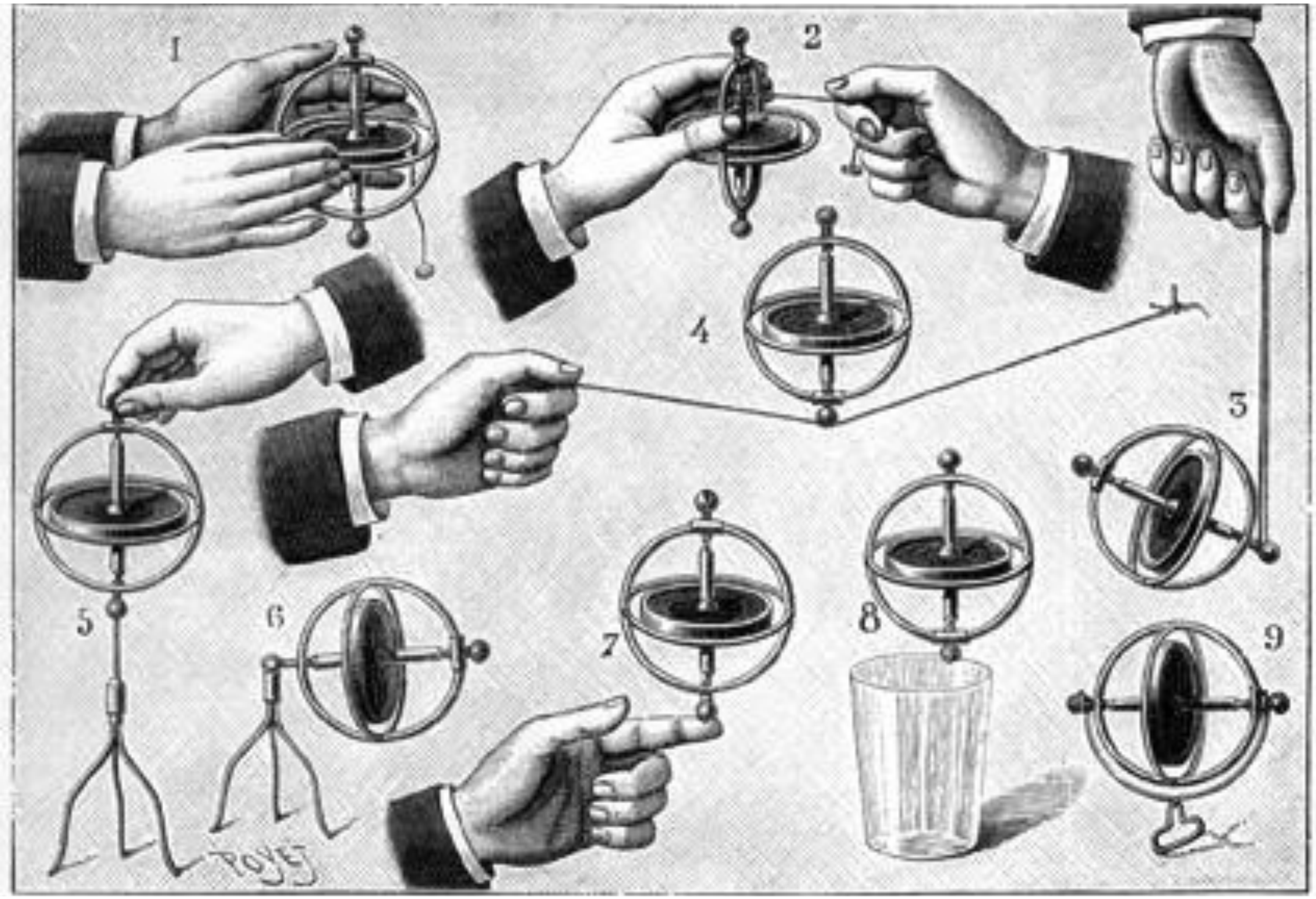
Collective Neutrino Oscillations



Georg G. Raffelt

3rd Schrödinger Lecture, Thursday 19 May 2011

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3300 citations

Neutrino oscillations in matter

L. Wolfenstein
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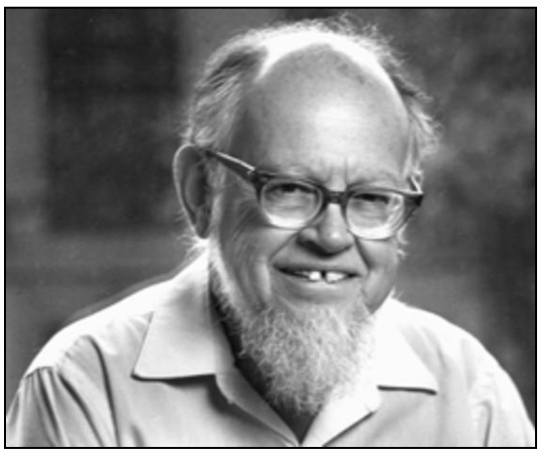
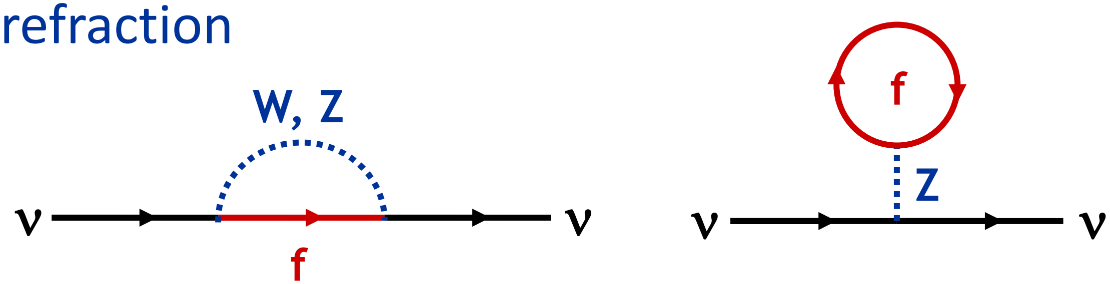
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

Neutrinos in a medium suffer flavor-dependent refraction



$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_{\mu} \end{cases}$$

Typical density of Earth: 5 g/cm^3

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

Neutrino Oscillations in Matter

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial z}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

With a 2×2 Hamiltonian matrix

Negative for $\overline{\nu}$

$$H = \frac{1}{2E}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \pm \sqrt{2}G_\mathrm{F}\begin{pmatrix}N_e - \frac{N_n}{2} & 0\\ 0 & -\frac{N_n}{2}\end{pmatrix}$$

Mass-squared matrix, rotated by mixing angle θ relative to interaction basis, drives oscillations

$$\frac{\Delta m^2}{2E} \sim \begin{cases} 4\ \mathrm{peV} & \text{for 12 mass splitting}\\ 120\ \mathrm{peV} & \text{for 13 mass splitting}\end{cases}$$

Solar, reactor and supernova neutrinos:
E ~ 10 MeV

Weak potential difference

$$\Delta V_\mathrm{weak} = \sqrt{2}G_\mathrm{F}N_e \sim 0.2\ \mathrm{peV}$$

for normal Earth matter, but large effect in SN core (nuclear density $3\times10^{14}\ \mathrm{g/cm^3}$)

$$\Delta V_\mathrm{weak} \sim 10\ \mathrm{eV}$$

Suppression of Oscillations in Supernova Core

Effective mixing angle in matter

$$\tan 2\theta_{\rm m} = \frac{\sin 2\theta}{\cos 2\theta - N_e 2E\sqrt{2}G_{\rm F}/\Delta m^2}$$

Supernova core

$$\rho = 3 \times 10^{14}\ {\rm g\,cm^{-3}}$$
$$Y_e = 0.35$$
$$N_e = 6 \times 10^{37}\ {\rm cm^{-3}}$$
$$E \sim 100\ {\rm MeV}$$

Solar mixing

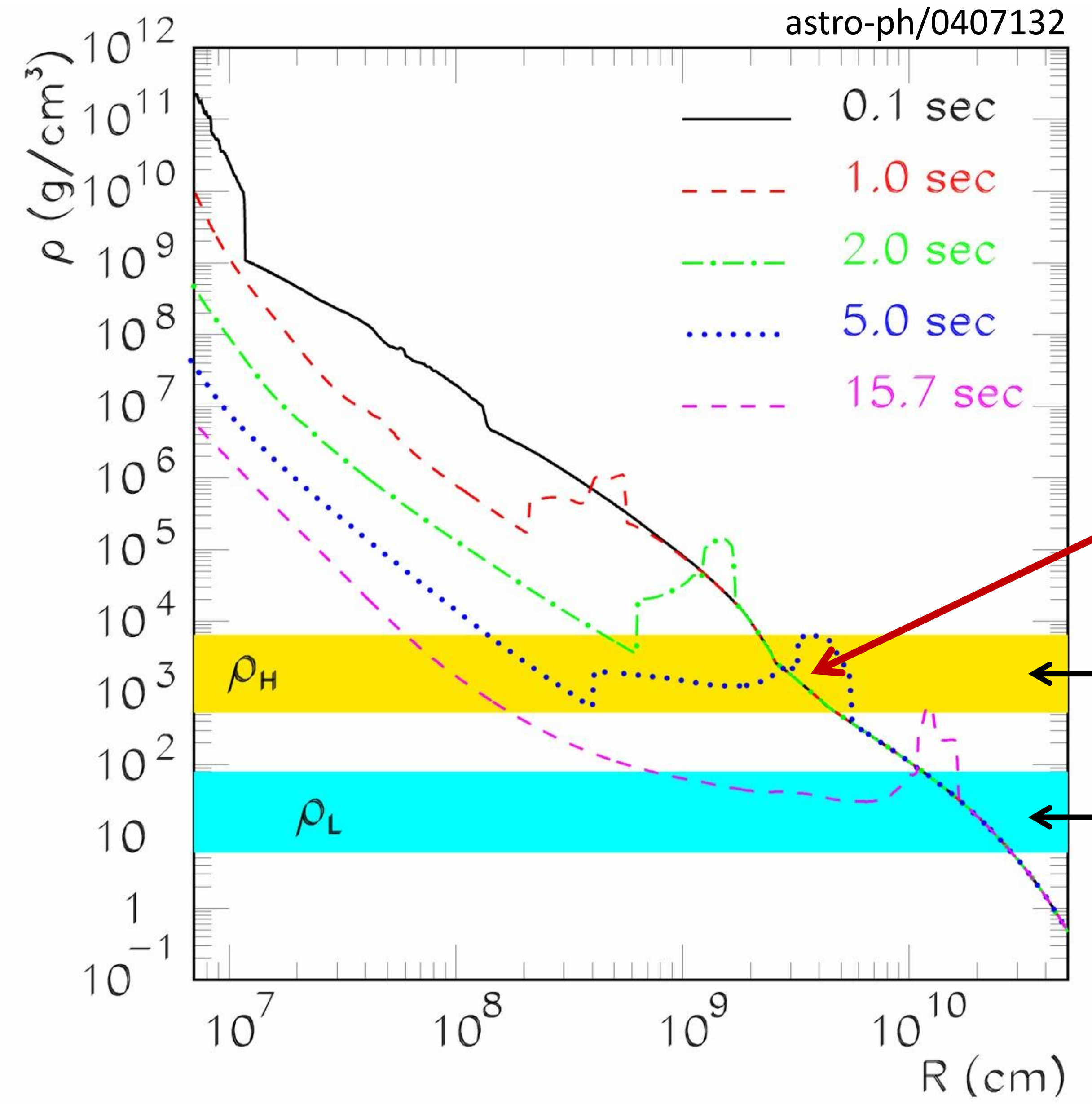
$$\Delta m^2 \sim 75\ {\rm meV}^2$$
$$\sin 2\theta \sim 0.94$$

Matter suppression effect

$$N_e 2E\sqrt{2}G_F/\Delta m^2 \sim 2 \times 10^{13}$$

- Inside a SN core, flavors are “de-mixed”
- Very small oscillation amplitude
- Trapped e-lepton number can only escape by diffusion

Snap Shots of Supernova Density Profiles



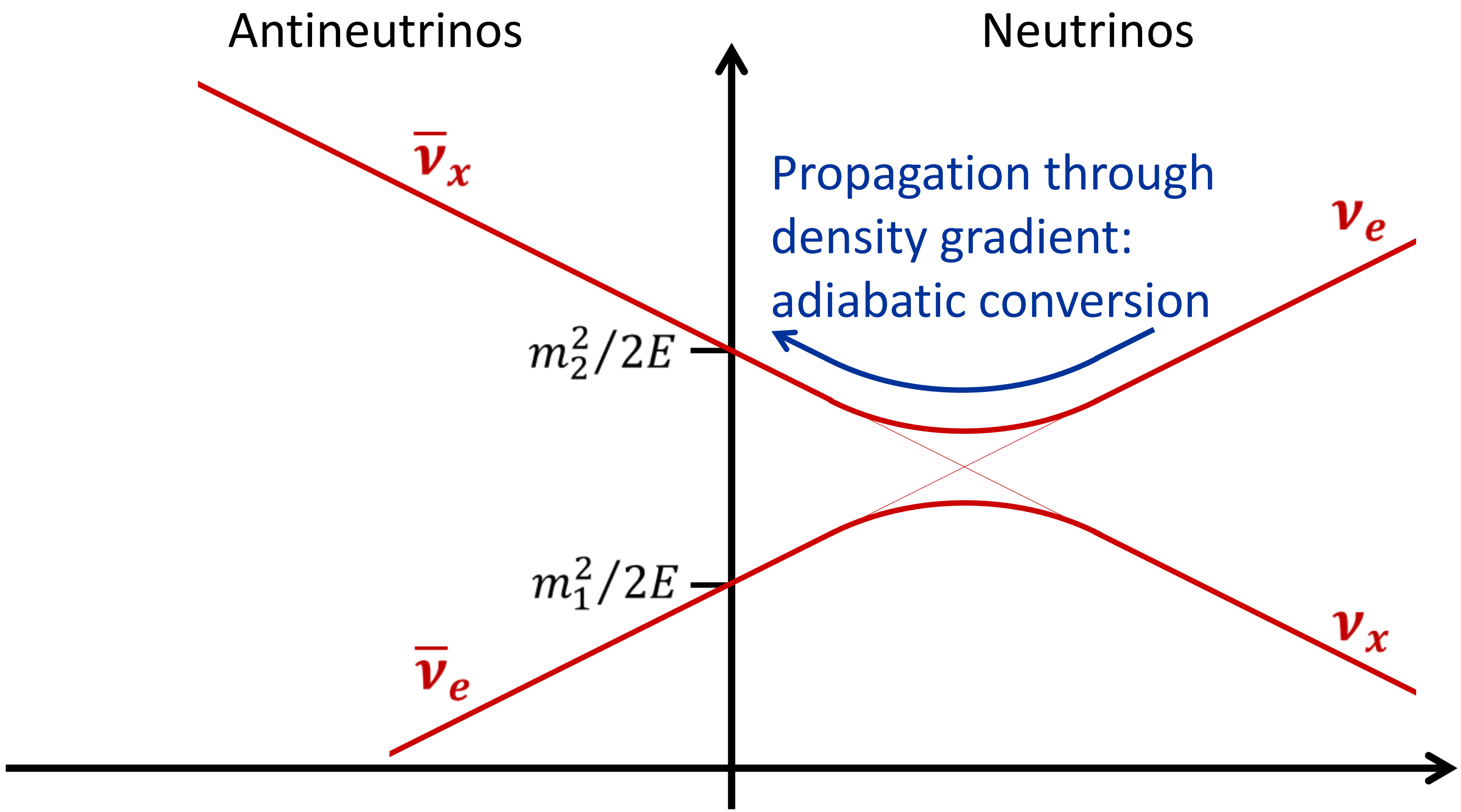
- $\bar{\nu}_e$-$\bar{\nu}_{\mu,\tau}$ conversions, driven by small mixing angle θ_{13} and "atmospheric" mass difference Δm^2_{13}
- May reveal neutrino mass hierarchy

← H-resonance (13 splitting)

← L-resonance (12 splitting)

Mikheev-Smirnov-Wolfenstein (MSW) effect

Eigenvalue diagram of 2×2 Hamiltonian matrix for 2-flavor oscillations



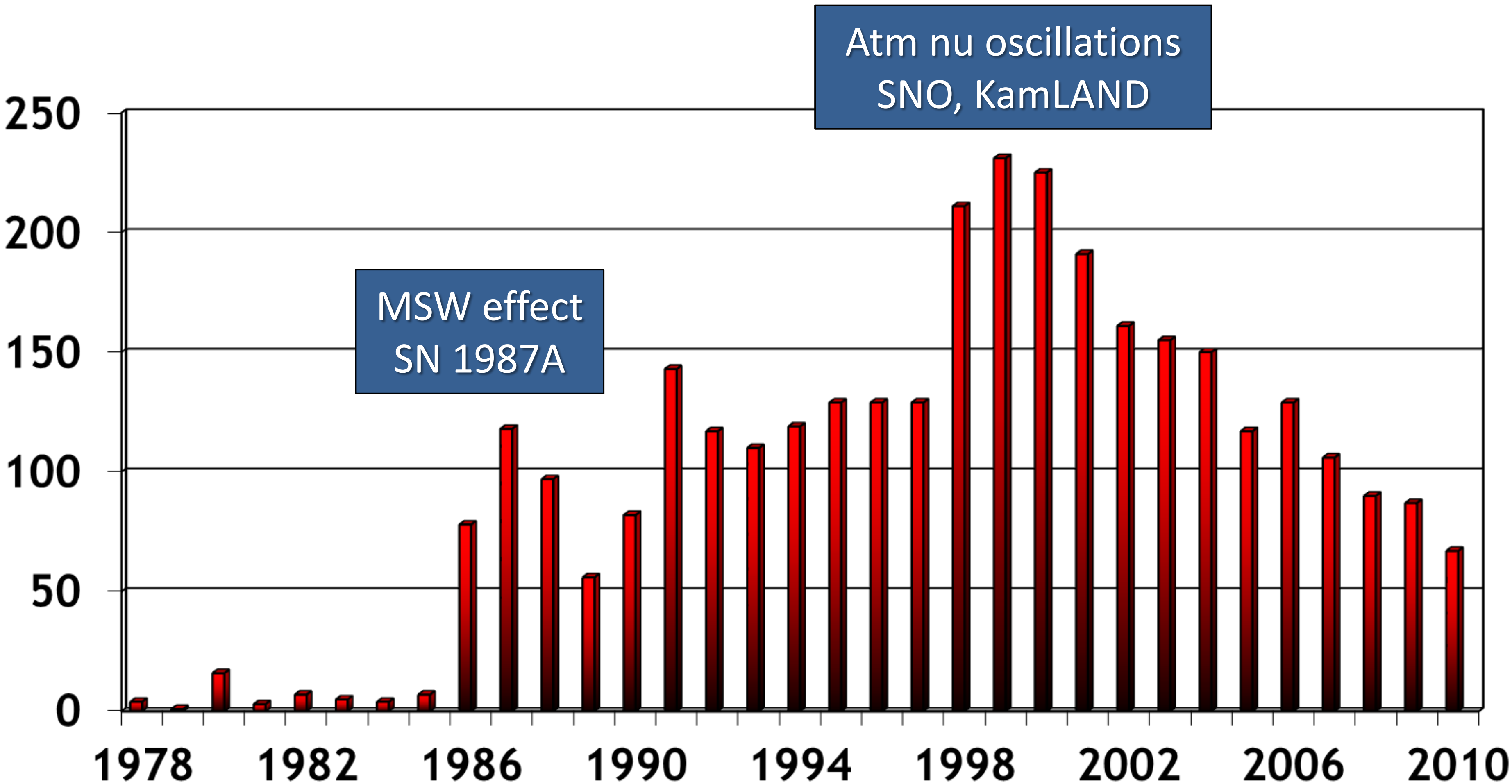
"Negative density" represents antineutrinos in the same diagram

Vacuum

Density

Stanislaw Mikheev († 23 April 2011)

Citations of Wolfenstein's Paper on Matter Effects

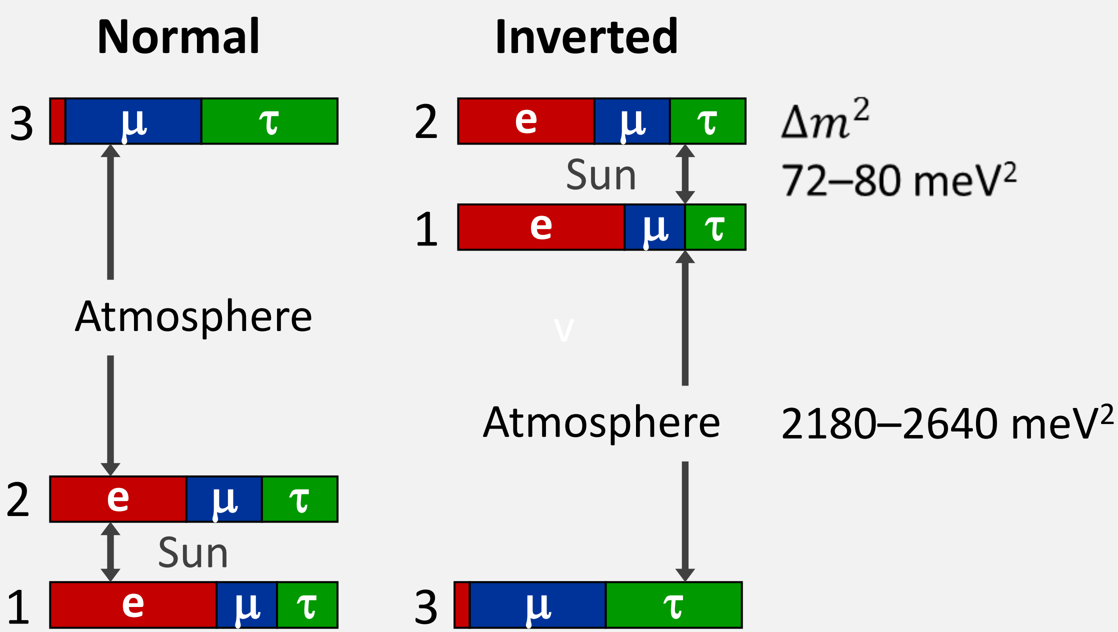


Annual citations of Wolfenstein, PRD 17:2369, 1978
in the SPIRES data base (total of 3278 citations 1978–2010)

Three-Flavor Neutrino Parameters

Three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ (Euler angles for 3D rotation), $c_{ij} = \cos\theta_{ij}$,
a CP-violating "Dirac phase" δ, and two "Majorana phases" α_2 and α_3

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}}_{\substack{39^\circ < \theta_{23} < 53^\circ\\ \text{Atmospheric/LBL-Beams}}} \underbrace{\begin{pmatrix}c_{13} & 0 & e^{-i\delta}s_{13}\\ 0 & 1 & 0\\ -e^{i\delta}s_{13} & 0 & c_{13}\end{pmatrix}}_{\substack{\theta_{13} < 11^\circ\\ \text{Reactor}}} \underbrace{\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}}_{\substack{33^\circ < \theta_{12} < 37^\circ\\ \text{Solar/KamLAND}}} \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\frac{\alpha_2}{2}} & 0\\ 0 & 0 & e^{i\frac{\alpha_3}{2}}\end{pmatrix}}_{\substack{\text{Relevant for}\\ 0\nu2\beta\text{ decay}}} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

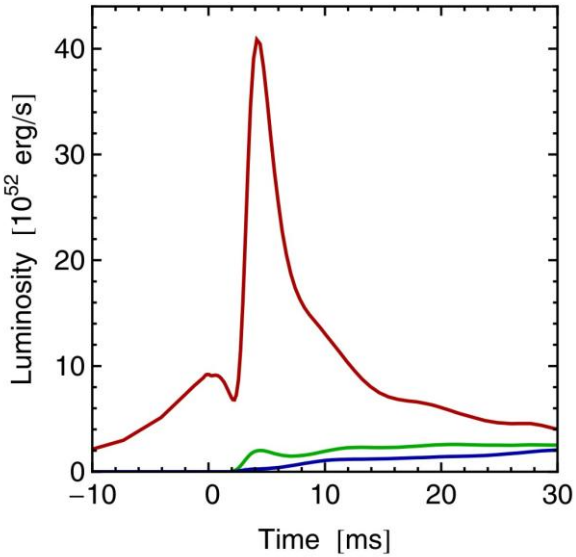


Tasks and Open Questions

- Precision for θ_{12} and θ_{23}
- How large is θ_{13}?
- CP-violating phase δ?
- Mass ordering? (normal vs inverted)
- Absolute masses? (hierarchical vs degenerate)
- Dirac or Majorana?

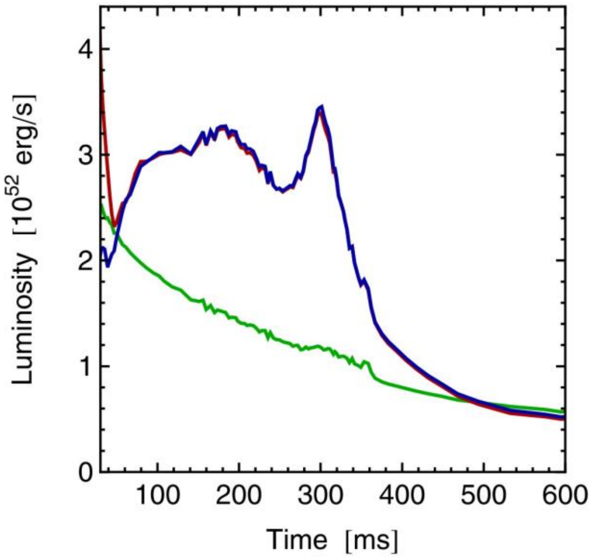
Three Phases of Neutrino Emission

Prompt ν_e burst



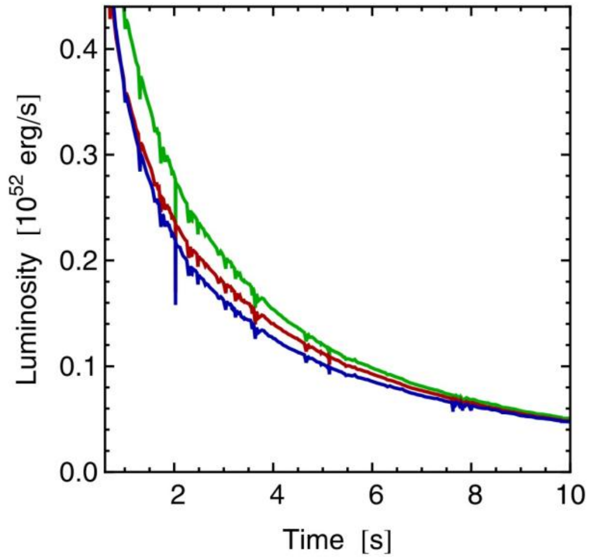
- Shock breakout
- De-leptonization of outer core layers

Accretion



- Shock stalls ~ 150 km
- Neutrinos powered by infalling matter

Cooling



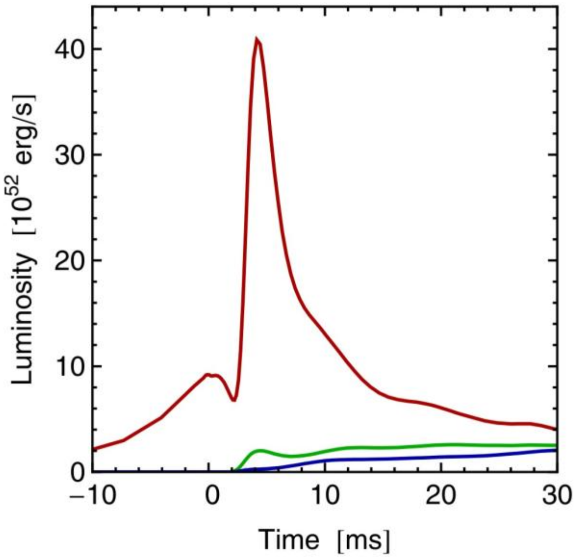
Cooling on neutrino diffusion time scale

- Spherically symmetric model (10.8 $M_\odot$) with Boltzmann neutrino transport
- Explosion manually triggered by enhanced CC interaction rate

Fischer et al. (Basel group), A&A 517:A80, 2010 [arxiv:0908.1871]

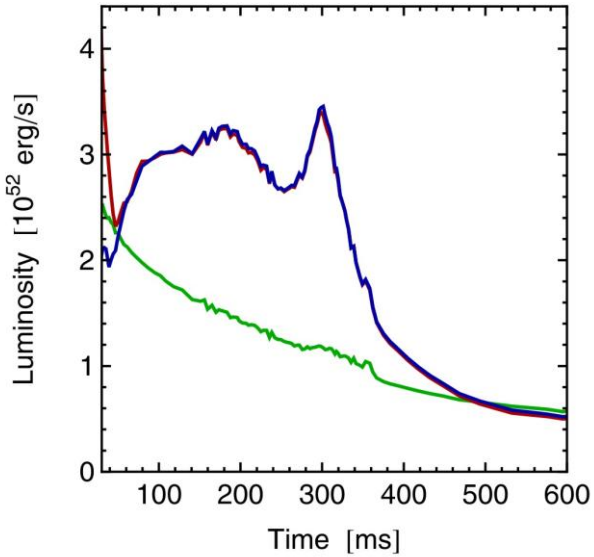
Three Phases of Neutrino Emission

Prompt ν_e burst



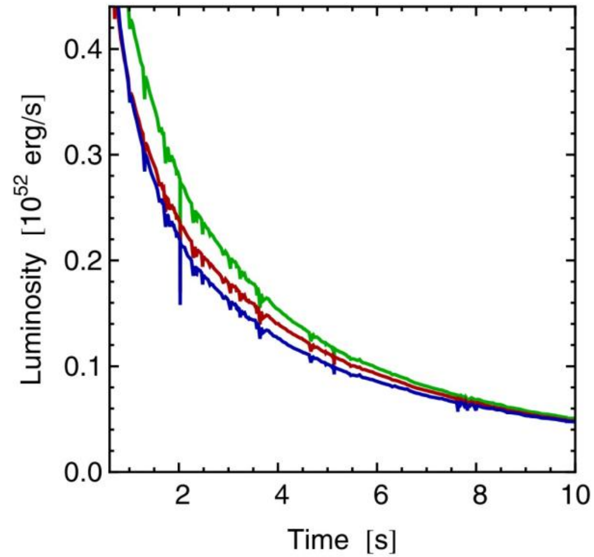
No large ν_e detector available

Accretion



Large fluxes and large $\bar{\nu}_e$-$\bar{\nu}_x$ flux differences

Cooling



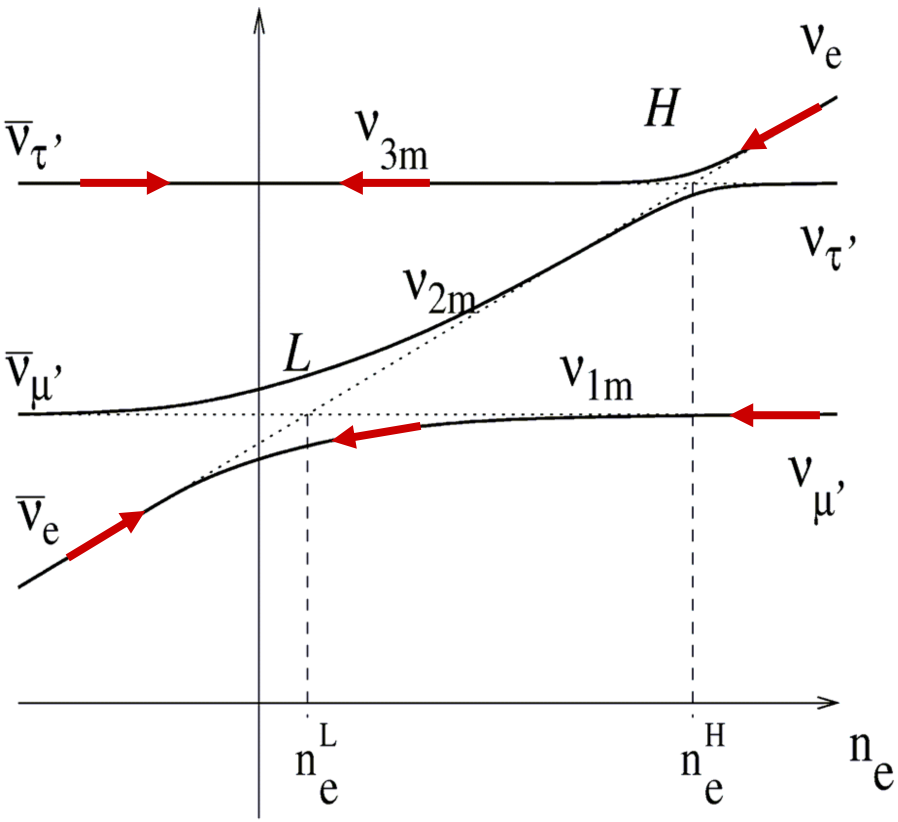
Smaller fluxes and small $\bar{\nu}_e$-$\bar{\nu}_x$ flux differences

- Spherically symmetric model (10.8 $M_\odot$) with Boltzmann neutrino transport
- Explosion manually triggered by enhanced CC interaction rate

Fischer et al. (Basel group), A&A 517:A80, 2010 [arxiv:0908.1871]

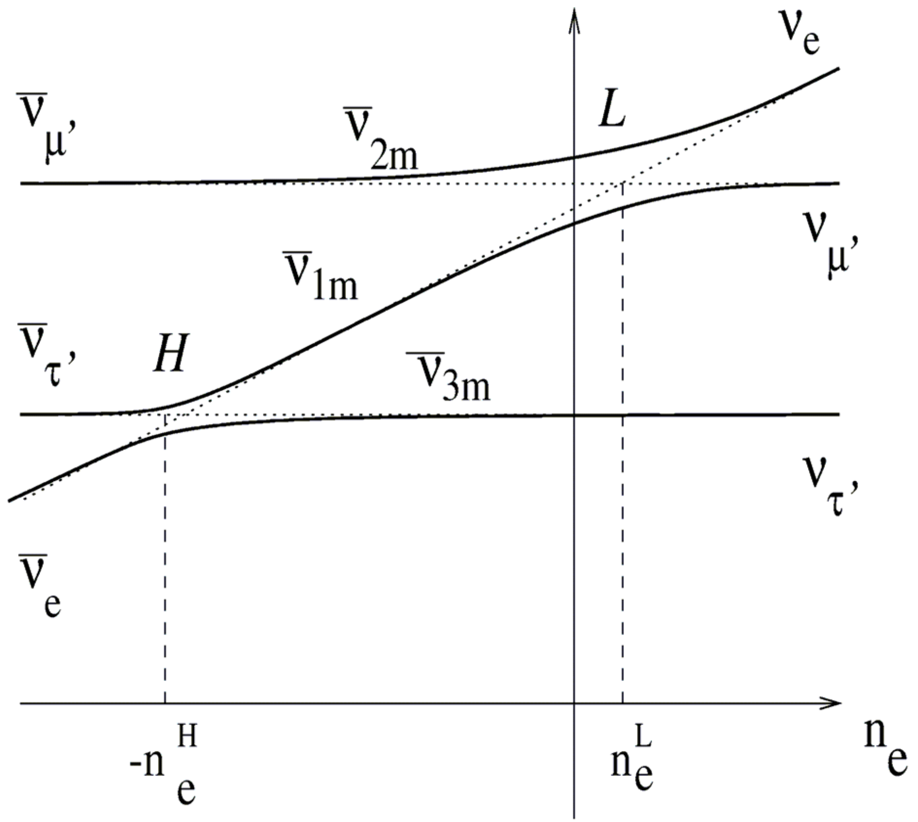
Level-Crossing Diagram in a Supernova Envelope

Normal mass hierarchy



Vacuum

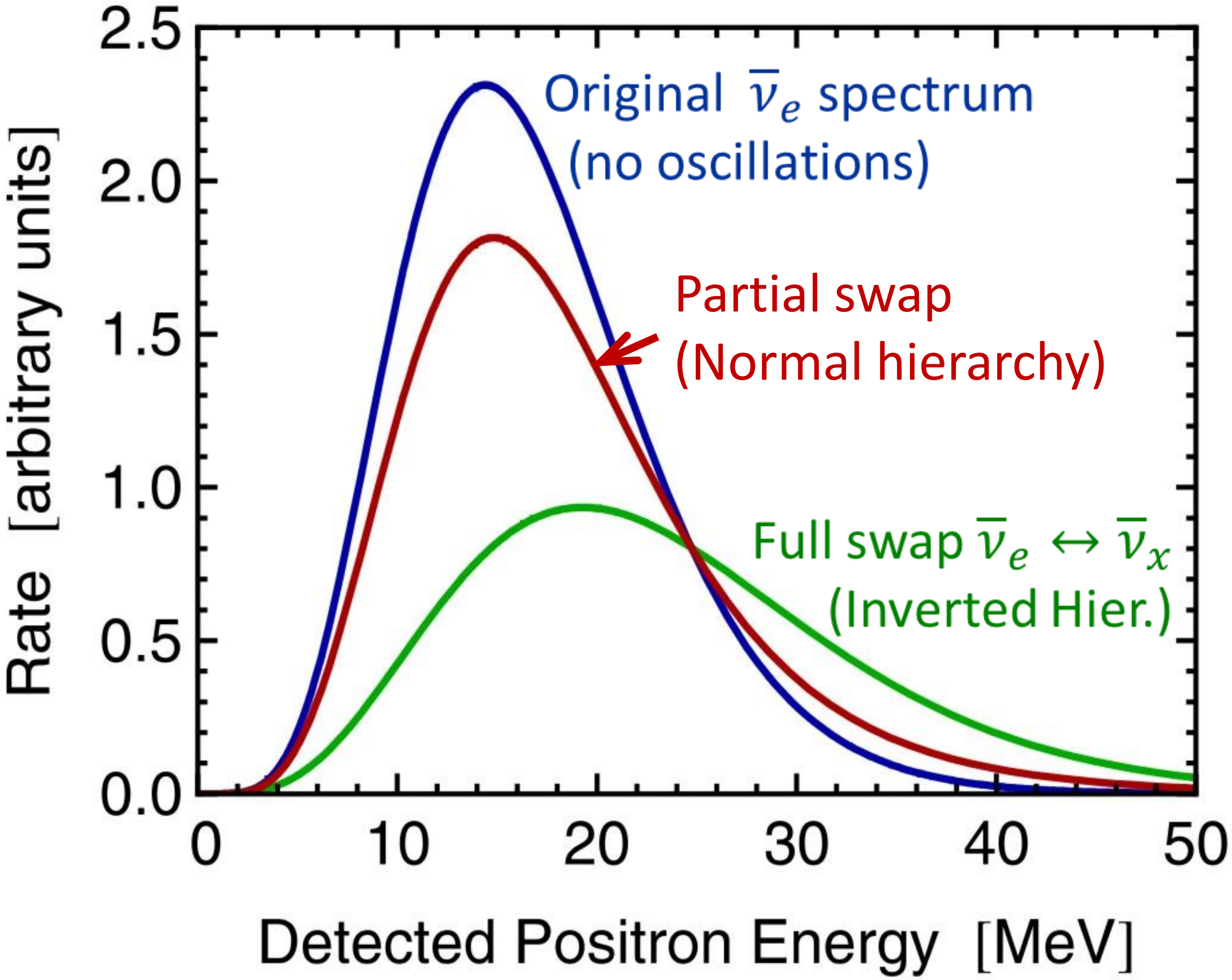
Inverted mass hierarchy



Vacuum

Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

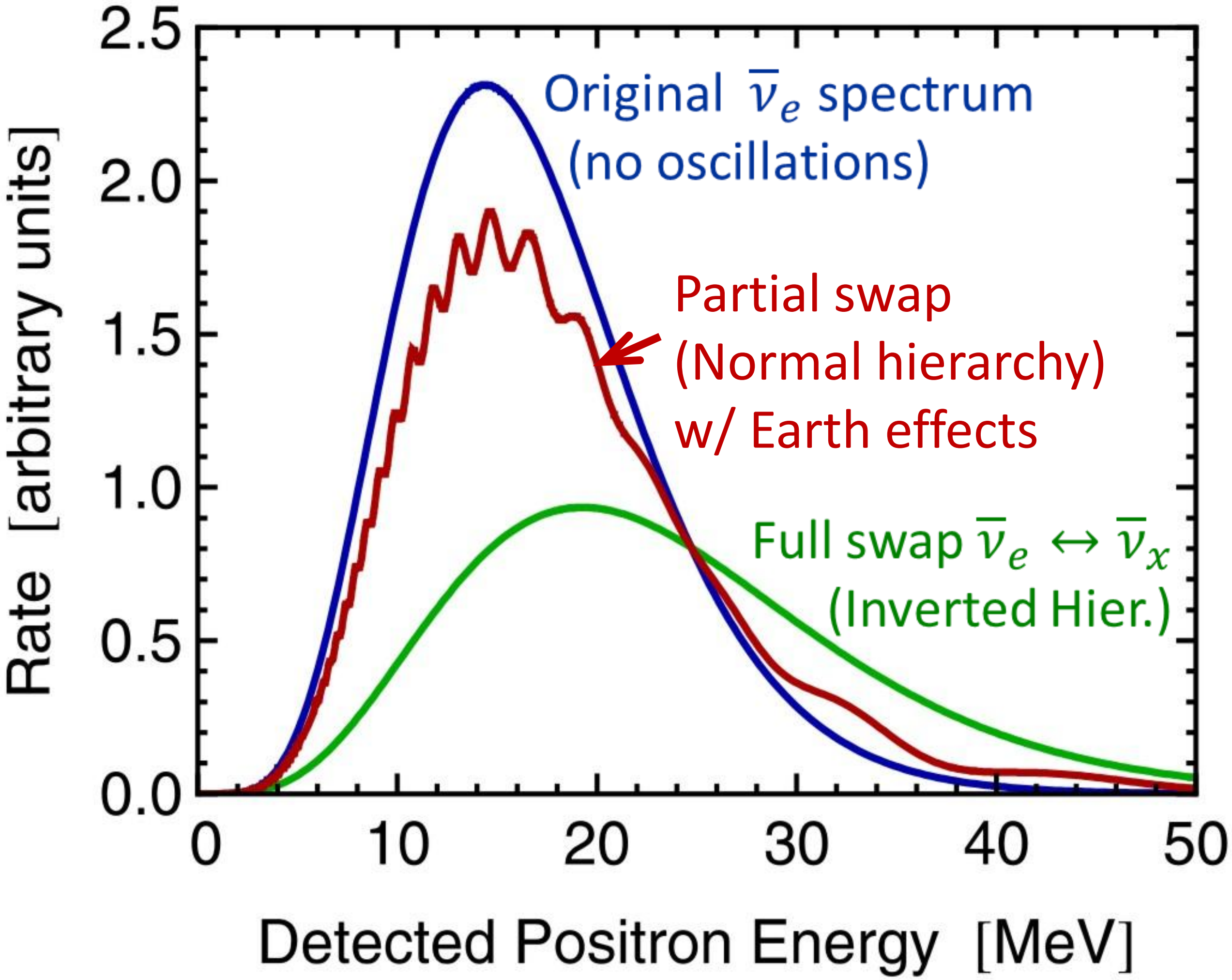
Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model (10.8 $M_\odot$)

Detection spectrum by $\overline{\nu}_e + p \to n + e^+$ (water Cherenkov or scintillator detectors)

Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model (10.8 $M_\odot$)

Detection spectrum by $\bar{\nu}_e + p \to n + e^+$ (water Cherenkov or scintillator detectors)

8000 km path length in Earth assumed

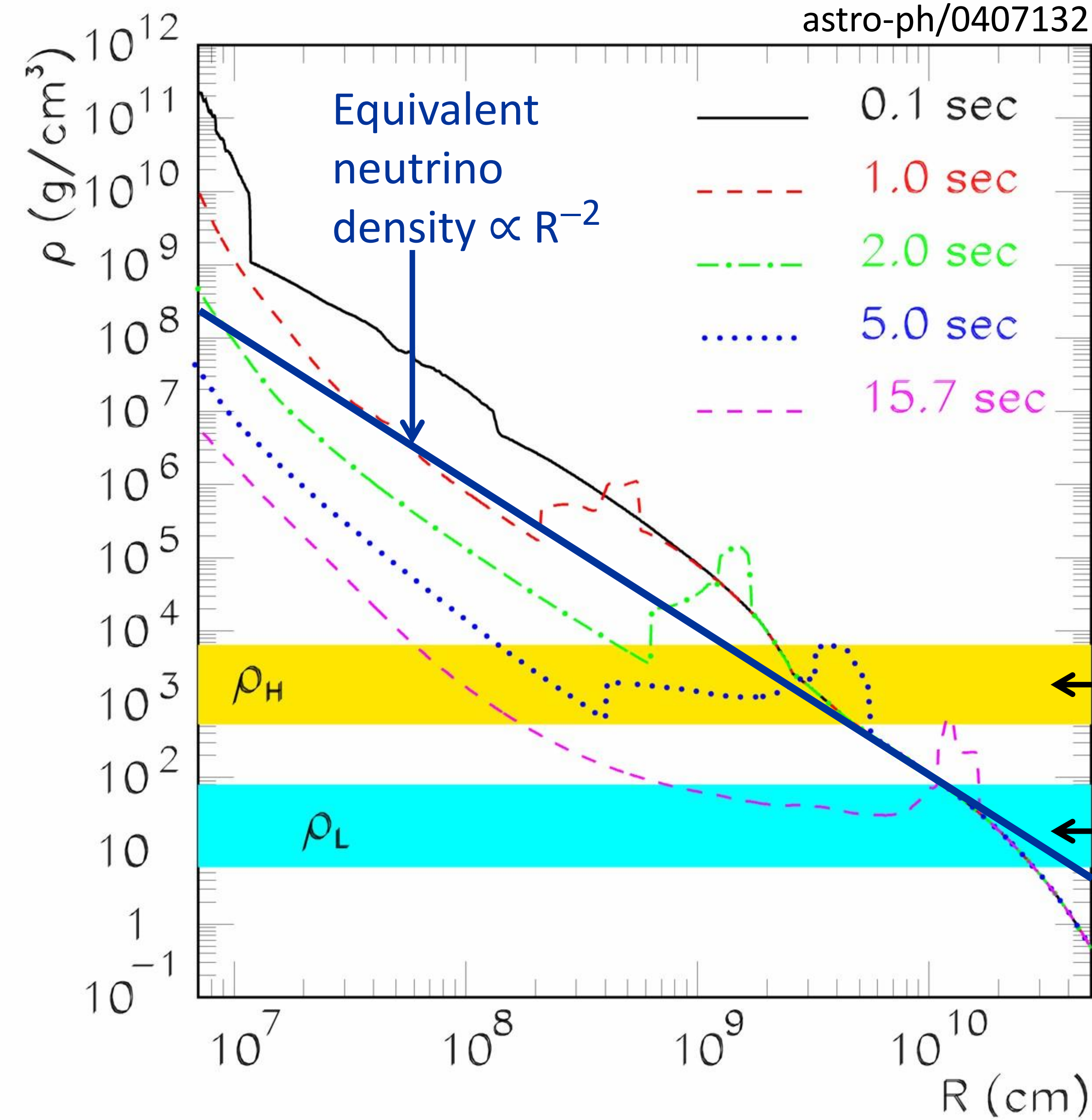
Detecting Earth effects requires good energy resolution
(Large scintillator detector, e.g. LENA, or megaton water Cherenkov)

Signature of Flavor Oscillations (Accretion Phase)

	1-3-mixing scenarios		
	A	B	C
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)
$\sin^2\theta_{13}$	$\gtrsim 10^{-3}$		$\lesssim 10^{-5}$
MSW conversion	adiabatic		non-adiabatic
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$	$\sin^2\theta_{12} \approx 0.3$
$\bar{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0	$\cos^2\theta_{12} \approx 0.7$
$\bar{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase
(Chakraborty et al., arXiv:1105.1130v1)

Snap Shots of Supernova Density Profiles



Accretion-phase luminosity

$$L_{\bar{\nu}_e} \sim 3 \times 10^{52}\,\mathrm{erg/s}$$

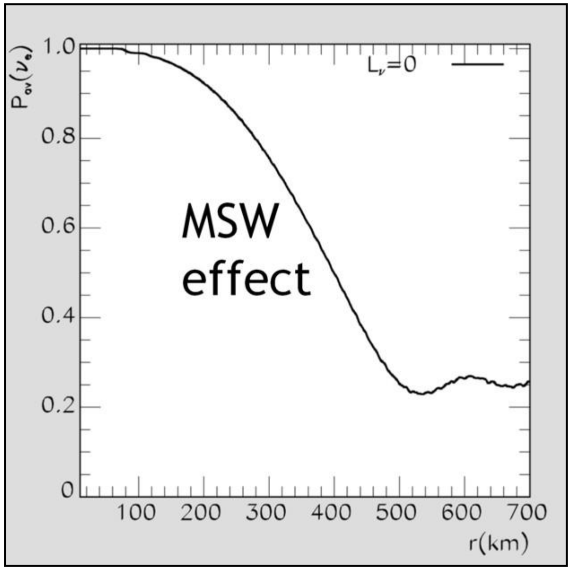
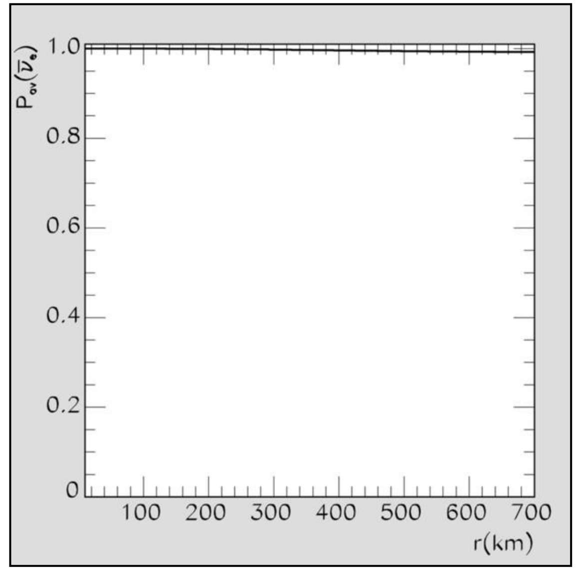
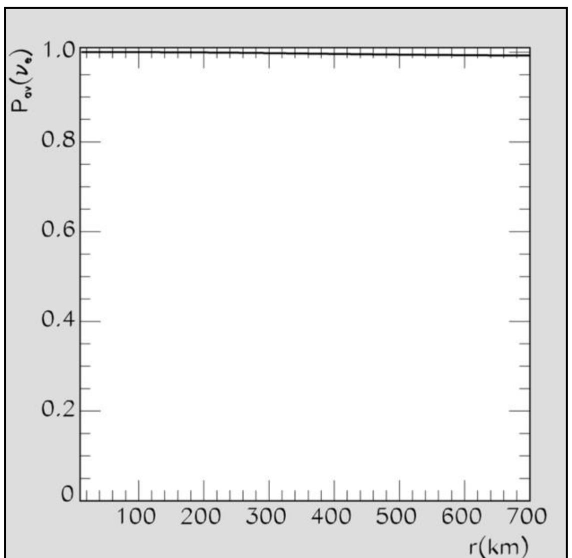
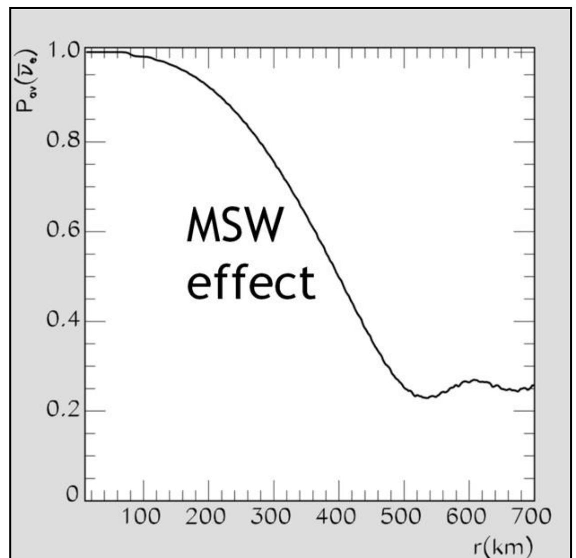
Corresponds to a neutrino number density of

$$3 \times 10^{33}\,\mathrm{cm}^{-3} \left(\frac{10\ \mathrm{km}}{R}\right)^2$$

← H-resonance (13 splitting)

← L-resonance (12 splitting)

Self-Induced Flavor Oscillations of SN Neutrinos

	Survival probability ν_e	Survival probability $\bar{\nu}_e$	
Normal Hierarchy **atm Δm^2** **θ_{13} close to Chooz limit**			No nu-nu effect
Inverted Hierarchy			No nu-nu effect

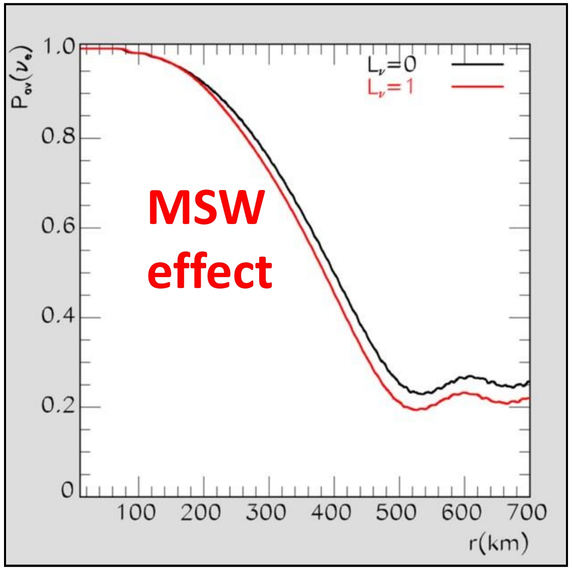
Self-Induced Flavor Oscillations of SN Neutrinos

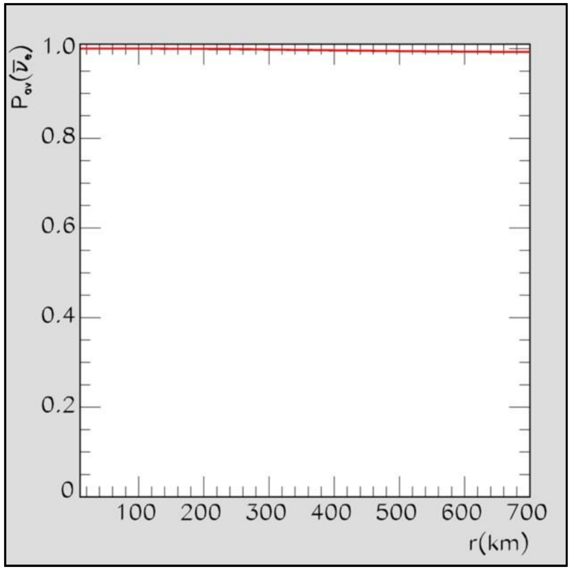
Survival probability ν_e | Survival probability $\bar{\nu}_e$

Normal Hierarchy

atm Δm^2

θ_{13} close to Chooz limit

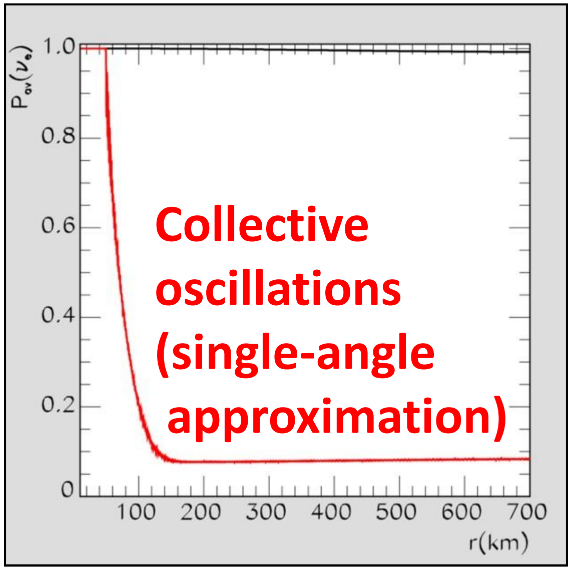


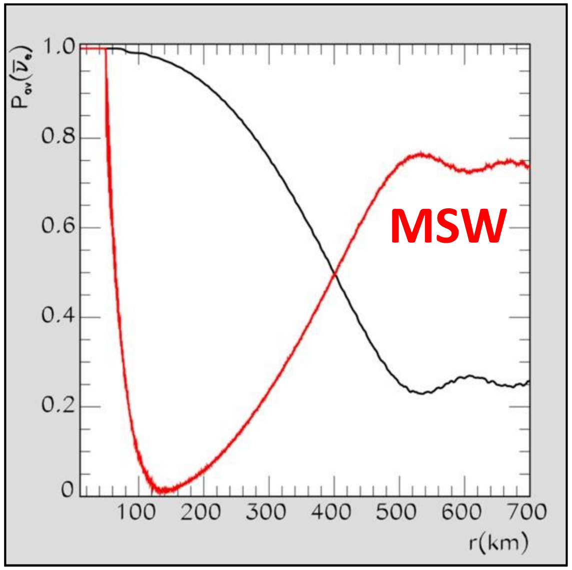


Realistic nu-nu effect

No nu-nu effect

Inverted Hierarchy





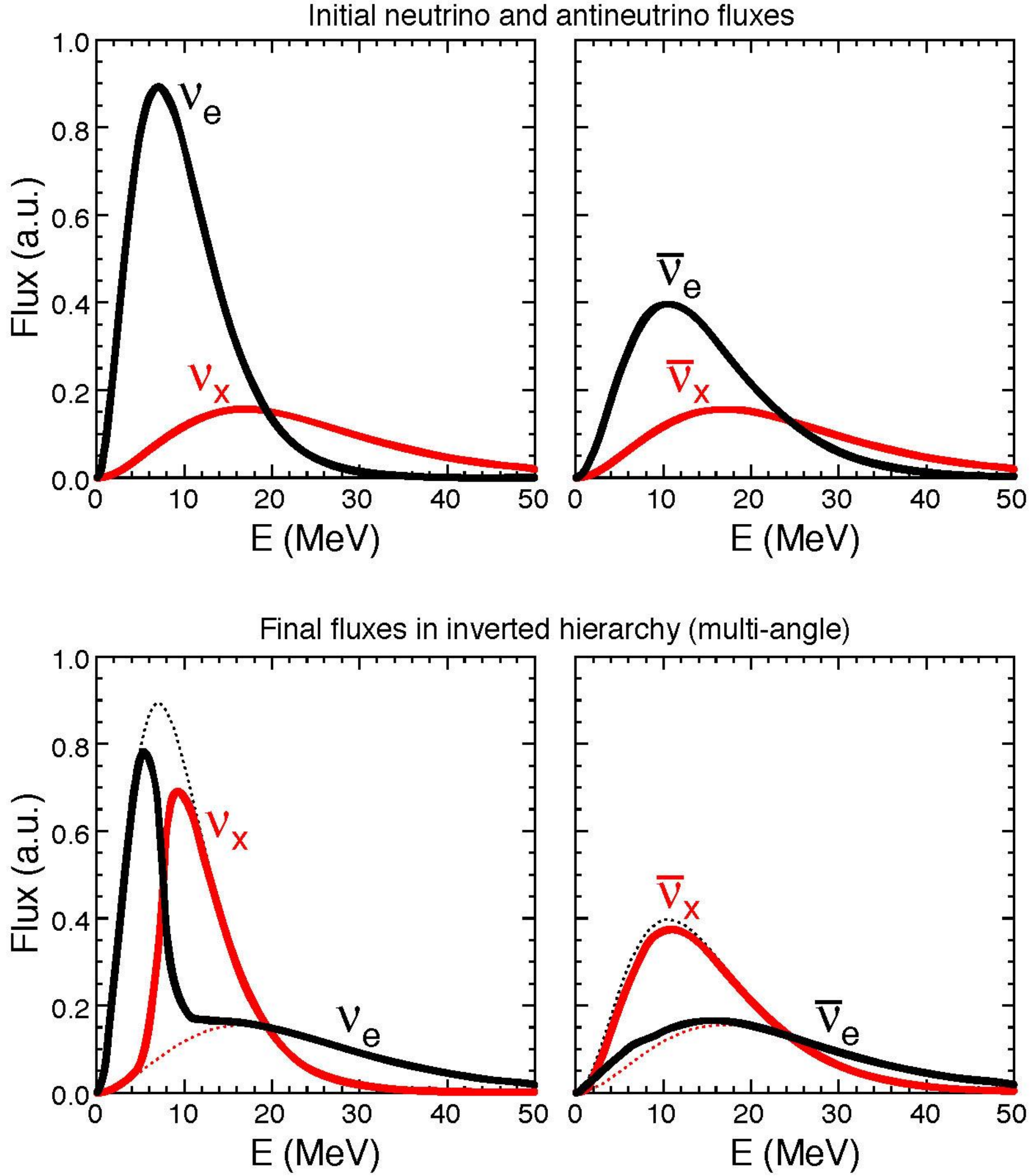
Realistic nu-nu effect

No nu-nu effect

Spectral Split

Initial fluxes at neutrino sphere

After collective trans-formation



Figures from

Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Collective Supernova Nu Oscillations since 2006

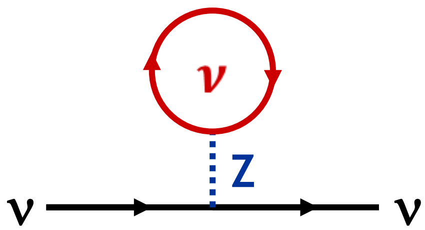
Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe, arXiv:0710.3112. Balantekin & Pehlivan, astro-ph/0607527. Blennow, Mirizzi & Serpico, arXiv:0810.2297. Cherry, Fuller, Carlson, Duan & Qian, arXiv:1006.2175. Chakraborty, Choubey, Dasgupta & Kar, arXiv:0805.3131. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs, arXiv:1104.4031, 1105.1130. Choubey, Dasgupta, Dighe & Mirizzi, arXiv:1008.0308. Dasgupta & Dighe, arXiv:0712.3798. Dasgupta, Dighe & Mirizzi, arXiv:0802.1481. Dasgupta, Dighe, Mirizzi & Raffelt, arXiv:0801.1660, 0805.3300. Dasgupta, Mirizzi, Tamborra & Tomàs, arXiv:1002.2943. Dasgupta, Dighe, Raffelt & Smirnov, 0904.3542. Dasgupta, Raffelt, Tamborra, arXiv:1001.5396. Duan, Fuller, Carlson & Qian, astro-ph/0608050, 0703776, arXiv:0707.0290, 0710.1271. Duan, Fuller & Qian, arXiv:0706.4293, 0801.1363, 0808.2046, 1001.2799. Duan, Fuller & Carlson, arXiv:0803.3650. Duan & Kneller, arXiv:0904.0974. Duan & Friedland, arXiv:1006.2359. Duan, Friedland, McLaughlin & Surman, arXiv:1012.0532. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0706.2498, 0712.1137. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659. Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998. Fogli, Lisi, Marrone & Tamborra, arXiv:0812.3031. Friedland, arXiv:1001.0996. Gava & Jean-Louis, arXiv:0907.3947. Gava & Volpe, arXiv:0807.3418. Galais, Kneller & Volpe, arXiv:1102.1471. Galais & Volpe, arXiv:1103.5302. Gava, Kneller, Volpe & McLaughlin, arXiv:0902.0317. Hannestad, Raffelt, Sigl & Wong, astro-ph/0608695. Wei Liao, arXiv:0904.0075, 0904.2855. Lunardini, Müller & Janka, arXiv:0712.3000. Mirizzi, Pozzorini, Raffelt & Serpico, arXiv:0907.3674. Mirizzi & Tomàs, arXiv:1012.1339. Pehlivan, Balantekin, Kajino, Yoshida, arXiv:1105.1182. Raffelt, arXiv:0810.1407, 1103.2891. Raffelt & Tamborra, arXiv:1006.0002. Raffelt & Sigl, hep-ph/0701182. Raffelt & Smirnov, arXiv:0705.1830, 0709.4641. Sawyer, hep-ph/0408265, 0503013, arXiv:0803.4319, 1011.4585. Wu & Qian, arXiv:1105.2068.

Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



Effective mixing Hamiltonian

$$H = \frac{M^2}{2E} + \sqrt{2}G_\mathrm{F}\begin{pmatrix}N_e - \frac{N_n}{2} & 0\\ 0 & -\frac{N_n}{2}\end{pmatrix} + \sqrt{2}G_\mathrm{F}\begin{pmatrix}N_{\nu_e} & N_{\langle\nu_e|\nu_\mu\rangle}\\ N_{\langle\nu_\mu|\nu_e\rangle} & N_{\nu_\mu}\end{pmatrix}$$

Mass term in flavor basis: causes vacuum oscillations

Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

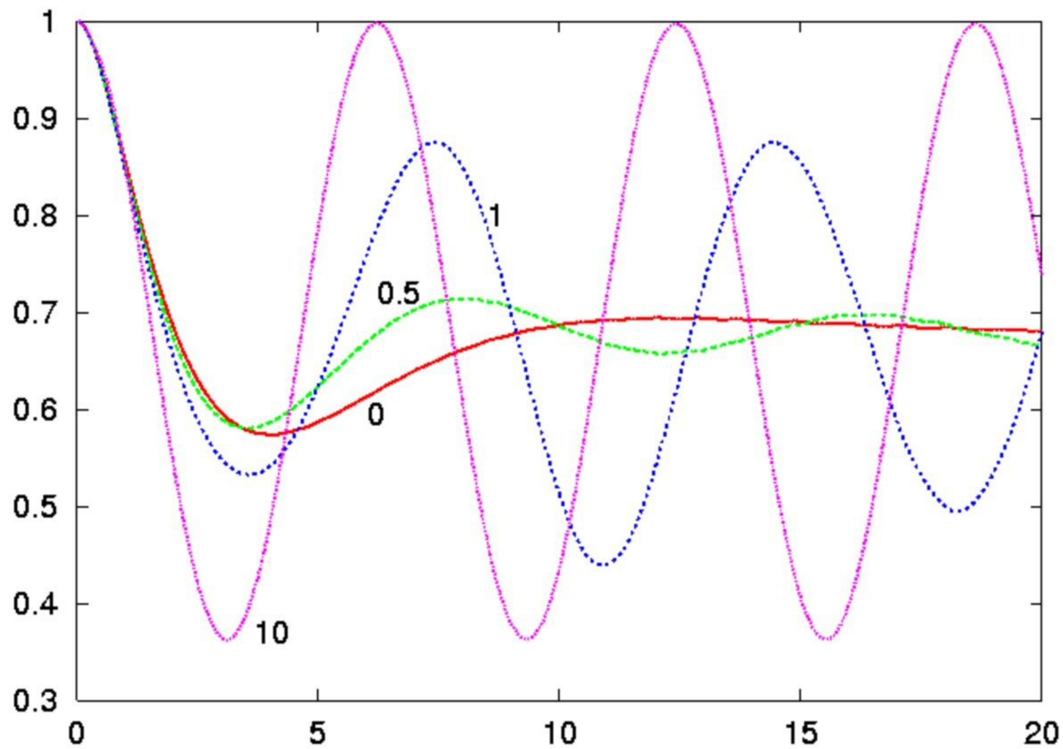
Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Synchronizing Oscillations by Neutrino Interactions

- Vacuum oscillation frequency depends on energy $\omega = \Delta m^2/2E$
- Ensemble with broad spectrum quickly decoheres kinematically
- ν-ν interactions "synchronize" the oscillations: $\omega_{\rm sync} = \langle \Delta m^2/2E \rangle$

Average e-flavor component of polarization vector



Time

Pastor, Raffelt & Semikoz, hep-ph/0109035

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$\mathrm{i}\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

$$\mathrm{i}\partial_t \rho = [H, \rho]$$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

$$\rho = \mathrm{Tr}(\rho) + \frac{1}{2}\mathbf{P}\cdot\boldsymbol{\sigma} \quad \text{and} \quad H = \frac{\Delta m^2}{2E}\mathbf{B}\cdot\boldsymbol{\sigma} \quad \text{with} \quad \mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$$

Equivalent spin-precession form of equation of motion

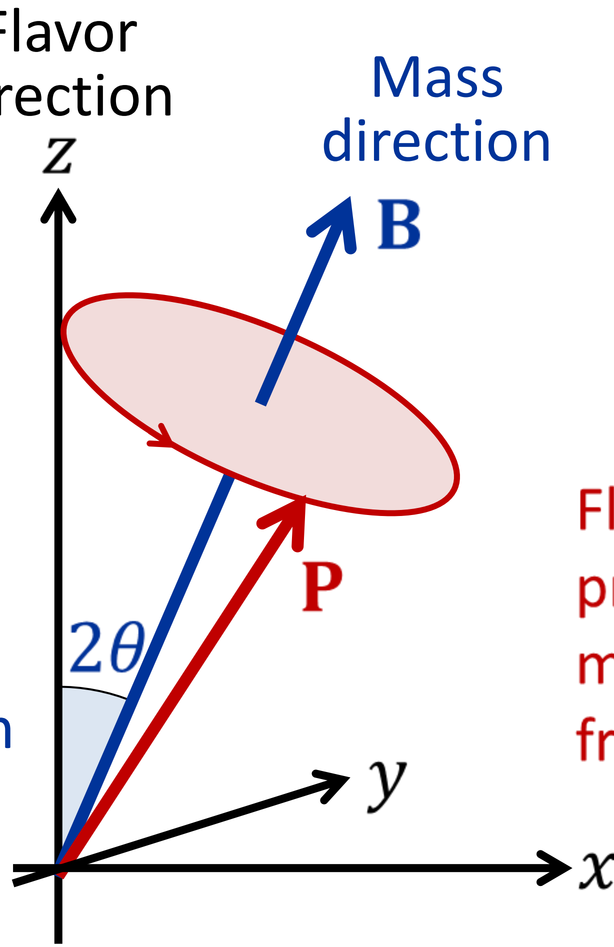
$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P} \quad \text{with} \quad \omega = \frac{\Delta m^2}{2E}$$

$\mathbf{P}$ is "polarization vector" or "Bloch vector"

Flavor Oscillation as Spin Precession

↑ Spin up ν_e

↓ Spin down ν_μ



Twice the vacuum mixing angle

Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2/2E$

Adding Matter

Schrödinger equation including matter

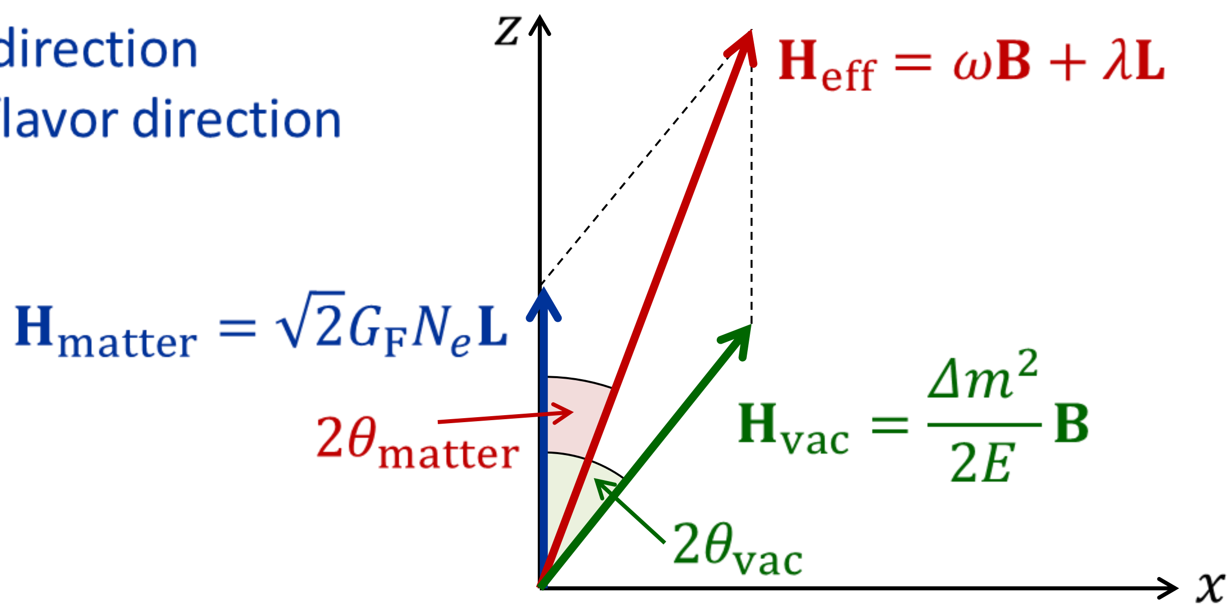
$$\mathrm{i}\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} + \sqrt{2} G_F \begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Corresponding spin-precession equation

$$\dot{\mathbf{P}} = \underbrace{(\omega \mathbf{B} + \lambda \mathbf{L})}_{\mathbf{H}_{\mathrm{eff}}} \times \mathbf{P} \quad \text{with} \quad \omega = \Delta m^2/2E \quad \text{and} \quad \lambda = \sqrt{2} G_{\mathrm{F}} N_e$$

$\mathbf{B}$ unit vector in mass direction

$\mathbf{L} = \mathbf{e}_z$ unit vector in flavor direction

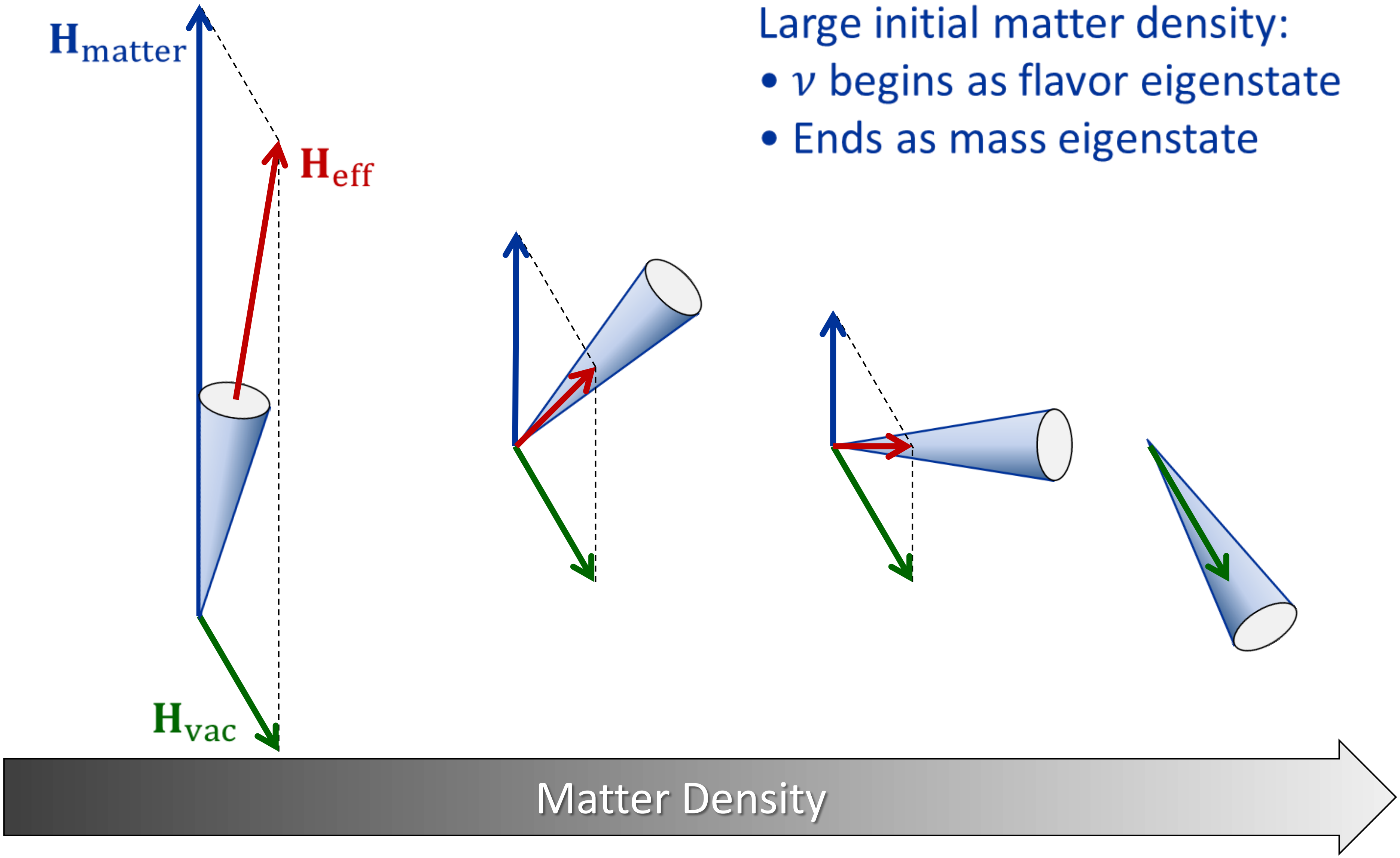


MSW Effect

Adiabatically decreasing density: Precession cone follows $\mathbf{H}_{\mathrm{eff}}$

Large initial matter density:

- ν begins as flavor eigenstate
- Ends as mass eigenstate



Adding Neutrino-Neutrino Interactions

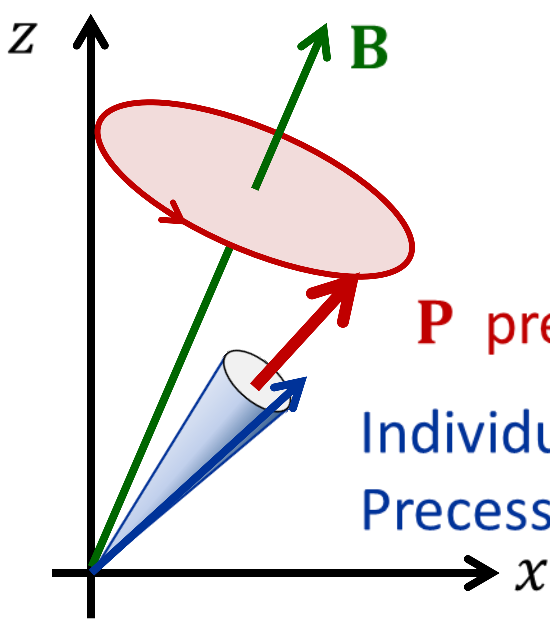
Precession equation for each ν mode with energy E, i.e. $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_\omega = \underbrace{(\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P})}_{\mathbf{H}_{\rm eff}} \times \mathbf{P}_\omega \quad \text{with} \quad \lambda = \sqrt{2}G_{\rm F}N_e \quad \text{and} \quad \mu = \sqrt{2}G_{\rm F}N_\nu$$

Total flavor spin of entire ensemble

$$\mathbf{P} = \textstyle\sum_\omega \mathbf{P}_\omega \qquad \text{normalize} \qquad |\mathbf{P}_{\mathbf{t=0}}| = 1$$

Individual spins do not remain aligned – feel "internal" field $\mathbf{H}_{\nu\nu} = \mu\mathbf{P}$



Synchronized oscillations for large neutrino density $\mu \gg \delta\omega$

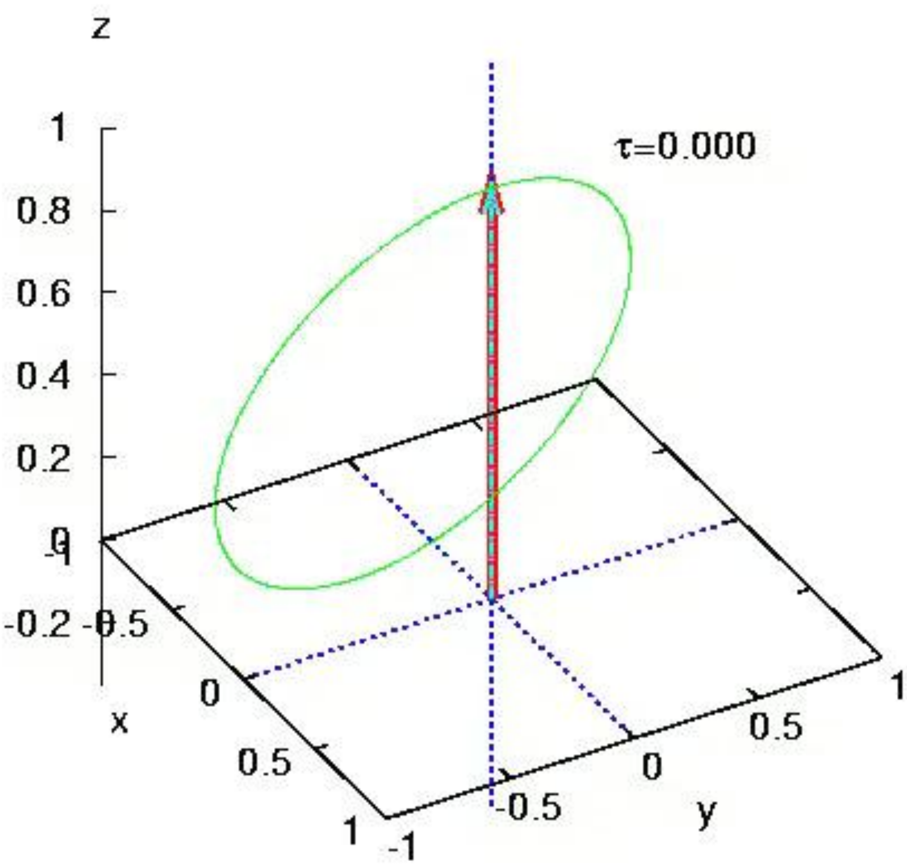
$\mathbf{P}$ precesses with $\omega_{\rm sync}$ for large ν density

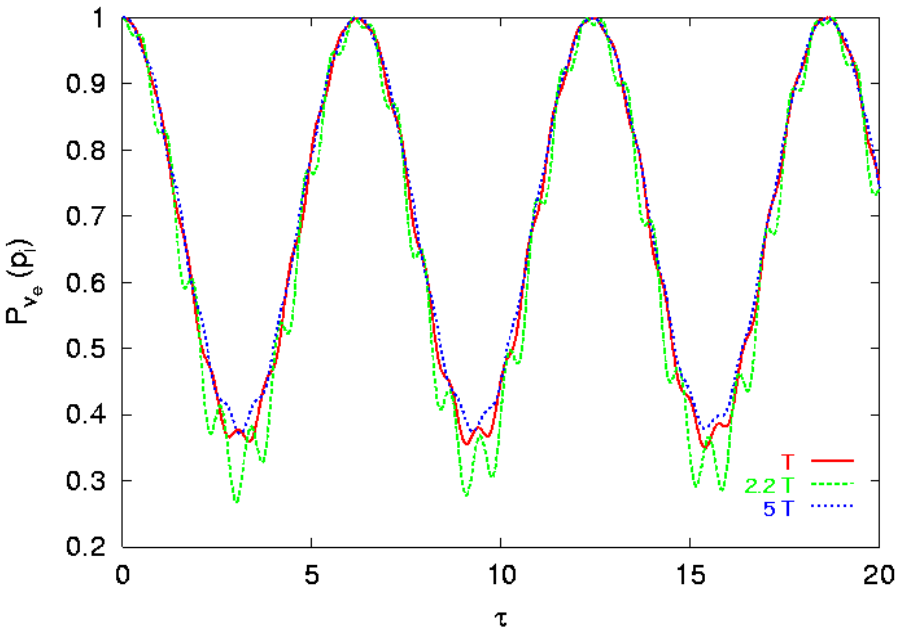
Individual $\mathbf{P}_\omega$ "trapped" on precession cones

Precess around $\mathbf{P}$ with frequency $\sim \mu$

Synchronized Oscillations by Nu-Nu Interactions

For large neutrino density, individual modes precess around large common dipole moment





Pastor, Raffelt & Semikoz, hep-ph/0109035

Two Spins Interacting with a Dipole Force

Simplest system showing ν-ν effects:
Isotropic neutrino gas with 2 energies E_1 and E_2, no ordinary matter

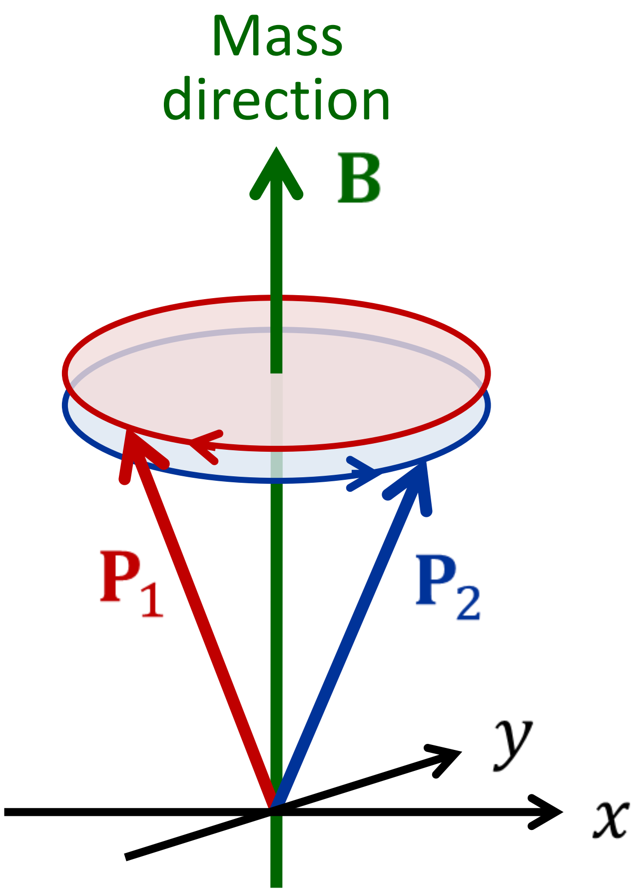
$$\dot{\mathbf{P}}_1 = (\omega_1 \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P_1} \quad \text{with} \quad \mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 \quad \text{and} \quad \omega_{1,2} = \Delta m^2/2E$$
$$\dot{\mathbf{P}}_2 = (\omega_2 \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P_2}$$

Go to "co-rotating frame" around **B** direction

$$\dot{\mathbf{P}}_1 = (\omega_c \mathbf{B} - \omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P_1}$$
$$\dot{\mathbf{P}}_2 = (\omega_c \mathbf{B} + \omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P_2}$$

with $\omega_c = \frac{1}{2}(\omega_2 + \omega_1)$ and $\omega = \frac{1}{2}(\omega_2 - \omega_1)$

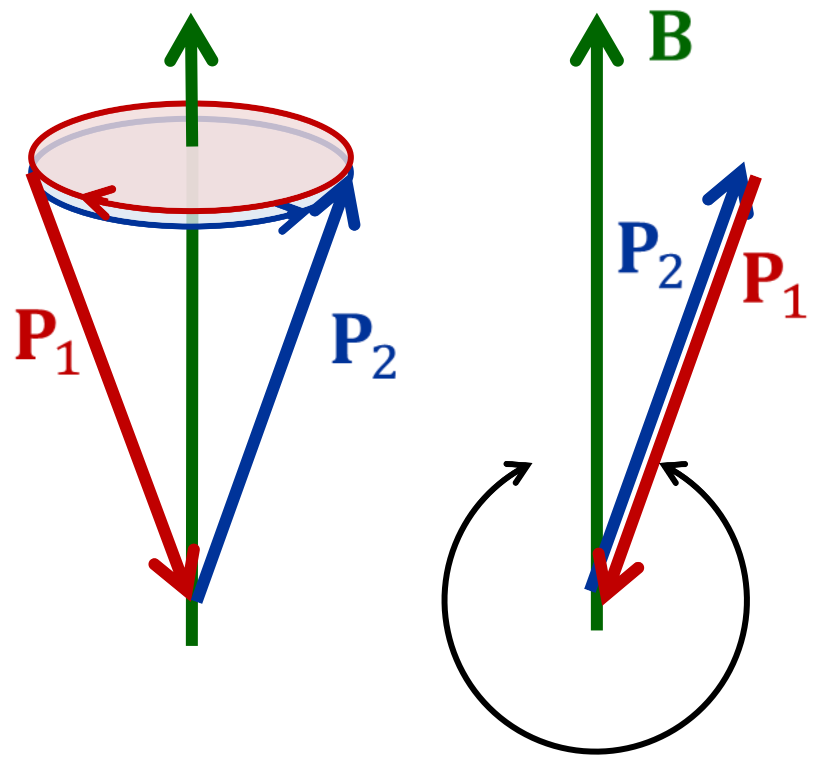
No interaction ($\mu = 0$)
$P_{1,2}$ precess in opposite directions
Strong interactions ($\mu \to \infty$)
$P_{1,2}$ stuck to each other
(no motion in co-rotating frame, perfectly synchronized in lab frame)

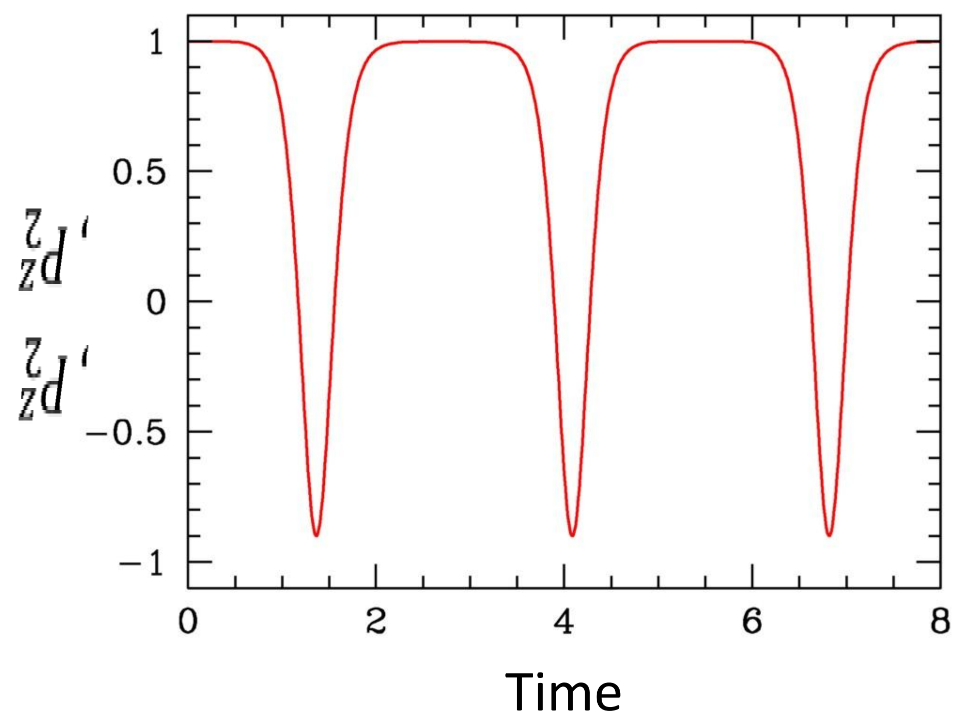


Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$)
Free precession in opposite directions

Even for very small mixing angle, large-amplitude flavor oscillations



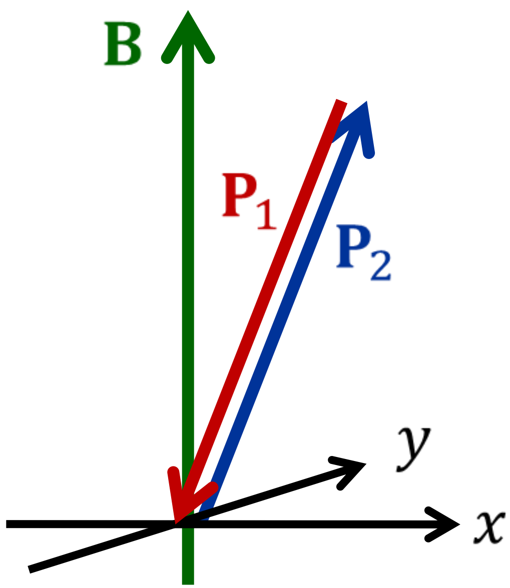


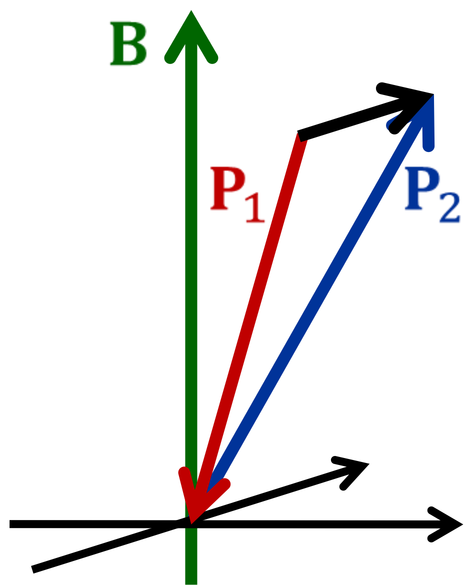
Strong interaction
($\mu \to \infty$)
Pendular motion

Instability in Flavor Space

Two-mode example in co-rotating frame, initially $\mathbf{P}_1 = \downarrow$, $\mathbf{P}_2 = \uparrow$ (flavor basis)

$$\dot{\mathbf{P}}_1 = [-\omega\mathbf{B} + \mu\,(\mathbf{P}_1 + \mathbf{P}_2)] \times \mathbf{P}_1$$
$$\dot{\mathbf{P}}_2 = [+\omega\mathbf{B} + \mu\,\underbrace{(\mathbf{P}_1 + \mathbf{P}_2)}_{\text{0 initially}}] \times \mathbf{P}_2$$





$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$

Matter effect transverse to mass and flavor directions

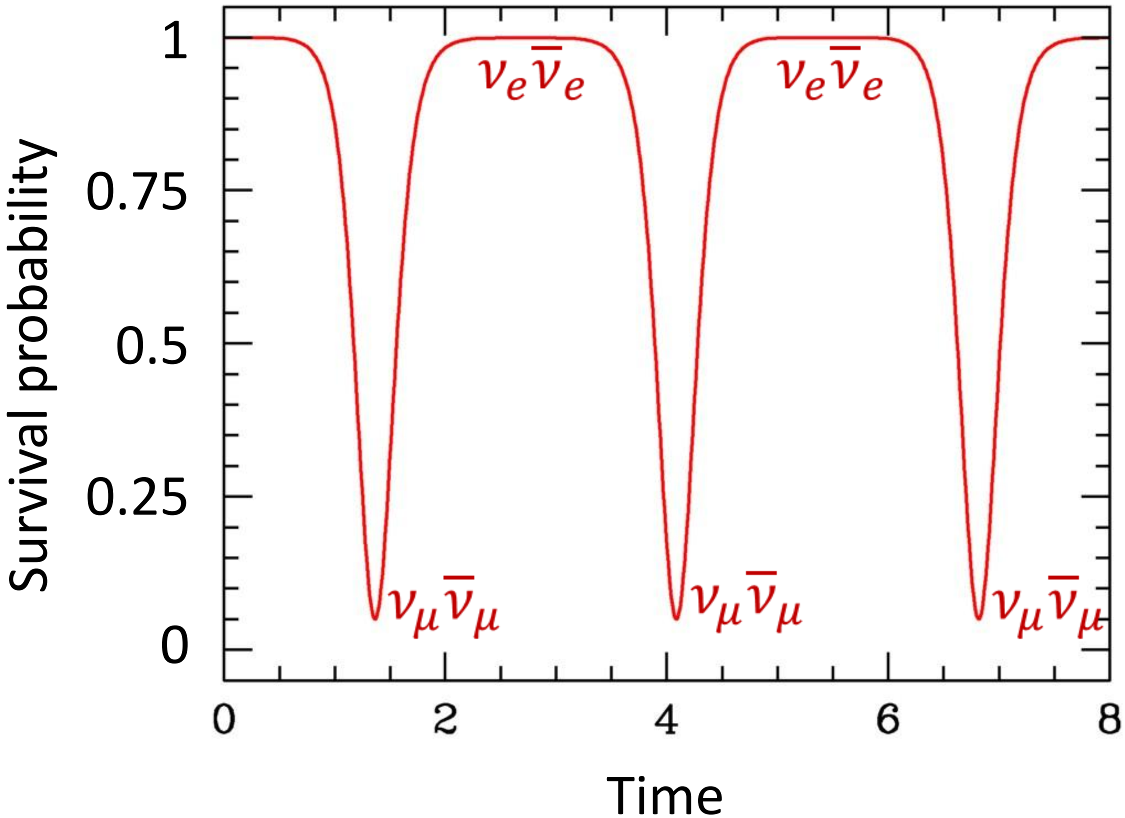
Both $\mathbf{P}_1$ and $\mathbf{P}_2$ tilt around $\mathbf{P}$ if μ is large

- Initially aligned in flavor direction and $\mathbf{P} = 0$
- Free precession $\pm\omega$

After a short time, transverse $\mathbf{P}$ develops by free precession

Collective Pair Annihilation

Gas of equal abundances of ν_e and $\overline{\nu}_e$, inverted mass hierarchy
Small effective mixing angle (e.g. made small by ordinary matter)



Dense neutrino gas unstable in flavor space: $\nu_e\overline{\nu}_e \leftrightarrow \nu_\mu\overline{\nu}_\mu$
Complete pair conversion even for a small mixing angle

Flavor Matrices of Occupation Numbers

Neutrinos described by Dirac field

$$\Psi(t,x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[a_\mathbf{p}(t)u_\mathbf{p} + b^\dagger_{-\mathbf{p}}(t)\, v_{-\mathbf{p}}\right] e^{\mathbf{i}\mathbf{p}\cdot\mathbf{x}}$$

in terms of the spinors in flavor space, providing spinor of flavor amplitudes

$$\Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix},\quad a = \begin{pmatrix}a_1\\ a_2\\ a_3\end{pmatrix},\quad b = \begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix} \quad \text{and} \quad \left|\begin{matrix}\nu_1(t,\mathbf{p})\\ \nu_2(t,\mathbf{p})\\ \nu_3(t,\mathbf{p})\end{matrix}\right\rangle = \begin{pmatrix}a_1^\dagger(t,\mathbf{p})\\ a_2^\dagger(t,\mathbf{p})\\ a_3^\dagger(t,\mathbf{p})\end{pmatrix} |0\rangle$$

Measurable quantities are expectation values of field bi-linears $\langle\Psi^\dagger\Psi\rangle$, therefore use "occupation number matrices" to describe the ensemble and its kinetic evolution (Boltzmann eqn for oscillations and collisions)

(ν) $\quad \rho_{ij} = \langle a_j^\dagger a_i\rangle$

$(\bar\nu)$ $\quad \bar\rho_{ij} = \langle b_j b_i^\dagger\rangle = 1 - \langle b_i^\dagger b_j\rangle$

Drops out in commutators

Describe $\bar\nu$ with negative occupation numbers, reversed order of flavor indices (holes in Dirac sea)

Flavor Pendulum

Classical Hamiltonian for two spins interacting with a dipole force μ

$$H = \omega\mathbf{B}\cdot(\mathbf{P}_2 - \mathbf{P}_1) + \frac{\mu}{2}\mathbf{P}^2$$

Angular-momentum Poisson brackets

$$\{P_i, P_j\} = \epsilon_{ijk}P_k$$

Total angular momentum

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$$

Precession equations of motion

$$\dot{\mathbf{P}}_{1,2} = (\mp\omega\mathbf{B} + \mu\mathbf{P})\times\mathbf{P}_{1,2}$$

Lagrangian top (spherical pendulum with spin), moment of inertia I

$$H = \omega\mathbf{B}\cdot\mathbf{Q} + \frac{\mathbf{P}^2}{2I}$$

Total angular momentum $\mathbf{P}$, radius vector $\mathbf{Q}$, fulfilling

$$\{P_i, P_j\} = \epsilon_{ijk}P_k, \quad \{Q_i, Q_j\} = 0$$

$$\{P_i, Q_j\} = \epsilon_{ijk}Q_k$$

Pendulum EoMs

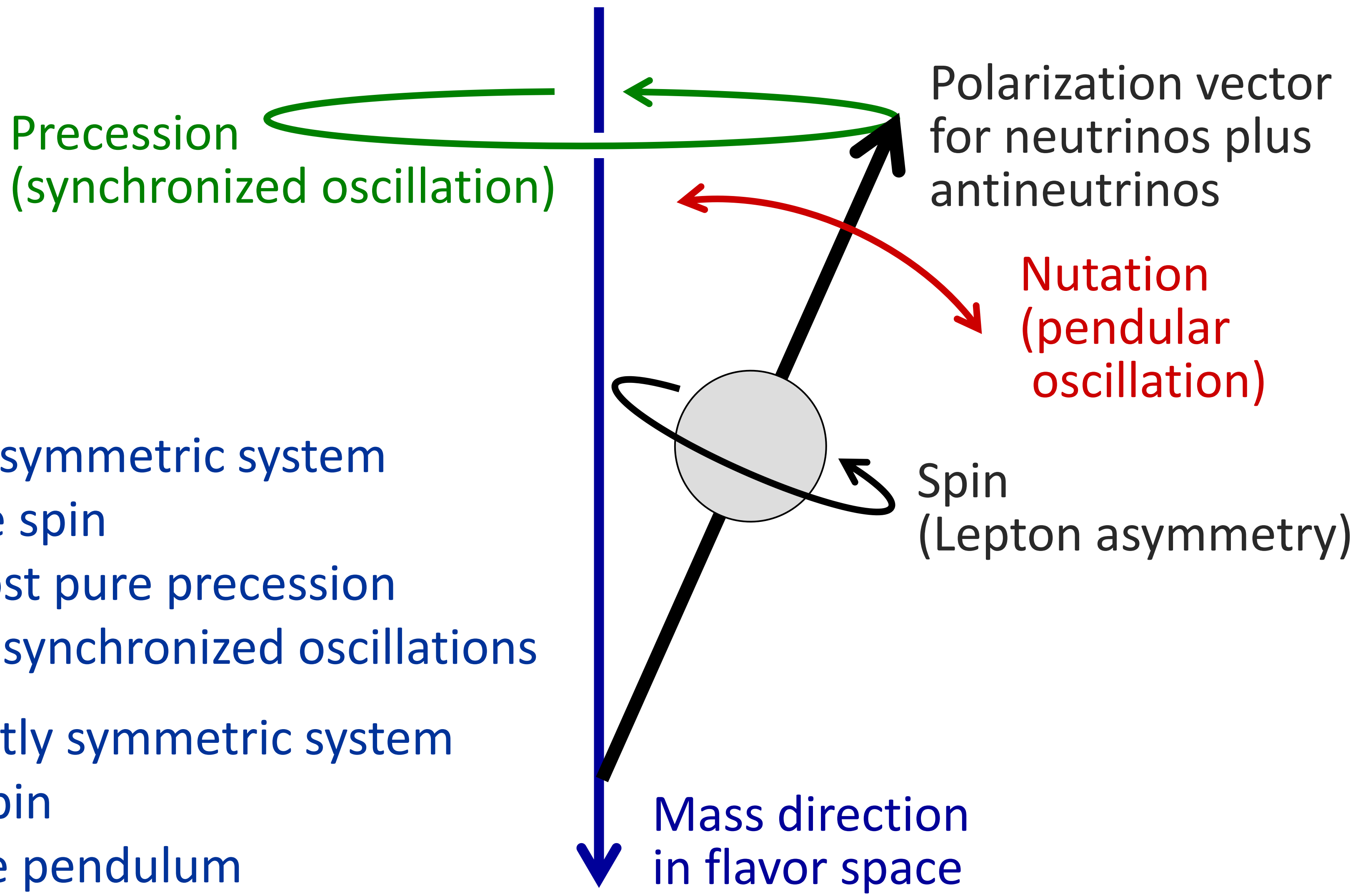
$$\dot{\mathbf{Q}} = I^{-1}\mathbf{P}\times\mathbf{Q} \quad \text{and} \quad \dot{\mathbf{P}} = \omega\mathbf{B}\times\mathbf{Q}$$

EoMs and Hamiltonians identical (up to a constant) with the identification

$$\mathbf{Q} = \mathbf{P}_2 - \mathbf{P}_1 - \frac{\omega}{\mu}\mathbf{B} \quad \text{and} \quad \mu = I^{-1}$$

Constants of motion: $\mathbf{P}_1^2$, $\mathbf{P}_2^2$, $\mathbf{B}\cdot\mathbf{P}$, $\mathbf{P}\cdot\mathbf{Q}$, $\mathbf{Q}^2$ and H

Pendulum in Flavor Space

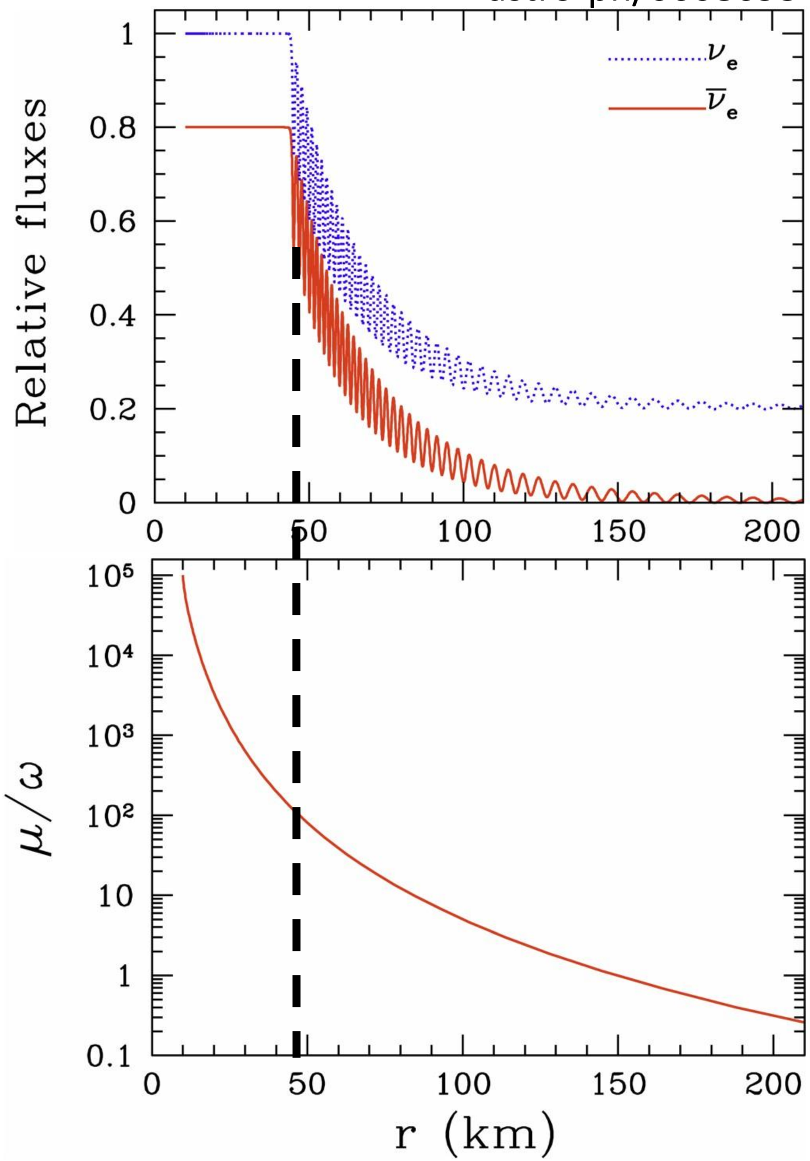


- Very asymmetric system
  - Large spin
  - Almost pure precession
  - Fully synchronized oscillations
- Perfectly symmetric system
  - No spin
  - Plane pendulum

Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695]

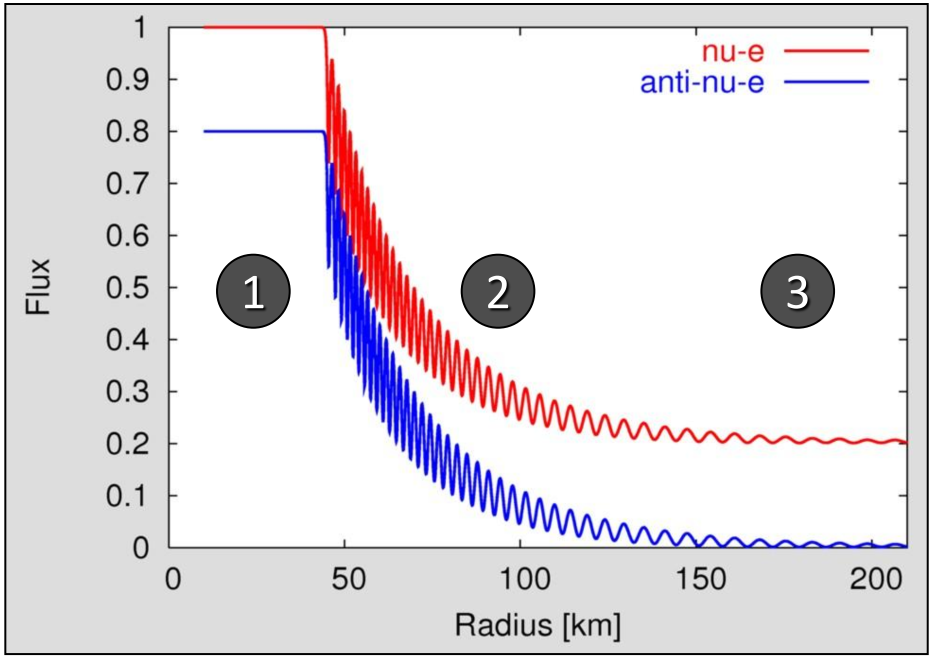
Flavor Conversion in a Toy Supernova

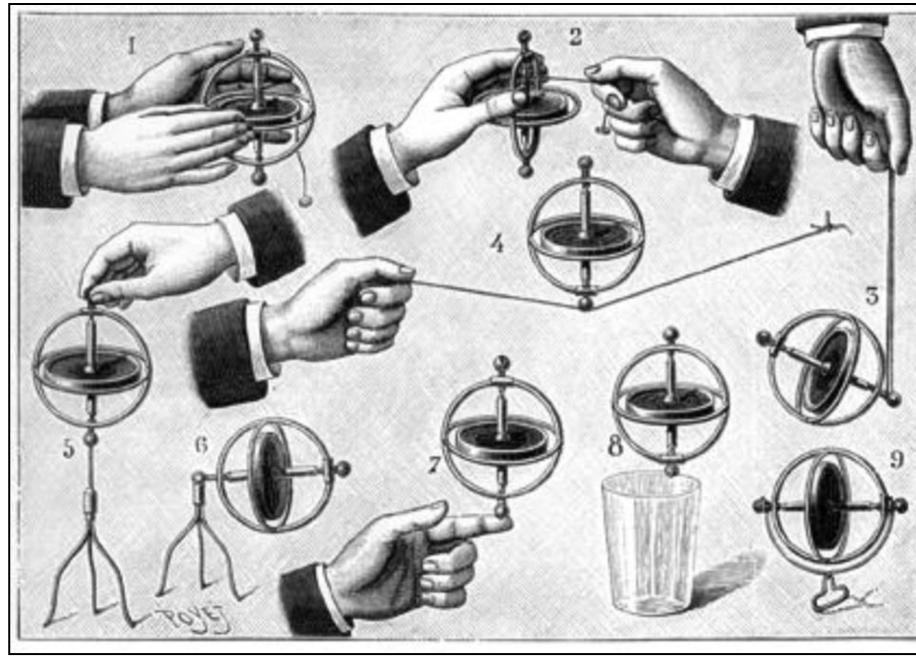
astro-ph/0608695

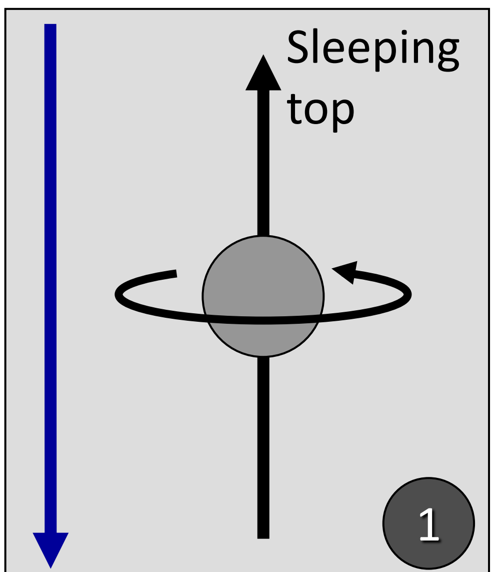


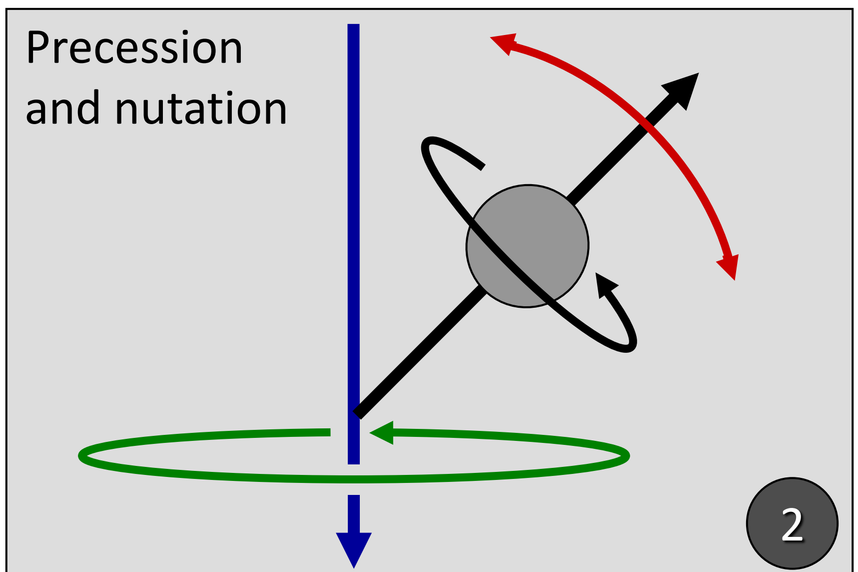
- Two modes with $\omega = \pm 0.3\ \mathrm{km}^{-1}$
- Assume 80% anti-neutrinos
- Sharp onset radius
- Oscillation amplitude declining
- Neutrino-neutrino interaction energy at nu sphere $(r = 10\ \mathrm{km})$
  $\mu = 0.3 \times 10^5\ \mathrm{km}^{-1}$
- Falls off approximately as r^{-4} (geometric flux dilution and nus become more co-linear)

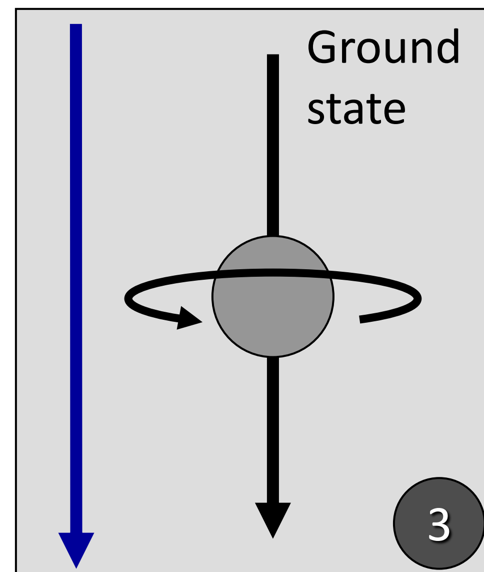
Neutrino Conversion and Flavor Pendulum











Fermi-Dirac Spectrum

Fermi-Dirac energy spectrum

$$\frac{dN}{dE} \propto \frac{E^2}{e^{E/T-\eta}+1}$$

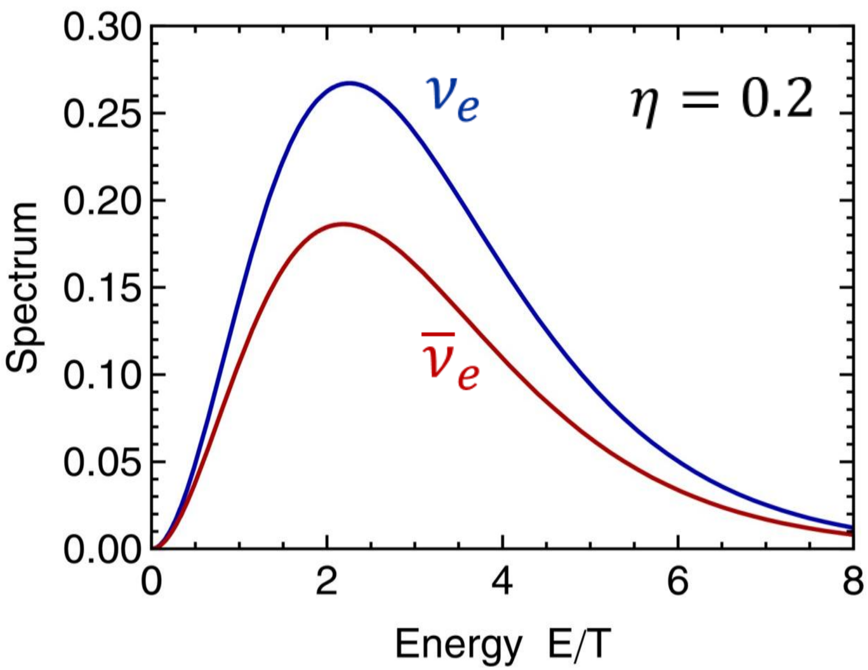
η degeneracy parameter, $-\eta$ for $\overline{\nu}$

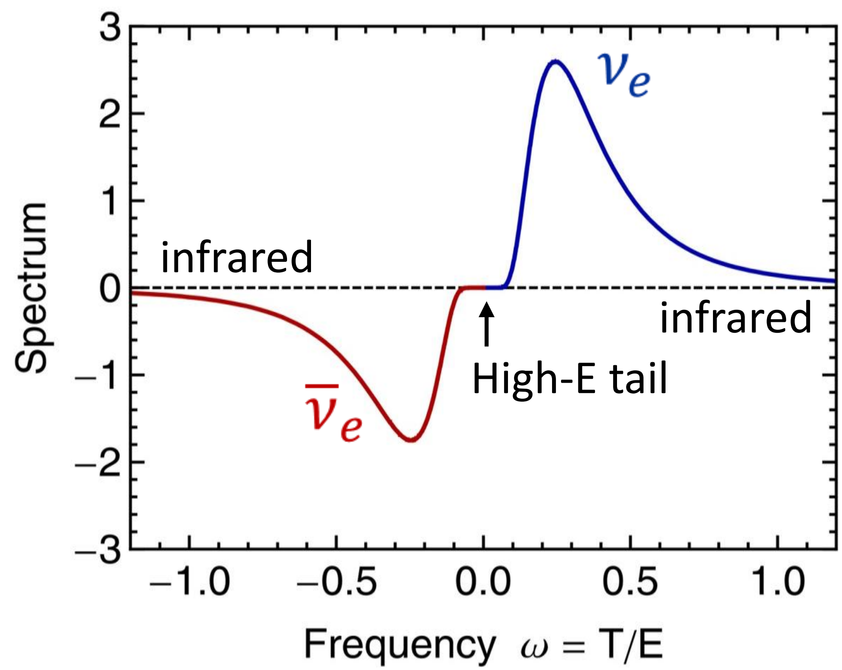
Same spectrum in terms of $\omega = T/E$

- Antineutrinos $E \to -E$
- and dN/dE negative (flavor isospin convention)

$\omega > 0$: $\nu_e = \uparrow$ and $\nu_\mu = \downarrow$

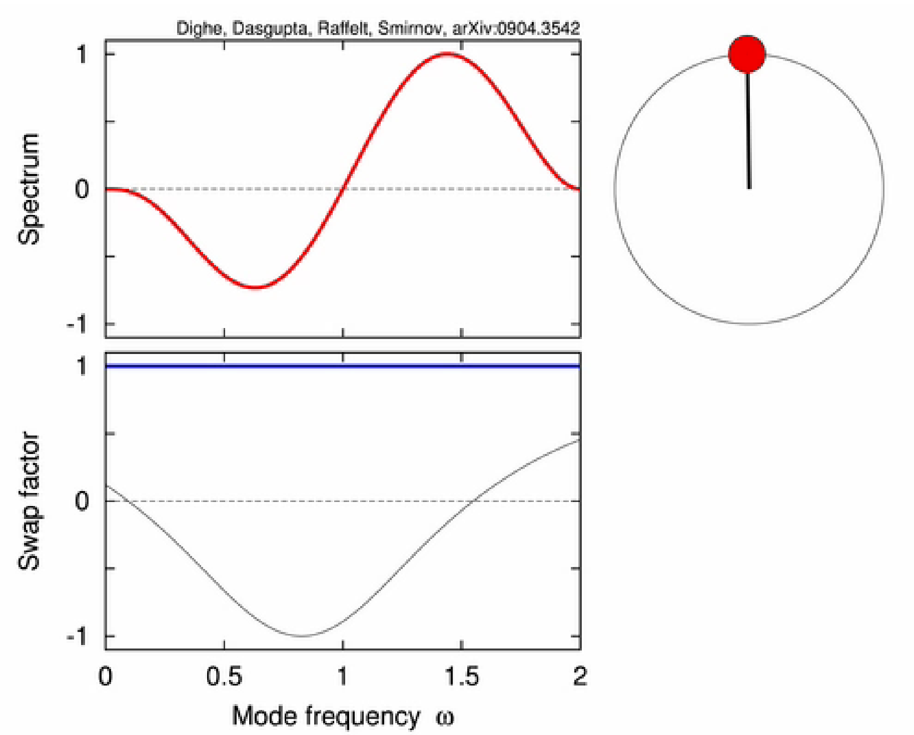
$\omega < 0$: $\overline{\nu}_e = \downarrow$ and $\overline{\nu}_\mu = \uparrow$



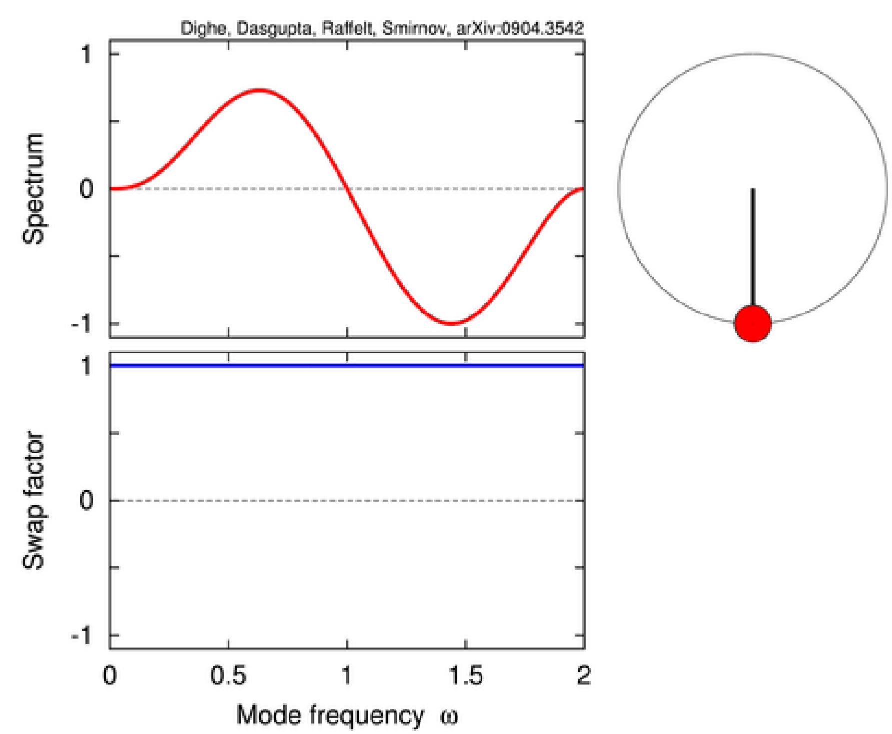


Flavor Pendulum

Single "positive" crossing
(potential energy at a maximum)



Single "negative" crossing
(potential energy at a minimum)



Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

General Stability Condition

Spin-precession equations of motion for modes with $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_\omega = \omega \mathbf{B} \times \mathbf{P}_\omega$$

Small-amplitude expansion: x-y-component described as complex number S (off-diagonal ρ element), linearized EoMs

$$-\mathrm{i}\dot{S}_\omega = \omega S_\omega - \mu \int d\omega'\, g_{\omega'} S_{\omega'}$$

Fourier transform $S_\omega = F_\omega e^{i\Omega t}$, with $\Omega = \gamma + i\kappa$ a complex frequency

$$(\omega - \Omega) F_\omega = \mu \int d\omega'\, g_{\omega'} S_{\omega'}$$

Eigenfunction is $F_\omega \propto (\omega - \Omega)^{-1}$ and eigenvalue $\Omega = \gamma + i\kappa$ is solution of

$$\mu^{-1} = \int d\omega \frac{g_\omega}{\omega - \Omega}$$

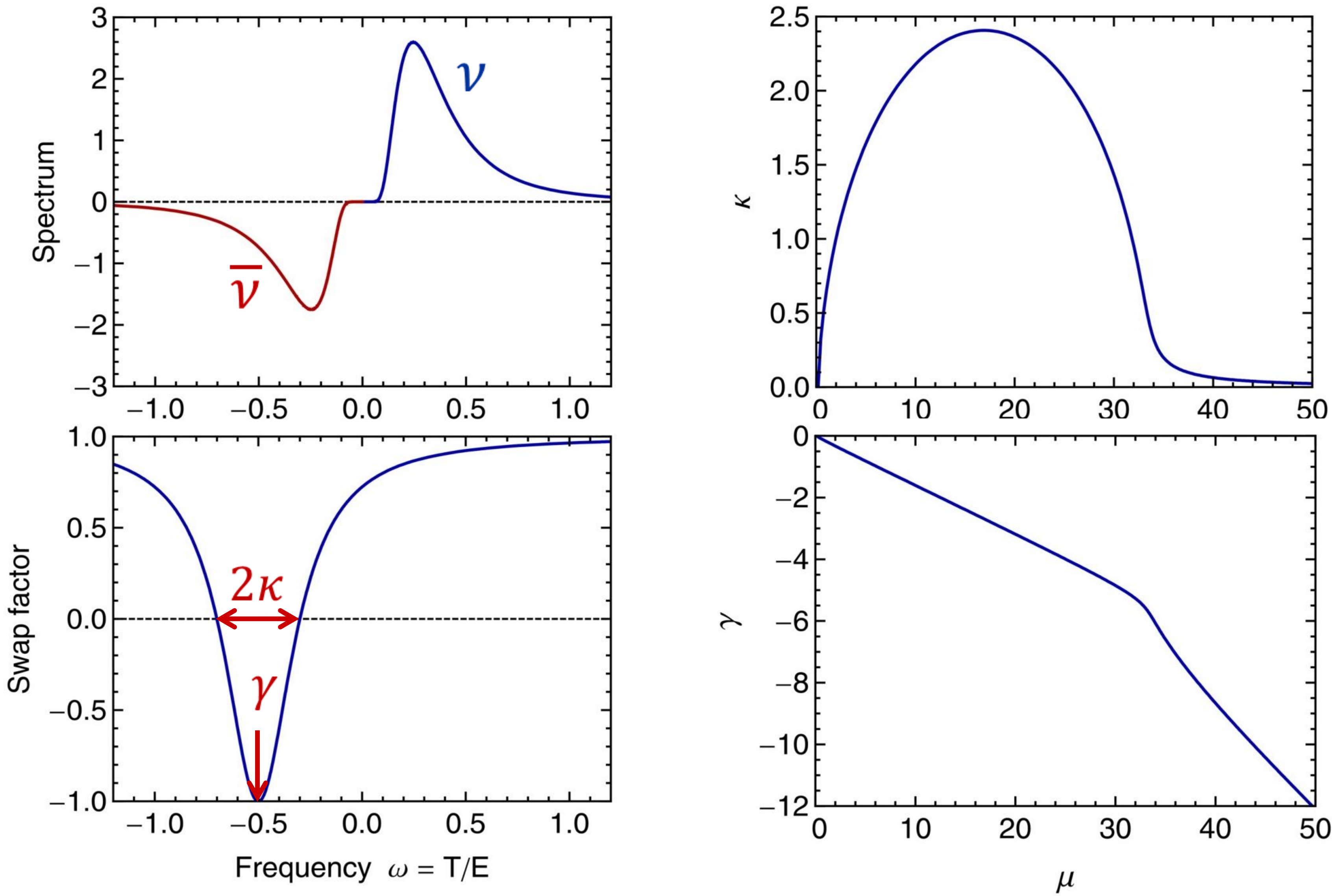
Instability occurs for

$$\kappa = \mathrm{Im}\,\Omega \neq 0$$

Exponential run-away solutions become pendulum for large amplitude.

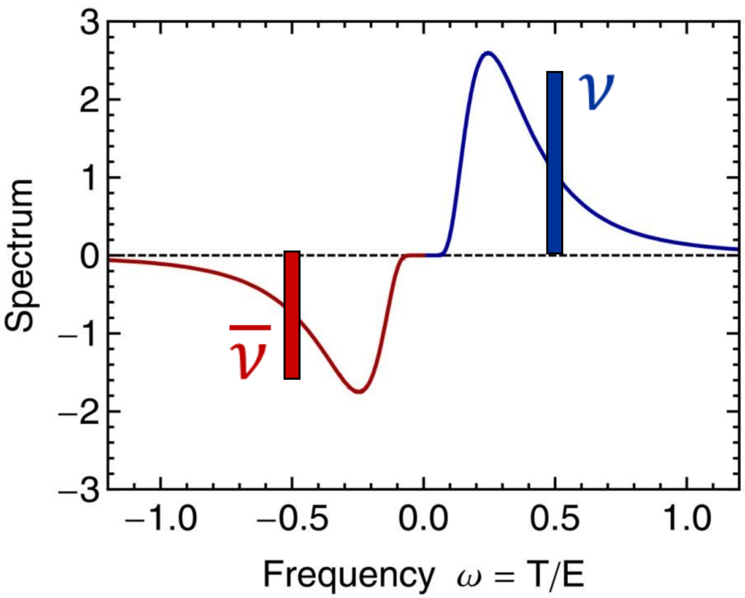
Banerjee, Dighe & Raffelt, work in progress

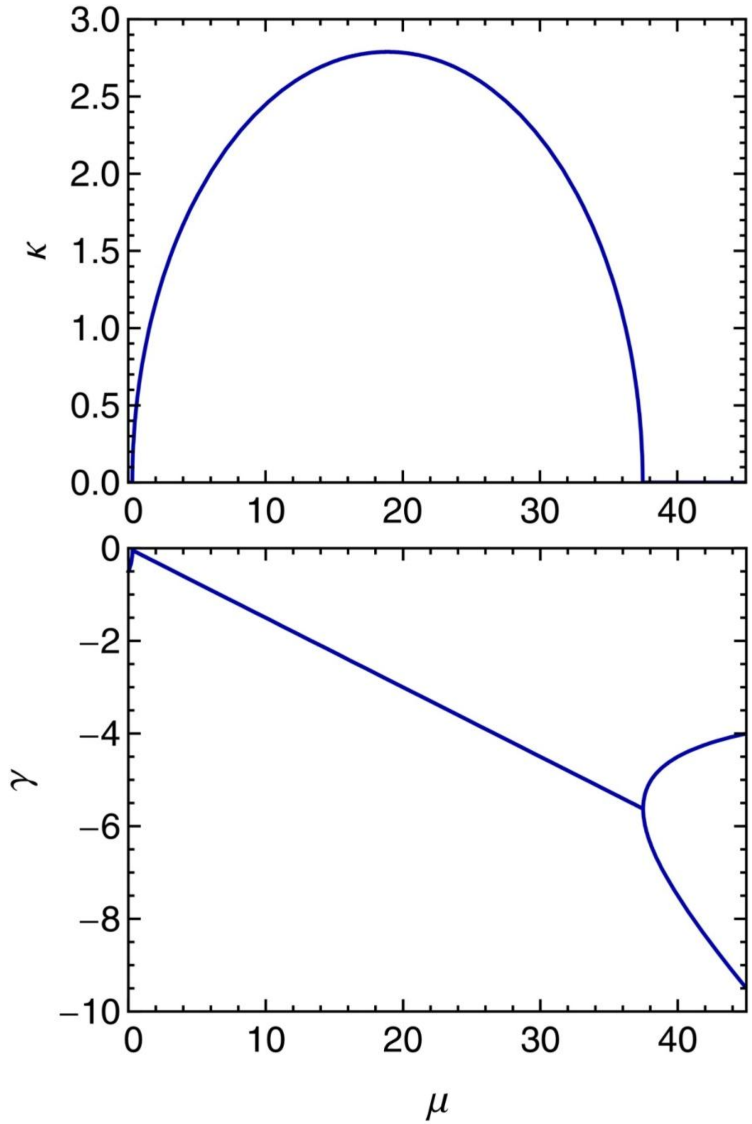
Stability of Fermi-Dirac Spectrum



Banerjee, Dighe & Raffelt, work in progress

Stability of Schematic Double Peak





Spectrum is unstable in the range

$$\frac{1}{(\sqrt{a}+1)^2} < \mu < \frac{1}{(\sqrt{a}-1)^2}$$

where $a = N_{\overline{\nu}}/N_{\nu}$.

With $a = 0.7$, system is unstable for

$$0.296 < \mu < 37.5$$

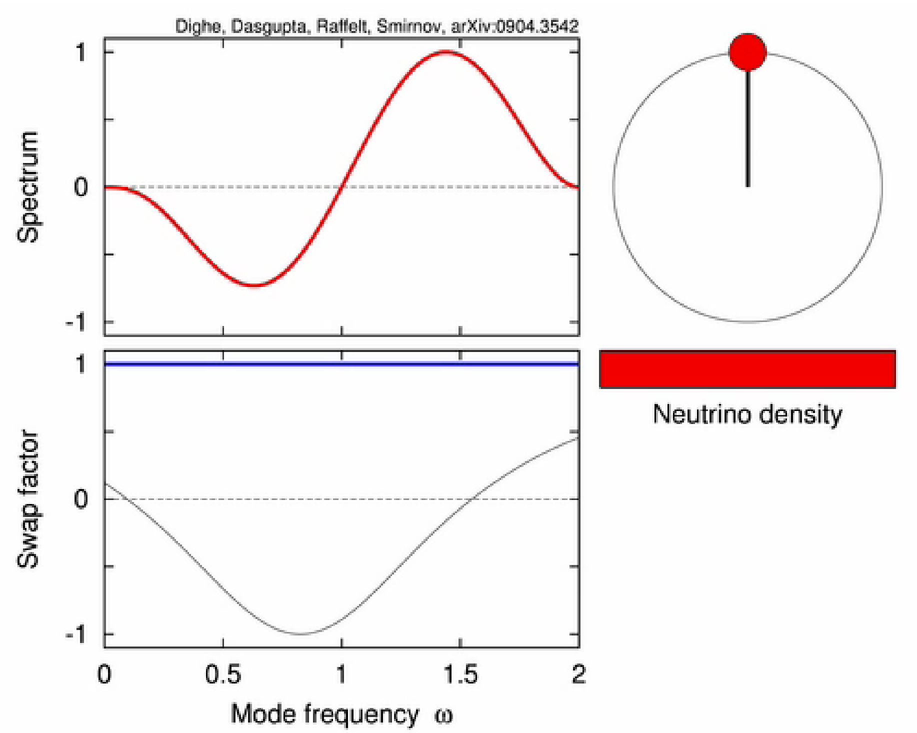
Otherwise the system is stuck.

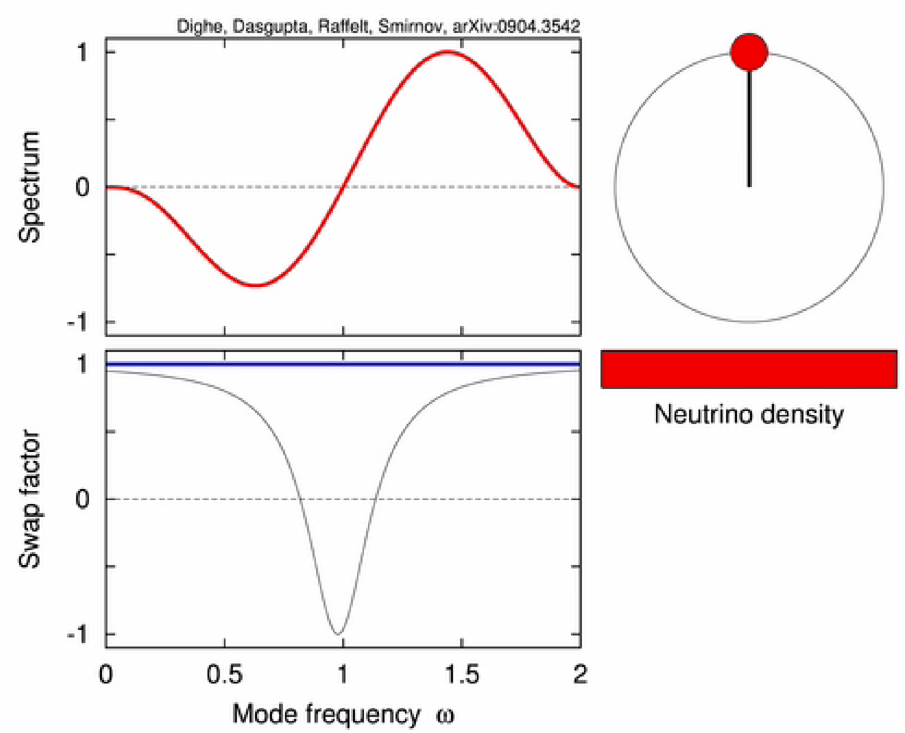
Banerjee, Dighe & Raffelt, work in progress

Decreasing Neutrino Density

Certain initial neutrino density

Four times smaller initial neutrino density

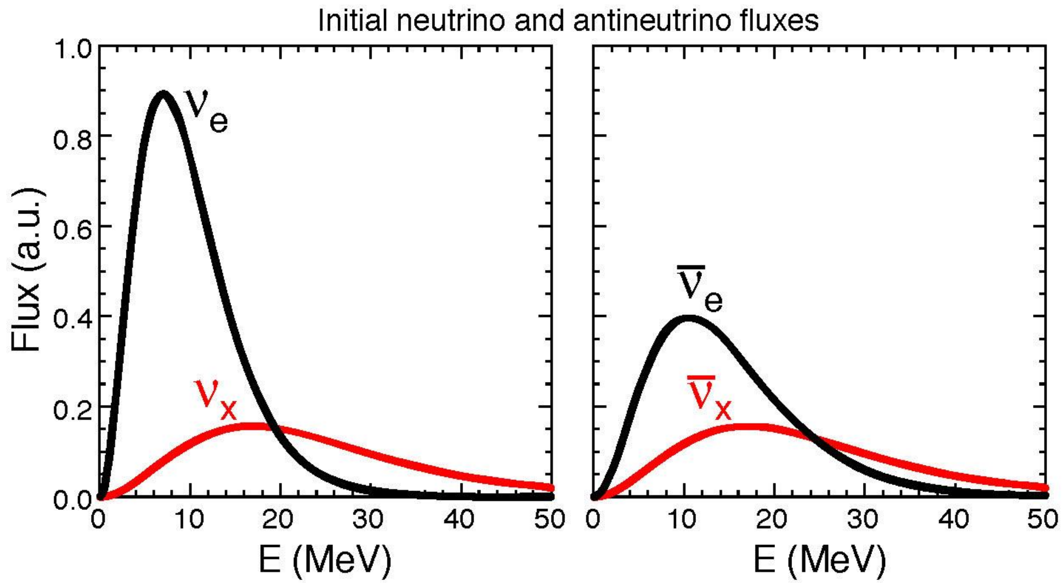




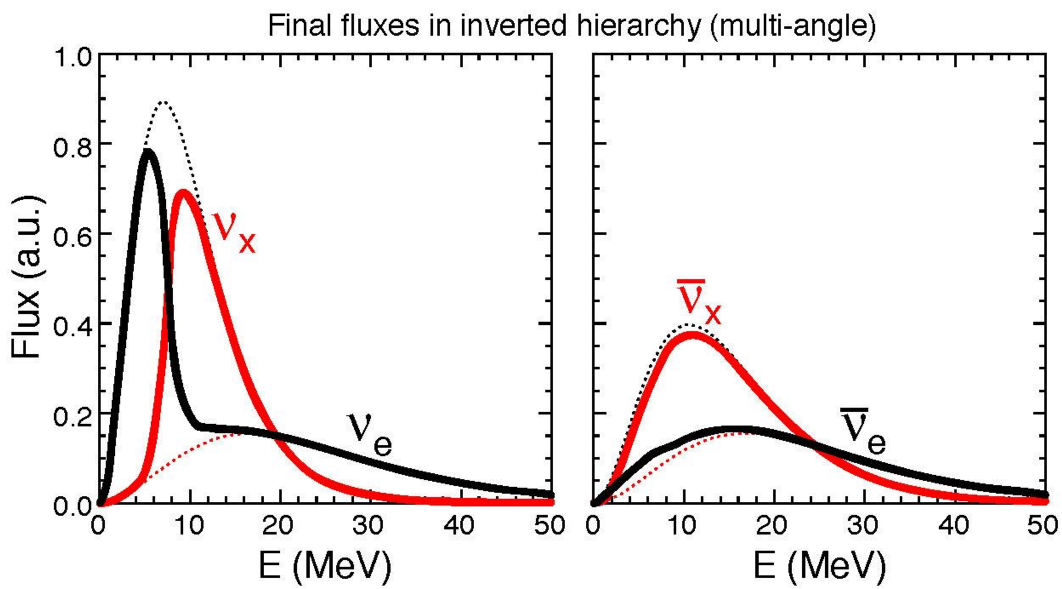
Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Spectral Split

Initial fluxes at neutrino sphere



After collective trans-formation



Figures from

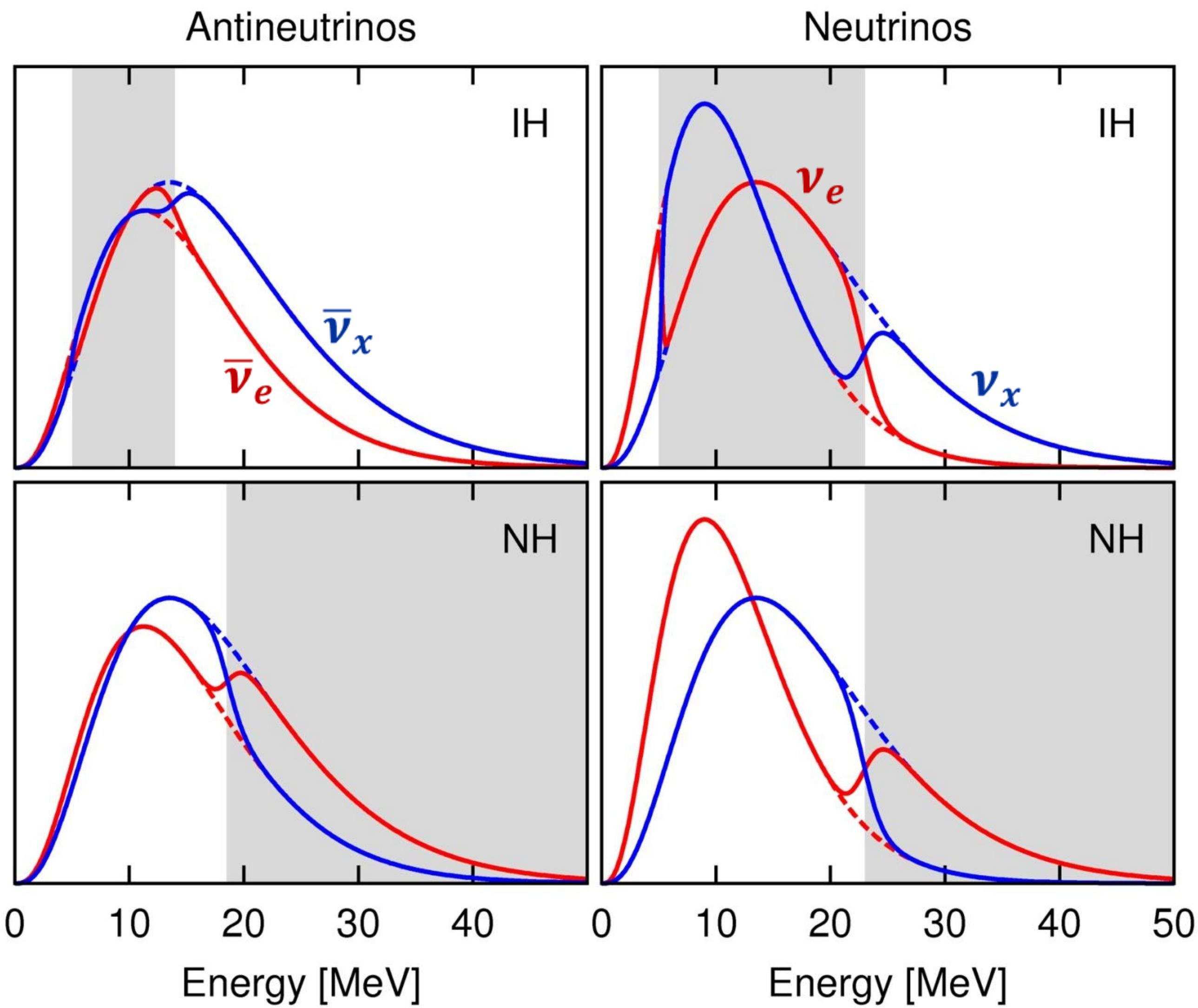
Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Multiple Spectral Splits

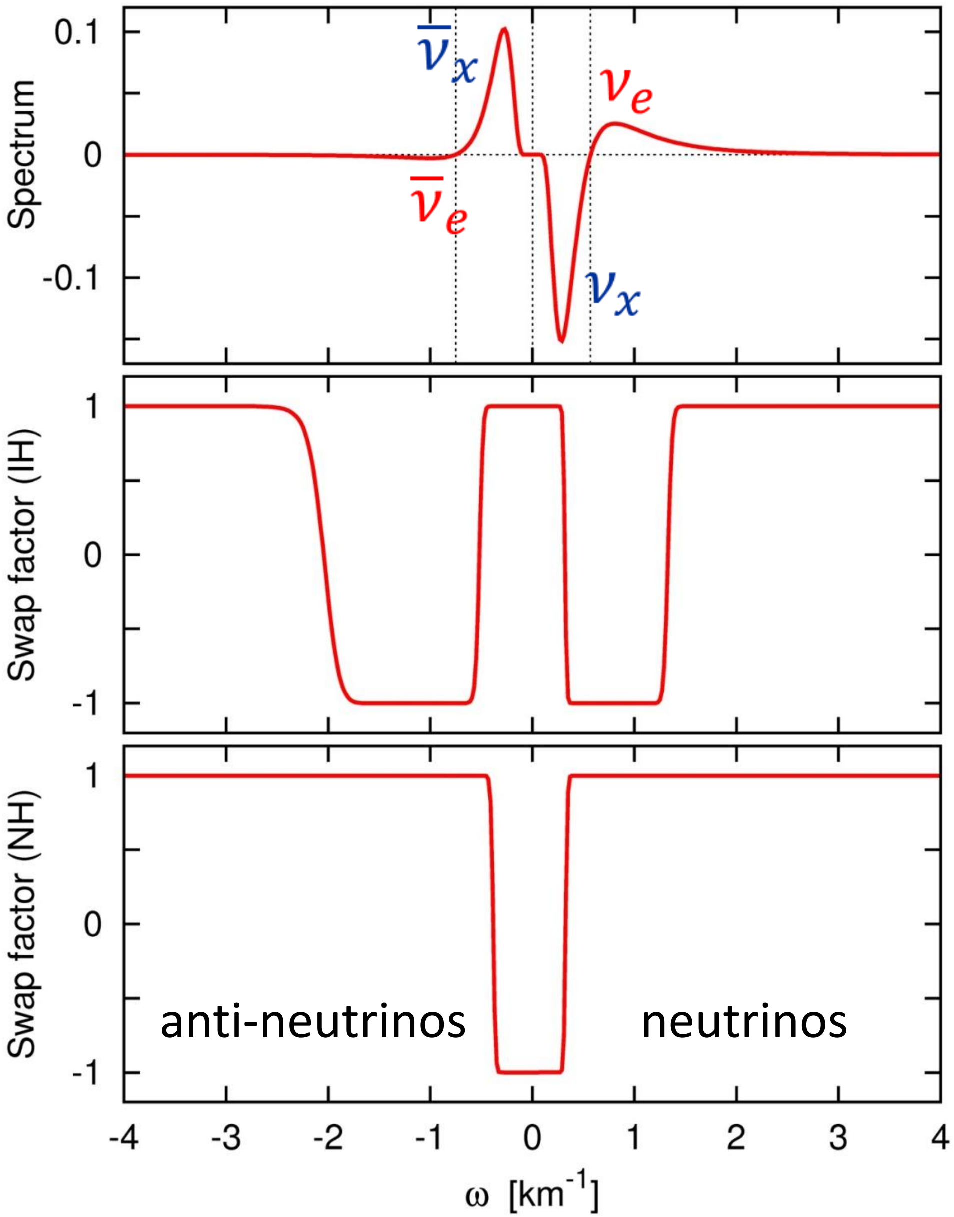


Inverted Hierarchy

Normal Hierarchy

Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542

Multiple Spectral Splits in the ω Variable



- Given is the flux spectrum f(E) for each flavor
- Use $\omega = \Delta m^2/2E$ to label modes
- Label anti-neutrinos with $-\omega$
- Define "spectrum" as

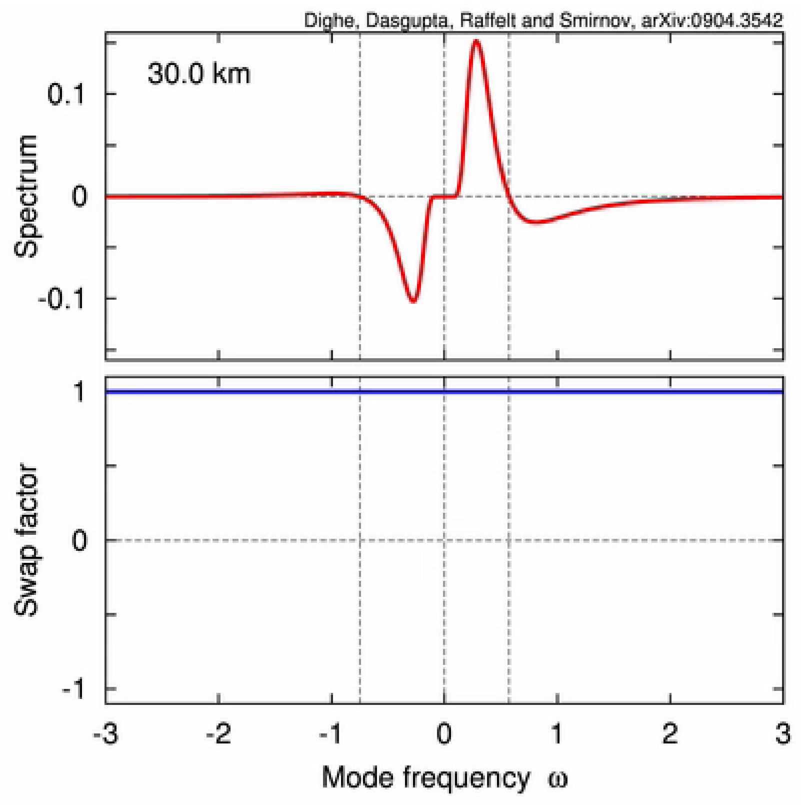
$$g(\omega) \propto \begin{cases} f_{\nu_e}(E) - f_{\nu_x}(E) \\ f_{\overline{\nu}_x}(E) - f_{\overline{\nu}_e}(E) \end{cases}$$

- Swaps develop around every "postive" spectral crossing
- Each swap flanked by two splits

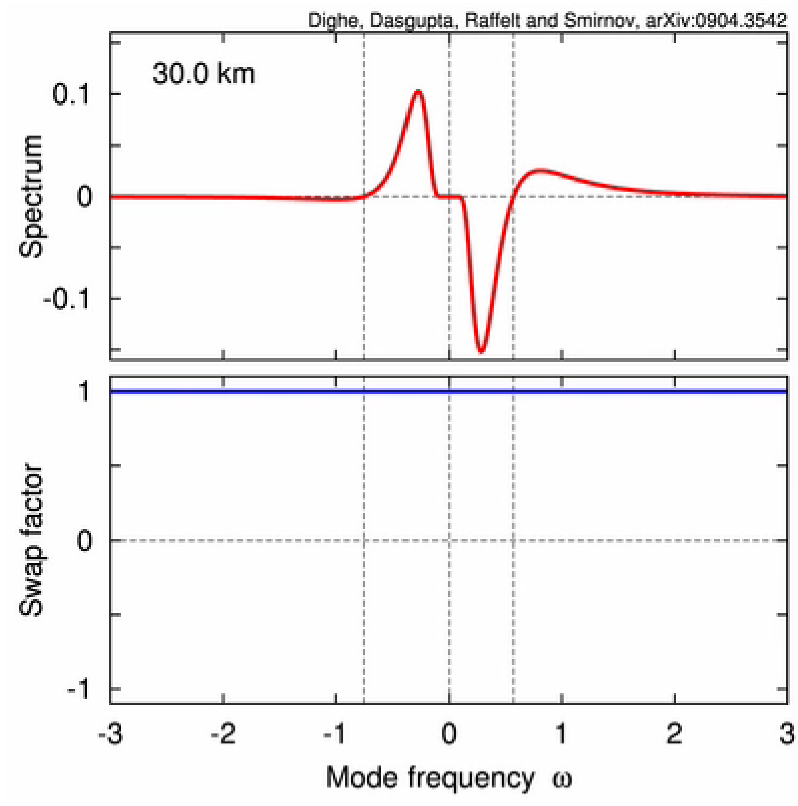
Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542

Supernova Cooling-Phase Example

Normal Hierarchy



Inverted Hierarchy



Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Multi-Angle Decoherence

Precession eqns for modes with vacuum oscillation frequency $\omega = \Delta m^2/2E$ (negative for $\overline{\nu}$), and velocity $\mathbf{v}$ (direction of motion), homogeneous system

$$\dot{\mathbf{P}}_{\omega,\mathbf{v}} = \omega\mathbf{B}\times\mathbf{P}_{\omega,\mathbf{v}} + \mu\sum_{\omega',\mathbf{v}'}(1-\cos\theta_{\mathbf{v}\mathbf{v}'})\,\mathbf{P}_{\omega',\mathbf{v}'}\times\mathbf{P}_{\omega,\mathbf{v}}$$

Axial symmetry around some direction (e.g. SN radial direction), measure velocities against that direction: $v = \cos\theta$

$$\dot{\mathbf{P}}_{\omega,v} = \omega\mathbf{B}\times\mathbf{P}_{\omega,v} + \mu\underbrace{\sum_{\omega',v'}\mathbf{P}_{\omega',v'}}_{\mathbf{P}}\times\mathbf{P}_{\omega,v} - \mu\underbrace{\sum_{\omega',v'}v'\,\mathbf{P}_{\omega',v'}}_{\mathbf{F}\ \text{(Flux)}}\times v\mathbf{P}_{\omega,v}$$

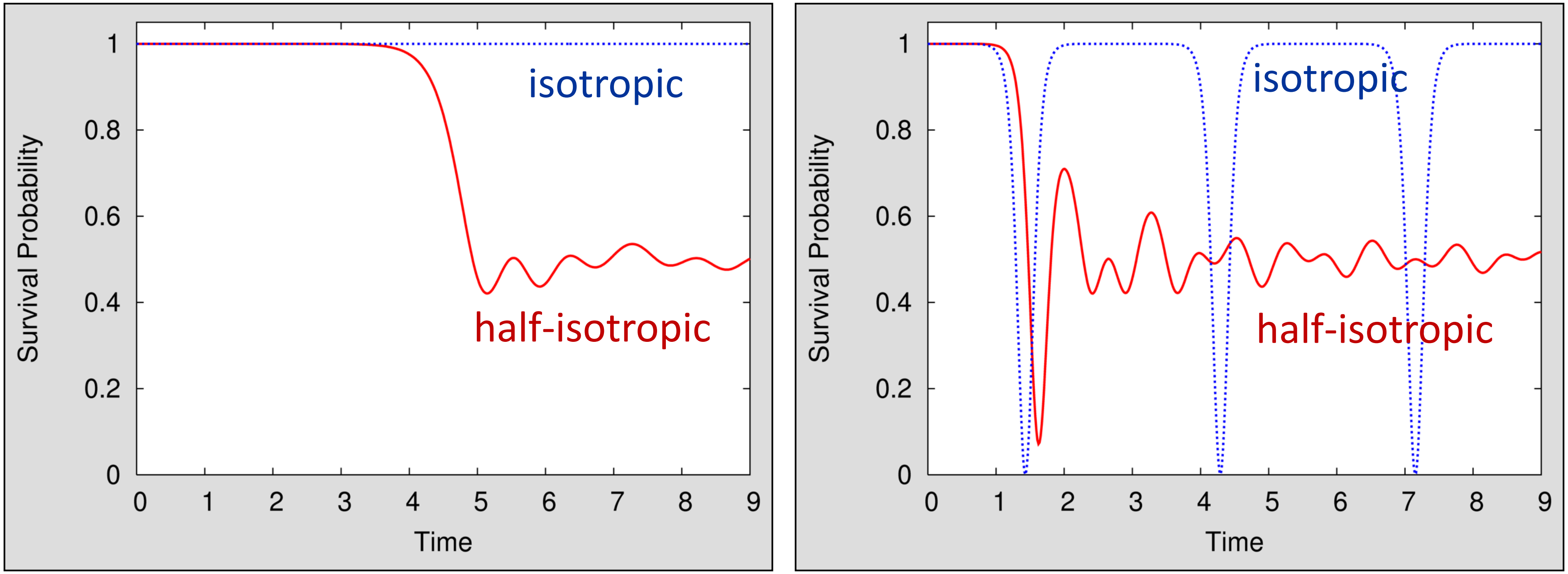
- Flux term vanishes in isotropic gas
- Can grow exponentially even with only a small initial seed fluctuation
- Symmetric $\nu\overline{\nu}$ system decoheres in both hierarchies

Raffelt & Sigl, Self-induced decoherence in dense neutrino gases, hep-ph/0701182

Multi-Angle Decoherence

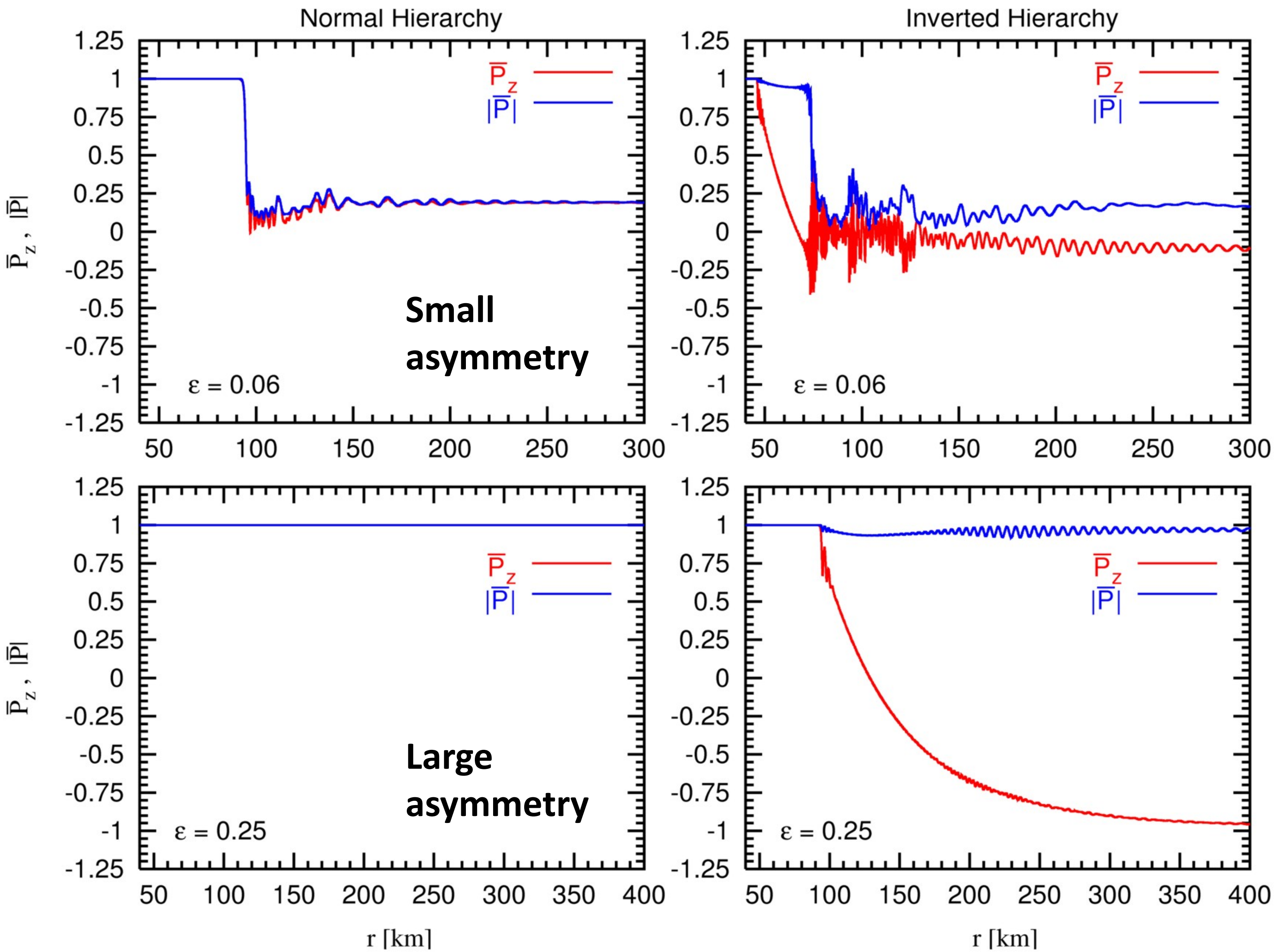
Homogeneous ensemble of $\nu_e \bar{\nu}_e$ (symmetric distribution)

Normal hierarchy | Inverted hierarchy



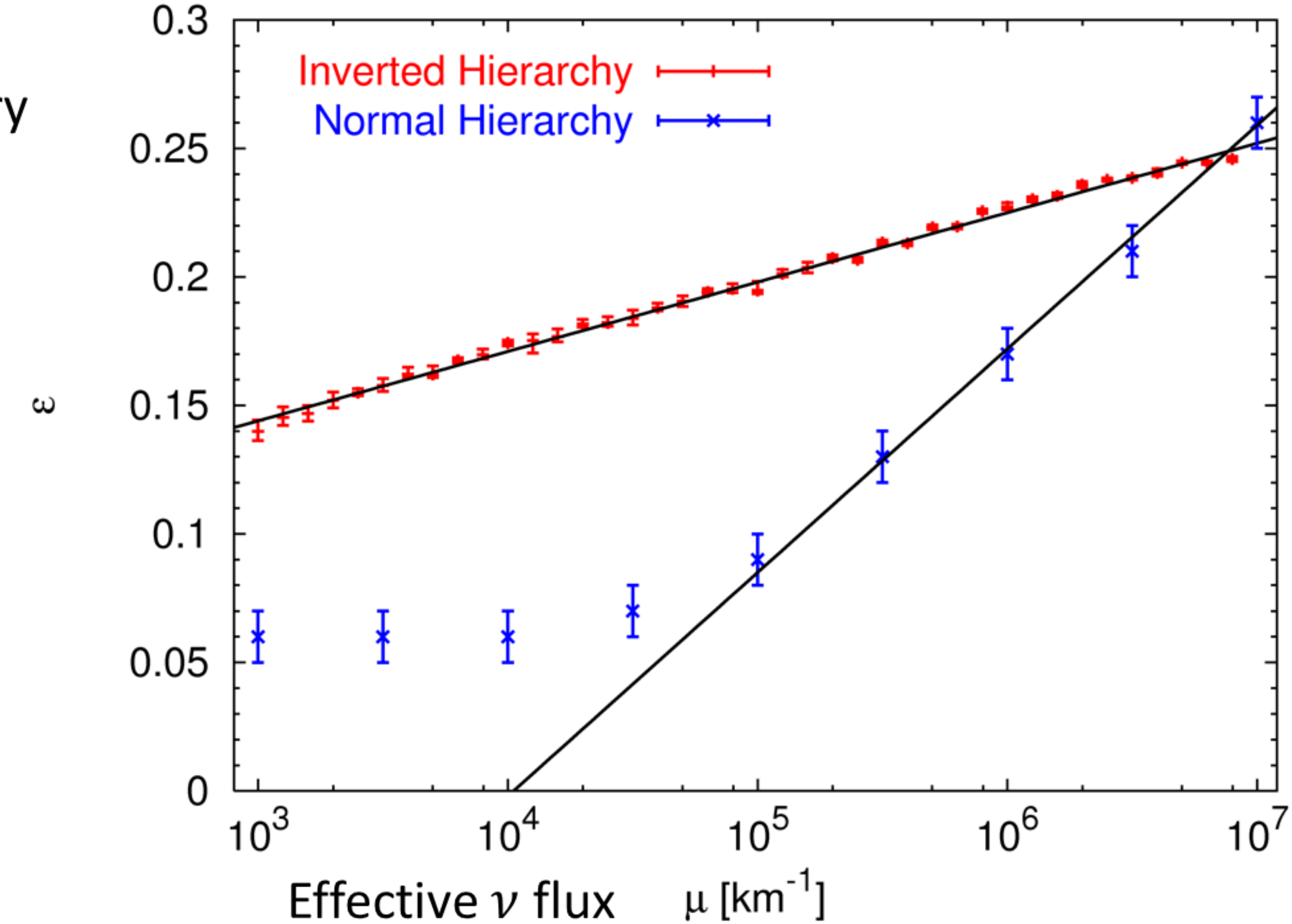
Raffelt & Sigl, Self-induced decoherence in dense neutrino gases, hep-ph/0701182

Multi-Angle Decoherence of Supernova Neutrinos



Critical Asymmetry for Multi-Angle Decoherence

Required $\nu_e \overline{\nu}_e$ asymmetry to suppress multi-angle decoherence



Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl: Decoherence in supernova neutrino transformations suppressed by deleptonization, astro-ph/0706.2498

Multi-Angle Matter Effect

Precession equation in a homogeneous ensemble

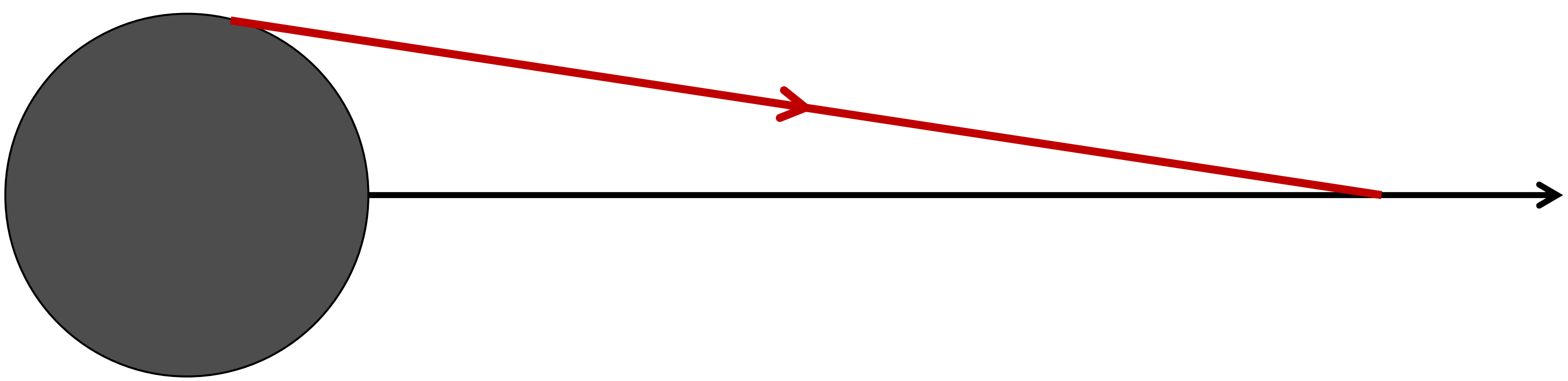
$$\partial_t \mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}},$$ where $\lambda = \sqrt{2}G_\mathrm{F}N_e$ and $\mu = \sqrt{2}G_\mathrm{F}N_\nu$

Matter term is "achromatic", disappears in a rotating frame

Neutrinos streaming from a SN core, evolution along radial direction

$$(\mathbf{v}\cdot\nabla_r)\,\mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}}$$

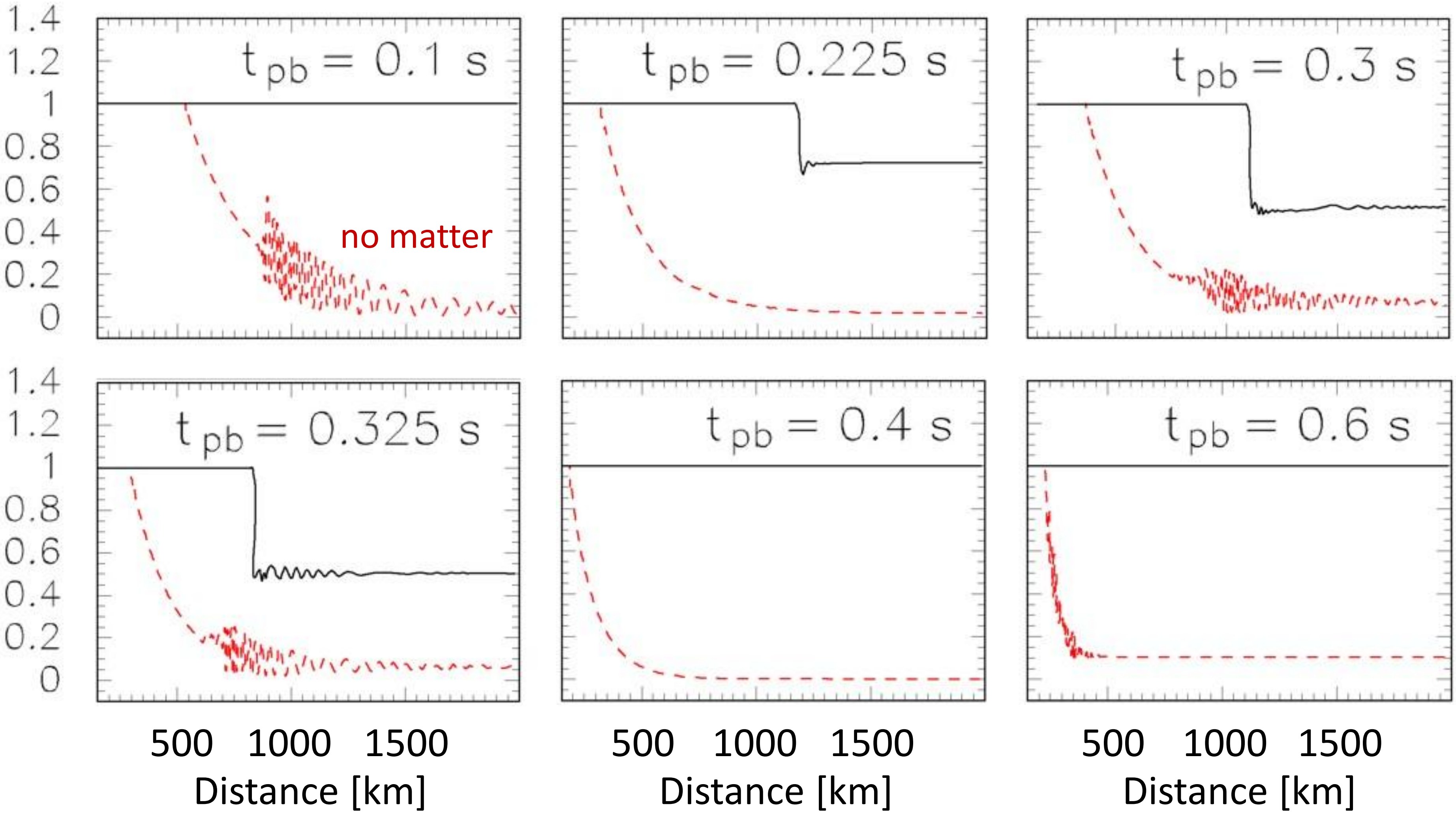
Projected on the radial direction, oscillation pattern compressed:
Accrues vacuum and matter phase faster than on radial trajectory
Matter effect can suppress collective conversion unless $N_\nu \gtrsim N_e$



Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Multi-Angle Matter Effect in Basel (10.8 M_{sun}) Model

Schematic single-energy, multi-angle simulations with realistic density profile



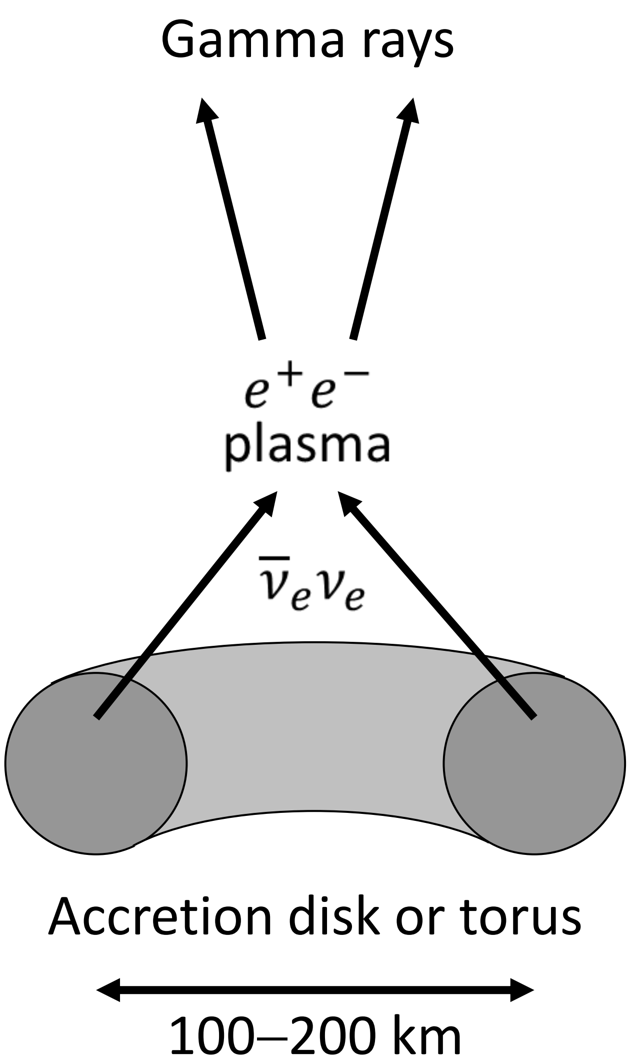
Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130

Signature of Flavor Oscillations (Accretion Phase)

	1-3-mixing scenarios		
	A	B	C
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)
$\sin^2\theta_{13}$	$\gtrsim 10^{-3}$		$\lesssim 10^{-5}$
MSW conversion	adiabatic		non-adiabatic
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$	$\sin^2\theta_{12} \approx 0.3$
$\bar{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0	$\cos^2\theta_{12} \approx 0.7$
$\bar{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130v1)

Coalescing Neutron Stars and Short Gamma-Ray Bursts



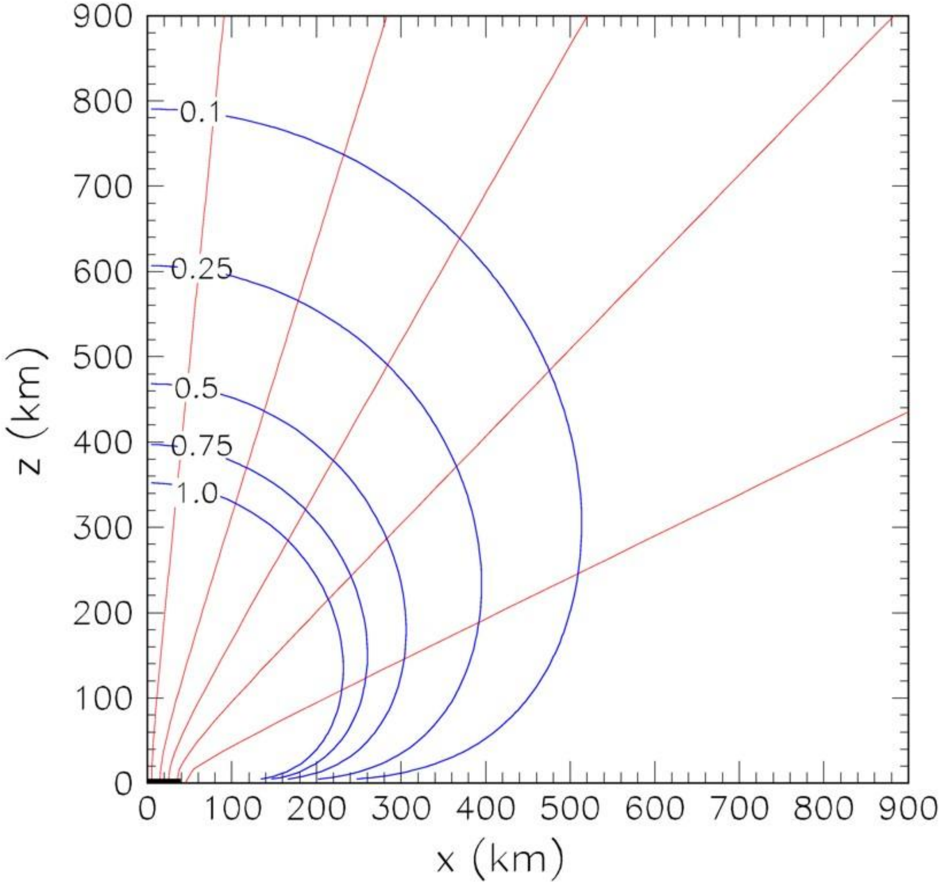
- Annihilation rate strongly suppressed if $\nu_e\bar{\nu}_e$ pairs transform to $\nu_x\bar{\nu}_x$ pairs
- Collective effects important?

Density of torus relatively small:

- ν_μ and ν_τ not efficiently produced
- Large $\nu_e\bar{\nu}_e$ pair abundance

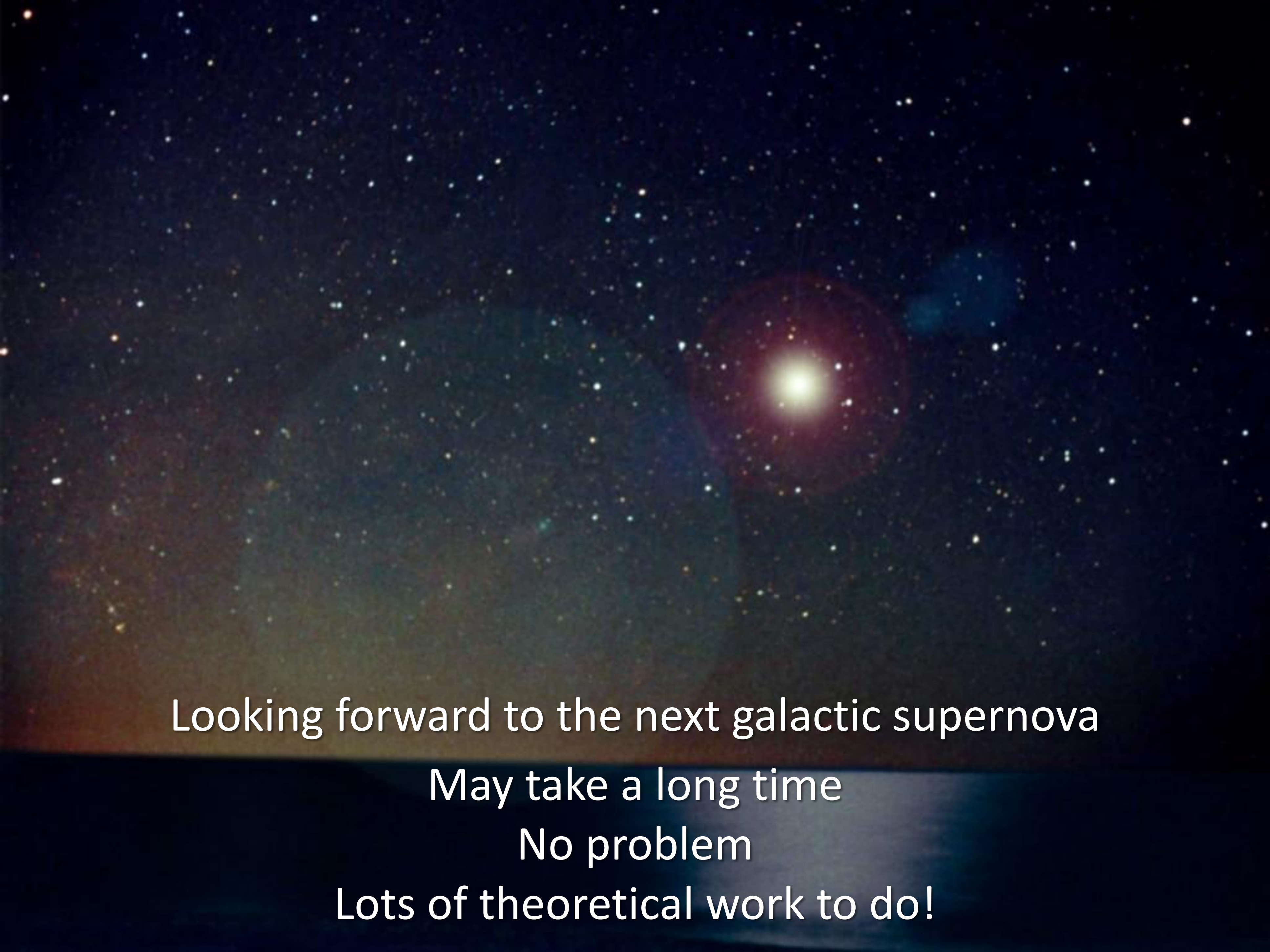
Oscillation Along Streamlines

ν_e survival probability for a disk-like source (coalescing neutron stars)



- No neutrino-neutrino interactions: Oscillations along trajectories like a "beam"
- Self-maintained coherence: Oscillation along "flux lines" of overall neutrino flux.

Dasgupta, Dighe, Mirizzi & Raffelt, arXiv:0805.3300



Looking forward to the next galactic supernova

May take a long time

No problem

Lots of theoretical work to do!