quad : \text{localisation near IR brane}
\end{aligned} \]

Higgs

effective 4D Yukawa couplings:

\[ \left( Y_{u,d} \right)_{ij} = \left( \lambda_{u,d} \right)_{ij} f_i^Q f_{j}^{u,d} \]

• \( \lambda_{u,d} \sim 0(1) \) anarchic complex 3 x 3 matrices \( \equiv Y_{5D} \)

• hierachical structure of quark masses and CKM parameters can be naturally generated by exponential suppression of \( f^{Q,u,d} \)
at IR brane.
Bulk Profiles of SM Gauge Bosons

- Gluons and Photon: flat (protection by Gauge symmetry)
- \( W^\pm, Z \): flat before EWSB but distorted near the IR brane after EWSB

Equivalently: Mixing of KK gauge bosons with \( W^\pm, Z \) in the process of EWSB modifies the couplings of mass eigenstates \( W^\pm, Z \)

- Recall: All KK gauge bosons live close to the IR brane
- All KK fermions live close to the IR brane
First Implications for Phenomenology

1. Gauge-Fermion Interactions: Overlaps of shape functions

2. Impact on Electroweak Precision Observables

Non-universalities in Gauge Couplings

of \{\text{KK-gauge bosons}\}

\{W^\pm, Z\}

to \{\text{SM fermions}\}

S parameter: \(M_{KK} \geq (2-3)\) TeV

T parameter: \(M_{KK} \geq 10\) TeV

Csaki, Grojean, Pilo, Terning (2003)

Also problems with \(Zb_L \bar{b}_L\)
Tree Level FCNC mediated by KK gauge bosons and $Z$ (breakdown of standard GIM mechanism)

$$d \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\mathcal{O}\left(\frac{v^2}{M_{KK}^2}\right)$$

But RS-GIM helps in avoiding disaster.

Gherghetta, Pomarol
Huber, Shafi
Agashe, Soni, Perez
4. Mixing of KK fermions with SM fermions and mixing of KK gauge bosons with SM gauge bosons

Breakdown of Unitarity of the CKM matrix

5. Tree level exchanges of $G_{KK}$ and $Z$ $ightarrow$ Contributions of new operators. In particular $Q_{LR}$ operators in addition to $Q_{LL}, Q_{RR}$

6. The presence of three 3 x 3 hermitian bulk matrices $c^q, c^u, c^d$ in addition to usual Yukawa couplings $ightarrow$ New flavour and CP violating parameters:

- $3 \times 6 = 18$ real phases
- $3 \times 3 = 9$ phases

Non-MFV
A RS Model with Custodial Protection

\[ \text{SU} (3)_C \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \otimes \text{U}(1)_X \otimes \text{P}_{\text{LR}} \]

Gauge Group in the Bulk

\[ \text{P}_{\text{LR}} : \text{SU}(2)_L \leftrightarrow \text{SU}(2)_R \]

\text{P}_{\text{LR}} \text{ symmetric fermion representations}
A Realistic Model in the Reach of the LHC

\[ \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_X \]

\[ \text{SU}(2)_R \times \text{U}(1)_X \rightarrow \text{U}(1)_Y \]
by boundary conditions

\[ \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2)_V \]
by Higgs VEV

+ \( (L \leftrightarrow R) \)-symmetric fermion representations

**low energy theory:** \( \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}} \)
What is protected in this Model?

(up to small symmetry breaking due to UV boundry conditions)

**A.** T-Parameter

Agashe, Delgado, May, Sundrum (0308036)
Csaki, Grojean, Pilo, Terning (0308038)

**B.** $Z \bar{b}_L^i b_L^i$

Agashe, Contino, Rold, Pomarol (0605341)

**C.** $Z \bar{d}_L^i d_L^j$

Blanke, AJB, Duling, Gori, Weiler (0809.1073)
Blanke, AJB, Duling, Gemmler, Gori (0812.3803)

**D.** $Z \bar{u}_R^i u_R^j$

AJB, Duling, Gori (0905.2318)

But: $Z \bar{d}_R^i d_R^i$, $Z \bar{u}_L^i u_L^j$, $W^+ \bar{u}_L^i d_L^j$ not protected
Particle Content of the Model

Albrecht, Blanke, AJB, Duling, Gemmler (0903.2415)

Gauge sector

\[ W^\pm, \quad W_H^\pm, \quad W'^\pm \quad A, \quad A^{(\ell)} \]

\[ Z^0, \quad Z_H, \quad Z' \]

KK

Quark sector

\[ (2,2) = \begin{pmatrix} \chi^{u_i}(-+)_{5/3} \\ \chi^{d_i}(-+)_{2/3} \end{pmatrix} \quad q^{u_i}(++)_{2/3} \]

\[ (1,1) = u_R^{i}(++)_{2/3} \]

\[ (3,1) = \begin{pmatrix} \psi'^i(-+)_{5/3} \\ U'^i(-+)_{2/3} \\ D'^i(-+)_{-1/3} \end{pmatrix}_R \oplus \begin{pmatrix} \psi''i(-+)_{5/3} \\ U''i(-+)_{2/3} \\ D''i(++)_{-1/3} \end{pmatrix}_R = (1,3) \]

SU(2)_L \otimes SU(2)_R

\[ Q=\frac{5}{3} \]

Fermions!

(Feynman rules worked out for SM and n=1 KK modes)
6. Patterns of Flavour Violation Beyond the SM
Three Strategies in Waiting for NP

1. **Precision Calculations**
   - **Background to NP**
     \[
     (B \rightarrow X_s\gamma, \ K^+ \rightarrow \pi^+\nu\bar{\nu}, \ B \rightarrow X_s\ell^+\ell^-)\]
   - Powerful in Electroweak Precision Physics

2. **Bottom-Up Approach**
   - Study of patterns of flavour violation in concrete NP models.
   - Correlations between observables!

3. **Top-Down Approach**
   - Powerful in Electroweak Precision Physics
   - In Flavour Physics less useful due to the presence of many operators (Buchmüller, Wyler: 1990)
   - Exception: Minimal Flavour Violation Hypothesis
To search for NP in rare $K$, $B_d$, $B_s$, $D$ decays, $\mathcal{CP}$ in $B_s$, $D$ decays, Lepton Flavour Violations

Specific Plots (Correlations)

- $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ vs $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$
- $\text{Br}(B_s \to \mu^+ \mu^-)$ vs $S_{\psi\phi}$
- $\text{Br}(B_s \to \mu^+ \mu^-)$ vs $\text{Br}(B_d \to \mu^+ \mu^-)$
- $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ vs $S_{\psi\phi}$
- $d_n$ vs $S_{\phi K_s}$
- $A_{\mathcal{CP}}(B \to X_S \gamma)$ vs $S_{\varphi K_s}$
- $\text{Br}(\tau \to \mu \gamma)$ vs $\Delta (g-2)_{\mu}$
- $\text{Br}(\tau \to \mu \mu \mu)$ vs $\text{Br}(\tau \to \mu \gamma)$
- $\text{Br}(\mu \to 3e)$ vs $\text{Br}(\mu \to e\gamma)$

Correlations will be crucial to distinguish various NP scenarios

Patterns of Flavour Violations in specific NP Models
Superstars of 2010 – 2015
(Flavour Physics)

\[ S_{\psi\phi} \] (\( B_s \rightarrow \phi\phi \))

\[ B_s \rightarrow \mu^+\mu^- \] (\( B_d \rightarrow \mu^+\mu^- \))

\[ K^+ \rightarrow \pi^+\nu\bar{\nu} \] (\( K_L \rightarrow \pi^0\nu\bar{\nu} \))

\[ B_d \rightarrow K^*\mu^+\mu^- \]

\[ \gamma \] from Tree Level Decays

\[ \mu \rightarrow e\gamma \]
\[ \tau \rightarrow \mu\gamma \]
\[ \tau \rightarrow e\gamma \]
\[ \mu \rightarrow 3e \]
\[ \tau \rightarrow 3 \text{ leptons} \]

\[ \varepsilon'/\varepsilon \]
(Lattice)

EDM's
\( (g-2)_\mu \)
Superstars enter the Scene

in the context of

SUSY, LHT, RS, 4G, 2HDM, RHMFV

(flavour models)
## Number of new Flavour Parameters

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<th>(Quark Sector)</th>
<th>(physical)</th>
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Some sensitivity to UV
Prima Donna of 2010 – Flavour Physics

Mixing Induced CP Asymmetry in $B_s \rightarrow \psi \phi$ ($S_{\psi \phi}$) ($A_{SL}^S$)

(TH very clean; Analog of $S_{\psi K_s}$)

$S_{\psi \phi} = \sin \left( 2|\beta_s| - 2\phi_s^{new} \right)_{SM} \cong 0.035$

$V_{ts} = -|V_{ts}|e^{-\beta_s}$

$\beta_s = -1^\circ$

CDF: Hints for a much larger value

D0: New Phase in $B_s^0 - \overline{B}_s^0$ mixing

*) See however Faller, Fleischer, Mannel (08)
Prima Donna of 2010 – Flavour Physics

Mixing Induced CP Asymmetry in $B_s \rightarrow \psi \phi$ ($S_{\psi \phi}$) ($A_{SL}^s$)

(TH very clean; Analog of $S_{\psi K_s}$)

$$S_{\psi \phi} = \sin \left( 2|\beta_s| - 2\phi_{s_{\text{new}}} \right)_{\text{SM}} \approx 0.035$$

CDF : Hints for a much larger value
D0

Without latest CDF result !

Preliminary result from Lenz, Nierste + CKM fitters

(soon!)

$S_{\psi \phi} = 0.78^{+0.12}_{-0.17}$

$3\sigma : [0.07, 1]$ range

Louise Oakes´s talk in Torino

$V_{ts} = -|V_{ts}| e^{-\beta_s}$

($\beta_s = -1^\circ$)

But CDF cannot exclude values above 0.5 !

*) See however Faller, Fleischer, Mannel (08)
Patterns of Deviations from CPV – SM Predictions

\[ K^0 - \bar{K}^0 \quad (\varepsilon_K) \quad \left| \varepsilon_K \right|_{SM} \approx 0.83 \pm 0.10 \]
\[ B_d^0 - \bar{B}_d^0 \quad (S_{\psi K_s}) \quad (S_{\psi K_s}) \approx 0.74 \pm 0.04 \quad (SM) \quad (UTfit) \]
\[ B_s^0 - \bar{B}_s^0 \quad (S_{\psi \phi}) \quad \left( \frac{S_{\psi \phi}}{S_{\psi \phi}} \right)_{SM} \approx 10 - 20 \]

Do these deviations signal non-MFV interactions at work?
General 2HDM with MFV and Flavour Blind CPV Phases

(AJB, Carlucci, Gori, Isidori)

Provides correct pattern

\[
\varepsilon_K : \quad \frac{m_d m_s}{M_H^2} m_t^4 (\tan \beta)^2 \left( V_{ts}^* V_{td} \right)^2 \quad \text{(tiny)}
\]

\[
S_{\psi K_s} : \quad \frac{m_b m_d}{M_H^2} m_t^4 (\tan \beta)^2 \left( V_{tb}^* V_{td} \right)^2 e^{i\phi_{\text{new}}}
\]

\[
S_{\psi \phi} : \quad \frac{m_b m_s}{M_H^2} m_t^4 (\tan \beta)^2 \left( V_{tb}^* V_{ts} \right)^2 e^{i\phi_{\text{new}}}
\]

\[
S_{\psi K_s} = \sin(2\beta - \theta_d^H) \quad S_{\psi \phi} \approx \sin(\theta_s^H)
\]

\[
\sin 2\beta > S_{\psi K_s} \quad (|\varepsilon_K| \text{ enhanced})
\]

\[
\frac{\theta_d^H}{\theta_s^H} \approx \frac{m_d}{m_s} \approx \frac{1}{17}
\]

\[
\tan \beta \approx 10 - 20 \quad M_H \approx 250\text{GeV}
\]

Large RG QCD effects $Q_{LR}$
$S_{\Psi_{Ks}}$ vs $S_{\Psi_{\phi}}$ and $|\varepsilon_K|$ vs $S_{\Psi_{\phi}}$

in a General 2HDM with MFV and Flavour Blind CPV

(AJB, Carlucci, Gori, Isidori)

Correct pattern of NP effects
**Z-Penguin (SM + Boxes CMFV)**

- $B_s \rightarrow \mu^+\mu^-$ and $B_d \rightarrow \mu^+\mu^-$

**SM**

$$\text{Br}(B_s \rightarrow \mu^+\mu^-) = (3.2 \pm 0.2) \cdot 10^{-9}$$

$$\text{Br}(B_d \rightarrow \mu^+\mu^-) = (1.0 \pm 0.1) \cdot 10^{-10}$$

**AJB (03)**

**CMFV**

- "Golden Relation"

$$\frac{\text{Br}(B_s \rightarrow \mu^+\mu^-)}{\text{Br}(B_d \rightarrow \mu^+\mu^-)} = \frac{\hat{B}_d \tau(B_s) \Delta M_s}{\hat{B}_s \tau(B_d) \Delta M_d}$$

$(\Delta B = 1)$

$(0.95 \pm 0.03)$ Lattice

$(\Delta B = 2)$

Valid in all CMFV models

Can be strongly violated in SUSY, LHT, RS, 4G

95% CL

**Fleischer et al**

- $\text{Br}(B_s \rightarrow \mu^+\mu^-) \leq \begin{cases} 3.3 \cdot 10^{-8} & \text{(CDF)} \\ 5.3 \cdot 10^{-8} & \text{(D0)} \end{cases}$

- $\text{Br}(B_d \rightarrow \mu^+\mu^-) \leq 1 \cdot 10^{-8}$ (CDF)

LHC should be able to discover $B_s \rightarrow \mu^+\mu^-$ even at the SM level

Error dominated by $\hat{B}_{d,s}$
**B_{s,d} \rightarrow \mu^+\mu^- in Various Models**

Babu, Kolda (99), … +100

**Can reach CDF and DØ bounds**

\[
\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-) \sim \frac{(\tan \beta)^6}{M_A^4}
\]

\[
\frac{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)_{4G}} \leq 4
\]

\[
\frac{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)_{SM}} \leq 20
\]

\[
\frac{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)_{LHT}} \leq 1.3
\]

\[
\frac{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)_{SM}} \leq 1.1
\]

(Z-penguin) (Blanke et al) (09)

(Z-penguin + Z-tree with r.h. couplings) (Custodial protection at work) (Gori et al) (08)
Br\( (B_s \rightarrow \mu^+ \mu^-) \) vs \( S_{\psi\phi} \)

Solution 3 to \( \varepsilon_K \)-Anomaly
Abelian (AC)

Solution 1 to \( \varepsilon_K \)-Anomaly
Non-Abelian (RVV)

(List Effects in D\(^0\)-\( \bar{D}^0 \))

(Small Effects in D\(^0\)-\( \bar{D}^0 \))
\[ \text{Br}(B_s \rightarrow \mu^+\mu^-) \text{ vs } S_{\psi\phi} \]

4G has hard time to describe simultaneously $\epsilon'/\epsilon$ and $S_{\psi\phi} > 0.2$ if $B_{6,8}$ within 20% from large N values
\[ \text{Br}(B_d \rightarrow \mu^+ \mu^-) \text{ vs } \text{Br}(B_s \rightarrow \mu^+ \mu^-) \]

MFV  
AJB; Hurth, Isidori, Kamenik, Mescia

RVV2  (RH currents)

LH currents
$B_{s,d} \to \mu^+\mu^- \text{ in 2HDM - MFV} \approx (\tan \beta)^4 / M_A^4$

(AJB, Carlucci, Gori, Isidori)

$10^9 \cdot \text{Br}(B_s \to \mu\mu)$

within few% determined by

\[
\frac{\Delta M_s}{\Delta M_d}
\]
\[ \text{Br}\left( B_d \to \mu^+ \mu^- \right) \text{ vs } \text{Br}\left( B_s \to \mu^+ \mu^- \right) \]

Very different patterns compared with SUSY, 2HDM, MFV
**K^+ → π^+ νν and K_L → π^0 νν (Z°-penguins)**

(TH cleanest FCNC decays in Quark Sector)

**Extensive TH efforts over 20 years**:
- Buchalla, AJB; Misiak, Urban (NLO QCD)
- A JB, Gorbahn, Haisch, Nierste (NNLO QCD)
- Brod, Gorbahn (QED, EW two loop)
- Isidori, Mescia, Smith (several LD analyses)
- Buchalla, Isidori (LD in K_L → π^0 νν)

<table>
<thead>
<tr>
<th>SM</th>
<th>Br(K^+ → π^+ νν) = (8.4 ± 0.7) \cdot 10^{-11}</th>
<th>Br(K_L → π^0 νν) = (2.6 ± 0.4) \cdot 10^{-11}</th>
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<td>Exp</td>
<td>Br(K^+ → π^+ νν) = \left(17^{+11}_{-10}\right) \cdot 10^{-11}</td>
<td>Br(K_L → π^0 νν) ≤ 6.8 \cdot 10^{-8}</td>
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</table>

(E787, E949 Brookhaven) (E391a, KEK)

**Future**: NA62
- Project X (FNAL)
  - Both very sensitive to New Physics

CP-conserving TH uncertainty 2-3%  

J-PARC KOTO
- CP-Violation in Decay
  - TH uncertainty 1-2%
$K_L \to \pi^0 \nu \bar{\nu}$ vs. $K^+ \to \pi^+ \nu \bar{\nu}$

Blanke et al (08)
Goto et al (08)
Recksiegel et al (09)

$\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$

(E787, E949 Brookhaven)

$\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$

SM, AC, RVV

RS

LHT

(SUSYf)
$K_L \to \pi^0 \nu\bar{\nu}$ vs. $K^+ \to \pi^+ \nu\bar{\nu}$ in 4G

BDFHPR

Much larger enhancements than in LHT, RS, SUSYf possible

With $\epsilon'/\epsilon$ Constraint
$K^+ \rightarrow \pi^+ \nu\bar{\nu}$ vs. $S_{\psi\phi}$

(Simultaneous Large Enhancements unlikely)
\[ K^+ \rightarrow \pi^+ \nu \bar{\nu} \text{ vs } S_{\psi\phi} \] (4G)

(Simultaneous Large Enhancements Possible)
### DNA Tests of Flavour Models

\[ O_i : \text{Observables} \quad M_i : \text{Models beyond SM} \]

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- ⭐⭐⭐⭐⭐ Very large New Physics effect
- ⭐⭐⭐⭐ Moderate New Physics effect
- ⭐⭐⭐ Very small New Physics effect
## DNA Tests of Flavour Models

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Vienna2010

110
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<tr>
<th>Parameter</th>
<th>NEW SM</th>
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<tr>
<td>$D^0 - ar{D}^0$</td>
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<tr>
<td>$\epsilon_K$</td>
<td>★★</td>
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<tr>
<td>$S_{\psi\phi}$</td>
<td>★★★</td>
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<tr>
<td>$S_{\phi K_S}$</td>
<td>★★</td>
</tr>
<tr>
<td>$A_{CP}(B \to X_s \gamma)$</td>
<td>★</td>
</tr>
<tr>
<td>$A_{7,8}(B \to K^* \mu^+ \mu^-)$</td>
<td>★★</td>
</tr>
<tr>
<td>$A_9(B \to K^* \mu^+ \mu^-)$</td>
<td>★</td>
</tr>
<tr>
<td>$B \to K^{(*)} \nu \bar{\nu}$</td>
<td>★★★</td>
</tr>
<tr>
<td>$B_s \to \mu^+ \mu^-$</td>
<td>★★★</td>
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<tr>
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<td>$(g - 2)_\mu$</td>
<td>★★</td>
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</tbody>
</table>
7. Outlook
In our search for a more fundamental theory we need to improve our understanding of Flavour
LHC, Tevatron
B, K, D Physics
Neutrino Physics
Lepton Flavour Violation ($\mu \rightarrow e \gamma$)
$\Delta m_n, \Delta m_e, (g-2)_\mu$

SM → 2010 → MFV → SUSY → ILC
   → SLHC → SFF → SB → SK
   → Little Higgs
   → 4th G
   → RS
   → ??
Final Messages of this Talk

**Flavour Physics (Quarks and Leptons):** Many observables (decays) not measured yet or measured poorly. Flavour Physics only now enters the precision era.

- Spectacular deviations from SM still possible
- Direct searches at Tevatron, LHC, ILC

Interplay
Final Messages of this Talk

Flavour Physics (Quarks and Leptons): Many observables (decays) not measured yet or measured poorly. Flavour Physics only now enters the precision era.

Spectacular deviations from SM still possible

DNA Flavour Test of NP models

Correlations between various observables can distinguish NP scenarios easier than LHC!

Direct searches at Tevatron, LHC, ILC

Interplay

Great discoveries and goals are just ahead of us!
Superstars of 2010 – 2015
(Flavour Physics)

\[ S_{\psi\phi} \quad B_s \to \mu^+\mu^- \quad K^+ \to \pi^+\nu\bar{\nu} \]
\[ (B_s \to \phi\phi) \quad (B_d \to \mu^+\mu^-) \quad (K_L \to \pi^0\nu\bar{\nu}) \]
\[ B^+ \to \tau^+\nu_\tau \quad (B_d \to K^*\mu^+\mu^-) \]

\[ \gamma \text{ from Tree Level Decays} \]
\[ \mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma \]
\[ \mu \to 3e \quad \tau \to 3 \text{ leptons} \]

\[ \epsilon'/\epsilon \quad \text{(Lattice)} \]
\[ \text{EDM's (g-2)}_\mu \]
Backup
$\frac{\varepsilon'}{\varepsilon} \sim 10^{-4} \left[ \frac{\text{Im} \lambda_t}{1.20 \cdot 10^{-4}} \right] F \left( m_t, \Lambda_{\text{MS}}^{(4)}, m_s, B_6, B_8, \Omega_{\text{IB}} \right) $

$F \approx 16 \cdot \left[ \frac{110 \text{ MeV}}{m_s (2 \text{ GeV})} \right]^2 \left[ B_6 \left( 1 - \Omega_{\text{IB}} \right) - \tilde{Z}(m_t) B_8 \right] \left( \frac{\Lambda_{\text{MS}}^{(4)}}{340 \text{ MeV}} \right)$

$\tilde{Z}(m_t) \approx 0.4 \left[ \frac{m_t}{165 \text{ GeV}} \right]^{2.5}$

$\Omega_{\text{IB}} = \text{Isospin Breaking}$

Basic Parameters: $\text{Im} \lambda_t, \Lambda_{\text{MS}}^{(4)}, B_6, B_8, m_s, \Omega_{\text{IB}}$
First Round of Measurements

\[ \frac{\varepsilon'}{\varepsilon} = \begin{cases} (23 \pm 6.5) \cdot 10^{-4} \quad \text{(NA31)} \\ (7.4 \pm 5.9) \cdot 10^{-4} \quad \text{(E731)} \end{cases} \]

Second Round of Measurements

\[ \frac{\varepsilon'}{\varepsilon} = \begin{cases} (14.7 \pm 2.2) \cdot 10^{-4} \quad \text{(NA48)} \\ (20.7 \pm 2.8) \cdot 10^{-4} \quad \text{(KTeV)} \end{cases} \]

Grand Average:

\[ \frac{\varepsilon'}{\varepsilon} = (16.6 \pm 1.6) \cdot 10^{-4} \]

Waiting for KLOE

Direct CP Violation firmly established
Starting Point:
\[ \mathcal{L} = \mathcal{L}_{SM}(g_i, m_i, V_{CKM}) + \mathcal{L}_{NP}(g_i^{NP}, m_i^{NP}, V_i^{NP}) \]

Goal:
Identify the effects of \( \mathcal{L}_{NP} \) in weak decays in the presence of the background from \( \mathcal{L}_{SM} \)

First Implication from \( \mathcal{L} \):

Feynman Diagrams

- \( B_d^0 - \bar{B}_d^0 \) Mixing
- \( K^+ \to \pi^+ \nu \bar{\nu}, \ K_L \to \pi^0 \nu \bar{\nu} \)
Putting S0(10)-SUSY-GUT of Dermisek-Raby into difficulties


1. The Model gives a nice description of quark and lepton masses, PMNS and most of CKM elements.

2. But fails to describe simultaneously the data on

\[ B_{s,d} \rightarrow \mu^+\mu^-, \ B \rightarrow X_s\gamma, \ B \rightarrow X_s l^+l^-, \ B_u \rightarrow \tau\nu \]

3. Gives

\[ |V_{ub}| \approx 3.2 \cdot 10^{-3} \]

\[ < \left( 4.2 \pm 0.3 \right) \cdot 10^{-3} \]

Exp.

Also SUSY Spectrum

Generally too low

Some recent solutions: Altmannshofer et al.
Very strong Constraints on New Physics

\[
\text{Br}(B \rightarrow X_S \gamma)_{\text{exp}} = (3.52 \pm 0.24) \cdot 10^{-4}
\]

\[
\text{Br}(B \rightarrow X_S \gamma)_{\text{SM}} = \begin{cases} 
(3.15 \pm 0.23) \cdot 10^{-4} & \text{(Misiak et al)} \\
(2.98 \pm 0.26) \cdot 10^{-4} & \text{(Becher, Neubert)} 
\end{cases}
\]

\[
\text{Br}(B \rightarrow X_S l^+ l^-)_{\text{exp}} = \begin{cases} 
(1.6 \pm 0.5) \cdot 10^{-6} & \text{(low } q^2) \\
(4.4 \pm 1.3) \cdot 10^{-7} & \text{(high } q^2) 
\end{cases}
\]

\[
\text{Br}(B \rightarrow X_S l^+ l^-)_{\text{SM}} = \begin{cases} 
(1.6 \pm 0.1) \cdot 10^{-6} & \text{(low } q^2) \\
(2.3 \pm 0.8) \cdot 10^{-6} & \text{(high } q^2) 
\end{cases}
\]

\[
A_{\text{CP}}(B \rightarrow X_S \gamma)_{\text{exp}} = 0.004 \pm 0.036
\]

\[
A_{\text{CP}}(B \rightarrow X_S \gamma)_{\text{SM}} = 0.004 \pm 0.002
\]

(Still factor 10 enhancement possible !)

Isidori et al. (incl.)
Gorbahn et al. (incl.)
Feldmann et al. (excl.)

Zero in \( A_{\text{FB}} \)
\( \hat{s}_0 = (3.50 \pm 0.12) \text{GeV}^2 \)

\( \star \)

TH
very clean

All this can be improved at Super-B
Super-Belle
\[ \text{Br}(B^+ \rightarrow \tau^+ \nu)_{\text{exp}} = (1.4 \pm 0.4) \cdot 10^{-4} \]  
(Belle, BaBar)

\[ \text{Br}(B^+ \rightarrow \tau \nu)_{SM} \approx G_F^2 F_B^2 |V_{ub}|^2 = (0.95 \pm 0.20) \cdot 10^{-4} \]

\[ \frac{\text{Br}(B^+ \rightarrow \tau \nu)_{MSSM}}{\text{Br}(B^+ \rightarrow \tau \nu)_{SM}} = \left[ 1 - \left( \frac{m_B}{m_{H^\pm}} \right)^2 \frac{\tan^2 \beta}{1 + \varepsilon_0 \tan \beta} \right]^2 \]  
(Hou)

This decay could be problematic for MSSM-MFV with large tan\(\beta\)

Altmannshofer, AJB, Guadagnoli, Wick (07)

Tree-Level

H\(^+\) exchange
1. The light Higgs is interpreted as a Goldstone boson of a spontaneously broken global symmetry (G).

2. Gauge and Yukawa couplings of the Higgs are introduced by gauging a subgroup of G.

3. "Dangerous" quadratic corrections are avoided at one-loop through Collective Symmetry Breaking (the Higgs becomes massive only when two couplings are non-vanishing).

- The Higgs dynamics is described (similarly to ChPT) by a non-linear sigma model up to $\Lambda \sim 10\text{TeV}$.
- The UV completion is unknown (another LH?, SUSY?, ED?).
Maximal Enhancements of $S_{\psi\phi}$, $\text{Br}(B_s \rightarrow \mu^+\mu^-)$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$

(without taking correlation between them)

<table>
<thead>
<tr>
<th>Model</th>
<th>Upper Bound on $(S_{\psi\phi})$</th>
<th>Enhancement of $\text{Br}(B_s \rightarrow \mu^+\mu^-)$</th>
<th>Enhancement of $\text{Br}(K^+ \rightarrow \pi^+\nu\bar{\nu})$</th>
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<tr>
<td>CMFV</td>
<td>0.04</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>MFV</td>
<td>0.04</td>
<td>1000%</td>
<td>30%</td>
</tr>
<tr>
<td>LHT</td>
<td>0.30</td>
<td>30%</td>
<td>150%</td>
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<tr>
<td>RS</td>
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<td>300%</td>
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<tr>
<td>AC</td>
<td>0.75</td>
<td>1000%</td>
<td>2%</td>
</tr>
<tr>
<td>RVV</td>
<td>0.50</td>
<td>1000%</td>
<td>10%</td>
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Large RH Currents

RS = RS with custodial protections
AC = Agashe, Carone
RVV = Ross, Velaso-Sevilla, Vives (04)

U(1)$_F$  
SU(3)$_F$
Lepton Flavour Violation, $\Delta(g - 2)_\mu$ and EDM's

$$S_{\phi K_s} = 0.44 \pm 0.17 \quad \left( S_{\phi K_s} \right)_{\text{SM}} \approx \left( S_{\psi K_s} \right)_{\text{SM}} + 0.02 \approx 0.70$$

(Beneke)

(MEGA) $\text{Br}(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11} \quad \rightarrow \quad 10^{-13} (\text{MEG})$ $\text{SM:} 10^{-54}$

$$\left( a_\mu \right)_{\text{SM}} < \left( a_\mu \right)_{\text{exp}} \quad (3.1\sigma)$$

$$a_\mu = \frac{1}{2} (g - 2)_\mu$$

(Regan et al) $d_e < 1.6 \cdot 10^{-27} \quad \rightarrow \quad 10^{-31}$ $\quad (d_e)_{\text{SM}} \approx 10^{-38}$

(Baker et al) $d_n < 2.9 \cdot 10^{-26} \quad \rightarrow \quad 10^{-28}$ $\quad (d_n)_{\text{SM}} \approx 10^{-32}$
Simultaneous Solution to $S_{\phi K_s}$ and $S_{\psi \phi}$ Anomalies in 4G Model

BaBar
Belle

CDF D0
**μ → eγ: State of the Art**

- **SM (+ Dirac ν_R):**
  very much suppressed due to the smallness of m_ν

\[ Br(\mu \rightarrow e\gamma)_SM \approx 10^{-54} \]

- **Experimental bound:**

\[ Br(\mu \rightarrow e\gamma)_{exp} < 1.2 \cdot 10^{-11} \quad (90\%C.L.) \]

It will be improved to \(~10^{-13}\) by MEG in 2008

- **MSSM and LHT could explain such high values.**

WED too (Agashe et al.)
Other interesting Processes

- $\mu^- \rightarrow e^- e^+ e^-$: even more constrained than $\mu \rightarrow e\gamma$
  
  \[
  Br(\mu^- \rightarrow e^- e^+ e^-)_{\text{exp}} < 1.0 \cdot 10^{-12}
  \]
  [SINDRUM Collaboration]

- $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$: similar to $\mu \rightarrow e\gamma$
  
  \[
  \begin{align*}
  Br(\tau \rightarrow \mu\gamma)_{\text{exp}} &< 1.6 \cdot 10^{-8} \\
  Br(\tau \rightarrow e\gamma)_{\text{exp}} &< 9.4 \cdot 10^{-8}
  \end{align*}
  \]
  [Belle, BaBar] \quad [BaBar, Belle]

- $\tau \rightarrow \mu\pi$: semileptonic decay
  
  \[
  Br(\tau \rightarrow \mu\pi)_{\text{exp}} < 5.8 \cdot 10^{-8}
  \]
  (Future: Super B) \quad [Belle, BaBar]

- $\mu \rightarrow e$ conversion
  
  \[
  R(\mu T_i \rightarrow e T_i) < 4.3 \cdot 10^{-12}
  \]
  \[10^{-18} \text{ (J-Parc)}\]

- $K_L \rightarrow \mu e$: flavour violating in both quark and lepton sectors
  
  \[
  Br(K_L \rightarrow \mu e)_{\text{exp}} < 4.7 \cdot 10^{-12}
  \]
  [BNL E871 Collaboration]
Simultaneous Solution to $\Delta a_\mu$ and $S_{\psi\phi}$ Anomalies

Solution 3 to $\varepsilon_K$-Anomaly
Abelian (AC)

Solution 1 to $\varepsilon_K$-Anomaly
Non-Abelian (RVV)

(Large Effects in $D^0-\bar{D}^0$)

(Small Effects in $D^0-\bar{D}^0$)
Correlations in the SU(3) Flavour Model (RVV2)

Solution to \((g-2)_\mu\) anomaly
Clear Distinction between MSSM and LHT

\[
\frac{\text{Br}(\mu^- \rightarrow e^- e^+ e^-)}{\text{Br}(\mu^- \rightarrow e^- \gamma)} \approx \frac{1}{161}
\]

\[
\frac{\text{Br}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{\text{Br}(\tau^- \rightarrow \mu^- \gamma)} \approx \frac{1}{435}
\]

MSSM : (Ellis, Hisano, Raidal, Shimizu; Arganda, Herrero; Paradisi) (Brignole, Rossi)


Both can reach MEGA’s \(\mu \rightarrow e\gamma\) bound