CHIRAL DYNAMICS WITH STRANGE QUARKS IN THE LIGHT OF RECENT LATTICE SIMULATIONS

in collaboration with S. Descotes-Genon and G. Toucas
and E. Passemar

- **Introduction**
- **Two-loop χPT calculation of the $K\pi$ form factor**
  - why is it interesting?
  - results
- **Resummed χPT**
  - method
  - results
- **Conclusion**

QCD AT LOW ENERGY

– NON PERTURBATIVE

– IMPORTANT PROPERTY: CHIRAL SYMMETRY in the limit of massless quarks

2 MODEL INDEPENDENT WAYS TO SOLVE QCD

↙ ↘

LATTICE QCD

EFT: CHPT

$M_\pi \geq 140$ MeV
$L \geq 3$ fm
$a < 0.1$ fm

← CHIRAL EXTRAPOLATION

→ LECs

SOME DETERMINATIONS FOR MESONS

LATTICE: HINT OF PROBLEMS WITH SU(3)$\times$SU(3) IN CALC. AT NLO in CHPT

WHAT WITH A CALCULATION AT NNLO in CHPT ?

- Brief Reminder: CHIRAL PERTURBATION THEORY: Weinberg, Gasser and Leutwyler $\cdots$

EFT OF THE STANDARD MODEL

PROPERTIES

- MOST GENERAL EFFECTIVE LAGRANGIAN in agreement with SYMMETRIES OF QCD

$$\mathcal{L}_{EFF}[U, \partial_\mu U, \cdots, \underbrace{\mathcal{M}, v_\mu, a_\mu,}\cdots, N]$$

π fields ↗ external sources ↗ matter fields ↖

- EXPANSION
  - in external momenta p: interaction between GB is weak
  - in quark masses m_q: small compared to $\Lambda_\chi \simeq 1$ GeV

$$\mathcal{L}_{EFF} = \sum_i \mathcal{L}_{\pi N}^{(i)} + \sum_j \mathcal{L}_{\pi\pi}^{(2j)}$$

i, j power in small parameter $q = \{p, m_q\}$

EFFECTIVE LAGRANGIAN AT TWO LOOP ORDER

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(6)}_{\pi\pi} + \cdots$$

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}$$

$$\mathcal{L}^{(4)}_{\pi\pi} = \sum_{i=1}^{10} L_i O_4^i$$

$$\mathcal{L}^{(6)}_{\pi\pi} = \sum_{i=1}^{90} C_i O_6^i$$

LECs: L_is and C_is

DETERMINATION OF THE LECs

- not constrained by symmetries
- in general scale dependent (renormalization): $b_i = b_i^r(\lambda) + \kappa_i \lambda^{d-4}(1/(d-4) + \cdots)$
- describe influence of “heavy” degrees of freedom not contained explicitely in χ Lagrangians
- RELATE MANY OBSERVABLES
- naturalness: should be of order one

TWO CLASSES

- dynamical LECs: ∂_μ^n

govern momentum dependence $\rightarrow$ accessible EXPERIMENTALLY

- symmetry breaking LECS: $\sim m_q^n, \;\; m_q^n \partial_\mu^m$

specify quark mass dependence of amplitudes $\rightarrow$ more difficult to extract from experiments

ADDITIONAL INPUT FROM THEORY

large-N_c method
lattice QCD
} RESULTS IN MESON SECTOR FOR L_i's

	Fit 10 Bijnens '01	πK Roy Steiner Büttiker et al '04	Prelim. Fit All(*) Bijnens &Jemos '09	Lattice Allton '08
$10^3 L_4^r$	0	0.53 ± 0.39	0.70 ± 0.66	$0.33(0.13)$
$10^3 L_5^r$	0.97	3.19 ± 2.40	0.56 ± 0.11	$0.93(0.073)$
$10^3 L_6^r$	0		0.14 ± 0.70	-
$10^3 L_8^r$	0.6		0.38 ± 0.17	-
$10^3(2L_6^r - L_4^r)$				0.032 (0.062)
$10^3(2L_8^r - L_5^r)$				0.050(0.043)

- LARGE N_c: L_5 L_8: $\mathcal{O}(N_c)$

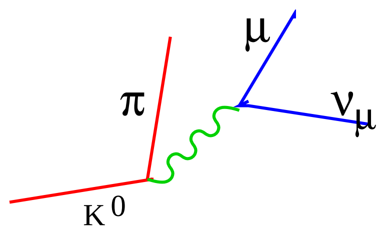
while L_4 L_6: $\mathcal{O}(1) \longrightarrow$ CONTRIBUTION FROM m_s to M_π^2, F_π^2 and $\langle 0|\bar{u}u|0\rangle$ SUPPRESSED

CONCENTRATE ON THE $K\pi$ STRANGENESS CHANGING FORM FACTORS
AND ON THE RATIO OF KAON TO PION DECAY CONSTANTS

TWO VERY IMPORTANT QUANTITIES FOR TESTING THE SM

$K\pi$ STRANGENESS CHANGING FORM FACTORS

- MEASURED in $K_{\ell 3}$



and INFORMATION from τ decay: $\tau \to K\pi\nu_\tau$

- Definition of VECTOR FORM FACTORS

$$\langle \pi^-(p')|\bar{s}\gamma_\mu u|K^0(p)\rangle = (p+p')_\mu f_+^{K^0\pi^-}(t) + (p-p')_\mu f_-^{K^0\pi^-}(t)$$

- $f_0(t)$ SCALAR FORM FACTOR

$$f_0(t) = f_+^{K^0\pi^-}(t) + \frac{t}{m_K^2 - m_\pi^2} f_-^{K^0\pi^-}(t), \qquad f_0(0) = f_+(0)$$

In the following: $\bar{f}_0(t) \equiv f_0(t)/f_0(0)$

THREE RELEVANT POINTS

- $t = 0$

$$\Gamma \propto |f_+(0) V_{us}|^2 F(\bar{f}_+, \bar{f}_0)$$

IF $f_+(0)$ KNOWN $\longrightarrow$ $|V_{us}|$ $\longrightarrow$ TEST of UNITARITY of CKM matrix: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

- CALLAN TREIMAN THEOREM: $SU(2) \times SU(2)$ theorem

CURRENT ALGEBRA (one of the first application):

$$\langle \pi^-(p') | \bar{s}\gamma_\mu u | K^0(p) \rangle \underset{p' \to 0}{\longrightarrow} = \frac{i}{F_\pi} \langle 0 | [Q_5^3, \bar{s}\gamma_\mu u] | K^0(p) \rangle = \frac{F_K}{F_\pi} p_\mu$$

$$\Rightarrow f_0(m_K^2) = \frac{F_K}{F_\pi}$$

M_π PHYSICAL: CORRECTIONS Δ_{CT} OF ORDER m_u, m_d:

$$\bar{f}_0(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi} \frac{1}{f_+^{K^0\pi^-}(0)} + \Delta_{CT}$$

$\nearrow$

Callan Treiman point $\Delta_{K\pi}$

- SOFT KAON ANALOG

$$\bar{f}_0(-m_K^2 + m_\pi^2) = \frac{F_\pi}{F_K} \frac{1}{f_+^{K^0\pi^-}(0)} + \tilde{\Delta}_{CT}$$

DETERMINATION OF Δ_{CT} and $\tilde{\Delta}_{CT}$:

CHPT:

- TO ONE LOOP: PURE LOOP EFFECTS, NO LECs AND NO CHIRAL LOGS

$\rightsquigarrow$ in ISOSPIN LIMIT: : Gasser & Leutwyler: Nucl. Phys. B250 (1985) 517

$\Delta_{CT} = -3.5 \cdot 10^{-3}$ $\longleftarrow$ ALLOWS FOR A NICE TEST OF THE SM

$\tilde{\Delta}_{CT} = 0.03$ $\longleftarrow$ $\sim M_K^2/M_\pi^2\, \Delta_{CT}$: UNUSUALLY SMALL FOR SU(3)$\times$ SU(3) BREAKING TO 1st ORDER

- TO TWO LOOPS: TWO LOW ENERGY CONSTANTS TO BE DETERMINED.

$$\Delta_{CT}|_{C_i} = -M_\pi^2/M_K^2\, \tilde{\Delta}_{CT}|_{C_i} = 16 F_\pi^4 (2C_{12} + C_{34}) M_\pi^2 (M_K^2 - M_\pi^2)$$

ESTIMATES FROM RESONANCE SATURATION + ISOSPIN BREAKING: $\Delta_{CT}^{K^0\pi^-} \leq 10^{-2}$

Bijnens and Ghorbani '08, Kastner & Neufeld '08

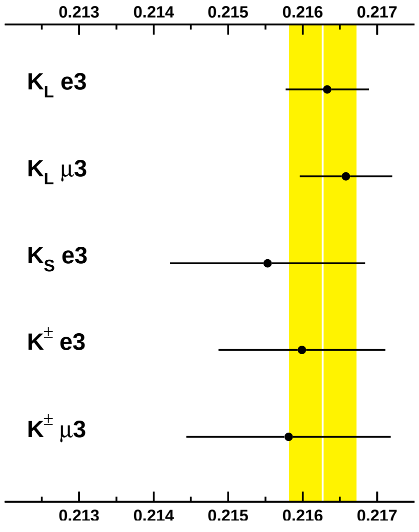
CALLAN TREIMAN POINT REVISITED in terms of MEASURED QUANTITIES

EXPERIMENTALLY: INFORMATION FROM SEMI-LEPTONIC DECAYS $\Rightarrow$ VALUE DEPENDS ON UNDERLYING THEORY FOR WEAK INTERACTIONS

IN THE SM:

$$\Rightarrow \bar{f}_0(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi}\left|\frac{V_{us}}{V_{ud}}\right| \frac{1}{\left|f_+^{K^0}(0)V_{us}\right|} V_{ud} + \Delta_{CT}$$

K_{l2}	$K_{\ell 3}$	$0^+ \to 0^+$ nuclear β decays
PDG	FLAWG	Towner & Hardy



- $f_+(0)V_{us} = 0.2163(5)$
- $V_{ud} = 0.97425(22)$

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IN THE SM:

$$\Rightarrow \bar{f}_0(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi}\left|\frac{V_{us}}{V_{ud}}\right| \frac{1}{|f_+^{K^0}(0) V_{us}|} V_{ud} + \Delta_{CT}$$

K_{l2}	$K_{\ell 3}$	$0^+ \to 0^+$ nuclear β decays
PDG	FLAWG	Towner & Hardy

$$\ln C|_{SM} = 0.2188 \pm 0.0035 + \Delta_{CT}$$

$$F_K/F_\pi|_{SM} = 1.192 \pm 0.006$$

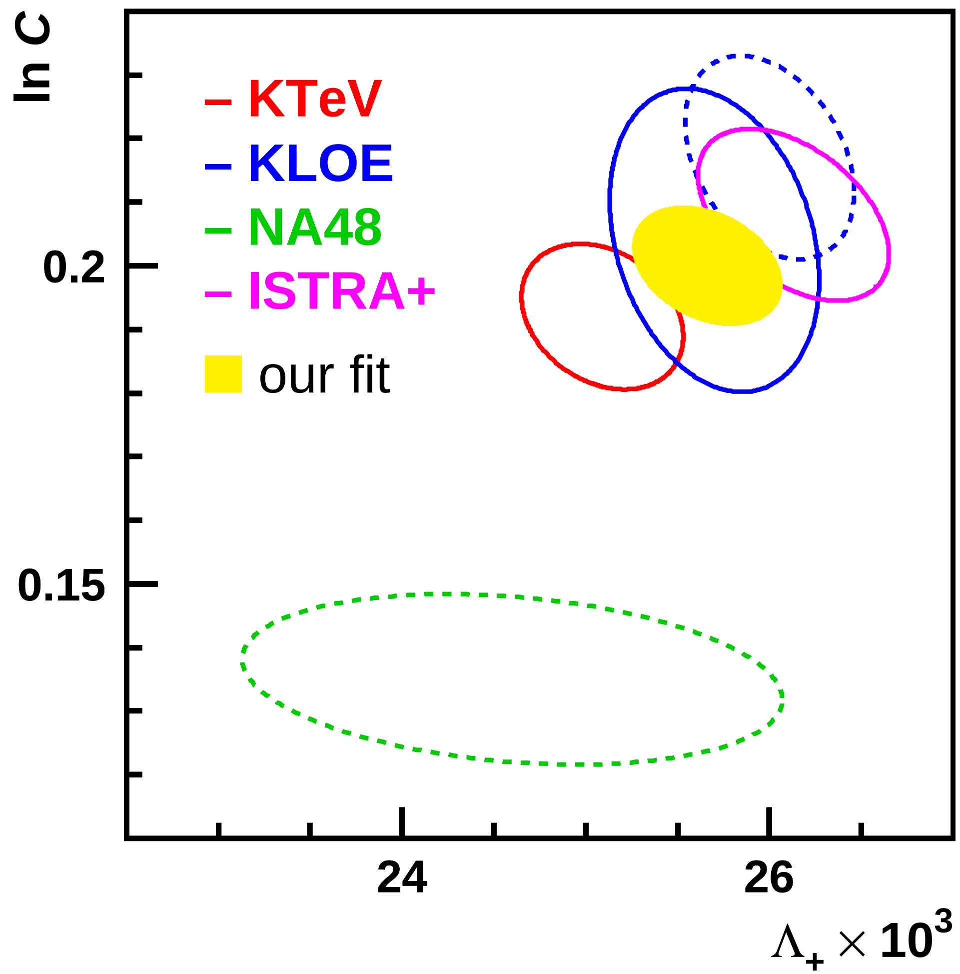
$$f_+(0)|_{SM} = 0.959 \pm 0.005$$

$$C \equiv \bar{f}_0(\Delta_{K\pi})$$

DISAGREEMENTS WITH THESE VALUES: TESTS OF NEW PHYSICS

charged Higgs (see for ex M. Antonelli et al '08), right handed currents (Stern et al) $\cdots$

EXPERIMENTALLY: M. Antonelli et al: arXiv:1005.2323



- Λ_+: slope of the vector form factor
- KLOE, KTeV, ISTRA(K^+) marginal/good agreement between them and with the SM
- NA48 4.5σ away
- FlaviaNet Working group : $\ln C = 0.2004 \pm 0.0091$

FORM FACTOR TO TWO LOOPS: J. Bijnens and P. Talavera: Nucl. Phys. B669(2003) 341

Introduce $\tilde{f}_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2}\left(f_-(t) + 1 - F_K/F_\pi\right) = f_0(t) + \frac{t}{m_K^2 - m_\pi^2}\left(1 - F_K/F_\pi\right)$

Advantage: NO DEPENDENCE ON LECs at $\mathcal{O}(p^4)$

$$\tilde{f}_0(t) = 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) - \frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)$$

- $\overline{\Delta}(t)$ and $\Delta(0)$ DEPEND ON F_π and LECs L_i at $\mathcal{O}(p^6)$
- at $t = 0$: $\tilde{f}_0(0) = f_+(0)$
  - Ademollo Gatto theorem + current conservation in SU(3) limit: $f_+(0) = 1 + \mathcal{O}(m_s - \hat{m})^2$

    $\longrightarrow$ no contributions from the $O(p^4) L_i$'s

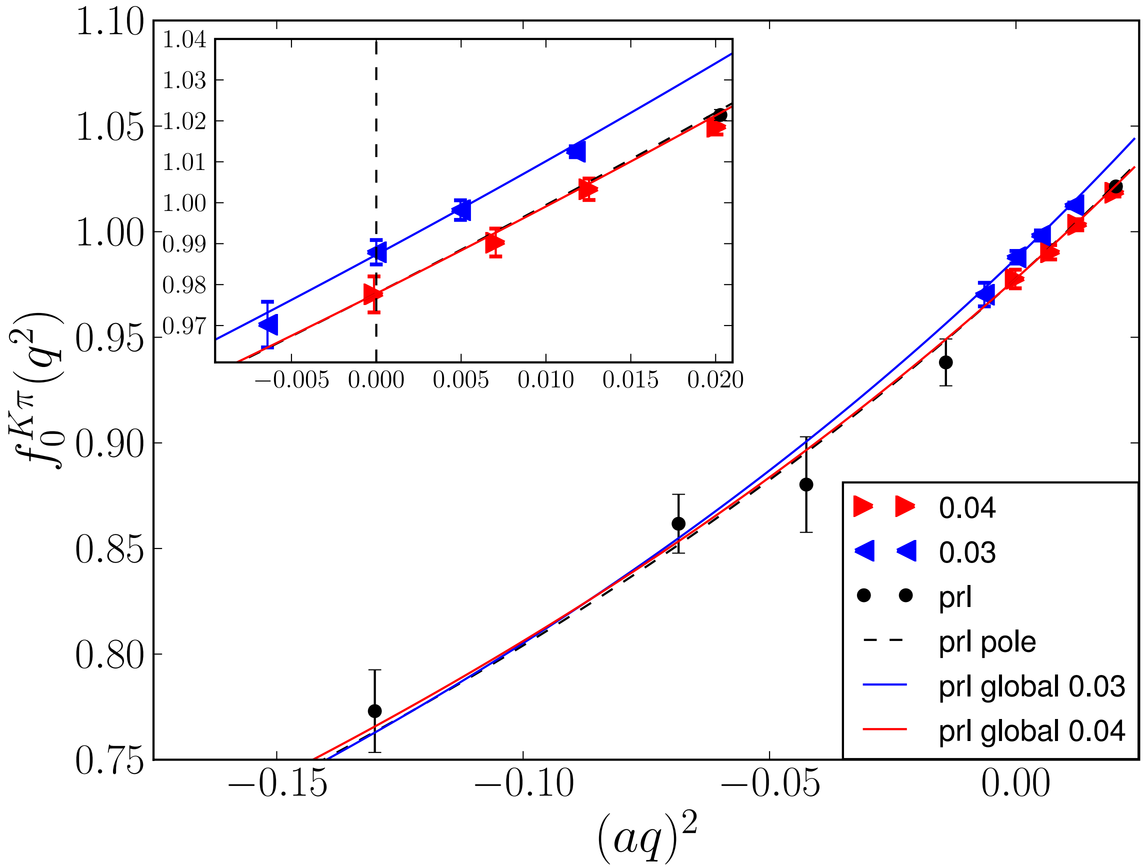
    $\longrightarrow$ NLO chiral logs fully determined in terms of M_K, M_π and F_π

    usual value at NLO: $f_{NLO} = -0.023$

    However depends on treatment of decay constant: F_0^2, F_π^2, $F_\pi F_K$?

    related to treatment of NNLO
  - NNLO: Logs known but one combination of 2 $O(p^6)$ LECs: $C_{12} + C_{34}$

LATTICE RESULTS



$\leftarrow t_{max} = (M_K - M_\pi)^2$

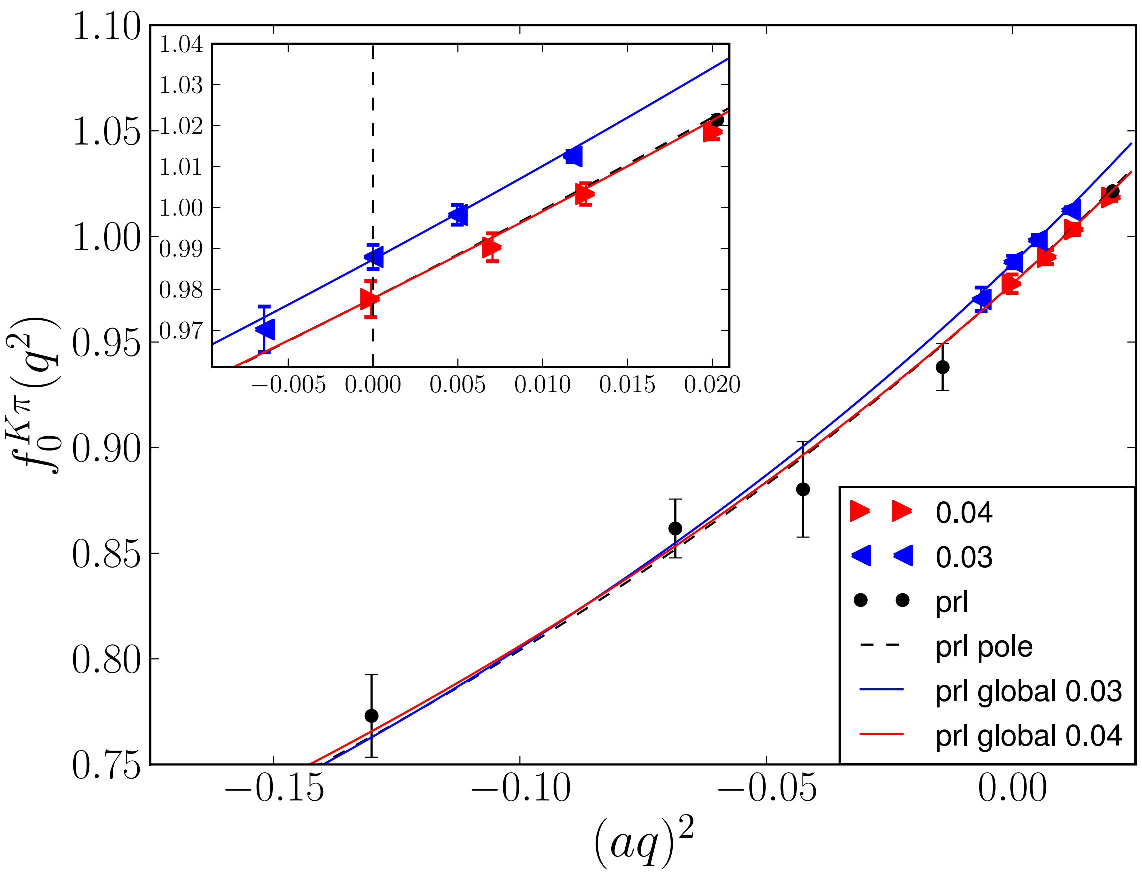
- Point at $t_{\max}$ very accuratelly determined from a double ratio of three point correlation functions

- $f_+(0) = 1 - 0.023 + \Delta f$

- CALCULATE AT DIFFERENT VALUES OF t AND DETERMINE Δf FROM EXTRAPOLATION

RBC/UKQCD: $$f_0^{K\pi}(q^2) = \frac{1+f_2+(m_K^2-m_\pi^2)^2\left(A_0+A_1(m_K^2+m_\pi^2)\right)}{1-q^2/\left(M_0+M_1(m_K^2+m_\pi^2)\right)^2}$$

LATTICE RESULTS



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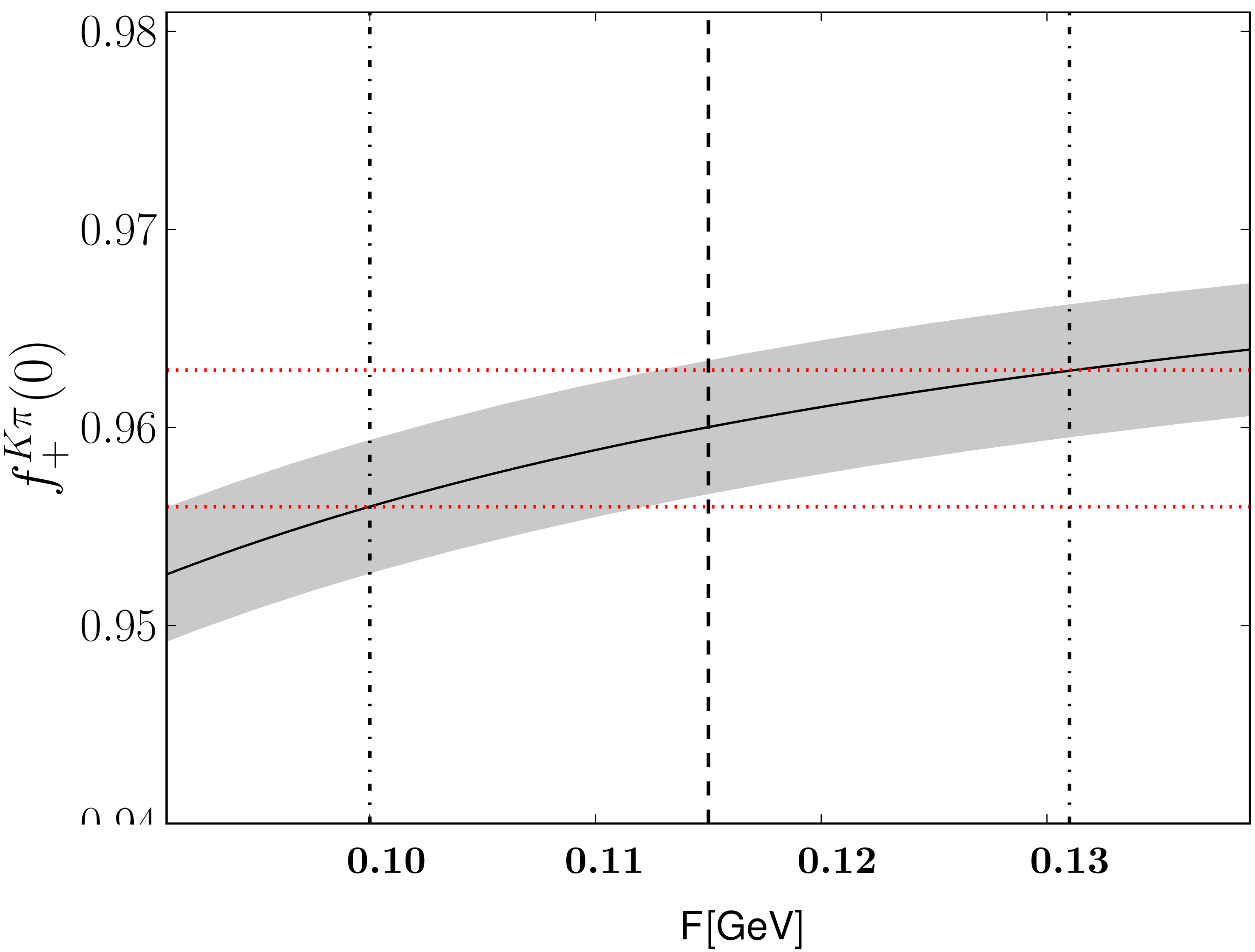
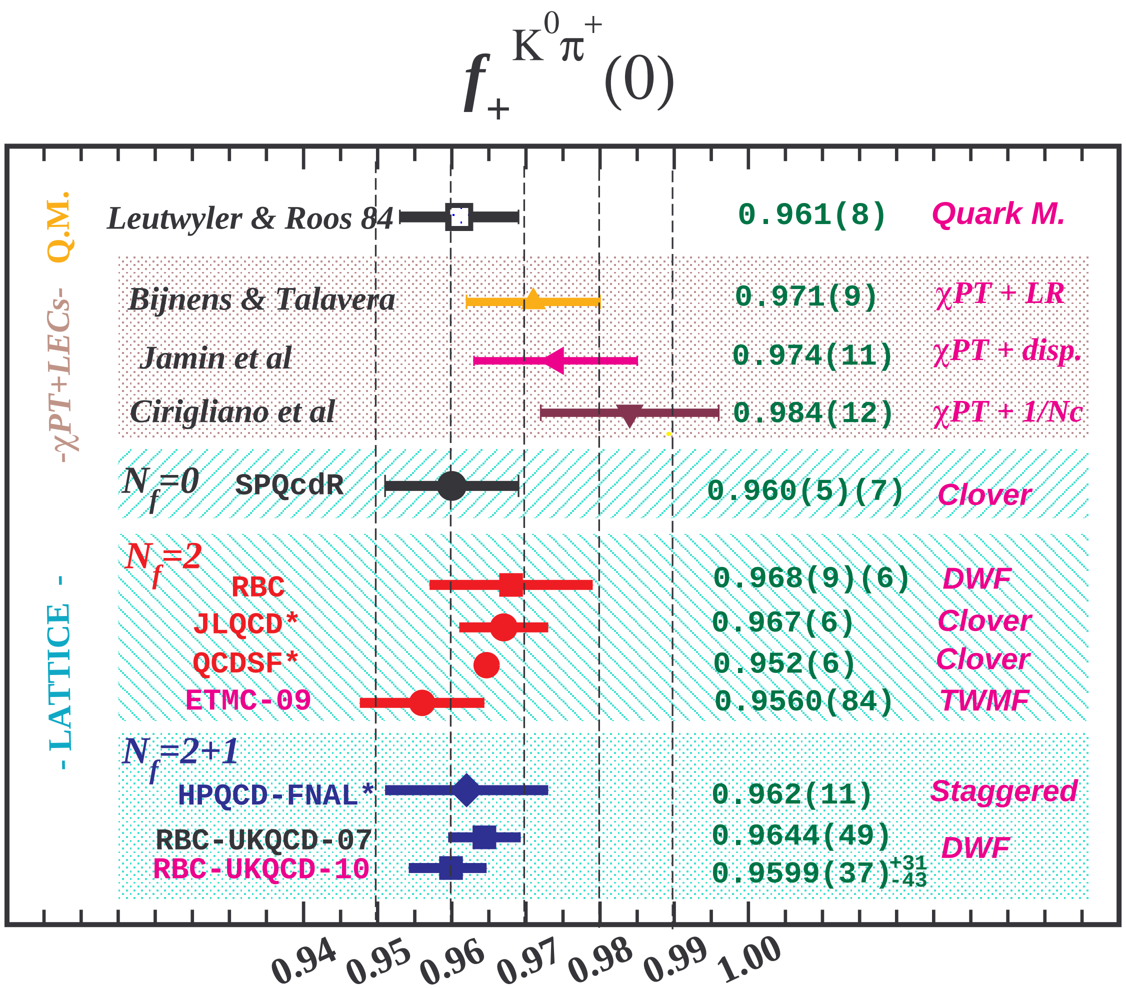
- CALCULATE AT DIFFERENT VALUES OF t AND DETERMINE Δf FROM EXTRAPOLATION

- Very recently: CALCULATION EXACTLY AT $q^2 = 0 \longleftarrow$ Twisted boundary conditions

ILLUSTRATION OF DEPENDENCE OF THE RESULTS ON CHOICE FOR F_π: Boyle et al '10

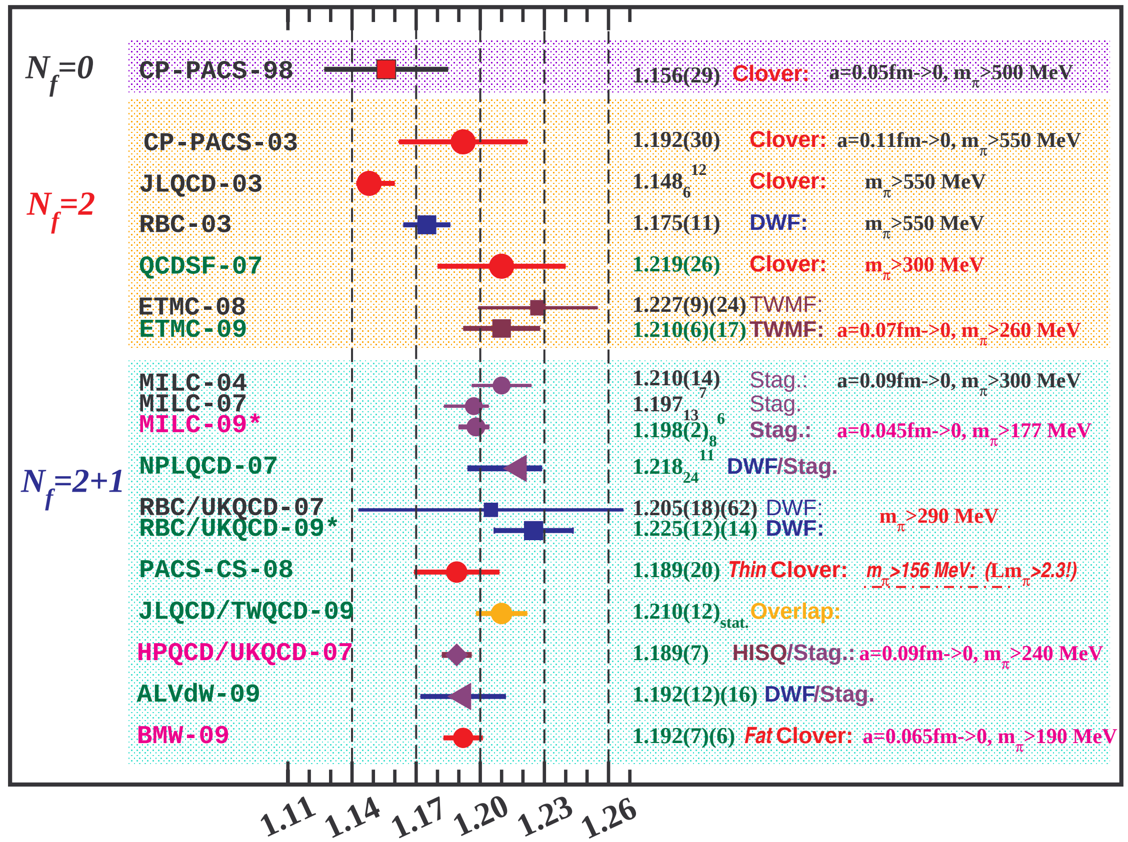


Here $F_\pi = 0.135$ GeV



- $f_+(0) = 1 - 0.023 + \Delta f$
- $\Delta f = -0.016(8) \longleftarrow$ ref. results of quark model from Leutwyler & Roos'85:
- $f_+(0)|_{SM} = 0.959 \pm 0.005$

f_K/f_π



- WHEN a LISTED $\longrightarrow$ EXTRAPOLATION TO CONTINUUM LIMIT DONE
- TWO TYPES OF LATTICE FERMIONS $\longrightarrow$ VALENCE AND SEA QUARKS DISCRETIZED DIFFERENTLY
- $F_K/F_\pi|_{SM} = 1.192 \pm 0.006$

RATIO OF KAON AND PION DECAY CONSTANTS TO TWO LOOPS

$$F_K/F_\pi = 1 + \frac{4}{F_\pi^2}(m_K^2 - m_\pi^2)L_5^r + \frac{8}{F_0^2}\Big[- m_\pi^4(C_{15}^r + 2C_{17}^r) + 2m_\pi^2 m_K^2\left(-(C_{14}^r + C_{15}^r) + \frac{1}{2}(C_{15}^r + 2C_{17}^r)\right) + 2m_K^4(C_{14} + C_{15}) + \mathbf{LOOPS}\Big]$$

PERFORM A FIT TO RBC/UKQCD:

VB and E. Passemar '09

only available SU(3) results for SCALAR FORM FACTOR

- Allton et al: Phys ReV D78 (2008) 114509; Boyle et al: JHEP 0705 (2007)
- Lattice data for 4 sets of π and K masses:

  (0.329,0.575) GeV, (0.416,0.604) GeV, (0.556,0.663) GeV, (0.671,0.719) GeV
- Five different values of t from ~ -0.4 GeV^2 to $t_{\rm max} = (M_K - M_\pi)^2$
- USE two lower mass sets and three smallest absolute values $t \leq 0.1$ GeV^2
- NO CORRECTION FROM FINITE VOLUME EFFECTS OR LATTICE ARTEFACTS

  $\longrightarrow$ ONLY STATISTICAL ERRORS INCLUDED

• L_i's

	Fit 10 set a	πK Roy Steiner	Prelim. Fit All(*) set b	Lattice
$10^3 L_1^r$	0.432	1.05 ± 0.12	0.99 ± 0.13	—
$10^3 L_2^r$	0.735	1.32 ± 0.03	0.60 ± 0.21	—
$10^3 L_3^r$	-2.35	-4.53 ± 0.14	-3.08 ± 0.47	—
$10^3 L_4^r$	0	0.53 ± 0.39	0.70 ± 0.66	$0.33(0.13)$
$10^3 L_5^r$	0.97	3.19 ± 2.40	0.56 ± 0.11	$0.93(0.073)$
$10^3 L_6^r$	0		0.14 ± 0.70	-
$10^3 L_7^r$	-0.31		-0.21 ± 0.15	-
$10^3 L_8^r$	0.6		0.38 ± 0.17	-
$10^3(2L_6^r - L_4^r)$				$0.032(0.062)$
$10^3(2L_8^r - L_5^r)$				$0.050(0.043)$

IN THE FOLLOWING USE L_is from FIT 10 set(a) and Prelim. Fit All set(b)

Fit 10: Amoros, Bijnens and Talavera: $\nu\phi$ B602 (2001) 87 (masses and K_{l4})

Prelim. Fit All: Bijnens and Jemos: arXiv:0909.4477 + talk at Flavianet '09 (as FIT 10 + πK scattering)

	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
$C_{12}\cdot 10^4$	5.77 ± 0.56	7.84 ± 0.58	4.81 ± 0.94	5.74 ± 0.95	4.69 ± 0.56	4.43 ± 0.88
$C_{34}\cdot 10^4$	2.54 ± 0.43	-1.28 ± 0.44	3.60 ± 0.96	1.07 ± 0.96	3.76 ± 0.43	3.50 ± 0.94
$C_{14}\cdot 10^4$	0^*	0^*	0.72 ± 1.37	0.71 ± 1.42	0.65^*	-0.93 ± 0.67
$C_{17}\cdot 10^4$	0^*	0^*	0.42 ± 3.31	1.92 ± 3.36	0.31^*	4.16 ± 1.56
F_0	89.8 ± 0.1	69.2 ± 0.0	89.8 ± 0.1	69.3 ± 0.0	89.8^*	89.8 ± 0.1
$f_+(0)$	0.956	0.963	0.956	0.961	0.956	0.958
F_K/F_π	1.20	1.19	1.20	1.19	1.20	1.19
$\ln C$	0.22	0.20	0.22	0.21	0.22	0.21
$f_0(\tilde{\Delta}_{K\pi})$	0.75	0.75	0.75	0.76	0.75	0.77
$10^3\Delta_{CT}$	1.00	-2.14	0.27	-3.65	0.18	-0.32
$10^2\tilde{\Delta}_{CT}$	-9.00	-9.86	-8.24	-8.18	-8.11	-7.03
$10^3\lambda_0$	18.08	17.77	18.24	17.66	18.18	16.71
χ^2	1.40/4	0.96/4	1.67/4	1.29/4	3.01/4	4.8/7

- BIJNENS CODE
- FIT OF f_0 FIT I/II 3 PARAMETERS FIT different sets of L_i's sets (a)/(b)
- COMBINED FIT OF f_0 and F_K/f_π

  FIT III/IV 5 PARAMETERS FIT (a)/(b)

  FIT VI set(a) + (0.556,0.663) GeV from LATTICE
- COMBINED FIT OF $\tilde{f}_0$ and F_K/f_π

  FIT V 2 PARAMETERS FIT set (a)

	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
$C_{12} \cdot 10^4$	5.77 ± 0.56	7.84 ± 0.58	4.81 ± 0.94	5.74 ± 0.95	4.69 ± 0.56	4.43 ± 0.88
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$C_{14} \cdot 10^4$	0^*	0^*	0.72 ± 1.37	0.71 ± 1.42	0.65^*	-0.93 ± 0.67
$C_{17} \cdot 10^4$	0^*	0^*	0.42 ± 3.31	1.92 ± 3.36	0.31^*	4.16 ± 1.56
F_0	89.8 ± 0.1	69.2 ± 0.0	89.8 ± 0.1	69.3 ± 0.0	89.8^*	89.8 ± 0.1
$f_+(0)$	0.956	0.963	0.956	0.961	0.956	0.958
F_K/F_π	1.20	1.19	1.20	1.19	1.20	1.19
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$10^3 \lambda_0$	18.08	17.77	18.24	17.66	18.18	16.71
χ^2	1.40/4	0.96/4	1.67/4	1.29/4	3.01/4	4.8/7

- VERY GOOD χ^2
- F_0/F_π SMALL FOR FIT II and IV since L_4 LARGE

$\Longrightarrow$ ERRORS IN REPLACING F_0 BY F_π AS USUALLY DONE ?

$$O = O_{LO} + O_{NLO}/F_\pi^2 + O_{NNLO}/F_0^4 \longrightarrow O_{LO} + O_{NLO}/F_\pi^2 + O_{NNLO}/F_\pi^4$$

same up to higher order terms

	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
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$C_{14} \cdot 10^4$	0^*	0^*	0.72 ± 1.37	0.71 ± 1.42	0.65^*	-0.93 ± 0.67
$C_{17} \cdot 10^4$	0^*	0^*	0.42 ± 3.31	1.92 ± 3.36	0.31^*	4.16 ± 1.56
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$f_0(\tilde{\Delta}_{K\pi})$	0.75	0.75	0.75	0.76	0.75	0.77
$10^3 \Delta_{CT}$	1.00	-2.14	0.27	-3.65	0.18	-0.32
$10^2 \tilde{\Delta}_{CT}$	-9.00	-9.86	-8.24	-8.18	-8.11	-7.03
$10^3 \lambda_0$	18.08	17.77	18.24	17.66	18.18	16.71
χ^2	1.40/4	0.96/4	1.67/4	1.29/4	3.01/4	4.8/7

- SCALAR FORM FACTOR with $\Delta S = 0$ and $\Delta S = 1$: C_{12} and C_{34} between -10^{-3} and 10^{-4}
- C_i: RESONANCE SATURATION $\longrightarrow$ $C_{14} \sim c_d c_m d_m / M_S^4 \sim -4.3 \cdot 10^{-3}$ GeV^{-2}
- C_i: G. Ecker, H. Neufeld and P. Masjuan, '10: BMW LATTICE DATA + F_K/F_π + SOME APPROX.

$(C_{14} + C_{15}) \cdot 10^3 = 0.37 \pm 0.08$ GeV^{-2} $\quad$ $(C_{15} + 2C_{17}) \cdot 10^3 = 1.29 \pm 0.16$ GeV^{-2}

$(C_{14} + C_{15}) \cdot 10^3 = 0.20 \pm 0.07$ GeV^{-2} $\quad$ $(C_{15} + 2C_{17}) \cdot 10^3 = 0.71 \pm 0.15$ GeV^{-2}

- QUARK MODEL: $C_{15} = 0$, $C_{14} = -8.3 \cdot 10^{-4}$ GeV^{-2}, $C_{17} = 0.1 \cdot 10^{-4}$ GeV^{-2}

	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
$C_{12}\cdot 10^4$	5.77 ± 0.56	7.84 ± 0.58	4.81 ± 0.94	5.74 ± 0.95	4.69 ± 0.56	4.43 ± 0.88
$C_{34}\cdot 10^4$	2.54 ± 0.43	-1.28 ± 0.44	3.60 ± 0.96	1.07 ± 0.96	3.76 ± 0.43	3.50 ± 0.94
$C_{14}\cdot 10^4$	0^*	0^*	0.72 ± 1.37	0.71 ± 1.42	0.65^*	-0.93 ± 0.67
$C_{17}\cdot 10^4$	0^*	0^*	0.42 ± 3.31	1.92 ± 3.36	0.31^*	4.16 ± 1.56
F_0	89.8 ± 0.1	69.2 ± 0.0	89.8 ± 0.1	69.3 ± 0.0	89.8^*	89.8 ± 0.1
$f_+(0)$	0.956	0.963	0.956	0.961	0.956	0.958
F_K/F_π	1.20	1.19	1.20	1.19	1.20	1.19
$\ln C$	0.22	0.20	0.22	0.21	0.22	0.21
$f_0(\tilde\Delta_{K\pi})$	0.75	0.75	0.75	0.76	0.75	0.77
$10^3\Delta_{CT}$	1.00	-2.14	0.27	-3.65	0.18	-0.32
$10^2\tilde\Delta_{CT}$	-9.00	-9.86	-8.24	-8.18	-8.11	-7.03
$10^3\lambda_0$	18.08	17.77	18.24	17.66	18.18	16.71
χ^2	$1.40/4$	$0.96/4$	$1.67/4$	$1.29/4$	$3.01/4$	$4.8/7$

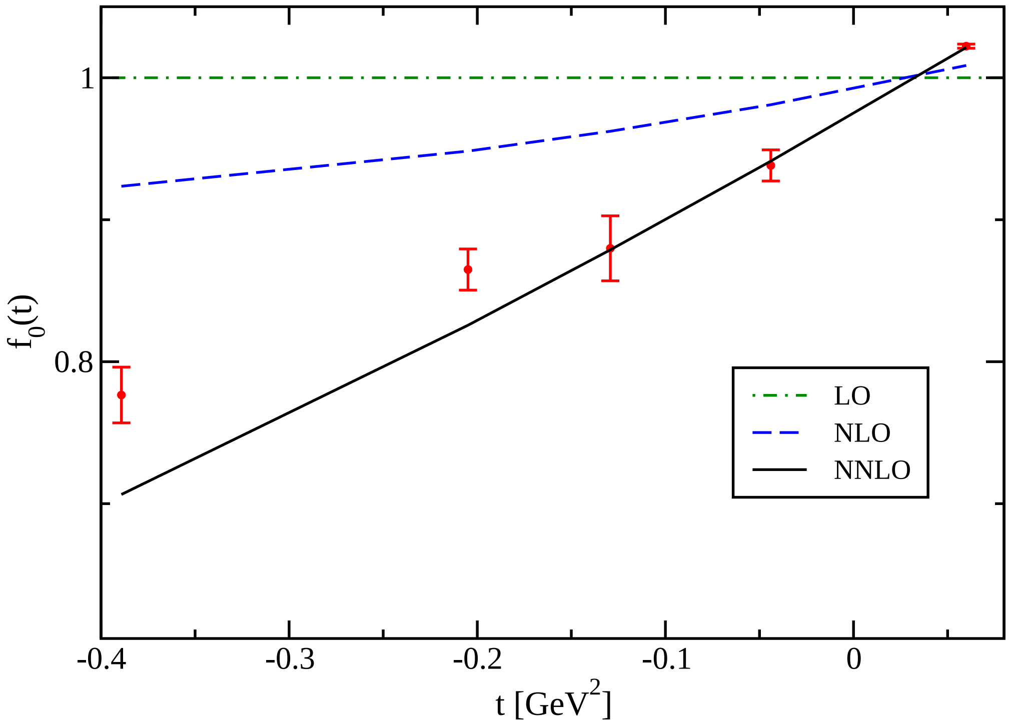
- COMPATIBLE WITH SM
- FROM DISPERSIVE REP. $\lambda_0 = \frac{M_\pi^2}{M_K^2-M_\pi^2}(\ln C - G(0))$ $\quad G(0) = 0.0398 \pm 0.0044$ (VB et al '06)

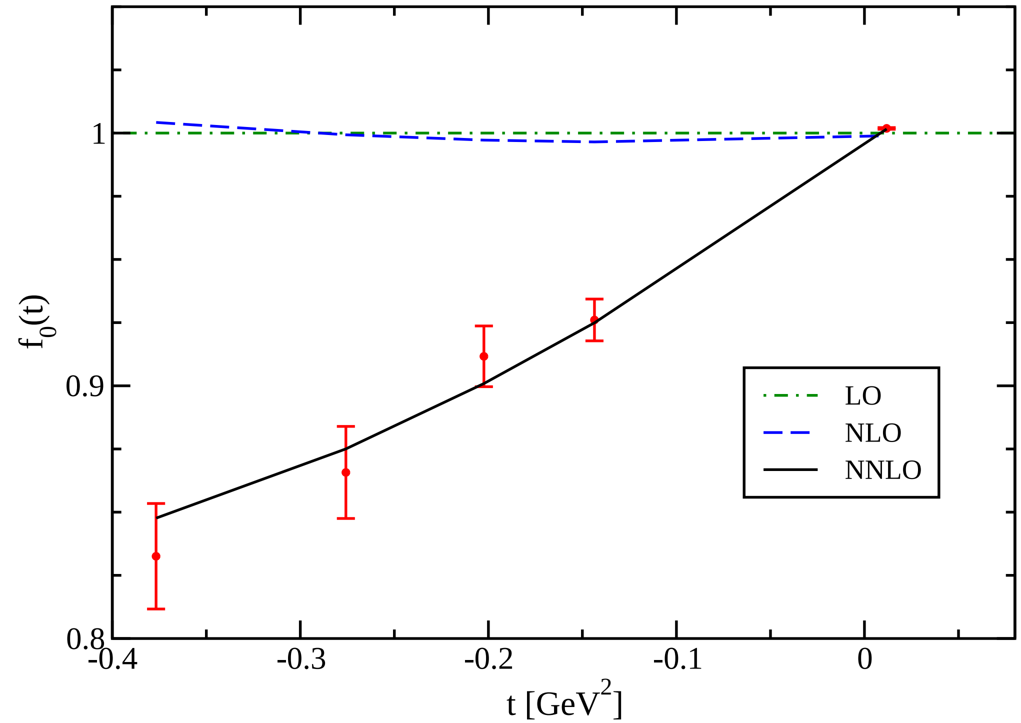
  FLAWG: $\lambda_0 = (15.94 \pm 0.79)10^{-3}$ (no NA48) and $\lambda_0' \sim 6\cdot 10^{-4}$

  SLOPE TOO BIG $\longrightarrow$ COMBINATION $2C_{12}+C_{34}$ TOO BIG

  $$\lambda_0 = \frac{M_\pi^2}{M_K^2-M_\pi^2}\left(\frac{F_K}{F_\pi}-1\right) + 8\frac{M_\pi^2}{F_\pi^4}(2C_{12}+C_{34})(M_K^2+M_\pi^2) + M_\pi^2\frac{d}{dt}\bar\Delta(t)$$

  HOWEVER CURVATURE 5 TIMES TOO SMALL: C_{12} TOO LARGE POSITIVE





$M_\pi = 329$ MeV, $M_K = 575$ MeV $\qquad$ $M_\pi = 556$ MeV, $M_K = 663$ MeV

- LATTICE: VERY ACCURATE DETERMINATION AT $t_{\mathbf{max}} = (M_K - M_\pi)^2$
- FIT ONLY OF THE THREE POINTS OF LARGEST VALUES $\rightarrow$ PREDICTIONS OTHERWISE
- CONVERGENCE WORSENS AS $|t|$ INCREASES

CONVERGENCES

AT PHYSICAL POINTS

Fit(III)

$$f_+(0) = 1 - 0.019 - 0.026 + \ldots$$
$$F_K/F_\pi = 1 + 0.140 + 0.061 + \ldots$$
$$f_0(\Delta_{K\pi}) = 1 + 0.139 + 0.063 + \ldots$$
$$\Delta_{CT} = 0 - 0.0025 + 0.0028 + \ldots$$
$$\tilde{\Delta}_{CT} = 0 + 0.024 - 0.106 + \ldots$$

Fit(IV)

$$f_+(0) = 1 - 0.019 - 0.019 + \ldots$$
$$F_K/F_\pi = 1 + 0.113 + 0.081 + \ldots$$
$$f_0(\Delta_{K\pi}) = 1 + 0.110 + 0.081 + \ldots$$
$$\Delta_{CT} = 0 - 0.0033 - 0.0003 + \ldots$$
$$\tilde{\Delta}_{CT} = 0 + 0.021 - 0.103 + \ldots$$

- AT FIRST SIGHT NOT VERY GOOD

  F_K/F_π and $f_0(\Delta_{K\pi})$ WORSE FOR set(b) where SMALL F_0

- BUT SYMMETRY ARGUMENTS: Ademollo Gato, Callan Treiman ,$\cdots$

- AND NNLO of the EXPECTED SIZE

DECOMPOSITION OF $f_+(0)$ set(b):

$$f_+(0) = 1 + (-0.019 + 0.000) + (0.012 + 0.002 - 0.033)$$

LARGE CONTRIBUTION FROM C_i: MOCK UP SOME HIGHER ORDER EFFECTS ?

CONVERGENCES

AT PHYSICAL POINTS

Fit(III)

$$f_+(0) = 1 - 0.019 - 0.026 + \ldots$$

$$F_K/F_\pi = 1 + 0.140 + 0.061 + \ldots$$

$$f_0(\Delta_{K\pi}) = 1 + 0.139 + 0.063 + \ldots$$

$$\Delta_{CT} = 0 - 0.0025 + 0.0028 + \ldots$$

$$\tilde{\Delta}_{CT} = 0 + 0.024 - 0.106 + \ldots$$

Fit(IV)

$$f_+(0) = 1 - 0.019 - 0.019 + \ldots$$

$$F_K/F_\pi = 1 + 0.113 + 0.081 + \ldots$$

$$f_0(\Delta_{K\pi}) = 1 + 0.110 + 0.081 + \ldots$$

$$\Delta_{CT} = 0 - 0.0033 - 0.0003 + \ldots$$

$$\tilde{\Delta}_{CT} = 0 + 0.021 - 0.103 + \ldots$$

- AT FIRST SIGHT NOT VERY GOOD

  F_K/F_π and $f_+(0)$ WORSE FOR set(b) where SMALL F_0
- BUT SYMMETRY ARGUMENTS: Ademollo Gato, Callan Treiman ,$\cdots$
- AND NNLO of the EXPECTED SIZE
- CONVERGENCE WORSENS AS PSEUDOSCALAR MASSES INCREASE

CONCLUSIONS

- GOOD FITS AND RESULTS IN GOOD AGREEMENT WITH STANDARG COUPLINGS OF QUARKS TO W
- CONVERGENCE NOT OPTIMUM
- WHAT IF WE CONSIDER OTHER OBSERVABLES AS MASSES AND PION DECAY CONSTANTS?

WHICH SCENARIO FOR $N_f = 2$ and $N_f = 3$?

CONDENSATE: $\Sigma(N_f) = \lim_{m_{i,i=1..n_f} \to 0} -\langle 0|\bar{u}u(x)|0\rangle$

$$\Sigma(2) = \Sigma(3) + m_s^{phys} \lim_{m_u, m_d \to 0} i \int d^4x < 0|\bar{u}u(x)\bar{s}s(0)|0 > +\mathcal{O}(m_s^2)$$

STRANGE SEA QUARKS EFFECT (scalar $1/N_c$ suppressed)

CHPT: $\Sigma(2) - \Sigma(3) \propto m_s L_6$, $F^2(2) - F^2(3) \propto m_s L_4$

STANDARD: ZWEIG RULE OK $L_4, L_6 \sim 0$

HOWEVER IN THE SCALAR SECTOR STRONG INDICATIONS THAT ZWEIG RULE AND LARGE N_c BADLY VIOLATED

VIOLATION OF ZWEIG RULE AND LARGE N_c

- LOW ENERGY πK scattering (Büttiker et al '04 + NEW FITS Bijnens et al '09)
- DISPERSIVE ESTIMATES OF SCALAR FORM FACTORS (Moussallam'99 & '00)
- J/Ψ DECAY INTO A VECTOR MESON AND TWO PSEUDOSCALARS

- but L_6 compatible with zero (Lahde and Meißner '06)

- LATTICE with 2+1 :

  MILC: $\Sigma(2)/\Sigma(3) \sim 1.52$

  PROBLEM TO FIT WITH STANDARD CHPT AT NLO (NNLO?)

POSSIBILITY OF WEAK CONVERGENCE OF THE CHIRAL SERIE

SOME WAYS OUT

- USE DIFFERENT REGULARIZATIONS
- HEAVY KAON CHPT, Roessl '99 $\rightsquigarrow$ Kaon treated as a heavy source
- RESUMMED CHPT, S. Descotes-Genon et al, '04

RESUMMED CHPT

RULES

- DEFINE "GOOD" OBSERVABLES $\rightarrow$ QCD CORRELATORS ARE NATURAL CANDIDATES
- WRITE THEM IN TERMS OF THREE PARAMETERS RELEVANT FOR SU(3) CHIRAL DYN.

$X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}$ $\quad Z(3) = \frac{F^2(3)}{F_\pi^2}$ $\quad r = \frac{m_s}{m}$ use instead of X(3): $Y(3) = \frac{2mB_0}{M_\pi^2} = \frac{X(3)}{Z(3)}$

- PSEUDOSCALAR MASSES AT LO $\rightarrow$ PHYSICAL ONES ONLY WHEN IMPOSED BY PHYSICAL PRINCIPLE $\rightarrow$ UNITARITY
- KEEP THE REMAINDER: $O = O_{LO} + O_{NLO} + O_{rem}$

  IF GOOD OBSERVABLES WELL DEFINED: O_{rem} SMALL
- EXPRESS THE LECs IN TERMS OF KNOWN PHYSICAL QUANTITIES,

  L_4 & $L_5 \rightarrow F_\pi$ & F_K $\quad L_6$ & $L_8 \rightarrow F_\pi^2 M_\pi^2$ & $F_K^2 M_K^2$

  $X(3), Z(3), r$ AND REMAINDERS $\quad L_i = \mathcal{F}(X(3), Z(3), r,$ OBSERV., remainders)

RECOVER "STANDARD" CHPT IN CERTAIN LIMITS

STANDARD vs RESUMMED

FROM M_P^2 AND $F_P^2 M_P^2$, $(P = K, \pi)$

$$\begin{aligned} X(3) &= 1 - \epsilon(r) - k[Y(3)]^2 2\Delta L_6 - d \\ Z(3) &= 1 - \eta(r) - kY(3)\Delta L_4 - e \end{aligned}$$

$$\epsilon(r) = 2\frac{r_2 - r}{r^2 - 1}, \quad r_2 = 2\left(\frac{F_K M_K}{F_\pi M_\pi}\right)^2 - 1 \sim 36\,, \quad \eta(r) = \frac{2}{r-1}\left(\frac{F_K^2}{F_\pi^2} - 1\right)\,,$$

$$\Delta L_i = L_i + \text{chiral logs} \qquad k = 32\frac{M_\pi^2}{F_\pi^2}(r+2)$$

$$\begin{aligned} X(3) &= 1 - 0.04 - 950Y(3)^2\Delta L_6 + \ldots \\ Z(3) &= 1 - 0.04 - 475Y(3)\Delta L_4 + \ldots \end{aligned} \qquad r \sim 25$$

- STANDARD CHPT $L_4^r = L_6^r \sim 0 \longrightarrow X(3) \sim 1$, $Z(3) \sim 1$, $\Sigma(3) \sim \Sigma(2)$ and $F(3) \sim F(2)$
- LARGE PREFACTOR $\rightsquigarrow$ SMALL DEVIATION OF ΔL_6 and ΔL_4 FROM ZERO LEADS TO DEVIATION FROM 1

STANDARD vs RESUMMED

FROM M_P^2 AND $F_P^2 M_P^2$, $(P = K, \pi)$

$$\begin{aligned} X(3) &= 1 - \epsilon(r) - k[Y(3)]^2 2\Delta L_6 - d \\ Z(3) &= 1 - \eta(r) - kY(3)\Delta L_4 - e \end{aligned}$$

$$\epsilon(r) \sim 2\frac{36 - r}{r^2 - 1} \sim 10^{-2}, \quad \eta(r) = \frac{2}{r-1}\left(\frac{F_K^2}{F_\pi^2} - 1\right) \sim 10^{-2}, \quad k \sim 18(r+2) \sim 5 \cdot 10^3$$

$$Y(3) = \frac{X(3)}{Z(3)} = \frac{2[1 - \epsilon(r) - d]}{1 - \eta(r) - e + \sqrt{[1 - \eta(r) - e]^2 + k[2\Delta L_6 - \Delta L_4][1 - \epsilon(r) - d]}}$$

- STANDARD CHPT: SMALL FLUCTUATIONS, $L_4^r = L_6^r \sim 0 \to Y(3) \sim 1$
- IF LARGE FLUCTUATIONS WITH $\Delta L_4 \neq 2\Delta L_6$ THEN DUE TO LARGE PREFACTOR $Y(3)$, $X(3)$ and $Z(3)$ DIFFERS FROM 1
- IF LARGE FLUCTUATIONS AND $\Delta L_4 \sim 2\Delta L_6$ THEN $Y(3) \sim 1$ BUT NOT $X(3)$ and $Z(3)$

STANDARD vs RESUMMED

FROM M_P^2 AND $F_P^2 M_P^2$, $(P = K, \pi)$

$$\begin{aligned} X(3) &= 1 - \epsilon(r) - k[Y(3)]^2 2\Delta L_6 - d \\ Z(3) &= 1 - \eta(r) - kY(3)\Delta L_4 - e \end{aligned}$$

$$\epsilon(r) \sim 2\frac{36-r}{r^2-1} \sim 10^{-2}, \quad \eta(r) = \frac{2}{r-1}\left(\frac{F_K^2}{F_\pi^2} - 1\right) \sim 10^{-2}, \quad k \sim 18(r+2) \sim 5 \cdot 10^3$$

$$Y(3) = \frac{X(3)}{Z(3)} = \frac{2[1-\epsilon(r)-d]}{1-\eta(r)-e+\sqrt{[1-\eta(r)-e]^2 + k[2\Delta L_6 - \Delta L_4][1-\epsilon(r)-d]}}$$

- $X(3)$, $Z(3)$, $Y(3) \sim 1 \rightsquigarrow$ PERTURBATIVE TREATMENT FINE
- $X(3)$, $Z(3)$, $Y(3)$ SMALL $\rightsquigarrow$ RESUMMATION OF VACUUM FLUCTUATIONS NECESSARY

Example: VECTOR FORM FACTOR

$$F_\pi F_K f_+(t) = \frac{F_\pi^2+F_K^2}{2} + \frac{3}{2}[tM^r_{K\pi}(t) + tM^r_{K\eta}(t) - L_{K\pi}(t) - L_{K\eta}(t)]$$
$$+ 2t\Big(\frac{1}{32\pi^2}\left[\frac{1}{6}\log\frac{\overset{\circ}{M}{}_\pi^2}{\mu^2} + \frac{1}{12}\log\frac{\overset{\circ}{M}{}_K^2}{\mu^2}\right] + \frac{F_\pi^2}{12}\langle r^2\rangle_V^\pi[1-e_V^\pi] + \frac{1}{32\pi^2}\Big[\frac{1}{12}+\frac{1}{9}Y(3)$$
$$+ \frac{M_\pi^2}{36M_K^2}(r+1)Y(3)\Big]\Big)$$
$$+ F_\pi F_K d_+ + te_+$$

- dependance on L_4 and L_5 in F_π and F_K
- $M(t)$, $L(t)$: USUAL ONE LOOP SCALAR UNITARITY INTEGRALS

  INVOLVE LOOP FUNCTION $\bar{J}_{PQ}(t)$: $\longrightarrow$ cuts at physical masses
- CONTRIBUTION FROM $L_9 \rightarrow \langle r^2\rangle_V^\pi$: e.m. square radius, exp: $\langle r^2\rangle_V^\pi = 0.451 \pm 0.031$ fm^2
- REMAINDERS
- Similar results for $F_K F_\pi f_0$

  $\longrightarrow$ CALLAN-TREIMAN THEOREM SATISFIED

WHAT ABOUT THE GELL-MANN OKUBO RELATION: $M_\eta^2 = (4M_K^2 - M_\pi^2)/3$

- IN NATURE SMALL DEVIATION $\Delta_{\rm GMO} = -3$ (in units of M_π^2)
- NATURALLY EXPLAINED IN STANDARD CHPT:

  EXACT AT LO, NLO IN PRINCIPLE SUPPRESSED COMPARED TO LO
- HOWEVER NEED A FINE TUNING OF THE LEC's

$$\Delta_{\rm GMO} = \mathbf{LOOPS} + \mathbf{LECS}$$
$$\sim -0.7 + 20 \cdot 10^3 (2L_7 + L_8 - L_5/6))$$

LEC's of typical order $10^{-3} \rightsquigarrow \Delta_{\rm GMO}$ TOO LARGE $\longrightarrow (2L_7 + L_8)$ SMALL

- IN RESUMMED CHPT RELEVANT QUANTITY:

$$F_\eta^2 M_\eta^2 = (4F_K^2 M_K^2 - F_\pi^2 M_\pi^2)/3$$

- IN NATURE ALSO WELL SATISFIED: $\tilde{\Delta}_{\rm GMO} = 4$ (in units of $F_\pi^2 M_\pi^2$)

  EXACT AT LO

  NLO:
$$\tilde{\Delta}_{\rm GMO} = 16 \frac{M_\pi^2}{F_\pi^2} (r-1)^2 Y(3)^2 (2L_7 + \Delta L_8) + d_{GO}$$
$$\sim 20 \cdot 10^3 Y(3)^2 (2L_7 + \Delta L_8) + d_{GO}$$

- MAGNITUDE DEPENDS ON Y(3).
- IN PRACTICE USE THE GMO TYPE RELATION TO DETERMINE L_7:

FIT TO LATTICE DATA: VB, S. Descotes-Genon and G. Toucas, '10

- LATTICE HAVE DIFFERENT QUARK MASSES

  INTRODUCE $p = \frac{\tilde{m}_s}{m_s}, \; q = \frac{\tilde{m}}{\tilde{m}_s}$

- REMAINDERS: $O_{\rm rem}$

  NNLO estimates: $\mathcal{O}(m_s^2)$ and $\mathcal{O}(m m_s)$

  MUST BE SCALED ACCORDINGLY WHEN USE OF LATTICE DATA:

$$O_{\rm rem} \sim \mathcal{O}(m_s^2) \to \tilde{O}_{\rm rem} = p^2 O_{\rm rem}$$

- USE OF THE GELL-MANN OKUBO TYPE RELATION FOR THE η

$$\tilde{F}_\eta^2 = \frac{4}{3}\tilde{F}_K^2 - \frac{1}{3}\tilde{F}_\pi^2 \,, \qquad \tilde{F}_\eta^2 \tilde{M}_\eta^2 = \frac{4}{3}\tilde{F}_K^2 \tilde{M}_K^2 - \frac{1}{3}\tilde{F}_\pi^2 \tilde{M}_\pi^2$$

- FIT $F_P, \; M_P, \; f_0(t), (P = \pi, K)$

- OBTAIN: $\mathcal{O}(p^4)$ LEC's, CURRENT MASSES, CONDENSATE AND DECAY CONSTANT IN CHIRAL LIMIT, SU(2) RELATED QUANTITIES, $f_+(0)$, Δ_{CT}, $\tilde{\Delta}_{CT}$

- 9 (11) FIT PARAMETERS

  $r, \; X(3)\,\text{or}\,Y(3), \; Z(3), \; F_K/F_\pi, \; \tilde{m}_{s,ref}/m_s$ + 4(6) REMAINDERS

- USE 2+1 SIMULATION BY RBC/UKQCD AND PACS-CS

PACS-CS	RBC/UKQCD
$\mathcal{O}$(a)-improved Wilson quark	Domain wall fermions
1 spacing, 1 volume	1 spacing, 2 volumes
One-loop perturbative renormalization	Non-perturbative renormalization
$F_P, \ M_P$	$F_P, \ M_P, \ f_0(t)$
POOR FITS WITH NLO SU(3) χPT	POOR FITS WITH NLO SU(3) for $F_P, \ M_P$
USE SU(2) χPT	USE SU(2) χPT
$(0.156, 0.553), (0.296, 0.594), (0.384, 0.581)$	$(0.329, 0.575), (0.416, 0.604)$
NO RESTRICTION ON REMAINDERS	REMAINDERS RESTRICTED TO 10%

RESULTS: DEVIATION FROM STANDARD χPT NOT IMPOSED

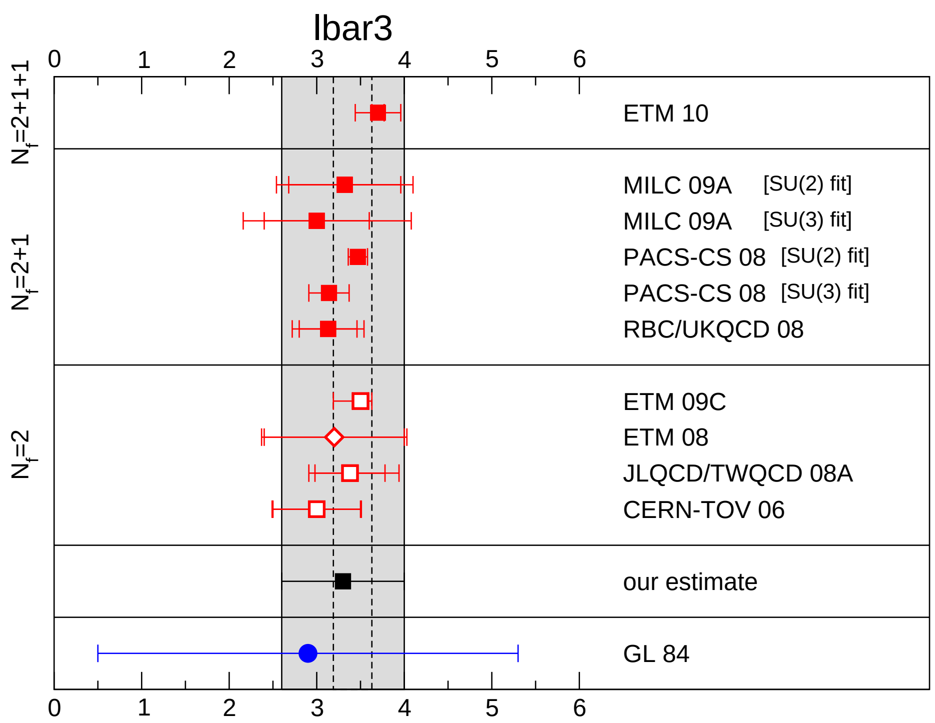
	PACS – CS Subset Without $K_{\ell 3}$	RBC/UKQCD Subset With $K_{\ell 3}$
r	26.5 ± 2.3	23.2 ± 1.5
$X(3)$	0.59 ± 0.21	0.20 ± 0.14
$Y(3)$	0.90 ± 0.22	0.43 ± 0.30
$Z(3)$	0.66 ± 0.09	0.46 ± 0.04
F_K/F_π	1.237 ± 0.025	1.148 ± 0.015
$\tilde{m}_{s,ref}/m_s$	1.24 ± 0.08	$1.15^\star$
$m_s(2\text{ GeV})[\text{MeV}]$	70 ± 4	107
$m(2\text{ GeV})[\text{MeV}]$	2.6 ± 0.3	4.6 ± 0.3
$B_0(2\text{ GeV})[\text{GeV}]$	3.34 ± 1.18	0.92 ± 0.67
$F_0[\text{MeV}]$	74.8 ± 4.9	62.2 ± 2.5
$L_4(\mu) \cdot 10^3$	-0.1 ± 0.2	2.4 ± 2.0
$L_5(\mu) \cdot 10^3$	1.8 ± 0.4	1.8 ± 1.6
$L_6(\mu) \cdot 10^3$	0.1 ± 0.4	4.7 ± 7.1
$L_8(\mu) \cdot 10^3$	0.8 ± 0.7	4.4 ± 7.1
$L_9(\mu) \cdot 10^3$	$\times$	4.4 ± 2.8
χ^2/N	$0.9/3$	$4.4/8$

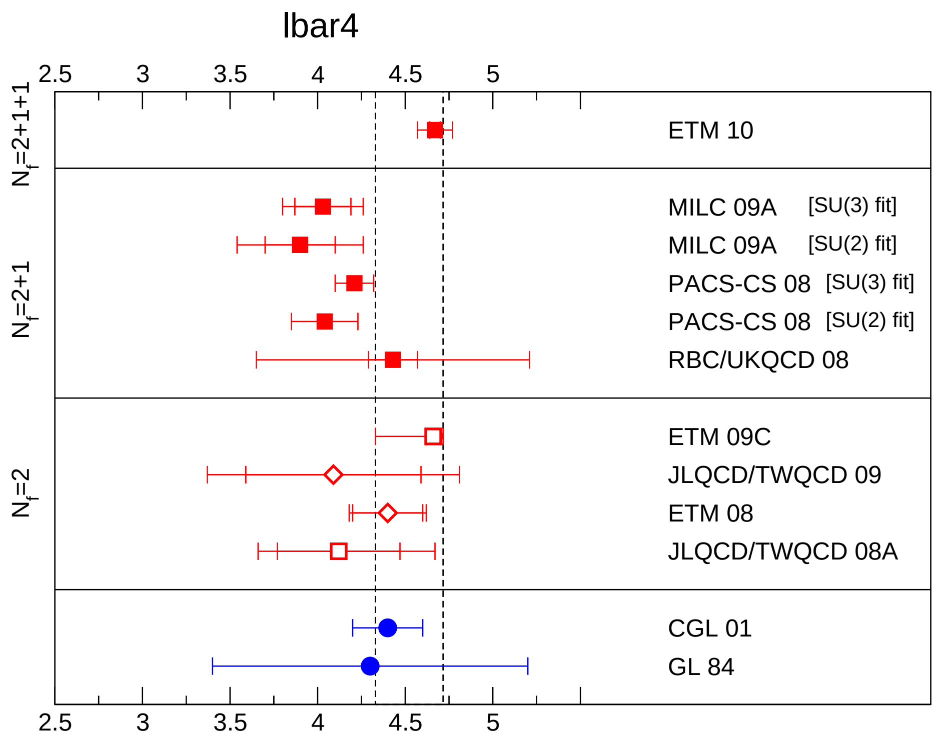
- GOOD χ^2/N
- FLUCTUATIONS: MILD AND $\Delta L_4 \sim 2\Delta L_6$ for PACS-CS/ LARGE FOR RBC/UKQCD
- $F_K/F_\pi|_{SM} = 1.192 \pm 0.006$
- $F_K/F_\pi|_{PACS} = 1.189 \pm 0.020, \quad F_K/F_\pi|_{RBC} = 1.205 \pm 0.018 \pm 0.062$

RESULTS:

	PACS – CS Subset Without $K_{\ell 3}$	RBC/UKQCD Subset With $K_{\ell 3}$
$X(2)$	0.90 ± 0.01	0.90 ± 0.02
$Y(2)$	1.04 ± 0.02	1.00 ± 0.03
$Z(2)$	0.87 ± 0.02	0.90 ± 0.02
$B(2\ \mathrm{GeV})[\mathrm{GeV}]$	3.83 ± 0.50	2.09 ± 0.19
$F[\mathrm{MeV}]$	85.8 ± 0.7	87.7 ± 0.8
$\bar{\ell}_3$	5.0 ± 2.1	-0.6 ± 3.7
$\bar{\ell}_4$	4.5 ± 0.5	3.3 ± 0.5
Σ/Σ_0	1.51 ± 0.51	4.52 ± 2.83
B/B_0	1.15 ± 0.26	2.28 ± 1.39
F/F_0	1.15 ± 0.08	1.41 ± 0.06

- ChPT: Gasser and Leutwyler $\bar{\ell}_3 = 2.9 \pm 2.4$
- ChPT at NNLO and the Roy equation analysis of $\pi\pi$ and F_S: Colangelo et al: Nucl; Phys. B 603 (2001) $\bar{\ell}_4 = 4.4 \pm 0.2$
- Lattice: VERY PRECISE ESTIMATE FROM FLAG $\bar{\ell}_3 = 3.3(7)$
- $L_4 \& L_6$ SMALL for PACS-CS,LARGER FOR RBC/UKQCD





- Figure from FLAG: arXiv:1011.4408
- our estimate means FLAG estimate

RESULTS

	PACS − CS Subset Without $K_{\ell 3}$	RBC/UKQCD Subset With $K_{\ell 3}$
$f_+(0)$	1.004 ± 0.149	0.985 ± 0.008
$\Delta_{CT} \cdot 10^3$	$\times$	-0.2 ± 12.1
$\Delta'_{CT} \cdot 10^3$	$\times$	-126 ± 104
$\langle r^2 \rangle_V^{K^+}$ [fm^2]	$\times$	0.248 ± 0.156
$\langle r^2 \rangle_V^{K^0}$ [fm^2]	$\times$	-0.027 ± 0.106
F_π^2	$0.66 + 0.22 + 0.12$	$0.45 + 0.69 - 0.14$
F_K^2	$0.44 + 0.48 + 0.08$	$0.34 + 0.76 - 0.10$
$F_\pi^2 M_\pi^2$	$0.60 + 0.30 + 0.10$	$0.20 + 0.95 - 0.15$
$F_K^2 M_K^2$	$0.42 + 0.50 + 0.08$	$0.14 + 0.97 - 0.11$
$F_\pi F_K f_+(0)$	$\times$	$0.40 + 0.75 - 0.15$

- LO SMALL BUT LO + NLO $>$ NNLO $+ \cdots$
- $f_+(0)|_{SM} = 0.959 \pm 0.005$

CONCLUSIONS

- STANDARD CHPT works well for SU(2) CONFIRMED
- MORE PROBLEMATIC FOR SU(3)

  Two LOOPS CALC. NECESSARY

  Resummed CHPT if numerical competition between LO and NLO expected from significant vacuum fluctuations of $\bar{s}s$ sea pairs

  could nicely reproduce PACS and RBC/UKQCD
- MORE STUDIES
- MORE LATTICE DATA AT SMALLER VALUES OF m_s NEEDED
- INDIRECT TEST OF THE SM BY MEASUREMENT OF $K_{\ell 3}$ DECAYS AND τ DECAY
- NEEDS OF VERY PRECISE DETERMINATION OF $f_+(0)$ and CT DISCREPANCY

$\rightsquigarrow$ VERY PRECISE DETERMINATION OF $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ LEC's