α_s FROM 5-JET CROSS SECTIONS

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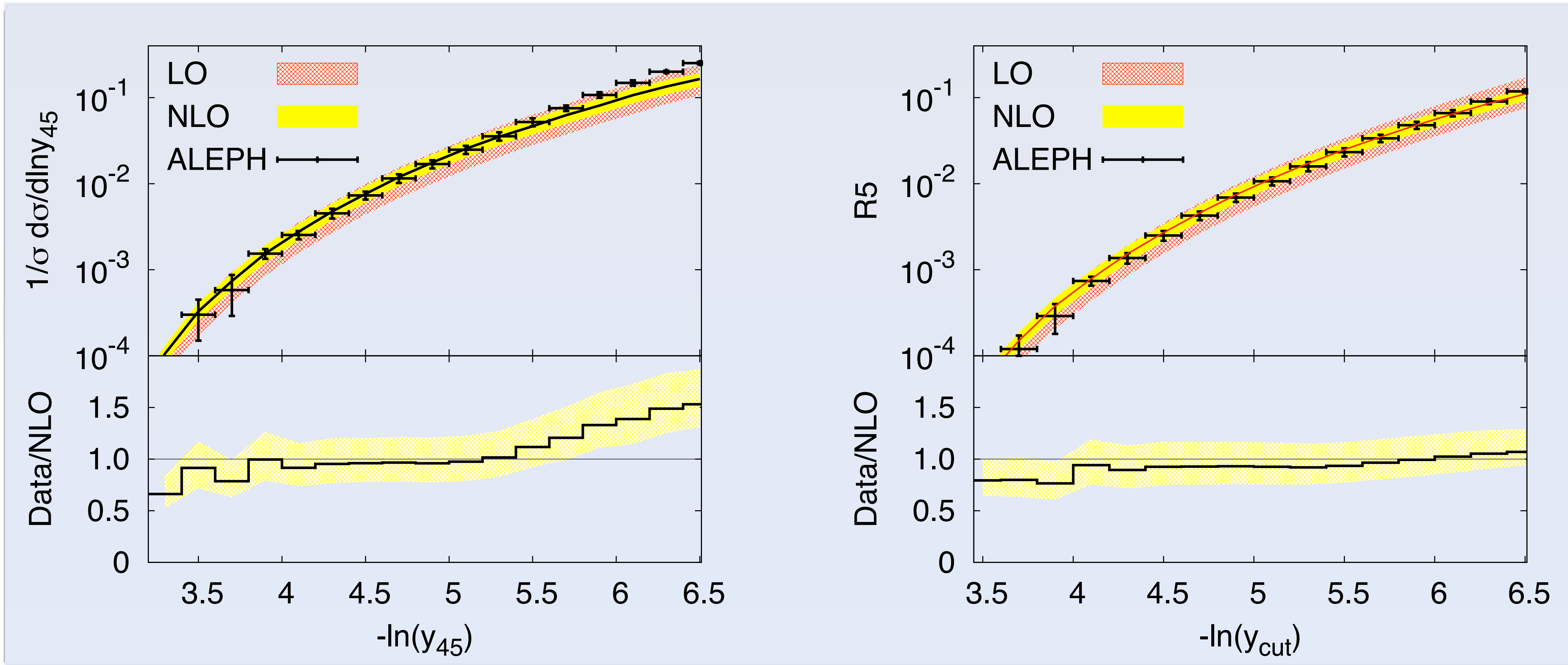
Results



LO
NLO
ALEPH
1/σ dσ/dlny45
10^{-1}
10^{-2}
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Data/NLO
1.5
1.0
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3.5 4 4.5 5 5.5 6 6.5
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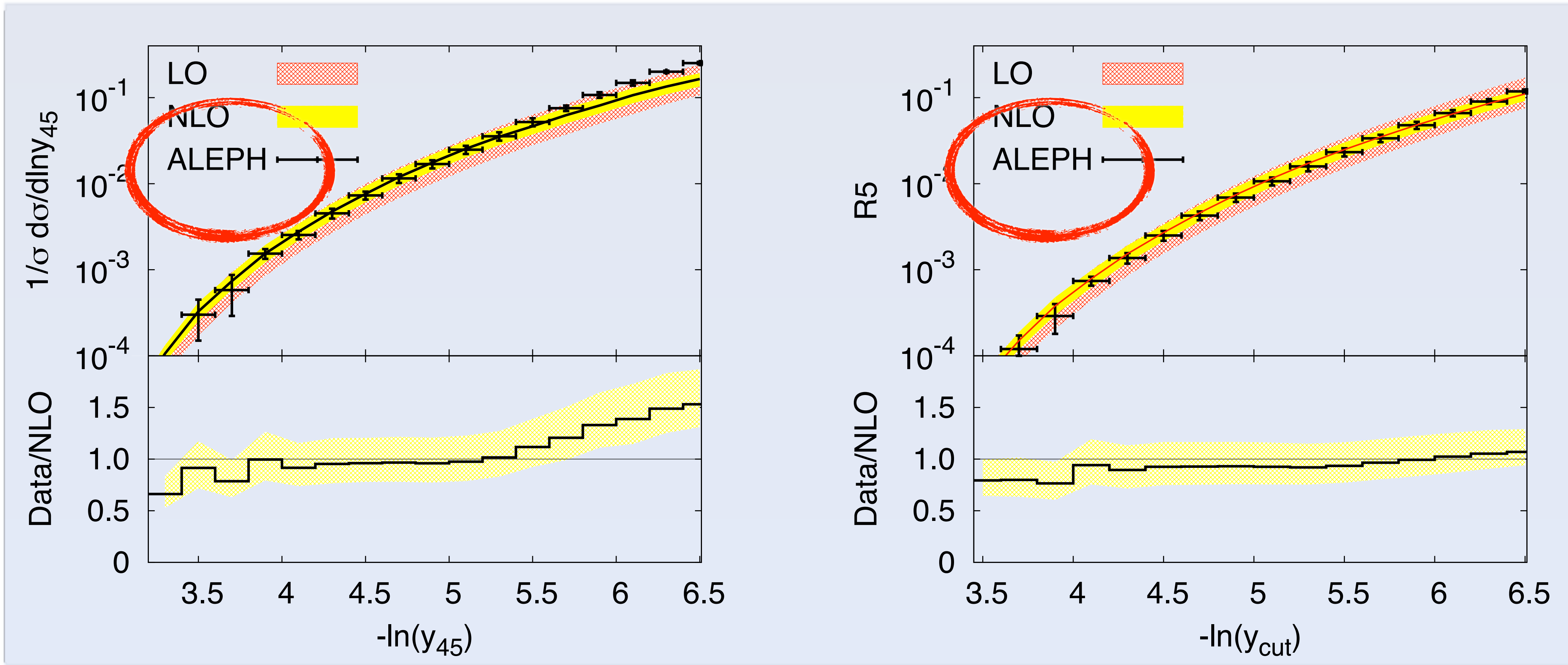
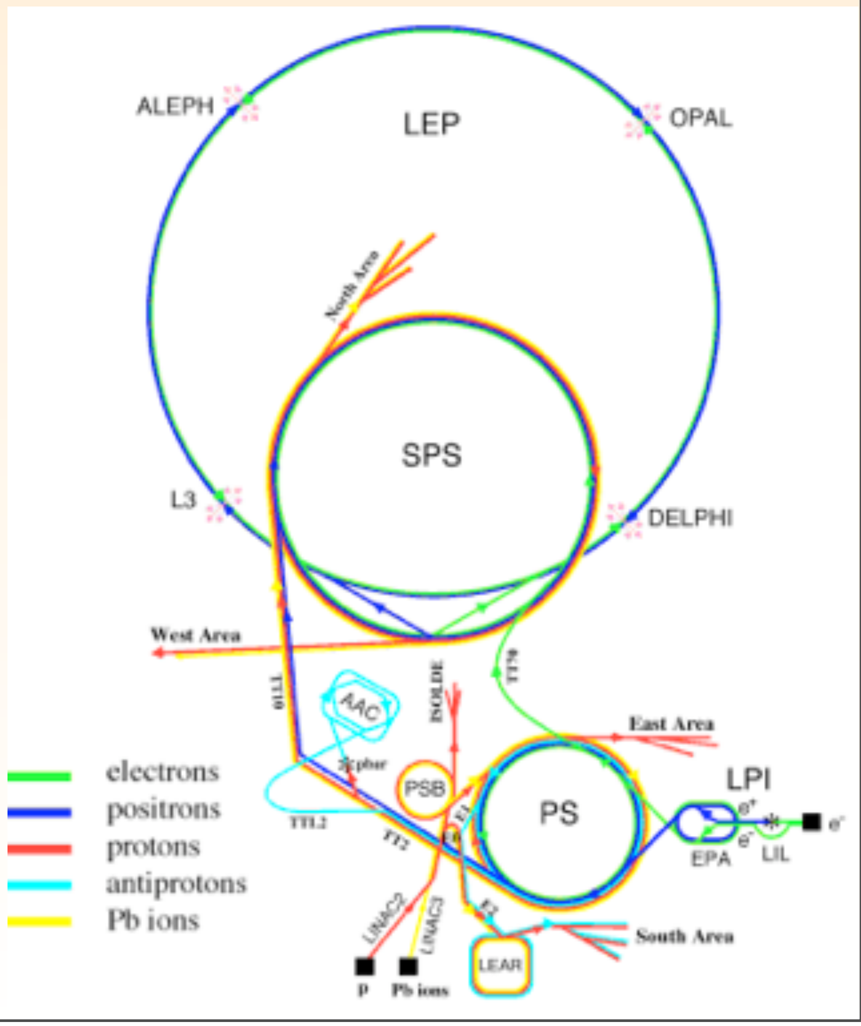


- ALEPH data points

ALEPH @ LEP

- ALEPH was one of the detectors at the LEP particle collider at CERN
- LEP (Large Electron-Positron collider) was build at CERN in the tunnel where now the LHC is running
- Starting at 1989 it collided leptons at a center-of-momentum energy equal to the Z-boson mass (~91 GeV) and finished operation running at 209 GeV at the end of 2000
- It still holds the particle accelerator speed record; a Lorentz boost factor close to 200000.



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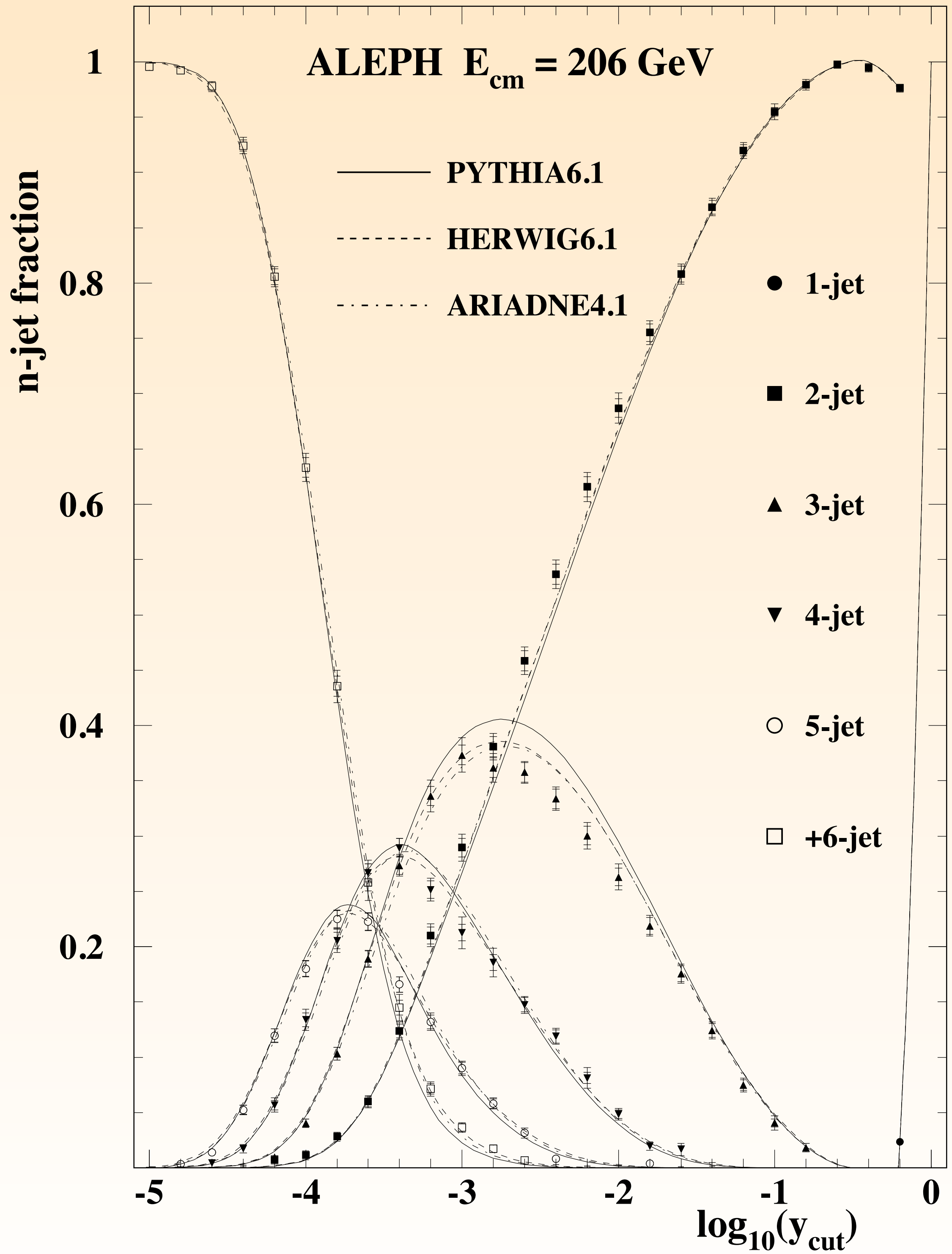
Analyses

ALEPH: Heister et al., Eur.J.Phys.C35,457 (2004)



- By now, most of the data has been analyzed in detail
- and not many new results are appearing since 2004-2005
- Distributions are in general well-described by MC simulation
- Data points are available on-line
- We are interested in purely QCD events with 5 jets

Figure 7: Measured n-jet fractions for $n = 1, 2, 3, 4, 5$ and $n \geq 6$ and the predictions of Monte Carlo models, at a centre-of-mass energy of 206 GeV.

ANALYSES

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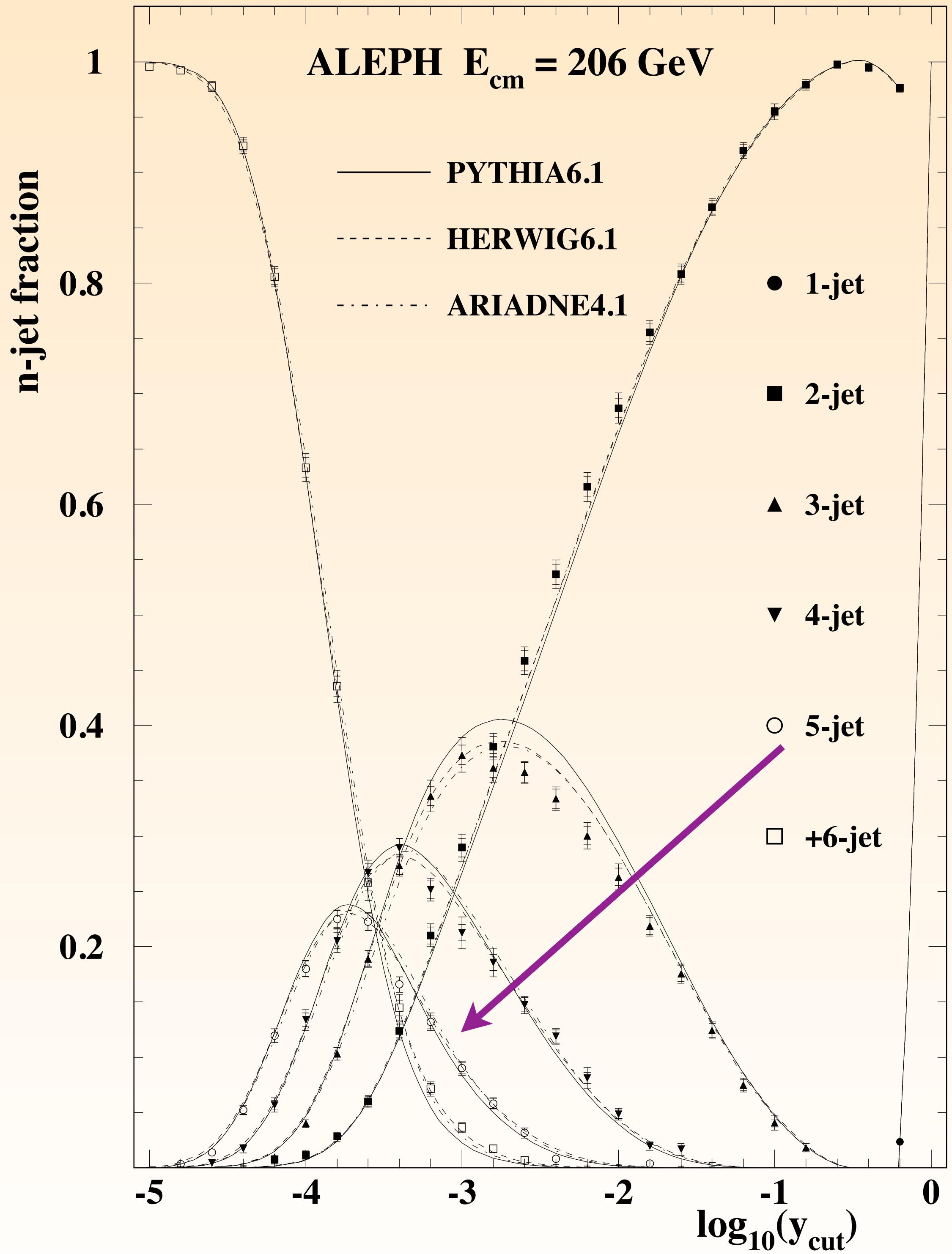

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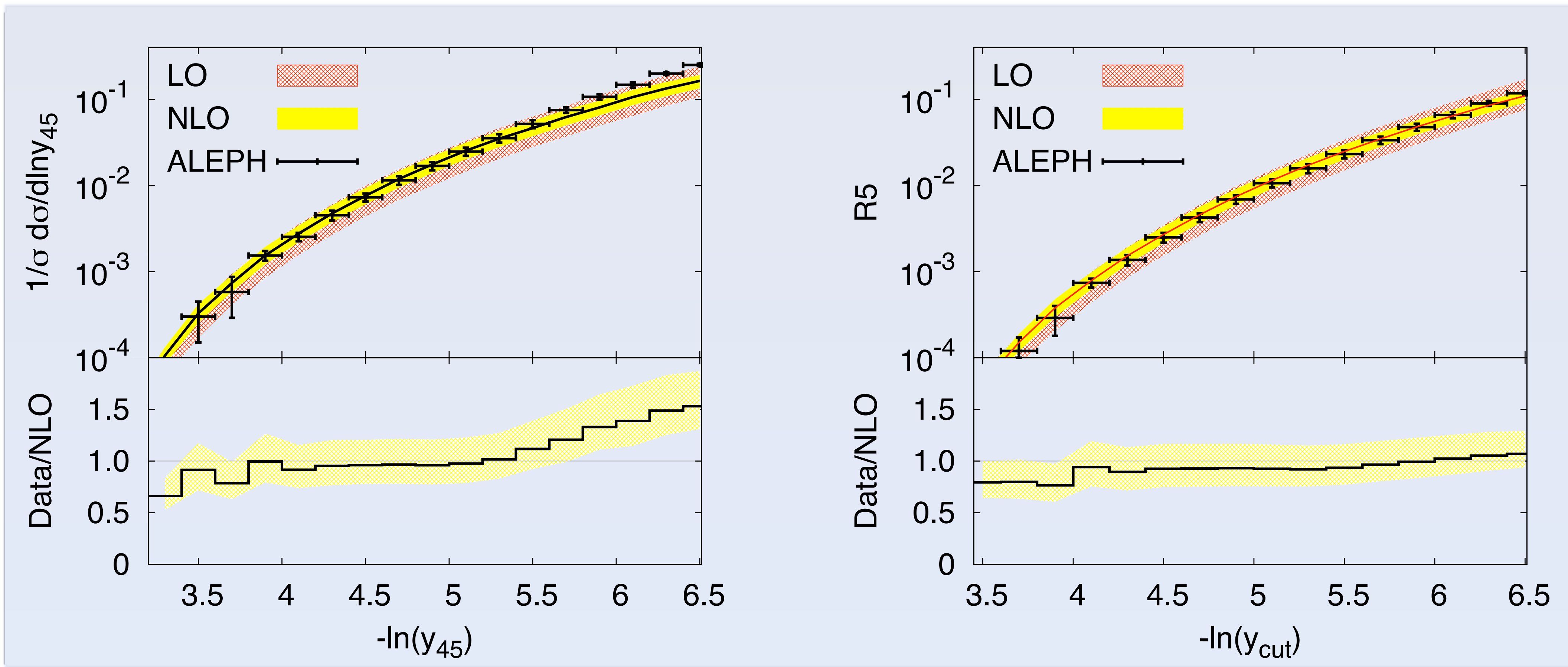

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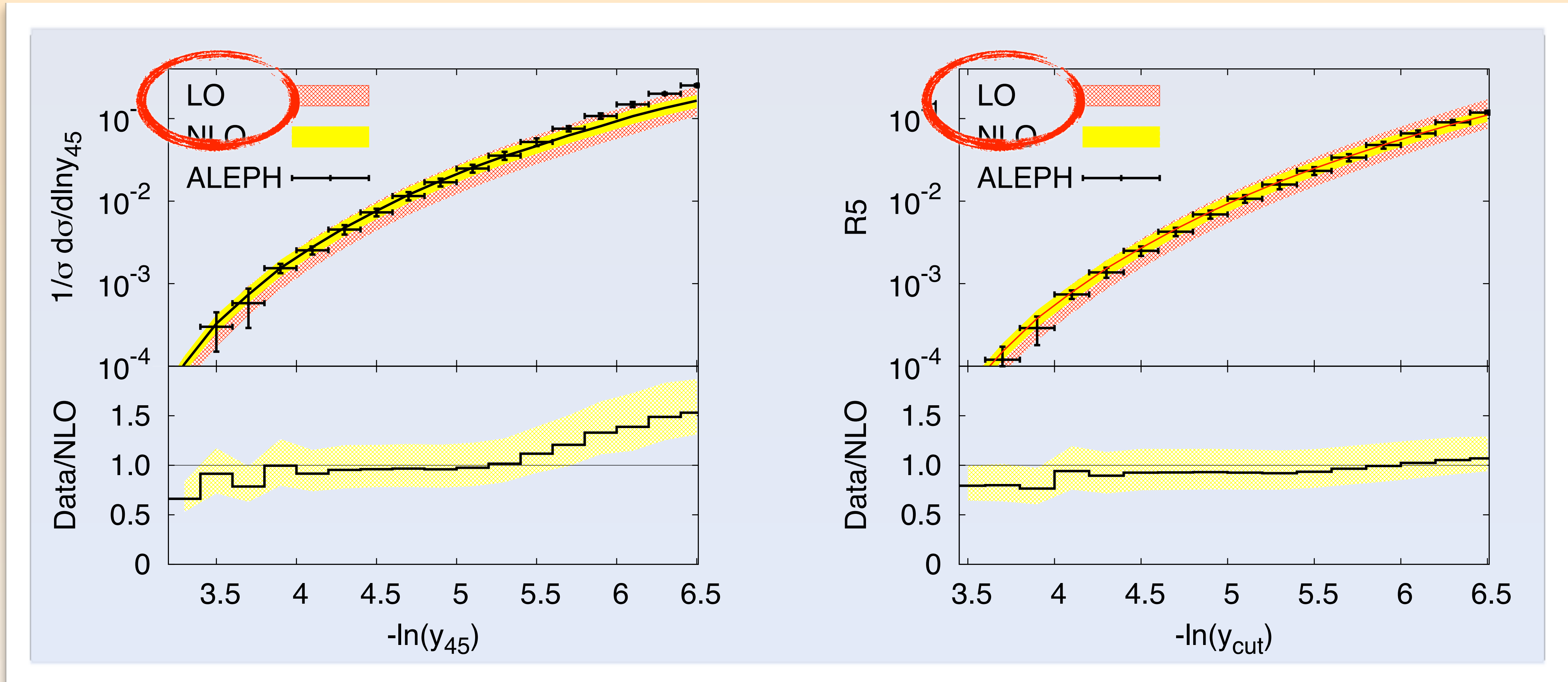
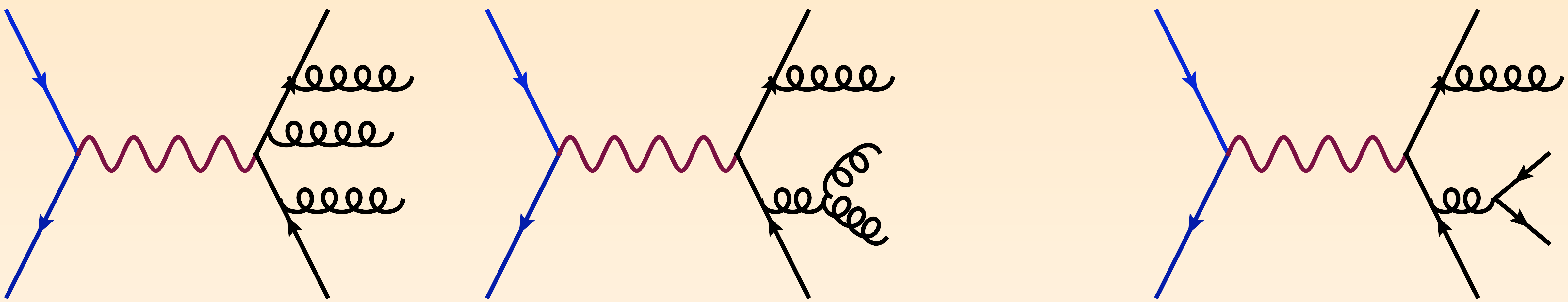


- Leading order predictions

Lowest order predictions



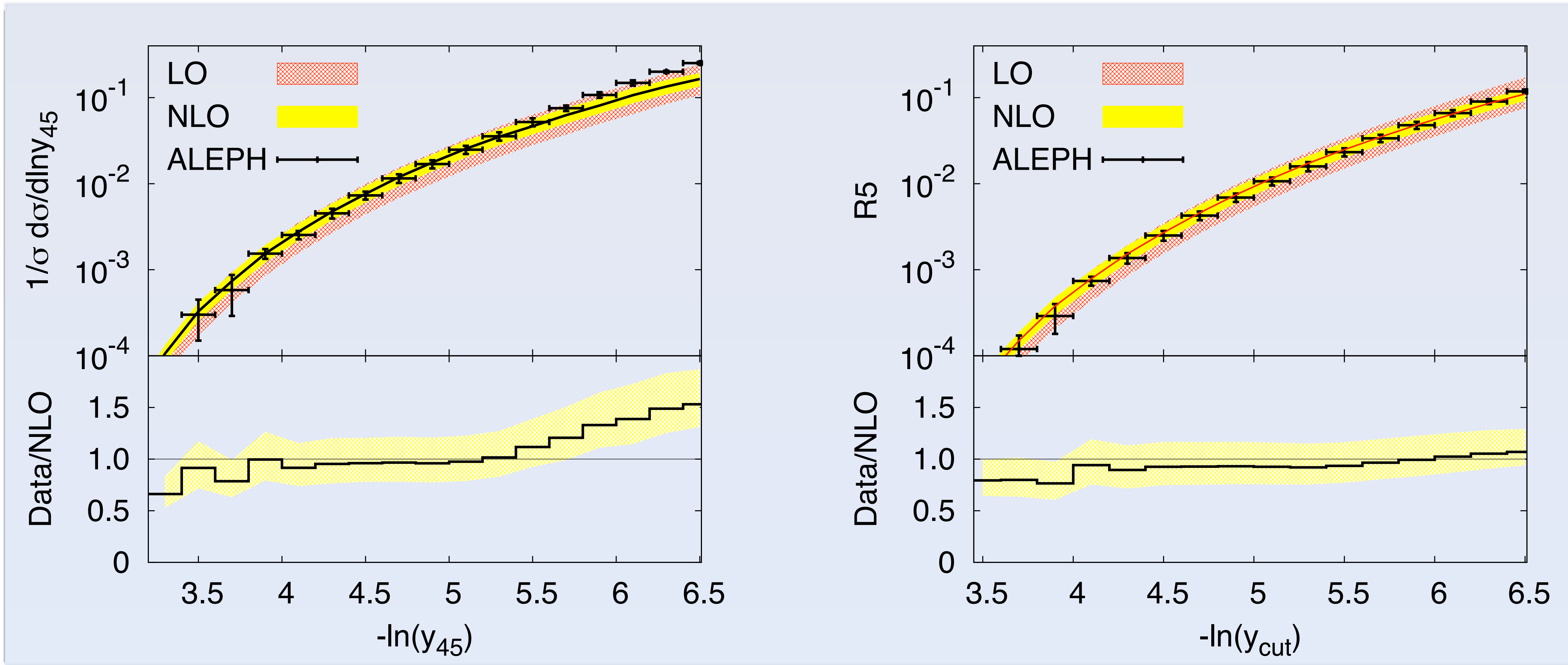
- Two distinct subprocesses:

$e^+e^- \rightarrow Z/\gamma^* \rightarrow$ qqbar ggg

$e^+e^- \rightarrow Z/\gamma^* \rightarrow$ qqbar qqbar g

- Generated with MadGraph

Results



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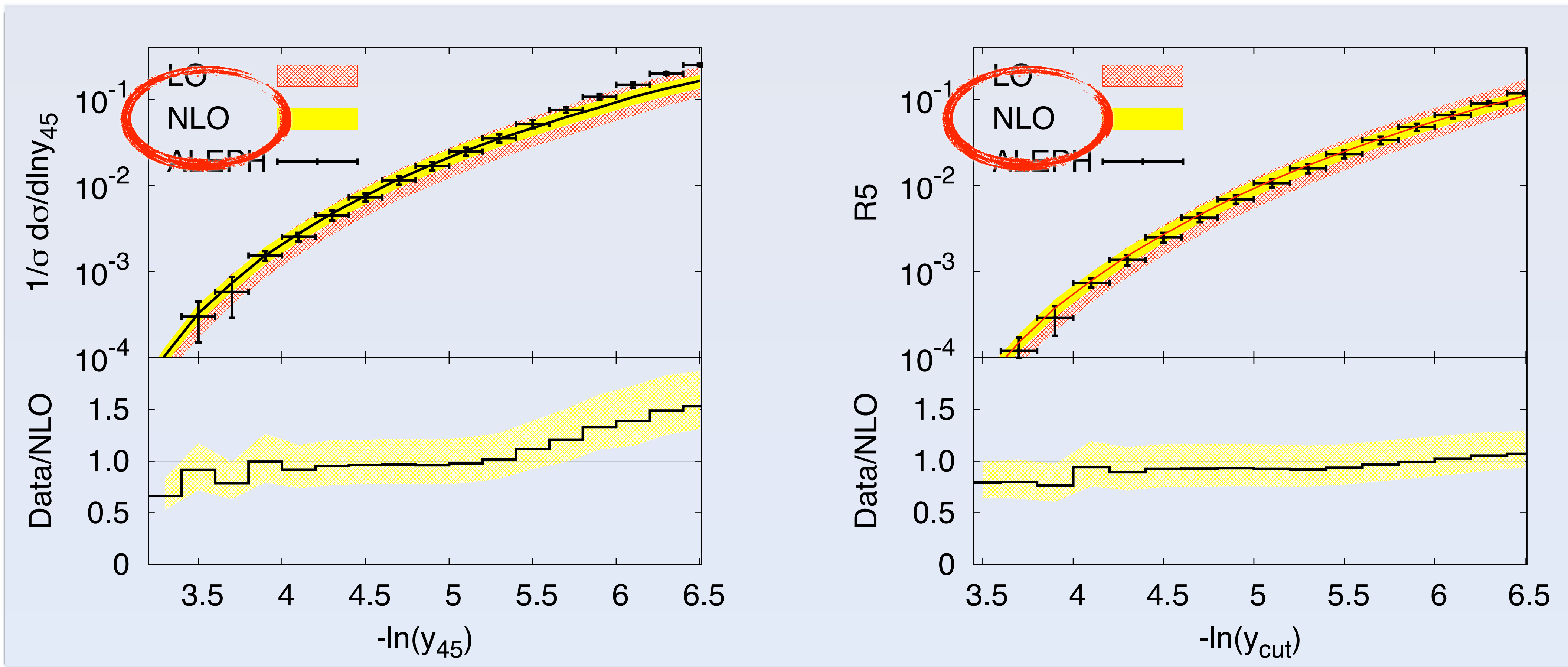

- Next-to-leading order predictions

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Next-to-leading order

$$\sigma^{\mathrm{NLO}} = \int_{m+1} d^{(d)}\sigma^{R} + \int_{m} d^{(d)}\sigma^{V} + \int_{m} d^{(4)}\sigma^{B}$$

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'Born' or 'LO' contribution

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'Born' or 'LO' contribution

'Virtual' or 'one-loop' NLO corrections

Next-to-leading order

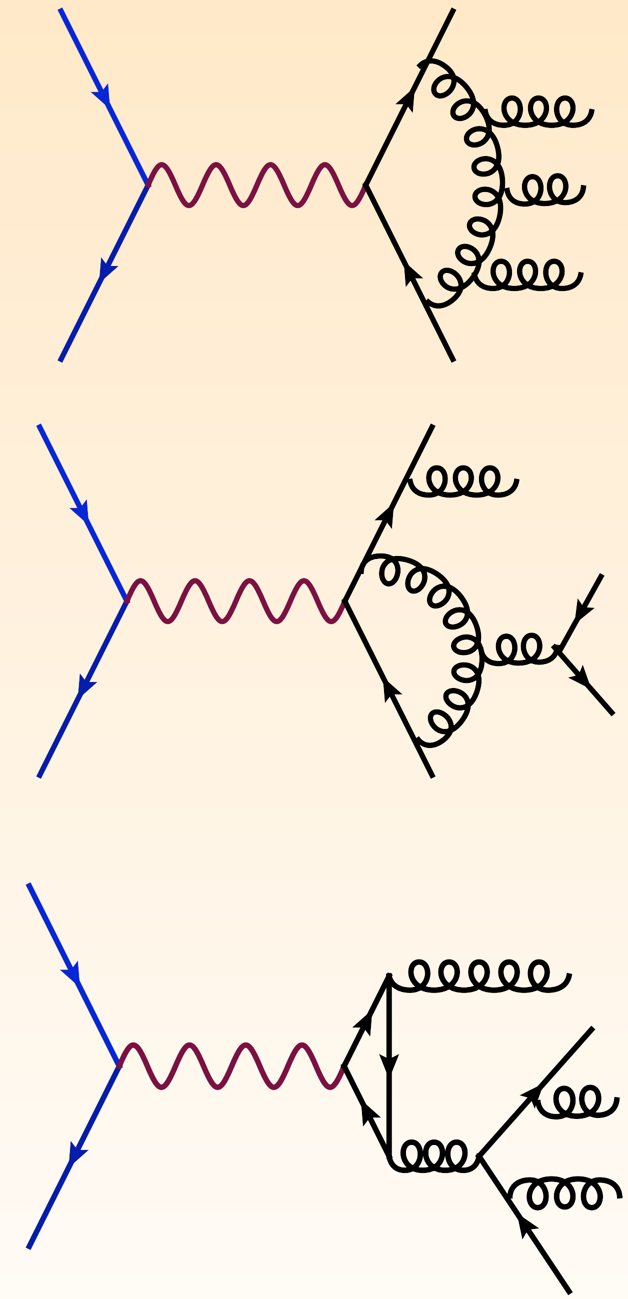
$$\sigma^{\mathrm{NLO}} = \int_{m+1} d^{(d)}\sigma^{R} + \int_{m} d^{(d)}\sigma^{V} + \int_{m} d^{(4)}\sigma^{B}$$

'Real emission' NLO corrections

'Born' or 'LO' contribution

'Virtual' or 'one-loop' NLO corrections

Virtual corrections



- Based on generalized D-dimensional unitarity
- Similar to the calculation for pp ➞ W+3j at NLO
  Ellis, Giele, Kunszt, Melnikov & Zanderighi (2009)
  Main differences:
  - Crossing of initial and final state particles
  - Coupling of EW boson is different
  - Closed fermion loops attached to EW boson (Checked to be small and neglected)
- Checked against the BlackHat code and agreement was found *Berger et al. (2009)*

IR divergence (of the real emission)

$$\sigma^{\mathrm{NLO}} = \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(d)}\sigma^V + \int_m d^{(4)}\sigma^B$$

- (UV-renormalized) virtual corrections ➞ IR divergent
- Real emission ➞ IR divergent
  - After integration, the sum of all contributions is finite (for infrared-safe observables)

Subtraction terms

$$\sigma^{\mathrm{NLO}} = \int_{m+1} d^{(d)}\sigma^{R} + \int_{m} d^{(d)}\sigma^{V} + \int_{m} d^{(4)}\sigma^{B}$$

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$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A\right] + \int_m \left[d^{(4)}\sigma^B + \int_{\mathrm{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A\right]_{\epsilon=0}$$

- Include subtraction terms to make real emission contributions and virtual contributions separately finite
- All can be integrated numerically

FKS subtraction

- FKS subtraction: Frixione, Kunszt & Signer 1996.
- Also known as "residue subtraction"
- Based on using plus-distributions to regulate the infrared divergences of the real emission matrix elements

FKS FOR BEGINNERS

- Easiest to understand by starting from real emission:

$$d\sigma^R = |M^{n+1}|^2 d\phi_{n+1}$$

- $|M^{n+1}|^2$ blows up like $\dfrac{1}{\xi_i^2}\dfrac{1}{1-y_{ij}}$ with $\begin{aligned}\xi_i &= E_i/\sqrt{\hat{s}}\\ y_{ij} &= \cos\theta_{ij}\end{aligned}$

- Partition the phase space in such a way that each partition has at most one soft and one collinear singularity

$$d\sigma^R = \sum_{ij} S_{ij}|M^{n+1}|^2 d\phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

- Use plus distributions to regulate the singularities

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M^{n+1}|^2 d\phi_{n+1}$$

FKS for beginners

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i (1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

- Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \, \frac{f(\xi)-f(0)}{\xi}$$

- One event has maximally three counter events:
  - Soft: $\xi_i \to 0$
  - Collinear: $y_{ij} \to 1$
  - Soft-collinear: $\xi_i \to 0 \qquad y_{ij} \to 1$

Subtraction terms

$$\sigma^{\rm NLO} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[d^{(4)}\sigma^B + \int_{\rm loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

- This defines the subtraction terms for the reals
- They need to be integrated over the one-parton phase space (analytically) and added to the virtual corrections
  - these are process-independent terms proportional to the (color-linked) Borns
- All formulae can be found in the paper arXiv:0908.4247
  [RF, Frixione, Maltoni, Stelzer]

MadFKS

RF, Frixione, Maltoni, Stelzer (2009)

- Automatic FKS subtraction within the MadGraph/ MadEvent framework
- Given the (n+1) process, it generates the real, all the subtraction terms and the Born processes
- For a NLO computation, only the finite parts of the virtual corrections are needed from the user
- Phase-space integration integrates (n) and (n+1) body processes can be done at the same time, or separately
- Any physics model: massive particles have only soft singularities, which are spin independent: MadFKS works also for BSM physics, e.g. squarks

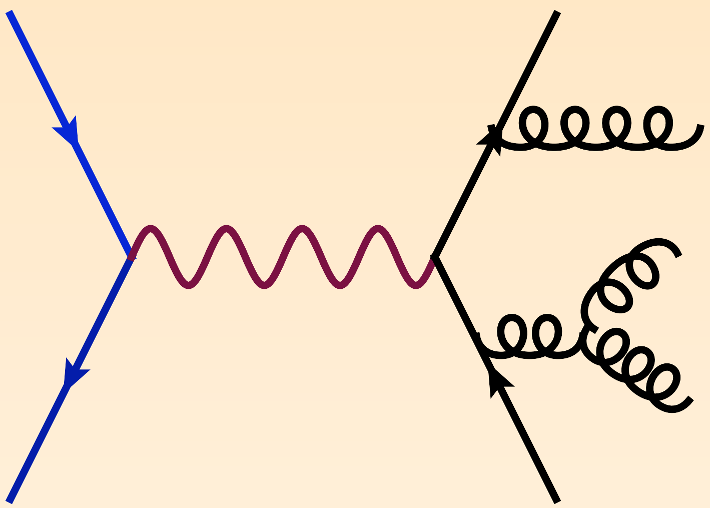
$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[d^{(4)}\sigma^B + \int_{\mathrm{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

Virtual corrections

$$\sigma^{\rm NLO} = \int_{m+1}\left[d^{(4)}\sigma^R - d^{(4)}\sigma^A\right] + \int_m \left[d^{(4)}\sigma^B + \int_{\rm loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A\right]_{\epsilon=0}$$

MadFKS

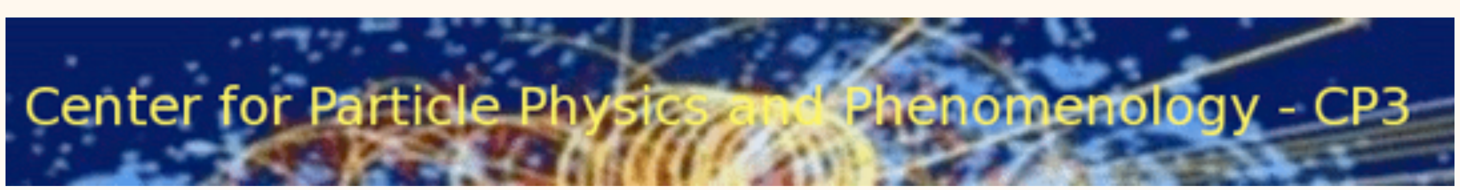
SCALE CHOICE



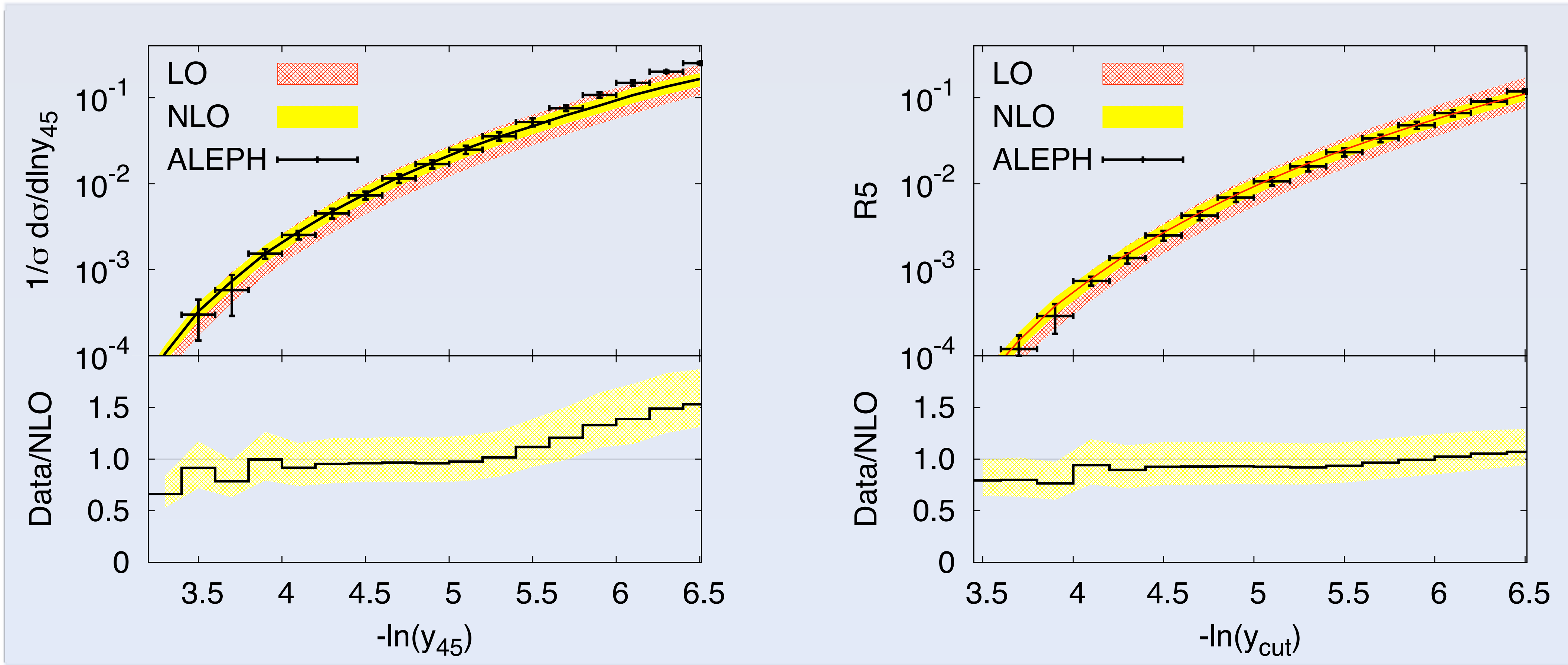
- The most important input parameter is the renormalization scale
- Any calculation in perturbative QCD depends on this non-physical scale
- For multi-scale processes there is in general a large dependence on this scale and the optimal choice is difficult to find
- Happily, going from LO to NLO, this dependence is reduced (in fact, it's one of the reasons to go to NLO!)
- We would like to take the scale of the hardest branching in the process as our default scale
- On average this is $\mu_R = 0.3\sqrt{s}$
- To address the uncertainties, we vary it by a factor 2 up and down

Running

- The computation has been run on the MadGraph cluster at the CP3 institute in Louvain-la-Neuve
- A week of running on ~300 machines
  - Born: couple of hours
    Real emission: 2 days
    Virtual corrections: 5 days



Results



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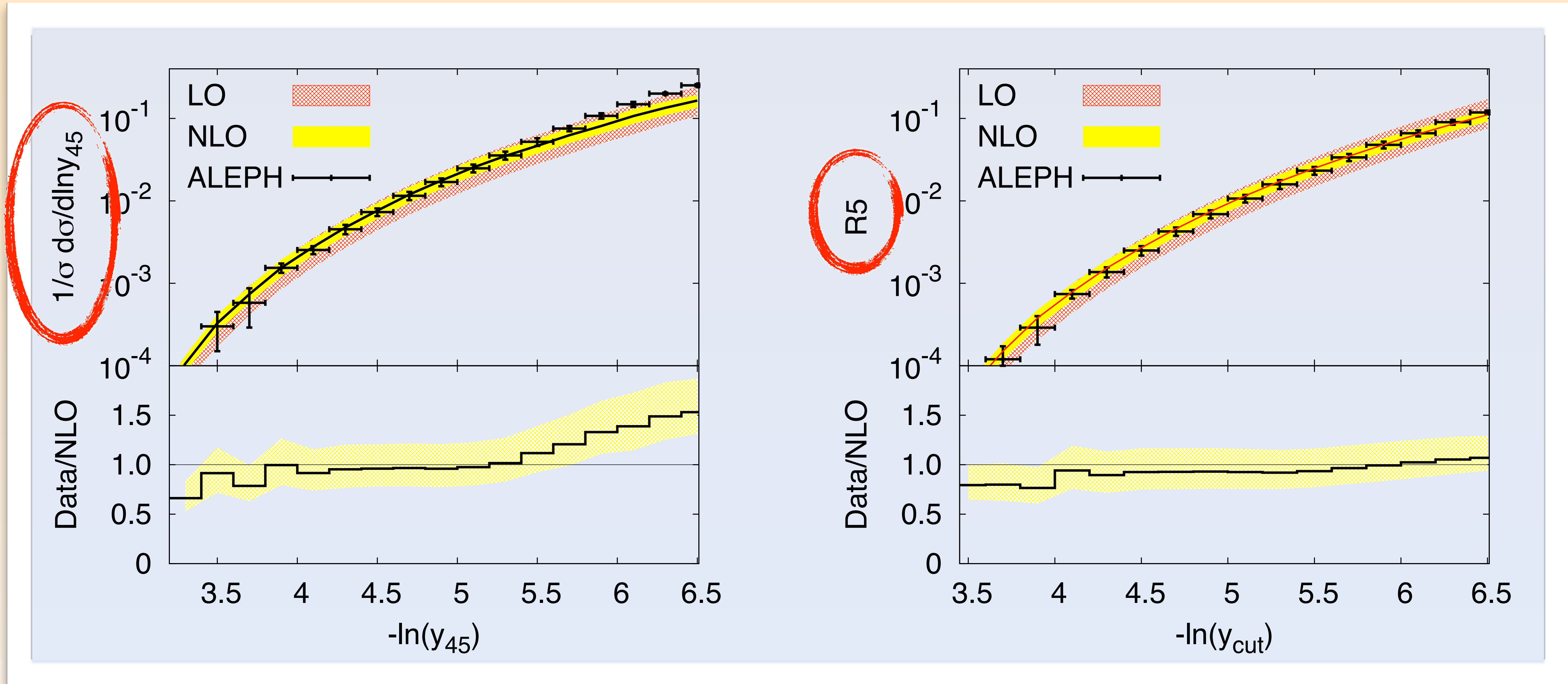

- What are we plotting...?

Observables

- We are looking at processes with $e^+e^- \rightarrow$ jets
- Jets are defined using the Durham jet algorithm
- Five-jet resolution parameter $\sigma_{\rm tot}^{-1}\dfrac{{\rm d}\sigma}{{\rm d}\ln y_{45}^{-1}}$

  where y_{45} is the maximum value of y_{cut} for which the event is classified as a five-jet event by the jet algorithm
- Five-jet rate $R_5(y_{\rm cut}) = \dfrac{\sigma_{\rm excl}^{5-{\rm jet}}(y_{\rm cut})}{\sigma_{\rm tot}}$

  where $\sigma_{\rm excl}^{5-{\rm jet}}(y_{\rm cut})$ is the exclusive five-jet production cross section, defined by running the Durham jet algorithm to the given events and by requiring that exactly five are reconstructed

Durham jet algo.

- Define the distance between each pair of particles as

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\theta_{ij})$$

- The pair of particles with the smallest distance is clustered together (by adding their 4-momentum) if $y_{ij} < y_{cut}$
- Iterate until all distances are larger than y_{cut} and the recombination stops
- The number of (pseudo)-particles left is equal to the number of jets in the event

DURHAM JET ALGO.

- Define the distance between each pair of particles as

Energy of parton i

Angle between parton i and j

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\theta_{ij})$$

Collision energy

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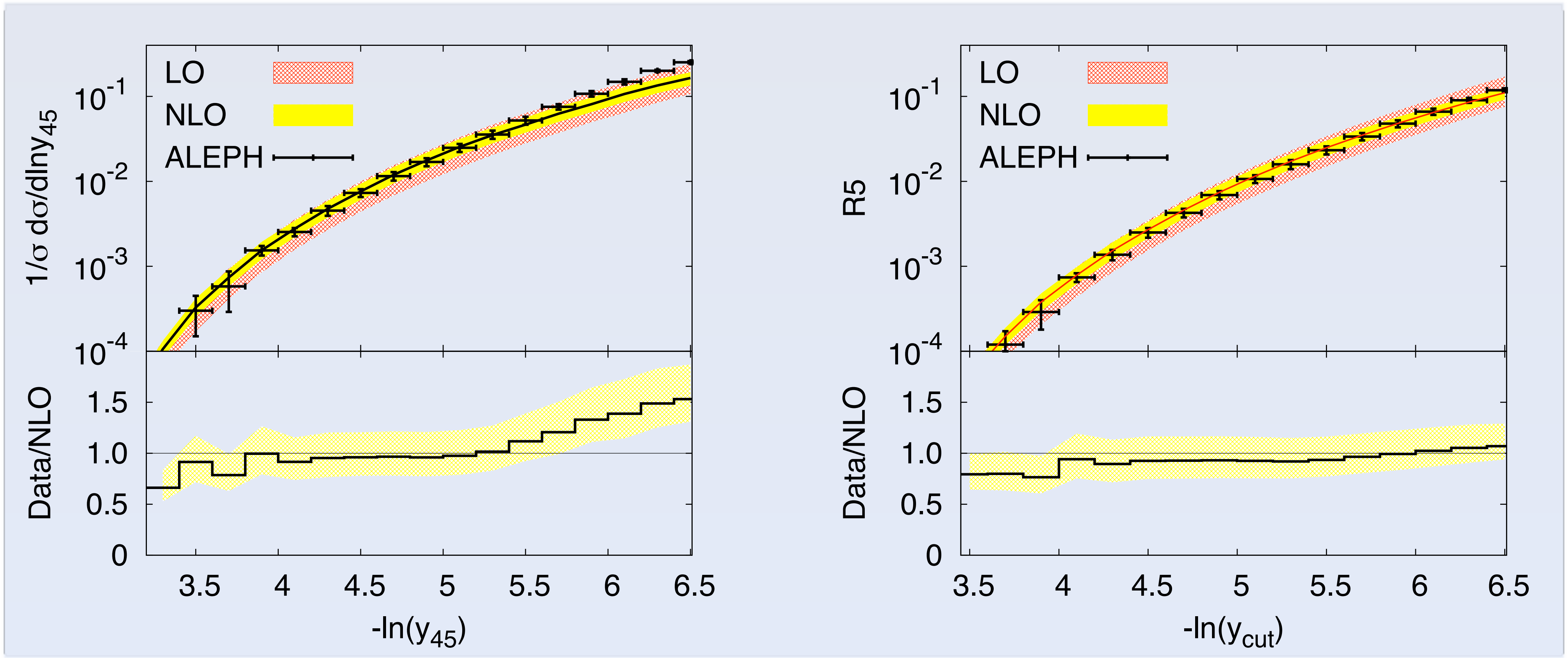
Functional form

$$\sigma_{\rm tot}^{-1}\frac{d\sigma}{d\ln y_{45}^{-1}} = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 A_{45}(y_{45}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 \left(B_{45}(y_{45}) + 3b_0 A_{45}(y_{45})\ln\frac{\mu}{\sqrt{s}}\right)$$

$$R_5(y_{\rm cut}) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 A_5(y_{\rm cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 \left(B_5(y_{\rm cut}) + 3b_0 A_5(y_{\rm cut})\ln\frac{\mu}{\sqrt{s}}\right)$$

- 5 light flavors, top mass is considered infinitely heavy
- All particles, except the Z boson are treated as massless
- Vector and axial currents do not interfere for these sufficiently inclusive observables
- Triangle fermion loops that lead to the axial anomaly are neglected
- **Therefore, A_{45}, B_{45}, A_5 and B_5 are independent of the electroweak parameters and the center-of-mass energy squared**

Results



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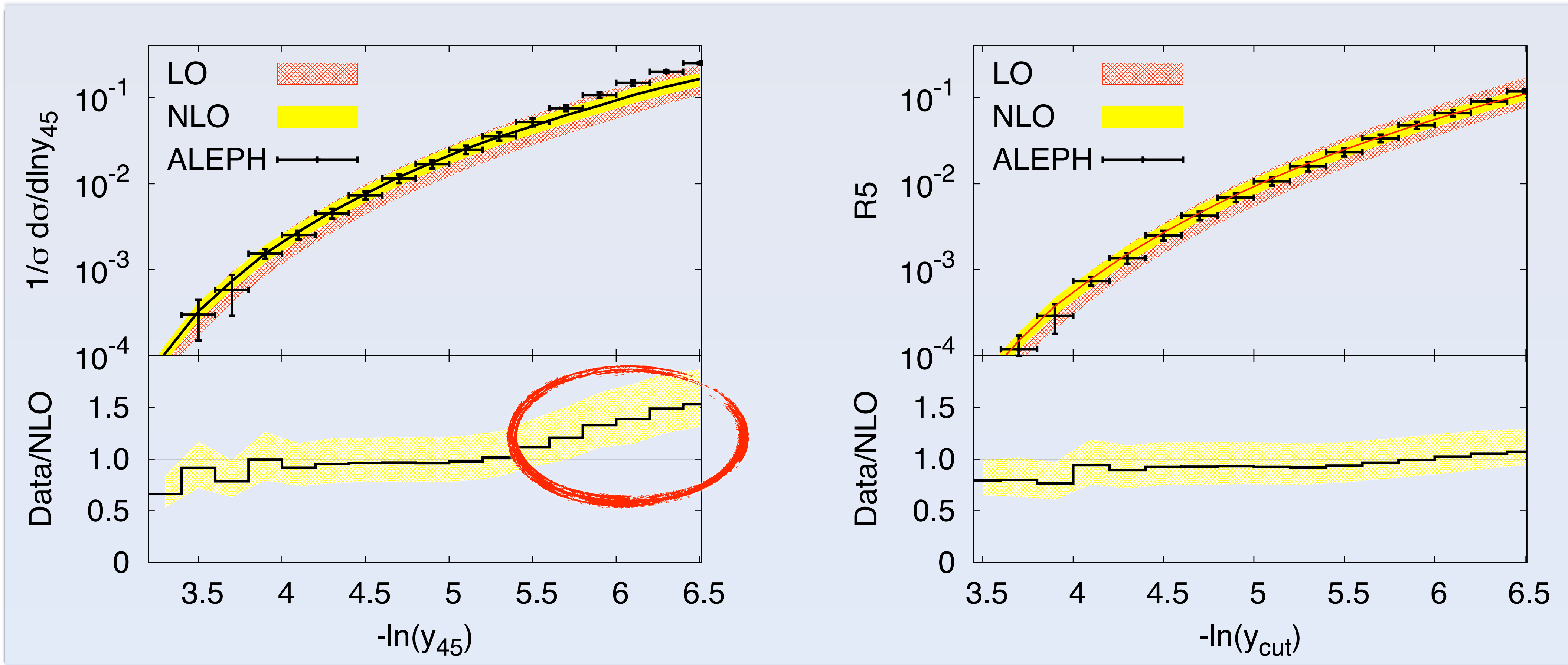
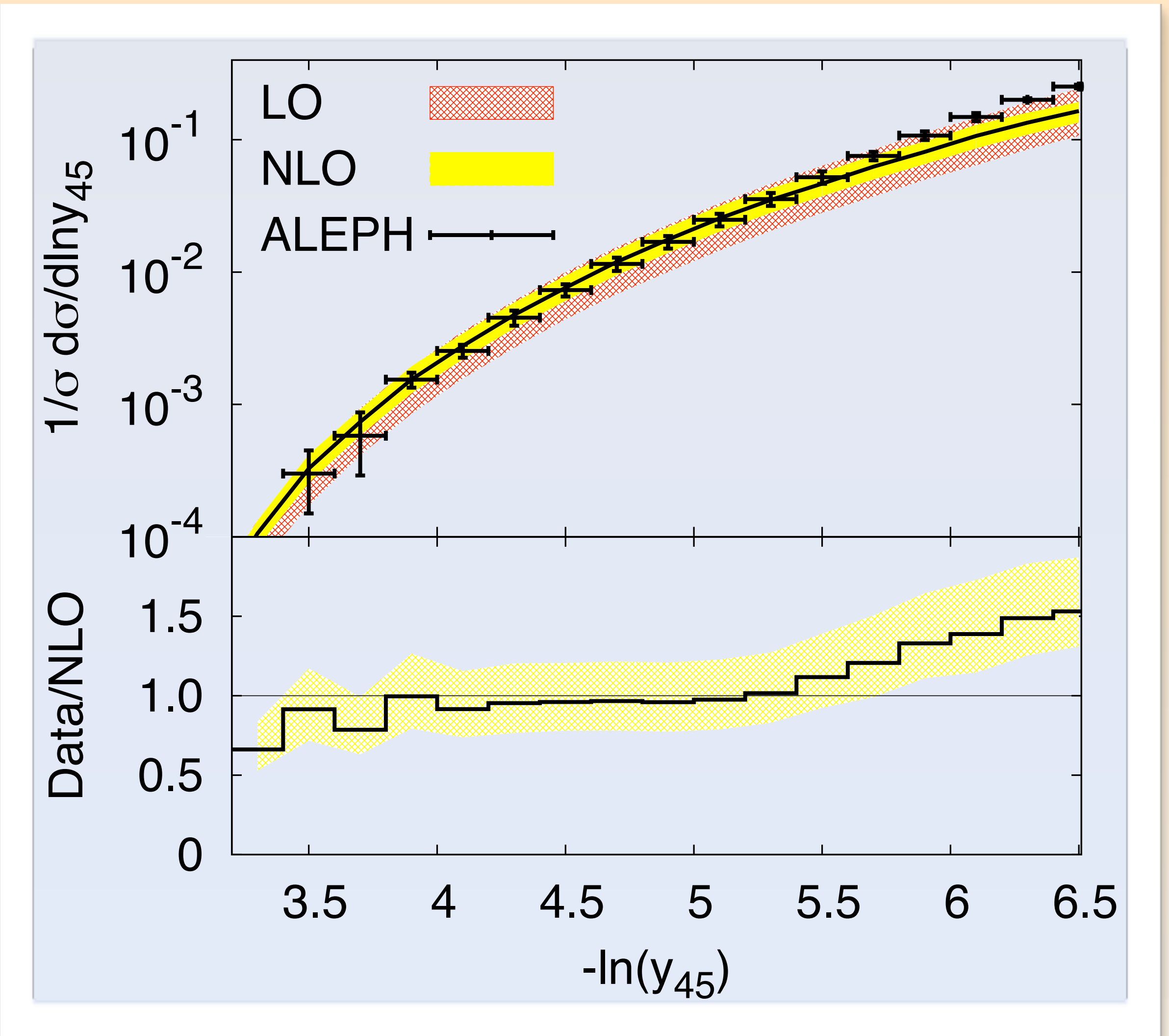


- Where is this discrepancy coming from?

Large Logarithms



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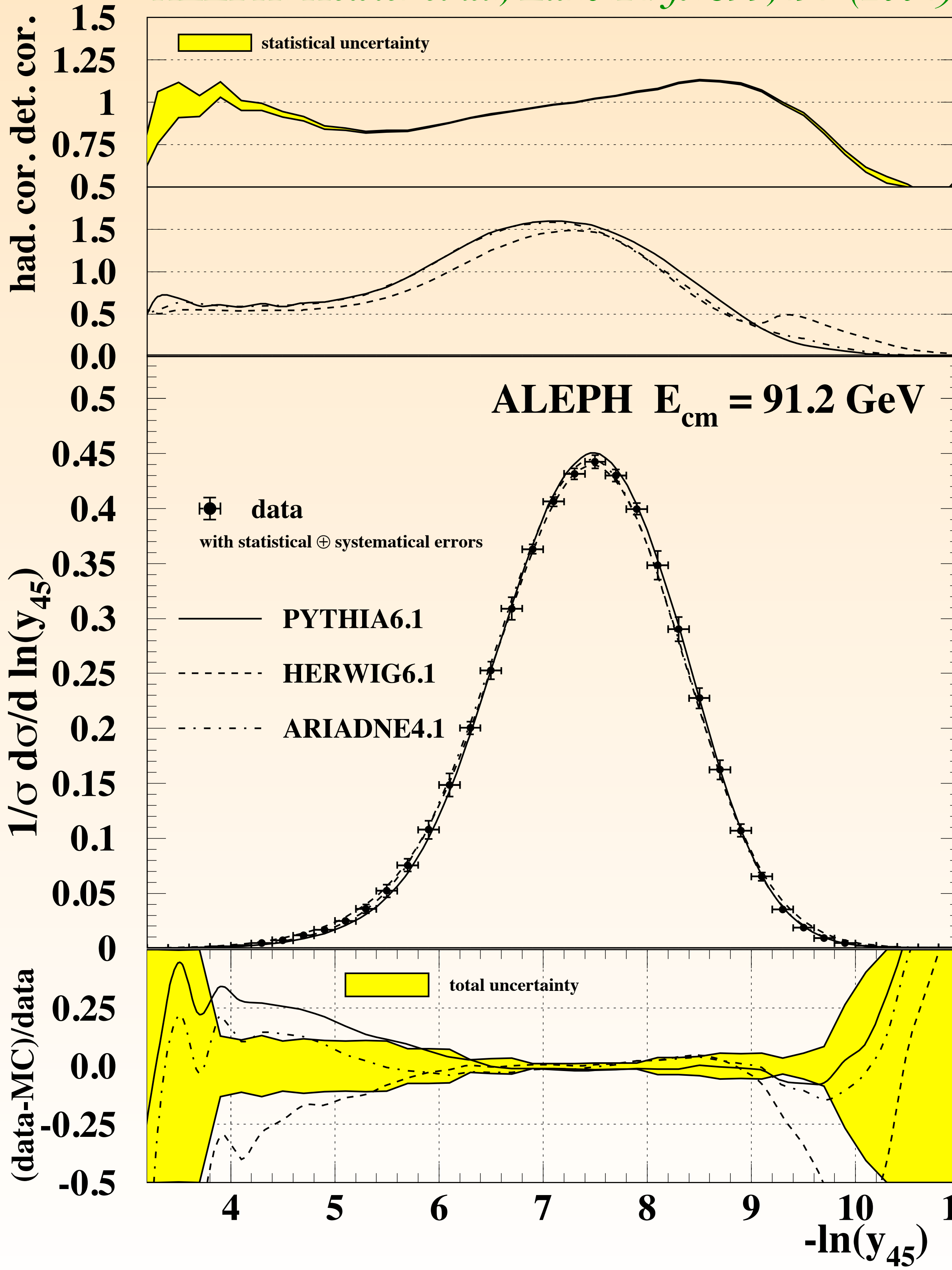

- Fixed order breaks down for $-\ln(y_{45})>6$
- Resummation, finite b-quark mass effects and non-perturbative corrections needed

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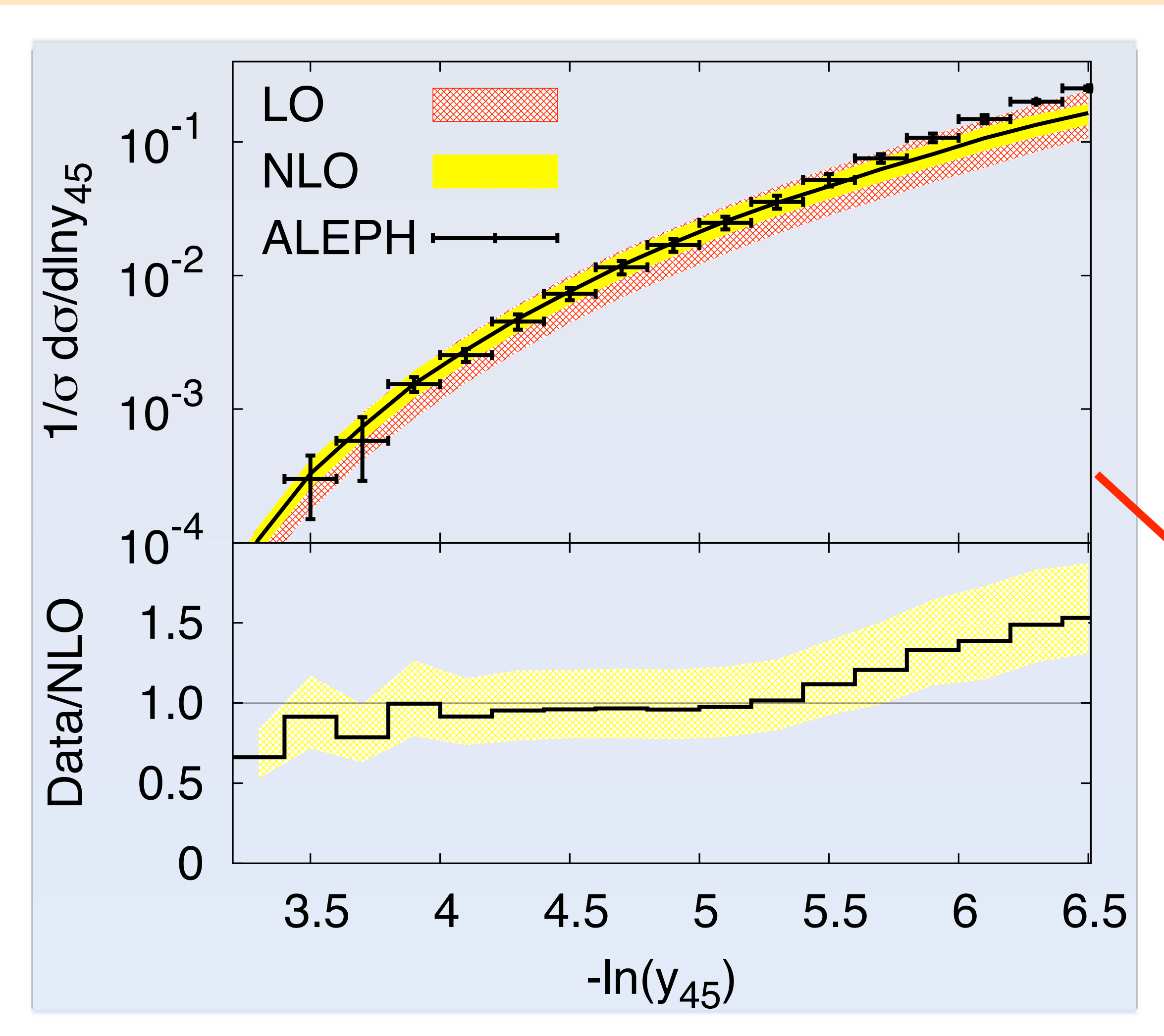

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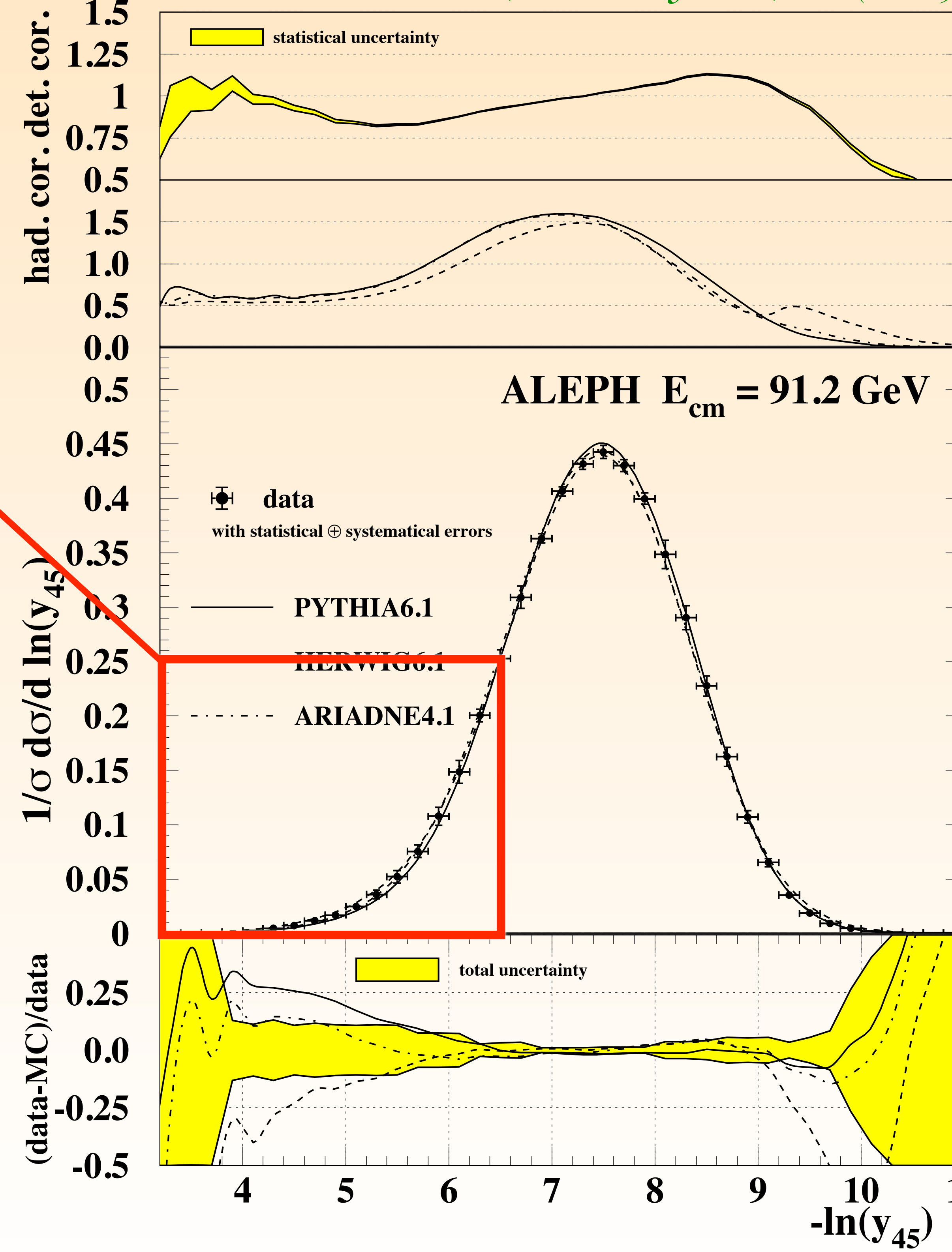

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Hadronization cor.

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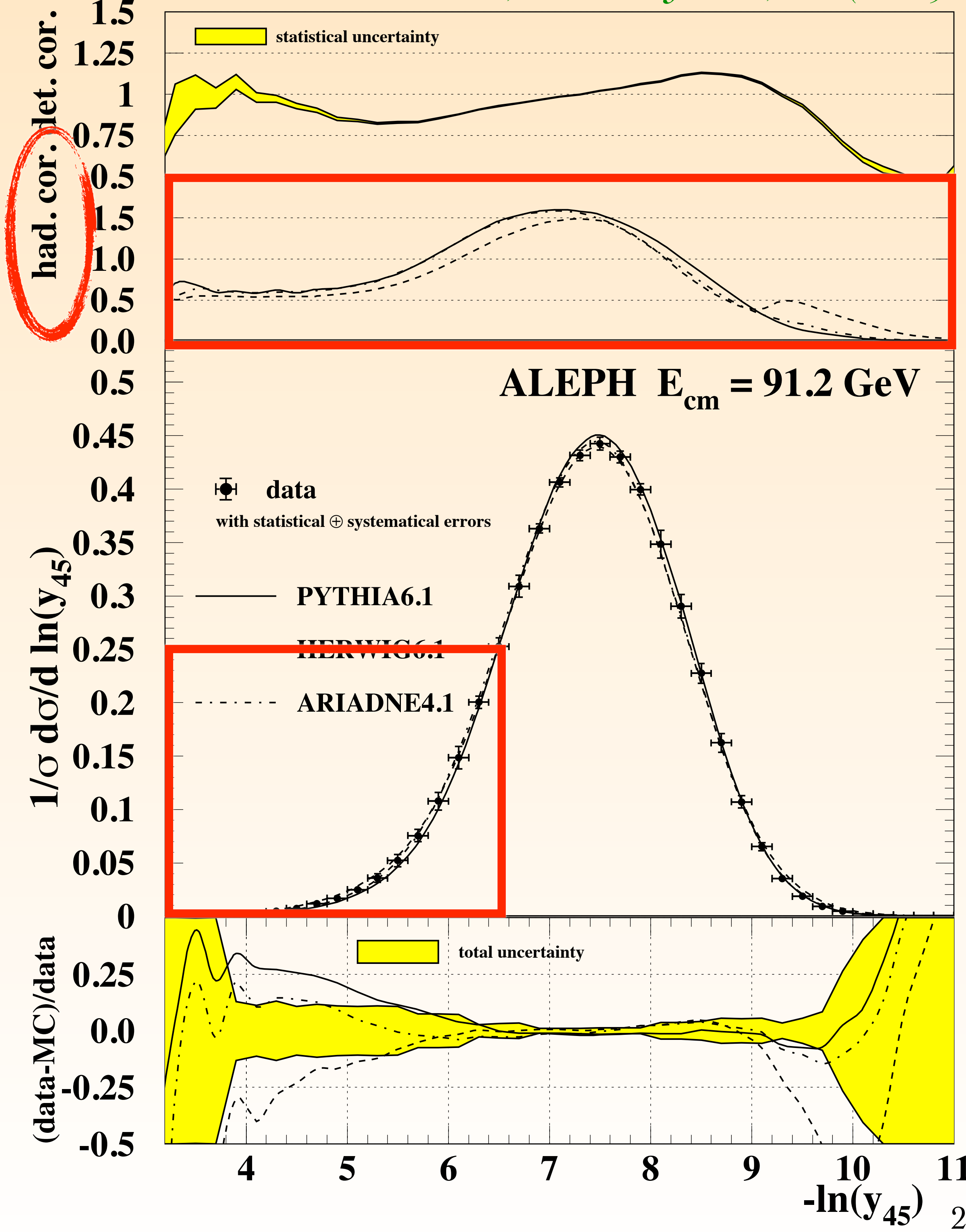
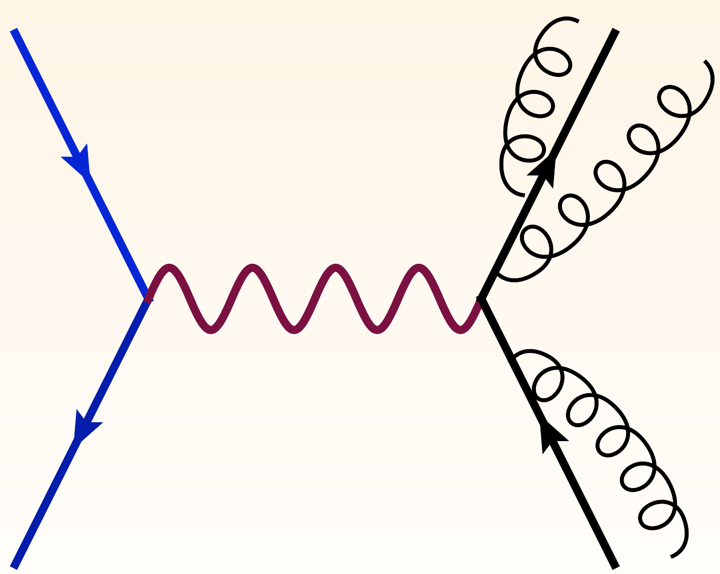


- Also in the perturbatively well-defined region, the hadronization corrections, as estimated by the ALEPH collaboration are large: ~100% for $3.5 < -\ln(y_{45}) < 5.5$

Hadronization cor.

- Over the years, the hadronization corrections have been estimate using Pythia, Herwig and Ariadne
- These are based on $2 \rightarrow 2$ (and $2 \rightarrow 3$) LO hard processes and where the extra radiation/jets are generated by the parton shower
- The parton shower uses a collinear approximation of the higher multiplicity matrix elements and gives the correct prescription of collinear partons
- Okay to describe event (shapes) based on 2-3 (maybe 4) well-separated partons
- For harder, well-separated partons it underestimates the rate
- To compensate for this, the hadronization corrections have been tuned in such a way that these tools describe the data

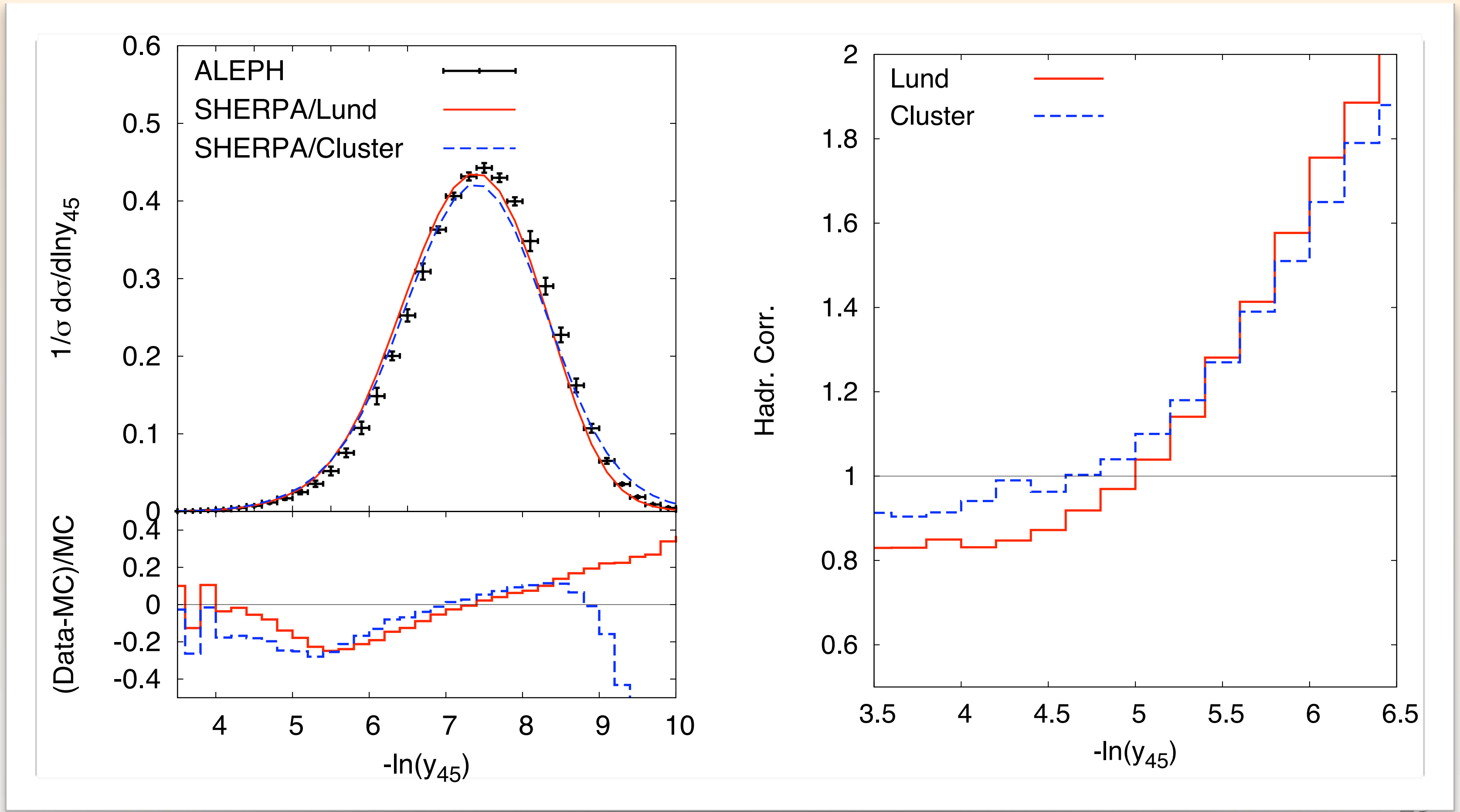


Hadronization cor.

- Sherpa uses CKKW matching for the merging of the parton shower with higher multiplicity matrix elements: it includes the $2 \rightarrow 5$ LO matrix elements consistently

- The hadronization corrections are much smaller, in particular in the region where perturbation theory is supposed to work best: ~25% for $3.5 < -\ln(y_{45}) < 5.5$ (compared to 100% in the traditional approach)

- We estimate the uncertainty in the hadronization corrections by using the two different models within sherpa



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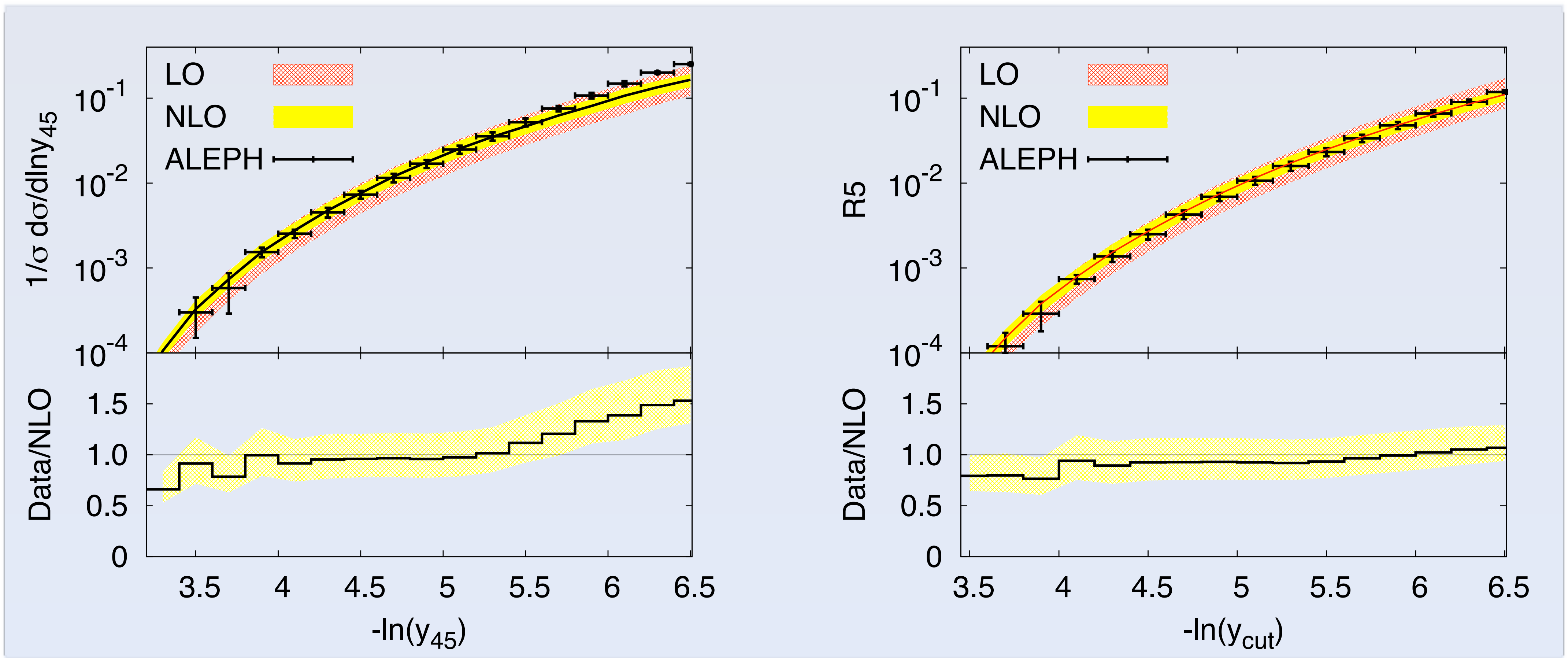

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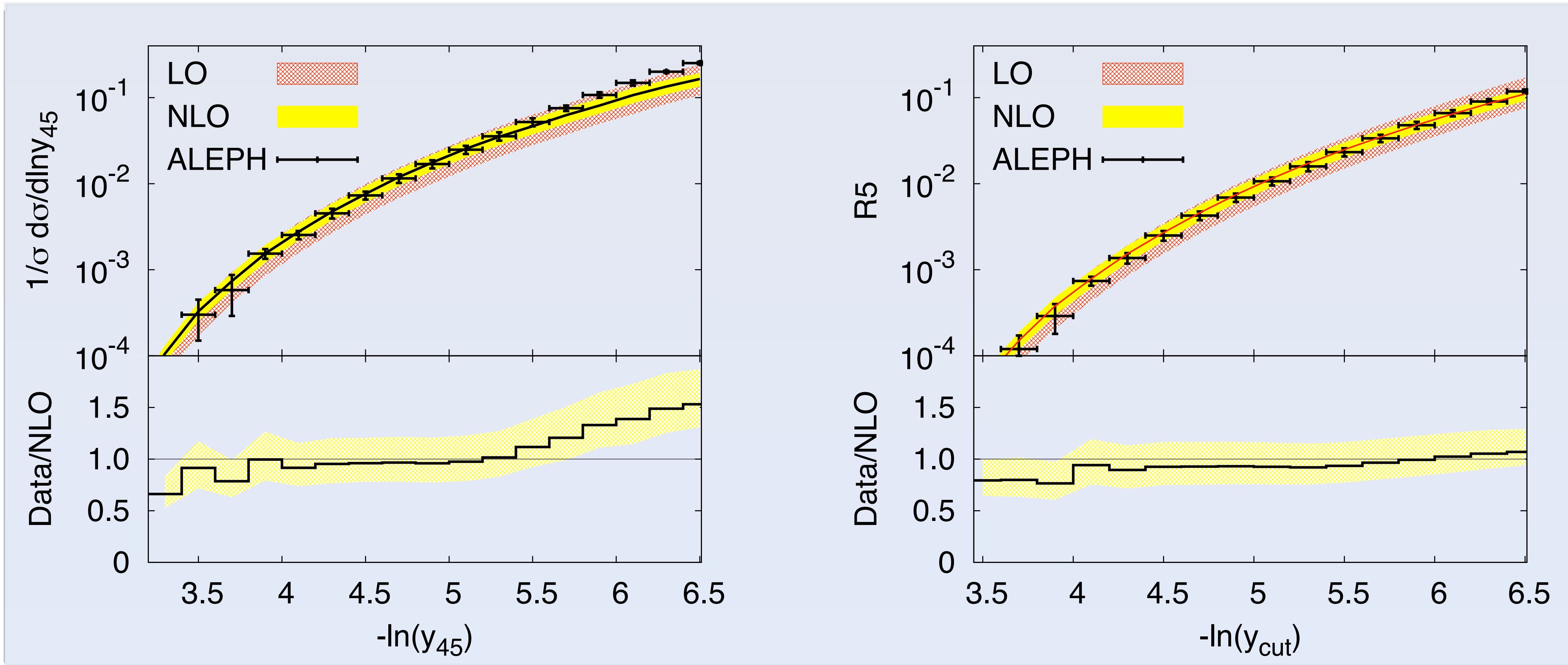

- What to do with it...?

$\alpha_s(M_Z)$ FIT

- Using the world average for the strong coupling, gives a good agreement between the NLO computation and the data
- We can turn this consideration around: using the data and the predictions we can extract the value for the strong coupling

$$\sigma_{\rm tot}^{-1}\frac{{\rm d}\sigma}{{\rm d}\ln y_{45}^{-1}} = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 A_{45}(y_{45}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 \left(B_{45}(y_{45}) + 3b_0 A_{45}(y_{45})\ln\frac{\mu}{\sqrt{s}}\right)$$

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High power of strong coupling: very sensitive to its value

Data used in the fit

Collision energy (GeV)		$1/\sigma\, d\sigma/dy_{45}$		R_5	
		range	data points*	range	data points*
LEP1	91	$3.8 < -\ln(y_{45}) < 5.3$	7 (11)	$4.0 < -\ln(y_{cut}) < 5.6$	8 (13)
LEP2	183	$4.8 < -\ln(y_{45}) < 6.4$	2 (1)	$2.1 < -\log_{10}(y_{cut}) < 2.9$	4 (2)
	189	$4.8 < -\ln(y_{45}) < 6.4$	2 (1)	$2.1 < -\log_{10}(y_{cut}) < 2.9$	4 (2)
	200	Not available		$2.1 < -\log_{10}(y_{cut}) < 2.9$	4 (2)
	206	$4.8 < -\ln(y_{45}) < 6.4$	2 (1)	$2.1 < -\log_{10}(y_{cut}) < 2.9$	4 (2)

- All data points are assumed to be separate measurements and treated as such (taking correlations in uncertainties into account)
- Number of data points between brackets is used to determine the "fit-range uncertainty"

Uncertainties

- Fit range
  The difference in α_s obtained by changing the range of data points that are included in the fit "in a reasonable way"
- Statistical (experimental)
  Assumed to be fully uncorrelated between different center-of-mass energies. Also the data points for $1/\sigma\, d\sigma/dy_{45}$ at a given energy are uncorrelated, but they are correlated for R_5 and between R_5 and $1/\sigma\, d\sigma/dy_{45}$. We have explicitly computed the correlation matrix
- Systematic (experimental)
  Assumed to be 100% correlated for all data points, except between LEP1 and LEP2 results (for which it's assumed to be uncorrelated)
- Perturbative
  Varying the renormalization scale used in the theoretical predictions by a factor two up and down around the central value. Assumed to be 100% correlated for all data points
- Hadronization
  Difference for the value of α_s using the *Lund* and *Cluster* models for hadronization as available in Sherpa. Assumed to be 100% correlated for all data points

LEP1 RESULTS

- LEP1
- Agreement with and without hadronization is remarkable
- Smaller perturbative uncertainty when including hadr. corr. due to smaller central value for α_s
- Five-jet observables are very sensitive to α_s which is reflected in the very small statistical error

	LEP1, hadr. $\sigma_{\rm tot}^{-1}\mathrm{d}\sigma/\mathrm{d}y_{45}$, R_5	LEP1, no hadr. $\sigma_{\rm tot}^{-1}\mathrm{d}\sigma/\mathrm{d}y_{45}$, R_5
stat.	$^{+0.0002}_{-0.0002}$	$^{+0.0002}_{-0.0002}$
syst.	$^{+0.0027}_{-0.0029}$	$^{+0.0027}_{-0.0029}$
pert.	$^{+0.0062}_{-0.0043}$	$^{+0.0068}_{-0.0047}$
fit range	$^{+0.0014}_{-0.0014}$	$^{+0.0005}_{-0.0005}$
hadr.	$^{+0.0012}_{-0.0012}$	–
$\alpha_s(M_Z)$	$0.1159^{+0.0070}_{-0.0055}$	$0.1163^{+0.0073}_{-0.0055}$

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- Agreement
  hadronizat
- Smaller pe
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  hadr. corr.
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- Five-jet observables are very
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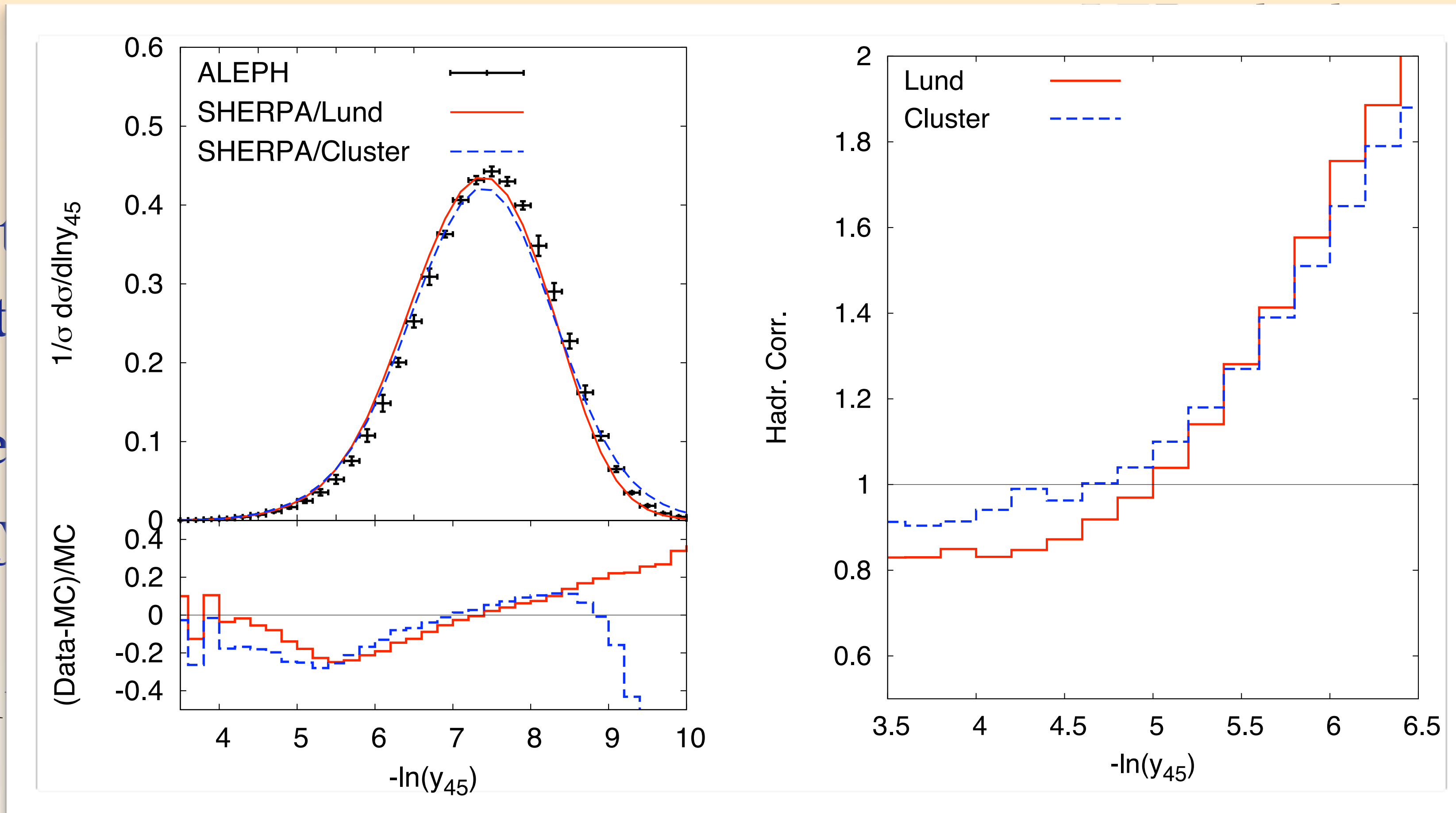

		LEP1, no hadr. $\sigma_{\rm tot}^{-1}d\sigma/dy_{45}$, R_5
		$^{+0.0002}_{-0.0002}$
		$^{+0.0027}_{-0.0029}$
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hadr.	$^{+0.0012}_{-0.0012}$	–
$\alpha_s(M_Z)$	$0.1159\,^{+0.0070}_{-0.0055}$	$0.1163\,^{+0.0073}_{-0.0055}$

LEP2

- LEP2
- Hadronization effects are negligible at LEP1: not even considered for LEP2 (they decrease with energy)
- Larger fit-range uncertainties, because we need to include data at smaller y_{cut}: more affected by large logarithms
- Statistical uncertainties are also larger due to less luminosity collected

	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$	LEP2, no hadr. R_5	LEP2, no hadr. $\sigma_{\rm tot}^{-1}{\rm d}\sigma/{\rm d}y_{45}$, R_5
stat.	$^{+0.0020}_{-0.0022}$	$^{+0.0022}_{-0.0025}$	$^{+0.0015}_{-0.0016}$
syst.	$^{+0.0008}_{-0.0009}$	$^{+0.0012}_{-0.0012}$	$^{+0.0008}_{-0.0008}$
pert.	$^{+0.0049}_{-0.0034}$	$^{+0.0029}_{-0.0020}$	$^{+0.0029}_{-0.0020}$
fit range	$^{+0.0038}_{-0.0038}$	$^{+0.0030}_{-0.0030}$	$^{+0.0028}_{-0.0028}$
$\alpha_s(M_Z)$	$0.1189^{+0.0066}_{-0.0057}$	$0.1120^{+0.0050}_{-0.0047}$	$0.1155^{+0.0044}_{-0.0039}$

- Perturbative uncertainty is much smaller due to smaller effective strong coupling at the higher energies

Final estimate

- Our final estimate for the value of the strong coupling by combining LEP1 and LEP2 data
- Statistical and systematic uncertainties are assumed to be uncorrelated, while perturbative uncertainties are assumed to be fully correlated
- We find

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$

$\alpha_s(M_Z^2)$

S. Alekhin, J.B., S. Klein, S. Moch, Phys. Rev. D81 (2010) 014032

$\delta\alpha_s(M_Z^2)/\alpha_s(M_Z^2) \approx 1\%$

	$\alpha_s(\mathrm{M}_Z^2)$	
ABKM	0.1135 ± 0.0014	HQ: FFS $N_f = 3$
A.Hoang et al.	$0.1135 \pm 0.0011 \pm 0.0006$	e^+e^- thrust
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach
BBG (2006)	$0.1134 \begin{array}{l} +0.0019 \\ -0.0021 \end{array}$	valence analysis, NNLO
JR (2008)	0.1124 ± 0.0020	dynamical approach
MSTW (2008)	0.1171 ± 0.0014	
H1/ZEUS (2010)	0.1145 ± 0.0042	(combined H1/ZEUS data, prelimiary)
ABM (2010)	0.1147 ± 0.0012	(FFN, combined H1/ZEUS data in)
BBG (2006)	$0.1141 \begin{array}{l} +0.0020 \\ -0.0022 \end{array}$	valence analysis, N^3LO
WA (2009)	0.1184 ± 0.0007	

J. Blümlein Status of DIS and PDFs for the LHC Wien, January 13th 2011 – p.33

- Our result: 0.1156 + 0.0041 - 0.0034 ($e^+e^- \rightarrow 5$ jets)

Conclusions

- We have calculated the process $e^+e^- \rightarrow 5$ jets for the first time at the Next-to-Leading Order
- We used the automated MadFKS code to set-up the calculation and we interfaced the virtual (loop) corrections by re-using amplitudes computed for $pp \rightarrow W+3j$ at NLO
- The predictions agree very well with the LEP data in the region of phase-space where fix-order perturbation theory is supposed to work
- Hadronization corrections are negligible when estimated with the Sherpa event generator, which uses CKKW matching to incorporate higher order matrix elements, in contrast to the traditional approaches using Pythia, Herwig and Ariadne

Conclusions

- Given that this process starts at α_s^3 at the Born level, it's very sensitive to the value of the strong coupling
- The five-jet observables R_5 and $1/\sigma\, d\sigma/dy_{45}$ have been used to extract a value of the strong coupling

  $$\alpha_s = 0.1156 + 0.0041 - 0.0034$$

  consistent with known results, but with a sizable uncertainty
- The total uncertainty is mainly due to remaining dependence on the renormalization scale as reflected in the "perturbative uncertainty"
- This calculation essentially closes the pure perturbative QCD studies of exclusive jetty final states at LEP